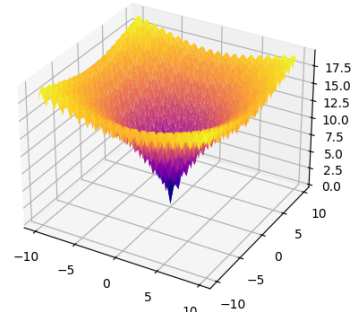
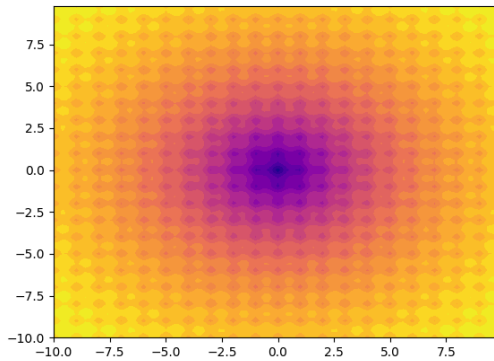


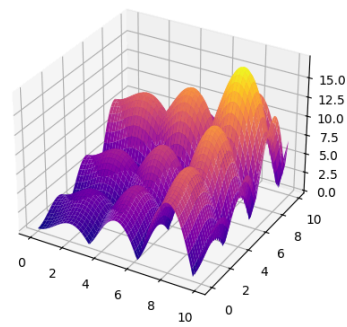
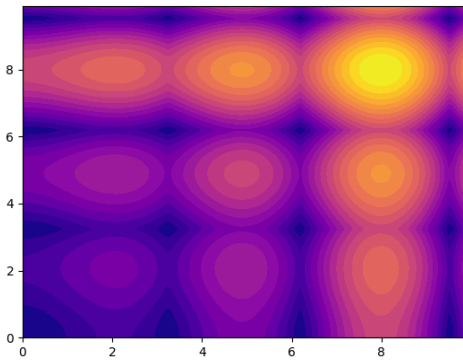
## 1. Ackley



$$f(\mathbf{x}) = -a \exp \left( -b \sqrt{\frac{1}{d} \sum_{i=1}^d x_i^2} \right) - \exp \left( \frac{1}{d} \sum_{i=1}^d \cos(cx_i) \right) + a + \exp(1)$$

<https://www.sfu.ca/~ssurjano/ackley.html>

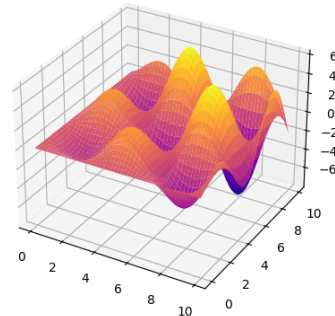
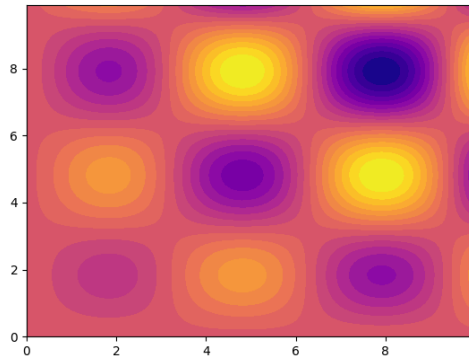
## 2. Alpine



$$f(\mathbf{x}) = \sum_{i=1}^d |x_i \sin(x_i)| + 0.1|x_i|$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

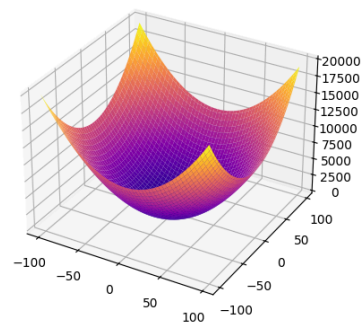
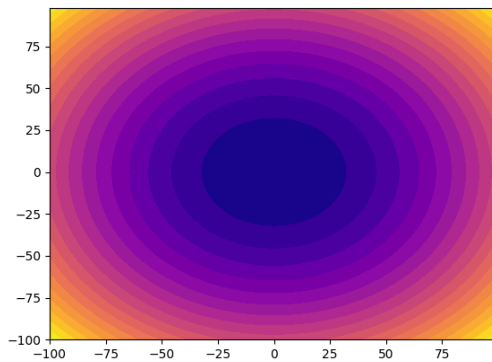
### 3. Alpine 2



$$f(x) = - \prod_{i=1}^d \sqrt{x_i} \sin(x_i)$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

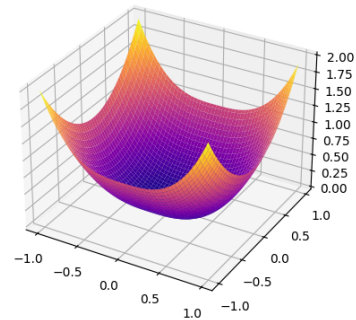
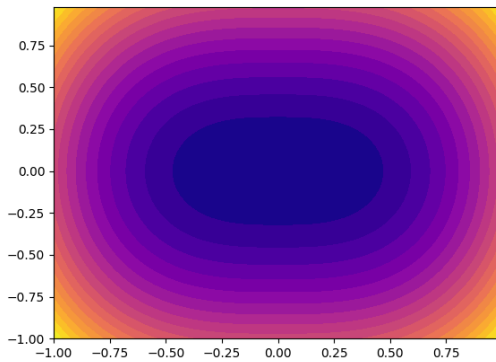
### 4. DeJong Sphere



$$f(\mathbf{x}) = \sum_{i=1}^d x_i^2$$

<https://www.sfu.ca/~ssurjano/spheref.html>

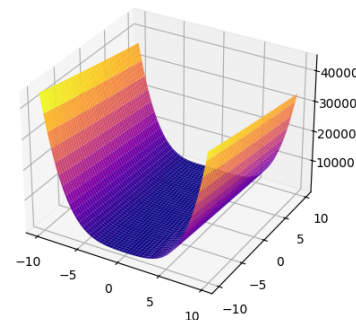
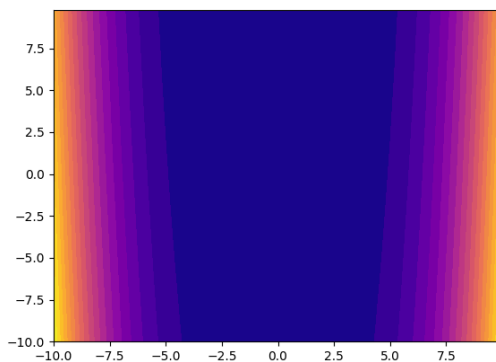
## 5. Different Powers Sum



$$f(\mathbf{x}) = \sum_{i=1}^d |x_i|^{i+1}$$

<https://www.sfu.ca/~ssurjano/sumpow.html>

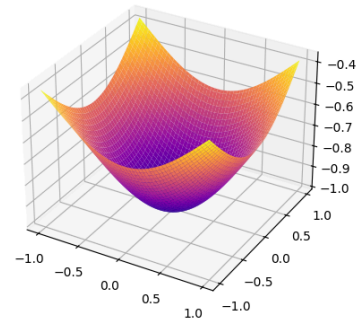
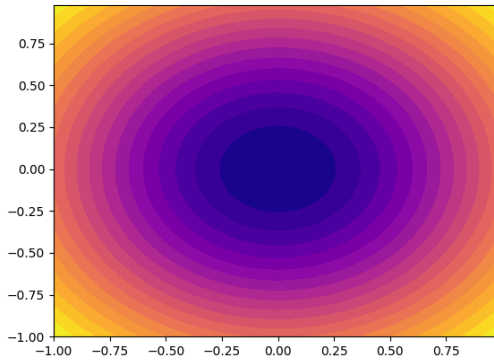
## 6. Dixon Price



$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^d i(2x_i^2 - x_{i-1})^2$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

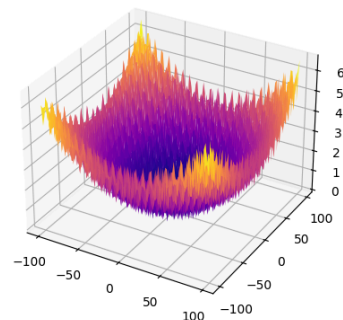
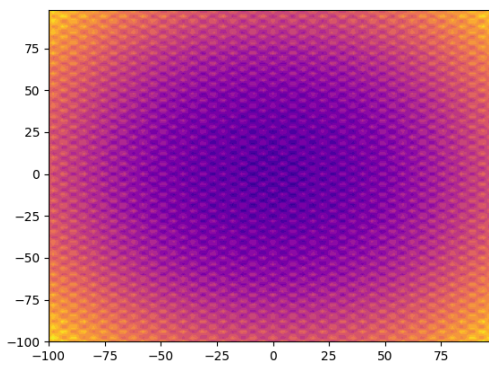
## 7. Exponential



$$f(x) = -\exp\left(-0.5 \sum_{i=1}^d x_i^2\right)$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

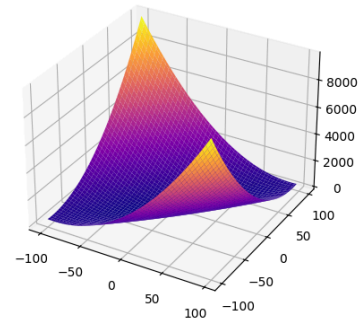
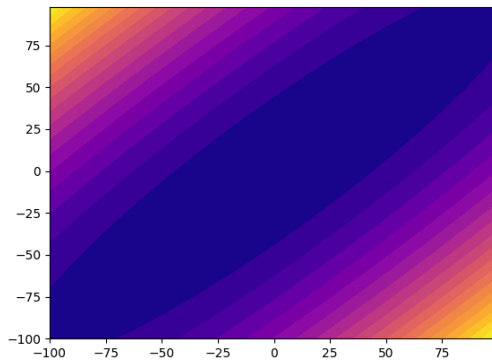
## 8. Griewank



$$f(\mathbf{x}) = \sum_{i=1}^d \frac{x_i^2}{4000} - \prod_{i=1}^d \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

<https://www.sfu.ca/~ssurjano/griewank.html>

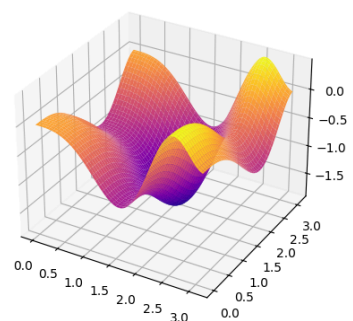
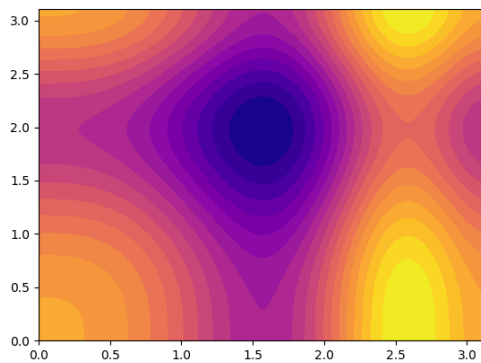
## 9. Matyas



$$f(\mathbf{x}) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$$

<https://www.sfu.ca/~ssurjano/matya.html>

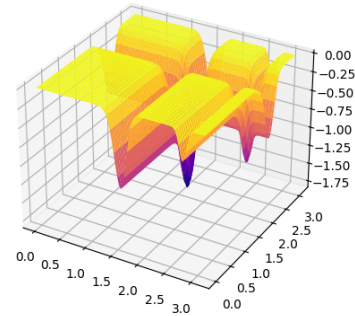
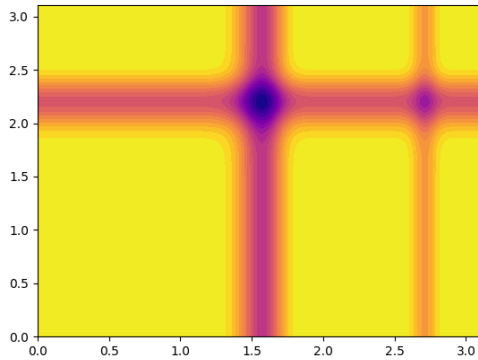
## 10. Michalewicz



$$f(X) = - \sum_{i=1}^n \sin(x_i) \left[ \sin \left( \frac{jx_i^2}{\pi} \right) \right], m = 1$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/197-michalewicz-s-function>

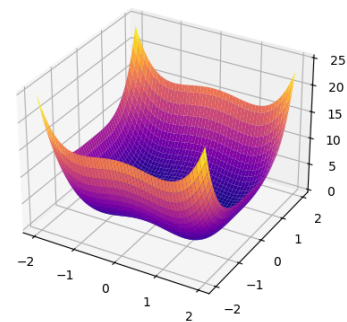
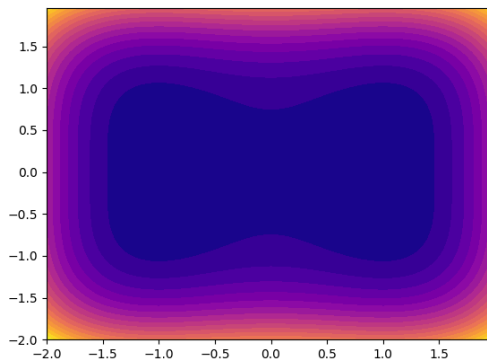
### 11. Michalewicz 2



$$f(\mathbf{x}) = - \sum_{i=1}^d \sin(x_i) \sin^{2m} \left( \frac{ix_i^2}{\pi} \right), m = 10$$

<https://www.sfu.ca/~ssurjano/michal.html>

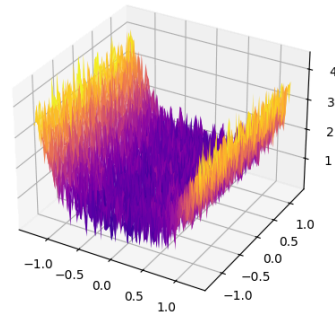
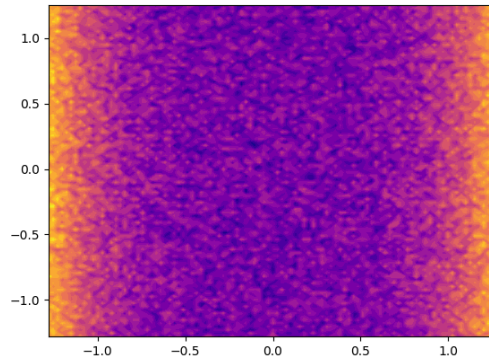
### 12. Qing



$$f(\mathbf{x}) = \sum_{i=1}^d (x_i^2 - i)^2$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

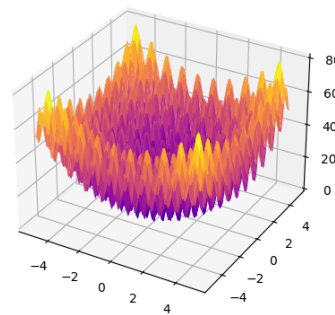
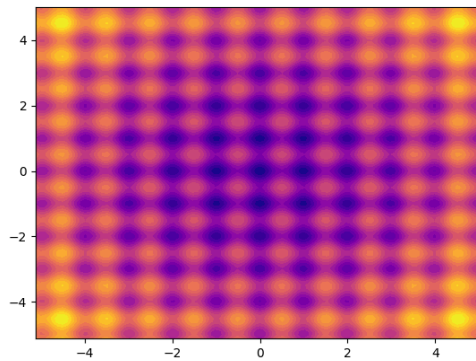
### 13. Quartic



$$f(\mathbf{x}) = \sum_{i=1}^n ix_i^4 + \text{random}[0, 1]$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

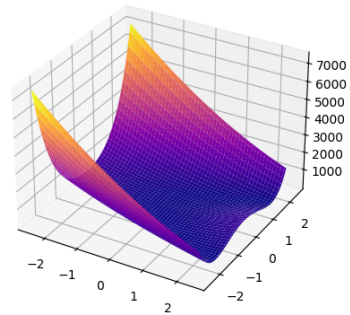
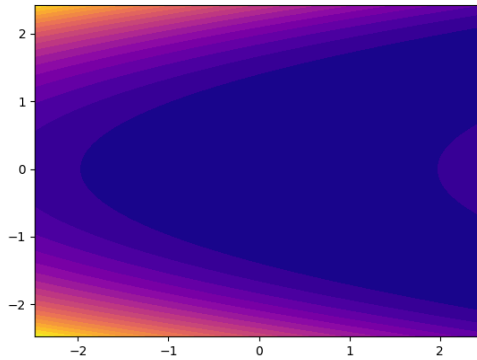
### 14. Rastrigin



$$f(\mathbf{x}) = 10d + \sum_{i=1}^d [x_i^2 - 10 \cos(2\pi x_i)]$$

<https://www.sfu.ca/~ssurjano/rastr.html>

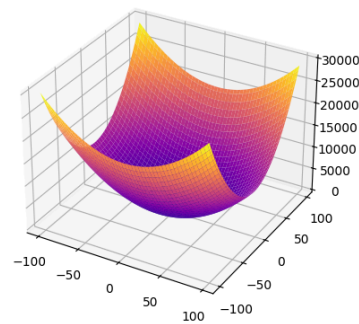
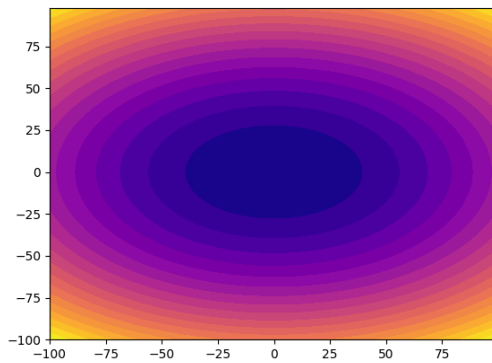
### 15. Rosenbrock



$$f(\mathbf{x}) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$$

<https://www.sfu.ca/~ssurjano/rosen.html>

### 16. Rotated Hyper Ellipsoid

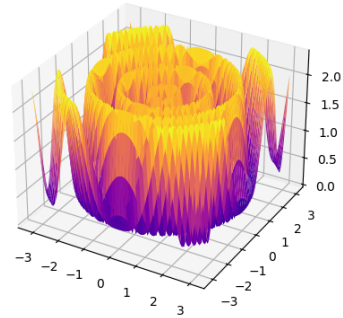
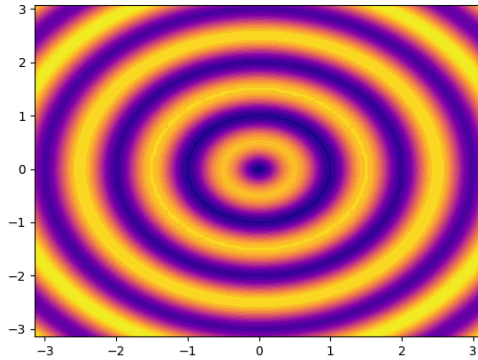


$$f(\mathbf{x}) = \sum_{i=1}^d \sum_{j=1}^i x_j^2$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>



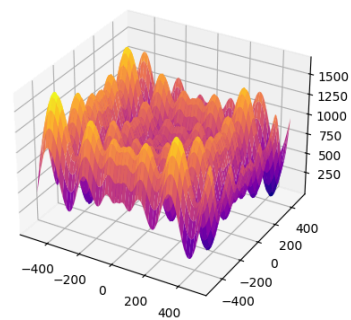
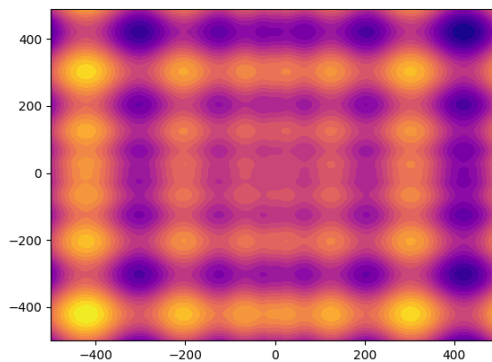
### 17. Salomon



$$f(\mathbf{x}) = 1 - \cos \left( 2\pi \sqrt{\sum_{i=1}^d x_i^2} \right) + 0.1 \sqrt{\sum_{i=1}^d x_i^2}$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

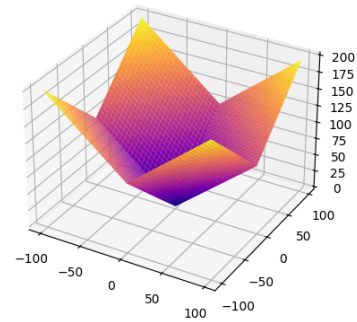
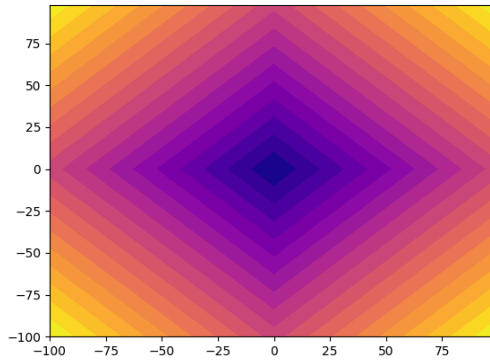
### 18. Schwefel



$$f(x) = 418.9829d - \sum_{i=1}^d x_i \sin(\sqrt{|x_i|})$$

<https://www.sfu.ca/~ssurjano/schwef.html>

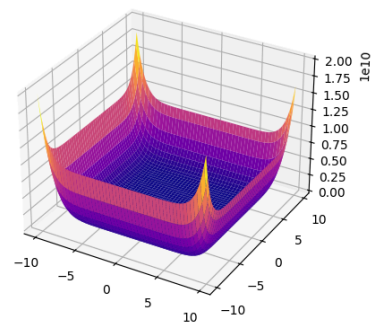
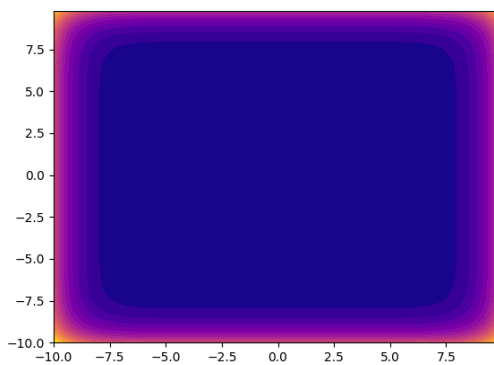
### 19. Schwefel 2.20



$$f(\mathbf{x}) = \sum_{i=1}^d |x_i|$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

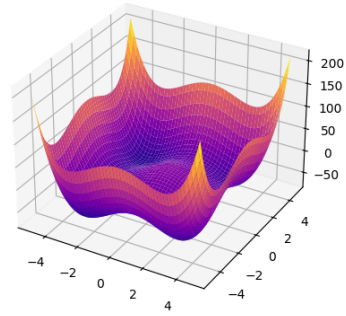
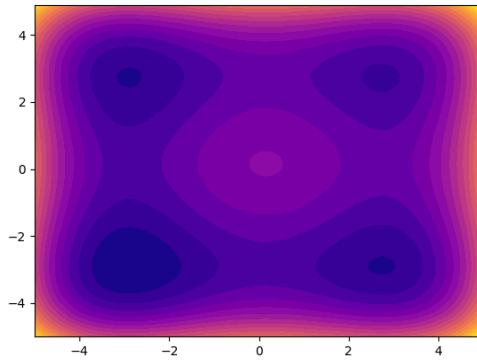
### 20. Schwefel 2.23



$$f(\mathbf{x}) = \sum_{i=1}^d x_i^{10}$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

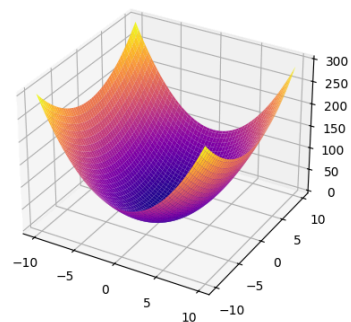
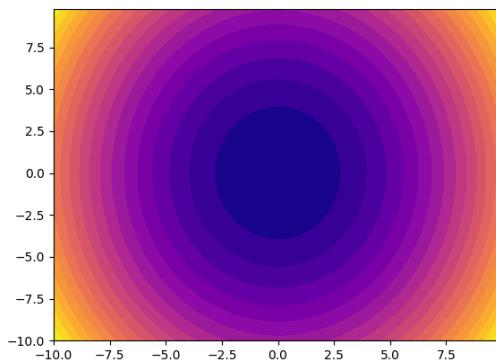
## 21. Styblinski



$$f(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^d (x_i^4 - 16x_i^2 + 5x_i)$$

<https://www.sfu.ca/~ssurjano/stybtang.html>

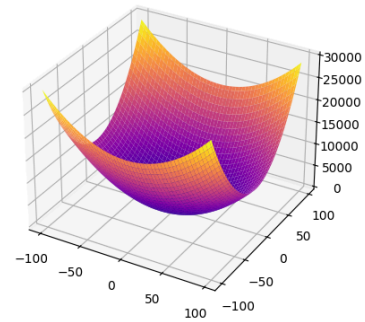
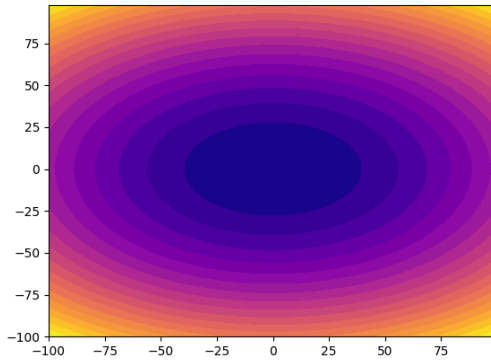
## 22. Sum Squares



$$f(\mathbf{x}) = \sum_{i=1}^d ix_i^2$$

<https://www.sfu.ca/~ssurjano/sumsqu.html>

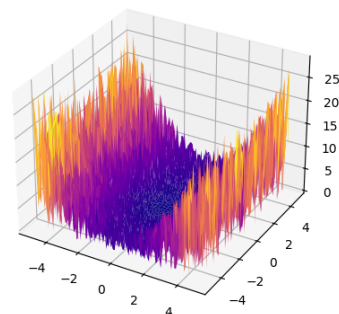
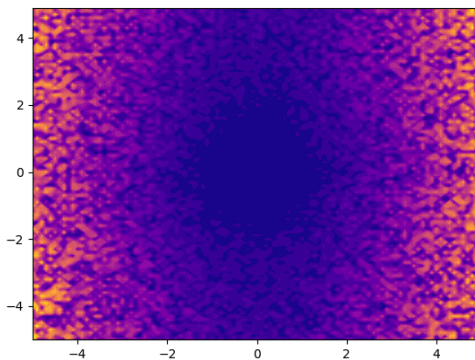
### 23. Trid



$$f(\mathbf{x}) = \sum_{i=1}^d (x_i - 1)^2 - \sum_{i=2}^d (x_i - x_{i-1})$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

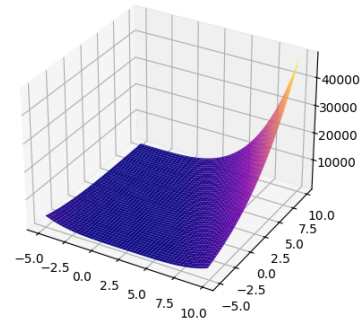
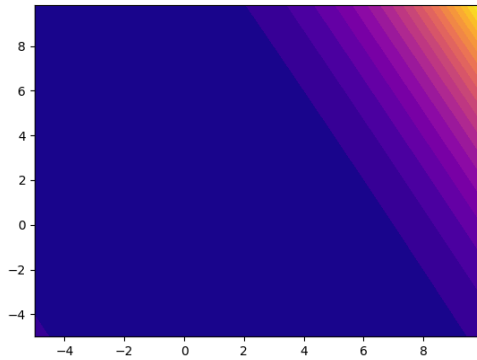
### 24. Xin-She Yang



$$f(\mathbf{x}) = \sum_{i=1}^d \text{random}_i[0, 1) |x_i|^i$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

## 25. Zakharov



$$f(\mathbf{x}) = \sum_{i=1}^n x_i^2 + \left( \sum_{i=1}^n 0.5ix_i \right)^2 + \left( \sum_{i=1}^n 0.5ix_i \right)^4$$

<https://towardsdatascience.com/optimization-eye-pleasure-78-benchmark-test-functions-for-single-objective-optimization-92e7ed1d1f12>

## Výsledky benchmarků pro 2 dimenze

Benchmark	DE rand/1/bin	DE best/1/bin	PSO	SOMA AtO	SOMA AtA
Ackley	5.63 e-16 (1)	0.15688 (3)	0.20000 (4)	6.17 e-09 (2)	1.77567 (5)
Alpine	3.26 e-16 (1)	0.03901 (4)	1.27 e-03 (3)	3.14 e-05 (2)	0.08692 (5)
Alpine2	-7.88560 (2)	-7.26075 (4)	-6.59552 (5)	-7.88560 (1)	-7.75353 (3)
DeJong Sphere	1.01 e-58 (1)	0.71070 (3)	1.16669 (4)	2.71 e-14 (2)	8.17741 (5)
DifferentPowersSum	1.90 e-21 (3)	3.44 e-06 (4)	5.24 e-35 (1)	2.45 e-22 (2)	1.49 e-04 (5)
DixonPrice	1.08 e-31 (2)	0.08996 (4)	3.04 e-32 (1)	2.16 e-15 (3)	0.10263 (5)
Exponential	-1.00000 (1)	-0.99964 (4)	-1.00000 (2)	-1.00000 (3)	-0.99949 (5)
Griewank	0.01813 (2)	0.12786 (5)	0.03444 (3)	0.01192 (1)	0.10152 (4)
Matyas	3.13 e-03 (2)	0.02217 (3)	0.03348 (4)	4.97 e-13 (1)	0.71780 (5)
Michalewicz	-1.87005 (1)	-1.86964 (4)	-1.87005 (2)	-1.87005 (3)	-1.86721 (5)
Michalewicz2	-1.80130 (3)	-1.72169 (5)	-1.80130 (1)	-1.80130 (2)	-1.75526 (4)
Qing	6.51 e-32 (2)	1.03 e-03 (5)	1.22 e-41 (1)	3.54 e-21 (3)	2.77 e-04 (4)
Quartic	0.02402 (2)	0.02483 (3)	0.01493 (1)	0.03057 (4)	0.04641 (5)
Rastrigin	0.23227 (2)	1.55742 (5)	1.02997 (3)	0.13440 (1)	1.24855 (4)
Rosenbrock	5.81 e-07 (1)	0.04584 (4)	0.03333 (3)	2.11 e-03 (2)	0.06377 (5)
RotatedHyperEllipsoid	4.89 e-29 (1)	0.08403 (3)	0.90777 (4)	3.65 e-16 (2)	15.69752 (5)
Salomon	0.07470 (4)	0.10249 (5)	0.06992 (3)	0.06991 (2)	0.06531 (1)
Schwefel	19.73975 (3)	55.27131 (5)	3.46694 (1)	11.84386 (2)	32.71650 (4)
Schwefel2	1.85 e-29 (1)	0.24575 (3)	0.30000 (4)	1.41 e-08 (2)	2.32014 (5)
Schwefel23	1.72 e-93 (2)	5.77 e-08 (4)	4.64 e-158 (1)	1.14 e-74 (3)	1.47 e-05 (5)
Styblinski	-78.33233 (1)	-74.53726 (5)	-75.97621 (4)	-77.86111 (3)	-77.87614 (2)
SumSquares	4.46 e-60 (1)	1.18 e-03 (4)	1.15 e-06 (3)	9.82 e-18 (2)	0.06528 (5)
Trid	1.77 e-58 (1)	6.91116 (4)	1.16667 (3)	4.01 e-15 (2)	13.44420 (5)
Xin-She Yang	9.58 e-05 (3)	1.29 e-04 (4)	7.43 e-05 (1)	7.71 e-05 (2)	1.84 e-03 (5)
Zakharov	1.78 e-56 (1)	1.99 e-03 (4)	4.31 e-32 (2)	4.35 e-17 (3)	0.06371 (5)
Average rank	2.28	2.96	2.80	2.88	4.08

### Největší rozdíly

Alpine2, Michalewicz2, Salomon, Schwefel, Styblinski

Podle očekávání jsou největší rozdíly u benchmarků s mnoha lokálními minimy. Algoritmus SOMA AtA, si velice dobře poradil s funkcemi Salomon, Styblinski a Alpine2, přičemž u ostatních funkcí propadl. Naopak algoritmy DE u těchto funkcí dopadly hůře než jejich průměrný rank.

## Výsledky benchmarků pro 10 dimenzí

Benchmark	DE rand/1/bin	DE best/1/bin	PSO	SOMA AtO	SOMA AtA
Ackley	3.65518 (3)	7.98972 (5)	3.10174 (2)	0.84531 (1)	7.83672 (4)
Alpine	3.25237 (3)	9.38008 (5)	2.18521 (2)	8.02 e-03 (1)	8.06091 (4)
Alpine2	-9.59 e+03 (2)	-2.45 e+03 (4)	-6.40 e+03 (3)	-1.65 e+04 (1)	-2.18 e+03 (5)
DeJong Sphere	2.03 e+03 (3)	6.92 e+03 (5)	10.50000 (2)	6.96 e-06 (1)	3.39 e+03 (4)
DifferentPower sSum	0.02023 (4)	0.06509 (5)	5.59 e-06 (2)	1.77 e-13 (1)	0.01332 (3)
DixonPrice	8.34 e+03 (4)	2.10 e+04 (5)	9.60000 (2)	0.35245 (1)	3.09 e+03 (3)
Exponential	-0.95569 (3)	-0.81570 (5)	-0.99688 (2)	-1.00000 (1)	-0.84668 (4)
Griewank	0.92480 (3)	2.51425 (5)	0.44921 (2)	0.15678 (1)	1.93954 (4)
Matyas	8.01 e+02 (3)	1.95 e+03 (5)	9.90000 (2)	1.01227 (1)	1.19 e+03 (4)
Michalewicz	-7.28296 (3)	-6.13004 (5)	-8.36904 (2)	-9.09616 (1)	-6.85243 (4)
Michalewicz2	-5.21871 (3)	-3.93195 (5)	-7.13334 (2)	-8.96600 (1)	-4.74036 (4)
Qing	68.12631 (3)	86.76856 (4)	8.46667 (1)	55.02997 (2)	87.28921 (5)
Quartic	3.04139 (3)	4.96924 (5)	1.76412 (1)	1.99497 (2)	3.85943 (4)
Rastrigin	60.11182 (3)	84.33975 (5)	10.76667 (1)	10.87404 (2)	64.45846 (4)
Rosenbrock	1.40 e+02 (3)	5.63 e+02 (5)	9.80000 (2)	6.79109 (1)	2.40 e+02 (4)
RotatedHyperE llipsoid	6.28 e+03 (3)	2.24 e+04 (5)	9.46667 (2)	9.37 e-05 (1)	1.76 e+04 (4)
Salomon	0.20115 (3)	0.29495 (5)	0.18322 (2)	0.17987 (1)	0.25967 (4)
Schwefel	1.31 e+03 (3)	2.03 e+03 (5)	9.93333 (1)	4.67 e+02 (2)	1.92 e+03 (4)
Schwefel2	89.20752 (3)	1.62 e+02 (5)	9.76667 (2)	0.22304 (1)	1.55 e+02 (4)
Schwefel23	1.06 e+07 (4)	2.75 e+07 (5)	9.33333 (2)	9.16 e-16 (1)	2.80 e+05 (3)
Styblinski	-3.50 e+02 (2)	-2.96 e+02 (4)	-3.43 e+02 (3)	-3.76 e+02 (1)	-2.94 e+02 (5)
SumSquares	90.88492 (3)	2.43 e+02 (5)	7.87375 (2)	5.98 e-06 (1)	1.69 e+02 (4)
Trid	1.08 e+04 (3)	1.58 e+04 (5)	10.76667 (2)	6.25 e-05 (1)	1.51 e+04 (4)
Xin-She Yang	1.74975 (4)	12.56258 (5)	0.01343 (2)	3.53 e-03 (1)	0.74880 (3)
Zakharov	95.10548 (5)	64.45464 (4)	9.50000 (2)	1.36 e-03 (1)	49.29648 (3)
Average rank	3.16	4.84	1.92	1.16	3.92

## Největší rozdíly

Alpine2, Qing, Styblinski, Zakharov

S větším počtem dimenzí se již rozdíly mezi jednotlivými funkcemi stírají a odchylka od průměru je podstatně nižší než při dvou dimenzích (průměrná směrodatná odchylka ranků 0.49 proti 0.94).

# Výsledky benchmarků pro 30 dimenzí

Benchmark	DE rand/1/bin	DE best/1/bin	PSO	SOMA AtO	SOMA AtA
Ackley	9.64924 (3)	11.91879 (4)	4.41117 (2)	3.45000 (1)	13.80990 (5)
Alpine	26.06921 (3)	48.60994 (4)	13.49226 (2)	2.23961 (1)	63.66653 (5)
Alpine2	-2.80 e+08 (3)	-1.11 e+07 (4)	-2.56 e+10 (2)	-3.15 e+11 (1)	-2.42 e+04 (5)
DeJong Sphere	2.30 e+04 (3)	4.20 e+04 (4)	24.23333 (1)	95.13014 (2)	6.69 e+04 (5)
DifferentPower sSum	0.02232 (3)	0.20700 (4)	1.15 e-06 (2)	1.11 e-08 (1)	0.64476 (5)
DixonPrice	2.00 e+05 (3)	8.79 e+05 (4)	24.90000 (1)	59.76790 (2)	1.65 e+06 (5)
Exponential	-0.38208 (3)	-0.12791 (4)	-0.93814 (2)	-0.99639 (1)	-0.04205 (5)
Griewank	6.50800 (3)	12.08915 (4)	1.54872 (2)	0.64872 (1)	17.41329 (5)
Matyas	6.08 e+03 (3)	1.95 e+04 (4)	26.00000 (1)	1.64 e+02 (2)	2.78 e+04 (5)
Michalewicz	-12.92337 (3)	-10.21303 (4)	-21.47243 (2)	-24.30501 (1)	-7.72656 (5)
Michalewicz2	-10.06550 (3)	-7.84348 (4)	-17.32450 (2)	-22.51722 (1)	-6.45638 (5)
Qing	6.44 e+03 (3)	6.74 e+03 (4)	23.43333 (1)	5.54 e+03 (2)	7.00 e+03 (5)
Quartic	29.50310 (3)	67.75695 (4)	10.72881 (1)	10.87658 (2)	1.15 e+02 (5)
Rastrigin	3.09 e+02 (3)	3.66 e+02 (4)	21.93333 (1)	53.09348 (2)	4.41 e+02 (5)
Rosenbrock	2.87 e+03 (3)	7.78 e+03 (4)	26.43333 (1)	61.84932 (2)	1.39 e+04 (5)
RotatedHyperE llipsoid	2.36 e+05 (3)	6.38 e+05 (4)	24.96667 (1)	1.25 e+03 (2)	9.25 e+05 (5)
Salomon	0.53694 (3)	0.71915 (4)	0.37654 (1)	0.37654 (2)	0.90982 (5)
Schwefel	7.79 e+03 (3)	8.98 e+03 (4)	24.53333 (1)	3.17 e+03 (2)	1.02 e+04 (5)
Schwefel2	6.55 e+02 (3)	9.21 e+02 (4)	25.90000 (1)	33.33848 (2)	1.14 e+03 (5)
Schwefel23	3.80 e+08 (3)	1.87 e+09 (4)	17.83333 (1)	2.12 e+02 (2)	7.14 e+09 (5)
Styblinski	-7.42 e+02 (3)	-6.20 e+02 (4)	-9.55 e+02 (2)	-1.06 e+03 (1)	-4.74 e+02 (5)
SumSquares	2.75 e+03 (3)	6.14 e+03 (4)	24.66667 (2)	9.81181 (1)	9.20 e+03 (5)
Trid	3.21 e+05 (3)	6.47 e+05 (4)	26.90000 (1)	9.62 e+02 (2)	9.12 e+05 (5)
Xin-She Yang	2.65 e+08 (3)	5.59 e+11 (4)	17.00000 (1)	28.61643 (2)	3.35 e+13 (5)
Zakharov	3.48 e+04 (3)	1.75 e+07 (4)	21.70000 (2)	10.11380 (1)	1.36 e+09 (5)
Average rank	3.00	4.00	1.44	1.56	5.00

## Největší rozdíly

Při 30 dimenzích se již neprojevují zásadní rozdíly mezi algoritmy, což je patrné už z průměrných ranků (průměrná směrodatná odchylka ranků pouze 0.2). SOMA AtA, DE best/1/bin a DE rand/1/bin se pokaždé umístily stejně a to na pátém, čtvrtém a třetím místě. Algoritmy PSO a SOMA AtO se umísťovaly střídavě na prvním a druhém místě. Explorativní



charakter těchto algoritmů umožnil najít konzistentně lepší řešení i při takto vysokém počtu dimenzí.

## Závěr

Se zadanými parametry jsou oba algoritmy DE převážně exploitativního charakteru, upřednostňují již nalezené dobré řešení před hledáním nových náhodných řešení. Algoritmus PSO je již ze své podstaty vyvážený. SOMA all-to-one se choval explorativně a stejné chování jsem očekával od SOMA all-to-all. Podle výsledků, kdy se při všech třech testech ukázal jako nejhorší, však jeho agresivní explorace znamenala dosažení limitu ohodnocení dříve než dokázal začít konvergovat ke globálnímu minimu. Zvýšením parametru `stepSize` nicméně došlo k výraznému zlepšení. Explorativním algoritmům se vedlo dobře i u náročných funkcí s mnoha lokálními minimy a bez zjevných gradientů.