Physics 121: Astronomy
Lab Manual
University of Richmond
Fall 2020

Ted Bunn

December 24, 2020

Contents

Lab 1

Apparent Size and Distance

One of the most important things to know about an astronomical object is its distance from us. Unfortunately, measuring distances is often very hard. There's no one method that works for finding the distances to all astronomical objects, so astronomers have come up with a number of different methods that work in different circumstances.

One method is based on the familiar observation that a faraway object looks smaller than an identical nearby object. One way to gauge the distance to an object, then, it to measure its apparent size. The smaller the apparent size, the greater the distance.

For instance, suppose you wanted to know the distance to a faraway galaxy. Also, suppose that there's a much closer object of the same apparent size (like the Sun and the quarter Calvin's Dad is holding). If the two objects have the same apparent size, then the nearby one can just block the view of the faraway one, like this:

Let's give some names to the sizes and distances in this picture:

- s_1 is the size (diameter) of the nearby object.
- d_1 is the distance from the observer to the nearby object.
- s_2 is the size (diameter) of the distant galaxy.
- d_2 is the distance from the observer to the distant galaxy.

Indicate these four distances on the diagram above. Then write down an equation giving the mathematical relationship between these quantities. (Hints: You might want to think in terms of proportions and ratios. You might possibly want to think back to what you learned about similar triangles in geometry class all those years ago. If you're stuck on this — or anything else — ask me.)

If we can somehow manage to determine the size and distance of the nearby object, and the size of the faraway galaxy, we can use this relationship to find the galaxy's distance.

To see how this works in practice, you're going to use this technique to measure the distance to a nearby building on campus.

1. You have a sheet of transparent plastic with lines of different thicknesses marked on them. Take the sheet outside and find a place where you have a clear view of one of the windows of a nearby building (Gray Court and the Modlin Center are the best choices). Bring a length of string and a meter stick with you.

2. One partner should hold up the plastic sheet with the lines oriented horizontally and adjust the sheet so that one of the lines just blocks the window in the distant building (that is, so that the apparent thickness of the line is the same as the apparent height of the window). The other partner should measure the distance from the person's eye to the line. (I suggest you stretch the string between the eye and the plastic sheet, then measure the length of the string.) Write down those values here. Also, write down the thickness of the line that you used.

   Note: Whenever you write down a measurement, you must also write down its units.

3. To determine the distance to the other building, you'll need one more bit of information: the actual size of the window. We're going to make an assumption (which may not be correct) that the windows in the other building are the same size as the windows in our classroom. Go back to the classroom, measure the height of the window, and record it here.

4. Using the relationship between s_1, d_1, s_2, d_2, together with the values you just recorded, determine the distance from here to the other building:

   To see how well you did, determine the distance using the aerial photograph from Google maps that I'll hand out. Here's how.

5. The scale bar at the lower left of the photograph shows the distance on the page that corresponds to 50 meters of actual distance. Measure the length of this bar, and use the result to determine how many meters of actual distance correspond to 1 cm on the page.

6. Mark on the page your approximate location when you made the measurement and the location of the window. Measure the distance between the two points on the page in centimeters.

7. Use this information to determine the distance from your observation point to the other building in meters.

Are your two determinations of the distance significantly different? If so, what do you think the main reasons are? Which measurements might have been inaccurate? What assumptions might have been incorrect?

Lab 2

Angular Measurements

In astronomy, we often talk about the "angular size" of an object. This is just a way of expressing how big the object appears to be, or in other words how much of our field of view it takes up. For instance, in the picture below (which is the same as the picture in your previous lab), the two objects have the same angular size, because they "fill up" the same angle in the observer's field of view.

The angular size is related to the actual size of the object and the object's distance. That relationship is going to be extremely useful to us, so I want you to spend a little while right now working it out.

Here's a picture of an object. The angular size is the angle α on the left. The distance from the observer to the object is d, and the actual size of the object is s. Our goal is to relate the three numbers α, s, d.

(Note: Your textbook uses the symbol D instead of s for the size of the object. I don't like that, because I think it's too easy to mix up d and D, so I'm going to call it s instead.)

We can use trigonometry to relate these three numbers, but the mathematics is a bit annoying. Fortunately, as long as the angle α is very small (which it almost always is in astronomical observations), there is a simplified relationship between these numbers that doesn't involve any trig. Here's how it works.

In the picture below, the straight line on the right represents the object. The length of this line is the exact value of s, the object's size. The picture also shows a curved line. That curve is an arc of a circle centered on the observer. Because it's curved and not straight, the length of this arc is not quite the same as s. On the other hand, if the angle α is very small, then this curve is almost the same length as the straight line. For this reason, *we are going to allow ourselves to pretend that the length of that curved arc is the same as s.* To figure out the length of the straight line, we'd need trig, but we can figure out the length of the arc with just geometry.

Here's yet another picture of the situation. Now the observer is at the center. The solid arc represents the size of the object being observed, and d is the distance from observer to object. The dashed circle is an imaginary circle centered on the observer.

Well, let's get started.

1. Suppose that the angle α is one degree. What fraction of the whole circle is covered by the arc s?

2. Remember that the circumference of a circle is 2π times the radius. What is the length of the arc s, in terms of the distance d? Continue to assume that α is one degree. (Your answer here should say $s =$ something times d.)

Repeat steps 1 and 2, but this time assume that $\alpha = 2^\circ$.

Repeat steps 1 and 2, but this time don't assume any particular value for α. Instead, your answers will be algebraic expressions including α as an unknown variable. Your final result should say $s =$ something involving both α and d.

Once you've got this expression worked out, show it to me. This extremely useful fact is known as the "small-angle formula."

Now that you know the small-angle formula, what do you use it for? The main thing is that any time you know two of the numbers α, s, d, you can use the formula to get the third one. Try these out:

The Moon is 3476 kilometers in diameter, and it's 380 000 kilometers away. What is the angular size of the Moon?

The Sun's angular size is 0.53 degrees, and the Sun is 150 million kilometers away. What's the Sun's diameter in kilometers?

Most of the time, angles in astronomy are much smaller even than one degree. For this reason, astronomers usually measure angles in units called *arc-minutes* and *arc-seconds*. One arc-minute is one-sixtieth of a degree:

$$1' = \left(\frac{1}{60}\right)^{\circ}.$$

($'$ means "minute"; $^\circ$ means "degree.") One arc-second is one-sixtieth of a minute:

$$1'' = \left(\frac{1}{60}\right)'.$$

($''$ means "second.")

How many arc-seconds are in one degree?

A decent astronomical telescope can see things whose angular size is as small as $1''$. How far away would your lab partner have to be in order for the angular distance between their eyes to be equal to $1''$? You'll have to measure the distance between your partner's eyes to answer this. (If your partner is any further away than this, then the telescope would no longer be able to see your partner's eyes as two separate objects; they'd be blurred into one blob.)

Optional question: If you know and remember your trig and feel like it, try answering the last question using trig instead of the small-angle formula. Does it make a significant difference in the final result?

Lab 3

Resolution of the Human Eye

Everyone knows that there are limits to how well we can see. Some objects are too small or too close together for our eyes to distinguish them. The same goes for any telescope or other observing device we might use: even the Hubble Space Telescope has limits to the level of fine detail it can "see." We define the *resolution* of an observing device (an eye or a telescope, for instance) to be the *angular size* of the smallest details it can see. The smaller the resolution, the better the device is at seeing fine details. In this lab, you will measure the resolution of your own eyes.

You will be given a piece of paper that has a pair of black dots on it, along with a single oval-shaped blob that's about the same size as the pair of dots. If you look at the paper from a short distance, it's easy to tell the pair of dots apart from the single blob, but from a great distance you can't tell which is which. We say that you can *resolve* the pair of dots if you can tell that the two dots really are two dots, not one.

Measurements. Take the sheet with the dots on it and move quite far away (10 meters or so) from your lab partner. Hold the sheet up either right-side up or upside-down, but don't tell your partner which. From this distance, your partner should not be able to tell the difference between the pair of dots and the single blob. Your partner should then walk slowly toward you until he or she can tell which one is pair of dots. Measure the distance between the two of you at the moment your partner can first tell with confidence which is which.

Repeat this procedure at least three times. Switch the orientation of the paper randomly, so that your partner doesn't know the "right" answer. Switch roles, so that each partner is the "observer" at least three times. List the individual measurements below, and also compute the average distance for each partner.

The resolution of your eyes is the angular separation between the two dots when you can just barely tell that there are two of them. To figure out this number, you need to know the separation between the two dots (to be precise, the separation between their centers). Measure this separation.

Use the above information to calculate the resolution of each partner's eyes. Give your answer in degrees and also in arc-minutes. Remember that one arc-minute ($1'$) is $\frac{1}{60}$ of a degree.

Once you've got your answers, record them on the whiteboard, so that everyone can see the range of resolutions.

Now suppose that you repeated the experiment using dots that were twice as big, and twice as far apart. What would you expect the distance between partners to come out to be?

What would you expect the resolution to come out to be?

The back of the page contains dots that are twice as large. Try the experiment and see. (You don't need to do 3 repetitions per partner this time; just one is fine.) Are the results consistent with your predictions?

Questions. Here are some questions to test whether you know what you're doing with all this stuff. Some of these questions have to do with resolution, and some are other applications of the small-angle formula. You don't have to turn these in, but you just might have a quiz some time soon that asks questions very similar to these.

Note that I've obnoxiously used a variety of different units here. If you need to look up unit conversions, go ahead. In some cases, I may have deliberately left out a number you need, in which case you should estimate it as best you can.

(In case you're wondering, on a quiz or exam, I will not expect you to know numbers such as the number of meters in a light-year or the distance from Earth to Sun. You should know the basic metric-system prefixes, like the number of centimeters in a meter or meters in a kilometer.)

1. The Moon is 384 000 kilometers away. There are two large craters on its surface, which you can just barely distinguish from each other with the naked eye. How far apart are the craters?

2. At the Battle of Bunker Hill, the rebels were supposedly told, "Don't shoot until you see the whites of their eyes." How close would the British be at this point?

3. You're standing on a deserted road, late at night. A vehicle is approaching you. At first, you can't tell whether it is a car (with two headlights) or a motorcycle (with one). How close will the vehicle be before you can tell?

4. The resolution of the Hubble Space Telescope is about $0.1''$ (that's 0.1 arc-seconds). How many times better than your eye is this?

5. The two dwarf planets Pluto and Charon (formerly known as the planet Pluto and its moon) are about 20 000 kilometers apart. They are about 29 astronomical units from the Earth. What is the resolution of a telescope that can just barely resolve these two bodies? Give your answer in arc-seconds.

6. Suppose that another star somewhere nearby has a planet orbiting it at exactly the same distance as the Earth is from the Sun. How close would the star have to be in order for the Hubble Space Telescope to be able to see the planet and star as separate objects? Give your answer in light-years.[1] The closest star is 4.2 light-years away. Is this close enough?

7. One of the stars in the handle of the Big Dipper is actually a pair of stars called Alcor and Mizar. The two stars have an angular separation of about $12'$ and are about 1 light-year away from us. How far are the two stars from each other? Give your answer in meters and in astronomical units.

8. The supergiant star Betelgeuse has an angular diameter of $0.044''$ and is 427 light-years from Earth. What is Betelgeuse's diameter? If the Sun suddenly swelled up until it was as big as Betelgeuse, which of the planets of the solar system would it engulf?

9. A typical distance between neighboring galaxies is 2 million light-years. A good ground-based telescope has a resolution of about $2''$. How far away must a galaxy be if an observer using this telescope has difficulty resolving it (that is, telling it apart from its neighbors)? Because the Universe is not infinitely old, there's a maximum distance we can see of about 50 billion light-years (light from greater distances hasn't had time to reach us). Bearing this in mind, is there ever a problem telling galaxies apart from their neighbors with a telescope like this?

10. > "Riders!" cried Aragorn, springing to his feet. "Many riders on swift steeds are coming towards us!"
    >
    > "Yes," said Legolas, "there are one hundred and five. Yellow is their hair, and bright are their spears. Their leader is very tall."

[1] Actually, seeing other planets is even harder than this question suggests. Looking for something very faint right next to something very bright is extremely difficult.

> Aragorn smiled. “Keen are the eyes of the Elves,” he said.
>
> “Nay! The riders are little more than five leagues distant,” said Legolas.
>
> J.R.R. Tolkien, *The Two Towers*

Roughly what is the resolution of Legolas’s eyes?

Answers. For purposes of the answers below, I’ll assume that the resolution you found for your eye was $2'$ (two arc-minutes).

1. 220 km.
2. I’ll estimate that the white of your eye is about 1 centimeter across (from the edge of your iris to the edge of your eye). Something about that size can be resolved at a distance of about 20 meters.
3. $2'$ is $120''$, which is 1200 times bigger than $0.1''$, so the Hubble space telescope has a resolution about 1200 times better than your eye.
4. Assuming the headlights are about 2 meters apart, the distance comes out to about 3.4 kilometers.
5. About $0.95''$.
6. 33 light-years. So the nearest star is close enough.
7. 3.3×10^{13} m, or 220 AU.
8. The diameter is 8.6×10^{11} m or 5.7 AU. All planets within half this distance, or about 2.85 AU, of the Sun would be engulfed. That’s Mercury, Venus, Earth, and Mars. (Jupiter’s orbit is about 5 AU in radius.)
9. About 2×10^{11} light-years. This is about 200 billion light-years, which is much bigger than the maximum distance we can see, so telling galaxies apart from their neighbors isn’t generally a problem even for very distant galaxies.
10. We’ve got to make some estimates here. Let’s say that Legolas can resolve features that are as small as 0.1 meters. (If his resolution were much worse than this, then I don’t think he could tell a tall person from a short person. Naturally, if you used a somewhat different number, that’s fine.) Five leagues is about 15 miles, or about 25 km. The resolution corresponding to these numbers is $0.0002°$, or $0.85''$. Legolas does about as well as a good astronomical telescope.

Lab 4

Daily Motion of Stars

This lab has a few purposes:

- To get used to some features of the *Stellarium* program.
- To identify some prominent constellations and asterisms that can be seen from our location.
- To see how stars move in the night sky.

A. Messing around.

Start up *Stellarium*, and spend a few minutes playing around with some of its features. Appendix A lists a bunch of things you can do. Here are some specific things you should try. Refer to the Appendix for details about how do do these things.

1. Adjust the time and date. When you first start the program, it will show the sky at the current time. It's much more interesting to see the sky at night. You can also examine what things look like at different times of year.

2. Speed up the flow of time by a large amount, so that a day goes by in just a few seconds.

3. Initially, the program is showing you the view of the sky from here in Richmond. Switch to some other locations.

4. Click on a star or other astronomical object. Information about that body should appear on the screen. Some of that information may not make sense yet, but it will!

5. Drag the image around (hold down the left mouse button and move the mouse around) to look at different parts of the sky.

6. Zoom in to look at a small patch of the sky, then zoom back out (using the mouse wheel or the page-up/page-down keys). Note the display at the bottom that says "FOV." This stands for "field of view," and it indicates the size of the patch of sky that's visible on the screen at that moment. Watch how the FOV changes as you zoom in and out.

7. Click on the "find" icon and search for various celestial objects. Try Uranus, for example, or Polaris (which is the name of the North Star).

8. The pop-up menus in the lower left contain a bunch of buttons you can click to change the appearance in various ways. I've listed the ones I think are most useful in the Appendix. Try these out. Some are harder to understand than others.

B. Big Dipper, Mizar, and Alcor.

Once you're done messing around, set your location back to Richmond, looking north, with a nice, large field of view (say 90° or so). Set the time to 10:00 tonight.

You should see the Big Dipper in the northeast. Turn on the constellation labels and lines. Notice that the Big Dipper is not a full constellation; it's just part of the larger constellation Ursa Major (the Great Bear). A group of stars like the Big Dipper, which is easily identifiable and has a name, but which isn't a whole constellation, is called an "asterism."

The second star in the handle of the Big Dipper is called Mizar. Click on this star to select it, and move it into the center of the field of view. Then zoom in on this star until you can see another star called Alcor right near it. Find the angular separation between these two stars. What is the angular separation?

This separation is visible to the naked eye, if you have good eyesight. Next time you're out on a clear night, look to see if you can spot both Alcor and Mizar.

Zoom in on Mizar still further, and you'll eventually see that it splits into two stars. Amazingly enough, although *Stellarium* might not tell you this, each of those two stars is itself a double star system. Unfortunately, these two pairs are too close together for us to see them separately, even with our best telescopes. (How do we know that they're there, then? Good question! We'll answer it eventually.) In fact, in 2009 it was discovered that Alcor is actually a double star system as well. So when you look at Mizar you're really seeing four stars, and if you look at both Alcor and Mizar, you're seeing six stars.

What is the angular separation between Mizar and its companion (not Alcor – the closer one)?

Select Mizar by clicking on it (if it's not still selected). You'll see some information about it in the upper left corner, including the distance to the star in light-years. Use the small-angle formula to determine the separation between Mizar and its companion. The small-angle formula will give you an answer in light-years. Convert this to meters and to astronomical units.

Zoom back out to the largest field of view, look toward the north again.

C. Other Constellations.

Identifying constellations isn't a big part of this course (after all, there's no science involved in constellation-spotting). Still, it's good to know where a few prominent constellations are.

The two stars at the end of the bowl of the Big Dipper (Merak and Dubhe) are called the pointer stars. Draw an imaginary line from Merak to Dubhe, and extend it about six times the separation between those stars. You'll hit the star Polaris, which is the tail end of the Little Dipper (the constellation Ursa Minor). Polaris is also known as the North Star. Polaris isn't a terribly bright star – in fact, none of the stars in Ursa Minor are very bright – but it's still important. It's the one star in the sky that all the others seem to rotate around. (More on this later.)

Look to the West from Polaris to find the constellation Cassiopeia. It looks like a W (on its side at the moment). The stars in Cassiopeia are quite bright, and Cassiopeia is easy to spot in the night sky pretty much all the time. Cassiopeia and the Big Dipper are probably the most useful constellations to use when orienting yourself to the night sky.

In the winter, the other easy constellation to spot is Orion. Find it in the southern sky.

That's enough with constellations for now; we'll do more when we actually do some observing.

D. Daily Motion of Stars.

Make sure you're looking North, with a large field of view (at least 90°). Turn off the effects of Earth's atmosphere. This will make it so that the sky is dark during the day, so that you can see the stars all the time. (Astronomers would love it if this were possible in real life!)

Set time to go at much faster than the normal rate, so that a day takes only a a few seconds. Observe how the stars move. The stars move in circles, with the star Polaris at the center. Note that some of the stars move in circles that always stay above the horizon, while others rise and set below the horizon. Stars that never set below the horizon are called "circumpolar."

Of course, whether a star sets below the horizon or not depends on the exact shape of the horizon. To keep things simple, let's assume that we're looking at the stars from a location with a nice, flat horizon (no hills or trees to get in the way). Here's one way to make this happen in *Stellarium*: click on "Sky and viewing options", then the "Landscape" tab, then check the "Ocean" box. As you'll see, that shows you what things would look like if you were surrounded by a nice, flat ocean.

How many of the seven main stars in the Big Dipper are circumpolar? (Here I'm counting all of Alcor and Mizar as one star.)

Now change locations to Boston, Massachusetts. How many of the stars in the Big Dipper are circumpolar when viewed from Boston?

When Santa looks at the stars from the north pole, how do they appear to move? What percentage of the stars he sees are circumpolar? Make a prediction first, then try it.

Now switch your location to Santiago, Chile. Find the center of the circles that stars appear to move in from this location (hint: look South). What constellation is this in?

Note that there is no bright star right at the center of these circles: Polaris is the North Star, but there is no South Star.

Go back home to Richmond. Pick a star that's not circumpolar (that is, one that rises and sets). Record the time that this star rises on one day, and the time it rises on the next day. Make sure your times are accurate to the minute. How much time elapses between successive risings of the star?

Repeat for a few other stars. Is the result always the same?

The length of time between successive risings of a star is called a "sidereal day." How different is a sidereal day from an ordinary day?

Lab 5

Intensity of Solar Radiation

There are two main reasons winter is colder than summer:

1. The Sun is up for fewer hours per day in the winter.

2. The Sun is lower in the sky in winter. This means that sunlight strikes Earth's surface at a shallow angle, which lowers the intensity of the radiation striking the Earth's surface.

In this lab, you'll verify these statements. In particular, you'll find out the amount of time the Sun is up at different times of year, you'll see how high the Sun rises in the sky in summer and winter, and you'll check that radiation striking a surface at a shallow angle deposits less energy than radiation that is closer to perpendicular.

A. Amounts of sunlight at different times of year.

Start up *Stellarium*, and set the date to June 21, the first day of summer. Determine the times of sunrise and sunset on that day, by running time forward and backward and watching the position of the Sun. (Use a flat horizon.) How many hours of daylight are there on that day?

Repeat this procedure on December 21, the first day of winter.

What is the ratio of these two values? That is, how many times greater is the amount of sunlight on June 21 compared to December 21?

B. Angle made by the Sun in summer and winter.

Set the time to noon on June 21 (the first day of summer). Make *Stellarium* draw the *meridian*, which is a line in the sky going from due north, directly overhead, to due south. On any given day, the moment the Sun crosses the meridian is the moment when it's highest in the sky. At what time does the Sun crosses the meridian on this date?

Why doesn't the Sun cross the meridian at exactly noon? (There are several reasons.)

Select the Sun and view the information on it. Find the Sun's *altitude*. This is the angle indicating the Sun's height above the horizon. ($90°$ would be directly overhead, and $0°$ would be right on the horizon.) What is the Sun's maximum altitude on this date?

Repeat this procedure to find the Sun's maximum altitude on December 21, the first day of winter.

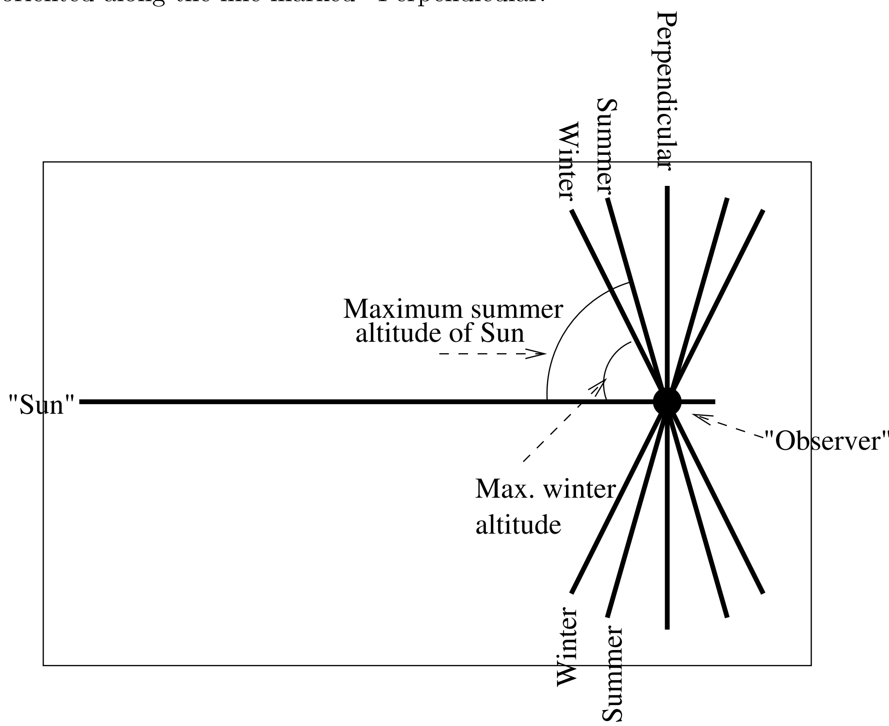
C. Effect of angle on the amount of solar radiation.

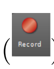
Next, you'll check to see how much of an effect this difference in angles makes to the amount of radiation striking the Earth. The light bulb on your table will represent the Sun. You will use a solar cell to represent a small piece of the Earth's surface. Your goal is to see how the amount of radiation from the bulb changes as the angle of the cell is varied. In particular, how much more radiation strikes the cell when the light is beating down on it from almost directly above (like sunlight in summer), compared to the amount when the light strikes it at a shallower angle (like sunlight in winter)?

Place the bulb at one end of the optical bench, and place the round platform with angle markings at the other end. We'll call the center of the round platform the "observer's location," and we'll call the bulb the "Sun."

Start the program called "solarcell.cap" on the lab computer. This program allows the computer to display the amount of radiation striking the solar cell.

Place the solar cell at the observer's location, facing directly towards the Sun. The surface of the cell should be perpendicular to a line going from the cell to the Sun. In the picture below, the cell should be oriented along the line marked "Perpendicular."



Turn on the light bulb, and hit the "Record" button () on the computer screen. You should see a graph showing the amount of radiation striking the solar cell as time passes. Try placing your hand in front of the solar cell to block the light. The reading should go down.

Let the computer record data for about 20-30 seconds. We want to know the average amount of light striking the solar cell. To find this out, you need to select a range of data where the graph looks reasonably flat. Hit the button that looks like this, near the top of your screen: . That'll create a rectangle that you can move to select the region of data you want to use. Once you've highlighted a stretch of data, the computer should display the average ("Mean") of those numbers. This is the average amount of radiation striking the solar cell during this time period. Record this value:

This represents the amount of power that strike the solar cell if it were lying flat on the ground, and the Sun was beating down on it from directly overhead. In fact, though, the Sun is never directly overhead (at least, not from a location like Richmond). The highest the Sun ever gets in the sky is the altitude you found for the first day of summer in part B.

Rotate the solar cell so that it the angle it makes with the line to the Sun corresponds to the altitude of the Sun on that date. (This means orienting the cell so that it lies along one of the lines marked "Summer" in the picture above.)

Repeat the procedure with the solar cell aligned on the other “summer” line.

The last two values should be close to the same. Average them together:

Repeat this procedure with the two winter lines to end up with an average amount of radiation striking the cell when it’s at the winter angle:

You should have found that the amount of radiation is greatest when the cell is perpendicular, a bit less when it’s at the summer angle, and much less when it’s at the winter angle.

How many times more intense is the midday solar radiation in summer than in winter?

Let me repeat the main point. Part A and this part show two different reasons why it’s hotter in summer: there are more hours of daylight in summer, and the sunlight is more intense.

Lab 6

Celestial Navigation

In the old days, before GPS, sailors figured out where they were by looking at the positions of the stars. In this lab, you'll see how that worked. To be specific, you'll look at the question of how to use the positions of celestial objects to determine your latitude and longitude.

Our reason for doing this is not that I expect you to find yourself lost at sea without access to GPS (although anything's possible, I guess). It's because figuring out how celestial navigation works is a good way to make sure you understand how things move in the night sky.

Latitude

As it turns out, determining your latitude (i.e., how far north or south of the equator you are) is much easier than determining your longitude. The easiest way is to observe the location of the star Polaris. As you know, Polaris is very close to the North Celestial Pole, or in other words almost directly above the Earth's North Pole. For purposes of the questions below, you can assume that Polaris is exactly at the North Celestial Pole.

Suppose you were standing at the Earth's North Pole, which is at a latitude of 90° north. In which direction would you have to look in order to see Polaris?

Now suppose you measured the *altitude* of Polaris. As we've seen, the altitude of an object is the angle that describes how far above the horizon that object is located. To be specific, draw an imaginary line from you to Polaris, and draw an imaginary horizontal line (pointing toward the horizon) directly below Polaris. The altitude means the angle between those two lines.

If you were at the north pole, what would the altitude of Polaris be?

Now suppose you were at the equator (which is at a latitude of 0°). In which direction would you have to look in order to see Polaris?

What would the altitude of Polaris be?

Based on the above considerations, you can guess that there might be a *very* simple rule relating the altitue of Polaris to the observer's latitude. What do you think that relationship is?

We could test this observation using actual observations of Polaris at night, but instead let's test it with *Stellarium*.

Start up *Stellarium*. Find the star Polaris and select it. Look at the information in the upper left, and find the star's altitude. What is the altitude of Polaris?

What is our latitude here in Richmond? (You can find this out by looking at "Location" in *Stellarium*, or I'm sure Google knows it.)

Do the two values you just found agree, at least roughly, with your expectations? (If the degrees agree but there's a discrepancy in the arc-minutes, that's close enough.)

If you're ever lost at sea, you now know how to find your latitude, by observing how high Polaris is in the sky.

Longitude

Suppose you wake up one night and find yourself in a boat tossing about in the middle of the ocean. Using the method above, you figure out your latitude. Now how are you going to find your longitude? As we'll see, this turns out to be a much harder problem than finding latitude.

Suppose your latitude comes out to be 20° north. To keep things (relatively) simple, let's suppose that there are just two possibilities for longitude: either you're at a longitude of 25° west, or you're at a longitude of 55° west.

Set the time in *Stellarium* to about 10:00 tonight, and set your location to be a latitude of 20° north and a longitude of 25° west. (You can type these numbers directly into the Location box. Just enter the number of degrees, and leave out the minutes and seconds.) Orient your view so that you're looking north, with the horizon at the bottom and a pretty large field of view (about 60° or so). Note a couple of landmarks to orient yourself in the sky. You should be able to see the Big Dipper (upside-down) and the bright star Capella, for instance.

Once you've got this set, take a screen shot by hitting control-S. Check that the screen shot was saved. (I think it end up in your Pictures folder.) Open it up and take a look at it to make sure it looks the way you expect. Once you see that the file looks right, go back to *Stellarium*.

Now change your location to 55° west longitude (keeping the latitude and the time the same). Take a screen shot of the sky from this new location.

Now advance the time by two hours, from 10:00 pm to midnight. Take one more screen shot.

At this point, you should have three screen shots.

1. Longitude: 25°. Time: 10:00 pm.
2. Longitude: 55°. Time: 10:00 pm.
3. Longitude: 55°. Time: midnight.

Incidentally, I should mention that the times in *Stellarium* are always times in our actual location (Richmond). That is, 10:00 pm means 10:00 pm Eastern time, regardless of the observer's location.

Two of these three images should look almost identical, and one should look significantly different. Which one is not like the others?

Now, let's get back to your plight as you sit bobbing in your boat in the middle of the ocean. Suppose that you have a timekeeping device (wristwatch cell phone, etc.) that is set to Eastern time. Using this, along with your observation of the night sky, can you tell which longitude you're at? If so, explain briefly how. If not, explain briefly why not.

Suppose that you *don't* have an accurate timekeeping device on board the boat. Can you tell what longitude you're at by observing the sky? If so, explain briefly how. If not, explain briefly why not.

In the 18th century, the British government offered large cash prizes for anyone who could figure out an accurate way to determine the longitude of a ship at sea. In 1765, John Harrison was awarded a £10 000 prize for solving this problem. (It's hard to figure out precise equivalents, but this is equivalent to millions of dollars today.) What do you think Harrison invented?

Back to you on your boat. Suppose that you had a clock with you on your boat, but it wasn't very accurate. Suppose that you use the clock to determine your longitude, but unbeknownst to you the clock is off by two hours. (For instance, you think it's midnight, when really it's 10:00 pm.) By how many degrees will your longitude determination be off?

Suppose that your clock were only off by one minute instead. How far off will your longitude determination be?

One degree of longitude corresponds to an actual distance of about 50 kilometers. If your clock were off by one minute of time, how far off would your determination of your location be (in kilometers)?

If you were a ship's captain trying to avoid hitting undersea rocks and shoals, this level of inaccuracy would be a real problem.

Lab 7

The Moon

In this lab, you'll check a bunch of things about the motion and phases of the Moon.

A. Sidereal and Synodic Months

A *sidereal month* is the length of time it takes the Moon to make one complete motion around the sky with respect to the stars. In other words, at the end of a sidereal month, the Moon appears in the sky next to the same stars as it did at the beginning of the month. A *synodic month* is the time it takes the Moon to go through one full cycle of phases (from full Moon to full Moon, for example). You'll use *Stellarium* to measure the length of both sidereal and synodic months.

Start up *Stellarium.* Make the ground transparent and turn off the Earth's atmosphere, so that we can see what the Moon is doing at all times. This makes our lives much easier than the real-life observers who first figured all this stuff out: they could only see the Moon when it was above the horizon at night. Again, consult the Appendix to see how to do these things, and of course ask me if you can't figure it out.

First, step forward in time until the Sun and Moon are right next to each other in the sky. This is the next time when the phase of the Moon will be new. Determine, to within an accuracy of about an hour or so, the moment when the Moon and Sun are closest to each other in the sky. Record the date and time:

Now let time run forward for a bit less than a month, until the phase of the Moon is nearly new again. Just as before, find the moment, accurate to the nearest hour, when the Moon and Sun are closest together. Record the date and time:

Based on these results, what is the length of a synodic month? Give your answer as a decimal number of days, with at least one digit after the decimal point.

Now you'll figure out the length of a sidereal month. Click on a star that's very near the Moon, and keep the field of view centered on that star. Record the date and time when that star passes closest to the Moon, accurate to the nearest hour.

Then let the time advance for a bit less than a month, until you see the marked star pass close to the Moon again. Record the time it passes closest to the Moon, accurate to the nearest hour.

Find the length of a sidereal month as a decimal number of days.

The year is 365.24 days long. How many synodic months are there in a year? (Give your answer as a decimal number with at least one digit after the decimal place).

How many sidereal months are there in a year?

You should find that the difference between these two numbers is very close to 1. (If you don't find this, let me know.) This is a general rule relating sidereal and synodic periods for all satellites. You might enjoy trying to figure out why it's true. (Then again, you might not.)

B. Phases of the Earth

Question: Suppose the Moon is in a waxing crescent phase as seen from the Earth. An astronaut on the Moon looks at the Earth. What phase of the Earth does she see?

I want you to figure out the answer to this question on your own first, and then test it with *Stellarium*.

Draw a diagram showing Earth, Sun, and Moon when the Moon is a waxing crescent. Indicate the directions of the Earth's orbit around the Sun and the Moon's orbit around the Earth.

Based on this diagram, what is the answer to the question above? (Your answer should be something like “full Earth,” “waning crescent Earth,” “waxing gibbous Earth.”)

Now test your answer. Adjust the time until the phase of the Moon is waxing crescent. Then click on the Location icon, and change the location to the Moon. Adjust the view until you see the Earth. Does its appearance agree with your prediction? (For instance, if you predicted the Earth would be in a crescent phase, is it?) Allow time to run forward. Is the Earth waxing or waning? Does this agree with your prediction?

Lab 8

Apparent Motion of the Planets

The motion of the stars in the night sky is pretty simple: they go in circles about the north celestial pole, making one complete circuit per sidereal day. The motion of the Sun and Moon are a bit more complicated: they share in the daily motion of the stars, but they gradually drift from west to east relative to them. In other words, as the stars race around the north celestial pole, the Sun and the Moon gradually fall behind.

The motion of the planets has a lot in common with the Sun and Moon, but it's a bit more complicated. One of the most important scientific advances in all of human history was Copernicus's correct explanation of why the planets seem to move the way they do. To understand what Copernicus figured out, we first have to examine how the planets appear to move in the sky.

Jupiter's daily motion. The first thing to realize is that, on any particular day, the planets seem to move in pretty much the same way as the stars: they circulate about the north celestial pole, rising in the east and setting in the west, taking about one day to go all the way around. Let's start by making sure of this.

Start up *Stellarium*. Locate the planet Jupiter. If it's not up above the horizon during the night time, shift time forward a month at a time until it is. Once you've reached a time when Jupiter is up at night, let time run rapidly for several days, and observe Jupiter's motion. You should find that, if you don't look too closely anyway, it moves in pretty much the same way as the stars it's next to.

Determine, to an accuracy of about a minute or so, the time when Jupiter sets on two successive days. How much time elapsed between these two occurrences?

If you did the same thing with a star (i.e., measured the difference between two times the star set), what would you find? You can try it if you want, but I hope you know what the answer is.

The two answers should be very similar. Depending on how carefully you measured, you may or may not have found a small difference between them. Now let's examine that small difference more carefully.

Jupiter's slow drift with respect to the stars. Over the course of a few days, Jupiter (and the other planets) seem to move in approximately the same ways as the stars, but there are small differences, which build up to become quite important over longer periods of time. To examine those differences, it helps to "turn off" the daily motion of everything. To do this, switch to the "equatorial mount." You should also turn off the effects of daylight, so that you can watch the planets at all times.

Set the time to the present, locate Jupiter, and center it in the field of view. Set time running forward *very* rapidly, so that a year goes by every few seconds. You should see Jupiter generally drift from right to left with respect to the stars, but sometimes reverse itself and drift from left to right.[1] When we look at the sky in the equatorial (stationary-sky) point of view, east is always on our left and west is on our right, so we say that Jupiter usually goes from west to east, but sometimes reverses direction and goes from east to west.

The times when Jupiter goes "backwards" (east to west) are called periods of *retrograde motion*. The next thing we want to do is look for patterns in when Jupiter goes into retrograde motion.

Set the time back to the present, and start it running forward rapidly. Find the beginning and end of the next three time periods when Jupiter is in retrograde motion. It's hard to spot the exact moment when retrograde begins or ends. You don't have to get the exact date right – just determine the month and year. To get you started, I'll tell you that Jupiter will next go into retrograde motion in about April 2019.

While you're at it, find all the times when Jupiter will be in *conjunction* with the Sun during this period (from now through the end of the third retrograde period). This means the time when it passes right next to the Sun in the sky.

List your results (three time periods when Jupiter is in retrograde and all the moments when it's in conjunction) below.

On a blank sheet of paper, make a time line covering the entire range of all these times, and the

[1] If you're keeping Jupiter centered in the field of view, of course, it won't actually move at all. When I say you'll see Jupiter go from right to left with respect to the stars, what I really mean is that you'll see the stars slip past it from left to right.

periods of retrograde and conjunction.

Summarize your results in a sentence or two.

Motion of other planets. Try the same thing with the planets Mars and Venus. To be specific, identify the next three times when the planet is in retrograde, and all the times of conjunction from now until the end of those three retrograde periods. Make a time line for each.

Which of these two planets shows a pattern very similar to Jupiter? Which is different?

Make a guess about why one of these planets is not like the others.

Lab 9

Sidereal and Synodic Periods

Part 1. An example. Imagine two runners, Ellen and Peter, running around a circular track, starting out out next to each other. Each runner runs at a constant speed, but one is faster than the other: Ellen takes 8 minutes to go around the track once, and Peter takes 10 minutes. Let's call these two times E and P respectively: $E = 8$ minutes and $P = 10$ minutes.

Eventually, Ellen will "lap" Peter – that is, she will have gone around the track one time more than him, so that she's right next to him again. I want you to figure out how much time it takes for this to happen. There are different ways to do this, but one thing that might help is to figure out where Ellen and Peter are after 10 minutes have gone by. By how much (what fraction of a lap) will Ellen will be ahead of Peter? How many times would you have to repeat this until Ellen had lapped Peter?

Your final answer should be a number of minutes. This is the amount of time it takes for the two runners to go from a particular configuration (right next to each other) to the same configuration again. If Peter and Ellen were planets, this would be called the "synodic period" of the two planets.

We're going to want a general relationship giving the synodic period in terms of the "sidereal periods" P and E. It turns out that the relationship is simplest if we express it in terms of the reciprocals of these numbers.

What are the reciprocals of the three numbers P, E, S for this situation? (Express them as decimals, not fractions.)

$\frac{1}{P} =$ $\qquad\qquad$ $\frac{1}{E} =$ $\qquad\qquad$ $\frac{1}{S} =$

Can you spot a relatively simple mathematical relationship between these three numbers? Write it down in the form of an equation involving $\frac{1}{P}$, $\frac{1}{E}$, $\frac{1}{S}$.

Part 2. The general rule. The rule you wrote down in the previous part turns out to apply no matter what P and E are. We're going to check that this is true algebraically now. In this section, you should forget the specific numerical values for P, E, S (that is, the 10 minutes, 8 minutes, etc.). We're going to work out an expression that is true for different possible values of these quantities.

As before, E stands for the time that Ellen takes to go around the track once, and P stands for the time that Peter takes. S stands for the "synodic period" – that is, the time it takes for Ellen to pull ahead of Peter by one full lap.

After a time S has gone by, how many laps has Ellen run? Your answer will be an algebraic expression involving E and S.[1]

After a time S has gone by, how many laps has Peter run? Your answer will be an expression involving S and P.

By definition, S is the time at which Ellen has run one more lap than Peter. That means that your answers to the two previous questions must differ by one. Write down that last sentence in the form of an algebraic equation.

[1] If you don't know what to do, try thinking it through with a particular number in mind. For instance, suppose that it takes Ellen 2 minutes to run a lap (so $E = 2$ minutes), and suppose $S = 20$ minutes. How many laps will Ellen have run in those 20 minutes? Write down whatever you did to get your answer, but use the letters S and E in place of 20 and 2, and you have the expression I want.

Do some algebraic manipulations on that equation until it looks like the equation you wrote down at the end of Part 1.

You're done! You've just derived the general rule relating sidereal and synodic periods. When we apply this in astronomy, E will be the sidereal period of the Earth (one year), P will be the sidereal period of another planet (the length of a year on that planet), and S will be the synodic period.

A couple of quick notes:

1. In this derivation, we assumed that Ellen was faster than Peter. That means that the formula we derived only works if the other planet has a longer period than the Earth. That turns out to be true for all of the planets further out than the Sun (Mars, Jupiter, etc.). The planets Mercury and Venus, go faster than the Earth, so the final formula looks slightly different. You could work it out by the same sort of reasoning if you wanted to, or you can look it up in your textbook.

2. In this expression, we usually give all of the periods in years, but you don't have to. You can use any unit of time you want, but it has to be the same unit for all three quantities.

3. The main reason this relationship is useful is in determining sidereal periods for other planets. After all, we know E (one year), and S is easy to measure – just observe the time it takes between two successive times the planet goes into opposition. So the most common use of this rule is to find the one remaining quantity, namely P.

Lab 10

Kepler's Third Law

Kepler's third law says that the period of a planet's orbit (the time to orbit once, called P) and the radius of its orbit (called a) are related like this:

$$P^2 = ka^3.$$

Here k is a constant. Its numerical value depends on what units we choose for P and a, but it's the same for all of the planets.

Jupiter and its moons look like a mini-solar system: the moons orbit Jupiter in pretty much the same way that the planets orbit the Sun. So it's natural to wonder whether Jupiter's moons obey Kepler's third law. In this lab, you'll use *Stellarium* to simulate observations of the moons to test Kepler's third law.

Start up *Stellarium*. Turn off the effects of Earth's atmosphere / daylight, and make the Earth transparent, so that we can see Jupiter all the time. (Once again, we're making our lives a bit easier than it is for real astronomers: in reality, we could only make the sort observations in this lab at certain times of year.) Set the program to the equatorial (stationary-sky) mount. Find Jupiter, center it in the field of view, and set the field of view to a size where you can clearly see Jupiter and its four Galilean moons (Io, Europa, Callisto, Ganymede).

Let time run forward at a pretty high speed and observe the orbits of the moons. The orbits are really very close to circular, but we're viewing them from the side, so the moons appear to move back and forth.

Pick one of the moons, and run time forwards and backwards until the moon is at its greatest separation from Jupiter. (Be as accurate as possible; you can probably determine the time to within about 10 minutes for the fastest-moving moon and less than an hour for the slowest one.) Measure the angular separation between Jupiter and the moon at this moment. Repeat the procedure for all four moons. Record your results in the table on the last page. You'll initially get the angular separation in arc-minutes and arc-seconds. Convert your answer to a number of arc-seconds. For instance, if you got a value of $2'$ and $20''$, you would record $140''$ (because $1'$ is $60''$, and $2 \times 60 + 20 = 140$)..

How far is Jupiter from us at the time these observations are being taken? Record the distance in A.U. here:

Use the small-angle formula to convert the angular separations for all four moons into actual separations. These will be the maximum distances the four moons get from Jupiter; in other words, they'll be the radii of the moons' orbits. Record them in the data table, along with their units.

Now that you know the radii of the orbits, the next step is to find the periods. The most accurate way to do this is to measure the time between moments when the moon passes in front of or behind Jupiter.

Set the date back to today, and then advance time forwards or backwards until one of the moons is just passing in front of the edge of Jupiter from the left. (Aim for an accuracy of about 5 minutes.) Record the the date and time, and also the "Modified Julian Date (MJD)." See Appendix A for details on how to do this.

Let time advance until that moon is just passing in front of the left edge of Jupiter again (one complete orbit later). Record the date, time, and MJD again.

Next, you should determine the time that elapsed between these two measurements. This is the period of the moon's orbit. Give the result as a decimal number of days. (You can do this by working out the difference between the two dates and times, but it's much easier using the MJDs. In fact, that's what MJDs are for!)

Repeat this procedure for all four moons. Each time you start working on a new Moon, set the time back to the present day.

Now that you know the period and the radius of all four moons, you can test Kepler's third law. Square all of the periods to get P^2, and cube all the radii to get a^3. Take the ratio P^2/a^3. Kepler's third law says that this should be the same for all four moons. Is it?

A couple of followup questions:

1. Is the numerical value of k for Jupiter's moons the same as the value for the planets orbiting the Sun? To answer this, figure out the value of k for the Earth's orbit around the Sun using the same units as you used for Jupiter's moons.

2. Why did we have to use the moons of Jupiter for this lab, instead of using Earth's Moon?

MOON	Io	Callisto	Ganymede	Europa
Max. angular separation from Jupiter ($'$ and $''$)				
Max. angular separation from Jupiter ($''$)				
Radius of orbit				
Date/time of first crossing of Jupiter				
MJD of first crossing				
Time of second crossing of Jupiter				
MJD of second crossing				
Orbital period				
P^2				
a^3				
k				

Lab 11

Phases of Venus

One of the most important things Galileo observed with his telescope is that Venus has phases like the Moon. This was the “smoking gun” that showed that the earth-centered (pre-Copernicus) system couldn’t possibly be right, so it went a long way toward convincing people that the Earth really did go around the Sun. We’re going to examine the phases of Venus to see what Galileo saw and why it mattered.

Start up *Stellarium*. Turn off the effects of Earth’s atmosphere (daylight) and make the ground transparent, so that it’s possible to see the stars and planets all the time. Use the equatorial (stationary-sky) point of view. Set the view to be centered on the Sun.

Keep the view centered on the Sun, and let time run forwards and backwards at high speed for a year or two. Observe motion of Venus. Note that it swings back and forth past the Sun, never getting more than a certain angular separation from the Sun in the sky.

Here’s one preliminary question before we look at the phases of Venus. If you wanted to observe Venus on September 15, 2020. At what time of night should you observe? Specifically, find a rough time when Venus is above the horizon but the Sun is below the horizon. In order to answer this question, you will probably find it useful to make the ground visible (as opposed to transparent).

What if you wanted to observe Venus on February 1, 2022? When would be the best time to see it?

What about in July 1, 2021?

If you had to choose one of those three dates on which to observe Venus, which one would be best? Which would be worst?

Venus spends quite a bit of its time very near the Sun in the sky, at which times it's hard to observe. The easiest time to observe Venus is when its as far away from the Sun in the sky as possible. This is called the time of "maximum elongation." Set the time back to today, and run time forwards until you find the next time of maximum elongation. (Don't worry about being incredibly precise; if you're within about a week or so of the correct date, that's fine.) Find the angular separation between Venus and the Sun at that time.

Maximum angular separation between Sun and Venus:

Here's a diagram showing the Earth's and Venus's orbits around the Sun. Suppose that the Earth is at the uppermost point in its orbit as shown, and suppose that Venus is at maximum elongation from the Sun (so that it appears as far from the Sun as possible in the sky). Draw a circle to mark the position of Venus in its orbit. (There are actually two possibilities. One corresponds to the case where Venus is to the East of the Sun and one to the case where it's to the West. If you assume everything orbits counterclockwise, you can figure out which is which, although it's a bit tricky. For the moment, it doesn't matter which one you choose.)

Venus, like the Moon, shines by reflected sunlight. That means that the only part of Venus we'll see is the part that's illuminated by the Sun. In the diagram above, shade in the half of Venus that's lit up by the Sun, and then use the diagram to predict the phase of Venus at this time. (That is, will Venus appear like a crescent, like a "full" Venus, or what?)

After you've made your prediction, use *Stellarium* to test it. Center the view on Venus and zoom in to enlarge the image of Venus. Does it have the phase you predicted?

Now set the date to July 15, 2019, and zoom in on Venus to observe its phase. What is the phase of Venus on that date?

Based on your observation, is Venus in front of the Sun or behind the Sun on that date?

Zoom back out again, and center the field of view on the Sun. Let time run forward until Venus has gone through about half of one orbit. You should see it pass by the Sun, go out to maximum elongation, and then come back until it approaches near the Sun again. At this point, when Venus's path is about to cross past the Sun again, is Venus in front of the Sun or behind the Sun? (Don't use the program to answer this; use what you know about the orbits.)

Based on your answer to the previous question, what do you expect the phase of Venus to be?

What do you expect about the angular size of Venus: should it be bigger or smaller than the last time you looked at it?

Now use *Stellarium* to test these predictions. Center the view on Venus and zoom in. Were your predictions right?

Keep the field of view centerd on Venus and zoomed all the way in. Let time run forward fast for a year or two. The main things to note are that Venus goes through a full set of phases (from crescent to full and back), and that its angular size changes along with its phase.

Earlier, I said that this observation provided strong proof that the old Earth-centered theory was wrong. Let's see why. Remember that in the old theory the Earth was at the center, the Sun went around the Earth, and Venus moved on an epicycle between the Earth and the Sun like this:

At each of the points A,B,C,D in the diagram, what would the phase of Venus be, according to this model?

A:

B:

C:

D:

What phases of Venus would *never* occur in the earth-centered model?

The fact that Venus is observed to have those phases is the "smoking gun" I referred to.

One more thing. Now that you've measured the maximum elongation of Venus, you can use it to figure out the radius of Venus's orbit. Sketch a picture showing Earth, Sun, and Venus at the moment of maximum elongation (this will look the same as the diagram on the second page of this lab).

Connect these three bodies with straight lines to form a triangle. When Venus is at maximum elongation, this triangle will have a right angle at Venus. The angle at the Earth is the angular separation between Venus and Sun (that is, the angle you determined on the second page). Indicate both of those angles on the diagram.

We also know the length of one side of the triange: the distance from the Earth to the Sun is 1 AU. Mark this on your triangle as well.

Now you have a right triangle, and you know one angle (other than the right angle) and one side. That means you can use trigonometry to find the lengths of the other sides. Determine the radius of Venus's orbit trigonometrically. (If you don't remember how to do this, ask me.)

Lab 12

Finding Masses with Kepler's Third Law

Newton's version of Kepler's third law goes like this:

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

Here P is the period of an orbiting body, a is the semimajor axis of the orbit, and m_1 and m_2 are the masses of the central body and the orbiting body. G is a constant whose value is

$$G = 6.67 \times 10^{-11}\,\mathrm{m}^3/(\mathrm{kg\,s}^2).$$

All the stuff after the 10^{-11} is just the units of G. It signifies that we're using seconds for our times, kilograms for our masses, and meters for our distances.

One incredibly useful thing about this form of Kepler's third law is that it lets you determine the masses of things. In fact, most of the time, if we know the mass of an astronomical object, we figured it out using this law.

The point of this little "lab" is to write this law in a convenient form for determining masses and then use it to figure out masses of some astronomical objects.

The first thing to realize is that usually the mass of the central body (m_1) is much, much larger than the mass of the orbiting body (m_2). That means that, to an excellent approximation, we can ignore m_2 in Kepler's third law and just write it

$$P^2 = \frac{4\pi^2}{Gm_1} a^3.$$

Since G is a universal constant (always the same known value), we can use this to determine the mass of any object, as long as we can measure the period and radius of the orbit of something that's going around that object.

Use this formula to figure out the mass of the Sun in kilograms. To do this, consider the orbit of the Earth around the Sun. What is P? What is a? (Remember that the value of G given above requires these quantities to be in seconds and meters.)

When using Kepler's third law, astronomers usually don't use kilograms, meters, and seconds. Instead, they usually measure periods in years, lengths in A.U., and masses in units of the Sun's mass ("solar masses"). In these units, the arithmetic becomes a bit simpler.

Consider the Earth's orbit around the Sun again. In these new units, what are P, a, and m_1? (Hint: you don't need your calculator for this step!)

In these units, what is the numerical value of the constant $\frac{4\pi^2}{G}$? (Plug the values you just wrote down into Kepler's law to see what happens.)

Remember that G is a universal constant, as are 4 and π. So the combination $\frac{4\pi^2}{G}$ will always have this simple numerical value, no matter what orbiting system we're considering (as long as we're using units of A.U., solar masses, and years).

For instance, let's use Kepler's third law to determine the mass of the Earth. Remember that the Moon's orbital period is 27.2 days. What is this in years?

The radius of the Moon's orbit is 384,000 km. What is this in A.U.?

Use these values to determine the Earth's mass in units of solar masses.

Earlier, you determined how many kilograms there are in a solar mass. Use this value to convert your answer for Earth's mass into kilograms.

The solar system orbits the center of the Milky Way Galaxy in a circular orbit with a radius of 8.5 kiloparsecs (kpc). How many meters is this?

How many A.U. is it?

The solar system's speed in its orbit around the Galaxy is about 220 km/s. How much time does it take the Sun to make one complete orbit? (Hint: how far does it have to travel during one orbit?) Give your answer in seconds and in years.

Using your values for the period and radius of the solar system's orbit, determine the mass of the Milky Way Galaxy in solar masses.

People have done surveys of all the visible material in the Milky Way Galaxy (stars, gas, and dust). They estimate that all the visible stuff has a combined mass of about 1.0×10^{10} solar masses. What percentage of the total mass of the Galaxy is visible stuff? What percentage is stuff we don't see? (The latter stuff is called "dark matter.")

Lab 13

Spectra of Light Sources

In this lab you will use a small portable spectroscope to examine several sources of light. The spectroscope consists of a slit at one end that lets light through and a "grating" at the other end that splits the light up into the various colors. We'll worry a bit later about exactly how the grating works.

To set up the spectroscope, hold it so that the slit is vertical. Point it at a bright light source and look through the other end. Rotate the grating end (keeping the slit vertical) until you see horizontal bands of color on either side of the slit. These are the colors that make up the light coming through the slit.

Examine the following light sources with the spectroscope:

1. The white light coming from the overhead projector.
2. The light from a candle flame.
3. The light from one of the two gas discharge lamps.
4. The light from the other gas discharge lamps.
5. The light from the fluorescent lights in the hallway.
6. The light from the Bunsen burner flame, while someone is placing a small amount of the white crystalline stuff in it.

Sketch the spectra of all the light sources, then answer the following questions:.

1. Which sources have "continuous" spectra, meaning that the light consists of all colors from red to blue?

2. For the sources with continuous spectra, can you notice a difference in the relative amounts of the various colors? For instance, does one have more red light relative to yellow or blue, as compared with the other? (This may be hard to discern, because the total intensity of the sources is different.)

3. Using a chart of spectra that I'll give you, try to determine what gases are in the two discharge tubes. Also try to determine what's in the white powder.

Lab 14

The Diffraction Grating

All sorts of waves, including light, do a couple of strange things:

1. Waves bend (a bit) around corners, and spread out (a bit) when passing through holes. This phenomenon is called *diffraction.*

2. When two waves meet each other, they can cancel each other out (if "peaks" of the first wave run into "valleys" from the second), or they can reinforce each other (if peaks meet up with peaks). This is called *interference.*

A diffraction grating is a device that takes advantage of both of these phenomena. They turn out to be very useful in measuring the wavelength of light and in splitting light up into its constituent wavelengths.

A diffraction grating is just a piece of glass (or something similar) with many thin parallel lines ruled on it. When light passes through the small gaps between these lines, it diffracts. Light that passed through the various gaps can then interfere with each other. In some directions, the waves that passed through the various gaps cancel each other out, and in other directions they reinforce each other. The result is a pattern of alternating bright and dark patches.

Suppose we take light with wavelength λ and shine it through a diffraction grating. We then project the light on a wall and measure the positions x of the bright spots (relative to the center of the diffraction pattern) as shown. The theory of interference tells us that

$$n\lambda = \frac{dx}{\sqrt{L^2 + x^2}}.$$

Here d is the separation between lines in the grating and L is the distance from the grating to the wall. n is called the "order" of the spot; it's just an integer: $n = 1, 2, 3, \ldots$.

Note that there are four different quantities in this expression that have units of length. One common source of error is to use incompatible units for these things. The safest thing is to use the same units for all four.

One nice thing we can do with all this is measure the wavelength of light.

1. Record the separation between grating lines: $d =$

2. Turn on the laser, being careful to avoid looking directly into the beam or shining it at anyone. Aim the light beam through the diffraction grating so that a horizontal series of dots appears on the wall. Adjust the positions of the laser and grating until you easily see at least two dots on either side of the brightest (central) dot.

3. Are the dots of the interference/diffraction pattern the same intensity? Describe the pattern you observe.

4. Measure the distance from the grating to the wall, L, as well as the distances from the central dot to the first dot to the right, x, and the first dot to the left, x'. Compute the average of these x values and record it as x_{ave}. Record all of these values in the table below.

5. Use the equation above to find the wavelength λ. Because these are the first two spots (other than the central dot), use the value $n = 1$.

6. Repeat the procedure using the second dot to the right of center and the second dot to the left. This time, use $n = 2$ when finding the wavelength.

7. If you can see the third pair of dots clearly, repeat the procedure with $n = 3$.

8. Move the laser to a new location (that is, change the value of L) and repeat the procedure for a couple of values of n. In the end, you should end up with at least five different determinations of λ.

9. Compute the average of your determinations of the laser light's wavelength and compare it to the expected value, which is printed on the laser.

L (cm)	x (cm)	x' (cm)	x_{ave} (cm)	λ (nm)

Suppose that we shone white light instead of laser light through the grating. What would the resulting pattern look like?

Lab 15

The Spectrum of Hydrogen

In this lab, you will use a spectrometer to measure the wavelengths of the spectral lines of hydrogen. The spectrometer consists of a slit through which light passes, a diffraction grating, and a small telescope to focus the light that comes out of the grating. The telescope can be rotated in a circle centered on the diffraction grating. By measuring the angles at which the spectral lines appear in the telescope, you can determine the wavelengths of the lines.

The relationship between the angle and the wavelength is

$$d \sin\theta = n\lambda.$$

Here d is the spacing between lines on the diffraction grating, and θ is the angle through which the light from a spectral line is bent by the grating. In this spectrometer, we are always seeing the "first-order" diffraction pattern, so $n = 1$. If you don't know or don't remember your trigonometry, don't worry about it: all you need to know for this lab is how to make your calculator tell you the sine of an angle.

(Incidentally, this formula is essentially the same as the formula you used in an earlier diffraction lab, although that one was written in non-trigonometric form.)

The first thing you'll need to know is d, the spacing between lines on the grating. The grating should be labeled with a number of lines per inch or a number of lines per millimeter. Use this to determine the distance between lines in either meters or centimeters. (If your grating is labeled in lines per inch, it may help you to know that one inch is 2.54 cm.)

Now that you know d, you can determine the wavelength of any spectral line just by measuring the angle θ through which the light is bent.

Set up the spectrometer with the hydrogen lamp right against the slit. Put the grating at the center, so that it is oriented perpendicular to the direction from the slit. Look through the telescope, and gradually move the telescope around until you see the spectral lines from the lamp. You don't want the bright spot that you see when the telescope is lined up pointing straight at the lamp; the spectral lines you're interested in are visible when it's off to the side.

Hydrogen has four spectral lines in the visible part of the spectrum. You should be able to see three of them clearly. The fourth one is fainter, but you may be able to see it.

For each of the three brightest spectral lines, determine the wavelength as follows.

1. Adjust the spectrometer so that each spectral line is centered in the telescope, and measure the angle θ corresponding to each spectral line on both the left and the right. Note that the angles marked on the spectrometer have 180° for light that is not deflected at all. We'd rather call that angle 0° instead of 180°, so you should subtract 180° from your angles.

   Measure the angles as accurately as possible. Note that there is a "Vernier scale" allowing you to measure an extra decimal place.

2. For each of the three lines, average the two measurements of θ together, and use the resulting value to determine λ.

3. For each of the spectral lines, record the energy of a photon, using the rule

$$E = \frac{hc}{\lambda}.$$

   Planck's constant has the value $h = 4.135 \times 10^{-15}$ eV s, and the speed of light is $c = 3.00 \times 10^8$ m/s. If you use these values, and if the wavelength is measured in meters, your energy will come out in units of electron volts (eV).

Line	θ_{left}	θ_{right}	$\theta_{average}$	Wavelength	Photon Energy
Line 1 (reddest)					
Line 2					
Line 3 (bluest)					

It turns out that there is a pattern in the energy levels of hydrogen. The energy levels are of the form

$$\begin{aligned} E_1 &= -\frac{A}{1^2}, \\ E_2 &= -\frac{A}{2^2}, \\ E_3 &= -\frac{A}{3^2}, \\ &\cdots \end{aligned}$$

Here A is a constant, whose value you are to determine.

The longest-wavelength visible spectral line of hydrogen (the red one) occurs when an atom jumps from energy level $n = 3$ down to $n = 2$. So the energy of one of those red photons should be

$$E_3 - E_2 = -\frac{A}{3^2} - \left(-\frac{A}{2^2}\right) = -\frac{A}{9} + \frac{A}{4}.$$

Set this expression equal to the energy you determined for the red spectral line, and solve for the value of the constant A. Show me the result when you're done.

Once you know the value of A, use it to determine the energy of a photon emitted when a hydrogen atom jumps from level 4 to level 2:

From level 5 to level 2:

These should look similar to the energies you meaured for the other two spectral lines. If they don't, talk to me.

What is the energy of a photon corresponding to the next spectral line (a jump from level 6 to level 2)? What is the wavelength of this spectral line?

Using the formula $d \sin\theta = \lambda$, predict the value of θ at which this spectral line should appear in the spectrometer. [Note to the trig-averse: to do this, you'll need to use the "inverse sine" (or "arcsine" or "$\sin^{-1}$") function on your calculator.] Now that you know where to look, see if you can see this spectral line in the spectrometer.

One final question: Why do all of these spectral lines correspond to jumps down to the second energy level? Why not the first or third, for instance?

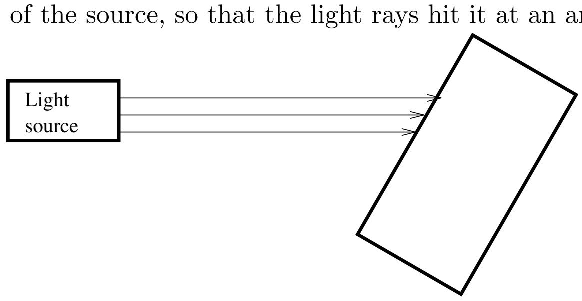
Lab 16

Refraction and Lenses

Over the next few classes, we're going to examine how telescopes work, but first we need to figure out some aspects of the behavior of light when it passes through transparent materials such as glass.

The apparatus for this lab consists of a light source that produces a set of parallel beams from lasers, along with a variety of transparent and reflective objects for the light to interact with. All of these objects can be attached magnetically to a small whiteboard, so that they can be easily positioned in various ways.

Part 1: Refraction. Plug in the light source and attach it to the whiteboard. Put the clear plastic rectangle in front of the source, so that the light rays hit it at an angle like this:



Put a piece of paper under the rectangle. Trace the outline of the rectangle on the paper. Also, trace the path of one of the rays as it enters the rectangle, and the same ray as it exits the other side. Use a straightedge to make sure your lines are straight. Then remove the paper, and draw a straight line connecting the points where the ray entered and exited the rectangle. You should end up with a picture like this:

The light ray bends when it enters the plastic. This phenomenon is called *refraction*. There is a rule called Snell's Law that describes the amount of bending:

$$\sin\theta_i = n\sin\theta_r.$$

In this law, n is a constant called the *index of refraction* of the material. The angles θ_i and θ_r are called the angle of incidence and angle of refraction. They're defined to be the angles the light ray makes with a line perpendicular to the surface where the light ray entered the material, as shown here:

It's easier to measure the angles that the light ray makes with the *edge* of the rectangle, instead of the angle that it makes with the *perpendicular line*. So measure the two angles indicated by the arrows in the picture, and use them to determine the angles θ_i and θ_r.

Now use these angles in Snell's Law (the equation above) to determine the value of the index of refraction n.

The value of n is supposed to be the same no matter what incident angle you choose. To test this, rotate the rectangle so that the light rays hit it at a different angle. Repeat the procedure above to determine θ_i, θ_r, and n. Do you get roughly the same value of n as before?

Part 2: Total internal reflection. Once the light ray has entered the surface, it has to bend again in order to get back out. (That's why the rays that leave the rectangle end up parallel to the ones that go in.) Sometimes the rays can't bend enough to make it back out, and in this case they are reflected, bouncing back and forth inside the material. To see this, take the long, thin rectangular piece of plastic, and arrange it so that one of the light rays enters at a slight angle like this:

If you adjust the angles right, you should be able to see the light ray bounce back and forth from side to side inside the plastic. Using total internal reflection, light signals can be transmitted over very great distances through fiber optic cables.

Part 3: Lenses. A curved piece of refracting material can act as a lens, bringing light to a sharp focus. Lenses are found in your eyes, as well as corrective lenses (eyeglasses or contact lenses), cameras, microscopes, etc. For this course, of course, the most important application of lenses is in telescopes. We'll examine how telescopes work in future classes. For the moment, let's just figure out some general properties of lenses.

Take one of the three lenses labeled 1,2,3, and place it in front of the light source. You should see the light rays bending to come (approximately) to a *focal point* on the far side of the lens. The *focal length* of a lens is defined to be the distance from the center of the lens to the focal point. Determine the focal lengths of all three lenses. (The easiest way to do this is to mark the focal point on the white board, and also to mark the edges of the lens. Then you can remove the lens and measure the distance from the focal point to the middle of the lens.)

These three lenses are all called *converging* lenses, because they cause the light to converge to a point. There are also *diverging* lenses, which generally have concave rather than convex surfaces. Put the lens labeled 5, which is a diverging lens, in front of the light source and see what it does. A diverging lens causes the light rays to spread out *as if* all the rays were coming from a focal point on the same side of the lens as the light source.

To see how this works, trace the paths of several of the rays after they exit the lens. Also, mark the positions of the edges of the lens. Now remove the lens and, using a straightedge, extend the rays *backwards* until they come together at a single point. The distance from the center of the lens to this focal point is called the focal length of the lens. Measure the focal length of this diverging lens.

Part 4: Combinations of lenses. Put one of the converging lenses (1,2,3) in front of the light source, and then put another one of the converging lenses right in front of the first one. The two lenses should behave approximately like a single lens. Is it a converging or a diverging lens? How does the focal length of the combined lenses compare with the focal length of the original lenses? (I'm not looking for a measurement here, just a general observation.)

Try the same thing with a combination of a converging and a diverging lens. Does the combination behave like a converging lens or a diverging lens?

Remove the other lenses and place the lens labeled 4 in front of the light source. Is this a converging lens or a diverging lens? Is its focal length larger or smaller than those of lenses 1,2,3,5?

Place the diverging lens (5) right in front of lens 4. Does the combination of lenses 4 and 5 behave like a converging lens or a diverging lens?

By now, you should have seen that sometimes a combination of a converging and a diverging lens acts like a converging lens, and sometimes it acts like a diverging lens. There is a general rule for predicting, for any given pair of lenses, which way the combination of the two will behave. Can you guess what this general rule might be?

Lab 17

Image formation by lenses

In this lab, you will examine how lenses form images. In particular, you will test the relationship between the location of the object whose image is being formed and the location of the image. Theoretically, we expect that relationship to be

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

Here p is the *object distance* (distance from object to lens), q is the *image distance* (distance from image to lens, and f is the *focal length* of the lens.

Part 1: Focal Length of a Lens.

1. Arrange the light source appartus to produce parallel rays of light across the surface of a piece of paper.

2. Place a convex lens in front of the light rays. Mark the lens position on a piece of paper, and trace the path of the rays.

3. What is the distance from the center of the lens to the focal point? This is the focal length of the lens.

Part 2: Image Formation

1. Attach the light source to one end of the optical bench, and place the lens partway down the bench. Arrange the light source so that the side with a circle and arrow drawn on it is facing the lens. This will be the "object" whose image we will be examining with the lens.

2. Measure the length of the arrow. We will call this h_o, meaning "the height of the object."

3. Adjust the lens and an index card until you see a clear, in-focus image of the object on the card. Measure the height of the image h_i, the distance from lens to object p, and the distance from lens to image q. Record them in the table below.

4. Move the lens about 5 cm toward or away from the object, and move the card until you get a clear image again. Measure h_i, p, q again. Repeat until you have five sets of measurements. For some, the image should be closer to the lens than the object ($q < p$) and some should be the other way around ($p < q$). (Note: make sure that the object is always further away from the lens than the focal length.)

p	q	h_i	$\frac{h_i}{h_0}$	$\frac{q}{p}$	f

5. For each observation, calculate and record the ratio of image and object heights, h_i/h_o, and the ratio of image and object distances, q/p. Record these in the table. What can you conclude about these quantities?

6. For each observation, use the formula at the beginning of this lab to calculate the focal length of the lens, and record it in the last column of the table. Do these results appear consistent with your measurement of the focal length in part 1?

7. Move the lens closer to the object, so that the object distance is less than the focal length. Note that there is nowhere you can place the card to get a clear image. Look through the lens at the object. Does the image appear to be larger or smaller than the actual size of the object? Does it appear to be closer or further away?

8. Suppose the object is extremely far away from the lens. Based on your results, where would you expect the image to form? (In other words, about what would you expect q to be?) Would you expect the image to be large or small?

9. If the Sun is shining, take the lens outside along with a piece of white paper, and try to form an image of the Sun on the paper. Are the results consistent with your predictions?

Lab 18

Refracting Telescopes

Part 1: Theory

Suppose that you have a telescope made of two lenses. The "objective" lens has a focal length of 10 centimeters, and the "eyepiece" lens has a focal length of 1 centimeter. A refracting telescope is constructed so that the two lenses are separated from each other by a distance equal to the sum of their focal lengths (11 cm in this case). That way, one of the focal points of the objective matches up with one of the focal points of the eyepiece The setup is something like this:

(Note that this picture is not to scale.) The objective lens is on the left, and the eyepiece is on the right. The observer looks through the eyepiece (not surprisingly) at a faraway object off to the left.

Suppose that you are using this telescope to look at an object that is 100 meters (10^4 centimeters) away. We will work out where the image of this object is produced, and how much it appears to be magnified.

The key facts that we'll need to do this are the rules you checked in your last lab. First, there's the relationship between the object position, the image position, and the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \tag{18.1}$$

Second, there's the rule for the *linear magnification* of an image:

$$\frac{h_i}{h_o} = \frac{q}{p}.$$

Here h_i is the height of the image produced by the telescope, and h_o is the height of the actual object. The ratio of these two is called the linear magnification M, so we'll write simply

$$M = \frac{q}{p}. \tag{18.2}$$

[Note: Often the expression for the magnification is written with a minus sign in it: $M = -\frac{q}{p}$. It doesn't really matter for our purposes, though.]

In the calculations you're about to do, you'll be using known values of p and f and trying to find q. So to make life simpler, take equation (1) and solve it for q in terms of f and p. (That is, rearrange it so that it says $q = \ldots$.) If you simplify this expression as much as you can, you'll be less likely to make mistakes later. (A good strategy is to isolate $\frac{1}{q}$ on one side of the equation, then put the stuff on the other side over a common denominator, and finally take the reciprocal.) If you're not sure whether you've got the right expression, show it to me.

Now suppose we have an object that is 100 m to the left of the telescope above (so $p = 10^4$ cm). The light passes through the objective lens and forms an image (ignore the eyepiece for now). Where is this image? Specifically, how far is it from the objective lens? Also, is it a real image or a virtual image? When figuring out the location of the image, record an answer in centimeters with at least three digits after the decimal point. (In other words, record your answer to an accuracy of 0.001 cm.)

Mark the approximate location of this image with an X on the diagram above. How far is this image from the eyepiece lens? Again, record your answer to an accuracy of 0.001 cm.

Calculate the linear magnification of this image using equation (2). This should be a number between 0 and 1. It indicates the factor by which the image is smaller than the original object.

Now we want to figure out the effect of the eyepiece lens. The big idea here is that we treat the *image* produced by the objective lens as if it were the *object* for the eyepiece lens. That is, we pretend that the image is a real thing that the eyepiece is examining. That means that the answer to the previous question (distance from the image to the eyepiece lens) becomes the new *object distance p*.

Is the image produced by the eyepiece a real image or a virtual image?

How far away is the image produced by the eyepiece? (When you calculate this, your answer for q will come out negative. That's OK: you can ignore the minus sign. It just indicates that the image is virtual.)

Now you know where the image will appear to be when you look through the eyepiece. Calculate the ratio of the distance to the original object to the distance to the final image. This is how many times closer the telescope makes the object appear.

Use equation (1) to calculate the linear magnification of the image produced by the eyepiece. This should be a number greater than 1. It indicates the factor by which the eyepiece has magnified the image.

The *overall linear magnification* of the telescope is the product of the magnification produced by the objective lens and the magnification produced by the eyepiece lens. What is the overall linear magnification of this telescope?

This number tells you how many times larger or smaller the final image is than the original object. You should have found that this number is less than 1, which means that the final image is actually *smaller* than the object. That may seem surprising, since we expect a telescope to make things look bigger, not smaller. What's going on here?

The point is that this linear magnification tells you how the actual size of the image and object compare. If you want to know how big an object *looks*, you need to consider their angular sizes.

The ratio of the angular size of the final image to the angular size of the original object is called the *angular magnification*. What is the angular magnification of this telescope?

Hint: The angular size depends on both the actual size and the distance. You've already worked out what effect the telescope has on the distance of the image (as compared to the object), and what effect it has on the size of the image (as compared to the object).

There is a general rule that says that the angular magnification of a telescope is equal to the ratio of the focal lengths $f_{\text{objective}}/f_{\text{eyepiece}}$. Is that true in this case?

Part 2: Constructing a Telescope

In this part of the lab, you will use two lenses and some cardboard tubes to construct a telescope.

Determine the focal lengths of the two lenses, by any means you can think of.

According to the rule at the end of the last part, the angular magnification of a telescope is the ratio of the focal lengths. What is the angular magnification of a telescope constructed with these lenses?

Insert the lenses into the tubes and adjust the distance between the lenses until the telescope gives a sharply-focused image of faraway objects.

What is the orientation of an object as seen in the telescope? Are they upside-down? Are they reversed left-to-right?

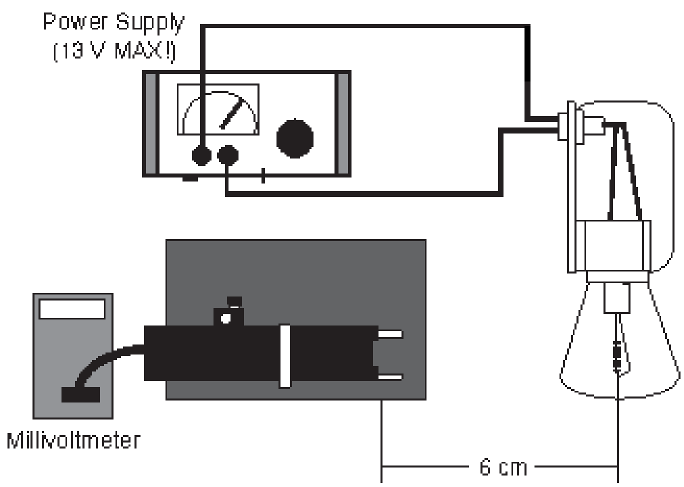
Stand in a location where you can see the eye chart posted on the wall. Adjust your distance from the eye chart until you can just barely read the F at the top of the chart (without using the telescope). Now look at the chart with the telescope. Which row is the smallest row you can read using the telescope? How many times smaller is this row than the F at the top? (The sizes of the letters in each row are printed at the right of the chart.) Compare this ratio with the angular magnification you calculated for the telescope. Do the results make sense?

Lab 19

Stefan-Boltzmann Law

Introduction. Stars (and other astronomical objects) emit a lot of their energy in the form of *thermal radiation*. To understand how these objects work, we need to know how much energy they emit. In this lab, you'll examine the relationship between the amount of *power* (that is, energy per time) emitted by an object and the object's *temperature*.

The apparatus for this experiment consists of a lamp and a radiation sensor. The lamp will be hooked up to a power supply, and the radiation sensor is attached to a meter that indicates the intensity of light striking it.



By changing the voltage being applied to the lamp, we can change its temperature. The radiation sensor then tells us how much power is being emitted at any given temperature.

We're going to want to know how the power depends on the temperature of the lamp filament. We can't conveniently attach a thermometer to the filament, so we have to get the temperature another way, namely by measuring the filament's *resistance*, which depends on the temperature in a known way. Here's how this works.

Suppose we apply a certain number of volts V to the lamp. That voltage will cause electric current to flow through the lamp. We'll measure the amount of current, which is traditionally called I (for some reason). The resistance is the ratio of these two:

$$R = \frac{V}{I}.$$

The value of R increases in a known way as the temperature goes up, so by keeping track of R we'll be able to keep track of the temperature.

Gathering the data. The first thing you'll need to know is the resistance R of the filament when it's at room temperature. We'll do this by applying a small voltage to the lamp, and recording the amount of current that flows. If we keep the voltage small, we can assume that the lamp won't heat up very much, so we can use these values to figure out the resistance at room temperature.

Connect the lamp to the power supply as shown in the diagram. Turn the voltage knob all the way to zero (counterclockwise), and turn the current knob up (clockwise). Now turn on the power supply. The voltage display on the power supply should read zero. Turn up the voltage knob gradually until it reads about 0.5 volts.

At this point, the power supply is supplying a small amount of energy to push a small amount of current through the lamp. It's not enough to heat up the filament significantly or to make the lamp glow. Record the voltage (V) and current (I) readings on the power supply here. The voltage should be 0.5 V, and the current is some number of "amperes" (A).

$$V = \qquad\qquad\qquad I =$$

Use the rule $R = \frac{V}{I}$ to determine the resistance. The unit of resistance is called the "ohm" and is written Ω.

$$R_{300\,\mathrm{K}} =$$

You might wonder why we call it by the strange name $R_{300\,\mathrm{K}}$. The reason is that this is the value of the resistance at room temperature, which corresponds to a temperature of about 300 kelvin.

Now that you have this, you can start taking measurements when the lamp is actually glowing. First, make sure that the lamp is at the same height as the radiation detector and the distance between the two is about 6 cm. The exact distance doesn't matter, but it's important that it not change once you start measuring. Make sure that the detector is facing directly toward the lamp and there are no other bright light sources in its path. The radiation sensor should be hooked up to the voltmeter's inputs labeled COMMON and V/Ω, and it should be set to read DC voltage. That setting on the meter's dial will probably look like a V with straight lines (not a wavy line) next to it or above it. Check with me to see if you've got everything wired up correctly.

Once you're ready, turn up the voltage dial gradually until it reads 2 volts. You should see the lamp glowing faintly.

In the data table at the end of this lab, record the voltage V (which is 2 V), the current I from the display on the power supply, and the reading on the voltmeter attached to the radiation sensor. The last number is a measure of the power being radiated by the lamp, so we'll call it P. Record these values in the first row of the table on the last page of this lab. (Leave the other columns in this table blank for now.)

Once you've done this, repeat for voltages of 4, 6, 8, 10, 12 V.

You should expose the radiation sensor to the lamp light only briefly when you're making each power measurement. In between eaurements, place sheets of insulating foam between the lamp and the sensor, with the silvered surface facing the lamp, so that the temperature of the radiation sensor stays fairly constant.

Analyzing the data. You'll need to determine the temperature of the filament during each of these measurements, by filling in the remaining columns in the data table. Here's how.

First, find the value of the resistance for each row, using the rule $R = \frac{V}{I}$.

The temperature is determined by how this resistance compares with its value at room temperature, so compute the ratio $\frac{R}{R_{300\,\mathrm{K}}}$ for each row.

The graph near the end of the lab shows how this ratio is related to temperature. For each row in the table, use the graph to determine the value of temperature corresponding to the resistance ratio you've determined. (Read across the graph at the height corresponding to the ratio. When you hit the curve, go straight down to get the temperature.)

Once you've done this, the data table should be filled in. Now we want to graph it, using *Excel*. Appendix B of your lab manual contains some information about making graphs in *Excel*. Create an *Excel* data table with two columns, T and P. (Put T on the left.) Following the instructions in Appendix B.2, make a graph showing how P depends on T.

You should find that the power goes up as the temperature goes up.

We want to see what sort of mathematical relationship matches this graph. Try adding a "trendline" to the chart, following the instructions in Appendix B.3. This shows the straight line that matches the data as well as possible. Does a straight line look like a good fit to this set of data?

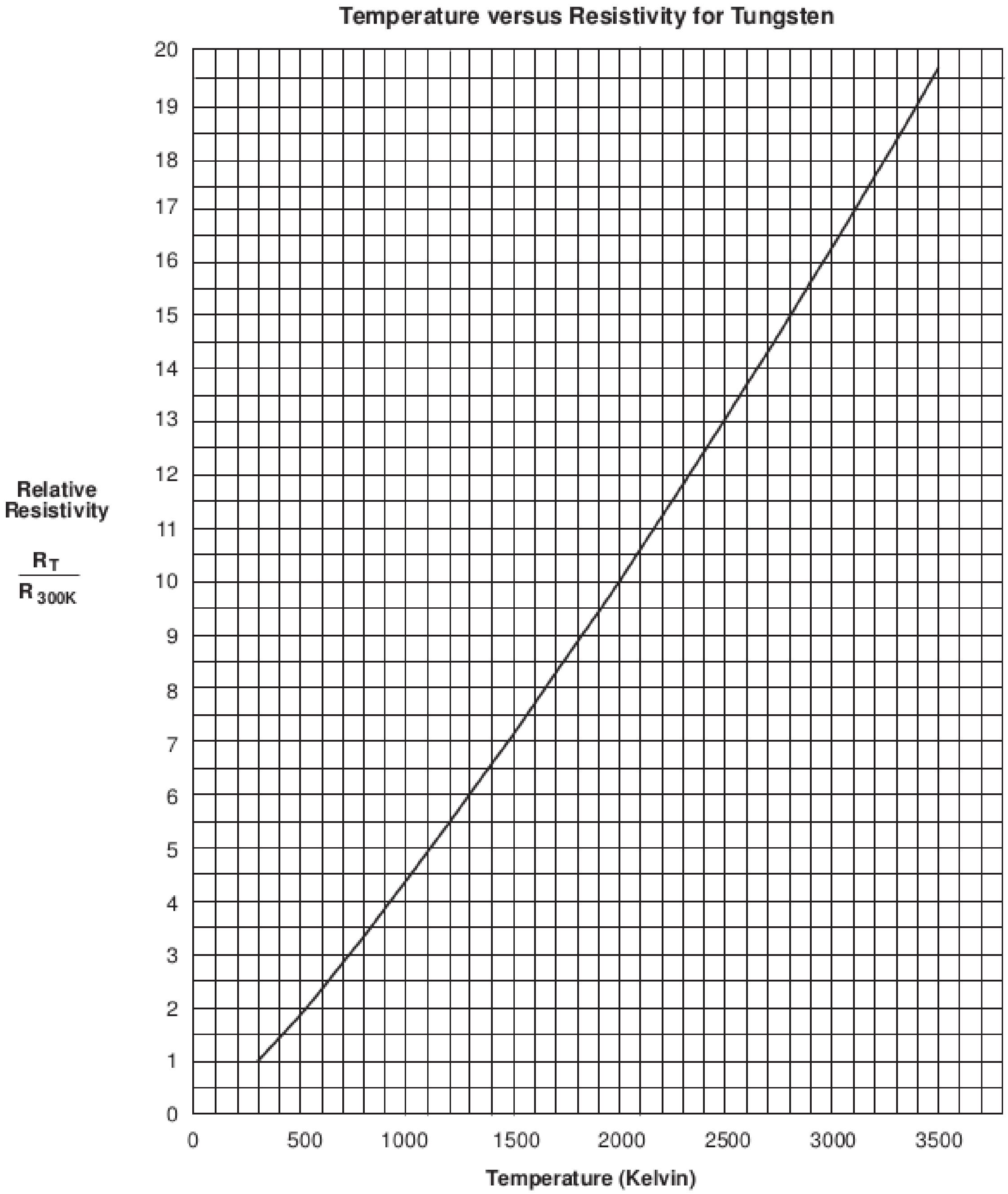
Double-click on the trendline to get various options to customize it. You should see various options such as "Exponential," "Linear," etc. These replace the straight-line relationship with various other mathematical relationships. You can try all of these out to see which ones look like a good match to the data.

In the end, we'll be most interested in the one called "Power." Select this trendline option, and also check the box that says "Display equation on chart." What this does is to find the best-fitting mathematical relationship of the form $y = (\text{something}) \times x^{(\text{something})}$ between the two quantities in the graph. In our case, x and y are T and P respectively.

According to *Excel*, what is the best-fitting value of the exponent (the power of x) in this relationship?

What do we expect this exponent to be, according to the Stefan-Boltzmann Law? (If you don't remember this from your reading, you can look it up.)

Does your data agree reasonably well with the Stefan-Boltzmann Law?


Temperature versus Resistivity for Tungsten
Relative Resistivity
$\frac{R_T}{R_{300K}}$
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
0
500
1000
1500
2000
2500
3000
3500
Temperature (Kelvin)

V (volts)	I (amps)	P (mV)	R (Ω)	$R/R_{300\,\mathrm{K}}$	T (K)

Lab 20

Solar Radiation and the Greenhouse Effect

The Earth (like the other planets) is warmed by sunlight. The Earth also radiates energy into space, mostly in the form of infrared radiation. On average, the amount of radiation received from the Sun must equal the amount of radiation emitted by the planet. After all, if the Earth emitted less radiation into space than it received from the Sun, then its temperature would keep going up; and if it emitted more radiation than it received, it would keep cooling down. Since the temperature of the Earth is fairly stable over the years, the radiation coming in from the Sun must balance the radiation being emitted.

Note: You've doubtless heard about global warming, so you may wonder how I can get away with saying that the temperature of the Earth is fairly stable. Global warming does mean there's an imbalance between the incoming and outgoing radiation, but it's very slight. Although that slight difference is enough to have major consequences for the climate, it's small enough that we can safely ignore it in the calculations we're about to do.

In this lab, you will use this idea to form an estimate of the temperature of the Earth. Specifically, you will calculate the total amount of solar power striking the Earth's surface. Then you will assume that the Earth radiates that same amount of power back into space. Assuming that the Earth radiates as a blackbody, you can use the Stefan-Boltzmann law to relate the radiated power to the temperature and so calculate the temperature of the Earth.

This calculation depends on one important assumption: that the Earth radiates like a blackbody. That assumption is wrong, so our calculated temperature will not match the actual temperature of the Earth. In particular, a phenomenon known as the "greenhouse effect" blocks some of the Earth's radiation from escaping into space. The results of this calculation will give an indication of how important the greenhouse effect is here on Earth.

Let's get started. Remember that the Sun's luminosity is

$$L_\odot = 3.86 \times 10^{26}\,\mathrm{W}.$$

A watt (W) is a unit of power equal to a joule per second. This is the total amount of power emitted by the Sun in all directions. The first thing we want to do is to figure out how much of this power strikes the Earth.

Consider an imaginary sphere centered on the Sun with a radius of 1 A.U. All of the radiation that leaves the Sun must pass through this sphere. Calculate the amount of radiation that strikes each square meter of this imaginary sphere's surface by dividing the Sun's luminosity by the sphere's surface area. Remember that the surface area of a sphere of radius R is

$$A = 4\pi R^2.$$

Remember that an A.U. is 1.49×10^{11} meters. Give your answer in units of watts per square meter ($\mathrm{W/m^2}$).

Now work out how much solar radiation strikes the Earth. To do this, note that the Earth covers up a small circular patch of that great big imaginary sphere. The amount of radiation striking the Earth will equal the amount striking that small circular patch. Recall that the area of a circle is πR^2. The radius of the Earth is 6380 kilometers. What is total amount of solar radiation striking the Earth? Give your answer in watts.

The Earth doesn't actually absorb all of that energy; some of it is reflected into space. The fraction of energy that is reflected by an object is called the object's *albedo*. Measurements of Earth from space have shown that the Earth's albedo is 0.35, meaning that it reflects 35% of all the radiation that hits it back into space and absorbs the rest.

What is the total amount of solar radiation *absorbed* by the Earth?

This number must equal the amount of radiation emitted by the Earth. Assume for the moment that the Earth emits radiation as a blackbody. That means that it obeys the Stefan-Boltzmann law,

$$F = \sigma T^4.$$

Recall that $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/(m^2\ K^4)}$. Also, remember that the flux F in the Stefan-Boltzmann law is the power radiated per area (watts per square meter). To get the total power radiated by the Earth, therefore, we should multiply the flux by the surface area of the Earth. As noted earlier, the surface area of a sphere is $4\pi R^2$, so the total power radiated by the Earth is

$$L_\oplus = 4\pi R^2 \sigma T^4.$$

Here $\oplus$ is the symbol for the Earth. L stands for "luminosity," which is what we always call the total amount of power emitted by an object. R is the radius of the Earth, and T is the temperature.

Set this expression equal to the total amount of solar power being absorbed by the Earth and solve for the temperature T. Your answer will come out in kelvin. Subtract 273 to convert it to degrees Celsius.

Does this seem like a reasonable estimate of the Earth's actual average temperature?

The main thing that went wrong in this calculation is the assumption that the Earth radiates as a blackbody, also known as a "perfect radiator." Suppose that something in the Earth's atmosphere is blocking radiation from getting out, so that it's not a perfect radiator. Then the amount of radiation leaving the Earth will be less than the amount calculated from the Stefan-Boltzmann law. This alters the balance between incoming and outgoing radiation, resulting in an increase in temperature.

Carbon dioxide and other gases in the Earth's upper atmosphere reflect some of the Earth's infrared radiation back down to the planet, preventing it from escaping into space. This "greenhouse effect" means that the Earth is not a perfect radiator, and it is why the Earth is warmer than your calculation above would indicate.

The greenhouse effect is responsible for climate change, so we think of it as a bad thing these days, but without it, the Earth would be uninhabitably cold! The problem we face now is that human activity has led to an *increase* in the greenhouse effect, altering the balance that existed for many millennia before we started altering the composition of the atmosphere.

Venus is an interesting example of the greenhouse effect. Follow the same steps as above to figure out what the surface temperature of Venus would be in the absence of a greenhouse effect. Here's some useful information about Venus:

- Distance from Sun to Venus: 0.723 A.U.
- Radius of Venus: 6050 km.
- Venus's albedo: 0.65.

You'll also need to know the luminosity of the Sun (3.86×10^{26} W) and the number of meters in an A.U. (1.49×10^{11} m).

Probes of Venus indicate that its surface temperature is about 700 K (430°C, or 800°F). Would you say that the greenhouse effect is more important on Venus than on Earth or vice versa?

Lab 21

Energy Flow in the Sun

In this lab, you will examine how the energy produced in the core of the Sun makes it out to the surface. The nuclear reactions in the core of the Sun produce photons, which eventually make their way to the surface of the Sun. This process takes a long time because the photons undergo frequent collisions with ionized atoms in the solar interior. These collisions cause the photons to bounce around in random directions instead of zooming straight out.

You will use simulation software to examine the process by which these photons make it out of the Sun. If it's not already running, start up the "energy flow out of the Sun" application on the lab PC and log in. It doesn't matter what you enter for names and table number.[1]

Part 1: Photons moving through a gas.

Select "Interaction" under the "Simulation" menu, and click "Run." This shows what happens to photons as they pass through a dense gas. The stationary circles represent atoms in the gas, and the smaller white circles represent photons. You should imagine that there is a source of photons off to the left somewhere, sending photons into the gas. These photons have the correct amount of energy to correspond to one of the spectral lines of the atoms. That means that when a photon hits an atom, it is very likely to be absorbed, sending the atom into an excited state. The atom quickly drops back down to its original state, re-emitting the photon in a new direction. The end result is that the photon undergoes a "random walk": instead of moving in a straight line, it bounces around making frequent changes in direction.

Stop the simulation. Under "Photon Type," select "Continuum," and restart the simulation. With this setting, the photons now have an energy that does *not* correspond to a spectral line of these atoms. This means that the atoms cannot absorb the photons, so the photons usually just pass right through.

On rare occasions, a photon with the wrong energy can bounce off of an atom, as you'll see if you let the simulation run for a while.

The *mean free path* is the average distance that a photon travels before it bounces off of an atom. Which has a longer mean free path, the continuum photons or the line photons?

[1]For boring technical reasons, this program has to run inside of a "virtual machine" that's set up on the lab computers. I'll make sure that it's all set up before class. This may mean that the application may run in a window inside of another window, which looks a little funny but doesn't make a difference in the end.

If the continuum photons *never* bounced off of atoms (as opposed to rarely doing so), what would their mean free path be?

Here's an important thing to realize for the rest of the lab: Throughout most of the interior of the Sun, the gas is ionized, meaning that the electrons are stripped off of the protons. Ionized gas is capable of interacting with photons of *all* wavelengths, not just specific ones. That means that *in the interior of the Sun, all photons behave like the "line" photons in this simulation.*

Part 2: Photon Diffusion in the Sun.

Click "Return" to get back to the main menu. Under "Simulation," go to "Flow" and then to "1 Photon." This simulation shows the path of a single photon as it makes its way out from the center of the Sun. Click "Run" a few times to see how the photons behave. Go to "Parameters" and check the "Yes" box under "Trails." This will cause the photon's path to be visible as it makes its way out.

In the "Parameters" menu, you can also vary the size of the Sun. Select # of Layers. It should initially be 20. Change that value to 40 and try running the simulation again. You should see that the Sun has gotten larger, and not surprisingly it takes more time for the photons to make their way out.

In this model of the Sun, each "layer" of the Sun has a thickness equal to the mean free path of the photons. That is, on average, a photon can make it in or out of the Sun by only one layer between bounces. The more layers there are, the more times a photon is likely to bounce on its way out, and so the longer it will take to get out. In fact, there is a general rule relating the number of layers to the time it takes photons to get out, which you will now determine.

Because the photons bounce around randomly, some get out faster than others. We're going to be interested in the time it takes the *average* photon to get out, so we'll want to run the simulation with a bunch of photons all at once. Go to the "Simulation" menu, and click "Flow" and then "Diffusion." This will behave just like the simulation, but with many photons.

Under "Parameters," you can set the number of layers and the number of photons. Set the number of layers to 10, and the number of photons to 200. Start the simulation, and let it keep going until all 200 photons have escaped from the Sun. record the average number of interactions required for a photon to make it out.

Repeat the process for a 20-layer Sun.

Do it one more time, this time for a 30-layer Sun. This one will take a little while!

Based on these results, take a guess about the mathematical relationship between the number of layers and the average number of interactions taken to get out:

Ask me if you're not sure whether your guess is right.

This relationship only tells you about the *average* time it takes a photon to get out. As you've seen, some photons are faster than average, and some are slower. Let's do one more simulation to get a feel for this.

Set the number of layers to 25, and the number of photons to 1000, and start the simulation running. After how many interactions does the first photon make it out?

Wait until most of the photons have escaped. What is the average number of interactions of all the photons? Is this consistent with your expectations based on the mathematical relationship you guessed above?

The graph in the lower part of the window shows the number of photons escaping the Sun as a function of time. The yellow line is the time when the number of interactions is n^2 (the square of the number of layers). Sketch a copy of the graph below.

This graph shows that quite a few photons (well over half) make it out in less than the average time, but some take much, much longer than that.

Part 3: Numbers for the actual Sun.

In these simulations, the number of layers is very small. In the actual Sun, the photons can travel only a very short distance before scattering (that is, the "mean free path" is small). Since a "layer" is supposed to be only as thick as the mean free path, a realistic model of the Sun would have many, many layers. In this portion of the lab, you'll work out some rough numbers for the actual Sun.

Based on the physical laws governing interactions between photons and electrons, scientists estimate that the mean free path of photons inside the Sun is about a tenth of a millimeter (10^{-4} meters). The Sun's radius is about 700,000 kilometers. If each "layer" has a size equal to the mean free path, how many layers does the Sun consist of?

Based on the relationship you found earlier between the number of layers and the number of interactions (bounces), how many interactions will an average photon undergo on its way out of the Sun?

The average distance a photon travels between interactions is equal to the mean free path, and of course photons travel at the speed of light (300,000 km/s). What is the average time between interactions for a photon in the Sun?

From the average time between interactions and the total number of interactions, determine how much time it takes the average photon to make it out of the Sun. Convert your answer into years.

If the nuclear fusion reactions powering the Sun were to somehow stop tomorrow, would we notice an immediate reduction in the Sun's luminosity?

Is there any way we could tell if the nuclear reactions in the Sun stopped tomorrow?

Lab 22

Parallax

Introduction.

Parallax is the method used to find the distances to the closest stars. It is the first step in the "distance ladder" used to find distances to other stars and ultimately to other galaxies. In this lab you will use the method of parallax to determine the distance to a nearby object: one of the classroom windows.

Here's the big idea behind parallax measurements. Suppose you want to measure the distance to the star in the drawing on the next page. You observe the apparent position of the star from two locations A and B. The directions you have to look in order to see the object from the two locations are slightly different, and the amount of difference can be represented by the angle θ. If you can determine θ, then you can use the small-angle formula to determine the distance to the object.

The small-angle formula says that

$$s = \left(\frac{2\pi}{360^\circ}\right)\theta D.$$

Here s is the distance between the two observation points, and D is the distance to the object.[1]

(By the way, when we first learned the small-angle formula, we said that s was the separation between two different objects being observed, not the separation between two different observation points. We're now applying the formula to a different situation, but since the geometrical setup is the same, the same formula works.)

The next question is how to measure θ. *In principle*, we could measure θ like this:

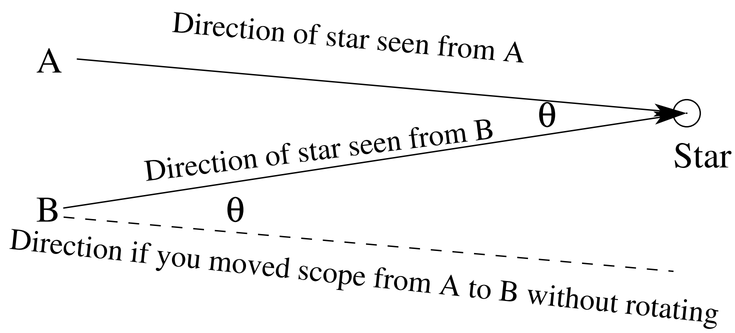
1. Put the telescope at point A and point it at the star.

2. Move the telescope from point A to point B, being *very careful* not to rotate it as you move it.

3. Once the telescope is at point B, rotate it until it is again pointing at the star. Measure the angle through which you had to turn the telescope. This is θ.

Here's a picture of the various angles:

[1] This is the version of the small-angle formula that has θ in degrees. That's the version that's most useful in this lab. If you measure angles in arc-seconds instead, then you use the version of the formula that has the number 206 265 in it.

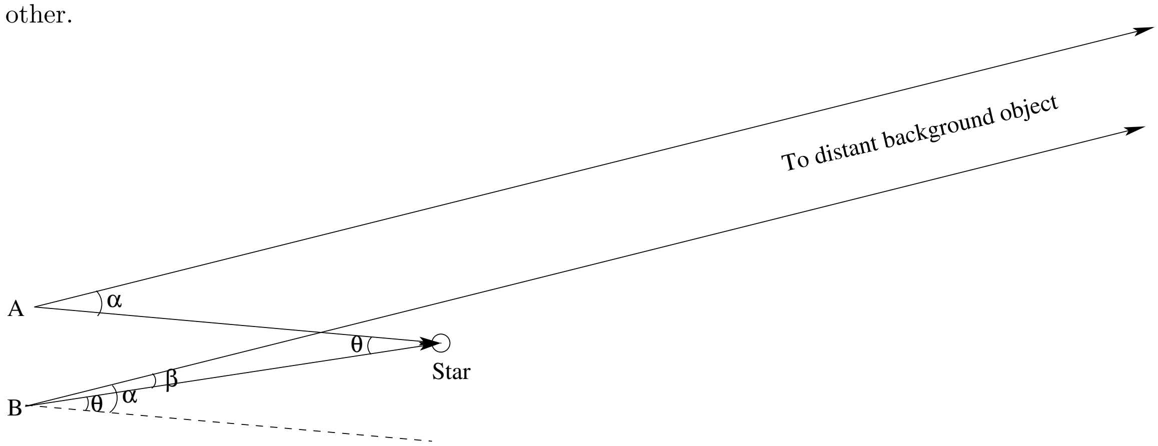


Although there is nothing wrong with this method in principle, in practice it's quite inaccurate. The reason is step 2: it's very hard to make sure that you didn't jostle the telescope and rotate it just a little bit when you were moving it. Since parallax angles are very small, even a slight error in this step will ruin the method. So *the method above is no good in practice.*

To make the method work, we need to measure the shift in the angular position of the star by comparing it to some much more distant background objects. Here's the idea:

1. Put the telescope at point A. Point the telescope at the star.
2. Rotate the telescope until it points at a particular *background object.* Measure the angle through which you had to turn the telescope to do this. The result is the *angular separation* between the star and the background object. Let's call this angle α.
3. Move the telescope to point B (without worrying about whether you're rotating it as you go).
4. Point the telescope at the star, then at the same background object as before. Measure the angular separation this time, and call it β.
5. The parallax angle θ is just the difference between α and β.

The picture below illustrates the various angles in this method. The main point is that the background object is presumed to be extremely far away (much further than the star). This means that the lines of sight to the background object from the two locations are essentially parallel to each other.



Procedure.

The "star" in this lab will be the vertical divider separating two panes of the windows at the side of the classroom. Arrange the telescopes so that you have a clear view of the "star." Put down a couple of pieces of masking tape to mark the location of the telescope. You'll need these later when you determine how far you moved the telescope. You can mark the locations of any convenient points on the telescope base.

(Incidentally, the telescopes we're using here are the ones on the spectroscopes. We're not using the "slit" side of the spectroscopes at all in this lab; we're just using the "telescope" side. So just set up the apparatus so that the slit is off out of the way, and ignore it.)

Rotate the telescope until the "star" is centered in the field of view. Use the crosshairs to make sure it's lined up accurately. Record the angle made by the telescope. If you forget how to read the Vernier scale on the telescope base, ask me.

Look through the telescope, and choose a faraway "reference object." This should be something easily identifiable and far away (outside of the building, and preferably across the street). It should also be close to, but not quite, the same direction as the "star." Good choices are tree trunks, signs, or lamp posts.

Rotate the telescope until the reference object is centered on the crosshairs, and record the angle made by the telescope now.

Subtract these two angles from each other. The result is α, the angular separation between the "star" and the reference object. (If I were you, I would convert the angles from degrees and minutes into a decimal number of degrees before subtracting. For instance, I'd convert $42^\circ\ 20'$, into 42.33°, by using the fact that a minute is $\frac{1}{60}$ of a degree.)

Move the telescope over about 15 centimeters or so. (The exact amount doesn't matter.) Move it in a direction perpendicular to the direction of the "star," not towards or away from the star. Mark the new position with masking tape again.

Look through the telescope. You should notice that the position of the "star" relative to the background objects has shifted a bit. That's parallax.

Repeat the procedure above: Measure the angular position of the star and the angular position of the reference object (same object as last time!).

What is the parallax angle θ?

Using a ruler and your masking-tape marks, measure the distance between the two observation points A and B.

Now you have all the information you need to determine the distance to the "star." What is it?

Use a tape measure to determine the actual distance.

Do you think your results are reasonably accurate? What do you think the main sources of error are in this method?

Lab 23

The Hertzsprung-Russell Diagram

In this lab, you'll use the data from the Hipparcos satellite to construct a Hertzsprung-Russell diagram. The data you'll use is the real stuff returned by the satellite; all I've done for you is to convert it from archaic astronomy units (such as magnitudes) into standard modern physics units (such as watts per square meter).

The full Hipparcos data set contains 100,000 stars, but you'll be working with only the approximately 1000 closest stars and the approximately 1000 brightest stars.

1. There are two Excel files on Blackboard (in the "Downloads for Labs" section) containing Hipparcos data on the closest and brightest stars. Download these files to the desktop, and open the file containing the brightest stars. It contains about 1000 rows and four columns: Catalog number, Parallax angle, temperature, and brightness. Note that the units of the various quantities are indicated on the second line (except for the catalog number, which doesn't have any units). You will be adding a bunch of new columns to the spreadsheet. As you do, use those first two lines indicate what quantity is in each column and what its units are.

2. For each of these stars, you will need to determine its luminosity and radius. Since there are 1000 stars, you wouldn't want to do it by hand, so you'll use Excel formulae instead. However, it's good to calculate at least one by hand in order to check the Excel calculation you're going to do. So for the first star in the list, calculate the following things:

   (a) Distance in parsecs.

   (b) Distance in meters.

   (c) Luminosity.

   (d) Radius.

3. Now use Excel formulae to repeat these calculations. Column E will contain the distances to the stars in parsecs. Indicate this in the first two rows of the column. Then, in cell E3, enter a formula that calculates the distance to the first star. (Appendix C contains advice on how to do this.) If your formula agrees with the result you calculated above, then drag the formula down to fill in the distances to the rest of the stars.

4. In the next three columns, use formulae to determine distance in meters, luminosity, and radius.

5. Create a Hertzsprung-Russell diagram for this set of stars. This is just a graph with temperature on the horizontal axis and luminosity on the vertical axis. (See Appendix C if you need advice on making graphs in Excel.)

   When you first make your graph it will probably not look right. One reason for this is that H-R diagrams traditionally have *logarithmic* y axes. This means that the numbers on the y axis are spaced out in powers of 10. Double-click on the y axis of your graph to get the "Format Axis" window, and check the box that says "Logarithmis scale." In that same window, you can adjust the minimum value on the axis so that the points aren't all crowded together near the top.

   Also, by annoying, stupid tradition, H-R diagrams are plotted with high temperatures on the left instead of the right. So double-click on the x axis and check the "values in reverse order" box.

   By the time you're done, your graph should look something like a normal H-R diagram. In particular, the main sequence should be clearly visible running from upper left to lower right. Also, the x and y axes should be labeled to indicate both what quantity is being plotted (*e.g.*, "Temperature") and what units it's in (*e.g.* "K").

6. Print out your graph. Mark one more point on it by hand to indicate the Sun. The Sun's temperature is 5800 K and its luminosity is 3.86×10^{26} W. The Sun should lie on the main sequence; if it doesn't, something has gone wrong.

7. Follow the same steps to create an H-R diagram for the data set consisting of the closest stars. Print out this graph as well.

8. Which of the two data sets contains more white dwarfs? Explain why this makes sense.

9. Which of the two data sets contains more giants? Explain why this makes sense.

10. For the closest stars, make a graph showing the temperatures of the stars on the x axis and the radii on the y axis. Use a logarithmic scale on the y axis, and make sure the ranges on the axes are set so that the points aren't all crowded together at one end. As in your previous graphs, be sure the axes are clearly labeled including units. Print out the graph.

11. What is the radius of a main-sequence star with a temperature of 5000 K? What is the radius of a white dwarf star with a temperature of 10,000 K? What is the radius of the largest star in this sample?

Lab 24

Spectroscopic Binary Stars

Part 1: Determining the masses of spectroscopic binary stars. For many binary star systems, we can't actually resolve the two stars (i.e, see them as separate objects); all we can do is observe the combined light from both of them. In this lab, you'll work through a calculation to figure out the masses of a pair of *spectroscopic binary* stars, in which we can observe the spectra of the pair of stars together but can't see each of the stars individually.

Suppose that you measure the spectra of such a binary star system, and you find that it has a pair of spectral lines, both very close to the wavelength 656.28 nm, which is the wavelength of a prominent spectral line in hydrogen. At any given moment, one is redshifted to a longer wavelength, and the other two is blueshifted to a shorter wavelength. As time passes, each line cycles back and forth between redshift and blueshift.

You make some measurements of the spectral lines and observe the following facts:

1. At a moment when the lines have the largest Doppler shifts, one of them has a wavelength of 656.72 nm, and the other has a wavelength of 655.40 nm.
2. Half a cycle later (i.e., 5 hours later), the wavelengths are 655.84 nm and 657.16 nm.
3. Another 5 hours later, the spectral lines are back at the wavelengths they were at at the first moment. In other words, the time it takes for the lines to cycle all the way around and back to where they started is 10 hours.

Sketch a picture showing the orbits of the two stars. Indicate where the stars are at Moment 1 and at Moment 2, and indicate the location of you the observer.

What are the speeds of the two stars? (You can use the wavelengths at either of the two moments described above. You might want to use both just to check your work.)

Suppose that you were standing (in your flame-proof suit) on the surface of one of the two stars, observing the other star. How fast would the other star move relative to you?

Suppose that, from this vantage point, you watched the other star make a complete circular revolution around you. What would the radius of this circle be? (Hint: you know how fast the star is moving, and you know how much time it takes to go around once.)

The number you just figured out is the semimajor axis of the orbit (the a in Kepler's third law).

Use Kepler's third law to determine the total mass $M_1 + M_2$ of the two stars. Give your answer in solar masses. I strongly recommend that you do this by determining a and P in units of AU and years respectively. Recall that Kepler's third law takes a relatively simple form in this case.

Now determine the individual masses of the stars. To do this, make use of the fact that the speeds at which the stars orbit are related to their masses like this: $M_1 v_1 = M_2 v_2$. Combine this fact with the value you found for the total mass, and solve for M_1, M_2.

In fact, what you've just discovered is the *minimum* possible mass for the two stars: they may in fact be heavier than this. Can you think of a reason that your calculations may have given masses that are smaller than the true values? (Can you think of an assumption you made in your calculations that may not be true?)

Part 2: Finding the masses of planets orbiting other stars. One of the hottest topics in astrophysics these days is the discovery of planets orbiting other stars. One of the ways people do this is by looking for the "wobble" of the star caused by the planet orbiting it. The planet itself is too small and dim to be observed, so we must infer its presence from the star's motion.

For example, observations of the star HD209458 reveal the folowing:

1. The velocity of the star wobbles back and forth in a regular way, repeating every 3.525 days.
2. The star's maximum speed towards us is 87.1 m/s, and its maximum speed away from us is 87.1 m/s in the other direction. (Note that these are meters per second, not kilometers per second.)
3. The star is a main-sequence star very similar to the Sun. Its mass is $1.13 M_\odot$.

You can use this information to figure out the mass of the planet orbiting this star, even though you can't see the planet or measure its spectral lines.

First, use Kepler's third law to determine the radius of the planet's orbit. I recommend that you use the AU-year-solar-mass version of the law. You can assume at this point that the mass of the planet is so small that it can be ignored in comparison with the star's mass – that is, $M_1 + M_2$ can be taken to be the same as the mass of the star.

Now that you know the radius of the planet's orbit, you can figure out the speed of the planet in its orbit. (How far does it travel during one orbit? How much time does this take?)

Now remember the rule $M_1 v_1 = M_2 v_2$ from above: the masses of the two bodies in an orbit are related to their orbital speeds. You know both speeds, so you can express the mass of the planet in terms of the star's mass. What is the mass of the planet in solar masses?

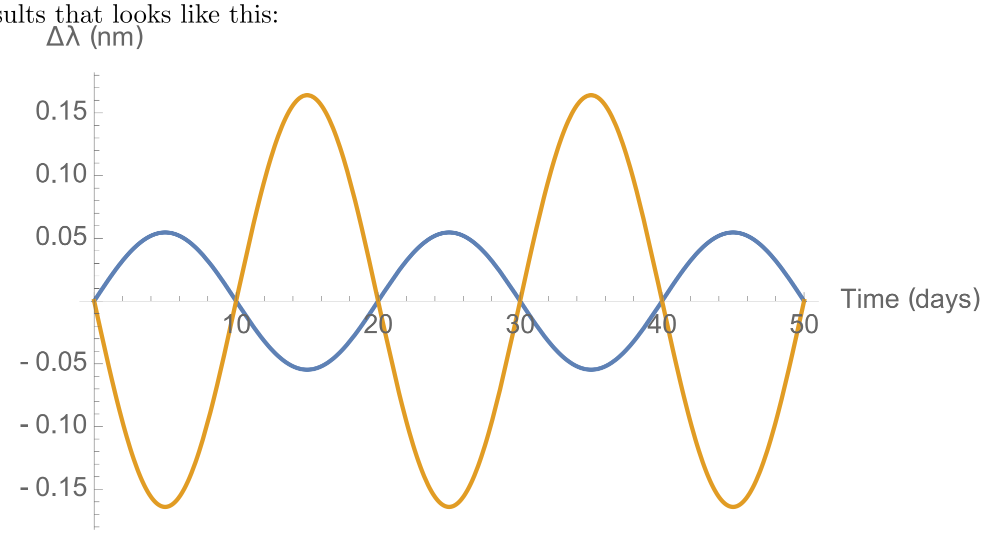
Planet-hunters often express the masses of the planets they find in units of Jupiter's mass. Jupiter's mass is about $\frac{1}{1000}$ that of the Sun. What is the mass of the planet in units of Jupiter masses?

By the way, just as in part 1, the mass you've found is actually the minimum possible mass, for the same reason. But subsequent observations show that in fact the problem that might result in this calculation giving an underestimate of the true mass isn't really a problem in this case: the mass you've found is about right. If you want to know how we know, I'll tell you.

Lab 25

More Fun with Binary Stars

An astronomer observes the spectrum of a star many times over a series of days. She observes that the Hα line of hydrogen usually appears split into two lines with slightly different wavelengths. She measures the wavelengths of those two lines during each of her observations and plots a graph of the results that looks like this:



What's plotted on this graph are the wavelength shifts of the spectral line. So when $t = 0$ days, there is no shift at all, and both lines appear right where they would be in the lab. A day or two later, the spectral line is split in two, with one line (represented by the blue curve) having a wavelength slightly longer than expected, and one (represented by the yellow curve) having a wavelength slightly shorter than expected. By 10 days, the two spectral lines have come back together again, and then they split apart again.

As you probably know, the reason this is happening is that this star is really a *double-line spectroscopic binary system*. The two graphs show the spectral lines due to the two stars in the system, which alternately move toward you and away from you as they orbit around each other.

To save you the trouble of reading numbers off the graph, I'll tell you that when $t = 5$ days, the blue curve is at a height of 0.055 nm, and the yellow curve is at -0.16 nm. This moment is when the two graphs are at their highest and lowest points, meaning that one star is moving directly toward you and one is moving directly away from you. You can assume that the stars are moving in circular orbits.

Let's call the star whose spectral line is given by the blue curve star number 1, and the other one star number 2. The wavelength of the Hα line, when measured in the lab, is 656.3 nm. Armed with this information, I want you to tell me the following about these stars and their orbits. (You may need some unit conversions. If so, look them up.)

1. How fast are the two stars moving?

2. Which star is heavier? What is the ratio of their masses M_1/M_2? (Remember that the masses and speeds of the orbits are related, so that $M_1 v_1 = M_2 v_2$.)

3. What is the period of the orbit? (That is, how much time does it take for the graph to repeat itself?)

4. What are the radii of the two stars' orbits? (Suggestion: How far does each star travel during one period of its motion? How is that number related to the radius?)

5. At the end of this document is a picture of the two stars' orbits. You, the observer, are way off to the left. Say both stars are moving counterclockwise. When $t = 5$ days, where are the two stars in this diagram? (Hint: is star 1 moving toward you or away from you at this moment? What about star 2?) Mark the locations on the diagram with a "5".

6. Mark the locations of the two stars when $t = 10, 15$, and 20 days.

7. What is the distance between the two stars at any given time? (Hint: How is it related to the radii of the two orbits, which you know?) The distance between the two stars is the a in Kepler's Third Law.

8. What is the total mass $M_1 + M_2$?

9. What are the masses M_1 and M_2?