CA1: EE6221 Robotics & Intelligent Sensors

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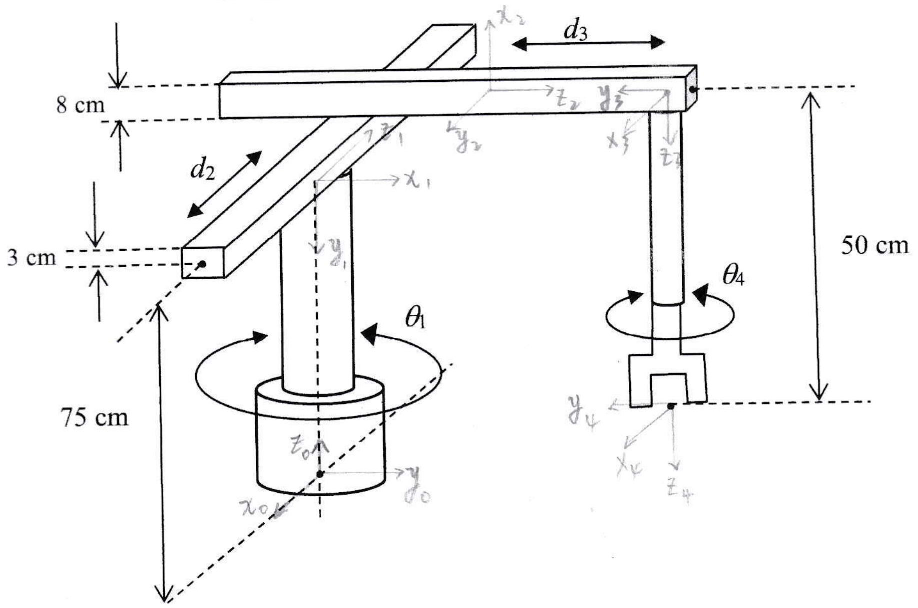
The following figure shows a robot with the joint variables $q = [\theta_1, d_2, d_3, \theta_4]^T$.

注意：建系位置/坐标系原点位置



i) Use Denavit-Hartenberg (D-H) algorithm to derive the kinematic parameters of the robot.

ii) Find the tool configuration vector of the robot. The link-coordinate homogeneous transformation matrix is given as below:

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $C\theta_k = \cos\theta_k$, $S\theta_k = \sin\theta_k$, and $\{\theta_k, d_k, a_k, \alpha_k\}$ are the kinematic parameters.

iii) Based on the tool configuration vector, comment on the limitations of the robot and suggest a method to improve on the design.

(70 Marks)

i)

axis	θ	d	a	α
1	θ_1	75cm	0	$-\frac{\pi}{2}$
2	$-\frac{\pi}{2}$	d_2	7cm	$-\frac{\pi}{2}$
3	$\frac{\pi}{2}$	d_3	0	$-\frac{\pi}{2}$
4	θ_4	50cm	0	0

ii) $T_{base}^{tool} = T_0^1 T_1^2 T_2^3 T_3^4$

$$= \begin{pmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -a_2 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} S\theta_1 & -C\theta_1 & 0 & d_3C\theta_1 - d_2S\theta_1 \\ -C\theta_1 & -S\theta_1 & 0 & d_3S\theta_1 + d_2C\theta_1 \\ 0 & 0 & -1 & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} S_{1-4} & -C_{1-4} & 0 & d_3C_1 - d_2S_1 \\ -C_{1-4} & S_{4-1} & 0 & d_3S_1 + d_2C_1 \\ 0 & 0 & -1 & 32cm \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \underline{w = \left(d_3C_1 - d_2S_1,\ d_3S_1 + d_2C_1,\ 32,\ 0,\ 0,\ -\exp\frac{\theta_4}{\pi}\right)^T}$$

iii)

(1): w_4 and w_5 both equal to zero, which means the robot is not enough flexible, adding more joints is better.

(2): w_3 is fixed, making it slideable is better to catch things on the direction of approaching vetor.

2. The dynamic model of a robot is given as

$$(2+3\cos\theta_2)\ddot{\theta}_1 + 5\ddot{\theta}_2 - (2\dot{\theta}_1^2 + 5\dot{\theta}_1\dot{\theta}_2)\sin\theta_2 + 50\cos\theta_1 + 20\cos(\theta_1+\theta_2) = u_1$$

$$7\ddot{\theta}_1 + (2+5\cos\theta_2)\ddot{\theta}_2 + 2\dot{\theta}_1^2\sin\theta_2 + 25\cos(\theta_1+\theta_2) = u_2$$

where θ_1, θ_2 are the joint variables, u_1, u_2 are the control inputs. The system possesses unmodelled resonance at 12.2 rad/sec. The desired trajectories for joint 1 and joints 2 are specified as $\theta_{1d}(t)$ and $\theta_{2d}(t)$ respectively.

(a) Design a computed torque PD controller such that joint 1 is critically damped and joint 2 is overdamped with a damping ratio of 1.15. $\xi = 1.15$ $\xi = 1$

(b) A student is considering the design a computed torque PI controller (i.e. there are also two control gains). Is this design feasible? Justify your answers.

(30 Marks)

(a) $M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = \tau$

$$\theta = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}, \quad \tau = \begin{bmatrix}u_1\\ u_2\end{bmatrix} \quad M(\theta) = \begin{bmatrix}2+3\cos\theta_2 & 5\\ 7 & 2+5\cos\theta_2\end{bmatrix}$$

$$C(\theta,\dot{\theta})\dot{\theta} = \begin{bmatrix}-(2\dot{\theta}_1^2+5\dot{\theta}_1\dot{\theta}_2)\sin\theta_2\\ 2\dot{\theta}_1^2\sin\theta_2\end{bmatrix} \quad g(\theta) = \begin{bmatrix}50\cos\theta_1 + 20\cos(\theta_1+\theta_2)\\ 25\cos(\theta_1+\theta_2)\end{bmatrix}$$

take this form : $\tau = \alpha v + \beta$, it gives:

$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = \alpha v + \beta$

$$\text{so,}\quad \alpha = M(\theta) = \begin{bmatrix}2+3\cos\theta_2 & 5\\ 7 & 2+5\cos\theta_2\end{bmatrix}$$

$$\beta = C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = \begin{bmatrix}-(2\dot{\theta}_1^2+5\dot{\theta}_1\dot{\theta}_2)\sin\theta_2 + 50\cos\theta_1 + 20\cos(\theta_1+\theta_2)\\ 2\dot{\theta}_1^2\sin\theta_2 + 25\cos(\theta_1+\theta_2)\end{bmatrix}$$

get the closed-loop equation: $\ddot{\theta} = v$ or $\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}v_1\\ v_2\end{bmatrix}$

the servo law becomes : $\begin{bmatrix}v_1\\ v_2\end{bmatrix} = \begin{bmatrix}\ddot{\theta}_{1d}\\ \ddot{\theta}_{2d}\end{bmatrix} + \begin{bmatrix}k_{v1} & 0\\ 0 & k_{v2}\end{bmatrix}\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\end{bmatrix} + \begin{bmatrix}k_{p1} & 0\\ 0 & k_{p2}\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}$ or

$$v = \ddot{\theta}_d + k_v\dot{E} + k_pE$$

so, the error equation would be: $\begin{bmatrix}\ddot{e}_1\\ \ddot{e}_2\end{bmatrix} + \begin{bmatrix}k_{v1} & 0\\ 0 & k_{v2}\end{bmatrix}\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\end{bmatrix} + \begin{bmatrix}k_{p1} & 0\\ 0 & k_{p2}\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix} = 0$

and the closed-loop characteristics equation is: $s^2 + k_{vi}s + k_{pi} = 0$

and the $\xi_1 = 1$ (for joint 1), $\xi_2 = 1.15$ (for joint 2)

$w_n \le 0.5 w_{res} = 0.5 \times 12.2 = 6.1\ \text{rad/s}$

$s^2 + 2\xi_1 w_n s + w_n^2 = s^2 + 12.2s + 37.21 = 0$

$s^2 + 2\xi_2 w_n s + w_n^2 = s^2 + 14.03s + 37.21 = 0$

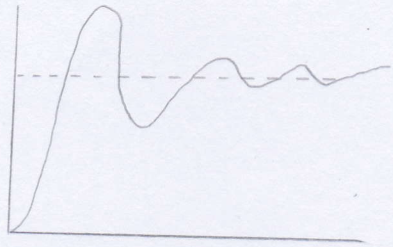
$k_{v1} = 12.2 \quad k_{p1} = 37.21$

$k_{v2} = 14.03 \quad k_{p2} = 37.21$

The overall control law is: $\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \begin{bmatrix}2+3\cos\theta_2 & 5\\ 7 & 2+5\cos\theta_2\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} + \begin{bmatrix}-(2\dot{\theta}_1^2+5\dot{\theta}_1\dot{\theta}_2) + 50\cos\theta_1 + 20\cos(\theta_1+\theta_2)\\ 2\dot{\theta}_1^2\sin\theta_2 + 25\cos(\theta_1+\theta_2)\end{bmatrix}$

(b) the formula of PI controller is: $\begin{cases} u(t) = k_p \cdot e(t) + k_i \cdot \int_0^t e(t)\, dt \\ u(t) = k_p \cdot e(t) + k_i \cdot \sum_{n=0}^{t} e(n) \end{cases}$

and the figure is:



the intergation can help to eliminate state error, but if the k_p and k_i are not suitable, it can take longer time to reach steady state, even never reach it. On robot, it can decrease efficiency and the error may cause harm to people or parts.

Besides, the characterstic equation is $s^3 + k_p s + k_i = 0$, the parameters of s^2 is zero, according to routh hurwitz stability criterion, this system is unstable.

Because $0 \times k_p \leq 1 \times k_i$