Physics 121: Astronomy
Lab Manual
University of Richmond
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Contents

Lab 1

Apparent Size and Distance

One of the most important things to know about an astronomical object is its distance from us. Unfortunately, measuring distances is often very hard. There's no one method that works for finding the distances to all astronomical objects, so astronomers have come up with a number of different methods that work in different circumstances.

One method is based on the familiar observation that a faraway object looks smaller than an identical nearby object. One way to gauge the distance to an object, then, it to measure its apparent size. The smaller the apparent size, the greater the distance.

For instance, suppose you wanted to know the distance to a faraway galaxy. Also, suppose that there's a much closer object of the same apparent size (like the Sun and the quarter Calvin's Dad is holding). If the two objects have the same apparent size, then the nearby one can just block the view of the faraway one, like this:

Distant Galaxy
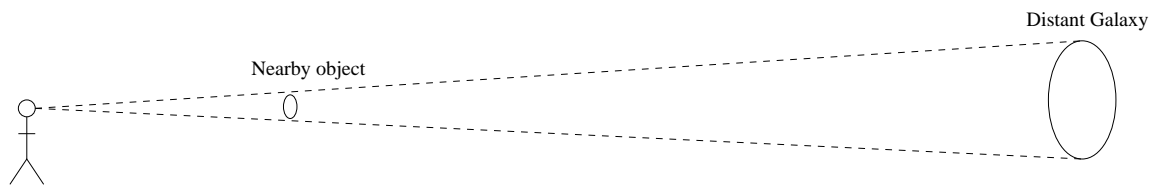
Nearby object

Let's give some names to the sizes and distances in this picture:

- s_1 is the size (diameter) of the nearby object.

- d_1 is the distance from the observer to the nearby object.

- s_2 is the size (diameter) of the distant galaxy.

- d_2 is the distance from the observer to the distant galaxy.

Indicate these four distances on the diagram above. Then write down an equation giving the mathematical relationship between these quantities. (Hints: You might want to think in terms of proportions and ratios. You might possibly want to think back to what you learned about similar triangles in geometry class all those years ago. If you're stuck on this — or anything else — ask me.)

If we can somehow manage to determine the size and distance of the nearby object, and the size of the faraway galaxy, we can use this relationship to find the galaxy's distance.

To see how this works in practice, you're going to use this technique to measure the distance to a nearby building on campus.

1. You have a sheet of transparent plastic with lines of different thicknesses marked on them. Take the sheet outside and find a place where you have a clear view of one of the windows of a nearby building (Gray Court and the Modlin Center are the best choices). Bring a length of string and a meter stick with you.

2. One partner should hold up the plastic sheet with the lines oriented horizontally and adjust the sheet so that one of the lines just blocks the window in the distant building (that is, so that the apparent thickness of the line is the same as the apparent height of the window). The other partner should measure the distance from the person's eye to the line. (I suggest you stretch the string between the eye and the plastic sheet, then measure the length of the string.) Write down those values here. Also, write down the thickness of the line that you used.

   Note: Whenever you write down a measurement, you must also write down its units.

3. To determine the distance to the other building, you'll need one more bit of information: the actual size of the window. We're going to make an assumption (which may not be correct) that the windows in the other building are the same size as the windows in our classroom. Go back to the classroom, measure the height of the window, and record it here.

4. Using the relationship between s_1, d_1, s_2, d_2, together with the values you just recorded, determine the distance from here to the other building:

   To see how well you did, determine the distance using the aerial photograph from Google maps that I'll hand out. Here's how.

5. The scale bar at the lower left of the photograph shows the distance on the page that corresponds to 50 meters of actual distance. Measure the length of this bar, and use the result to determine how many meters of actual distance correspond to 1 cm on the page.

6. Mark on the page your approximate location when you made the measurement and the location of the window. Measure the distance between the two points on the page in centimeters.

7. Use this information to determine the distance from your observation point to the other building in meters.

Are your two determinations of the distance significantly different? If so, what do you think the main reasons are? Which measurements might have been inaccurate? What assumptions might have been incorrect?

Lab 2

Angular Measurements

In astronomy, we often talk about the "angular size" of an object. This is just a way of expressing how big the object appears to be, or in other words how much of our field of view it takes up. For instance, in the picture below (which is the same as the picture in your previous lab), the two objects have the same angular size, because they "fill up" the same angle in the observer's field of view.
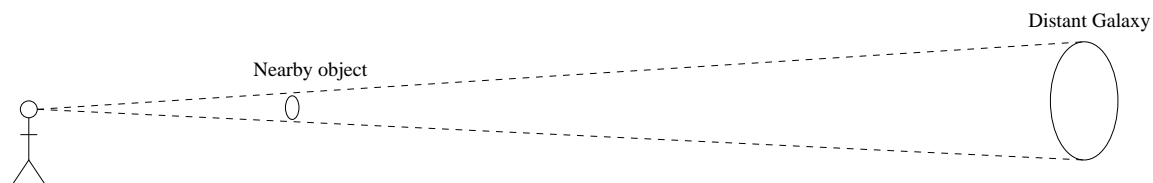
The angular size is related to the actual size of the object and the object's distance. That relationship is going to be extremely useful to us, so I want you to spend a little while right now working it out.

Here's a picture of an object. The angular size is the angle α on the left. The distance from the observer to the object is d, and the actual size of the object is s. Our goal is to relate the three numbers α, s, d.
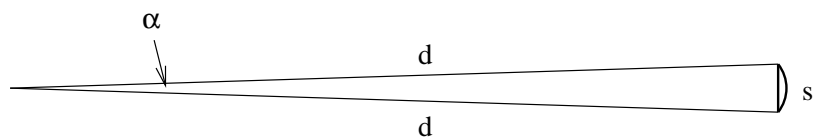
(Note: Your textbook uses the symbol D instead of s for the size of the object. I don't like that, because I think it's too easy to mix up d and D, so I'm going to call it s instead.)

We can use trigonometry to relate these three numbers, but the mathematics is a bit annoying. Fortunately, as long as the angle α is very small (which it almost always is in astronomical observations), there is a simplified relationship between these numbers that doesn't involve any trig. Here's how it works.

In the picture below, the straight line on the right represents the object. The length of this line is the exact value of s, the object's size. The picture also shows a curved line. That curve is an arc of a circle centered on the observer. Because it's curved and not straight, the length of this arc is not quite the same as s. On the other hand, if the angle α is very small, then this curve is almost the same length as the straight line. For this reason, *we are going to allow ourselves to pretend that the length of that curved arc is the same as s.* To figure out the length of the straight line, we'd need trig, but we can figure out the length of the arc with just geometry.

Exact value of s

Good enough approximation to the value of s
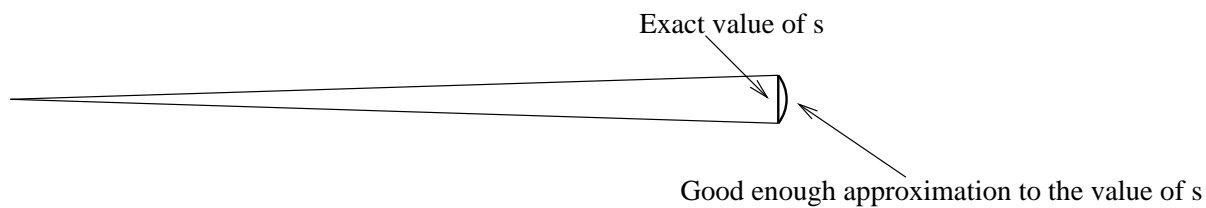
Here's yet another picture of the situation. Now the observer is at the center. The solid arc represents the size of the object being observed, and d is the distance from observer to object. The dashed circle is an imaginary circle centered on the observer.

d
α
Observer
d
s
Object
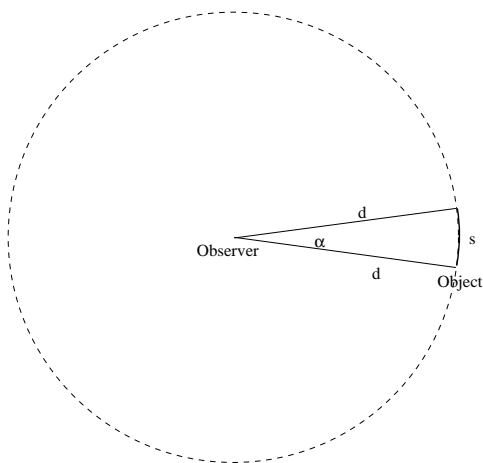
Well, let's get started.

1. Suppose that the angle α is one degree. What fraction of the whole circle is covered by the arc s?

2. Remember that the circumference of a circle is 2π times the radius. What is the length of the arc s, in terms of the distance d? Continue to assume that α is one degree. (Your answer here should say $s =$ something times d.)

Repeat steps 1 and 2, but this time assume that $\alpha = 2°$.

Repeat steps 1 and 2, but this time don't assume any particular value for α. Instead, your answers will be algebraic expressions including α as an unknown variable. Your final result should say $s =$ something involving both α and d.

Once you've got this expression worked out, show it to me. This extremely useful fact is known as the "small-angle formula."

Now that you know the small-angle formula, what do you use it for? The main thing is that any time you know two of the numbers α, s, d, you can use the formula to get the third one. Try these out:

The Moon is 3476 kilometers in diameter, and it's $380\,000$ kilometers away. What is the angular size of the Moon?

The Sun's angular size is 0.53 degrees, and the Sun is 150 million kilometers away. What's the Sun's diameter in kilometers?

Most of the time, angles in astronomy are much smaller even than one degree. For this reason, astronomers usually measure angles in units called *arc-minutes* and *arc-seconds*. One arc-minute is one-sixtieth of a degree:

$$1' = \left(\frac{1}{60}\right)^\circ.$$

($'$ means "minute"; $^\circ$ means "degree.") One arc-second is one-sixtieth of a minute:

$$1'' = \left(\frac{1}{60}\right)'.$$

($''$ means "second.")

How many arc-seconds are in one degree?

A decent astronomical telescope can see things whose angular size is as small as $1''$. How far away would your lab partner have to be in order for the angular distance between their eyes to be equal to $1''$? You'll have to measure the distance between your partner's eyes to answer this. (If your partner is any further away than this, then the telescope would no longer be able to see your partner's eyes as two separate objects; they'd be blurred into one blob.)

Optional question: If you know and remember your trig and feel like it, try answering the last question using trig instead of the small-angle formula. Does it make a significant difference in the final result?

Lab 3

Resolution of the Human Eye

Everyone knows that there are limits to how well we can see. Some objects are too small or too close together for our eyes to distinguish them. The same goes for any telescope or other observing device we might use: even the Hubble Space Telescope has limits to the level of fine detail it can "see." We define the *resolution* of an observing device (an eye or a telescope, for instance) to be the *angular size* of the smallest details it can see. The smaller the resolution, the better the device is at seeing fine details. In this lab, you will measure the resolution of your own eyes.

You will be given a piece of paper that has a pair of black dots on it, along with a single oval-shaped blob that's about the same size as the pair of dots. If you look at the paper from a short distance, it's easy to tell the pair of dots apart from the single blob, but from a great distance you can't tell which is which. We say that you can *resolve* the pair of dots if you can tell that the two dots really are two dots, not one.

Measurements. Take the sheet with the dots on it and move quite far away (10 meters or so) from your lab partner. Hold the sheet up either right-side up or upside-down, but don't tell your partner which. From this distance, your partner should not be able to tell the difference between the pair of dots and the single blob. Your partner should then walk slowly toward you until he or she can tell which one is pair of dots. Measure the distance between the two of you at the moment your partner can first tell with confidence which is which.

Repeat this procedure at least three times. Switch the orientation of the paper randomly, so that your partner doesn't know the "right" answer. Switch roles, so that each partner is the "observer" at least three times. List the individual measurements below, and also compute the average distance for each partner.

The resolution of your eyes is the angular separation between the two dots when you can just barely tell that there are two of them. To figure out this number, you need to know the separation between the two dots (to be precise, the separation between their centers). Measure this separation.

Use the above information to calculate the resolution of each partner's eyes. Give your answer in degrees and also in arc-minutes. Remember that one arc-minute (1′) is $\frac{1}{60}$ of a degree.

Once you've got your answers, record them on the whiteboard, so that everyone can see the range of resolutions.

Now suppose that you repeated the experiment using dots that were twice as big, and twice as far apart. What would you expect the distance between partners to come out to be?

What would you expect the resolution to come out to be?

The back of the page contains dots that are twice as large. Try the experiment and see. (You don't need to do 3 repetitions per partner this time; just one is fine.) Are the results consistent with your predictions?

Questions. Here are some questions to test whether you know what you're doing with all this stuff. Some of these questions have to do with resolution, and some are other applications of the small-angle formula. You don't have to turn these in, but you just might have a quiz some time soon that asks questions very similar to these.

Note that I've obnoxiously used a variety of different units here. If you need to look up unit conversions, go ahead. In some cases, I may have deliberately left out a number you need, in which case you should estimate it as best you can.

(In case you're wondering, on a quiz or exam, I will not expect you to know numbers such as the number of meters in a light-year or the distance from Earth to Sun. You should know the basic metric-system prefixes, like the number of centimeters in a meter or meters in a kilometer.)

1. The Moon is 384 000 kilometers away. There are two large craters on its surface, which you can just barely distinguish from each other with the naked eye. How far apart are the craters?

2. At the Battle of Bunker Hill, the rebels were supposedly told, "Don't shoot until you see the whites of their eyes." How close would the British be at this point?

3. You're standing on a deserted road, late at night. A vehicle is approaching you. At first, you can't tell whether it is a car (with two headlights) or a motorcycle (with one). How close will the vehicle be before you can tell?

4. The resolution of the Hubble Space Telescope is about $0.1''$ (that's 0.1 arc-seconds). How many times better than your eye is this?

5. The two dwarf planets Pluto and Charon (formerly known as the planet Pluto and its moon) are about 20 000 kilometers apart. They are about 29 astronomical units from the Earth. What is the resolution of a telescope that can just barely resolve these two bodies? Give your answer in arc-seconds.

6. Suppose that another star somewhere nearby has a planet orbiting it at exactly the same distance as the Earth is from the Sun. How close would the star have to be in order for the Hubble Space Telescope to be able to see the planet and star as separate objects? Give your answer in light-years.[1] The closest star is 4.2 light-years away. Is this close enough?

7. One of the stars in the handle of the Big Dipper is actually a pair of stars called Alcor and Mizar. The two stars have an angular separation of about $12'$ and are about 1 light-year away from us. How far are the two stars from each other? Give your answer in meters and in astronomical units.

8. The supergiant star Betelgeuse has an angular diameter of $0.044''$ and is 427 light-years from Earth. What is Betelgeuse's diameter? If the Sun suddenly swelled up until it was as big as Betelgeuse, which of the planets of the solar system would it engulf?

9. A typical distance between neighboring galaxies is 2 million light-years. A good ground-based telescope has a resolution of about $2''$. How far away must a galaxy be if an observer using this telescope has difficulty resolving it (that is, telling it apart from its neighbors)? Because the Universe is not infinitely old, there's a maximum distance we can see of about 50 billion light-years (light from greater distances hasn't had time to reach us). Bearing this in mind, is there ever a problem telling galaxies apart from their neighbors with a telescope like this?

10.   "Riders!" cried Aragorn, springing to his feet. "Many riders on swift steeds are coming towards us!"
        "Yes," said Legolas, "there are one hundred and five. Yellow is their hair, and bright are their spears. Their leader is very tall."

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[1] Actually, seeing other planets is even harder than this question suggests. Looking for something very faint right next to something very bright is extremely difficult.

Aragorn smiled. "Keen are the eyes of the Elves," he said.

"Nay! The riders are little more than five leagues distant," said Legolas.

J.R.R. Tolkien, *The Two Towers*

Roughly what is the resolution of Legolas's eyes?

Answers. For purposes of the answers below, I'll assume that the resolution you found for your eye was $2'$ (two arc-minutes).

1. 220 km.

2. I'll estimate that the white of your eye is about 1 centimeter across (from the edge of your iris to the edge of your eye). Something about that size can be resolved at a distance of about 20 meters.

3. $2'$ is $120''$, which is 1200 times bigger than $0.1''$, so the Hubble space telescope has a resolution about 1200 times better than your eye.

4. Assuming the headlights are about 2 meters apart, the distance comes out to about 3.4 kilometers.

5. About $0.95''$.

6. 33 light-years. So the nearest star is close enough.

7. 3.3×10^{13} m, or 220 AU.

8. The diameter is 8.6×10^{11} m or 5.7 AU. All planets within half this distance, or about 2.85 AU, of the Sun would be engulfed. That's Mercury, Venus, Earth, and Mars. (Jupiter's orbit is about 5 AU in radius.)

9. About 2×10^{11} light-years. This is about 200 billion light-years, which is much bigger than the maximum distance we can see, so telling galaxies apart from their neighbors isn't generally a problem even for very distant galaxies.

10. We've got to make some estimates here. Let's say that Legolas can resolve features that are as small as 0.1 meters. (If his resolution were much worse than this, then I don't think he could tell a tall person from a short person. Naturally, if you used a somewhat different number, that's fine.) Five leagues is about 15 miles, or about 25 km. The resolution corresponding to these numbers is $0.0002°$, or $0.85''$. Legolas does about as well as a good astronomical telescope.

Lab 4

Daily Motion of Stars

This lab has a few purposes:

- To get used to some features of the *Stellarium* program.

- To identify some prominent constellations and asterisms that can be seen from our location.

- To see how stars move in the night sky.

A. Messing around.

Start up *Stellarium*, and spend a few minutes playing around with some of its features. Appendix A lists a bunch of things you can do. Here are some specific things you should try. Refer to the Appendix for details about how do do these things.

1. Adjust the time and date. When you first start the program, it will show the sky at the current time. It's much more interesting to see the sky at night. You can also examine what things look like at different times of year.

2. Speed up the flow of time by a large amount, so that a day goes by in just a few seconds.

3. Initially, the program is showing you the view of the sky from here in Richmond. Switch to some other locations.

4. Click on a star or other astronomical object. Information about that body should appear on the screen. Some of that information may not make sense yet, but it will!

5. Drag the image around (hold down the left mouse button and move the mouse around) to look at different parts of the sky.

6. Zoom in to look at a small patch of the sky, then zoom back out (using the mouse wheel or the page-up/page-down keys). Note the display at the bottom that says "FOV." This stands for "field of view," and it indicates the size of the patch of sky that's visible on the screen at that moment. Watch how the FOV changes as you zoom in and out.

7. Click on the "find" icon and search for various celestial objects. Try Uranus, for example, or Polaris (which is the name of the North Star).

8. The pop-up menus in the lower left contain a bunch of buttons you can click to change the appearance in various ways. I've listed the ones I think are most useful in the Appendix. Try these out. Some are harder to understand than others.

B. Big Dipper, Mizar, and Alcor.

Once you're done messing around, set your location back to Richmond, looking north, with a nice, large field of view (say 90° or so). Set the time to 10:00 tonight.

You should see the Big Dipper in the northeast. Turn on the constellation labels and lines. Notice that the Big Dipper is not a full constellation; it's just part of the larger constellation Ursa Major (the Great Bear). A group of stars like the Big Dipper, which is easily identifiable and has a name, but which isn't a whole constellation, is called an "asterism."

The second star in the handle of the Big Dipper is called Mizar. Click on this star to select it, and move it into the center of the field of view. Then zoom in on this star until you can see another star called Alcor right near it. Find the angular separation between these two stars. What is the angular separation?

This separation is visible to the naked eye, if you have good eyesight. Next time you're out on a clear night, look to see if you can spot both Alcor and Mizar.

Zoom in on Mizar still further, and you'll eventually see that it splits into two stars. Amazingly enough, although *Stellarium* might not tell you this, each of those two stars is itself a double star system. Unfortunately, these two pairs are too close together for us to see them separately, even with our best telescopes. (How do we know that they're there, then? Good question! We'll answer it eventually.) In fact, in 2009 it was discovered that Alcor is actually a double star system as well. So when you look at Mizar you're really seeing four stars, and if you look at both Alcor and Mizar, you're seeing six stars.

What is the angular separation between Mizar and its companion (not Alcor – the closer one)?

Select Mizar by clicking on it (if it's not still selected). You'll see some information about it in the upper left corner, including the distance to the star in light-years. Use the small-angle formula to determine the separation between Mizar and its companion. The small-angle formula will give you an answer in light-years. Convert this to meters and to astronomical units.

Zoom back out to the largest field of view, look toward the north again.

C. Other Constellations.

Identifying constellations isn't a big part of this course (after all, there's no science involved in constellation-spotting). Still, it's good to know where a few prominent constellations are.

The two stars at the end of the bowl of the Big Dipper (Merak and Dubhe) are called the pointer stars. Draw an imaginary line from Merak to Dubhe, and extend it about six times the separation between those stars. You'll hit the star Polaris, which is the tail end of the Little Dipper (the constellation Ursa Minor). Polaris is also known as the North Star. Polaris isn't a terribly bright star – in fact, none of the stars in Ursa Minor are very bright – but it's still important. It's the one star in the sky that all the others seem to rotate around. (More on this later.)

Look to the West from Polaris to find the constellation Cassiopeia. It looks like a W (on its side at the moment). The stars in Cassiopeia are quite bright, and Cassiopeia is easy to spot in the night sky pretty much all the time. Cassiopeia and the Big Dipper are probably the most useful constellations to use when orienting yourself to the night sky.

In the winter, the other easy constellation to spot is Orion. Find it in the southern sky.

That's enough with constellations for now; we'll do more when we actually do some observing.

D. Daily Motion of Stars.

Make sure you're looking North, with a large field of view (at least 90°). Turn off the effects of Earth's atmosphere. This will make it so that the sky is dark during the day, so that you can see the stars all the time. (Astronomers would love it if this were possible in real life!)

Set time to go at much faster than the normal rate, so that a day takes only a a few seconds. Observe how the stars move. The stars move in circles, with the star Polaris at the center. Note that some of the stars move in circles that always stay above the horizon, while others rise and set below the horizon. Stars that never set below the horizon are called "circumpolar."

Of course, whether a star sets below the horizon or not depends on the exact shape of the horizon. To keep things simple, let's assume that we're looking at the stars from a location with a nice, flat horizon (no hills or trees to get in the way). Here's one way to make this happen in *Stellarium*: click on "Sky and viewing options", then the "Landscape" tab, then check the "Ocean" box. As you'll see, that shows you what things would look like if you were surrounded by a nice, flat ocean.

How many of the seven main stars in the Big Dipper are circumpolar? (Here I'm counting all of Alcor and Mizar as one star.)

Now change locations to Boston, Massachusetts. How many of the stars in the Big Dipper are circumpolar when viewed from Boston?

When Santa looks at the stars from the north pole, how do they appear to move? What percentage of the stars he sees are circumpolar? Make a prediction first, then try it.

Now switch your location to Santiago, Chile. Find the center of the circles that stars appear to move in from this location (hint: look South). What constellation is this in? (There are several constellations right near this point. Astronomers have defined precise boundaries between the constellations. To see which one this point is in, figure out how to turn on the "constellation boundaries" in *Stellarium*.)

Note that there is no bright star right at the center of these circles: Polaris is the North Star, but there is no South Star.

Go back home to Richmond. Pick a star that's not circumpolar (that is, one that rises and sets). Record the time that this star rises on one day, and the time it rises on the next day. Make sure your times are accurate to the minute. How much time elapses between successive risings of the star?

Repeat for a few other stars. Is the result always the same?

The length of time between successive risings of a star is called a "sidereal day." How different is a sidereal day from an ordinary day?

Lab 5

Intensity of Solar Radiation

There are two main reasons winter is colder than summer:

1. The Sun is up for fewer hours per day in the winter.

2. The Sun is lower in the sky in winter. This means that sunlight strikes Earth's surface at a shallow angle, which lowers the intensity of the radiation striking the Earth's surface.

In this lab, you'll verify these statements. In particular, you'll find out the amount of time the Sun is up at different times of year, you'll see how high the Sun rises in the sky in summer and winter, and you'll check that radiation striking a surface at a shallow angle deposits less energy than radiation that is closer to perpendicular.

A. Amounts of sunlight at different times of year.

Start up *Stellarium*, and set the date to June 21, the first day of summer. Determine the times of sunrise and sunset on that day, by running time forward and backward and watching the position of the Sun. (Use a flat horizon.) How many hours of daylight are there on that day?

Repeat this procedure on December 21, the first day of winter.

What is the ratio of these two values? That is, how many times greater is the amount of sunlight on June 21 compared to December 21?

B. Angle made by the Sun in summer and winter.

Set the time to noon on June 21 (the first day of summer). Make *Stellarium* draw the *meridian*, which is a line in the sky going from due north, directly overhead, to due south. On any given day, the moment the Sun crosses the meridian is the moment when it's highest in the sky. At what time does the Sun crosses the meridian on this date?

Why doesn't the Sun cross the meridian at exactly noon? (There are several reasons.)

Select the Sun and view the information on it. Find the Sun's *altitude*. This is the angle indicating the Sun's height above the horizon. (90° would be directly overhead, and 0° would be right on the horizon.) What is the Sun's maximum altitude on this date?

Repeat this procedure to find the Sun's maximum altitude on December 21, the first day of winter.

C. Effect of angle on the amount of solar radiation.

Next, you'll check to see how much of an effect this difference in angles makes to the amount of radiation striking the Earth. The light bulb on your table will represent the Sun. You will use a solar cell to represent a small piece of the Earth's surface. Your goal is to see how the amount of radiation from the bulb changes as the angle of the cell is varied. In particular, how much more radiation strikes the cell when the light is beating down on it from almost directly above (like sunlight in summer), compared to the amount when the light strikes it at a shallower angle (like sunlight in winter)?

Place the bulb at one end of the optical bench, and place the round platform with angle markings at the other end. We'll call the center of the round platform the "observer's location," and we'll call

the bulb the "Sun."

Start the program called "solarcell.cap" on the lab computer. This program allows the computer to display the amount of radiation striking the solar cell.

Place the solar cell at the observer's location, facing directly towards the Sun. The surface of the cell should be perpendicular to a line going from the cell to the Sun. In the picture below, the cell should be oriented along the line marked "Perpendicular."
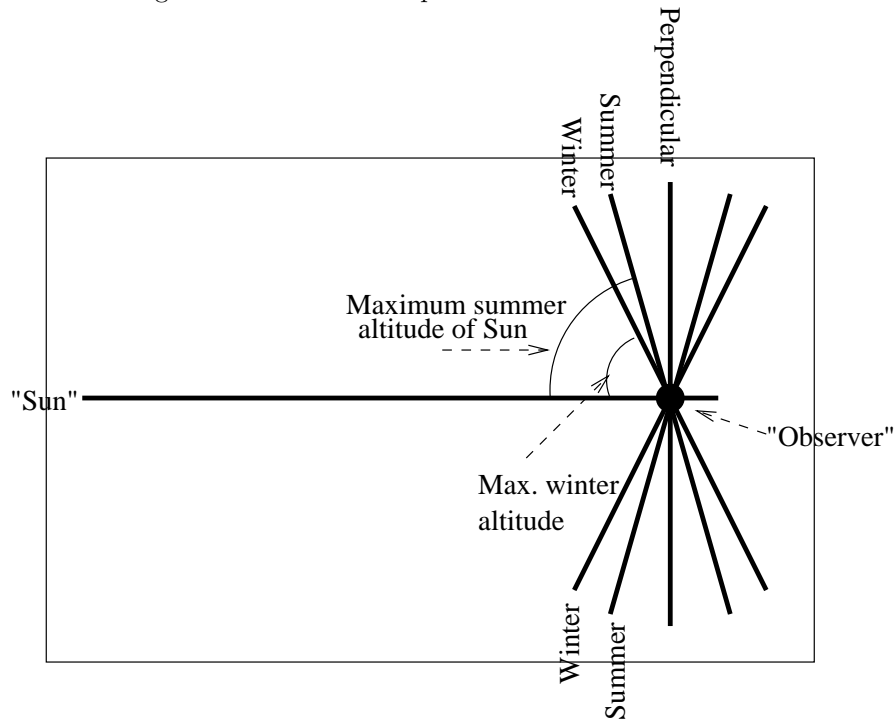
Turn on the light bulb, and hit the "Record" button ( ) on the computer screen. You should see a graph showing the amount of radiation striking the solar cell as time passes. Try placing your hand in front of the solar cell to block the light. The reading should go down.

Let the computer record data for about 20-30 seconds. We want to know the average amount of light striking the solar cell. To find this out, you need to select a range of data where the graph looks reasonably flat. Hit the button that looks like this, near the top of your screen: . That'll create a rectangle that you can move to select the region of data you want to use. Once you've highlighted a stretch of data, the computer should display the average ("Mean") of those numbers. This is the average amount of radiation striking the solar cell during this time period. Record this value:

This represents the amount of power that strike the solar cell if it were lying flat on the ground, and the Sun was beating down on it from directly overhead. In fact, though, the Sun is never directly overhead (at least, not from a location like Richmond). The highest the Sun ever gets in the sky is the altitude you found for the first day of summer in part B.

Rotate the solar cell so that it the angle it makes with the line to the Sun corresponds to the altitude of the Sun on that date. (This means orienting the cell so that it lies along one of the lines marked "Summer" in the picture above.)

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Repeat the procedure with the solar cell aligned on the other "summer" line.

The last two values should be close to the same. Average them together:

Repeat this procedure with the two winter lines to end up with an average amount of radiation striking the cell when it's at the winter angle:

You should have found that the amount of radiation is greatest when the cell is perpendicular, a bit less when it's at the summer angle, and much less when it's at the winter angle.

How many times more intense is the midday solar radiation in summer than in winter?

Let me repeat the main point. Part A and this part show two different reasons why it's hotter in summer: there are more hours of daylight in summer, and the sunlight is more intense.

Lab 6

Celestial Navigation

In the old days, before GPS, sailors figured out where they were by looking at the positions of the stars. In this lab, you'll see how that worked. To be specific, you'll look at the question of how to use the positions of celestial objects to determine your latitude and longitude.

Our reason for doing this is not that I expect you to find yourself lost at sea without access to GPS (although anything's possible, I guess). It's because figuring out how celestial navigation works is a good way to make sure you understand how things move in the night sky.

Latitude

As it turns out, determining your latitude (i.e., how far north or south of the equator you are) is much easier than determining your longitude. The easiest way is to observe the location of the star Polaris. As you know, Polaris is very close to the North Celestial Pole, or in other words almost directly above the Earth's North Pole. For purposes of the questions below, you can assume that Polaris is exactly at the North Celestial Pole.

Suppose you were standing at the Earth's North Pole, which is at a latitude of 90° north. In which direction would you have to look in order to see Polaris?

Now suppose you measured the *altitude* of Polaris. As we've seen, the altitude of an object is the angle that describes how far above the horizon that object is located. To be specific, draw an imaginary line from you to Polaris, and draw an imaginary horizontal line (pointing toward the horizon) directly below Polaris. The altitude means the angle between those two lines.

If you were at the north pole, what would the altitude of Polaris be?

Now suppose you were at the equator (which is at a latitude of 0°). In which direction would you

have to look in order to see Polaris?

What would the altitude of Polaris be?

Based on the above considerations, you can guess that there might be a *very* simple rule relating the altitue of Polaris to the observer's latitude. What do you think that relationship is?

We could test this observation using actual observations of Polaris at night, but instead let's test it with *Stellarium*.

Start up *Stellarium*. Find the star Polaris and select it. Look at the information in the upper left, and find the star's altitude. What is the altitude of Polaris?

What is our latitude here in Richmond? (You can find this out by looking at "Location" in *Stellarium*, or I'm sure Google knows it.)

Do the two values you just found agree, at least roughly, with your expectations? (If the degrees agree but there's a discrepancy in the arc-minutes, that's close enough.)

If you're ever lost at sea, you now know how to find your latitude, by observing how high Polaris is in the sky.

Longitude

Suppose you wake up one night and find yourself in a boat tossing about in the middle of the ocean. Using the method above, you figure out your latitude. Now how are you going to find your longitude? As we'll see, this turns out to be a much harder problem than finding latitude.

Suppose your latitude comes out to be 20° north. To keep things (relatively) simple, let's suppose that there are just two possibilities for longitude: either you're at a longitude of 25° west, or you're at a longitude of 55° west.

Set the time in *Stellarium* to about 10:00 tonight, and set your location to be a latitude of 20° north and a longitude of 25° west. (You can type these numbers directly into the Location box. Just enter the number of degrees, and leave out the minutes and seconds.) Orient your view so that you're looking north, with the horizon at the bottom and a pretty large field of view (about 60° or so). Note a couple of landmarks to orient yourself in the sky. You should be able to see the Big Dipper (upside-down) and the bright star Capella, for instance.

Once you've got this set, take a screen shot by hitting control-S. Check that the screen shot was saved. (I think it end up in your Pictures folder.) Open it up and take a look at it to make sure it looks the way you expect. Once you see that the file looks right, go back to *Stellarium*.

Now change your location to 55° west longitude (keeping the latitude and the time the same). Take a screen shot of the sky from this new location.

Now advance the time by two hours, from 10:00 pm to midnight. Take one more screen shot.

At this point, you should have three screen shots.

1. Longitude: 25°. Time: 10:00 pm.

2. Longitude: 55°. Time: 10:00 pm.

3. Longitude: 55°. Time: midnight.

Incidentally, I should mention that the times in *Stellarium* are always times in our actual location (Richmond). That is, 10:00 pm means 10:00 pm Eastern time, regardless of the observer's location.

Two of these three images should look almost identical, and one should look significantly different. Which one is not like the others?

Now, let's get back to your plight as you sit bobbing in your boat in the middle of the ocean. Suppose that you have a timekeeping device (wristwatch cell phone, etc.) that is set to Eastern time. Using this, along with your observation of the night sky, can you tell which longitude you're at? If so, explain briefly how. If not, explain briefly why not.

Suppose that you *don't* have an accurate timekeeping device on board the boat. Can you tell what longitude you're at by observing the sky? If so, explain briefly how. If not, explain briefly why not.

In the 18th century, the British government offered large cash prizes for anyone who could figure out an accurate way to determine the longitude of a ship at sea. In 1765, John Harrison was awarded a £10 000 prize for solving this problem. (It's hard to figure out precise equivalents, but this is equivalent to millions of dollars today.) What do you think Harrison invented?

Back to you on your boat. Suppose that you had a clock with you on your boat, but it wasn't very accurate. Suppose that you use the clock to determine your longitude, but unbeknownst to you the clock is off by two hours. (For instance, you think it's midnight, when really it's 10:00 pm.) By how many degrees will your longitude determination be off?

Suppose that your clock were only off by one minute instead. How far off will your longitude determination be?

One degree of longitude corresponds to an actual distance of about 50 kilometers. If your clock were off by one minute of time, how far off would your determination of your location be (in kilometers)?

If you were a ship's captain trying to avoid hitting undersea rocks and shoals, this level of inaccuracy would be a real problem.

Lab 7

The Moon

In this lab, you'll check a bunch of things about the motion and phases of the Moon.

A. Sidereal and Synodic Months

A *sidereal month* is the length of time it takes the Moon to make one complete motion around the sky with respect to the stars. In other words, at the end of a sidereal month, the Moon appears in the sky next to the same stars as it did at the beginning of the month. A *synodic month* is the time it takes the Moon to go through one full cycle of phases (from full Moon to full Moon, for example). You'll use *Stellarium* to measure the length of both sidereal and synodic months.

Start up *Stellarium*. Make the ground transparent and turn off the Earth's atmosphere, so that we can see what the Moon is doing at all times. This makes our lives much easier than the real-life observers who first figured all this stuff out: they could only see the Moon when it was above the horizon at night. Again, consult the Appendix to see how to do these things, and of course ask me if you can't figure it out.

First, step forward in time until the Sun and Moon are right next to each other in the sky. This is the next time when the phase of the Moon will be new. Determine, to within an accuracy of about an hour or so, the moment when the Moon and Sun are closest to each other in the sky. Record the date and time:

Now let time run forward for a bit less than a month, until the phase of the Moon is nearly new again. Just as before, find the moment, accurate to the nearest hour, when the Moon and Sun are closest together. Record the date and time:

Based on these results, what is the length of a synodic month? Give your answer as a decimal number of days, with at least one digit after the decimal point.

Now you'll figure out the length of a sidereal month. Click on a star that's very near the Moon, and keep the field of view centered on that star. Record the date and time when that star passes closest to the Moon, accurate to the nearest hour.

Then let the time advance for a bit less than a month, until you see the marked star pass close to the Moon again. Record the time it passes closest to the Moon, accurate to the nearest hour.

Find the length of a sidereal month as a decimal number of days.

The year is 365.24 days long. How many synodic months are there in a year? (Give your answer as a decimal number with at least one digit after the decimal place).

How many sidereal months are there in a year?

You should find that the difference between these two numbers is very close to 1. (If you don't find this, let me know.) This is a general rule relating sidereal and synodic periods for all satellites. You might enjoy trying to figure out why it's true. (Then again, you might not.)

B. Phases of the Earth

Question: Suppose the Moon is in a waxing crescent phase as seen from the Earth. An astronaut on the Moon looks at the Earth. What phase of the Earth does she see?

I want you to figure out the answer to this question on your own first, and then test it with *Stellarium*.

Draw a diagram showing Earth, Sun, and Moon when the Moon is a waxing crescent. Indicate the directions of the Earth's orbit around the Sun and the Moon's orbit around the Earth.

Based on this diagram, what is the answer to the question above? (Your answer should be something like "full Earth," "waning crescent Earth," "waxing gibbous Earth.")

Now test your answer. Adjust the time until the phase of the Moon is waxing crescent. Then click on the Location icon, and change the location to the Moon. Adjust the view until you see the Earth. Does its appearance agree with your prediction? (For instance, if you predicted the Earth would be in a crescent phase, is it?) Allow time to run forward. Is the Earth waxing or waning? Does this agree with your prediction?

Lab 8

Apparent Motion of the Planets

The motion of the stars in the night sky is pretty simple: they go in circles about the north celestial pole, making one complete circuit per sidereal day. The motion of the Sun and Moon are a bit more complicated: they share in the daily motion of the stars, but they gradually drift from west to east relative to them. In other words, as the stars race around the north celestial pole, the Sun and the Moon gradually fall behind.

The motion of the planets has a lot in common with the Sun and Moon, but it's a bit more complicated. One of the most important scientific advances in all of human history was Copernicus's correct explanation of why the planets seem to move the way they do. To understand what Copernicus figured out, we first have to examine how the planets appear to move in the sky.

Jupiter's daily motion. The first thing to realize is that, on any particular day, the planets seem to move in pretty much the same way as the stars: they circulate about the north celestial pole, rising in the east and setting in the west, taking about one day to go all the way around. Let's start by making sure of this.

Start up *Stellarium*. Locate the planet Jupiter. If it's not up above the horizon during the night time, shift time forward a month at a time until it is. Once you've reached a time when Jupiter is up at night, let time run rapidly for several days, and observe Jupiter's motion. You should find that, if you don't look too closely anyway, it moves in pretty much the same way as the stars it's next to.

Determine, to an accuracy of about a minute or so, the time when Jupiter sets on two successive days. How much time elapsed between these two occurrences?

If you did the same thing with a star (i.e., measured the difference between two times the star set), what would you find? You can try it if you want, but I hope you know what the answer is.

The two answers should be very similar. Depending on how carefully you measured, you may or may not have found a small difference between them. Now let's examine that small difference more carefully.

Jupiter's slow drift with respect to the stars. Over the course of a few days, Jupiter (and the other planets) seem to move in approximately the same ways as the stars, but there are small differences, which build up to become quite important over longer periods of time. To examine those differences, it helps to "turn off" the daily motion of everything. To do this, switch to the "equatorial mount." You should also turn off the effects of daylight and make the ground transparent, so that you can watch the planets at all times.

Set the time to the present, locate Jupiter, and center it in the field of view. Set time running forward *very* rapidly, so that a year goes by every few seconds. You should see Jupiter generally drift from right to left with respect to the stars, but sometimes reverse itself and drift from left to right.[1] When we look at the sky in the equatorial (stationary-sky) point of view, east is always on our left and west is on our right, so we say that Jupiter usually goes from west to east, but sometimes reverses direction and goes from east to west.

The times when Jupiter goes "backwards" (east to west) are called periods of *retrograde motion*. The next thing we want to do is look for patterns in when Jupiter goes into retrograde motion.

Set the time back to the present, and start it running forward rapidly. Find the beginning and end of the next three time periods when Jupiter is in retrograde motion. It's hard to spot the exact moment when retrograde begins or ends. You don't have to get the exact date right – just determine the month and year. To get you started, I'll tell you that Jupiter will next go into retrograde motion in about June 2021.

While you're at it, find all the times when Jupiter will be in *conjunction* with the Sun during this period (from now through the end of the third retrograde period). This means the time when it passes right next to the Sun in the sky. (Some conjunctions will be closer to the Sun than others; any time one goes past the other counts as a conjunction.)

List your results (three time periods when Jupiter is in retrograde and all the moments when it's in conjunction) below.

---

[1]If you're keeping Jupiter centered in the field of view, of course, it won't actually move at all. When I say you'll see Jupiter go from right to left with respect to the stars, what I really mean is that you'll see the stars slip past it from left to right.

On a blank sheet of paper, make a time line covering the entire range of all these times, and the periods of retrograde and conjunction.

Summarize your results in a sentence or two.

Motion of other planets.   Try the same thing with the planets Mars and Venus. To be specific, identify the next three times when the planet is in retrograde, and all the times of conjunction from now until the end of those three retrograde periods. Make a time line for each.

Which of these two planets shows a pattern very similar to Jupiter? Which is different?

Make a guess about why one of these planets is not like the others.

Lab 9

Sidereal and Synodic Periods

Part 1. An example. Imagine two runners, Ellen and Peter, running around a circular track, starting out out next to each other. Each runner runs at a constant speed, but one is faster than the other: Ellen takes 8 minutes to go around the track once, and Peter takes 10 minutes. Let's call these two times E and P respectively: $E = 8$ minutes and $P = 10$ minutes.

Eventually, Ellen will "lap" Peter – that is, she will have gone around the track one time more than him, so that she's right next to him again. I want you to figure out how much time it takes for this to happen. There are different ways to do this, but one thing that might help is to figure out where Ellen and Peter are after 10 minutes have gone by. By how much (what fraction of a lap) will Ellen will be ahead of Peter? How many times would you have to repeat this until Ellen had lapped Peter?

Your final answer should be a number of minutes. This is the amount of time it takes for the two runners to go from a particular configuration (right next to each other) to the same configuration again. If Peter and Ellen were planets, this would be called the "synodic period" of the two planets.

We're going to want a general relationship giving the synodic period in terms of the "sidereal periods" P and E. It turns out that the relationship is simplest if we express it in terms of the reciprocals of these numbers.

What are the reciprocals of the three numbers P, E, S for this situation? (Express them as decimals, not fractions.)

$$\frac{1}{P} = \qquad\qquad\qquad \frac{1}{E} = \qquad\qquad\qquad \frac{1}{S} =$$

Can you spot a relatively simple mathematical relationship between these three numbers? Write it down in the form of an equation involving $\frac{1}{P}$, $\frac{1}{E}$, $\frac{1}{S}$.

Part 2. The general rule. The rule you wrote down in the previous part turns out to apply no matter what P and E are. We're going to check that this is true algebraically now. In this section, you should forget the specific numerical values for P, E, S (that is, the 10 minutes, 8 minutes, etc.). We're going to work out an expression that is true for different possible values of these quantities.

As before, E stands for the time that Ellen takes to go around the track once, and P stands for the time that Peter takes. S stands for the "synodic period" – that is, the time it takes for Ellen to pull ahead of Peter by one full lap.

After a time S has gone by, how many laps has Ellen run? Your answer will be an algebraic expression involving E and S.[1]

After a time S has gone by, how many laps has Peter run? Your answer will be an expression involving S and P.

By definition, S is the time at which Ellen has run one more lap than Peter. That means that your answers to the two previous questions must differ by one. Write down that last sentence in the form of an algebraic equation.

---

[1] If you don't know what to do, try thinking it through with a particular number in mind. For instance, suppose that it takes Ellen 2 minutes to run a lap (so $E = 2$ minutes), and suppose $S = 20$ minutes. How many laps will Ellen have run in those 20 minutes? Write down whatever you did to get your answer, but use the letters S and E in place of 20 and 2, and you have the expression I want.

Do some algebraic manipulations on that equation until it looks like the equation you wrote down at the end of Part 1.

You're done! You've just derived the general rule relating sidereal and synodic periods. When we apply this in astronomy, E will be the sidereal period of the Earth (one year), P will be the sidereal period of another planet (the length of a year on that planet), and S will be the synodic period.

A couple of quick notes:

1. In this derivation, we assumed that Ellen was faster than Peter. That means that the formula we derived only works if the other planet has a longer period than the Earth. That turns out to be true for all of the planets further out than the Sun (Mars, Jupiter, etc.). The planets Mercury and Venus, go faster than the Earth, so the final formula looks slightly different. You could work it out by the same sort of reasoning if you wanted to, or you can look it up in your textbook.

2. In this expression, we usually give all of the periods in years, but you don't have to. You can use any unit of time you want, but it has to be the same unit for all three quantities.

3. The main reason this relationship is useful is in determining sidereal periods for other planets. After all, we know E (one year), and S is easy to measure – just observe the time it takes between two successive times the planet goes into opposition. So the most common use of this rule is to find the one remaining quantity, namely P.

Lab 10

Kepler's Third Law

Kepler's third law says that the period of a planet's orbit (the time to orbit once, called P) and the radius of its orbit (called a) are related like this:

$$P^2 = ka^3.$$

Here k is a constant. Its numerical value depends on what units we choose for P and a, but it's the same for all of the planets.

Jupiter and its moons look like a mini-solar system: the moons orbit Jupiter in pretty much the same way that the planets orbit the Sun. So it's natural to wonder whether Jupiter's moons obey Kepler's third law. In this lab, you'll use *Stellarium* to simulate observations of the moons to test Kepler's third law.

Start up *Stellarium*. Turn off the effects of Earth's atmosphere / daylight, and make the Earth transparent, so that we can see Jupiter all the time. (Once again, we're making our lives a bit easier than it is for real astronomers: in reality, we could only make the sort observations in this lab at certain times of year.) Set the program to the equatorial (stationary-sky) mount. Find Jupiter, center it in the field of view, and set the field of view to a size where you can clearly see Jupiter and its four Galilean moons (Io, Europa, Callisto, Ganymede).

Let time run forward at a pretty high speed and observe the orbits of the moons. The orbits are really very close to circular, but we're viewing them from the side, so the moons appear to move back and forth.

Pick one of the moons, and run time forwards and backwards until the moon is at its greatest separation from Jupiter. (Be as accurate as possible; you can probably determine the time to within about 10 minutes for the fastest-moving moon and less than an hour for the slowest one.) Measure the angular separation between Jupiter and the moon at this moment. Repeat the procedure for all four moons. Record your results in the table on the last page. You'll initially get the angular separation in arc-minutes and arc-seconds. Convert your answer to a number of arc-seconds. For instance, if you got a value of $2'$ and $20''$, you would record $140''$ (because $1'$ is $60''$, and $2 \times 60 + 20 = 140$)..

How far is Jupiter from us at the time these observations are being taken? Record the distance in A.U. here:

Use the small-angle formula to convert the angular separations for all four moons into actual sep-

arations. These will be the maximum distances the four moons get from Jupiter; in other words, they'll be the radii of the moons' orbits. Record them in the data table, along with their units. (Be sure to keep at least a couple of significant figures for these numbers.)

Now that you know the radii of the orbits, the next step is to find the periods. The most accurate way to do this is to measure the time between moments when the moon passes in front of or behind Jupiter.

Set the date back to today, and then advance time forwards or backwards until one of the moons is just passing in front of the edge of Jupiter from the left. (Aim for an accuracy of about 5 minutes.) Record the the date and time, and also the "Modified Julian Date (MJD)." See Appendix A for details on how to do this.

Let time advance until that moon is just passing in front of the left edge of Jupiter again (one complete orbit later). Record the date, time, and MJD again.

Next, you should determine the time that elapsed between these two measurements. This is the period of the moon's orbit. Give the result as a decimal number of days. (You can do this by working out the difference between the two dates and times, but it's much easier using the MJDs. In fact, that's what MJDs are for!)

Repeat this procedure for all four moons. Each time you start working on a new Moon, set the time back to the present day.

Now that you know the period and the radius of all four moons, you can test Kepler's third law. Square all of the periods to get P^2, and cube all the radii to get a^3. Take the ratio P^2/a^3. Kepler's third law says that this should be the same for all four moons. Is it?

A couple of followup questions:

1. Is the numerical value of k for Jupiter's moons the same as the value for the planets orbiting the Sun? To answer this, figure out the value of k for the Earth's orbit around the Sun using the same units as you used for Jupiter's moons.

2. Why did we have to use the moons of Jupiter for this lab, instead of using Earth's Moon?

MOON	Io	Callisto	Ganymede	Europa
Max. angular separation from Jupiter ($'$ and $''$)				
Max. angular separation from Jupiter ($''$)				
Radius of orbit				
Date/time of first crossing of Jupiter				
MJD of first crossing				
Time of second crossing of Jupiter				
MJD of second crossing				
Orbital period				
P^2				
a^3				
k				

Lab 11

Phases of Venus

One of the most important things Galileo observed with his telescope is that Venus has phases like the Moon. This was the "smoking gun" that showed that the earth-centered (pre-Copernicus) system couldn't possibly be right, so it went a long way toward convincing people that the Earth really did go around the Sun. We're going to examine the phases of Venus to see what Galileo saw and why it mattered.

Start up *Stellarium*. Turn off the effects of Earth's atmosphere (daylight) so that it's possible to see the stars and planets all the time. Use the equatorial (stationary-sky) point of view. Set the view to be centered on the Sun.

Keep the view centered on the Sun, and let time run forwards and backwards at high speed for a year or two. Observe motion of Venus. Note that it swings back and forth past the Sun, never getting more than a certain angular separation from the Sun in the sky.

Here's one preliminary question before we look at the phases of Venus. If you wanted to observe Venus on September 15, 2020. At what time of night should you observe? Specifically, find a rough time when Venus is above the horizon but the Sun is below the horizon. In order to answer this question, you will probably find it useful to make the ground visible (as opposed to transparent).

What if you wanted to observe Venus on February 1, 2021? When would be the best time to see it?

What about in July 1, 2021?

If you had to choose one of those three dates on which to observe Venus, which one would be best? Which would be worst?

Venus spends quite a bit of its time very near the Sun in the sky, at which times it's hard to observe. The easiest time to observe Venus is when its as far away from the Sun in the sky as possible. This is called the time of "maximum elongation." Set the time back to today, and run time forwards until you find the next time of maximum elongation. You'll probably find it convenient to make the ground transparent when you're doing this. Don't worry about being incredibly precise; if you're within about a week or so of the correct date, that's fine. Find the angular separation between Venus and the Sun at that time.

Maximum angular separation between Sun and Venus:

Here's a diagram showing the Earth's and Venus's orbits around the Sun. Suppose that the Earth is at the uppermost point in its orbit as shown, and suppose that Venus is at maximum elongation from the Sun (so that it appears as far from the Sun as possible in the sky). Draw a circle to mark the position of Venus in its orbit. (There are actually two possibilities. One corresponds to the case where Venus is to the East of the Sun and one to the case where it's to the West. If you assume everything orbits counterclockwise, you can figure out which is which, although it's a bit tricky. For the moment, it doesn't matter which one you choose.)

Earth

Sun

Venus's Orbit

Earth's Orbit
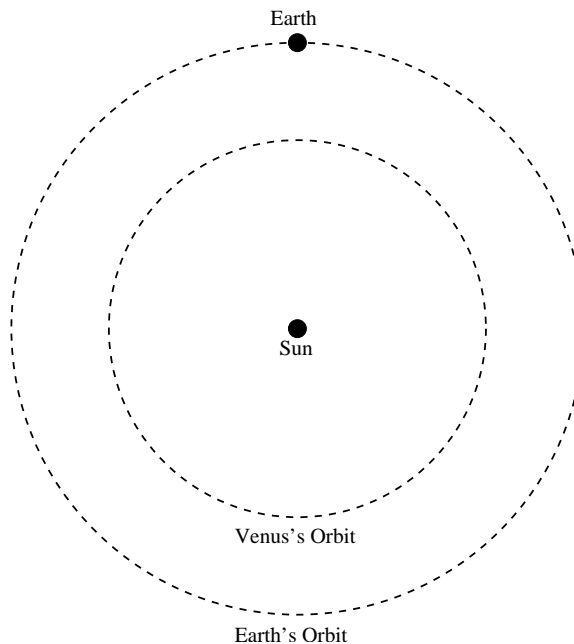
Venus, like the Moon, shines by reflected sunlight. That means that the only part of Venus we'll see is the part that's illuminated by the Sun. In the diagram above, shade in the half of Venus that's lit up by the Sun, and then use the diagram to predict the phase of Venus at this time. (That is, will Venus appear like a crescent, like a "full" Venus, or what?)

After you've made your prediction, use *Stellarium* to test it. Center the view on Venus and zoom in to enlarge the image of Venus. Does it have the phase you predicted?

Now set the date to July 15, 2019, and zoom in on Venus to observe its phase. What is the phase of Venus on that date?

Based on your observation, is Venus in front of the Sun or behind the Sun on that date?

Zoom back out again, and center the field of view on the Sun. Let time run forward until Venus has gone through about half of one orbit. You should see it pass by the Sun, go out to maximum elongation, and then come back until it approaches near the Sun again. At this point, when Venus's path is about to cross past the Sun again, is Venus in front of the Sun or behind the Sun? (Don't use the program to answer this; use what you know about the orbits.)

Based on your answer to the previous question, what do you expect the phase of Venus to be?

What do you expect about the angular size of Venus: should it be bigger or smaller than the last time you looked at it?

Now use *Stellarium* to test these predictions. Center the view on Venus and zoom in. Were your predictions right?

Keep the field of view centerd on Venus and zoomed all the way in. Let time run forward fast for a year or two. The main things to note are that Venus goes through a full set of phases (from crescent to full and back), and that its angular size changes along with its phase.

Earlier, I said that this observation provided strong proof that the old Earth-centered theory was wrong. Let's see why. Remember that in the old theory the Earth was at the center, the Sun went around the Earth, and Venus moved on an epicycle between the Earth and the Sun like this:

Sun

A

D

B

C

Venus's
Epicycle

Earth

Earth's Orbit
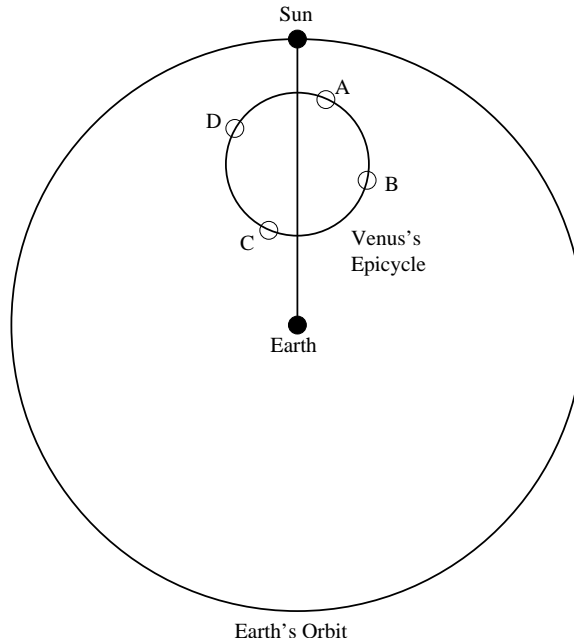
At each of the points A,B,C,D in the diagram, what would the phase of Venus be, according to this model?

A:

B:

C:

D:

What phases of Venus would *never* occur in the earth-centered model?

The fact that Venus is observed to have those phases is the "smoking gun" I referred to.

One more thing. Now that you've measured the maximum elongation of Venus, you can use it to figure out the radius of Venus's orbit. Sketch a picture showing Earth, Sun, and Venus at the moment of maximum elongation (this will look the same as the diagram on the second page of this lab).

Connect these three bodies with straight lines to form a triangle. When Venus is at maximum

elongation, this triangle will have a right angle at Venus. The angle at the Earth is the angular separation between Venus and Sun (that is, the angle you determined on the second page). Indicate both of those angles on the diagram.

We also know the length of one side of the triange: the distance from the Earth to the Sun is 1 AU. Mark this on your triangle as well.

Now you have a right triangle, and you know one angle (other than the right angle) and one side. That means you can use trigonometry to find the lengths of the other sides. Determine the radius of Venus's orbit trigonometrically. (If you don't remember how to do this, ask me.)

Lab 12

Finding Masses with Kepler's Third Law

Newton's version of Kepler's third law goes like this:

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

Here P is the period of an orbiting body, a is the semimajor axis of the orbit, and m_1 and m_2 are the masses of the central body and the orbiting body. G is a constant whose value is

$$G = 6.67 \times 10^{-11} \, \mathrm{m^3/(kg\,s^2)}.$$

All the stuff after the 10^{-11} is just the units of G. It signifies that we're using seconds for our times, kilograms for our masses, and meters for our distances.

One incredibly useful thing about this form of Kepler's third law is that it lets you determine the masses of things. In fact, most of the time, if we know the mass of an astronomical object, we figured it out using this law.

The point of this little "lab" is to write this law in a convenient form for determining masses and then use it to figure out masses of some astronomical objects.

The first thing to realize is that usually the mass of the central body (m_1) is much, much larger than the mass of the orbiting body (m_2). That means that, to an excellent approximation, we can ignore m_2 in Kepler's third law and just write it

$$P^2 = \frac{4\pi^2}{Gm_1} a^3.$$

Since G is a universal constant (always the same known value), we can use this to determine the mass of any object, as long as we can measure the period and radius of the orbit of something that's going around that object.

Use this formula to figure out the mass of the Sun in kilograms. To do this, consider the orbit of the Earth around the Sun. What is P? What is a? (Remember that the value of G given above requires these quantities to be in seconds and meters. You may need to look up a number or two. That's OK.)

When using Kepler's third law, astronomers usually don't use kilograms, meters, and seconds. Instead, they usually measure periods in years, lengths in A.U., and masses in units of the Sun's mass ("solar masses"). In these units, the arithmetic becomes a bit simpler.

Consider the Earth's orbit around the Sun again. In these new units, what are P, a, and m_1? (Hint: you don't need your calculator for this step!)

In these units, what is the numerical value of the constant $\frac{4\pi^2}{G}$? (Plug the values you just wrote down into Kepler's law to see what happens.)

Remember that G is a universal constant, as are 4 and π. So the combination $\frac{4\pi^2}{G}$ will always have this simple numerical value, no matter what orbiting system we're considering (as long as we're using units of A.U., solar masses, and years).

For instance, let's use Kepler's third law to determine the mass of the Earth. Remember that the Moon's orbital period is 27.2 days. What is this in years?

The radius of the Moon's orbit is 384,000 km. What is this in A.U.?

Use these values to determine the Earth's mass in units of solar masses.

Earlier, you determined how many kilograms there are in a solar mass. Use this value to convert your answer for Earth's mass into kilograms.

The solar system orbits the center of the Milky Way Galaxy in a circular orbit with a radius of 8.5 kiloparsecs (kpc). How many meters is this?

How many A.U. is it?

The solar system's speed in its orbit around the Galaxy is about 220 km/s. How much time does it take the Sun to make one complete orbit? (Hint: how far does it have to travel during one orbit?) Give your answer in seconds and in years.

Using your values for the period and radius of the solar system's orbit, determine the mass of the Milky Way Galaxy in solar masses.

People have done surveys of all the visible material in the Milky Way Galaxy (stars, gas, and dust). They estimate that all the visible stuff has a combined mass of about 1.0×10^{10} solar masses. What percentage of the total mass of the Galaxy is visible stuff? What percentage is stuff we don't see? (The latter stuff is called "dark matter.")

Lab 13

Spectra of Light Sources

In this lab you will use a small portable spectroscope to examine several sources of light. The spectroscope consists of a slit at one end that lets light through and a "grating" at the other end that splits the light up into the various colors. We'll worry a bit later about exactly how the grating works.

To set up the spectroscope, hold it so that the slit is vertical. Point it at a bright light source and look through the other end. Rotate the grating end (keeping the slit vertical) until you see horizontal bands of color on either side of the slit. These are the colors that make up the light coming through the slit.

Examine the following light sources with the spectroscope:

1. The white light coming from the overhead projector.

2. The light from a candle flame.

3. The light from one of the two gas discharge lamps.

4. The light from the other gas discharge lamps.

5. The light from the fluorescent lights in the hallway.

6. The light from the Bunsen burner flame, while someone is placing a small amount of the white crystalline stuff in it.

Sketch the spectra of all the light sources, then answer the following questions:.

1. Which sources have "continuous" spectra, meaning that the light consists of all colors from red to blue?

2. For the sources with continuous spectra, can you notice a difference in the relative amounts of the various colors? For instance, does one have more red light relative to yellow or blue, as compared with the other? (This may be hard to discern, because the total intensity of the sources is different.)

3. Using a chart of spectra that I'll give you, try to determine what gases are in the two discharge tubes. Also try to determine what's in the white powder.

Lab 14

The Diffraction Grating

All sorts of waves, including light, do a couple of strange things:

1. Waves bend (a bit) around corners, and spread out (a bit) when passing through holes. This phenomenon is called *diffraction*.

2. When two waves meet each other, they can cancel each other out (if "peaks" of the first wave run into "valleys" from the second), or they can reinforce each other (if peaks meet up with peaks). This is called *interference*.

A diffraction grating is a device that takes advantage of both of these phenomena. They turn out to be very useful in measuring the wavelength of light and in splitting light up into its constituent wavelengths.

A diffraction grating is just a piece of glass (or something similar) with many thin parallel lines ruled on it. When light passes through the small gaps between these lines, it diffracts. Light that passed through the various gaps can then interfere with each other. In some directions, the waves that passed through the various gaps cancel each other out, and in other directions they reinforce each other. The result is a pattern of alternating bright and dark patches.

Suppose we take light with wavelength λ and shine it through a diffraction grating. We then project the light on a wall and measure the positions x of the bright spots (relative to the center of the diffraction pattern) as shown. The theory of interference tells us that

$$n\lambda = \frac{dx}{\sqrt{L^2 + x^2}}.$$

Here d is the separation between lines in the grating and L is the distance from the grating to the wall. n is called the "order" of the spot; it's just an integer: $n = 1, 2, 3, \ldots$.

Note that there are four different quantities in this expression that have units of length. One common source of error is to use incompatible units for these things. The safest thing is to use the same units for all four.

One nice thing we can do with all this is measure the wavelength of light.

L

Laser

Grating

Various x's.

Wall
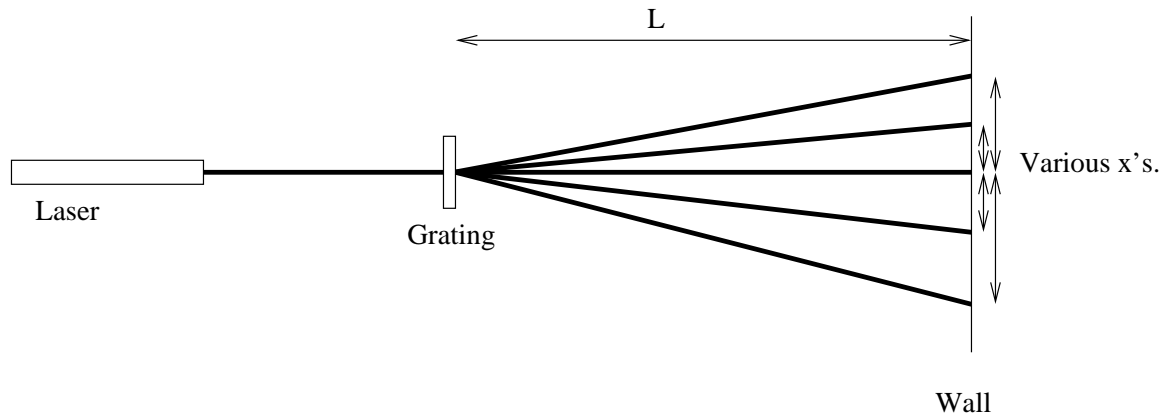
1. Record the separation between grating lines: $d =$

2. Turn on the laser, being careful to avoid looking directly into the beam or shining it at anyone. Aim the light beam through the diffraction grating so that a horizontal series of dots appears on the wall. Adjust the positions of the laser and grating until you easily see at least two dots on either side of the brightest (central) dot.

3. Are the dots of the interference/diffraction pattern the same intensity? Describe the pattern you observe.

4. Measure the distance from the grating to the wall, L, as well as the distances from the central dot to the first dot to the right, x, and the first dot to the left, x'. Compute the average of these x values and record it as x_{ave}. Record all of these values in the table below.

5. Use the equation above to find the wavelength λ. Because these are the first two spots (other than the central dot), use the value $n = 1$.

6. Repeat the procedure using the second dot to the right of center and the second dot to the left. This time, use $n = 2$ when finding the wavelength.

7. If you can see the third pair of dots clearly, repeat the procedure with $n = 3$.

8. Move the laser to a new location (that is, change the value of L) and repeat the procedure for a couple of values of n. In the end, you should end up with at least five different determinations of λ.

9. Compute the average of your determinations of the laser light's wavelength and compare it to the expected value, which is printed on the laser.

L (cm)	x (cm)	x' (cm)	x_{ave} (cm)	λ (nm)

Suppose that we shone white light instead of laser light through the grating. What would the resulting pattern look like?

Lab 15

The Spectrum of Hydrogen

In this lab, you will use a spectrometer to measure the wavelengths of the spectral lines of hydrogen. The spectrometer consists of a slit through which light passes, a diffraction grating, and a small telescope to focus the light that comes out of the grating. The telescope can be rotated in a circle centered on the diffraction grating. By measuring the angles at which the spectral lines appear in the telescope, you can determine the wavelengths of the lines.

The relationship between the angle and the wavelength is

$$d \sin \theta = n\lambda.$$

Here d is the spacing between lines on the diffraction grating, and θ is the angle through which the light from a spectral line is bent by the grating. In this spectrometer, we are always seeing the "first-order" diffraction pattern, so $n = 1$. If you don't know or don't remember your trigonometry, don't worry about it: all you need to know for this lab is how to make your calculator tell you the sine of an angle.

(Incidentally, this formula is essentially the same as the formula you used in an earlier diffraction lab, although that one was written in non-trigonometric form.)

The first thing you'll need to know is d, the spacing between lines on the grating. The grating should be labeled with a number of lines per inch or a number of lines per millimeter. Use this to determine the distance between lines in either meters or centimeters. (If your grating is labeled in lines per inch, it may help you to know that one inch is 2.54 cm.)

Now that you know d, you can determine the wavelength of any spectral line just by measuring the angle θ through which the light is bent.

Set up the spectrometer with the hydrogen lamp right against the slit. Put the grating at the center, so that it is oriented perpendicular to the direction from the slit. Look through the telescope, and

gradually move the telescope around until you see the spectral lines from the lamp. You don't want the bright spot that you see when the telescope is lined up pointing straight at the lamp; the spectral lines you're interested in are visible when it's off to the side.

You should be able to see three spectral lines clearly: one red, one greenish-blue, and one blue one. I'm going to call the red one "line 1," etc., but note that if you start from the center and move out from there, you'll see them in reverse order.

For each of the three brightest spectral lines, determine the wavelength as follows.

1. Adjust the spectrometer so that each spectral line is centered in the telescope, and measure the angle θ corresponding to each spectral line on both the left and the right. Note that the angles marked on the spectrometer have $180°$ for light that is not deflected at all. We'd rather call that angle $0°$ instead of $180°$, so you should subtract $180°$ from your angles.

   Measure the angles as accurately as possible. Note that there is a "Vernier scale" allowing you to measure an extra decimal place. I'll explain in class how to read it.

2. For each of the three lines, average the two measurements of θ together, and use the resulting value to determine λ.

3. For each of the spectral lines, record the energy of a photon, using the rule

$$E = \frac{hc}{\lambda}.$$

   Planck's constant has the value $h = 4.135 \times 10^{-15}$ eV s, and the speed of light is $c = 3.00 \times 10^8$ m/s. If you use these values, and if the wavelength is measured in meters, your energy will come out in units of electron volts (eV).

Line	θ_{left}	θ_{right}	$\theta_{average}$	Wavelength	Photon Energy
Line 1 (reddest)					
Line 2					
Line 3 (bluest)					

It turns out that there is a pattern in the energy levels of hydrogen. The energy levels are of the form

$$\begin{aligned} E_1 &= -\frac{A}{1^2}, \\ E_2 &= -\frac{A}{2^2}, \\ E_3 &= -\frac{A}{3^2}, \\ &\cdots \end{aligned}$$

Here A is a constant, whose value you are to determine.

The longest-wavelength visible spectral line of hydrogen (the red one) occurs when an atom jumps from energy level $n = 3$ down to $n = 2$. So the energy of one of those red photons should be

$$E_3 - E_2 = -\frac{A}{3^2} - \left(-\frac{A}{2^2}\right) = -\frac{A}{9} + \frac{A}{4}.$$

Set this expression equal to the energy you determined for the red spectral line, and solve for the value of the constant A. Show me the result when you're done.

Once you know the value of A, use it to determine the energy of a photon emitted when a hydrogen atom jumps from level 4 to level 2:

From level 5 to level 2:

These should look similar to the energies you meaured for the other two spectral lines. If they don't, talk to me.

What is the energy of a photon corresponding to the next spectral line (a jump from level 6 to level 2)? What is the wavelength of this spectral line?

Using the formula $d \sin \theta = \lambda$, predict the value of θ at which this spectral line should appear in the spectrometer. [Note to the trig-averse: to do this, you'll need to use the "inverse sine" (or "arcsine" or "$\sin^{-1}$") function on your calculator.] Now that you know where to look, see if you can see this spectral line in the spectrometer.

One final question: Why do all of these spectral lines correspond to jumps down to the second energy level? Why not the first or third, for instance?

Lab 16

Refraction and Lenses

Over the next few classes, we're going to examine how telescopes work, but first we need to figure out some aspects of the behavior of light when it passes through transparent materials such as glass.

The apparatus for this lab consists of a light source that produces a set of parallel beams from lasers, along with a variety of transparent and reflective objects for the light to interact with. All of these objects can be attached magnetically to a small whiteboard, so that they can be easily positioned in various ways.

Part 1: Refraction. Plug in the light source and attach it to the whiteboard. Put the clear plastic rectangle in front of the source, so that the light rays hit it at an angle like this:
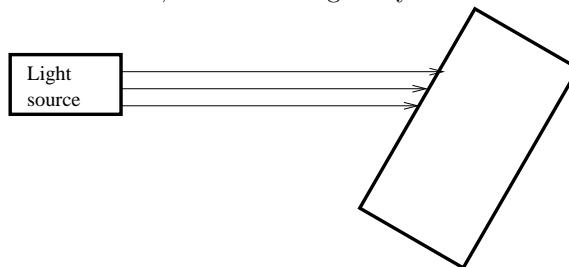
Put a piece of paper under the rectangle. Trace the outline of the rectangle on the paper. Also, trace the path of one of the rays as it enters the rectangle, and the same ray as it exits the other side. Use a straightedge to make sure your lines are straight. Then remove the paper, and draw a straight line connecting the points where the ray entered and exited the rectangle. You should end up with a picture like this:
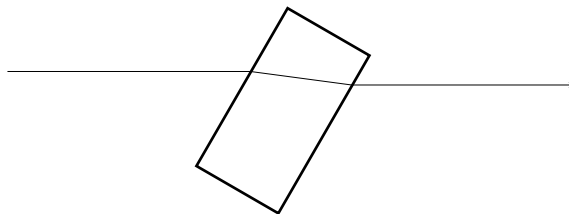

The light ray bends when it enters the plastic. This phenomenon is called *refraction*. There is a rule called Snell's Law that describes the amount of bending:

$$\sin \theta_i = n \sin \theta_r.$$

In this law, n is a constant called the *index of refraction* of the material. The angles θ_i and θ_r are called the angle of incidence and angle of refraction. They're defined to be the angles the light ray

makes with a line perpendicular to the surface where the light ray entered the material, as shown here:
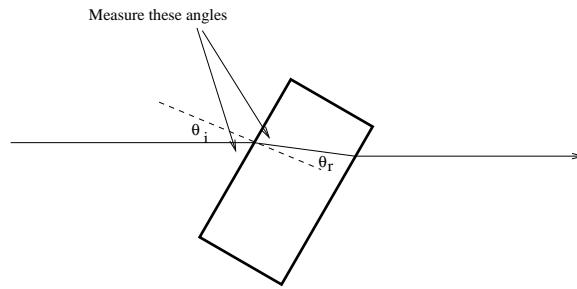
It's easier to measure the angles that the light ray makes with the *edge* of the rectangle, instead of the angle that it makes with the *perpendicular line*. So measure the two angles indicated by the arrows in the picture, and use them to determine the angles θ_i and θ_r.

Now use these angles in Snell's Law (the equation above) to determine the value of the index of refraction n.

The value of n is supposed to be the same no matter what incident angle you choose. To test this, rotate the rectangle so that the light rays hit it at a different angle. Repeat the procedure above to determine θ_i, θ_r, and n. Do you get roughly the same value of n as before?

Part 2: Total internal reflection. Once the light ray has entered the surface, it has to bend again in order to get back out. (That's why the rays that leave the rectangle end up parallel to the ones that go in.) Sometimes the rays can't bend enough to make it back out, and in this case they are reflected, bouncing back and forth inside the material. To see this, take the long, thin rectangular piece of plastic, and arrange it so that one of the light rays enters at a slight angle like this:
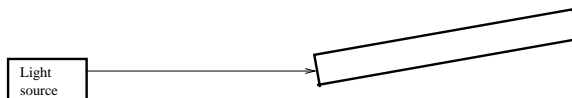
If you adjust the angles right, you should be able to see the light ray bounce back and forth from side to side inside the plastic. Using total internal reflection, light signals can be transmitted over very great distances through fiber optic cables.

Part 3: Lenses. A curved piece of refracting material can act as a lens, bringing light to a sharp focus. Lenses are found in your eyes, as well as corrective lenses (eyeglasses or contact lenses), cameras, microscopes, etc. For this course, of course, the most important application of lenses is in telescopes. We'll examine how telescopes work in future classes. For the moment, let's just figure out some general properties of lenses.

Take one of the three lenses labeled 1,2,3, and place it in front of the light source. You should see the light rays bending to come (approximately) to a *focal point* on the far side of the lens. The *focal length* of a lens is defined to be the distance from the center of the lens to the focal point. Determine the focal lengths of all three lenses. (The easiest way to do this is to mark the focal point on the white board, and also to mark the edges of the lens. Then you can remove the lens and measure the distance from the focal point to the middle of the lens.)

These three lenses are all called *converging* lenses, because they cause the light to converge to a point. There are also *diverging* lenses, which generally have concave rather than convex surfaces. Put the lens labeled 5, which is a diverging lens, in front of the light source and see what it does. A diverging lens causes the light rays to spread out *as if* all the rays were coming from a focal point on the same side of the lens as the light source.

To see how this works, trace the paths of several of the rays after they exit the lens. Also, mark the positions of the edges of the lens. Now remove the lens and, using a straightedge, extend the rays *backwards* until they come together at a single point. The distance from the center of the lens to this focal point is called the focal length of the lens. Measure the focal length of this diverging lens.

Part 4: Combinations of lenses. Put one of the converging lenses (1,2,3) in front of the light source, and then put another one of the converging lenses right in front of the first one. The two lenses should behave approximately like a single lens. Is it a converging or a diverging lens? How does the focal length of the combined lenses compare with the focal length of the original lenses? (I'm not looking for a measurement here, just a general observation.)

Try the same thing with a combination of a converging and a diverging lens. Does the combination behave like a converging lens or a diverging lens?

Remove the other lenses and place the lens labeled 4 in front of the light source. Is this a converging lens or a diverging lens? Is its focal length larger or smaller than those of lenses 1,2,3,5?

Place the diverging lens (5) right in front of lens 4. Does the combination of lenses 4 and 5 behave like a converging lens or a diverging lens?

By now, you should have seen that sometimes a combination of a converging and a diverging lens acts like a converging lens, and sometimes it acts like a diverging lens. There is a general rule for predicting, for any given pair of lenses, which way the combination of the two will behave. Can you guess what this general rule might be?

Lab 17

Image formation by lenses

In this lab, you will examine how lenses form images. In particular, you will test the relationship between the location of the object whose image is being formed and the location of the image. Theoretically, we expect that relationship to be

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

Here p is the *object distance* (distance from object to lens), q is the *image distance* (distance from image to lens, and f is the *focal length* of the lens.

Part 1: Focal Length of a Lens.

1. Arrange the light source appartus to produce parallel rays of light across the surface of a piece of paper.

2. Place a convex lens in front of the light rays. Mark the lens position on a piece of paper, and trace the path of the rays.

3. What is the distance from the center of the lens to the focal point? This is the focal length of the lens.

Part 2: Image Formation

1. Attach the light source to one end of the optical bench, and place the lens partway down the bench. Arrange the light source so that the side with a circle and arrow drawn on it is facing the lens. This will be the "object" whose image we will be examining with the lens.

2. Measure the length of the arrow. We will call this h_o, meaning "the height of the object."

3. Adjust the lens and the white screen until you see a clear, in-focus image of the object on the card. Measure the height of the image h_i, the distance from lens to object p, and the distance from lens to image q. Record them in the table below.

4. Move the lens about 5 cm toward or away from the object, and move the card until you get a clear image again. Measure h_i, p, q again. Repeat until you have five sets of measurements. For some, the image should be closer to the lens than the object ($q < p$) and some should be the other way around ($p < q$). (Note: make sure that the object is always further away from the lens than the focal length.)

p	q	h_i	$\frac{h_i}{h_0}$	$\frac{q}{p}$	f

5. For each observation, calculate and record the ratio of image and object heights, h_i/h_o, and the ratio of image and object distances, q/p. Record these in the table. What can you conclude about these quantities?

6. For each observation, use the formula at the beginning of this lab to calculate the focal length of the lens, and record it in the last column of the table. Do these results appear consistent with your measurement of the focal length in part 1?

7. Move the lens closer to the object, so that the object distance is less than the focal length. Note that there is nowhere you can place the card to get a clear image. Look through the lens at the object. Does the image appear to be larger or smaller than the actual size of the object? Does it appear to be closer or further away?

8. Suppose the object is extremely far away from the lens. Based on your results, where would you expect the image to form? (In other words, about what would you expect q to be?) Would you expect the image to be large or small?

9. If the Sun is shining, take the lens outside along with a piece of white paper, and try to form an image of the Sun on the paper. Are the results consistent with your predictions?

Lab 18

Refracting Telescopes

Part 1: Theory

Suppose that you have a telescope made of two lenses. The "objective" lens has a focal length of 10 centimeters, and the "eyepiece" lens has a focal length of 1 centimeter. A refracting telescope is constructed so that the two lenses are separated from each other by a distance equal to the sum of their focal lengths (11 cm in this case). That way, one of the focal points of the objective matches up with one of the focal points of the eyepiece The setup is something like this:
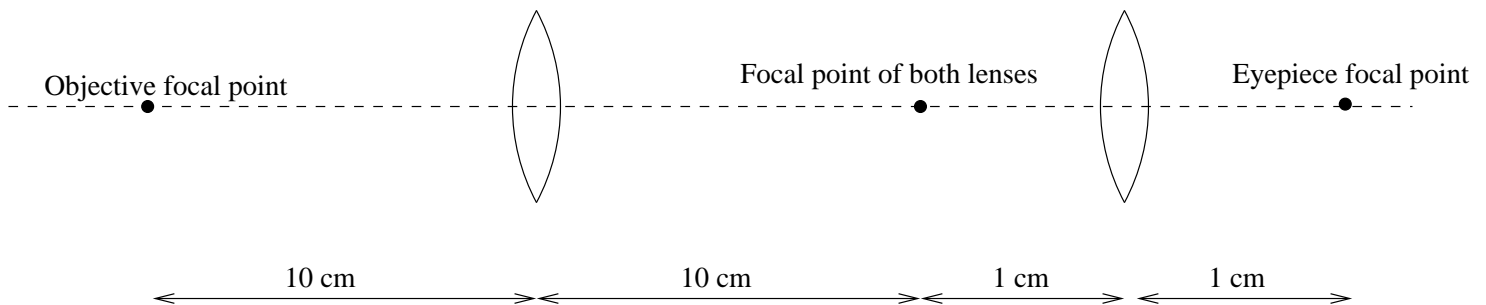
(Note that this picture is not to scale.) The objective lens is on the left, and the eyepiece is on the right. The observer looks through the eyepiece (not surprisingly) at a faraway object off to the left.

Suppose that you are using this telescope to look at an object that is 100 meters (10^4 centimeters) away. We will work out where the image of this object is produced, and how much it appears to be magnified.

The key facts that we'll need to do this are the rules you checked in your last lab. First, there's the relationship between the object position, the image position, and the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \tag{18.1}$$

Second, there's the rule for the *linear magnification* of an image:

$$\frac{h_i}{h_o} = \frac{q}{p}.$$

Here h_i is the height of the image produced by the telescope, and h_o is the height of the actual object. The ratio of these two is called the linear magnification M, so we'll write simply

$$M = \frac{q}{p}. \tag{18.2}$$

[Note: Often the expression for the magnification is written with a minus sign in it: $M = -\frac{q}{p}$. It doesn't really matter for our purposes, though.]

In the calculations you're about to do, you'll be using known values of p and f and trying to find q. So to make life simpler, take equation (1) and solve it for q in terms of f and p. (That is, rearrange it so that it says $q = \ldots$.) If you simplify this expression as much as you can, you'll be less likely to make mistakes later. (A good strategy is to isolate $\frac{1}{q}$ on one side of the equation, then put the stuff on the other side over a common denominator, and finally take the reciprocal.) If you're not sure whether you've got the right expression, show it to me.

Now suppose we have an object that is 100 m to the left of the telescope above (so $p = 10^4$ cm). The light passes through the objective lens and forms an image (ignore the eyepiece for now). Where is this image? Specifically, how far is it from the objective lens? Also, is it a real image or a virtual image? When figuring out the location of the image, record an answer in centimeters with at least three digits after the decimal point. (In other words, record your answer to an accuracy of 0.001 cm.)

Mark the approximate location of this image with an X on the diagram above. How far is this image from the eyepiece lens? Again, record your answer to an accuracy of 0.001 cm.

Calculate the linear magnification of this image using equation (2). This should be a number between 0 and 1. It indicates the factor by which the image is smaller than the original object.

Now we want to figure out the effect of the eyepiece lens. The big idea here is that we treat the *image* produced by the objective lens as if it were the *object* for the eyepiece lens. That is, we pretend that the image is a real thing that the eyepiece is examining. That means that the answer to the

previous question (distance from the image to the eyepiece lens) becomes the new *object distance p*.

Is the image produced by the eyepiece a real image or a virtual image?

How far away is the image produced by the eyepiece? (When you calculate this, your answer for q will come out negative. That's OK: you can ignore the minus sign. It just indicates that the image is virtual.)

Now you know where the image will appear to be when you look through the eyepiece. Calculate the ratio of the distance to the original object to the distance to the final image. This is how many times closer the telescope makes the object appear.

Use equation (1) to calculate the linear magnification of the image produced by the eyepiece. This should be a number greater than 1. It indicates the factor by which the eyepiece has magnified the image.

The *overall linear magnification* of the telescope is the product of the magnification produced by the objective lens and the magnification produced by the eyepiece lens. What is the overall linear magnification of this telescope?

This number tells you how many times larger or smaller the final image is than the original object. You should have found that this number is less than 1, which means that the final image is actually *smaller* than the object. That may seem surprising, since we expect a telescope to make things look bigger, not smaller. What's going on here?

The point is that this linear magnification tells you how the actual size of the image and object compare. If you want to know how big an object *looks*, you need to consider their angular sizes.

The ratio of the angular size of the final image to the angular size of the original object is called the *angular magnification*. What is the angular magnification of this telescope?

Hint: The angular size depends on both the actual size and the distance. You've already worked out what effect the telescope has on the distance of the image (as compared to the object), and what effect it has on the size of the image (as compared to the object).

There is a general rule that says that the angular magnification of a telescope is equal to the ratio of the focal lengths $f_{\mathrm{objective}}/f_{\mathrm{eyepiece}}$. Is that true in this case?

Part 2: Constructing a Telescope

In this part of the lab, you will use two lenses and some cardboard tubes to construct a telescope.

Determine the focal lengths of the two lenses, by any means you can think of.

According to the rule at the end of the last part, the angular magnification of a telescope is the ratio of the focal lengths. What is the angular magnification of a telescope constructed with these lenses?

Insert the lenses into the tubes and adjust the distance between the lenses until the telescope gives a sharply-focused image of faraway objects.

What is the orientation of an object as seen in the telescope? Are they upside-down? Are they reversed left-to-right?

Stand in a location where you can see the eye chart posted on the wall. Adjust your distance from the eye chart until you can just barely read the F at the top of the chart (without using the telescope). Now look at the chart with the telescope. Which row is the smallest row you can read using the telescope? How many times smaller is this row than the F at the top? (The sizes of the letters in each row are printed at the right of the chart.) Compare this ratio with the angular magnification you calculated for the telescope. Do the results make sense?

Lab 19

Stefan-Boltzmann Law

Introduction. Stars (and other astronomical objects) emit a lot of their energy in the form of *thermal radiation.* To understand how these objects work, we need to know how much energy they emit. In this lab, you'll examine the relationship between the amount of *power* (that is, energy per time) emitted by an object and the object's *temperature.*

The apparatus for this experiment consists of a lamp and a radiation sensor. The lamp will be hooked up to a power supply, and the radiation sensor is attached to a meter that indicates the intensity of light striking it.
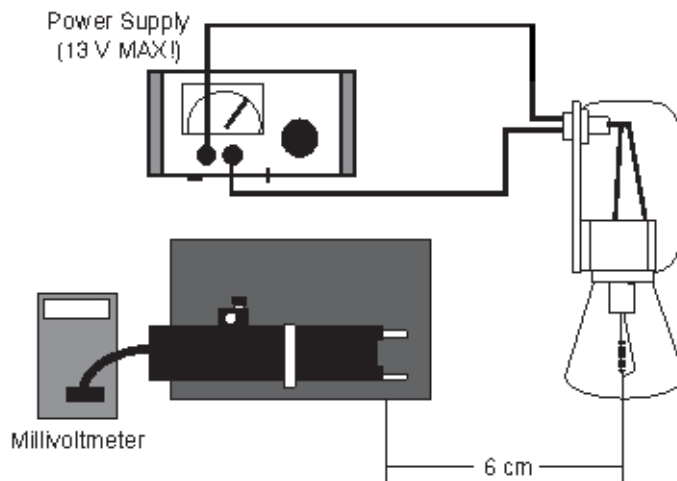
By changing the voltage being applied to the lamp, we can change its temperature. The radiation sensor then tells us how much power is being emitted at any given temperature.

We're going to want to know how the power depends on the temperature of the lamp filament. We can't conveniently attach a thermometer to the filament, so we have to get the temperature another way, namely by measuring the filament's *resistance*, which depends on the temperature in a known way. Here's how this works.

Suppose we apply a certain number of volts V to the lamp. That voltage will cause electric current to flow through the lamp. We'll measure the amount of current, which is traditionally called I (for some reason). The resistance is the ratio of these two:

$$R = \frac{V}{I}.$$

The value of R increases in a known way as the temperature goes up, so by keeping track of R we'll

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be able to keep track of the temperature.

Gathering the data. The first thing you'll need to know is the resistance R of the filament when it's at room temperature. We'll do this by applying a small voltage to the lamp, and recording the amount of current that flows. If we keep the voltage small, we can assume that the lamp won't heat up very much, so we can use these values to figure out the resistance at room temperature.

Connect the lamp to the power supply as shown in the diagram. Turn the voltage knob all the way to zero (counterclockwise), and turn the current knob up (clockwise). Now turn on the power supply. The voltage display on the power supply should read zero. Turn up the voltage knob gradually until it reads about 0.5 volts.

At this point, the power supply is supplying a small amount of energy to push a small amount of current through the lamp. It's not enough to heat up the filament significantly or to make the lamp glow. Record the voltage (V) and current (I) readings on the power supply here. The voltage should be 0.5 V, and the current is some number of "amperes" (A).

$$V = \qquad\qquad I =$$

Use the rule $R = \frac{V}{I}$ to determine the resistance. The unit of resistance is called the "ohm" and is written Ω.

$$R_{300\,\mathrm{K}} =$$

You might wonder why we call it by the strange name $R_{300\,\mathrm{K}}$. The reason is that this is the value of the resistance at room temperature, which corresponds to a temperature of about 300 kelvin.

Now that you have this, you can start taking measurements when the lamp is actually glowing. First, make sure that the lamp is at the same height as the radiation detector and the distance between the two is about 6 cm. The exact distance doesn't matter, but it's important that it not change once you start measuring. Make sure that the detector is facing directly toward the lamp and there are no other bright light sources in its path. The radiation sensor should be hooked up to the voltmeter's inputs labeled COMMON and V/Ω, and it should be set to read DC voltage. That setting on the meter's dial will probably look like a V with straight lines (not a wavy line) next to it or above it. Check with me to see if you've got everything wired up correctly.

Once you're ready, turn up the voltage dial gradually until it reads 2 volts. You should see the lamp glowing faintly.

In the data table at the end of this lab, record the voltage V (which is 2 V), the current I from the display on the power supply, and the reading on the voltmeter attached to the radiation sensor. The last number is a measure of the power being radiated by the lamp, so we'll call it P. Record

these values in the first row of the table on the last page of this lab. (Leave the other columns in this table blank for now.)

Once you've done this, repeat for voltages of 4, 6, 8, 10, 12 V.

You should expose the radiation sensor to the lamp light only briefly when you're making each power measurement. In between eaurements, place sheets of insulating foam between the lamp and the sensor, with the silvered surface facing the lamp, so that the temperature of the radiation sensor stays fairly constant.

Analyzing the data. You'll need to determine the temperature of the filament during each of these measurements, by filling in the remaining columns in the data table. Here's how.

First, find the value of the resistance for each row, using the rule $R = \frac{V}{I}$.

The temperature is determined by how this resistance compares with its value at room temperature, so compute the ratio $\frac{R}{R_{300\,\mathrm{K}}}$ for each row.

The graph near the end of the lab shows how this ratio is related to temperature. For each row in the table, use the graph to determine the value of temperature corresponding to the resistance ratio you've determined. (Read across the graph at the height corresponding to the ratio. When you hit the curve, go straight down to get the temperature.)

Once you've done this, the data table should be filled in. Now we want to graph it, using *Excel*. Appendix B of your lab manual contains some information about making graphs in *Excel*. Create an *Excel* data table with two columns, T and P. (Put T on the left.) Following the instructions in Appendix B.2, make a graph showing how P depends on T.

You should find that the power goes up as the temperature goes up.

We want to see what sort of mathematical relationship matches this graph. Try adding a "trendline" to the chart, following the instructions in Appendix B.3. This shows the straight line that matches the data as well as possible. Does a straight line look like a good fit to this set of data?

Double-click on the trendline to get various options to customize it. You should see various options such as "Exponential," "Linear," etc. These replace the straight-line relationship with various other mathematical relationships. You can try all of these out to see which ones look like a good match to the data.

In the end, we'll be most interested in the one called "Power." Select this trendline option, and also check the box that says "Display equation on chart." What this does is to find the best-fitting mathematical relationship of the form $y = (\text{something}) \times x^{(\text{something})}$ between the two quantities in the graph. In our case, x and y are T and P respectively.
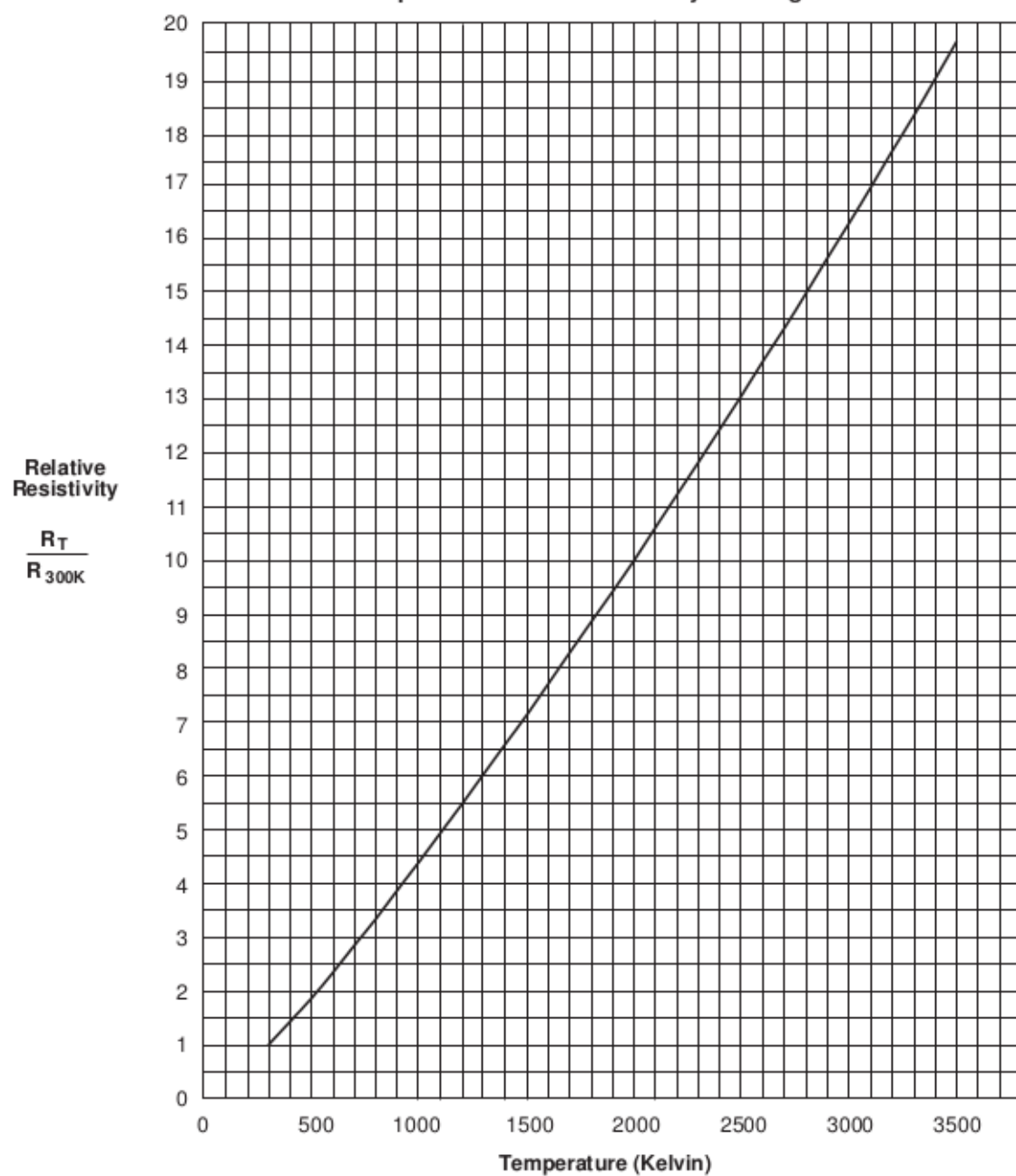
According to *Excel*, what is the best-fitting value of the exponent (the power of x) in this relationship?

What do we expect this exponent to be, according to the Stefan-Boltzmann Law? (If you don't

remember this from your reading, you can look it up.)

Does your data agree reasonably well with the Stefan-Boltzmann Law?

Temperature versus Resistivity for Tungsten

Relative
Resistivity

$$\frac{R_T}{R_{300K}}$$

Temperature (Kelvin)

V (volts)	I (amps)	P (mV)	R (Ω)	$R/R_{300\,\mathrm{K}}$	T (K)

Lab 20

Solar Radiation and the Greenhouse Effect

The Earth (like the other planets) is warmed by sunlight. The Earth also radiates energy into space, mostly in the form of infrared radiation. On average, the amount of radiation received from the Sun must equal the amount of radiation emitted by the planet. After all, if the Earth emitted less radiation into space than it received from the Sun, then its temperature would keep going up; and if it emitted more radiation than it received, it would keep cooling down. Since the temperature of the Earth is fairly stable over the years, the radiation coming in from the Sun must balance the radiation being emitted.

Note: You've doubtless heard about global warming, so you may wonder how I can get away with saying that the temperature of the Earth is fairly stable. Global warming does mean there's an imbalance between the incoming and outgoing radiation, but it's very slight. Although that slight difference is enough to have major consequences for the climate, it's small enough that we can safely ignore it in the calculations we're about to do.

In this lab, you will use this idea to form an estimate of the temperature of the Earth. Specifically, you will calculate the total amount of solar power striking the Earth's surface. Then you will assume that the Earth radiates that same amount of power back into space. Assuming that the Earth radiates as a blackbody, you can use the Stefan-Boltzmann law to relate the radiated power to the temperature and so calculate the temperature of the Earth.

This calculation depends on one important assumption: that the Earth radiates like a blackbody. That assumption is wrong, so our calculated temperature will not match the actual temperature of the Earth. In particular, a phenomenon known as the "greenhouse effect" blocks some of the Earth's radiation from escaping into space. The results of this calculation will give an indication of how important the greenhouse effect is here on Earth.

Let's get started. Remember that the Sun's luminosity is

$$L_\odot = 3.86 \times 10^{26} \, \text{W}.$$

A watt (W) is a unit of power equal to a joule per second. This is the total amount of power emitted by the Sun in all directions. The first thing we want to do is to figure out how much of this power strikes the Earth.

Consider an imaginary sphere centered on the Sun with a radius of 1 A.U. All of the radiation that leaves the Sun must pass through this sphere. Calculate the amount of radiation that strikes each

square meter of this imaginary sphere's surface by dividing the Sun's luminosity by the sphere's surface area. Remember that the surface area of a sphere of radius R is

$$A = 4\pi R^2.$$

Remember that an A.U. is 1.49×10^{11} meters. Give your answer in units of watts per square meter (W/m^2).

Now work out how much solar radiation strikes the Earth. To do this, note that the Earth covers up a small circular patch of that great big imaginary sphere. The amount of radiation striking the Earth will equal the amount striking that small circular patch. Recall that the area of a circle is πR^2. The radius of the Earth is 6380 kilometers. What is total amount of solar radiation striking the Earth? Give your answer in watts.

The Earth doesn't actually absorb all of that energy; some of it is reflected into space. The fraction of energy that is reflected by an object is called the object's *albedo*. Measurements of Earth from space have shown that the Earth's albedo is 0.35, meaning that it reflects 35% of all the radiation that hits it back into space and absorbs the rest.

What is the total amount of solar radiation *absorbed* by the Earth?

This number must equal the amount of radiation emitted by the Earth. Assume for the moment that the Earth emits radiation as a blackbody. That means that it obeys the Stefan-Boltzmann law,

$$F = \sigma T^4.$$

Recall that $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4). Also, remember that the flux F in the Stefan-Boltzmann law is the power radiated per area (watts per square meter). To get the total power radiated by the Earth, therefore, we should multiply the flux by the surface area of the Earth. As noted earlier, the surface area of a sphere is $4\pi R^2$, so the total power radiated by the Earth is

$$L_\oplus = 4\pi R^2 \sigma T^4.$$

Here $\oplus$ is the symbol for the Earth. L stands for "luminosity," which is what we always call the total amount of power emitted by an object. R is the radius of the Earth, and T is the temperature.

Set this expression equal to the total amount of solar power being absorbed by the Earth and solve for the temperature T. Your answer will come out in kelvin. Subtract 273 to convert it to degrees Celsius.

Does this seem like a reasonable estimate of the Earth's actual average temperature?

The main thing that went wrong in this calculation is the assumption that the Earth radiates as a blackbody, also known as a "perfect radiator." Suppose that something in the Earth's atmosphere is blocking radiation from getting out, so that it's not a perfect radiator. Then the amount of radiation leaving the Earth will be less than the amount calculated from the Stefan-Boltzmann law. This alters the balance between incoming and outgoing radiation, resulting in an increase in temperature.

Carbon dioxide and other gases in the Earth's upper atmosphere reflect some of the Earth's infrared radiation back down to the planet, preventing it from escaping into space. This "greenhouse effect" means that the Earth is not a perfect radiator, and it is why the Earth is warmer than your calculation above would indicate.

The greenhouse effect is responsible for climate change, so we think of it as a bad thing these days, but without it, the Earth would be uninhabitably cold! The problem we face now is that human activity has led to an *increase* in the greenhouse effect, altering the balance that existed for many millennia before we started altering the composition of the atmosphere.

Venus is an interesting example of the greenhouse effect. Follow the same steps as above to figure out what the surface temperature of Venus would be in the absence of a greenhouse effect. Here's some useful information about Venus:

- Distance from Sun to Venus: 0.723 A.U.

- Radius of Venus: 6050 km.

- Venus's albedo: 0.65.

You'll also need to know the luminosity of the Sun (3.86×10^{26} W) and the number of meters in an A.U. (1.49×10^{11} m).

Probes of Venus indicate that its surface temperature is about 700 K (430°C, or 800°F). Would you say that the greenhouse effect is more important on Venus than on Earth or vice versa?

Lab 21

Energy Flow in the Sun

In this lab, you will examine how the energy produced in the core of the Sun makes it out to the surface. The nuclear reactions in the core of the Sun produce photons, which eventually make their way to the surface of the Sun. This process takes a long time because the photons undergo frequent collisions with ionized atoms in the solar interior. These collisions cause the photons to bounce around in random directions instead of zooming straight out.

You will use simulation software to examine the process by which these photons make it out of the Sun. If it's not already running, start up the "energy flow out of the Sun" application on the lab PC and log in. It doesn't matter what you enter for names and table number.[1]

Part 1: Photons moving through a gas.

Select "Interaction" under the "Simulation" menu, and click "Run." This shows what happens to photons as they pass through a dense gas. The stationary circles represent atoms in the gas, and the smaller white circles represent photons. You should imagine that there is a source of photons off to the left somewhere, sending photons into the gas. These photons have the correct amount of energy to correspond to one of the spectral lines of the atoms. That means that when a photon hits an atom, it is very likely to be absorbed, sending the atom into an excited state. The atom quickly drops back down to its original state, re-emitting the photon in a new direction. The end result is that the photon undergoes a "random walk": instead of moving in a straight line, it bounces around making frequent changes in direction.

Stop the simulation. Under "Photon Type," select "Continuum," and restart the simulation. With this setting, the photons now have an energy that does *not* correspond to a spectral line of these atoms. This means that the atoms cannot absorb the photons, so the photons usually just pass right through.

On rare occasions, a photon with the wrong energy can bounce off of an atom, as you'll see if you let the simulation run for a while.

The *mean free path* is the average distance that a photon travels before it bounces off of an atom. Which has a longer mean free path, the continuum photons or the line photons?

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[1]For boring technical reasons, this program has to run inside of a "virtual machine" that's set up on the lab computers. I'll make sure that it's all set up before class. This may mean that the application may run in a window inside of another window, which looks a little funny but doesn't make a difference in the end.

If the continuum photons *never* bounced off of atoms (as opposed to rarely doing so), what would their mean free path be?

Here's an important thing to realize for the rest of the lab: Throughout most of the interior of the Sun, the gas is ionized, meaning that the electrons are stripped off of the protons. Ionized gas is capable of interacting with photons of *all* wavelengths, not just specific ones. That means that *in the interior of the Sun, all photons behave like the "line" photons in this simulation.*

Part 2: Photon Diffusion in the Sun.

Click "Return" to get back to the main menu. Under "Simulation," go to "Flow" and then to "1 Photon." This simulation shows the path of a single photon as it makes its way out from the center of the Sun. Click "Run" a few times to see how the photons behave. Go to "Parameters" and check the "Yes" box under "Trails." This will cause the photon's path to be visible as it makes its way out.

In the "Parameters" menu, you can also vary the size of the Sun. Select # of Layers. It should initially be 20. Change that value to 40 and try running the simulation again. You should see that the Sun has gotten larger, and not surprisingly it takes more time for the photons to make their way out.

In this model of the Sun, each "layer" of the Sun has a thickness equal to the mean free path of the photons. That is, on average, a photon can make it in or out of the Sun by only one layer between bounces. The more layers there are, the more times a photon is likely to bounce on its way out, and so the longer it will take to get out. In fact, there is a general rule relating the number of layers to the time it takes photons to get out, which you will now determine.

Because the photons bounce around randomly, some get out faster than others. We're going to be interested in the time it takes the *average* photon to get out, so we'll want to run the simulation with a bunch of photons all at once. Go to the "Simulation" menu, and click "Flow" and then "Diffusion." This will behave just like the simulation, but with many photons.

Under "Parameters," you can set the number of layers and the number of photons. Set the number of layers to 10, and the number of photons to 200. Start the simulation, and let it keep going until all 200 photons have escaped from the Sun. record the average number of interactions required for a photon to make it out.

Repeat the process for a 20-layer Sun.

Do it one more time, this time for a 30-layer Sun. This one will take a little while!

Based on these results, take a guess about the mathematical relationship between the number of layers and the average number of interactions taken to get out:

Ask me if you're not sure whether your guess is right.

This relationship only tells you about the *average* time it takes a photon to get out. As you've seen, some photons are faster than average, and some are slower. Let's do one more simulation to get a feel for this.

Set the number of layers to 25, and the number of photons to 1000, and start the simulation running. After how many interactions does the first photon make it out?

Wait until most of the photons have escaped. What is the average number of interactions of all the photons? Is this consistent with your expectations based on the mathematical relationship you guessed above?

The graph in the lower part of the window shows the number of photons escaping the Sun as a function of time. The yellow line is the time when the number of interactions is n^2 (the square of the number of layers). Sketch a copy of the graph below.

This graph shows that quite a few photons (well over half) make it out in less than the average time, but some take much, much longer than that.

Part 3: Numbers for the actual Sun.

In these simulations, the number of layers is very small. In the actual Sun, the photons can travel only a very short distance before scattering (that is, the "mean free path" is small). Since a "layer"

is supposed to be only as thick as the mean free path, a realistic model of the Sun would have many, many layers. In this portion of the lab, you'll work out some rough numbers for the actual Sun.

Based on the physical laws governing interactions between photons and electrons, scientists estimate that the mean free path of photons inside the Sun is about a tenth of a millimeter (10^{-4} meters). The Sun's radius is about 700,000 kilometers. If each "layer" has a size equal to the mean free path, how many layers does the Sun consist of?

Based on the relationship you found earlier between the number of layers and the number of interactions (bounces), how many interactions will an average photon undergo on its way out of the Sun?

The average distance a photon travels between interactions is equal to the mean free path, and of course photons travel at the speed of light (300,000 km/s). What is the average time between interactions for a photon in the Sun?

From the average time between interactions and the total number of interactions, determine how much time it takes the average photon to make it out of the Sun. Convert your answer into years.

If the nuclear fusion reactions powering the Sun were to somehow stop tomorrow, would we notice an immediate reduction in the Sun's luminosity?

Is there any way we could tell if the nuclear reactions in the Sun stopped tomorrow?

Lab 22

Parallax

Introduction.

Parallax is the method used to find the distances to the closest stars. It is the first step in the "distance ladder" used to find distances to other stars and ultimately to other galaxies. In this lab you will use the method of parallax to determine the distance to a nearby object: one of the classroom windows.

Here's the big idea behind parallax measurements. Suppose you want to measure the distance to the star in the drawing on the next page. You observe the apparent position of the star from two locations A and B. The directions you have to look in order to see the object from the two locations are slightly different, and the amount of difference can be represented by the angle θ. If you can determine θ, then you can use the small-angle formula to determine the distance to the object.

The small-angle formula says that

$$s = \left(\frac{2\pi}{360°} \right) \theta D.$$

Here s is the distance between the two observation points, and D is the distance to the object.[1]

(By the way, when we first learned the small-angle formula, we said that s was the separation between two different objects being observed, not the separation between two different observation points. We're now applying the formula to a different situation, but since the geometrical setup is the same, the same formula works.)

The next question is how to measure θ. *In principle*, we could measure θ like this:

1. Put the telescope at point A and point it at the star.

2. Move the telescope from point A to point B, being *very careful* not to rotate it as you move it.

3. Once the telescope is at point B, rotate it until it is again pointing at the star. Measure the angle through which you had to turn the telescope. This is θ.

Here's a picture of the various angles:

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[1] This is the version of the small-angle formula that has θ in degrees. That's the version that's most useful in this lab. If you measure angles in arc-seconds instead, then you use the version of the formula that has the number 206 265 in it.
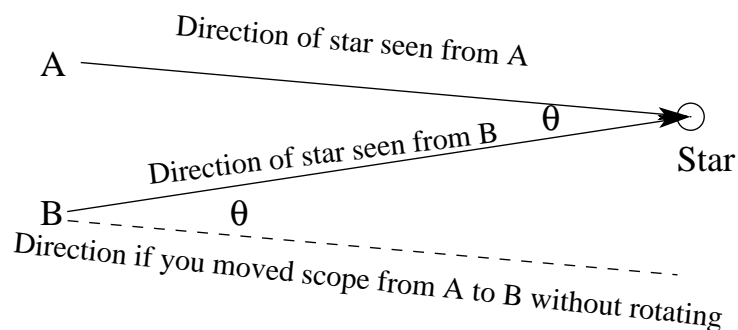
Although there is nothing wrong with this method in principle, in practice it's quite inaccurate. The reason is step 2: it's very hard to make sure that you didn't jostle the telescope and rotate it just a little bit when you were moving it. Since parallax angles are very small, even a slight error in this step will ruin the method. So *the method above is no good in practice.*

To make the method work, we need to measure the shift in the angular position of the star by comparing it to some much more distant background objects. Here's the idea:

1. Put the telescope at point A. Point the telescope at the star.

2. Rotate the telescope until it points at a particular *background object.* Measure the angle through which you had to turn the telescope to do this. The result is the *angular separation* between the star and the background object. Let's call this angle α.

3. Move the telescope to point B (without worrying about whether you're rotating it as you go).

4. Point the telescope at the star, then at the same background object as before. Measure the angular separation this time, and call it β.

5. The parallax angle θ is just the difference between α and β.

The picture below illustrates the various angles in this method. The main point is that the background object is presumed to be extremely far away (much further than the star). This means that the lines of sight to the background object from the two locations are essentially parallel to each other.
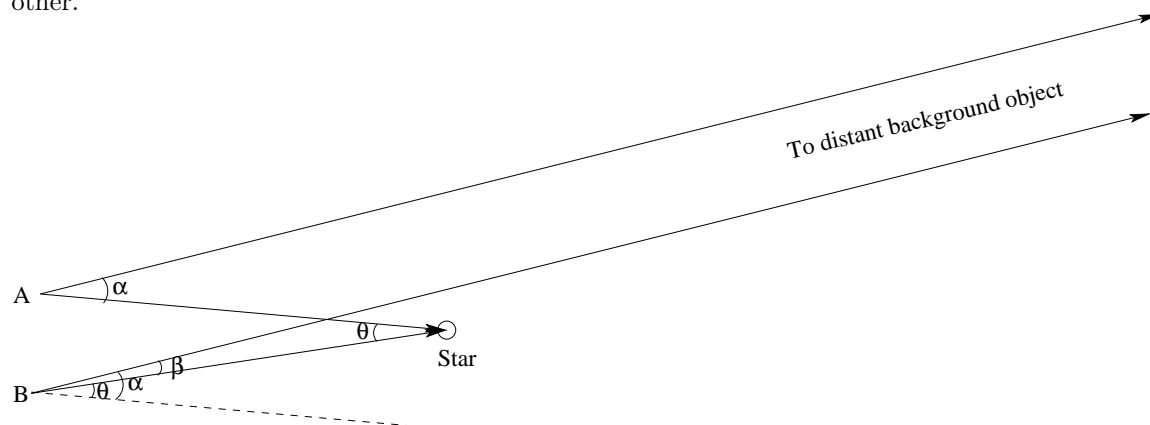
Procedure.

The "star" in this lab will be the vertical divider separating two panes of the windows at the side of the classroom. Arrange the telescopes so that you have a clear view of the "star." Put down a couple of pieces of masking tape to mark the location of the telescope. You'll need these later

when you determine how far you moved the telescope. You can mark the locations of any convenient points on the telescope base.

(Incidentally, the telescopes we're using here are the ones on the spectroscopes. We're not using the "slit" side of the spectroscopes at all in this lab; we're just using the "telescope" side. So just set up the apparatus so that the slit is off out of the way, and ignore it.)

Rotate the telescope until the "star" is centered in the field of view. Use the crosshairs to make sure it's lined up accurately. Record the angle made by the telescope. If you forget how to read the Vernier scale on the telescope base, ask me.

Look through the telescope, and choose a faraway "reference object." This should be something easily identifiable and far away (outside of the building, and preferably across the street). It should also be close to, but not quite, the same direction as the "star." Good choices are tree trunks, signs, or lamp posts.

Rotate the telescope until the reference object is centered on the crosshairs, and record the angle made by the telescope now.

Subtract these two angles from each other. The result is α, the angular separation between the "star" and the reference object. (If I were you, I would convert the angles from degrees and minutes into a decimal number of degrees before subtracting. For instance, I'd convert 42° 20′, into 42.33°, by using the fact that a minute is $\frac{1}{60}$ of a degree.)

Move the telescope over about 15 centimeters or so. (The exact amount doesn't matter.) Move it in a direction perpendicular to the direction of the "star," not towards or away from the star. Mark the new position with masking tape again.

Look through the telescope. You should notice that the position of the "star" relative to the background objects has shifted a bit. That's parallax.

Repeat the procedure above: Measure the angular position of the star and the angular position of the reference object (same object as last time!).

What is the parallax angle θ?

Using a ruler and your masking-tape marks, measure the distance between the two observation points A and B.

Now you have all the information you need to determine the distance to the "star." What is it?

Use a tape measure to determine the actual distance.

Do you think your results are reasonably accurate? What do you think the main sources of error are in this method?

Lab 23

The Hertzsprung-Russell Diagram

In this lab, you'll use the data from the Hipparcos satellite to construct a Hertzsprung-Russell diagram. The data you'll use is the real stuff returned by the satellite; all I've done for you is to convert it from archaic astronomy units (such as magnitudes) into standard modern physics units (such as watts per square meter).

The full Hipparcos data set contains 100,000 stars, but you'll be working with only the approximately 1000 closest stars and the approximately 1000 brightest stars.

1. There are two Excel files on Blackboard (in the "Downloads for Labs" section) containing Hipparcos data on the closest and brightest stars. Download these files to the desktop, and open the file containing the brightest stars. It contains about 1000 rows and four columns: Catalog number, Parallax angle, temperature, and brightness. Note that the units of the various quantities are indicated on the second line (except for the catalog number, which doesn't have any units). You will be adding a bunch of new columns to the spreadsheet. As you do, use those first two lines indicate what quantity is in each column and what its units are.

2. For each of these stars, you will need to determine its luminosity and radius. Since there are 1000 stars, you wouldn't want to do it by hand, so you'll use Excel formulae instead. However, it's good to calculate at least one by hand in order to check the Excel calculation you're going to do. So for the first star in the list, calculate the following things:

   (a) Distance in parsecs.

   (b) Distance in meters.

   (c) Luminosity.

   (d) Radius.

3. Now use Excel formulae to repeat these calculations. Column E will contain the distances to the stars in parsecs. Indicate this in the first two rows of the column. Then, in cell E3, enter a formula that calculates the distance to the first star. (Appendix C contains advice on how to do this.) If your formula agrees with the result you calculated above, then drag the formula down to fill in the distances to the rest of the stars.

4. In the next three columns, use formulae to determine distance in meters, luminosity, and radius.

5. Create a Hertzsprung-Russell diagram for this set of stars. This is just a graph with temperature on the horizontal axis and luminosity on the vertical axis. (See Appendix C if you need advice on making graphs in Excel.)

   When you first make your graph it will probably not look right. One reason for this is that H-R diagrams traditionally have *logarithmic* y axes. This means that the numbers on the y axis are spaced out in powers of 10. Double-click on the y axis of your graph to get the "Format Axis" window, and check the box that says "Logarithmis scale." In that same window, you can adjust the minimum value on the axis so that the points aren't all crowded together near the top.

   Also, by annoying, stupid tradition, H-R diagrams are plotted with high temperatures on the left instead of the right. So double-click on the x axis and check the "values in reverse order" box.

   By the time you're done, your graph should look something like a normal H-R diagram. In particular, the main sequence should be clearly visible running from upper left to lower right. Also, the x and y axes should be labeled to indicate both what quantity is being plotted (*e.g.*, "Temperature") and what units it's in (*e.g.* "K").

6. Print out your graph. Mark one more point on it by hand to indicate the Sun. The Sun's temperature is 5800 K and its luminosity is 3.86×10^{26} W. The Sun should lie on the main sequence; if it doesn't, something has gone wrong.

7. Follow the same steps to create an H-R diagram for the data set consisting of the closest stars. Print out this graph as well.

8. Which of the two data sets contains more white dwarfs? Explain why this makes sense.

9. Which of the two data sets contains more giants? Explain why this makes sense.

10. For the closest stars, make a graph showing the temperatures of the stars on the x axis and the radii on the y axis. Use a logarithmic scale on the y axis, and make sure the ranges on the axes are set so that the points aren't all crowded together at one end. As in your previous graphs, be sure the axes are clearly labeled including units. Print out the graph.

11. What is the radius of a main-sequence star with a temperature of 5000 K? What is the radius of a white dwarf star with a temperature of 10,000 K? What is the radius of the largest star in this sample?

Lab 24

Spectroscopic Binary Stars

Part 1: Determining the masses of spectroscopic binary stars. For many binary star systems, we can't actually resolve the two stars (i.e, see them as separate objects); all we can do is observe the combined light from both of them. In this lab, you'll work through a calculation to figure out the masses of a pair of *spectroscopic binary* stars, in which we can observe the spectra of the pair of stars together but can't see each of the stars individually.

Suppose that you measure the spectra of such a binary star system, and you find that it has a pair of spectral lines, both very close to the wavelength 656.28 nm, which is the wavelength of a prominent spectral line in hydrogen. At any given moment, one is redshifted to a longer wavelength, and the other two is blueshifted to a shorter wavelength. As time passes, each line cycles back and forth between redshift and blueshift.

You make some measurements of the spectral lines and observe the following facts:

1. At a moment when the lines have the largest Doppler shifts, one of them has a wavelength of 656.72 nm, and the other has a wavelength of 655.40 nm.

2. Half a cycle later (i.e., 5 hours later), the wavelengths are 655.84 nm and 657.16 nm.

3. Another 5 hours later, the spectral lines are back at the wavelengths they were at at the first moment. In other words, the time it takes for the lines to cycle all the way around and back to where they started is 10 hours.

Sketch a picture showing the orbits of the two stars. Indicate where the stars are at Moment 1 and at Moment 2, and indicate the location of you the observer.

What are the speeds of the two stars? (You can use the wavelengths at either of the two moments described above. You might want to use both just to check your work.)

You know the speeds of the two stars, and you know how long it takes them to go around once. From this information, determine the radii of the two stars' orbits. You can assume that the orbits are circles. (If you're not sure what to do, I'll give the one-word hint "circumference". If you still don't know, of course, ask!)

The quantity called a in Kepler's Third Law is the distance between the two stars. How is this related to the two radii? (You may want to refer to your picture at the beginning of this lab.) Now that you know that relationship, tell me the value of a for this system.

Use Kepler's third law to determine the total mass $M_1 + M_2$ of the two stars. Give your answer in solar masses. I strongly recommend that you do this by determining a and P in units of AU and years respectively. Recall that Kepler's third law takes a relatively simple form in this case.

Now determine the individual masses of the stars. To do this, make use of the fact that the speeds at which the stars orbit are related to their masses like this: $M_1v_1 = M_2v_2$. Combine this fact with the value you found for the total mass, and solve for M_1, M_2.

In fact, what you've just discovered is the *minimum* possible mass for the two stars: they may in fact be heavier than this. Can you think of a reason that your calculations may have given masses that are smaller than the true values? (Can you think of an assumption you made in your calculations that may not be true?)

Part 2: Finding the masses of planets orbiting other stars. One of the hottest topics in astrophysics these days is the discovery of planets orbiting other stars. One of the ways people do this is by looking for the "wobble" of the star caused by the planet orbiting it. The planet itself is too small and dim to be observed, so we must infer its presence from the star's motion.

For example, observations of the star HD209458 reveal the folowing:

1. The velocity of the star wobbles back and forth in a regular way, repeating every 3.525 days.

2. The star's maximum speed towards us is 87.1 m/s, and its maximum speed away from us is 87.1 m/s in the other direction. (Note that these are meters per second, not kilometers per second.)

3. The star is a main-sequence star very similar to the Sun. Its mass is $1.13M_\odot$.

You can use this information to figure out the mass of the planet orbiting this star, even though you can't see the planet or measure its spectral lines.

First, use Kepler's third law to determine the radius of the planet's orbit. I recommend that you use the AU-year-solar-mass version of the law. You can assume at this point that the mass of the planet is so small that it can be ignored in comparison with the star's mass – that is, $M_1 + M_2$ can be taken to be the same as the mass of the star.

Now that you know the radius of the planet's orbit, you can figure out the speed of the planet in its orbit. (How far does it travel during one orbit? How much time does this take?)

Now remember the rule $M_1 v_1 = M_2 v_2$ from above: the masses of the two bodies in an orbit are related to their orbital speeds. You know both speeds, so you can express the mass of the planet in terms of the star's mass. What is the mass of the planet in solar masses?

Planet-hunters often express the masses of the planets they find in units of Jupiter's mass. Jupiter's mass is about $\frac{1}{1000}$ that of the Sun. What is the mass of the planet in units of Jupiter masses?

By the way, just as in part 1, the mass you've found is actually the minimum possible mass, for the same reason. But subsequent observations show that in fact the problem that might result in this calculation giving an underestimate of the true mass isn't really a problem in this case: the mass you've found is about right. If you want to know how we know, I'll tell you.

Lab 25

More Fun with Binary Stars

An astronomer observes the spectrum of a star many times over a series of days. She observes that the Hα line of hydrogen usually appears split into two lines with slightly different wavelengths. She measures the wavelengths of those two lines during each of her observations and plots a graph of the results that looks like this:
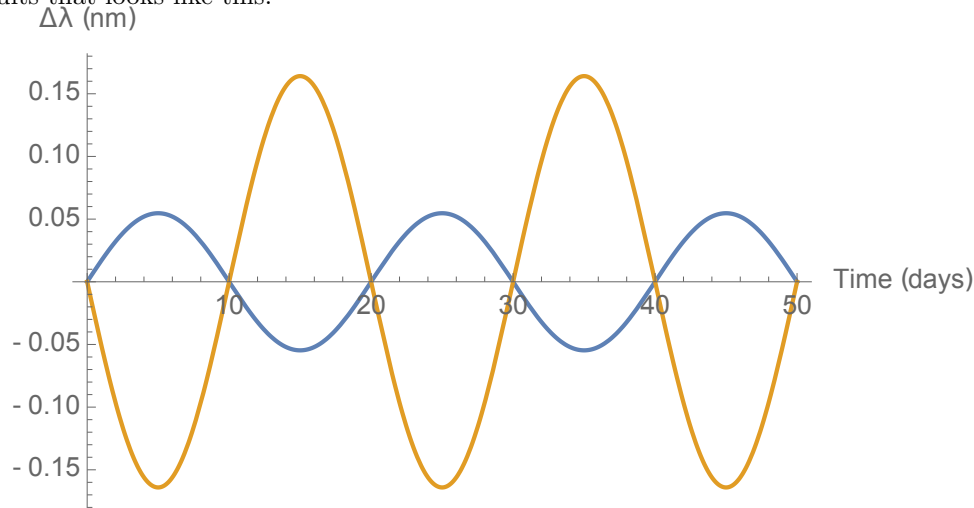
What's plotted on this graph are the wavelength shifts of the spectral line. So when $t = 0$ days, there is no shift at all, and both lines appear right where they would be in the lab. A day or two later, the spectral line is split in two, with one line (represented by the blue curve) having a wavelength slightly longer than expected, and one (represented by the yellow curve) having a wavelength slightly shorter than expected. By 10 days, the two spectral lines have come back together again, and then they split apart again.

As you probably know, the reason this is happening is that this star is really a *double-line spectroscopic binary system*. The two graphs show the spectral lines due to the two stars in the system, which alternately move toward you and away from you as they orbit around each other.

To save you the trouble of reading numbers off the graph, I'll tell you that when $t = 5$ days, the blue curve is at a height of 0.055 nm, and the yellow curve is at -0.16 nm. This moment is when the two graphs are at their highest and lowest points, meaning that one star is moving directly toward you and one is moving directly away from you. You can assume that the stars are moving in circular orbits.

Let's call the star whose spectral line is given by the blue curve star number 1, and the other one star number 2. The wavelength of the Hα line, when measured in the lab, is 656.3 nm. Armed with

this information, I want you to tell me the following about these stars and their orbits. (You may need some unit conversions. If so, look them up.)

1. How fast are the two stars moving?

2. Which star is heavier? What is the ratio of their masses M_1/M_2? (Remember that the masses and speeds of the orbits are related, so that $M_1 v_1 = M_2 v_2$.)

3. What is the period of the orbit? (That is, how much time does it take for the graph to repeat itself?)

4. What are the radii of the two stars' orbits? (Suggestion: How far does each star travel during one period of its motion? How is that number related to the radius?)

5. At the end of this document is a picture of the two stars' orbits. You, the observer, are way off to the left. Say both stars are moving counterclockwise. When $t = 5$ days, where are the two stars in this diagram? (Hint: is star 1 moving toward you or away from you at this moment? What about star 2?) Mark the locations on the diagram with a "5".

6. Mark the locations of the two stars when $t = 10, 15$, and 20 days.

7. What is the distance between the two stars at any given time? (Hint: How is it related to the radii of the two orbits, which you know?) The distance between the two stars is the a in Kepler's Third Law.

8. What is the total mass $M_1 + M_2$?

9. What are the masses M_1 and M_2?

To observer
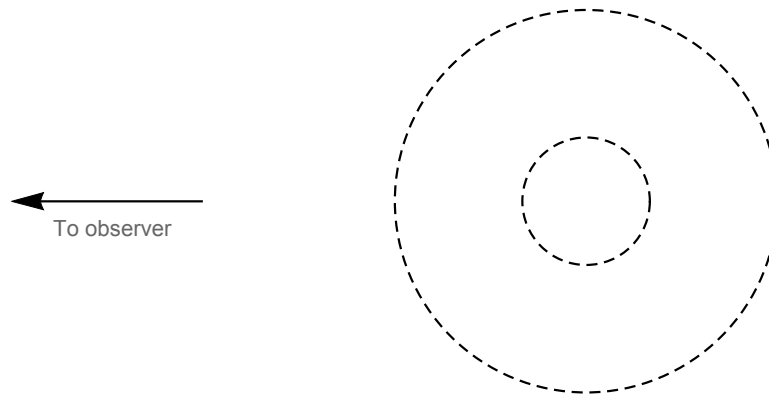
Lab 26

Planetary Nebulae

This lab is available at http://web.williams.edu/astronomy/research/PN/nebulae/exercise1.php .

Exercise 1: Emission Lines and Central Star Temperature in Planetary Nebulae

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Planetary nebulae are hot glowing gas clouds ejected by dying low- to intermediate-mass stars. They glow because they are heated by energetic ultraviolet photons from the exposed stellar core. According to Kirchhoff's laws, the light produced by a planetary nebula should be an emission spectrum, with spikes of emission at specific wavelengths corresponding to the elements in the gas. A spectrum can be displayed as a picture showing stripes of color at the wavelength of each emission line, or as a graph, plotting the amount of light at each wavelength.

In this exercise, you will learn how to

- use this database to plot a spectrum
- identify the elements in a planetary nebula's spectrum
- interpret the spectra of several planetary nebulae to rank the temperatures of their central stars.

Ionization in a Planetary Nebula

The central star in a planetary nebula is the exposed core of the original star. The temperature of the central star in a planetary nebula can be quite high, sometimes exceeding 200,000 K. (Eventually, all central stars will cool and become white dwarfs, and the planetary nebulae will expand and fade from view.) Typically, central star temperatures range from about 30,000 K to 100,000 K. At these high temperatures, a star will emit a great deal of radiation; the amount of radiation at each wavelength depends on the temperature, according to the Planck Law, otherwise known as "blackbody radiation," energetic enough to ionize the atoms in the nebula.

Of particular interest is the amount of ultraviolet radiation emitted; the energy of some ultraviolet photons is so high that they can ionize the atoms in the nebula, stripping off one or more electrons. The amount of energy required to produce the next higher level of ionization in an atom is called its ionization potential, usually expressed in electron volts. In general, heavier atoms are more easily ionized for the first time than lighter atoms. If an atom is already ionized, the remaining electrons are held more tightly, and it becomes even harder to remove the next electron to ionizing the atom more highly. The degree of ionization of atoms in a planetary nebula depends on the temperature of the central star. The hotter the star, the more photons of all energies it emits, **and** the greater the proportion higher-energy photons. Therefore a hotter star is capable of ionizing more atoms to higher ionization states than a cooler star. So, by examining the spectrum of a planetary nebula to see what ionization states of the various elements are present, you can get an idea of the temperature of the central star.

Plotting a Spectrum

All spectra in the database are listed on the Browse page. Clicking on the name of any planetary nebula takes you to the "Spectrum Display" page for that nebula. To expand any region of the graphed spectrum, hold the left mouse button down at one corner of the region you wish to enlarge, drag the mouse to the opposite corner of that region, and then release the mouse button. You can do this repeatedly to keep enlarging. To get back to the full plot, click on the "Zoom Out" radio button under the graph display. The horizontal axis of these graphs is the wavelength in Angstroms, and the vertical axis is the flux (in ergs cm^{-2} s^{-1} Angstrom^{-1}.

Identifying Emission Lines

The Templates page contains a set of spectra labelled with the wavelengths of emission lines seen in planetary nebulae and identifying the ion producing each emission line. The name of the element is given using the standard chemical symbol from the periodic table (e.g., H=hydrogen, N=nitrogen, Ne=neon, etc.). The ionization state of the element is indicated by a Roman numeral suffix in the following way: neutral=I, singly ionized=II, doubly ionized=III (i.e. ionization state = Roman numeral -1). For example, O III means doubly ionized oxygen, O^{+2}. Certain electron transitions involve energy levels that are said to be *metastable*; the resulting emission lines are called *forbidden lines*, which really only means that they are less likely to occur than emission lines from the ordinary kind of transitions. Conditions in planetary nebulae, as it turns out, are extremely conducive to the production of this kind of emission line, and in fact, most of the emission lines you will see in these spectra are forbidden lines, which are denoted by brackets around the ion designation (i.e. a forbidden line produced by doubly ionized oxygen would be written as [O III].

The Exercise

You may find it helpful to print out this page of instructions.

Listed below are three planetary nebulae whose central stars have very different temperatures. You will examine the spectra of each nebula and by noticing the presence or absence of certain emission lines, be able to rank them in order of the temperature of the central star.

<div align="center">M1-57   IC 3568   NGC 6210</div>

1. Print out the data table for this exercise. Notice that there is an identical table for each planetary nebula, containing a selection of forbidden lines produced by highly ionized atoms in the nebular gas. The first column gives the wavelength of the emission line, and the second identifies the element and the ionization stage. The third column lists the ionization potential of the preceding ionization stage. For example, for [K IV] (which means K^{+3}) the relevant ionization potential is that of K^{+2}, since we are interested in the energy required to ionize K^{+2} one step further to K^{+3}.Click on the name of one of the planetary nebulae above; this will take you to its spectrum display. Expand the spectrum around each of the wavelengths listed in the data table, and look for that particular emission line. Note its absence or presence and fill in the appropriate table. Because of the relative motion between the Earth and each nebula, the wavelengths may be Doppler-shifted slightly from their nominal values. If you are not sure whether the line is really there, write "?" in the table.
2. Repeat for the other two nebulae.
3. Examine each table. Of the elements producing emission lines that you detected, note which has the highest ionization potential, meaning which element requires the most energy to reach its observed ionization state. The higher the maximum ionization potential, the hotter the central star must be.
4. You can now rank these three planetary nebulae on that basis. Fill in the "results" portion of the data table in descending order of stellar temperature.
5. You might want to search the literature for determinations of the stellar temperatures for these planetary nebulae to see if your relative ranking is correct. And you also might like to try this for some of the other nebulae on the Browse page.

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Data Tables for Exercise 1
Emission Lines and Central Star Temperature
in Planetary Nebulae

Table 1

NEBULA NAME: M1-57

Wavelength (Angstroms)	Ion	Ionization Potential of next lowest ionization state	Present (Y) Absent (N)
6087	[Ca V]	67.1 eV	
7005	[Ar V]	59.8 eV	
6434	[Ar V]	59.8 eV	
6101	[K IV]	45.7 eV	
8046	[Cl IV]	39.6 eV	
7531	[Cl IV]	39.6 eV	

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Table 2

NEBULA NAME: IC 3568

Wavelength (Angstroms)	Ion	Ionization Potential of next lowest ionization state	Present (Y) Absent (N)
6087	[Ca V]	67.1 eV	
7005	[Ar V]	59.8 eV	
6434	[Ar V]	59.8 eV	
6101	[K IV]	45.7 eV	
8046	[Cl IV]	39.6 eV	
7531	[Cl IV]	39.6 eV	

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Table 3

NEBULA NAME:NGC 6210

Wavelength (Angstroms)	Ion	Ionization Potential of next lowest ionization state	Present (Y) Absent (N)
6087	[Ca V]	67.1 eV	
7005	[Ar V]	59.8 eV	
6434	[Ar V]	59.8 eV	
6101	[K IV]	45.7 eV	
8046	[Cl IV]	39.6 eV	
7531	[Cl IV]	39.6 eV	

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Results

Rank (1=hottest central star)	Nebula
1	
2	
3	

Lab 27

Planetary Nebulae 2

This lab is available at http://web.williams.edu/astronomy/research/PN/nebulae/exercise2.php .

Exercise 2: Detecting Interstellar Reddening

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In this exercise, you will learn how interstellar reddening affects the spectrum of a planetary nebula. Comparing the spectra of several nebulae, you will be able to determine which are more or less affected by interstellar reddening. Combining these results with the galactic latitudes of these planetary nebulae, you will be able to conclude something about the distribution of interstellar dust in our Milky Way Galaxy.

The Balmer Decrement

In the Bohr model of the hydrogen atom there are many distinct energy levels, between which electrons can transfer if they emit or absorb the proper amount of energy. Upward moves require absorption of energy, while downward ones release energy. Downward electron transitions that end on the second energy level are called the Balmer series, and are important in optical astronomy, since these are the only transitions that involve visible light. The first three of these are called Hα, Hβ, and Hγ, for the transitions from 3-2, 4-2, and 5-2, respectively. When many ionized hydrogen atoms are recombining, as in a planetary nebula where atoms are being ionized and recombining all the time, the captured electrons cascade down through the energy levels, emitting photons of the appropriate wavelengths as they fall. The likelihood of any particular downward jump is dictated by atomic constants, and thus the ratios of all possible transitions can be calculated. This leads to the "Balmer decrement," the well known ratios among the intensities of the Balmer lines, where Hα is the strongest line, Hβ is weaker, Hγ is weaker still, and so on. Under typical conditions in planetary nebulae these ratios are (from Osterbrock, Astrophysics of Planetary Nebulae and Active Galactic Nuclei, University Science Books, 1989):

$$H\alpha/H\beta = 2.86 \text{ and } H\gamma/H\beta = 0.47$$

The Phenomenon of Interstellar Reddening

Thus, the Balmer decrement, the intensity ratios of Balmer lines in all planetary nebulae, should be roughly the same. However, this is not what is observed. Interstellar reddening produced by micron-sized dust particles selectively dims shorter-wavelength, bluer light more than it does longer-wavelength, redder light, leading to Balmer line ratios that differ systematically from the theoretical predictions. A planetary nebula lying behind a cloud of interstellar dust will be observed to have the intensity ratios Hα/Hβ **more** than 2.86, and Hγ/Hβ **less** than 0.47. The more dust, the greater the disparity between the observed and theoretical Balmer decrements. Turning this concept around, from the size of the discrepancy between observed and theoretical Balmer decrements, astronomers can infer the amount of interstellar reddening, and therefore, dust, between us and a given planetary nebula.

The Milky Way Galaxy and Galactic Coordinates

Our solar system and all of the planetary nebulae in this database reside in the Milky Way Galaxy. The Milky Way is a flattened spiral of stars, gas, and dust, surrounded by a more spheroidal, extended, and much more diffuse region, called the galactic halo. Locations in the Milky Way are conveniently specified by galactic coordinates, similar to latitude and longitude as seen by someone viewing from the center of the Earth. The origin of the galactic coordinate system, though, is not at the center of the Milky Way, but rather, is located at the sun's position, because that's where we are as we view the heavens.

The Browse page of this website lists galactic coordinates for each planetary nebula as "lll.l (sign)bb.b," where lll.l is the galactic longitude in degrees, and bb.b is the galactic latitude in degrees. The plus or minus sign before the galactic latitude indicates whether the object is above or below the galactic plane, respectively.

The Exercise

Listed below are eight planetary nebulae. For each of them, you will estimate the relative intensities of the Hα and Hβ lines and compare them with the theoretical prediction of 2.86 and with each other. Finally, you will be able to come to some conclusions about the distribution of dust in the Milky Way.

H4-1
IC 2448
J900
NGC 2022
NGC 6578
NGC 7026
NGC 7662
Pe1-18

Data Collection

1. Print out a copy of the data table for this exercise. You may also find it helpful to print out this page as well.
2. The wavelength of Hα is 6563 Angstroms, and for Hβ it is 4861 Angstroms. Take a moment to look at the relevant templates for wavelength identifications to familiarize yourself with the appearance of the spectrum in the vicinity of these Balmer lines. You will find it helpful to print out a copy of the templates containing them.
3. Review the Help page to make sure you know how to view and expand a spectrum.
4. Click on one of the nebulae listed above. This will get you to the "View Spectrum" page where you will see the full spectrum of the nebula.
5. Locate the Hα line and expand the spectrum to zoom in on this wavelength region. Repeat if necessary to give a good view of the line. (It is usually bracketed on either side by two lines that come from nitrogen.) Now zoom in on the upper part of the line so that you can get a good estimate of the maximum intensity (height) at the center. Write this maximum height down in the data table, remembering to include the scale exponent, given at the top left of the graph window (e.g., x10^{-12}).
6. Zoom back out to the full spectrum by pressing on the "zoom out" button under the spectrum display. Now zoom in on the bottom of the Hα line. Read the level of the continuum, or base level of the spectrum near the Hα line, and record it on the data sheet.
7. Subtract the continuum level from the maximum height and record the Hα net height in the data table. Zoom back out to the full spectrum.
8. Repeat steps 5-7 for the Hβ line.
9. Now calculate the observed ratio of Hα/Hβ using the net heights and write the result in the last column of the data table.
10. Repeat steps 5-9 for all remaining nebulae.

Data Analysis

1. Fill in Table 2 of the data table, listing the nebulae in order from highest Hα/Hβ to lowest.
2. Fill in the last column with each nebula's galactic latitude, **without** the plus or minus sign (i.e., use the absolute value of the galactic latitude).
3. What trend do you notice between the galactic latitude and the value of the Hα/Hβ ratio?
4. Remember that the amount of reddening depends how much dust you are looking through along the line of sight to any nebula. This in turn depends on the thickness of the dust and the distance to the nebula. If all of these nebulae were at the same distance from us (**which they are not**, although most of the objects in this sample are at similar distances), what does your conclusion from step 3 imply about the distribution of dust in the Milky Way? Can you find images or other material from your textbooks or reliable websites to support your conclusion?

Data Tables for Exercise 2
Detecting Interstellar Reddening

Table 1

NEBULA	galactic latitude	Hα maximum	Hα continuum	Hα net height	Hβ maximum	Hβ continuum	Hβ net height	Hα/Hβ
H4-1								
IC 2448								
J 900								
NGC 2022								
NGC 6578								
NGC 7026								
NGC 7662								
Pe1-18								

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Table 2

NEBULA	Hα/Hβ (highest to lowest)	Galactic Latitude (without sign)

100

Lab 28

Radio Astronomy of Pulsars

Radio Astronomy of Pulsars

Student Manual
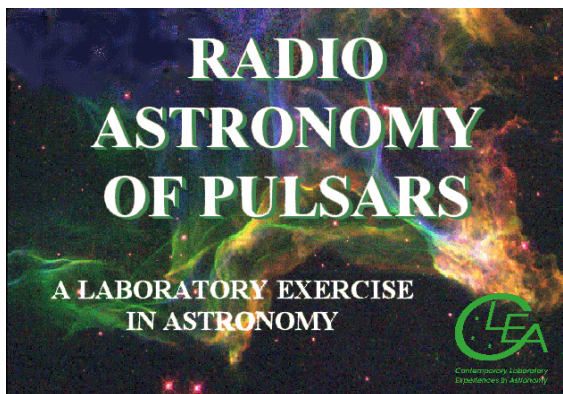
Goals

You should be understand the fundamental operation of a radio telescope and recognize how it is similar to, and different from, an optical telescope. You should understand how astronomers, using radio telescopes, recognize the distinctive properties of pulsars. You should understand what is meant by interstellar dispersion, and how it enables us to measure the distances to pulsars.

Procedure

The lab consists of the following parts

1. Familiarization with the radio telescope.

2. Observation of a pulsar with a single-channel radio receiver to learn about the operation of the receiver and the appearance of the radio signals from a pulsar at various receiver settings.

3. Determination of the pulse periods of several pulsars.

4. Measurement of the distance of a pulsars using the delay in arrival times of pulses at different frequencies due to interstellar dispersion.

5. Determine the distance of a pulsar using the techniques you have just learned.

Part 1: The Radio Telescope

1. Start up the Pulsar Lab program. Click **File** on the menu bar, select **Run** and then the **Radio Telescope** option.
  - The window should now show you the control panel for the CLEA radio telescope. A view screen at the center shows the telescope itself, a large steerable dish, which acts as the antenna to collect radio waves and send them to your receiver.
  - The Universal Time (UT) and the local sidereal time for your location are shown in the large digital displays on the left.
  - The coordinates at which the telescope is pointed, Right Ascension (RA) and Declination (Dec), are shown in the large displays at the bottom.

2. Just below and to the right of these coordinate displays is a button labeled **View**. **Click** on the View button, and screen in the center will show you a map of the sky, with the coordinate lines labeled.
  - A yellow square shows you where the telescope is pointed.

3. You can steer the telescope around the sky by clicking and holding down the **N-E-S-W** buttons at the left side of the window. Try it, and watch the square move, showing that the telescope is moving around the sky.
  - The coordinate readouts will also move.
  - You can change the pointing speed of the telescope by resetting the slew rate button at the lower left. Try setting it to 100, and see how much faster you can move the telescope around the sky.

4. You can move the telescope in two other ways:
  - by clicking on the set coordinates button at the bottom of the screen

• by selecting objects from the **Hot-List** pull-down menu on the menu bar at the top of the window. We will use the **Hot-List** in this exercise, because it is so convenient.

5. The telescope has a tracking motor designed to keep it pointed at the same spot in the heavens as the earth turns. Right now the motor is off, and, even if you are not moving the telescope with the N-E-S-W buttons, you will see the Right Ascension display changing, because the rotating earth is causing the telescope to sweep the heavens. You should turn on the tracking motor to remedy this. Just below the time displays on the left hand side of the screen is a button labeled **Tracking**. If you click on it, you will see the word *on* appear next to the button, and you will notice that the Right Ascension display stops changing. The telescope will now track any object it is pointed at.

6. You are now ready to receive signals from your first pulsar.
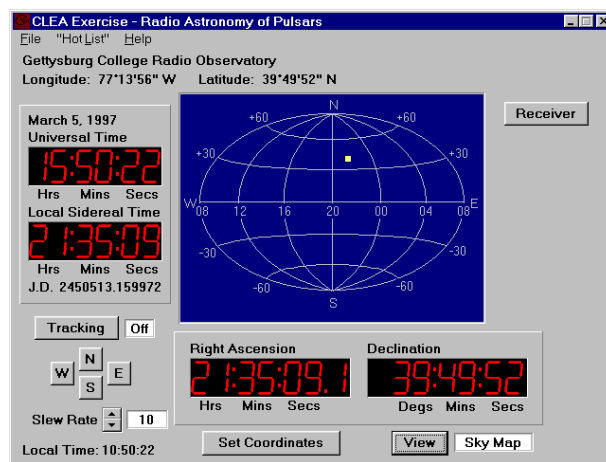
Figure 1: Sky View

Part 2: Observation of a Pulsar with a Single-Channel Radio Receiver

Let's begin by familiarizing yourself with the receiver and general properties of pulsars. In this part of the exercise you will point the telescope at a moderately strong pulsar and, using a radio receiver with a graphic display, look at the pulsing radio signal to get some idea of its overall characteristics. The radio waves we receive from pulsars are characterized by sharp pulses of short duration, very steady in their period of repetition, with periods of as short as a few hundredths of a second up to several seconds. The strength of individual pulses varies a bit, in a random fashion, as we shall see, but the overall strength of the signals depends most strongly on the frequency at which you observe them. Our radio receiver can be tuned to any frequency between 400 and 1400 Megahertz (MHz), and we will use this feature to see, qualitatively, how a pulsar's signal strength changes with frequency.

1. We want to point our radio telescope to pulsar **0628-28**. To move the telescope to the proper coordinates, we will use the **Hot List.** The **Hot List** is located on the menu bar. Click and pull down the **Hot List** menu and choose **View/Select from Lis**t. **Click** on the pulsar desired, **0628-08** (the name is in the leftmost column), and **click on the OK button** at the bottom.

   • After asking you for verification, the telescope will begin to move. You'll see the square on the sky map move, and the coordinate displays change, until the telescope is pointing at the object.
   • Write down, in the space provided , the Right Ascension and Declination you are pointing to:

   RA _____ Dec _____

2. Now that the big dish antenna is pointed in the right direction, you want to turn on your radio receiver. **Click** on the **Receiver** button in the upper right of the telescope control window.

> • A rectangular window will open which has the controls for your receiver on the right, and a graphic display of the signal strength versus time on the left. (See Figure 2.)
>
> • The frequency the receiver is set to is displayed in the window near the upper right. It is currently set to 600 MHz, and you should leave it there. Later, when you want to change frequency, there are buttons next to it to tune the receiver to different frequencies. Fine tuning can be accomplished by changing the **Freq. Incr**. (frequency increment), button to its right in conjunction with the main tuning button. There are also buttons to control the horizontal and vertical scale of the graphic display.
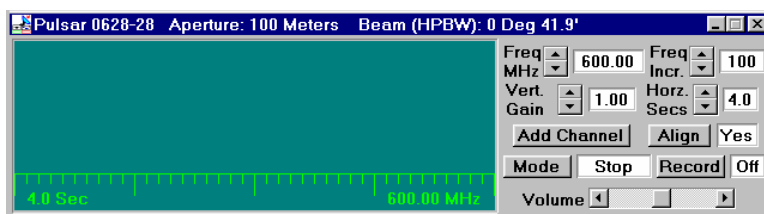
Figure 2: Main Receiver Window

3. Let's look at what the pulsar signal looks like. **Click** on the **Mode** button to **start** the receiver. You'll see a graphical trace begin at the left of the screen, tracing out the signal strength versus time on the graph. It looks like a random jiggle, which is the background static, with an occasional brief rise in signal strength, which is the pulsar signal. (If your computer is equipped with sound, you can also hear what the signal would sound like if you converted the signal to sound, like you do when listening to a radio station). Note how regularly the signal repeats.

4. **Click** on the **Mode** switch again to **turn off** the receiver. Note that it completes one scan of the screen before it stops.

5. Let's see what the other controls do. **Start** the receiver again. Now watch the trace as you change the **Vertical Gain** control by **clicking on the up and down buttons**. This is like the volume control on a radio, except it only controls the graphic display.

> • When the gain is high (you can turn it up to 8), the graphic trace is bigger, both the background and the pulsar signal are magnified.
>
> • When the gain is low (you can turn it down to 0.25) you can barely see the pulsar. You'll find that the best setting is one where the pulses are high, but don't rise above the top of the display.
>
> • Write down the gain setting you think is best here: _____. The setting will vary from pulsar to pulsar, and also is dependent on how you have set the Horz Sec control. (It should be set at 4, right now).

6. Let's try changing the horizontal scale (Horz Secs). *You can only set this control when the receiver is off.* **Click the Mode switch off**, and when the trace stops, reset Horz Sec to 2. This will make the graphic trace take 2 seconds to sweep across the screen. **Start the receiver again**.

> • You will see the trace race across the screen faster.
>
> • You may also note that the signal seems weaker, because your receiver is spending less time collecting radio waves before it displays them on the screen. (Astronomers would say the "integration time" is shorter.)

7. **Try resetting the Horz Sec to 0.5 sec**. The pulses seem so wide you may have trouble distinguishing them, and you may have to raise the vertical gain to make them out at all.

8. **Change the Horz Sec to 16**. The trace pokes across the screen, and because the receiver is collecting *more* signal at a slower pace, the signals seem stronger. You will have to turn down the gain to avoid having your pulse peaks out of range of the screen.

9. Now let's measure the period of the pulsar. Set the vertical gain to 4 and the horizontal seconds to 4, and make sure the frequency of the receiver is 600 MHz. **Start the receiver**. Let it run for a few seconds to see the pulses, then turn it off again. When the trace stops moving, you can measure the time between pulses on the screen.

- The computer has measuring cursor to aid you in doing this. Holding down the *left* mouse button produces a vertical blue line on the screen which you can move as you hold down the button. Center it in the middle of one of the pulses near the left side of the screen. Note the blue numbers on the screen that tell you the time in seconds at which the pulse arrived.
- You want to measure the time of arrival of the next pulse (time increases to the right)—so you can get another line, a white one, to appear by holding down the *right* mouse button. Position it over the next pulse. You can read this time from numbers on the screen, too. Now record the time of arrival of both pulses on the table below. The difference between these is the period of the pulsar!
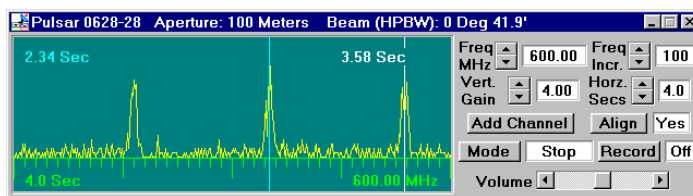

Figure 3: Main Receiver Window

- Time of First Pulse (T $_1$) _____

- Time of Next Pulse (T $_2$)_____

- Period of Pulsar (T $_2$ -T $_1$) _____

10. A more precise way of measuring the period is to measure the time elapsed between several pulses and then divide by the number of periods that have elapsed.

- Set the horizontal seconds control to 16, turn down the gain a notch or two, and turn on the receiver again so that you can see many pulses on one scan. Measure the time it takes for 10 periods, and use this to calculate the period of the pulsar. See if this gives you the same value. Record your measurements below.

- Time of first Pulse (T $_0$) _____

- Time 10 Periods later (T $_{10}$) _____
(Note that this is the 11th pulse)

- Period of Pulsar =(T $_{10}$ -T $_0$)/10 _____

11. Now let's look at the relationship between the pulsar period and the frequency. You can tune the receiver to different frequencies and, using whichever one of these methods is most convenient, measure the period at different frequencies. Since the signal strength changes with frequency, you may have to adjust the vertical gain or the horizontal seconds controls to see the pulses clearly at each frequency.

Fill in the 400 MHz, 800 MHz, and 1200 MHz rows of the following table. (Ignore the other rows: there's no point in repeating this exercise that many times!)

Frequency	Time of First Pulse	Time of Last Pulse	Number of Periods	Period of Pulsar
400 MHz				
600 MHz				
800 MHz				
1000 MHz				
1200 MHz				
1400 MHz				

12. From what you have measured, make state briefly how the period of the pulsar depends on the frequency:

13. How does the pulsar signal strength depend on frequency? Clearly it does, since you have to reset the vertical gain and horizontal seconds controls at different frequencies. But to see how it changes in a more systematic fashion, lets keep those controls set—for example, 4 for the vertical gain and 4 for the horizontal seconds. Then look at the signal at various frequencies from 1400 MHz back down to 400 MHz. Answer the following questions.
   - The pulsar signal is stronger at _____(lower/higher) frequencies.
   - If I am hunting for pulsars in the sky, or want to measure the period of a pulsar I've never investigated before, the best frequency to tune my receiver would be _____ MHz.

14. You can now click the **x** at the upper right of the receiver window to close the receiver and return to the telescope control, where you will investigate several other pulsars.

Part 3: Measurement of the Distance of Pulsars Using Dispersion

A. Method

Most pulsars can't be seen with optical telescopes, so we can't use their luminosities to determine distance. In addition, they are much too far for parallax measurements. How can we determine their distance then? One powerful method is to use the phenomenon of ***dispersion***.

All forms of electromagnetic radiation, including radio waves, travel at the same speed in a *vacuu*m. This speed is speed of light

$$c = 3 \times 10^8 \, meters/sec$$

However, interstellar space is not quite a vacuum. On the average the interstellar medium consists of a few atoms and a few free electrons in each cubic centimeter. It isn't much, but it's enough to slow down electromagnetic waves slightly. The lower the frequency, the slower the radiation travels. This means that, though the effect is small, pulses from a pulsar arrive a fraction of a second earlier at higher frequencies than at lower frequencies, because the higher frequency pulses travel faster through the interstellar medium.

You'll be able to see this easily using our radio telescope, since you can receive signals at up to three wavelengths simultaneously, and can compare the arrival times on the three graphic displays.

By measuring the times of arrival of pulses from the same pulsar at different frequencies you can determine the distance to the pulsar, as long as you know the speed of radio waves through the interstellar medium at different frequencies. We do in fact know how frequency affects the speed of electromagnetic radiation from the theory of electromagnetism developed over 100 years ago.

B. An Example from the Everyday World

In a simplified case unrelated to electromagnetism, we can look at how arrival time helps determine distance traveled by two athletes running a race. Suppose we have two runners (A and B) who are racing each other. Runner A runs a steady 5 kilometers an hour, and Runner B runs a steady 10 kilometers an hour.

We do not know ahead of time *HOW FAR* they are running, but we do know their speeds and we do know that they both start running at the same time. It's easy to see that the *difference* in the times they cross the finish line depends on the length of the race. (See FIGURE 4). Suppose the course is 10 kilometers long. Runner A finishes in two hours. Runner B finishes in 1 hour. So there is a 1 hour difference between them if the course is 10 kilometers long. If the course is 20 kilometers long, Runner A finishes in four hours, and Runner B finishes in 2 hours, or a 2 hour time difference between the two. You can, in principle, determine the length of the race from the difference in the finish times.
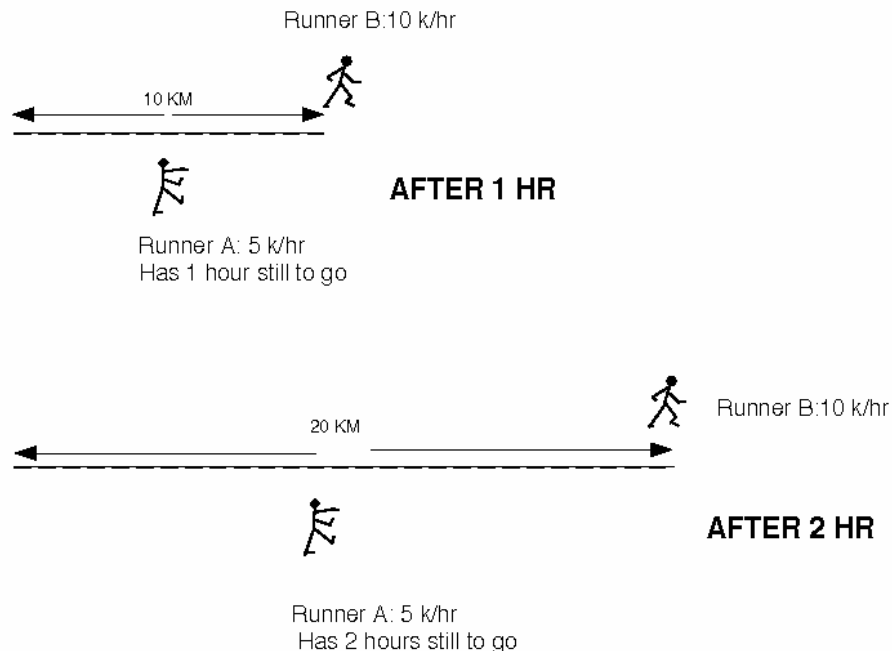

Figure 4: Illustration of the example

We can represent this mathematically by deriving a formula where the length of time it takes for runner A to finish the length of the course L is divided by her speed:

$$T_A = L / V_a$$

Similarly the length of time it takes for runner B to finish is the length of the course divided by his speed:

$$T_B = L/V_b$$

So the difference in times can be described as

$$T_B - T_A = L/V_b - L/V_a$$

or (factoring out **L** from both terms on the right)

$$T_B - T_A = L(1/V_b - 1/V_a)$$

And solving for **L**

$$L = \frac{T_B - T_A}{(1/V_b - 1/V_a)}$$

Try the equation using the following numbers taken from our above example:
$$V_a = 5 \text{ km/hr}$$
$$V_b = 10 \text{ km/hr}$$
$$T_B - T_A = 1 \text{ hour, or 2 hours,}$$
and you'll see you get L = 10 km for a 1 hour difference, and L= 20 km for a 2 hr difference.

C. The Dispersion Formula for the Interstellar Medium

The laws of physics enable us to calculate the speed of electromagnetic radiation in the interstellar medium and to derive a formula similar to the one above for the distance traveled in terms of the delay in arrival between radio pulses received at different frequencies. Lower frequencies travel slower, arriving later. So in order to determine the distance of a pulsar, we simply need to measure the time of arrival of a pulse from a pulsar at two different frequencies. We'll do that in the next part.

These differences in speed are incredibly tiny. All of the radio waves from a pulsar travel at almost exactly the speed of light (3×10^8 m/s). The interstellar medium slows them down by only about 0.002 m/s for radio waves with frequencies of 400 MHz, and even less (0.0005 m/s) for 800-MHz waves. Still, over the immense distances to pulsars those tiny differences add up to a significant difference in the arrival times of pulses of different frequencies.

As you can see from the previous section, the quantity we'll need to know to determine the distance to a pulsar is the difference in the reciprocals of the speeds, $(1/V_b - 1/V_a)$, where V_a and V_b are the speeds of radio waves at two different frequencies. This quantity depends on the amount of ionized gas in the interstellar medium. The physics that goes into calculating it is a bit involved, so I'll just tell you what that quantity turns out to be for the frequencies we'll be interested in:

Frequency a (MHz)	*Frequency b* (MHz)	$1/V_b - 1/V_a$ (seconds / parsec)
600	400	4.32×10^{-4}
800	400	5.84×10^{-4}
800	600	1.51×10^{-4}

In the next section, you'll measure the arrival times of pulses from a pulsar at several different frequencies. You'll then use these values, together with the relation

$$L = \frac{T_B - T_A}{(1/V_b - 1/V_a)}$$

from the previous section, to work out the distance (*L*).

D. Measuring The Distances Of Pulsars

1. Using the control panel of your radio telescope, go to pulsar **0628-28**. Open the radio receiver window, set the vertical gain for 4 and the horizontal seconds for 4, and tune the receiver to 400 MHz. Then turn on the receiver just to make sure you are getting strong pulses.

2. Stop the receiver now. Let's add a second receiver. **Click** on the **add channel** button and a second receiver display should appear below the first, aligned with it. Set the vertical, horizontal and frequency controls to the same values as the first receiver, a frequency of 400 MHz, 4 for the vertical gain and 4 for the horizontal secs.

3. Set the **Freq Incr**. button on the lower receiver to **10 MHz**, (making it possible to tune the second receiver 10 MHz at time).

4. Turn on the receivers by clicking the mode button located on the top receiver. Both receivers will start recording.
> • Because they are both receiving the same signal at the same frequency, the two traces should be exactly the same (except, perhaps for a slight random noise in each separate receiver).

How does the arrival times of pulses depend on frequency? Let's find out.

5. Turn on the channels by clicking the mode button in the first channel. While the receivers are running, tune the second receiver to 410 MHz . Do you notice a difference in the arrival times of the higher frequency pulse? Watch for a few seconds.

6. Tune the receiver up to 420 MHz, then 430 MHz. Is the behavior becoming clearer?

• Tune the second receiver *slowly* in 10 MHz increments up to 600 MHz, pausing now and then to watch the scans.

What can you say about the arrival times of pulses at higher frequencies? Do they arrive earlier or later than the pulses at lower frequencies? Write your conclusion as a brief statement below.

7. Turn off the receivers with the mode button. Now open up a *third* receiver using the **add channel** button and tune it to 800 MHz, and set both the vertical gain and horizontal gain to 4.

8. Turn on the mode button in the first receiver and watch the traces on the three receivers.
    Is the behavior you see in accord with what you now understand about the arrival times of the pulses at different wavelengths? Explain.

E. Measuring the Arrival Times Of the Pulses

We're now ready to measure the times of arrival of the pulses. First, we record the data from several screen scans, save it in a file, and finally analyze the file in a separate graphic analysis window.

1. Turn off the receivers with the mode switch and verify that the three receivers are set to get data at 400, 600, and 800 MHz simultaneously with the horizontal seconds set at 4 and the vertical gain at 4 in each receiver.

2. **Click** on the **record button** to enable the data recorder, and then turn the receivers **on** with the **mode switch**.

3. Let the receiver scan for five or six screens worth of data (only the first 4 will be saved). Then switch off the receivers.
    • The computer will tell you that 1600 data points have been saved.
    • It will then show you the name of the object for verification. **Click OK.**
    • A pop up a screen will appear showing you information about the object you just recorded, including the time when the recording began (in Julian days—a running date), the frequencies, and the time between each sample of the signal. See FIGURE 5 below.
    • You can not edit this data. This screen is for information *only*.

Figure 5
Data Properties Pop-Up Screen

4. **Click on OK**. The computer will then ask you if you want to save the data or analyze it immediately. Click on yes, and the computer prompts us for a file name. It will make one up using the name you logged

in with. If you want to change this name, you can do so. Record the name of the file you have created in the space provided:

FILE NAME_____.

Having saved the data in a file, let's now analyze it.

5. First **close** the receivers by **clicking on the x** in the upper right corner of the main receiver window.

6. Under the **File** menu on the main control screen, **choose Run...Data Analysis**. If you have not turned off the program, a window should appear showing you the general information on the data file you just created (otherwise, it will ask you to use the **Files..load** menu to choose from a menu of data files by name.)

7. **Click on the file containing the data you want to analyze**. When you are sure you have the right file loaded for analysis, you will see a measuring window with the three traces labeled. This is similar your three receivers, but it displays the stored data you just received.
   - You can set down measuring lines, and scroll back and forth in the display at your leisure.
   - Experiment with using the horizontal pan buttons, which move the traces right and left.
   - Experiment with the zoom in and zoom out buttons, which magnify the scale of the traces.

8. Now let's measure the times of arrival of a pulse at the three different frequencies you recorded. **Zoom in until you can see about two pulses across the screen**. This high magnification will let you measure the pulse arrival times more readily. Pan the screen so that there is a pulse at 400 MHz in the right half of the top screen.
You should see the same pulse arriving earlier (to the left, at an earlier time) at the 600 MHz frequency, and still earlier (even further to the left) at the 800 MHz frequency.

9. Now let's measure. **Holding down the *left hand* mouse button** while you're moving the mouse in a measuring window will move a vertical line back and forth across the screen. **Set one line in the middle of the 400 MHz pulse.** You can read the time of arrival in the blue window to the right of the measuring window. **Set measuring lines similarly in the 600 MHz and 800 MHz windows.**

10. Record the times of arrival of the pulse at the three frequencies T_{400}, T_{600} and T_{800} on the table below.

PULSAR 0628-28 Dispersion Data

T_{400} _____

T_{600} _____

T_{800} _____

11. Now, using the dispersion formula for radio waves described above, use the *difference* in arrival times at two different frequencies to calculate the distance to the pulsar. (Since there are three different pairs of frequencies, you can calculate this three ways, just to check you're doing it right.

PULSAR 0628-28

Dispersion Distance Analysis

Frequencies		$T_2 - T_1$	$1/V_b - 1/V_a$	D (pc)
f_1	f_2			
600	400			
800	400			
800	600			

If you've not made any numeric mistakes, your three distances should agree to at least two significant figures, and should be of the order of 1000 parsecs or so. Check over your work and your measurements if there's some disagreement.

Lab 29

The Crab Pulsar

In this lab you'll use the same software as last time to examine the rotation period of the Crab pulsar. In particular, you'll measure the rate at which the pulsar is slowing down.

Part 1: Period of the Crab Pulsar.

Start up the radio telescope, turn on tracking, and use the Hot List to point the telescope at the Crab pulsar (known to its friends as "0531+21"). Start up a receiver, and observe the pulses from this star. Because this star pulses very quickly, you'll need to set the **Horz. Secs** level to 2.0. Adjust the gain to some appropriate value. Turn on the **Record** button, and start the receiver. Let it run for at least four full traces across the screen (8 seconds). Once you have acquired your data, open it up in the data analysis window. You will use this data to determine the period of the pulsar as accurately as possible.

First, zoom in as far as you can on the leftmost portion of the data. Determine the time of the very first pulse (by dragging the vertical cursor over the peak of the pulse). The software is supposed to start the clock right at the moment of the first pulse, so the time of this pulse should be zero. If this is not the case, talk to me.

Let's call that very first pulse "pulse number zero." We'll number all the pulses after it 1,2,3, etc. Scroll forward to pulse number 20 and measure its time:

Time of Pulse # 20 =

Use this value to calculate the period of the pulsar:

Period of Pulsar =

Our goal today is to measure the *change* in the period of the pulsar over time. This change is very tiny, so we're going to need to be highly accurate. The number you just calculated for the period isn't quite good enough.

We could improve our measurement of the period if we count more pulses. For instance, if you had measured the time of pulse # 200 instead of pulse # 20, you'd get a better result. It's tedious to count 200 pulses, though, so we'll use a shortcut.

Based on the value you found for the period, estimate the time at which pulse number 50 should occur. (Don't count 50 pulses – just predict when it should occur.)

Predicted time of Pulse # 50 =

Now scroll ahead through your data to test this prediction. Position the cursor at the time you just calculated, and see if there's a pulse there. You should find that there's a pulse at almost, but not exactly, the time you predicted. Measure the actual time of that pulse as accurately as possible:

Actual time of pulse # 50 =

Note: you don't have to count the pulses to make sure this really is # 50. Since you made a pretty accurate prediction of when # 50 should occur, and you found a pulse at just about that time, you can be confident you've got the right pulse even without counting.

Using the value you just measured the time of pulse # 50, determine the period of the pulsar. This should be nearly the same as the value you got before, but the new value is more accurate:

Period of Pulsar (2nd estimate) =

Record this value to at least four significant figures (for instance, 0.03283 seconds, although this isn't the right value).

We just "leapfrogged" from a 20-pulse estimate of the period to a 50-pulse estimate. Let's play that game one more time to get a 200-pulse estimate. That'll be accurate enough for our purposes.

Use your most recent estimate of the period of the pulsar to predict when pulse # 200 should occur:

Predicted time of Pulse # 200 =

Then go to that portion of your data, and find the pulse that's closest to that time. This is the

Actual time of Pulse # 200 =

From this value, determine yet another estimate of the

Period of Pulsar (3rd estimate) =

Record this value to at least five significant figures.

In principle, we could continue this "leapfrog" procedure, but there's no need: this value is an accurate enough estimate. There's just one more bit of information we'll need: the uncertainty in this measurement of the period (that is, an estimate of how far off this number might be).

One way to determine this is to measure the period several different times and see how much it changes from measurement to measurement. You just determined the period by using the time of pulse # 200. Determine it again using pulse # 199 and pulse # 201:

$$
\begin{aligned}
\text{Time of Pulse \# 199} &= \\
\text{Period determined from Pulse \# 199} &= \\
\text{Time of Pulse \# 201} &= \\
\text{Period determined from Pulse \# 201} &=
\end{aligned}
$$

Now that you have three different determinations of the period (from pulses 199,200,201), average them together to get your best estimate of the true period. Then figure out the uncertainty associated with this value. The uncertainty is how far off the worst of your three measurements was from the mean.

Give your final answer here, in the form "(*Number* ± *Number*) seconds," with the two numbers being your best estimate and its uncertainty. Your best estimate should have five significant figures.

Final Value for Period (with uncertainty) =

Part 2. The Period of the Pulsar in the Future.

To see whether the Crab pulsar is slowing down, we need to wait a while and measure the period again. Since the slowdown is very gradual, we need to wait a long time. Five years works well. Rather than actually waiting five years, which would be inconvenient, we can tell the software to pretend that five years have gone by and simulate data from a future date.

Close down the data analysis window and the receiver window. Under the **File** menu, click on **Date/Time**, and change the date to five years from today. After you've done that, you'll need to tell the telescope to point toward the Crab pulsar again.

Now *repeat the entire procedure* to determine an accurate measurement of the period of the pulsar, together with its uncertainty.

Based on the values you determined, *and their uncertainties*, can you conclude that the Crab pulsar is slowing down? (Is the difference between the two periods so great that it lies outside the bounds of the measurement uncertainties?)

What is the rate at which the period is changing, in seconds per year?

The Crab pulsar was created in a supernova explosion in the year 1054. Assuming that it has been slowing down at a steady rate ever since that time, what was its period just after it was created?

Lab 30

Black Holes

Mass and radius. One of the key facts about black holes is the mathematical relationship between the radius of the black hole and its mass:

$$R_{\text{Sch}} = \frac{2GM}{c^2}.$$

The radius is called the "Schwarzschild radius" after Karl Scharzschild, who first worked out the mathematics of black holes. First, let's work out Schwarzschild radii for some different sized black holes. It's convenient to group all of the constants together like this:

$$R_{\text{Sch}} = \left(\frac{2G}{c^2}\right) M. \tag{30.1}$$

Remember that the gravitation constant is $G = 6.67 \times 10^{-11}\,\text{m}^3/(\text{kg}\,\text{s}^2)$ and the speed of light is $c = 3.00 \times 10^8$ m/s. What is the numerical value of the combination $(2G/c^2)$, and what are its units?

(Remember how to determine the units: substitute the values of G and c into the expression $2G/c^2$, with their units, and then manipulate the units algebraically just as if they were numbers or variables or anything else – for instance, cancel anything that appears in both numerator and denominator of a fraction.)

$$\frac{2G}{c^2} =$$

According to equation (1) above, you can convert any given mass into the corresponding Schwarzschild radius by simply multiplying by this value. Are the units you found above consistent with this statement? What units do you need to use for the mass? What about for the radius?

What is the Schwarzschild radius for a black hole the mass of the Earth (5.98×10^{24} kg)?

A typical black hole that formed from a star has a mass of about 5 times the mass of the Sun. What is the Schwarzschild radius for such a black hole? (The Sun's mass is 1.99×10^{30} kg.)

The black hole at the center of our Galaxy may have a mass as high as 4×10^{6} solar masses. What is its Schwarzschild radius?

Suppose we wanted to make a black hole whose radius was 10^{-10} meters (about the size of an atom). How much mass would we need?

Density. One way to get a feel for the extreme conditions required to form a black hole is to consider the density of matter involved. If we wanted to produce a black hole of a certain mass, we'd have to compress all of that matter into a sphere whose radius was equal to the Schwarzschild radius. Remember that density is mass divided by volume, and that the volume of a sphere is

$$V = \frac{4}{3}\pi R^3.$$

Using this information, write down an expression giving the density required to form a black hole in terms of the mass M. Your expression should just have M and constants such as G and c in it. In particular, it should not involve the radius R — if it does, use equation (1) to express the radius in terms of M.

Density =

Note: It's not really correct to say that this is the *density of the black hole* (although sometimes people do say this). Rather, it's the density of matter required to cause the black hole to form in the first place. After the black hole forms, all the mass gets concentrated into a very small volume at the center, so the final central density after the black hole has formed is much higher.

Does a very massive black hole require a larger or smaller density than a not-so-massive black hole?

What is the density of matter required to form a 5-solar-mass black hole? (Be sure to include units!)

What is the density of matter required to form a million-solar-mass black hole?

What would the mass have to be if the required density is equal to the density of water? (Note: the density of water is 1 gram per cubic centimeter, but grams per cubic centimeter are not the units you want to use in this expression! If you're not sure how to express the density in the correct units, ask me.)

You meet an alien from the Andromeda galaxy, who asks you the following question: "To make a black hole with a mass of 10 slurms, the required density is 50 quatloos. What density is required to make a black hole with a mass of 40 slurms?" You don't know how large a unit of mass a slurm is, nor do you know how large a unit of density a quatloo is. Nonetheless, you can answer the alien's question. What is the answer?

Orbital Periods. Black holes often have smaller objects orbiting around them. When they do, we can use the orbital periods to determine the black hole's mass using Kepler's third law. Remember Kepler's third law:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$$

For the next couple of calculations, it may help you to recall that 1 AU is 1.50×10^{11} meters and that 1 year is 3.155×10^7 seconds.

Suppose you observe a possible supermassive black hole at the center of a galaxy. A star is orbiting the center with an orbital radius of 1 AU and is going at a speed of 6×10^7 m/s.

What is the star's orbital period?

What is the mass of the black hole?

Express the radius of the star's orbit as a multiple of the black hole's Schwarzschild radius.

Lab 31

Galaxy Rotation Curves and Dark Matter

Introduction.   As you know by now, the way we (almost always) measure masses in astrophysics is by measuring orbits and applying Kepler's Third Law. When we apply this technique to galaxies, we get a total mass that is much larger than the mass of all of the visible stuff we see. This is the main way astrophysicists infer that galaxies contain large amounts of unseen "dark matter."

In this lab, you will use observations of the spiral galaxy NGC 3521 to figure out the mass distribution of the galaxy. The data you will use was taken by Y. Sofue and collaborators with a radio telescope in Japan.[1] (This is real data, unlike the various simulation labs we often do.)

Useful facts for this lab.   You'll need to use the small-angle formula,

$$s = \frac{\alpha d}{206,265}.$$

You'll also need Kepler's Third Law,

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}.$$

Remember that in units of years, AU, and solar masses, the constants $\frac{4\pi^2}{G}$ come out equal to one and so can be ignored. Finally, some unit conversions:

$$1\,\mathrm{pc} = 206,265\,\mathrm{AU}.$$

$$1\,\mathrm{AU} = 1.49 \times 10^8\,\mathrm{km}.$$

Procedure.

1. Download and open the Excel file **NGC 3521 rotation curve**, which is in the **Downloads for labs** section of the Blackboard site for this course. This file contains measurements of the radial velocity (the speed towards or away from us) of gas clouds in this galaxy as a function of the clouds' distance from the center of the galaxy. The first column is the apparent location of the cloud relative to the galaxy's center. The fourth column is the speed of that gas cloud towards or away from us. (The speed is towards us on one side of the galaxy and away on the other side. Both sides have been combined in this data set.)

---

[1]In case you're wondering, further information about the data is at http://www.ioa.s.u-tokyo.ac.jp/~sofue/h-rot.htm, but you don't need to do anything with the information at that site.

2. How do you think that the speed information was obtained?

3. You'll need to know the actual distances of the various gas clouds from the center of the galaxy. Use the small-angle formula to figure those out in both kiloparsecs and AU, entering the results in the second and third columns of the spreadsheet. The distance to the galaxy NGC 3521 is 8900 kpc. Since there are many rows of data, you'd be very unwise to do this step one row at a time; use an Excel formula to do them all at once. See Appendix C for reminders about how to do this sort of thing.

4. Plot a graph showing the distance from the center in kpc on the x axis and the rotation speed on the y axis. Be sure that the axes of your graph are clearly labeled including both the name of the quantity being plotted and its units. This graph is called the "rotation curve" of the galaxy.

5. Using another Excel formula, create a new column in the spreadsheet giving the period of the orbit of each gas cloud in years. (The orbits can all be assumed to be circles.) As usual, be sure the column is labeled with both a name and units.

   Your formula will probably give an error in the very first line, because the radius and speed are both zero there. That's OK, as long as it gives correct results for the other lines. The same thing is true for the next step.

6. Next you will use Kepler's Third Law to determine the mass contained within each gas cloud's orbit. Create a new column to contain this information, and use an appropriate Excel formula.

7. Plot a graph showing the distance from the galaxy's center in kiloparsecs on the x axis and the mass on the y axis.

8. From your graph, it should be clear that something has gone wrong at large distances. Explain how you know. (What does the graph do at large distances? Why is this impossible?)

   As it turns out, the data are reliable out to about 17 kpc.

9. Other measurements have been made to determine the distribution of light coming from this galaxy. They have determined that 90% of the total light from the galaxy comes from the innermost 9 kpc of the data. Assuming that the maximum value of mass you found is the total mass of the galaxy, what percentage of the total mass of the galaxy lies within 9 kpc of the center? Based on this information, would you conclude that the total mass in the galaxy is more spread out than the stars, or less spread out?

10. In answering the previous question, you assumed that the largest mass value you found was equal to the total mass of the galaxy. That might be wrong: the graph of mass might continue to go up if we had reliable data at greater distances. If you did discover that the total mass of the galaxy continued to go up as you observed out to greater distances, would it strengthen or weaken the conclusion you drew in the previous question?

11. The luminosity of this galaxy is about 1.5×10^{10} times the luminosity of the Sun. Calculate the "mass-to-light ratio" of this galaxy by dividing the total mass (in solar masses) by the total luminosity (in solar luminosities).

12. If the galaxy were made up of nothing but stars just like the Sun (with no gas, dust, or dark matter), what would the mass-to-light ratio be?

13. Astrophysicists think that a typical collection of gas, dust, and stars in a galaxy (with no dark matter) should have a mass-to-light ratio of about 2. Assuming that that's correct, use the luminosity to determine the mass of all of the gas, dust, and stars in this galaxy. How many times greater is the total mass of the galaxy than the mass of all the visible stuff?

Lab 32

Hubble's Law

The Hubble Redshift Distance Relation
Student Manual

CLEA

Contemporary Laboratory
Experiences in Astronomy
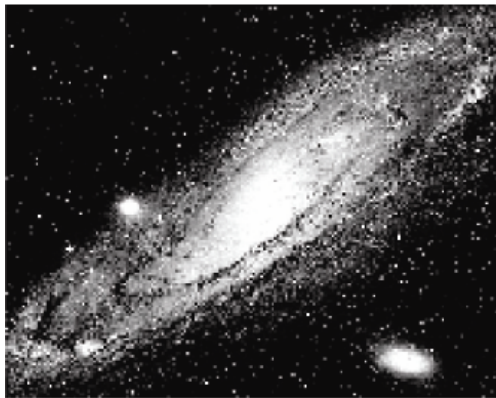
Introduction

The late biologist J.B.S. Haldane once wrote: "The universe is not only queerer than we suppose, but queerer than we can suppose." One of the queerest things about the universe is that virtually all the galaxies in it (with the exception of a few nearby ones) are moving away from the Milky Way. This curious fact was first discovered in the early 20th century by astronomer Vesto Slipher, who noted that absorption lines in the spectra of most spiral galaxies had longer wavelengths (were "redder") than those observed from stationary objects. Assuming that the redshift was caused by the Doppler shift, Slipher concluded that the red-shifted galaxies were all moving away from us.

In the 1920's, Edwin Hubble measured the distances of the galaxies for the first time, and when he plotted these distances against the velocities for each galaxy he noted something even queerer: The further a galaxy was from the Milky Way, the faster it was moving away. Was there something special about our place in the universe that made us a center of cosmic repulsion?

Astrophysicists readily interpreted Hubble's relation as evidence of a universal expansion. The distance between all galaxies in the universe was getting bigger with time, like the distance between raisins in a rising loaf of bread. An observer on *ANY* galaxy, not just our own, would see all the other galaxies traveling away, with the furthest galaxies traveling the fastest.

This was a remarkable discovery. The expansion is believed today to be a result of a "Big Bang" which occurred between 10 and 20 billion years ago, a date which we can calculate by making measurements like those of Hubble. The rate of expansion of the universe tells us how long it has been expanding. We determine the rate by plotting the velocities of galaxies against their distances, and determining the slope of the graph, a number called the Hubble Parameter, H_o, which tells us how fast a galaxy at a given distance is receding from us. So Hubble's discovery of the correlation between velocity and distance is fundamental in reckoning the history of the universe.

The software for the **CLEA** *Hubble Redshift Distance Relation* laboratory exercise puts you in control of a large optical telescope equipped with a TV camera and an electronic spectrometer. Using this equipment, you will determine the distance and velocity of several galaxies located in selected clusters around the sky. From these data you will plot a graph of velocity (the y-axis) versus distance (the x-axis).

How does the equipment work? The TV camera attached to the telescope allows you to see the galaxies, and "steer" the telescope so that light from a galaxy is focused into the slit of the spectrometer. You can then turn on the spectrometer, which will begin to collect photons from the galaxy. The screen will show the spectrum—a plot of the intensity of light collected versus wavelength. When a sufficient number of photons are collected, you will be able to see distinct spectral lines from the galaxy (the H and K lines of calcium), and you will measure their wavelength using the computer cursor. The wavelengths will be longer than the wavelengths of the H and K labs measured from a non-moving object (397.0 and 393.3 nanometers), because the galaxy is moving away. The spectrometer also measures the apparent magnitude of the galaxy from the rate at which it receives photons from the galaxy. So for each galaxy you will have recorded the wave-lengths of the H and K lines and the apparent magnitude.

From the data collected above, you can calculate the speed of the galaxy from the Doppler-shift formula. You will also estimate the distance to the galaxy by comparing its apparent brightness to an assumed value for its luminosity. (In a real experimental setup, we'd come up with some more complicated and more accurate method for determining distance.) The result is a velocity (in km/sec) and a distance (in megaparsecs, Mpc) for each galaxy. The galaxy clusters you will observe have been chosen to be at different distances from the Milky Way, giving you a suitable range to see the straight line relationship Hubble first determined. The slope of the straight line will give you the value of H_o, the **Hubble Parameter**, which is a measure of the rate of expansion of the universe. Once you have H_o, you can use it to estimate the age of the universe.

The details of the measurements and calculations are described in the following sections.

Using the Hubble Redshift Program

Welcome to the observatory! We will simulate an evening's observation during which we will collect data and draw conclusions on the rate of expansion of the universe. We will gain a proficiency in using the telescope to collect data by working together on the first object. Collecting data for the other four objects will be left to you to complete the evening's observing session. Then you will analyze the data, draw your conclusions, and use the information to predict the age of the universe.

Let's begin.

1. Open the Hubble Redshift program. Select **File…Log in** from the menu bar, and enter your names (or don't enter them – see if I care!). Click **OK** when ready. The title screen appears.

2. Select **File…Run** from the menu bar to begin the exercise. The screen shows the control panel and view window as found in the "warm room" at the observatory.

The **Hubble Redshift Distance Relation** program simulates the operation of a computer-controlled spectrometer attached to a telescope at a large mountain-top observatory. It is realistic in appearance, and is designed to give you a "hands-on" approach of how astronomers collect and analyze data for research.

Notice that the **dome** is closed and **tracking status** is off. Before you can access the telescope controls, a message appears notifying you that you have control of the telescope. Click **OK** to continue.

3. To begin our evening's work, first open the dome by clicking on the **dome** button. The dome opens and the view we see is from the finder scope. The finder scope is mounted on the side of the main telescope and points in the same direction. Because the field of view of the finder scope is much larger than the field of view of the main instrument, it is used to *locate* the objects we want to measure. The field of view is displayed on-screen by a CCD camera attached on the finder scope. (Note that it is not necessary for astronomers to view objects through an eyepiece.) Locate the **Change View** button on the control panel and note its status, i.e. finder scope. Also note that the stars are drifting in the view window. This is due to the rotation of the earth and is very noticeable under high magnification of the finder telescope. It is even more noticeable in the main instrument which has even a higher magnification. In order to have the telescope keep an object centered in the spectrometer opening (slit) to collect data, we need to turn on the drive control motors on the telescope.

4. We do this by clicking on the **tracking** button. The telescope will now track in sync with the stars. However, before we can collect data we need to do the following:

       (a) Select a field of view (one is currently selected).

       (b) Select an object to study (one from each field of view).

5. To review the fields of study for tonight's observing session, select **Field…** from the menu bar at the top of the control panel. The items you see listed are the fields that contain the objects we have selected to study tonight. An astronomer would have selected these fields in advance of going to the telescope by: selecting objects that will be well placed for observing during the time we will be at the telescope. This list in the **select star field dialog box** contains 5 fields for study tonight. You will need to select one galaxy from **each** field of view and collect data with the spectrometer (a total of 5 galaxies).

To see how the telescope works, change the field of view to Ursa Major II at RA 11 hour 0 minutes and Dec. 56 degrees 48 min. Press **OK** to move the telescope to the correct position.

Notice the telescope "slews" (moves rapidly) to the RA and DEC coordinates we have selected. The view window will show a portion of the sky that was electronically captured by the charge coupled device (CCD) camera attached to the telescope.

The view window has two magnifications (see Figure 1 below):

Finder View is the view through the finder scope that gives a wide field of view and has a red square which outlines the instrument field of view.

Instrument View is the view from the main telescope with red vertical lines that show the position of the slit of the spectrometer.

As in any image of the night sky, stars and galaxies are visible in the view window. It is easy to recognize bright galaxies in this lab simulation, since the shapes of the brighter galaxies are clearly different from the dot-like images of stars. But faint, distant galaxies can look similar to like stars, since we can't see their shape.
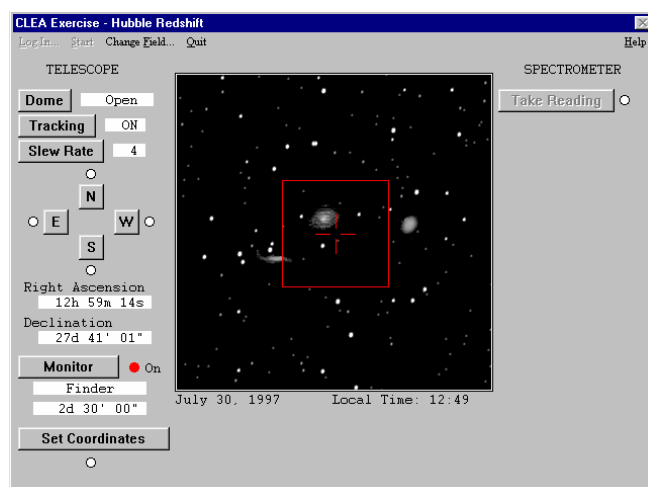
Figure 1:
Field of View from the
Finder Scope

6. Now change fields by clicking on Ursa Major I in the lists of selected galaxies to highlight the field. Then click the **OK** button.

7. Locate the **Change View** button in the lower left hand portion of the screen. **Click** on this button to change the view from the **Finder Scope** to the **Instrument**. The field of view is now smaller so that you can accurately position the galaxy in the slit of the spectrometer. Use the directional buttons (**N, S, E or W**), to "slew", or move, the telescope to carefully position, in the slit, the object you intend to use to collect data—any of the galaxies are suitable. To move continuously, **press and hold down** the left mouse button. Notice the red light comes on to indicate the telescope is "slewing" in that direction.

As in real observatories, it takes a bit of practice to move the telescope to an object. You can adjust the speed or "slew rate" of the telescope by using the mouse to press the **slew rate** button. (1 is the slowest and 16 is the fastest). When you have positioned the galaxy accurately in the slit, click on the **take reading** button to the right of the view screen.

The more light you get into your spectrometer, the stronger the signal it will detect, and the shorter the time required to get a usable spectrum. Try to position the spectrometer slit on the brightest portion of the galaxy. If you position it on the fainter parts of the galaxy, you are still able to obtain a good spectrum but the time required will be much longer. If you position the slit completely off the galaxy, you will just get a spectrum of the sky, which will be mostly random noise.

As we collect data from the object. We will be looking at the spectrum from the galaxy in the slit of the spectrometer. The spectrum of the galaxy will exhibit the characteristic H & K calcium lines which would normally appear at wavelengths 3968.5 Å and 3933.7 Å, respectively, if the galaxies were not moving. However, the H & K lines will be red shifted to longer wavelengths depending on how fast the galaxy is receding.

Photons are collected one by one. We must collect a sufficient number of photons to allow identification of the wavelength. Since an incoming photon could be of any wavelength, we need to integrate for some time before we can accurately measure the spectrum and draw conclusions. The more photons collected, the less the noise in the spectrum, making the absorption lines easier to pick out. To initiate the data collection, press **start/resume count.**

8. To check the progress of the spectrum, click the **stop count** button. The computer will plot the spectrum with the available data. Clicking the **stop count** button also places the cursor in the measurement mode. Using the mouse, place the arrow anywhere on the spectrum, press and hold the left mouse button. Notice the arrow changes to a cross hair and the wavelength data appears at the top of the display. As you hold the left mouse button, move the mouse along the spectrum. You are able to measure the wavelength and intensity at the position of the mouse pointer.
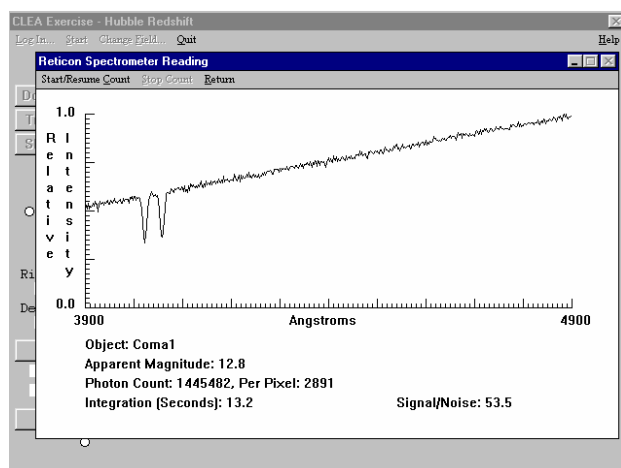
Figure 2:
Spectrometer Reading Window

Information that appears in the lower left hand portion of the window

Object: the name of the object being studied

Apparent magnitude: the visual magnitude of the object

Right Ascension: Displays the celestial coordinates of the center of the field of view. Right Ascension is displayed in hours, minutes and seconds.

Declination: Declination is displayed in degrees, minutes and seconds.

Information that appears in the lower right hand portion of the window

Integration (seconds): The number of seconds it took to collect data

Photon count: The total number of photons collected so far, and the average number per pixel

Avg. per Channel: The number of photons averaged from the total number of the channels of the spectrometer.

Signal-to-noise Ratio: A measurement of the quality of the data taken to distinguish the H and K lines of calcium from the noise. Try to get a signal-to-noise ratio of 10 to 1. For faint galaxies, this may take some time.

Information that appears in the top portion of the window

Wavelength (angstroms): Wavelength as read by the cursor in the measurement mode

Intensity: Relative intensity of light from the galaxy at the position marked by the cursor in the measurement mode

9. Click **start/resume count** from the menu bar in the Spectrometer Reading Window. Continue to collect photons until a *clear* spectrum of the H & K lines of calcium is displayed. These lines are approximately 40 Å apart. They should stand out from the noise. If not, continue to count photons. If you are not sure about the data, check with a lab instructor to help you interpret the data.

10. Record the object, photon count, apparent magnitude, and the measured wavelength of the H & K lines of calcium on the data sheet located at the end of this exercise. The H & K lines measured should be red shifted from the laboratory values depending on the galaxy's motion. Apparent magnitude is just another way of describing the apparent brightness of an object. We'll use it later in figuring out the distance to the galaxy.

11. To collect data for additional galaxies, press **Return**. Move the telescope to a new field of view, then follow steps 6 through 10. Repeat until you have measurements of one galaxy in each of five different fields of view.

Calculating the Hubble Parameter

Now that we see how to use our instrument to collect data, we can use this information to determine the distance and velocity of each galaxy.

To find the distance, we would normally use the inverse-square law, $b = L / (4\pi d^2)$. Astronomers often write this relationship in a different way, using *apparent magnitudes* and *absolute magnitudes*. Apparent magnitude, which is usually called *m*, is just a different way of talking about apparent brightness, and absolute magnitude, *M,* is just another way of talking about luminosity. Both apparent and absolute magnitudes run backwards (that is, higher values of magnitude correspond to fainter objects). They're also both *logarithmic* scales, meaning that *adding* a certain value to the magnitude is equivalent to *multiplying* the brightness by a certain value. All you really need to know about magnitudes is that the inverse-square law can be expressed in terms of magnitudes like this:

(A)   **M = m + 5 – 5 * log D**

or

$$\log D = \frac{m - M + 5}{5}$$

or

$$D = 10^{(m-M+5)/5}$$

All three of these equations are equivalent to each other, and all are just cleverly-disguised versions of the inverse-square law, $b = L / (4\pi d^2)$. In all of these expressions, m is the apparent magnitude, M is the absolute magnitude, and D is the distance *in parsecs*.

For purposes of this lab, we'll make the (not terribly realistic) assumption that all of the galaxies you observed have the same absolute magnitude (in other words, they all have the same intrinsic luminosity): $M = -22$. (Note the minus sign: the absolute value is a negative number in this case.) When you observed each galaxy, you recorded its apparent magnitude m, so you have enough information to find the distance to each galaxy.

You'll need to determine the speed of each galaxy in addition to its distance. Remember the Doppler Effect formula, which can be written

(B) $$\mathbf{v = c}\ \frac{\Delta\lambda}{\lambda_0}$$

Here's how to use your data to get a value for the Hubble parameter. You can either do steps 1-4 on paper or by entering your data into an Excel worksheet. Either way, once you get to step 5 (plotting a graph), you'll want to use Excel.

1. Use your measured magnitudes and the assumed absolute magnitude for each galaxy and derive the distance, D, to each galaxy using equation (A). Express your answer in both parsecs and mega parsecs in the appropriate places on your data table. Note that equation (A) tells you how to find the log of the distance.

2. Use your measured wavelengths to calculate the redshifts for each line, $\Delta\lambda_H$ and $\Delta\lambda_K$. Record each on your data table.

3. Use the Doppler shift formula, to determine the velocities as determined by both the H and K lines. There is a place on the data table for each of these figures:

4. Calculate and record the velocity of the galaxy. It is the average of the velocities determined from the H and K lines.

5. Now plot a Hubble diagram by graphing the velocity of a galaxy in km/sec (y-axis) vs. the distance in megaparsecs (x-axis). Draw a straight line through the origin (a *trendline* in Excel) that best fits all the data points. The slope of the line is the Hubble Parameter (H). To calculate the slope of the line, measure a value of **D** and **v** from a point near the **upper right** end of the trendline. Determine H using the following equation:

(C) $$\mathbf{H = \frac{v}{D}}$$

       where H is the Hubble Parameter in km/sec/Mpc
       v is the velocity measured from your line
       D is the distance measured from your line

Record your value for the Hubble Parameter on your data table on the line called the **Average Value of H.**

Determining the Age of the Universe

The Hubble Law, equation (C), can be used to determine the age of the universe. Using *your* average value of H, calculate the recessional velocity of a galaxy of a galaxy which is 800 Mpc away.

Velocity of a galaxy 800 Mpc away: _____km/sec

Verify your velocity by looking it up on your Hubble diagram.

You now have two important pieces of information:

       1. How far away is the galaxy.
       2. How fast it is moving away from us.

You can visualize the process if you think about a trip in your car. If you tell a friend that you are 120 miles away from your starting point and that you traveled 60 miles per hour, your friend would know you had been traveling TWO hours. That is your trip started two hours ago.

Now let's determine when this galaxy "started its trip". The distance is 800 Mpc, but first convert Mpc into km because the rate, or velocity, is in km/sec.

800 Mpc =   _____km

Use equation (E) to determine how many seconds ago the universe started:

               _____secs

There are about 3.15×10^7 seconds in one year. Convert your answer into years:

               _____years

This is an estimate of the age of the Universe. As we discussed in class, this estimate depends on some important assumptions, mainly that the galaxy was moving at the same speed in the past as today.

One final question: suppose that you had used a galaxy that was 400 Mpc away, instead of one at a distance of 800 Mpc, in making this calculation. Would the final answer have come out the same? Why or why not?

Galaxy Field Name	Abs Mag M	Photon Count	App Mag m	Dist in pc	Dist in Mpc	$\lambda_{K measured}$ K Line	$\lambda_{H measured}$ H Line	$\Delta\lambda_H$	$\Delta\lambda_K$	Velocity H	Velocity K	Velocity AVG
	-22											
	-22											
	-22											
	-22											
	-22											

Using the Graph: Average Value of H = _____ km/sec/Mpc

Useful Equations and Quantities

$M = m + 5 - 5 * \log D$ 　　　　 $v_K = c * \dfrac{\Delta\lambda_K}{\lambda_K}$ 　　　　 **1 light year = .306pc**

$\log D = \dfrac{m - M + 5}{5}$ 　　　　 $\Delta\lambda_H = \lambda_{H\ measured} - \lambda_H$ 　　　　 **1 MPC = 1 x 10^6 pc**

$v_H = c * \dfrac{\Delta\lambda_H}{\lambda_H}$ 　　　　 $\Delta\lambda_K = \lambda_{K\ measured} - \lambda_K$ 　　　　 **1pc = 3.26 light years**

Wavelength of K Line: 　　　　 **Wavelength of H Line:** 　　　　 **c = 3 x 10^5 km/sec**
　　$\lambda_K = 3968.47$ 　　　　　　　　$\lambda_H = 3968.47$

Lab 33

Are We Alone?

As far as we can tell, our Sun is a fairly typical star in a fairly typical galaxy. The Universe contains a huge number of stars very much like our own. Do any of them have life? Nobody knows, but that doesn't stop us from trying to find out!

First, let's get a feel for the number of stars in the Universe. Here are some useful numbers to get us started:

- There are about 300 billion stars in our own Milky Way Galaxy.

- The observable Universe is a sphere with a radius of about 5000 megaparsecs.

- There's about one large galaxy like our own for every 100 cubic megaparsecs (Mpc^3) of volume.

About how many large galaxies are there in the observable Universe?

Assuming all of those large galaxies have about the same number of stars as our own Galaxy, about how many stars are there in the observable Universe?

Suppose that a star has a one-in-a-million chance of having a planet that develops life. How many stars in our Galaxy have life?

Continuing to suppose that a star has a one-in-a-million chance of developing life, how many stars in the observable Universe have life?

Now suppose that life is less likely: say that only one in a trillion (10^{12}) stars develops life. How many stars in the observable Universe have life?

Of course, the problem is that we have no idea what the probability of life developing around any given star is. It could be one in a million, one in a trillion, or something much larger or much smaller!

Suppose that there are planets out there with life on them. How could we detect them? If we were really lucky, maybe the creatures on them would have developed a technological society similar to ours. In that case, they might actually send out radio broadcasts into the Galaxy, to see if anyone's out there. Even if they didn't do that, they might use radio waves to communicate with each other (just as we do). In that case, we might be able to detect their broadcast signals. (Just as an alien civilization 20 light-years away might just now be receiving our radio and television broadcasts from 20 years ago. I wonder if they like "Seinfeld.")

Back in 1961, the astronomer Frank Drake wrote down an equation that can be used to estimate the number of alien civilizations that we might hope to detect in this manner. Here's one way of writing the *Drake Equation*:
$$N_c = N_* f_p n_e f_l f_i f_c f_L.$$
Here's what this means.

The quantity N_c is the number of civilizations in our Galaxy from which we might possibly be able to detect signals. That's what we want to find. The things on the right are

- N_* is the number of stars in our Galaxy.

- f_p is the fraction of all of those stars that have planets.

- n_e is the average number of planets around each such star that can sustain life. (One way people often interpret this term is that it's the number of planets with liquid water. It's much easier for the complex chemical reactions that cause life to occur if there's liquid water around.)

- f_l is the fraction of all of such planets on which life actually evolves.

- f_i is the fraction of all such life-containing planets on which *intelligent* life arises.

- f_c is the fraction of all such intelligent-life-containing planets where the inhabitants are communicating with us. (This could mean either the fraction that are deliberately beaming signals out into space to let us know they're there, or the fraction that are using broadcast signals to communicate with each other.)

- f_L is the fraction of the Galaxy's life during which the civilization is communicating. The life of the Galaxy so far is about 10 billion years, so this last term is the lifetime of the technologcal civilization divided by 10 billion years.

Although this equation looks complicated and has a lot of terms in it, I hope it makes sense where all the terms come from. Most of the terms (all the ones called f) are fractions that could be anywhere from 0 to 1.

The problem is that we have very little idea what a lot of the numbers on the right side of this equation should be!

For instance, maybe life evolves pretty much every time it possibly can, so that every planet that can possibly sustain life does in fact evolve life. If that's true, then f_l would be about equal to 1 (that is, 100%). On the other hand, maybe it takes an incredible stroke of luck for the molecules on a planet to organize themselves in such a way that life gets started. If that's true, then f_l would be an extremely small number.

Similarly, it might be the case that intelligent life is very rare – maybe there are lots of planets oozing with bacteria but very few with big-brained creatures like us. In that case, f_i would be small. Or maybe once life gets started it's inevitable that it grows in complexity until something intelligent arises. In that case, f_i would be nearly equal to 1.

Using Google (or some other search engine), dig around the web for a while to come up with a range of plausible estimates for these numbers.[1] ("Drake equation" is a good search term. Be warned: there are slightly different ways of writing the equation, with different choices of symbols for the various quantities.) Feel free to use your own judgment along with anything you find on the web. Let's assume that the number of stars in our Galaxy is known to be 300 billion:

$$N_* = 3 \times 10^{11}.$$

For each of the other numbers, list two values: a "pessimistic" estimate and an "optimistic" one. There's not one right answer for these, of course!

- $f_p =$
- $n_e =$
- $f_l =$
- $f_i =$
- $f_c =$
- $f_L =$

Using all of your low numbers, estimate the number N_c of communicating civilizations. Then use all of your high numbers to estimate the number. Give a range of plausible values for N_c:

There is an ongoing project called the Search for Extraterrestrial Intelligence (SETI) looking for radio signals from intelligent beings on other planets. Based on your estimates, do you think SETI is likely to succeed?

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[1] Just in case there's any doubt, let me mention that "dig around the web" is not generally considered to be part of a good scientific methodology. In this case, though, we just don't know what the right answers are, and I just want to get an idea of a plausible range of answers.

Appendix A

Introduction to Stellarium

Stellarium is a software application that simulates the appearance of astronomical objects in the sky. The user can adjust lots of features of the view, including the observer's location, the time, etc.

Stellarium is free. I encourage you to download it onto your own computer by going to the Web site `http://stellarium.org`, so that you can play with it outside of class. You may find it helpful when studying or answering homework questions.

In this Appendix, I'll list some of the features of the program that will be most useful to us. The best way to learn about these features is not just to read about them here, though. Spend some time playing around, to learn what you can do.

Many of the useful features in *Stellarium* are in two menus that pop up on the bottom left of the screen when you move the mouse over to that region:

In the descriptions below, I'll refer quite a bit to the various icons in these menus.

Starting up and closing down the program. You start *Stellarium* in the usual way. When you're done with the program, you exit by clicking on this icon: ⏻.

When you first start *Stellarium*, position the mouse near the bottom of the screen so that the horizontal row of icons appears. You may find that the Angle Measure icon ∠ doesn't appear in this row. If you're going to want to measure angular separations (which is something we'll do pretty

often), you'll need to fix this before proceeding. Here's how. Click on the configuration icon , then click on "Plugins." The words "Angle Measure" should be highlighted at the upper left of the configuration box. There's a checkbox at the bottom of this box, next to the words "Load at startup." Check this box. Then exit *Stellarium* (), and start it again.

Moving around the sky. Hold down the left mouse button (starting anywhere on the screen) and drag to look at different parts of the sky.

Zooming in and out. Use the mouse wheel or the page-up and page-down keys to zoom in and out. Note that the "Field of View" (FOV) is indicated at the bottom of the screen. The FOV is the size (in degrees) of the patch of sky that's visible on the screen at that moment.

Selecting and centering on an object. If you click on a star or planet (or other celestial object), it will be highlighted, and some (possibly cryptic) information on the object will appear in the upper left corner. If you hit the space bar or click on , that object will be moved to the field of view.

Changing the observer's location. Click on to pull up a box that lets you choose where the observer is located. You can either enter latitude and longitude or choose a city from a list. You can even change your observation location to the Moon or another planet.

Changing time. There are two ways to adjust the time.

The first is to use these icons: . These let you adjust how fast time is flowing in the simulation. The second icon in is a play/pause button, which starts and stops the flow of time. The ones on the left and right adjust the speed of the playback. The third one resets the time to the present moment.

If you want to go to a specific moment in time, you're better off using the clock icon, . This causes a box to pop up in which you can adjust the date and time to anything you like. You can also use this box to jump forward in time by fixed increments (one hour, one day, etc.)

Note that the clock icon lets you display the date and time in two different ways: the default is the normal date and time, but you can also switch it so that it displays something called the *Julian Day (JD)* or the *Modified Julian Day (MJD)*. The JD is the number of days that have elapsed from an arbitrarily-chosen starting date, which happens to be noon on January 1, 4713 BCE. The MJD works the same way, but its starting date is midnight on November 17, 1858. Times of day are expressed as decimal portions of a day – for every hour that elapses, the JD or MJD goes up by $\frac{1}{24}$, or 0.416667.

The JD and MJD are very useful if you want to quickly calculate the time elapsed between two events: all you have to do is subtract them. If you record dates in the usual way, it's much more trouble to figure out the elapsed time: you have to remember that "30 days hath November," keep track of leap years, and all that.

Finding a specific object. Click .

Miscellaneous display options. These three icons help you identify constellations: . The first draws lines connecting the stars in each constellation. The second labels the constellations

by name. The third draws pictures of each constellation. (Personally, I don't think the last one is much use.) All three of these are toggles – that is, repeatedly clicking them flips the feature on and off.

Astronomers label points in the sky with two kinds of coordinates, equatorial and azimuthal. (Note: *Sky Safari* calls the latter coordinate system "horizon coordinates.") We'll talk about these in detail in class, but for the moment here are the basics. Both of these are like latitude and longitude on Earth. The difference is that the equatorial grid is "attached" to the sky, so that each star's coordinates stay the same as time passes. The azimuthal grid, on the other hand, is attached to the Earth. You can turn on grids showing these two coordinate systems using these icons: .

The icon turns on and off labels identifying the names of planets and their moons.

The icon switches the Earth's surface between being solid and transparent. If you turn off the Earth's surface, you can see objects even when they're below the horizon.

The icon switches on and off the effects of Earth's atmosphere. When the atmosphere is "turned off," you can see the stars even in the daytime.

Of course, the last two represent things we can't do in real life (no matter how much astronomers wish we could), but they're useful tools for helping to build intution about how objects move in the sky.

Measuring angles.   Often, we want to know the *angular separation* between two objects in the sky. Angular separation is the angle between a line from your eye to one object and a line from your eye to the other object; it's a measure of how far apart the objects appear to be. To determine angular separation, click on the icon , and then drag the mouse (holding down the left button) from one object to the other. When you're done measuring angles, click the icon again.

Switching between equatorial and azimuthal mounts.   As we'll see in detail, objects in the night sky appear to rotate approximately once per day about a point in the sky called the "north celestial pole." This apparent motion is really due to the Earth's rotation on its axis. Often, it's useful to examine what the sky would look like if the Earth's rotation could be taken away. Clicking on the icon switches back and forth between two ways of looking at the sky: the "azimuthal" view shows the sky as it really looks from a given point on surface of the rotating Earth, and the "equatorial" view shows what happens when the Earth's rotation is taken away.

As a matter of fact, astronomical telescopes are often placed on an "equatorial mount," which has a motor that rotates the telescope in just the right way to cancel out the Earth's motion. The equatorial point of view is what we actually see in this case.

Note: These two types of "mounts" are what I called the "stationary-Earth" and "stationary-sky" points of view in the *Sky Safari* appendix.

Appendix B

Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for some of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, I will briefly describe how to use Excel to do some useful tasks.

B.1 Data and formulae

The figure below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula "=B3∧3" (without the quotes) into cell C3. Note the equals sign, which indicates to Excel that a formula is coming. The ∧ sign stands for raising to a power, so this formula says "take the value in cell B3 and raise it to the power 3." After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column. This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C4 contains the formula "=B4∧3," and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D3 through D6). For those who don't know, the mean is just the average, and the standard deviation is a measure of how widely scattered the individual numbers are about the average. The standard deviation is often used to characterize the margin of error in a measurement. The mean and standard deviation were computed using the formulae "=average(D3:D6)" and "=stdev(D3:D6)".
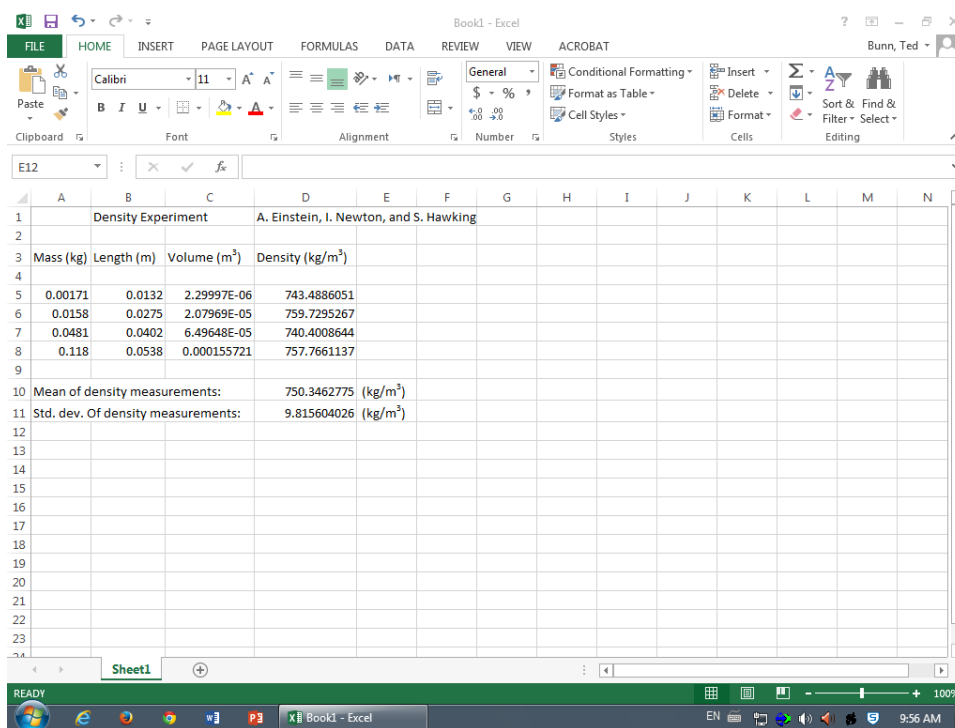
B.2 Graphs

Here's how to make graphs in Excel. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, select the first one, then hold down the control key while selecting the second one.) Then click on the "Insert" tab at the top of the screen. The graphs we'll want will always be "scatter plots," so click on the "scatter chart" item (right above the word "Charts").

Sometimes, you may want to make a graph in Excel where the x column is to the right of the y column in your worksheet. In these cases, Excel will make the graph with the x and y axes reversed. There are a couple of ways to fix this problem, but the simplest one is to make a copy of the y column in the worksheet and paste it so that it's to the right of the x column.

Once you've got your graph, you'll want to customize it. The most important thing is to label your axes. (For some inexplicable reason Excel, by default, doesn't do this.) Click on the chart, and then click on the big + sign that will appear to the right of the chart. This will bring up a list of checkboxes. Check the "Axis Titles" box. Now you can click on each of the two axis titles and edit them. **The axes on all graphs must be labeled to indicate both what quantity is on that axis and what its units are.** For instance, if a graph showed the distance to an object in meters, the axis label should say "Distance (m)."

You should probably also click on the title at the top of the graph, and replace the text there with something more descriptive than what's already there.

For instance, a graph showing the relationship between mass and volume for the data in the spreadsheet above would end up looking like this:
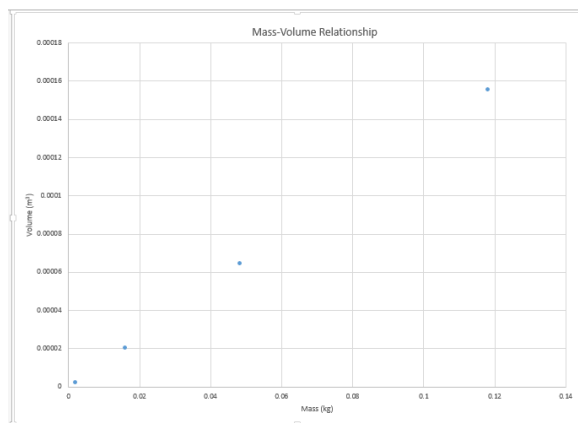
Here are a couple of other customizations you may want to do on occasion. If you double-click on either the horizontal or vertical axis of your graph, you'll pull up the "Format Axis" box. There are a couple of potentially useful things here. First, you can change the range of values shown on the graph by typing numbers into the Minimum and Maximum Bounds boxes. You can also make the axis run backwards (with high values on the left / bottom instead of the right / top) by checking "Values in reverse order." Finally, there's a checkbox to cause the axis to be logarithmic (meaning that the values on it go, say, 1,10,100,1000 instead of 1,2,3,4).

B.3   Fitting lines and curves

After you've made a graph, you can have Excel draw a straight line or curve that is a best fit to the data. Select the graph, then click the + sign at the upper right to bring up the list of chart elements. Check the "Trendline" box. This should cause a straight line to appear on your graph, which passes as nearly as possible through the points on the graph. You can double-click on the line to customize the line in various ways. One of the most useful is to check the box that says "Display Equation on chart." **Excel will not put the correct units on the numbers in this equation, but you should.**