7. (a) $G(s)=\dfrac{2s}{s^2+6s+5}$, bilinear transform: $s=\dfrac{2}{T}\dfrac{1-z^{-1}}{1+z^{-1}}$

$$= \frac{z-1}{z+1} \quad (T=2s)$$

$$\Rightarrow G(z)=\frac{2\left(\frac{z-1}{z+1}\right)}{\left(\frac{z-1}{z+1}\right)^2+6\left(\frac{z-1}{z+1}\right)+5}=\frac{2(z-1)(z+1)}{(z-1)^2+6(z-1)(z+1)+5(z+1)^2}$$

$$=\frac{2z^2-2}{z^2-2z+1+6z^2-6+5z^2+10z+5}=\frac{2z^2-2}{12z^2+8z}=\frac{z^2-1}{6z^2+4z}$$

$$G(s)=\frac{2s}{(s+1)(s+5)}=\frac{A}{s+1}+\frac{B}{s+5}, \quad (A+B)s+(5A+B)=2s$$

$$A+B=2, \quad 5A+B=0$$

$$=-\frac{1}{2}\frac{1}{s+1}+\frac{5}{2}\frac{1}{s+5} \qquad \Rightarrow A=-\frac{1}{2}, \quad B=\frac{5}{2}$$

$$E(s)=\frac{1}{s}, \quad C(s)=G(s)\cdot E(s)=\frac{1}{(s+1)(s+5)}=\frac{1}{4}\times\left(\frac{1}{s+1}-\frac{1}{s+5}\right)$$

$$\Rightarrow c(t)=\frac{1}{4}(e^{-t}-e^{-5t}), \quad c(kT)=\frac{1}{4}(e^{-kT}-e^{-5kT})$$

$$k=0: \ c(0)=0, \qquad k=1: \ c(T)=\frac{1}{4}(e^{-T}-e^{-5T})=\frac{1}{4}(e^{-2}-e^{-10})$$

$$k=2: \ c(2T)=\frac{1}{4}(e^{-4}-e^{-20}), \qquad k=3: \ c(3T)=\frac{1}{4}(e^{-6}-e^{-30})$$

(b) $U(z)=\dfrac{Y(z)}{G_{ZAS}(z)}=\dfrac{Y(z)}{R(z)}\dfrac{R(z)}{G_{ZAS}(z)}=G_{cl}(z)\dfrac{R(z)}{G_{ZAS}(z)}$, $R(z)=\dfrac{1}{1-z^{-1}}$

$$\Rightarrow U(z)=\frac{(1-0.25z^{-1})(1-0.92z^{-1})}{(1-z^{-1})(1+0.51z^{-1})z^{-1}}\times G_{cl}(z)$$

$$\Rightarrow G_{cl}(z)=K\times\cancel{(1-z^{-1})}(1+0.51z^{-1})z^{-1}$$

$$U(z)=K\times(1-0.25z^{-1})(1-0.92z^{-1})$$

$$G_{cl}(1)=1, \quad (1+0.51)K=1, \quad K=\frac{1}{1.51}\approx 0.6623$$

$$\Rightarrow G_{cl}(z)=0.6623(1+0.51z^{-1})z^{-1}$$

$$1-G_{cl}(z)=1-0.6623z^{-1}(1+0.51z^{-1})$$

$$\Rightarrow C(z)=\frac{1}{G_{ZAS}(z)}\frac{G_{cl}(z)}{1-G_{cl}(z)}=\frac{(1-0.25z^{-1})(1-0.92z^{-1})}{\cancel{z^{-1}(1+0.51z^{-1})}}\,\frac{0.6623\cancel{z^{-1}(1+0.51z^{-1})}}{1-0.6623z^{-1}(1+0.51z^{-1})}$$

$$=0.6623\times\frac{1-1.17z^{-1}+0.23z^{-2}}{1-0.1523z^{-1}-0.3378z^{-2}}$$

$$C(z) = 0.6623 \times \frac{1 - 1.17z^{-1} + 0.23z^{-1}}{1 - 0.1523z^{-1} - 0.3378z^{-2}}$$

$$C(z=1) = 0.6623 \times \frac{1 - 1.17 + 0.23}{1 - 0.1523 - 0.3378} = 0.0779$$

(c) final value theorem: $y(\infty) = \lim_{k\to\infty} y(kT) = \lim_{z\to 1}(1-z^{-1})Y(z)$

$$Y(z) = G_{cl}(z) \times R(z) = 0.6623z^{-1}(1+0.51z^{-1}) \times \frac{1}{1-z^{-1}}$$

$$\Rightarrow y(\infty) = \lim_{z\to 1} Y(z) = \lim_{z\to 1}(0.6623z^{-1}(1+0.51z^{-1}))$$

$$= 0.6623 \times 1.51 = 1$$

2. (a) $G(s) = \frac{2}{s(s+4)}$, $G(z) = (1-z^{-1})\mathcal{Z}\left[\frac{G(s)}{s}\right]$

$$\Rightarrow G_{ZAS}(z) = (1-z^{-1})\mathcal{Z}\left[\frac{2}{s^2(s+4)}\right] = \frac{1}{8}(1-z^{-1})\mathcal{Z}\left[\frac{4^2}{s^2(s+4)}\right]$$

$$= \frac{1}{8}(1-z^{-1})\frac{[(4T-1+e^{-4T}) + (1-e^{-4T}-4Te^{-4T})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-4T}z^{-1})}$$

$$= \frac{1}{8} \times \frac{[1+e^{-2} + (1-e^{-2}-2e^{-2})z^{-1}]z^{-1}}{(1-z^{-1})(1-e^{-2}z^{-1})}$$

$$= \frac{1}{8} \times \frac{[1.1353 + 0.5940z^{-1}]z^{-1}}{(1-z^{-1})(1-0.1353z^{-1})}$$

(b) $C(z) = \frac{1}{G_{ZAS}(z)}\frac{G_{cl}(z)}{1-G_{cl}(z)}$, $G_{ZAS}(z) = 0.1\frac{z-0.2}{(z-0.1)(z-1)}$

$G_{cl}(z) = \frac{z-0.2}{(z-1)^2}$, $1 - G_{cl}(z) = \frac{(z-1)^2 - z + 0.2}{(z-1)^2} = \frac{z^2-3z+1.2}{(z-1)^2}$

$$\Rightarrow C(z) = \frac{(z-0.1)(z-1)}{0.1(z-0.2)}\frac{z-0.2}{z^2-3z+1.2} = \frac{z^2-1.1z+0.1}{0.1z^2-0.3z+0.12}$$

(c) Comment

5. (a) $\ddot{y}(t)+2\dot{y}(t)=-y(t)+7u(t)$, $x_1(t)=y(t)$, $x_2(t)=\dot{y}(t)$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 7\end{bmatrix}u(t).$$

$\dot{x}_1(t)=\dot{y}(t)=x_2(t)$

$\dot{x}_2(t)=\ddot{y}(t)$

(ii) $x_1(t)=y(t)$, $\dot{x}_1(t)=\dot{y}(t)=x_2(t)-x_1(t)$

$x_2(t)=y(t)+\dot{y}(t)$, $\dot{x}_2(t)=\dot{y}(t)+\ddot{y}(t)=-\dot{y}(t)-y(t)+7u(t)$

$=-x_2(t)+7u(t)$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 7\end{bmatrix}u(t)$$

(b) $K=[0\ \ 1]\,W_c^{-1}\alpha_c(A)$, $\alpha_c(z)=z^2$,

$$W_c^{-1}=[B\ AB]^{-1}=\begin{bmatrix}0.1152 & 5.0081\\ 22.12 & 17.2271\end{bmatrix}^{-1}=\frac{\begin{bmatrix}17.2271 & -5.0081\\ -22.12 & 0.1152\end{bmatrix}}{-108.7946}$$

$$=\begin{bmatrix}-0.1583 & 0.0460\\ 0.2033 & -0.0011\end{bmatrix}$$

$$\alpha_c(A)=A^2=\begin{bmatrix}1 & 0.2212\\ 0 & 0.7788\end{bmatrix}\begin{bmatrix}1 & 0.2212\\ 0 & 0.7788\end{bmatrix}=\begin{bmatrix}1 & 0.3935\\ 0 & 0.6065\end{bmatrix}$$

$$\Rightarrow K=[0\ \ 1]\begin{bmatrix}-0.1583 & 0.0460\\ 0.2033 & -0.0011\end{bmatrix}\begin{bmatrix}1 & 0.3935\\ 0 & 0.6065\end{bmatrix}$$

$$=[0.2033\ \ -0.0011]\begin{bmatrix}1 & 0.3935\\ 0 & 0.6065\end{bmatrix}=[0.2033\ \ 0.0793]$$

$\Rightarrow u(k)=-Kx(k)=-[0.2033\ \ 0.0793]\,x(k)$

(ii) $L_o=\alpha_o(A)\,W_o^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}$, $\alpha_o(A)=A^2$

$$W_o^{-1}=\begin{bmatrix}C\\ CA\end{bmatrix}^{-1}=\begin{bmatrix}1 & 0\\ 1 & 0.2212\end{bmatrix}^{-1}=\frac{\begin{bmatrix}0.2212 & 0\\ -1 & 1\end{bmatrix}}{0.2212}=\begin{bmatrix}1 & 0\\ -4.5208 & 4.5208\end{bmatrix}$$

$$\Rightarrow L_o=\begin{bmatrix}1 & 0.3935\\ 0 & 0.6065\end{bmatrix}\begin{bmatrix}1 & 0\\ -4.5208 & 4.5208\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$=\begin{bmatrix}-0.7789 & 1.7789\\ -2.7419 & 2.7419\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1.7789\\ 2.7419\end{bmatrix}$$

4. (a) $\dot{x}(t) = A x(t) + B u(t)$

$$\Phi(T) = \mathcal{L}^{-1}\{[SI - A]^{-1}\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}^{-1}\right\} = \mathcal{L}^{-1}\left\{\frac{\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S(S+3)+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{S+3}{(S+1)(S+2)} & \frac{1}{(S+1)(S+2)} \\ \frac{-2}{(S+1)(S+2)} & \frac{S}{(S+1)(S+2)} \end{bmatrix}\right\}$$

S^2+3S+2

$(S+1)(S+2)$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{2}{S+1} - \frac{1}{S+2} & \frac{1}{S+1} - \frac{1}{S+2} \\ -2\left(\frac{1}{S+1} - \frac{1}{S+2}\right) & \frac{2}{S+2} - \frac{1}{S+1} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ -2e^{-T} + 2e^{-2T} & 2e^{-2T} - e^{-T} \end{bmatrix}$$

$$\Theta(T) = \int_0^T \Phi(\tau) B \, d\tau = \int_0^T \begin{bmatrix} 2e^{-\tau} - 2e^{-2\tau} \\ 4e^{-2\tau} - 2e^{-\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} -2e^{-\tau} + e^{-2\tau} \\ -2e^{-2\tau} + 2e^{-\tau} \end{bmatrix} \Bigg|_0^T$$

$$= \begin{bmatrix} -2e^{-T} + e^{-2T} + 2 - 1 \\ -2e^{-2T} + 2e^{-T} + 2 - 2 \end{bmatrix}$$

$$x(k+1) = \Phi(T) x(k) + \Theta(T) u(k)$$

$$W_c = [B \;\; AB]$$

(b) $S(2)=\begin{bmatrix}0&0\\0&0\end{bmatrix}$, $K(1)=\left(\begin{bmatrix}0.25&0.5\end{bmatrix}S(2)\begin{bmatrix}0.25\\0.5\end{bmatrix}+4\right)^{-1}B^TS(2)A$

$= 0$

$S(1)=\left(\begin{bmatrix}2&2\\0&2\end{bmatrix}-\begin{bmatrix}0.25\\0.5\end{bmatrix}0\right)^T\cdot 0+0+Q=\begin{bmatrix}2&0\\0&2\end{bmatrix}$

$K(0)=\left(\begin{bmatrix}0.25&0.5\end{bmatrix}\begin{bmatrix}2&0\\0&2\end{bmatrix}\begin{bmatrix}0.25\\0.5\end{bmatrix}+\underset{r}{4}\right)^{-1}\begin{bmatrix}0.25&0.5\end{bmatrix}\begin{bmatrix}2&0\\0&2\end{bmatrix}\begin{bmatrix}2&2\\0&2\end{bmatrix}$

$=\left(\begin{bmatrix}0.5&1\end{bmatrix}\begin{bmatrix}0.25\\0.5\end{bmatrix}+4\right)^{-1}\begin{bmatrix}0.5&1\end{bmatrix}\begin{bmatrix}2&2\\0&2\end{bmatrix}$

$=(0.625+4)^{-1}\begin{bmatrix}1&3\end{bmatrix}=\begin{bmatrix}0.2162&0.6486\end{bmatrix}$

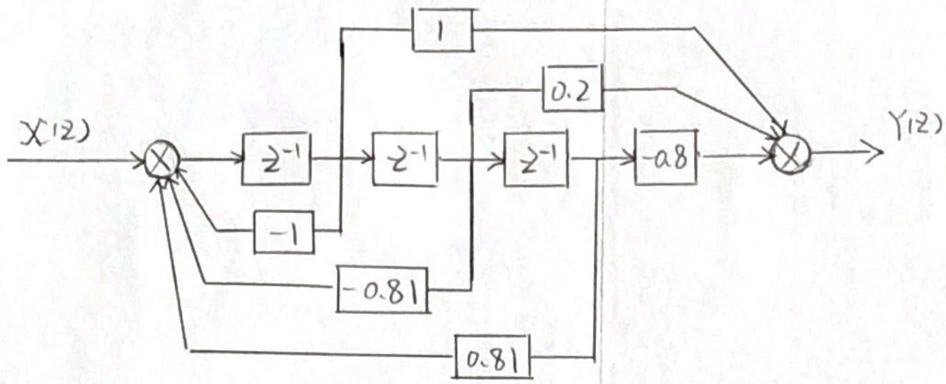
for $r=40$:

$K(0)=(0.625+40)^{-1}\begin{bmatrix}1&3\end{bmatrix}=\begin{bmatrix}0.0246&0.0738\end{bmatrix}$

comment: large K will speed up the convergence process.

5. (a) $C(z) = \dfrac{z^2+0.2z-0.8}{(z-1)(z^2-0.81)} = \dfrac{z^{-1}+0.2z^{-2}-0.8z^{-3}}{1-z^{-1}-0.81z^{-2}+0.81z^{-3}}$

$y(k) - y(k-1) - 0.81y(k-2) + 0.81y(k-3) = x(k-1) + 0.2x(k-2) - 0.8x(k-3)$



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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014 - 2015

EE6203 – COMPUTER CONTROL SYSTEMS

November/December 2014 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises EIGHT (8) pages.
2. Answer all FIVE (5) questions.
3. All questions carry equal marks.
4. The Transform Table is included in Appendix A on pages 6 to 8.

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1. (a) Consider the following second order system:

$$G(s) = \frac{C(s)}{E(s)} = \frac{2s}{s^2 + 6s + 5}$$

Assume that a sampling period T of 2 seconds is used. Discretise the system using the bilinear transformation method.

If a unit-step input signal $e(kT)$ is applied to the system at $k = 0$, determine the response $c(kT)$ for $k = 0,1,2,3$.

(10 Marks)

(b) A particular plant has the following transfer function:

$$G_{ZAS}(z) = \frac{(z + 0.51)}{(z - 0.25)(z - 0.92)}$$

Design a digital controller so that a zero steady-state error can be achieved with a step input. Justify your design.

(6 Marks)

(c) State the final value theorem and use it to show the steady-state value obtained in part 1(b).

(4 Marks)

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2. (a) A particular plant has the following transfer function:

$$G(s) = \frac{2}{s(s+4)}$$

Find its discretized transfer function $G_{ZAS}(z)$ if a zero-order-hold is used. Assume that the desired sampling period is 0.5 second.

(6 Marks)

(b) For a given plant with a transfer function $G_{ZAS}(z)$ and a digital controller $C(z)$, the corresponding close-loop transfer function $G_{cl}(z)$ can be written as:

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$$

Derive an expression for $C(z)$ in terms of $G_{ZAS}(z)$ and $G_{cl}(z)$.

Given that the plant in part 2(a) can be approximated as:

$$G_{ZAS}(z) = 0.1\frac{(z-0.2)}{(z-0.1)(z-1)}$$

Assuming that you want to achieve the following closed loop transfer function:

$$G_{cl}(z) = \frac{(z-0.2)}{(z-1)^2}$$

what would be the digital controller?

(10 Marks)

(c) Comment on the controller obtained in part 2(b).

(4 Marks)

3. (a) An industrial process is described by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} = -y(t) + 7u(t)$$

where $u(t)$ and $y(t)$ are the input and output variables, respectively.

Note: Question No. 3 continues on Page 3

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(i) If the state variables are defined as:

$$x_1(t) = y(t) \text{ and } x_2(t) = \frac{dy(t)}{dt}$$

Obtain a state-space model for the continuous-time system.

(ii) If the state variables are defined as:

$$x_1(t) = y(t) \text{ and } x_2(t) = y(t) + \frac{dy(t)}{dt}$$

Obtain a state-space model for the continuous-time system.

(8 Marks)

(b) A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}u(k)$$

$$y(k) = \mathbf{C}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 0.2212 \\ 0 & 0.7788 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.1152 \\ 22.12 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$ and $u(k)$ are the states and input variables, respectively.

(i) Design a deadbeat controller of the following form

$$u(k) = -\mathbf{K}\mathbf{x}(k)$$

(ii) Design a full-order prediction estimator of the following form

$$\overline{\mathbf{x}}(k+1) = \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_o(y(k) - \mathbf{C}\overline{\mathbf{x}}(k))$$

that will give deadbeat error response.

(12 Marks)

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4. (a) A continuous-time system has a state-space representation given by

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. Suppose that the system is sampled with a zero-order hold and a sampling period $T = 0.1$ second, obtain a discretised state-space model for the system.

Determine the controllability of the discretized system.

(9 Marks)

(b) A process is described by the following state-space representation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\,u(k)$$

where $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$. The performance index for the system is given by

$$J = \frac{1}{2}\mathbf{x}^T(N)\mathbf{S}(N)\mathbf{x}(N) + \frac{1}{2}\sum_{k=0}^{N-1}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + ru^2(k)\right)$$

and the design equations for $k = N-1, \ldots, 1, 0$ are

$$\mathbf{K}(k) = \left(\mathbf{B}^T\mathbf{S}(k+1)\mathbf{B} + r\right)^{-1}\mathbf{B}^T\mathbf{S}(k+1)\mathbf{A}$$

$$u^*(k) = -\mathbf{K}(k)\mathbf{x}(k)$$

$$\mathbf{S}(k) = \left(\mathbf{A} - \mathbf{B}\mathbf{K}(k)\right)^T\mathbf{S}(k+1)\left(\mathbf{A} - \mathbf{B}\mathbf{K}(k)\right) + r\mathbf{K}(k)^T\mathbf{K}(k) + \mathbf{Q}$$

If $\mathbf{Q} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $\mathbf{S}(N) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $N = 2$, find the gains required to minimize J for (i) $r = 4$ (ii) $r = 40$.

(8 Marks)

(c) Comment on the results obtained in part 4(b).

(3 Marks)

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5. (a) For a DC motor speed control system, the digital controller is designed such that it yields zero steady-state error to a step input. The controller can be described by the following transfer function:

$$C(z) = \frac{Y(z)}{X(z)} = \frac{(z-0.8)(z+1)}{(z-1)(z+0.9)(z-0.9)}$$

If it is desired to implement this controller using standard programming approach, show a block diagram of its representation.

(8 Marks)

(b) The denominator polynomial of the controller in part 5(a) can be written as

$$\begin{aligned} P(z^{-1}) &= (1-z^{-1})(1+0.9z^{-1})(1-0.9z^{-1}) \\ &= 1 - z^{-1} - 0.81z^{-2} + 0.81z^{-3} \end{aligned}$$

Perform a sensitivity analysis of the poles $p_1 = 1$ and $p_2 = -0.9$ with respect to the coefficient $b = -1$.

(10 Marks)

(c) Comment on the results obtained in part 5(b).

(2 Marks)