

$$T_A = L / V_a$$

Similarly the length of time it takes for runner B to finish is the length of the course divided by his speed:

$$T_B = L / V_b$$

So the difference in times can be described as

$$T_B - T_A = L / V_b - L / V_a$$

or (factoring out L from both terms on the right)

$$T_B - T_A = L(1/V_b - 1/V_a)$$

And solving for L

$$L = \frac{T_B - T_A}{(1/V_b - 1/V_a)}$$

Try the equation using the following numbers taken from our above example:

$$V_a = 5 \text{ km/hr}$$

$$V_b = 10 \text{ km/hr}$$

$$T_B - T_A = 1 \text{ hour, or } 2 \text{ hours,}$$

and you'll see you get $L = 10 \text{ km}$ for a 1 hour difference, and $L = 20 \text{ km}$ for a 2 hr difference.

C. The Dispersion Formula for the Interstellar Medium

The laws of physics enable us to calculate the speed of electromagnetic radiation in the interstellar medium and to derive a formula similar to the one above for the distance traveled in terms of the delay in arrival between radio pulses received at different frequencies. Lower frequencies travel slower, arriving later. So in order to determine the distance of a pulsar, we simply need to measure the time of arrival of a pulse from a pulsar at two different frequencies. We'll do that in the next part.

These differences in speed are incredibly tiny. All of the radio waves from a pulsar travel at almost exactly the speed of light ($3 \times 10^8 \text{ m/s}$). The interstellar medium slows them down by only about 0.002 m/s for radio waves with frequencies of 400 MHz, and even less (0.0005 m/s) for 800-MHz waves. Still, over the immense distances to pulsars those tiny differences add up to a significant difference in the arrival times of pulses of different frequencies.

As you can see from the previous section, the quantity we'll need to know to determine the distance to a pulsar is the difference in the reciprocals of the speeds, $(1/V_b - 1/V_a)$, where V_a and V_b are the speeds of radio waves at two different frequencies. This quantity depends on the amount of ionized gas in the interstellar medium. The physics that goes into calculating it is a bit involved, so I'll just tell you what that quantity turns out to be for the frequencies we'll be interested in: