Histogram and Its Applications to Image Analysis

1. Definition and Properties of Histogram

1.1 Definition

Histogram is a kind of tool to calculate the number or frequency of pixels with different grey levels of an image, to provide information about frequency [1], function can be written as:

$$P_i(i) = \frac{n_i}{N}$$

$P_i(i)$ is the frequency of occurrence of gray level i in this picture, and the i = [0, L]. N is the total amount of pixels, while n_i represents the figure for specific pixels with i grey level. Therefore, image histogram demonstrates the frequency distribution of gray-level.

1.2 Properties

According to statistics, the $\mathrm{P_i(i)}$ >= 0 and the sum of each $\mathrm{P_i(i)}$ is 1. Therefore, we can estimate the probability of occurrence of gray-level. Histogram has no information about the position of pixels, it can only show the distribution of grey-level and the characteristics, such as brightness and contrast. Besides, the number of modes in histogram can reflect the general structure of an image [1]. I use opencv to show the properties in figure 1 and 2.

Figure 1 color image and grey image

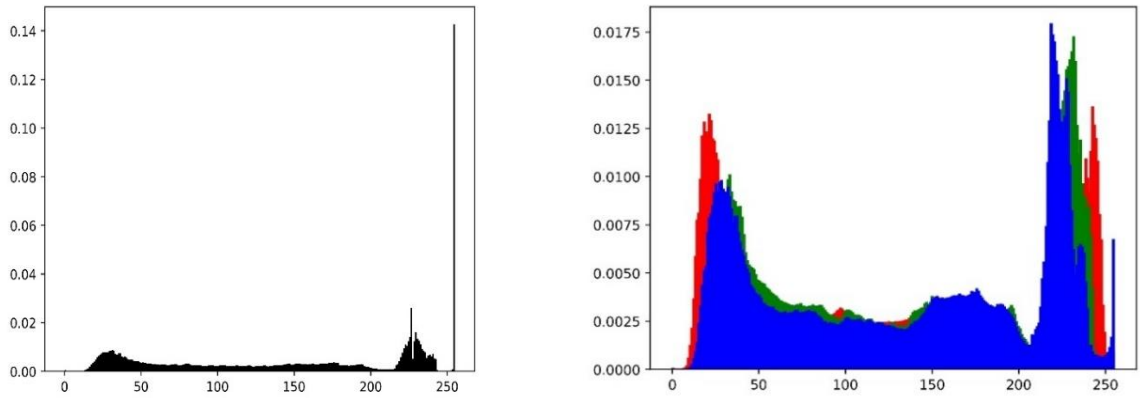

Figure 2 image histograms

2. Histogram Equalization

2.1 Principles

The aim of histogram equalization is acquiring a uniform histogram of input image by transforming grey-level. The function is:

$$c(f) = \sum_{t=0}^{i} P_i(i) = \sum_{t=0}^{i} \frac{n_i}{N} \mathrm{i} = 0,1,2, \dots , \quad \mathrm{L}$$

2.2 Merits and limitations

To acquire clear image, the idea histogram equalization is to expand the gray-level with more pixels while decrease the gray-level with less pixels. [2] Therefore, after histogram equalization, the image can better show the global contrast. However, in figure 4, overexposure can be seen, and some details lost in this process. That is the main

limitation of general histogram equalization, in other words, global histogram equalization.[2] Besides, to solve the problems above, we can use adaptive histogram equalization, and figure 5 shows the result.

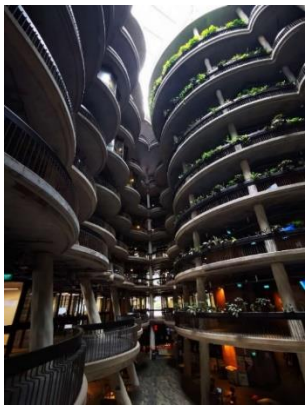

Figure 3 the photo of inside hive

Figure 4 before(left) and after(right) histogram equalization

Figure 5 global(middle) and adaptive(right) histogram equalization

There is also another way to solve overexposure and loss of details. Using adaptive entropy index to apply histogram equalization [3]. This approach use 3 sub-images to keep the details of image and then combine them by a parameter called adaptive entropy index after histogram equalization.

3. Other Application of Histogram

3.1 Hough Transform

Hough transform can be used to connect curves such as lines, circles, and ellipses in a picture. It transforms the points in rectangular coordinate system into polar coordinates, and the problem becomes finding the meeting point of the curve. Equation (1) is mapped into equation (2) while generalized Hough transform can be represented by equation (3). v is a vector of coordinates; c is a vector of coefficients.

$$y = ax + b \quad (1)$$

$$x\cos\theta + y\sin\theta = \rho \quad (2)$$

$$g(v,c) = 0 \quad (3)$$

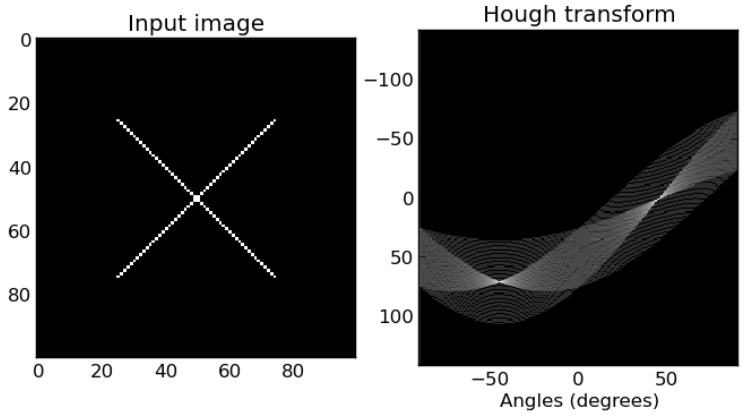

Figure 6 the coordinate transformation of Hough transform

Figure 7 is the result after detecting, almost every kind of curve are detected and signed in blue line, However, the result of detecting on the margin of shadow is not good because of the light, but to some degree, the margin of shadow is also a kind of curve.

Figure 7 apply hough transform to my card

Reference

[1] Gonzalez, R., Woods, R., & Masters, B. (2009). Digital Image Processing, Third Edition. Journal of Biomedical Optics, 14(2), 29901–. https://doi.org/10.1117/1.3115362

[2] S. Patel and M. Goswami, "Comparative analysis of Histogram Equalization techniques," 2014 International Conference on Contemporary Computing and Informatics (IC3I), Mysore, India, 2014, pp. 167-168, doi: 10.1109/IC3I.2014.7019808.

[3] S. H. Majeed and N. A. M. Isa, "Adaptive Entropy Index Histogram Equalization for Poor Contrast Images," in IEEE Access, vol. 9, pp. 6402-6437, 2021, doi: 10.1109/ACCESS.2020.3048148.