

# Quantum Phase Estimation - Report

## Exact Eigenvalue Estimation of Hermitian Matrices

Let  $H$  be a Hermitian matrix. Let  $|\lambda\rangle$  be an eigenvector of  $H$  with eigenvalue  $\lambda \implies e^{iHt}|\lambda\rangle = e^{i\lambda t}|\lambda\rangle$

Of course, the measurements of QPE will *not* give us a direct approximation of  $\lambda$ , but rather of some "phase"  $\theta$  such that:

$$e^{i\lambda t} = e^{i\theta} \implies \lambda = \frac{\theta}{t}$$

Recall that if the phase  $\theta$  happens to be of the form  $\frac{2\pi k}{2^n}$  then we can measure the ancilla qubits and get  $k$  exactly.

We can "force" this to happen. Let us say we pre-compute all eigenvalues  $\lambda_1, \dots, \lambda_n$  of  $H$  and want to estimate the eigenvalue  $\lambda_i$ . Well, we can simply choose  $t$  such that:

$$\lambda_i = \frac{2\pi k}{t2^n}$$

This is equivalent to choosing:

$$t = \frac{2\pi k}{\lambda_i 2^n}$$

### Hamiltonian used for this simulation

#### Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

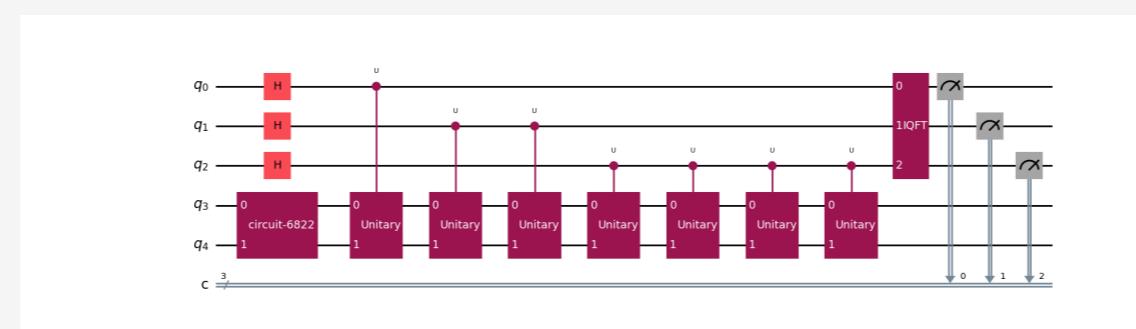
#### Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

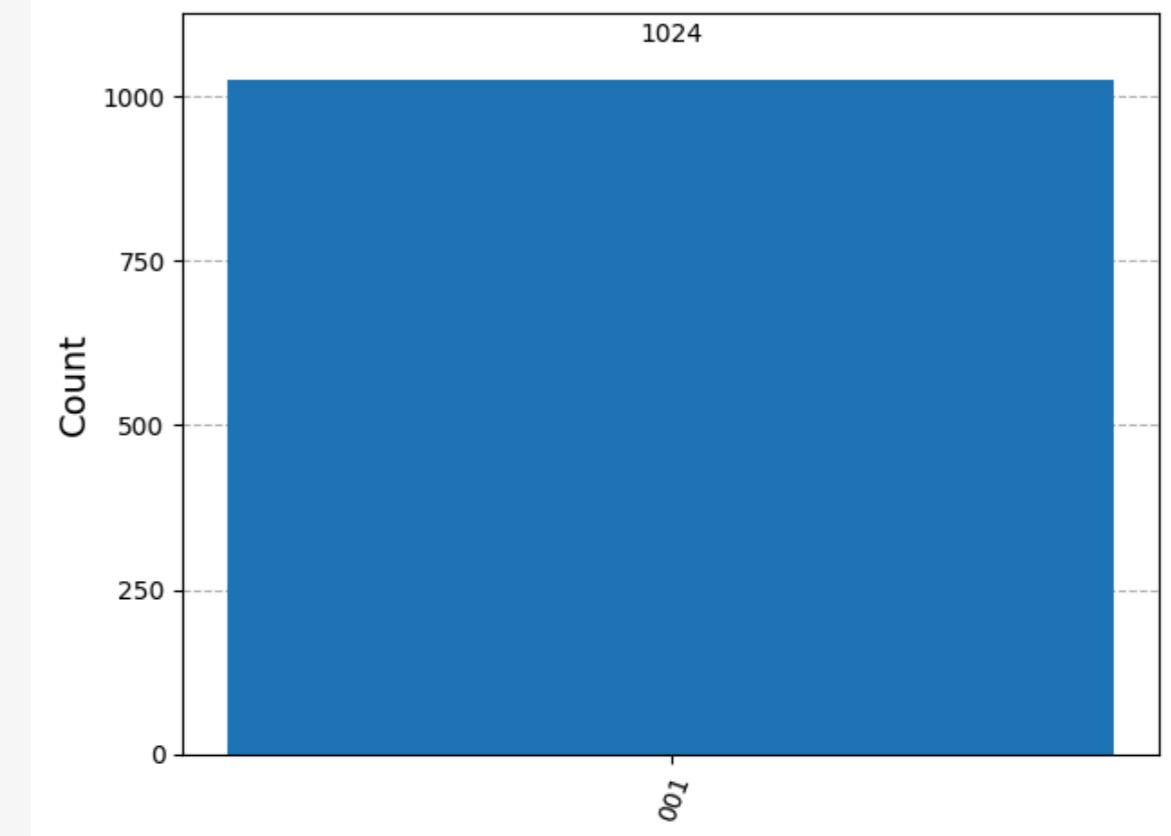
Time: -0.2513998258660903 | Ancilla: 3

Expected Binary: 001

Measured: 001



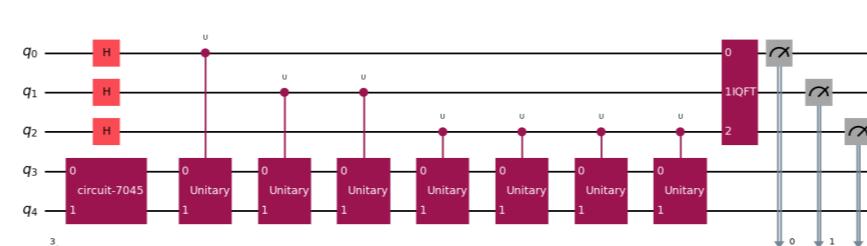
Quantum Circuit Diagram



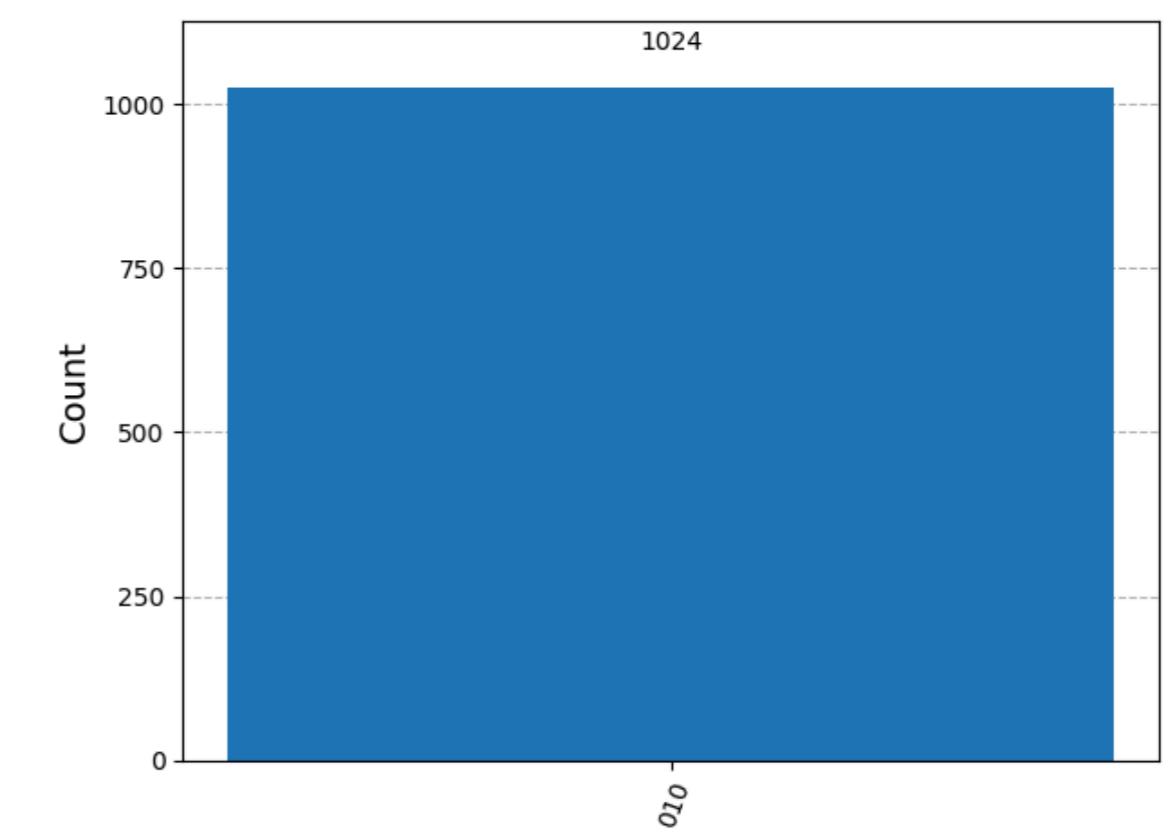
Time: -0.5027996517321806 | Ancilla: 3

Expected Binary: 010

Measured: 010



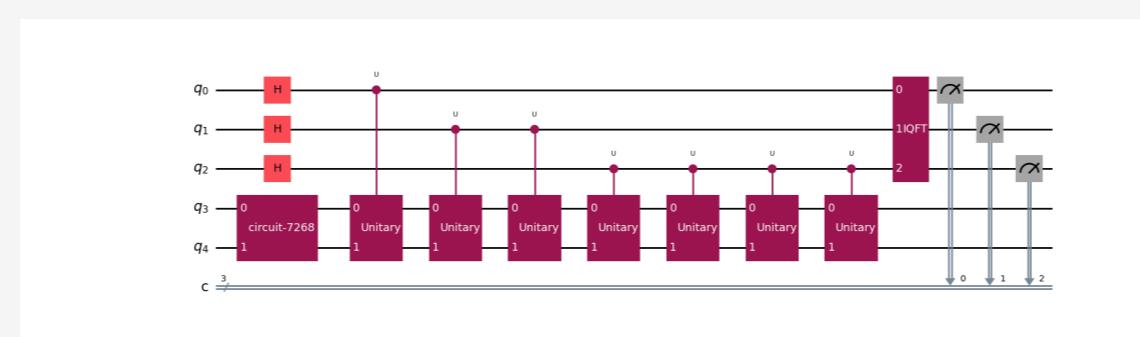
Quantum Circuit Diagram



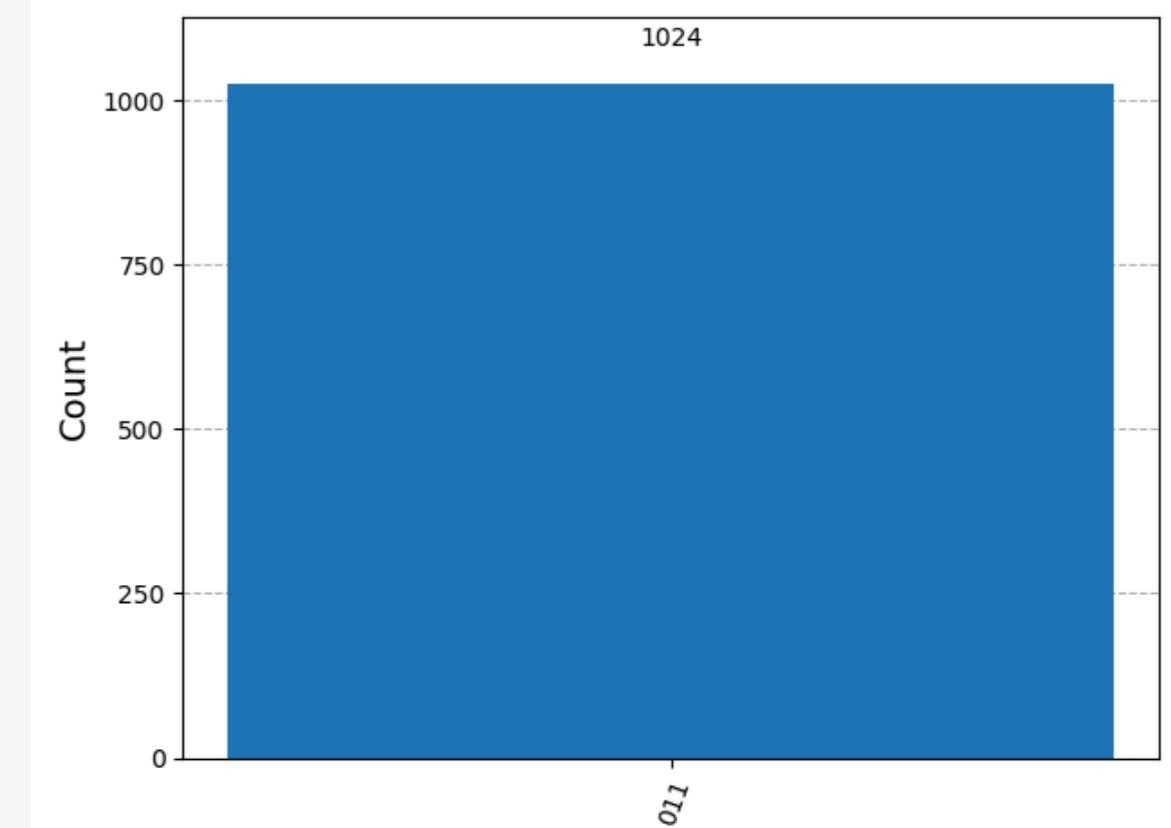
Time: -0.7541994775982708 | Ancilla: 3

Expected Binary: 011

Measured: 011



Quantum Circuit Diagram

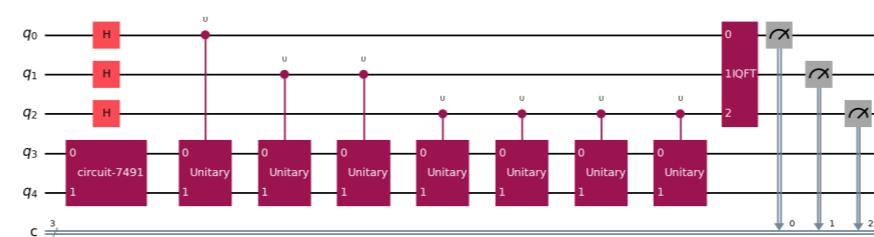


Results for Exact Phase Estimation

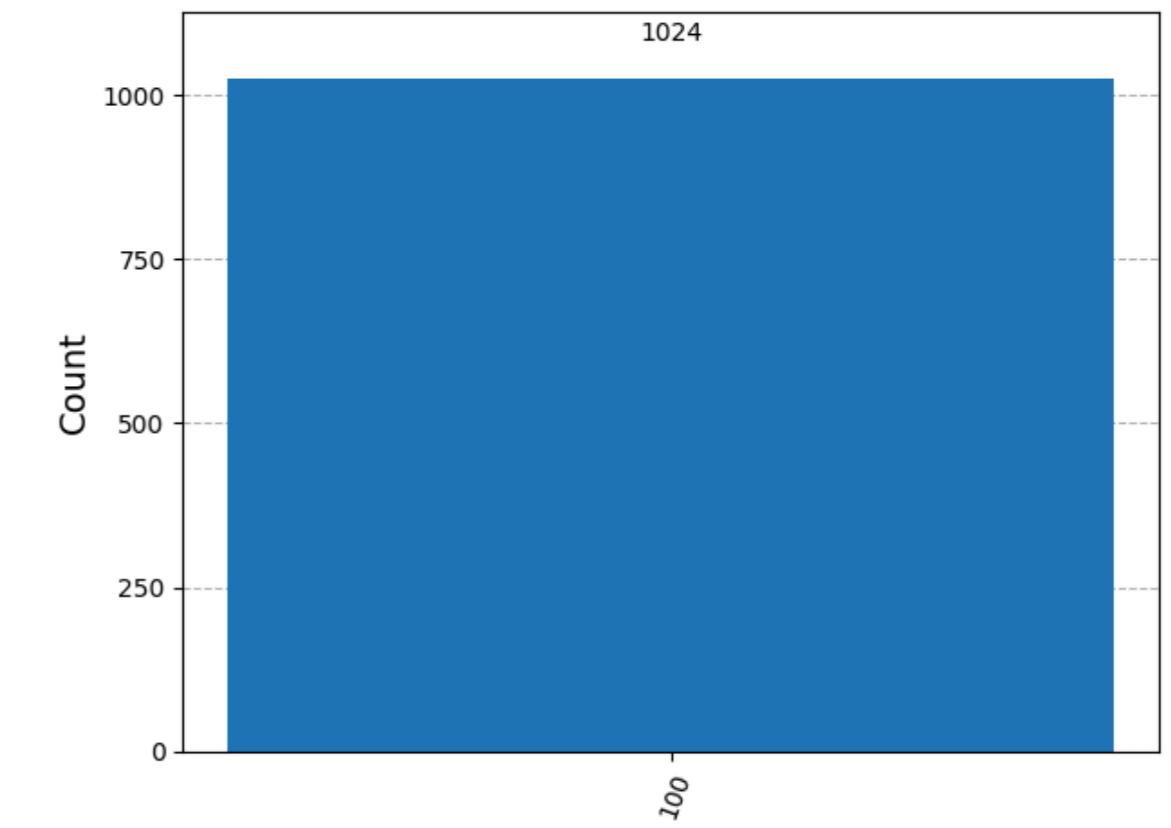
Time: -1.005599303464361 | Ancilla: 3

Expected Binary: 100

Measured: 100



Quantum Circuit Diagram



Results for Exact Phase Estimation

## Approximate Eigenvalue Estimation of Hermitian Matrices

We aim to estimate the eigenvalues of a Hermitian matrix  $H$  using QPE.

$$e^{iHt}|\lambda\rangle = e^{2\pi i\varphi}|\lambda\rangle$$

$$\lambda = \frac{2\pi\varphi}{t}$$

### Hamiltonian used for this simulation

Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

Time: 0.01 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

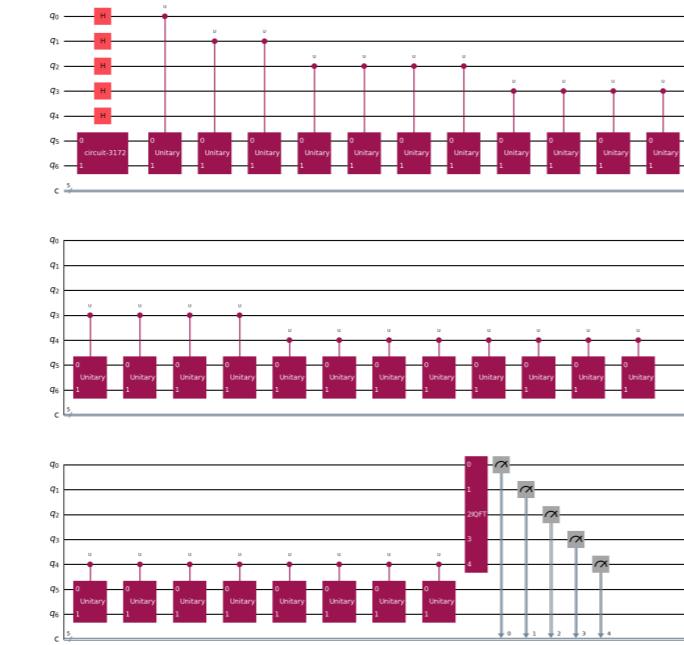
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.00382

Expected bitstring: (00000)

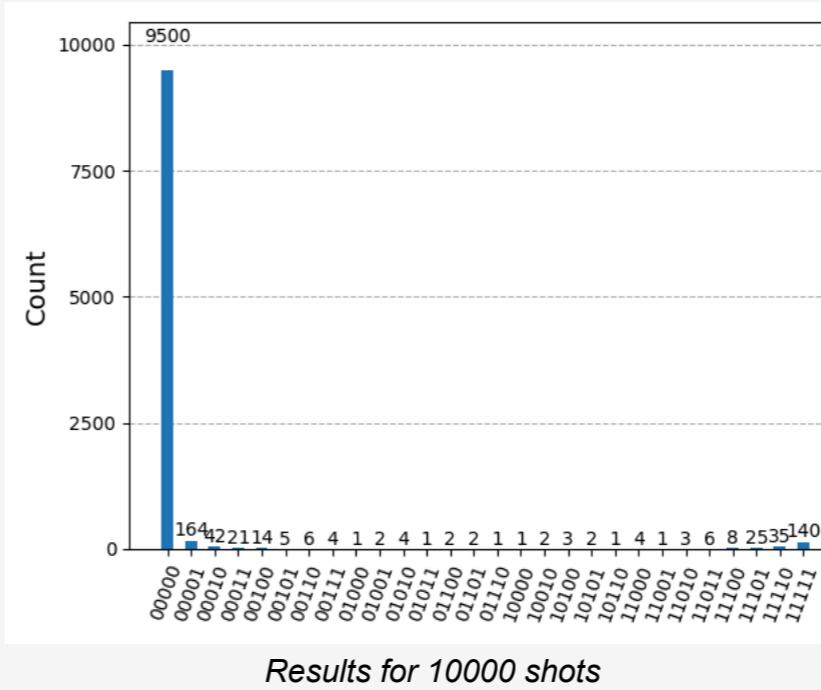
Most common measured bitstring ( $k$ ): 00000

Phase ( $\frac{k}{2^n}$ ): 0.0

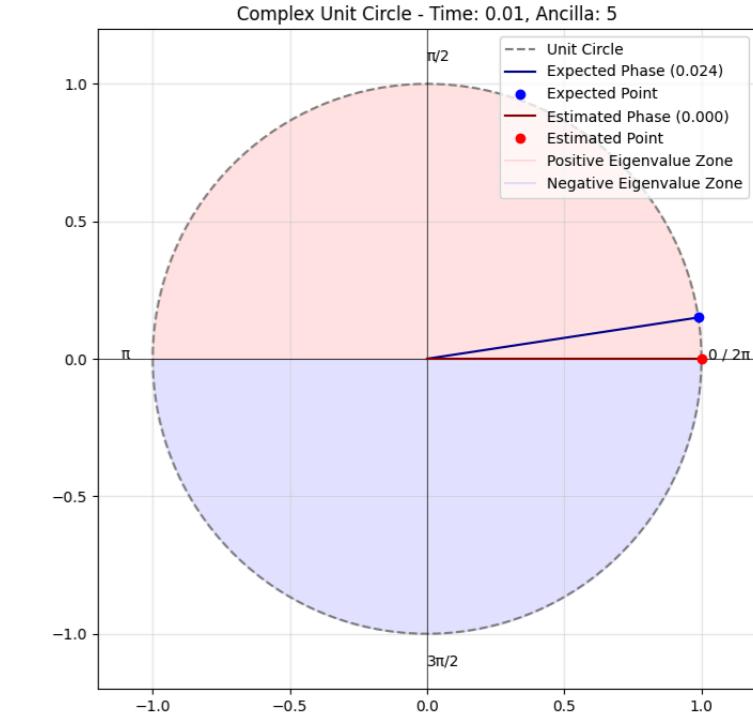
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 0.0



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.05 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

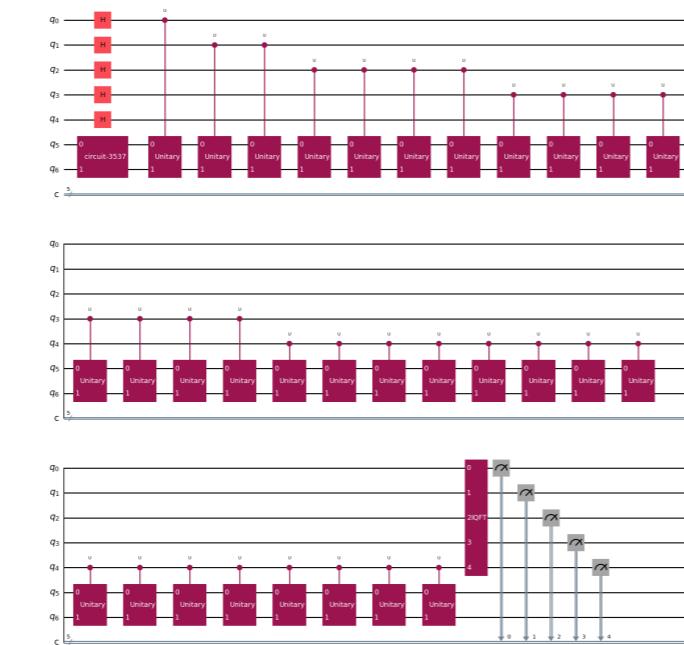
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0191

Expected bitstring: (00001)

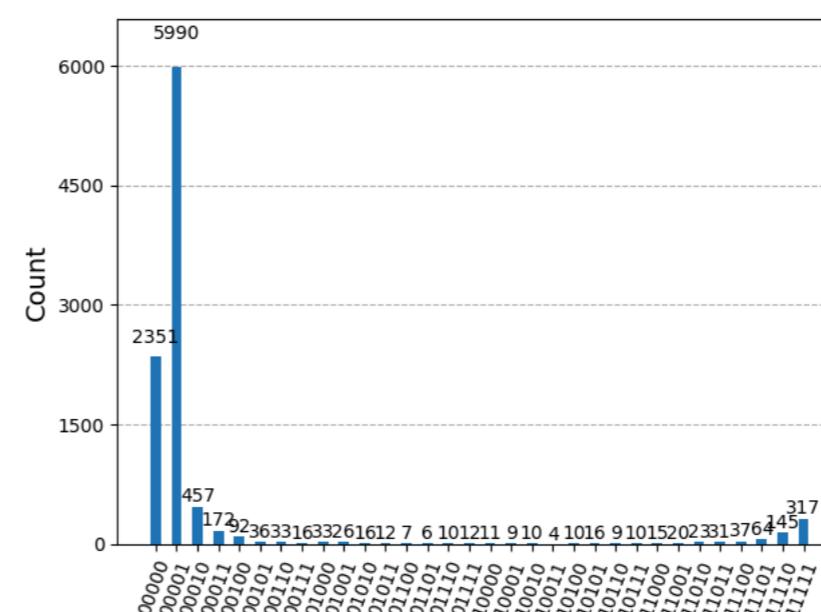
Most common measured bitstring ( $k$ ): 00001

Phase ( $\frac{k}{2^n}$ ): 0.03125

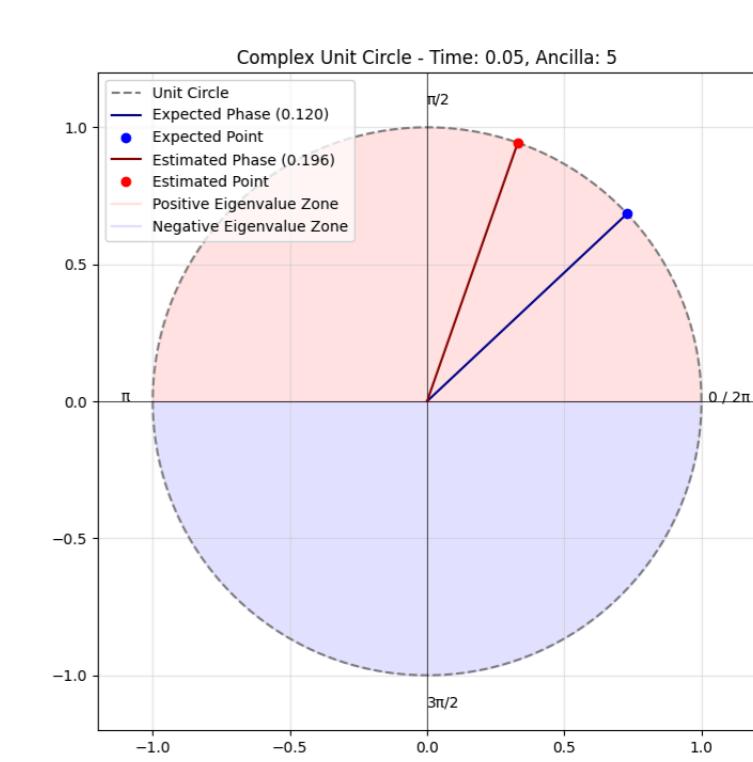
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

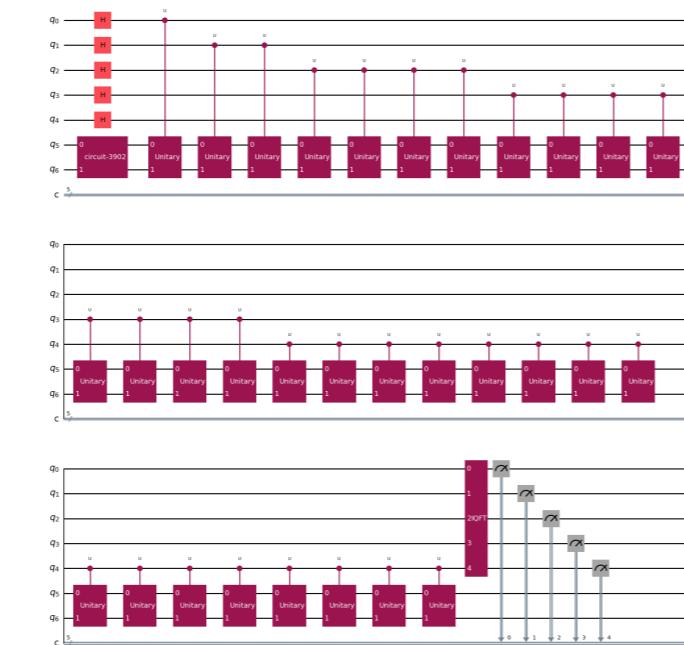
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0382

Expected bitstring: (00001)

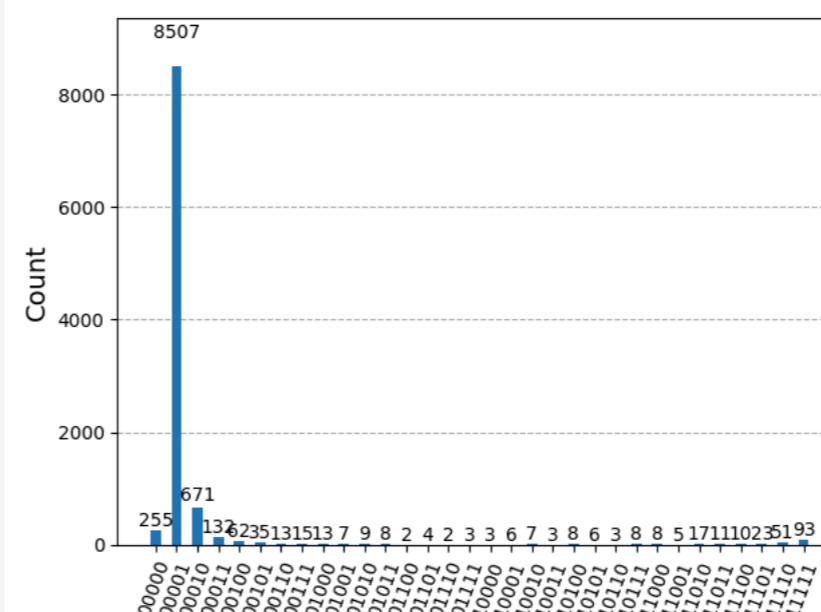
Most common measured bitstring ( $k$ ): 00001

Phase ( $\frac{k}{2^n}$ ): 0.03125

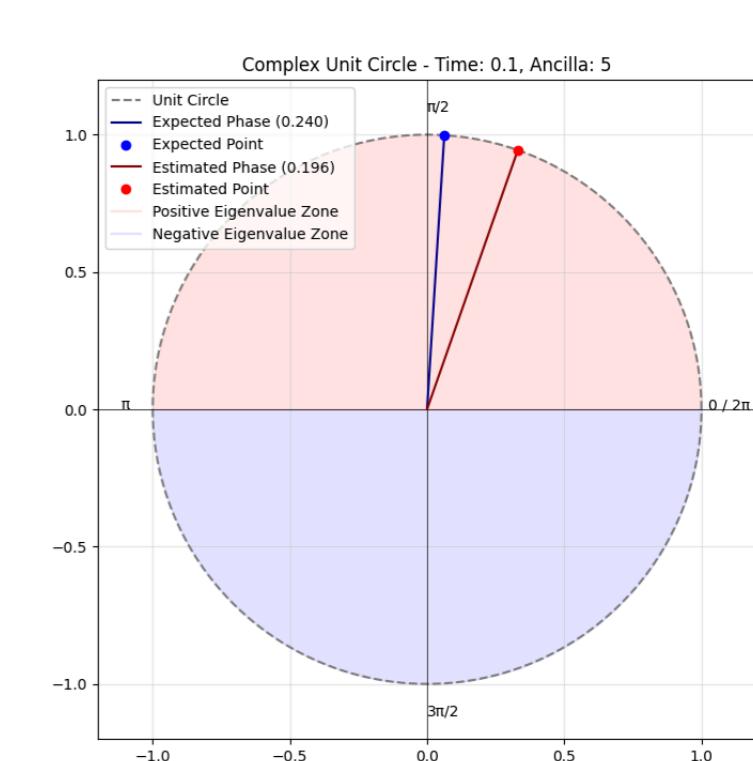
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 1.9635



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

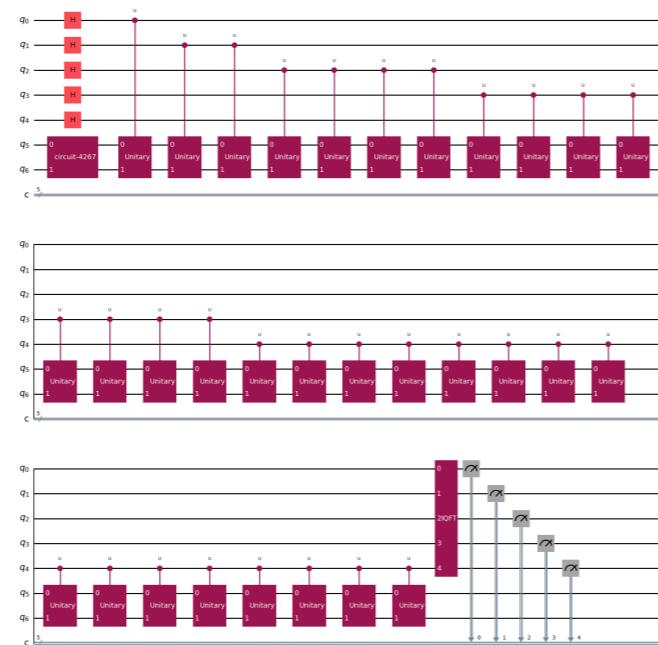
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.19099

Expected bitstring: (00110)

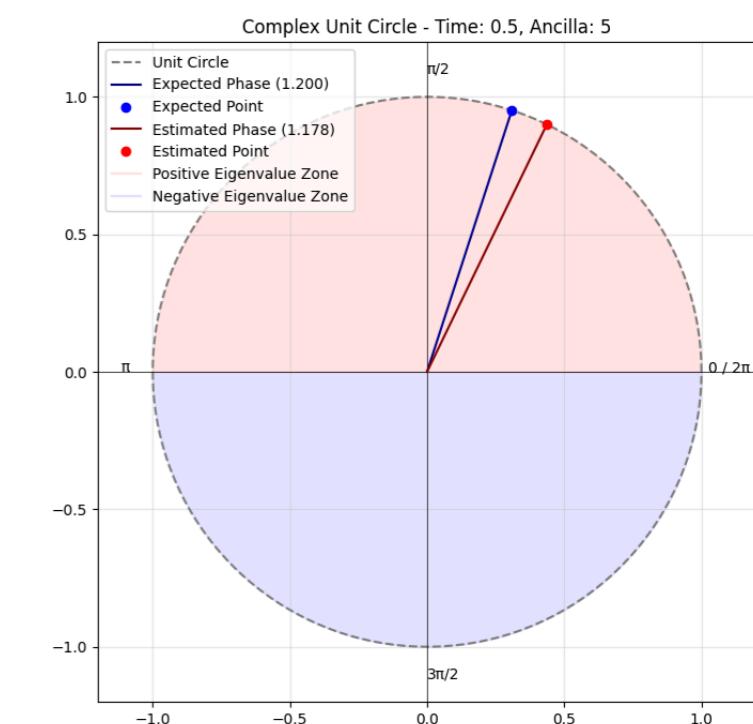
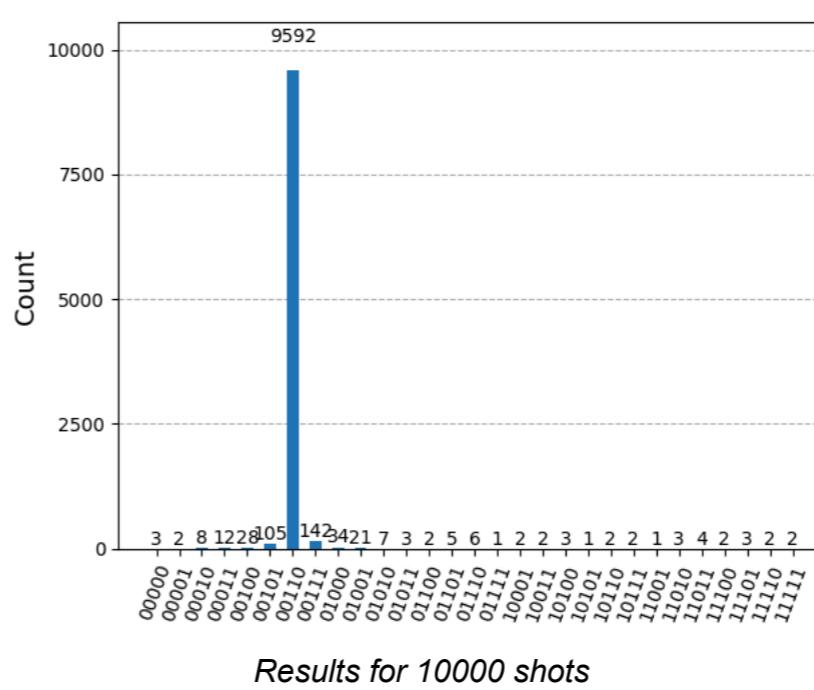
Most common measured bitstring ( $k$ ): 00110

Phase ( $\frac{k}{2^n}$ ): 0.1875

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 2.35619



Quantum Circuit Diagram



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

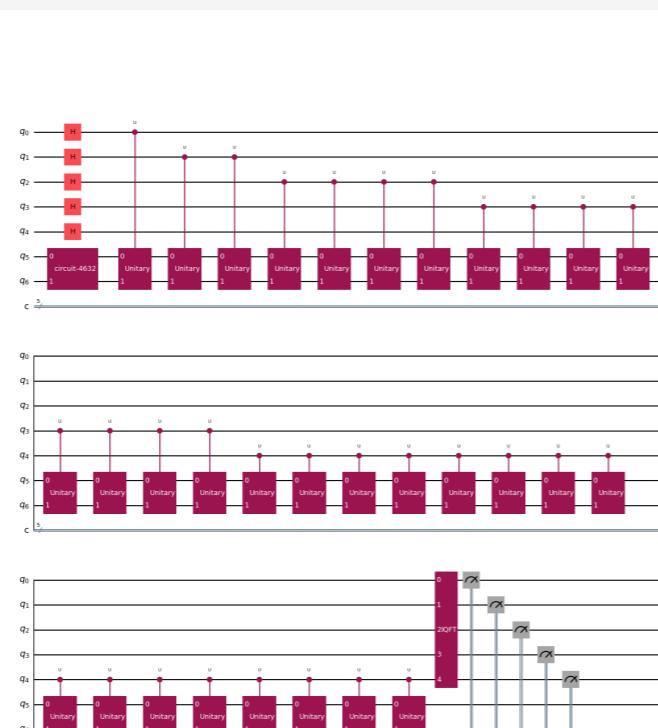
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.38197

Expected bitstring: (01100)

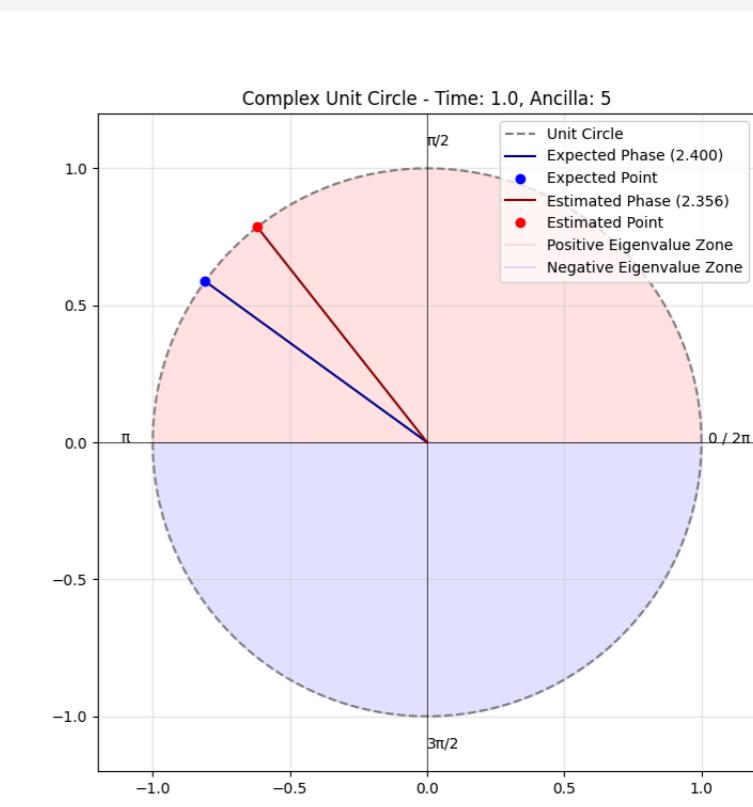
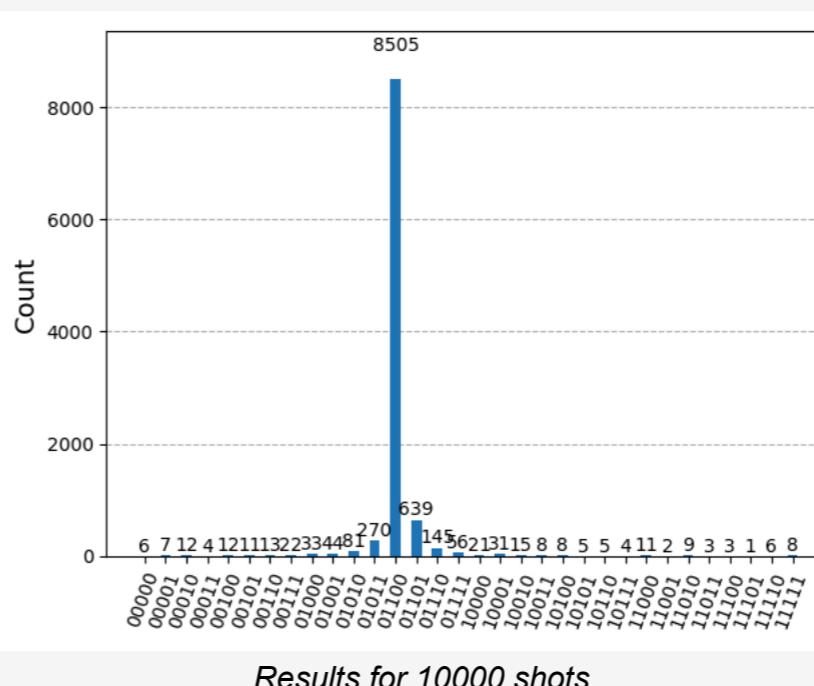
Most common measured bitstring ( $k$ ): 01100

Phase ( $\frac{k}{2^n}$ ): 0.375

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 2.35619



Quantum Circuit Diagram



Complex Unit Circle

Time: 0.01 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): -3.1241

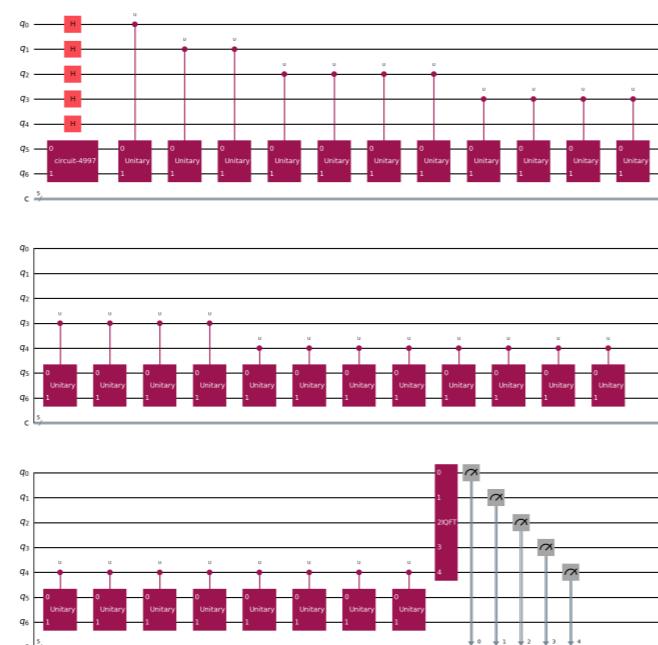
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.99503

Expected bitstring: (100000)

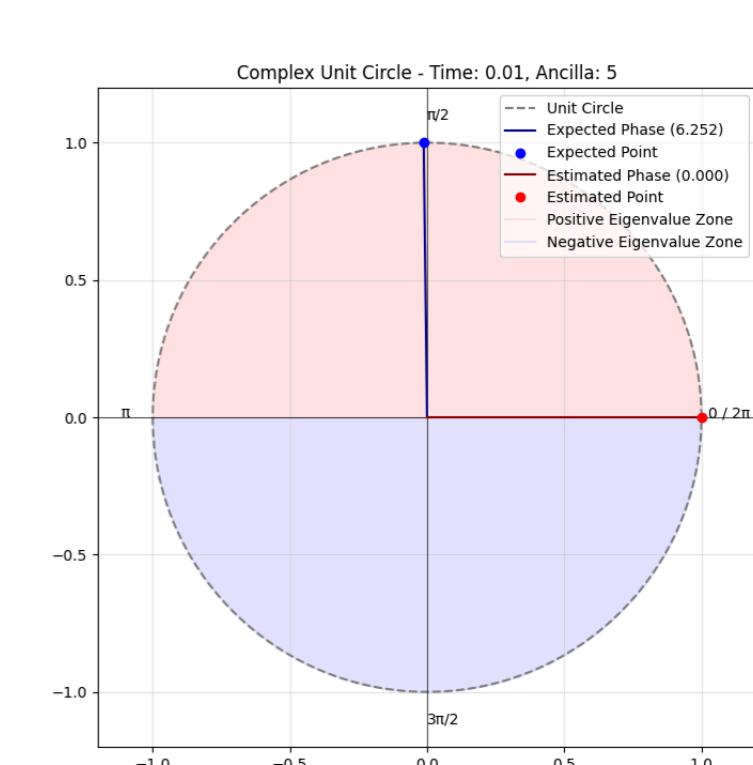
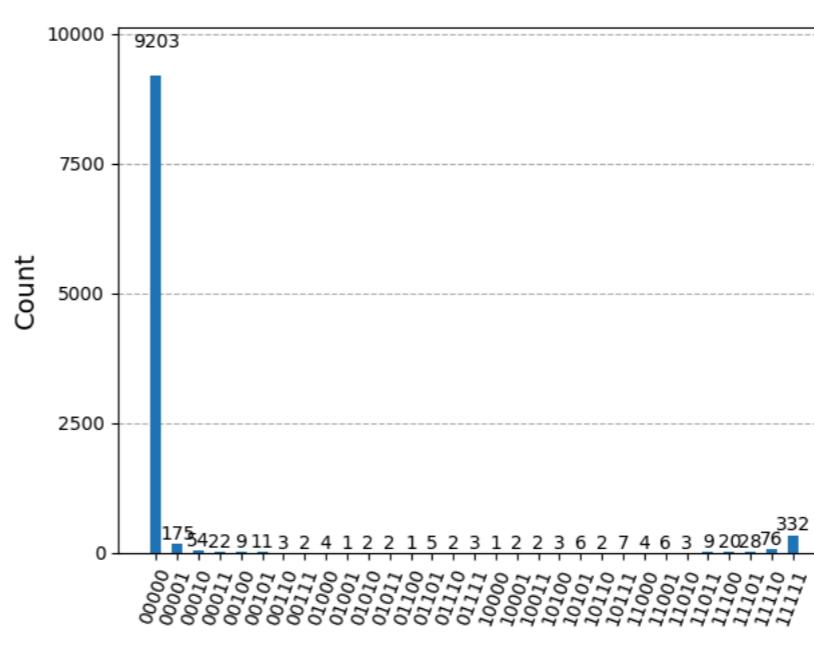
Most common measured bitstring ( $k$ ): 00000

Phase ( $\frac{k}{2^n}$ ): 0.0

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 0.0



Quantum Circuit Diagram



Complex Unit Circle

Time: 0.05 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): -3.1241

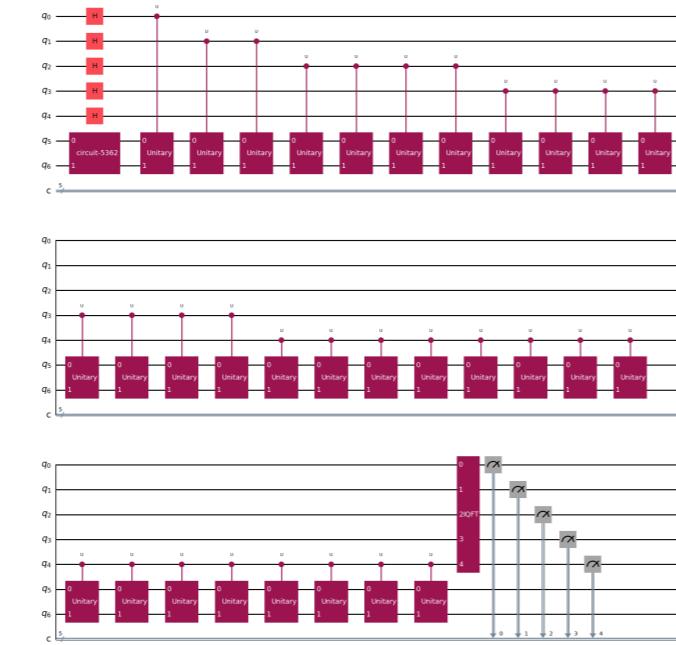
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.97514

Expected bitstring: (11111)

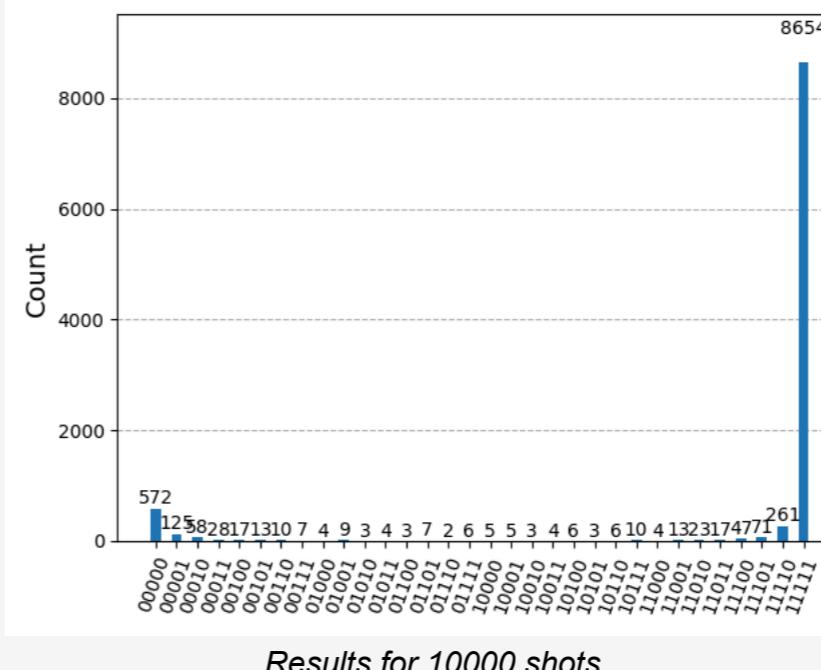
Most common measured bitstring ( $k$ ): 11111

Phase ( $\frac{k}{2^n}$ ): -0.03125

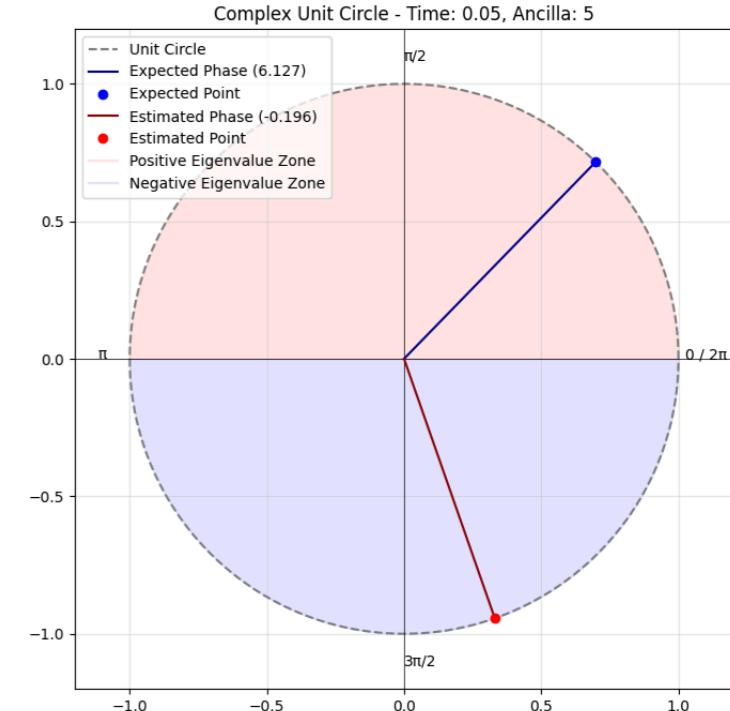
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): -3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): -3.1241

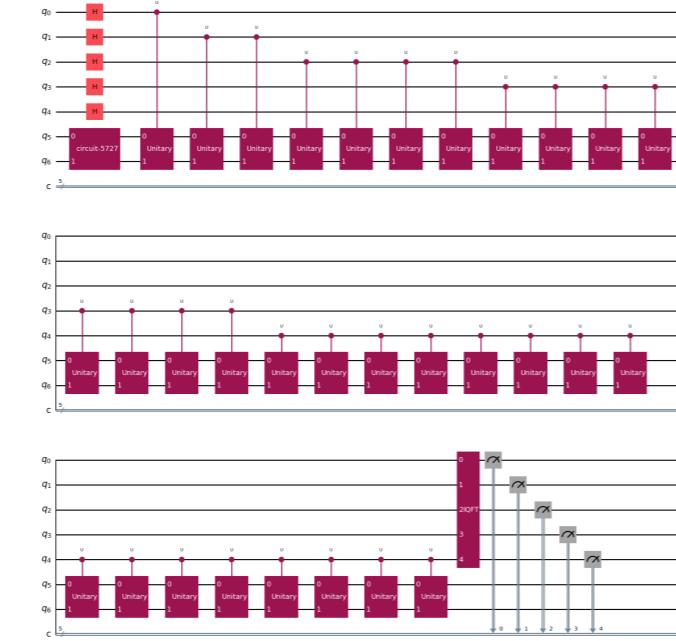
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.95028

Expected bitstring: (11110)

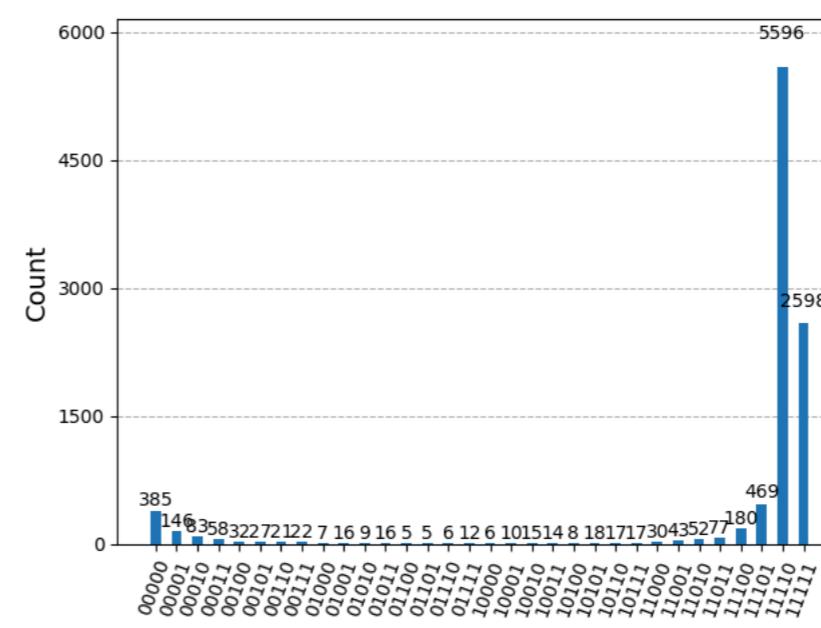
Most common measured bitstring ( $k$ ): 11110

Phase ( $\frac{k}{2^n}$ ): -0.0625

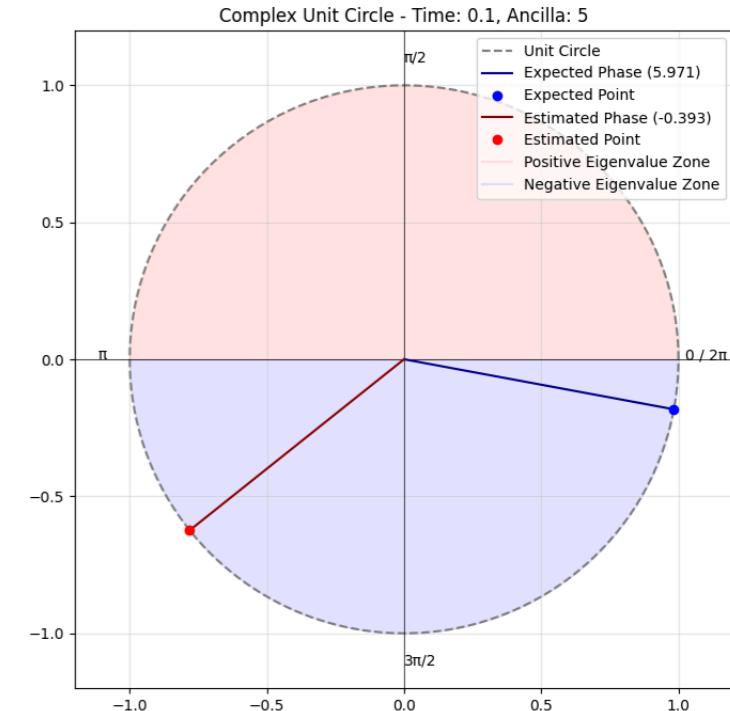
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): -3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): -3.1241

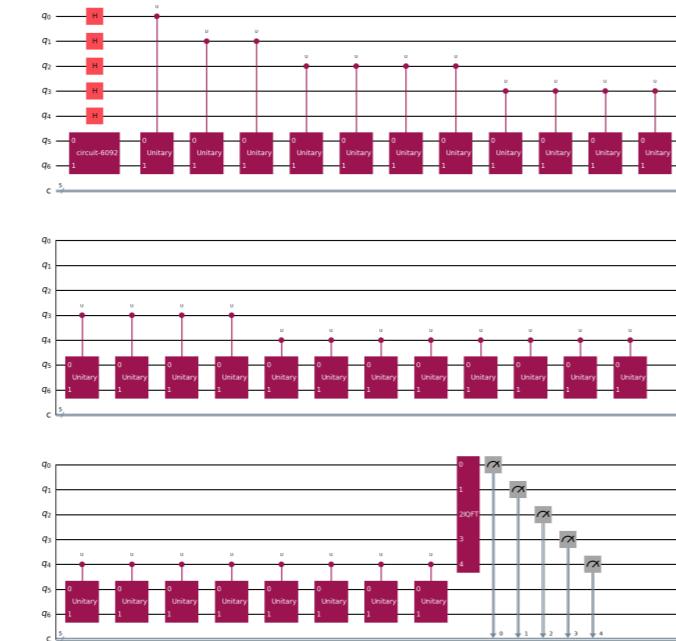
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.75139

Expected bitstring: (11000)

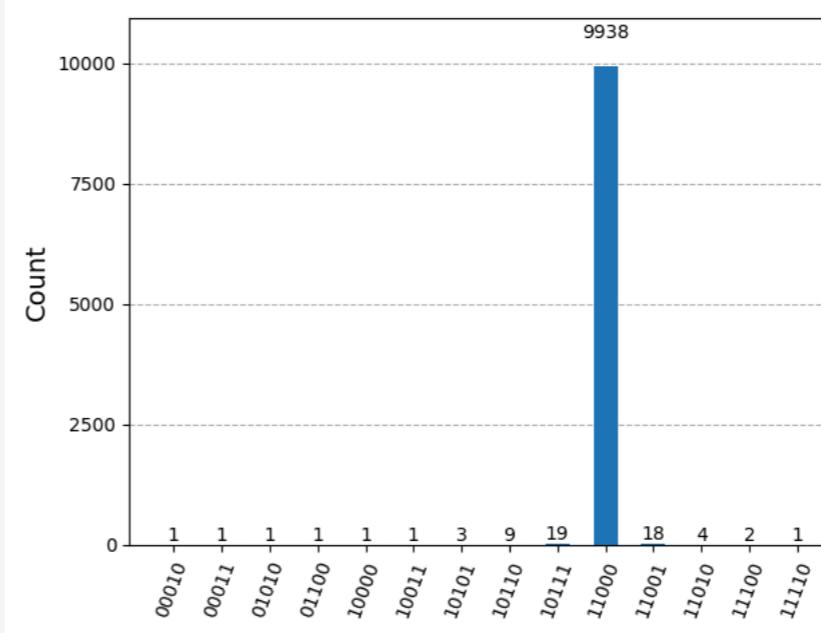
Most common measured bitstring ( $k$ ): 11000

Phase ( $\frac{k}{2^n}$ ): -0.25

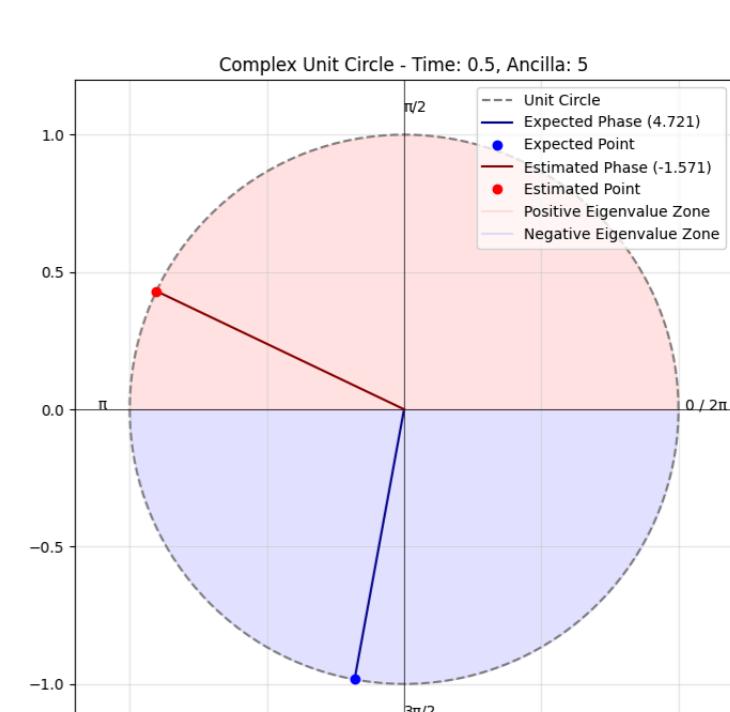
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): -3.14159



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): -3.1241

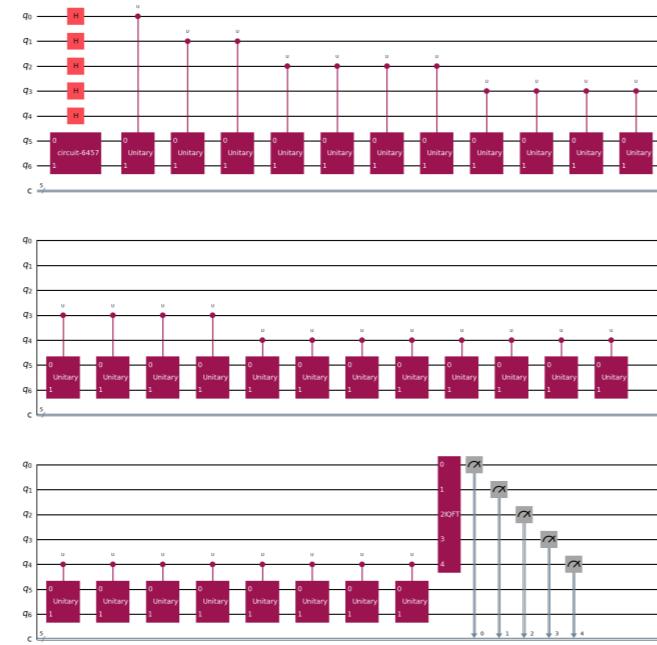
Exact expected phase ( $\frac{\lambda \cdot t}{2\pi} \bmod 1$ ): 0.50278

Expected bitstring: (10000)

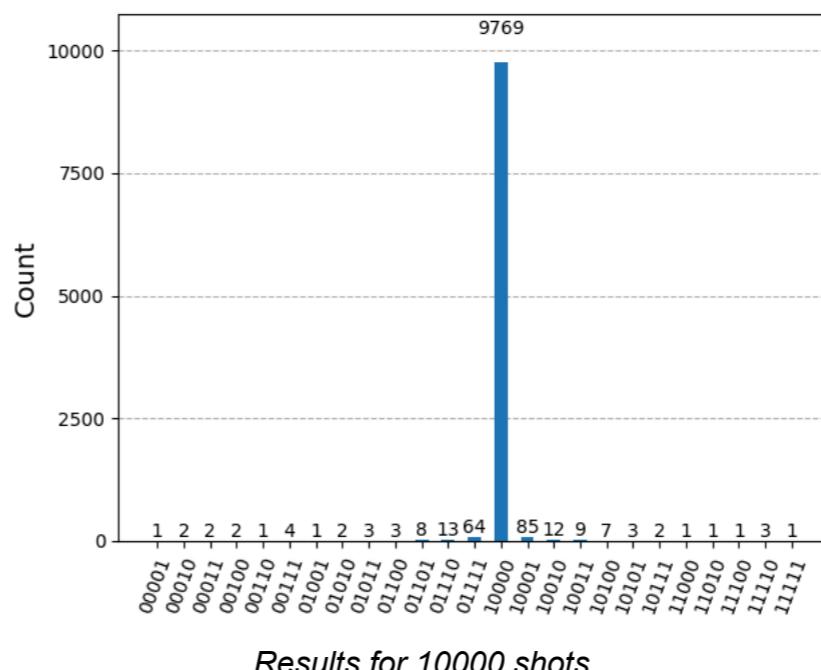
Most common measured bitstring ( $k$ ): 10000

Phase ( $\frac{k}{2^n}$ ): -0.5

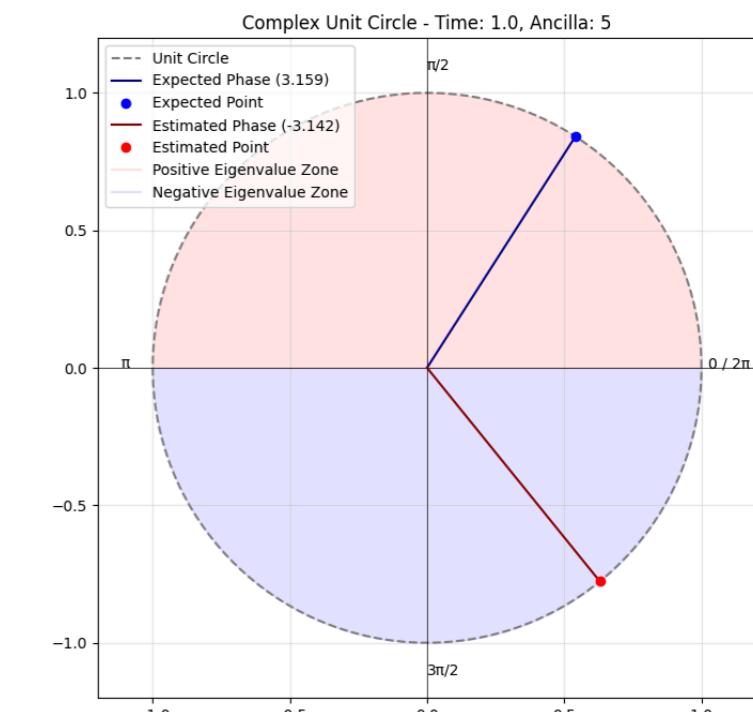
Estimated Energy ( $\frac{2\pi\varphi}{t}$ ): -3.14159



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

## qDRIFT Simulation

Hamiltonian used for this simulation

Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

Time: 0.01 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

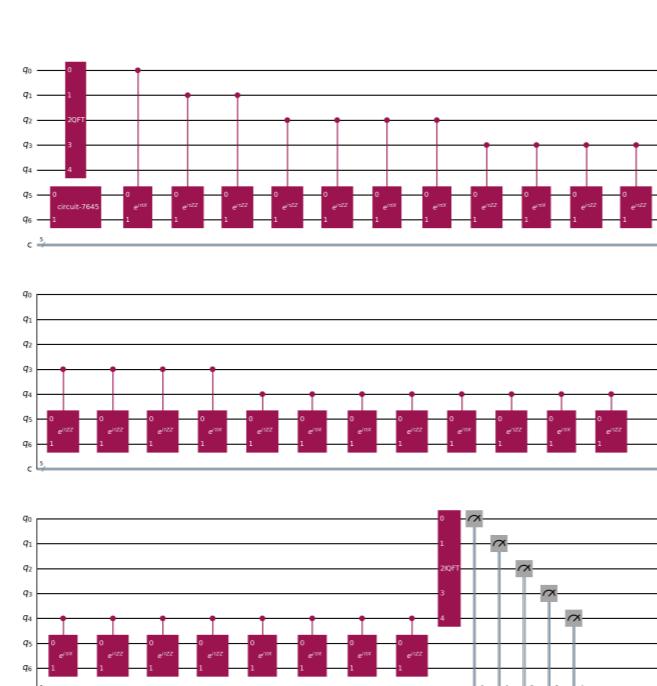
Exact expected phase ( $\frac{\lambda \cdot t}{2\pi} \bmod 1$ ): 0.00382

Expected bitstring: (00000)

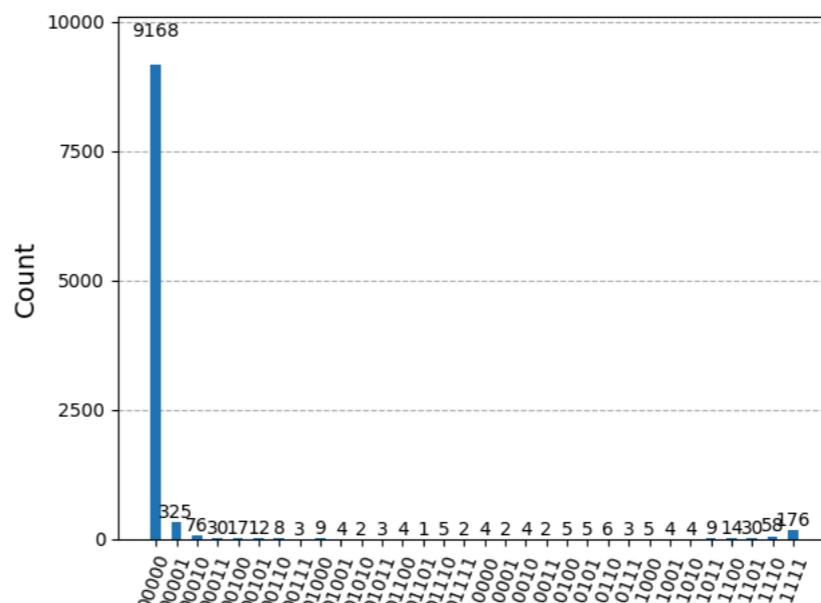
Most common measured bitstring ( $k$ ): 00000

Phase ( $\frac{k}{2^n}$ ): 0.0

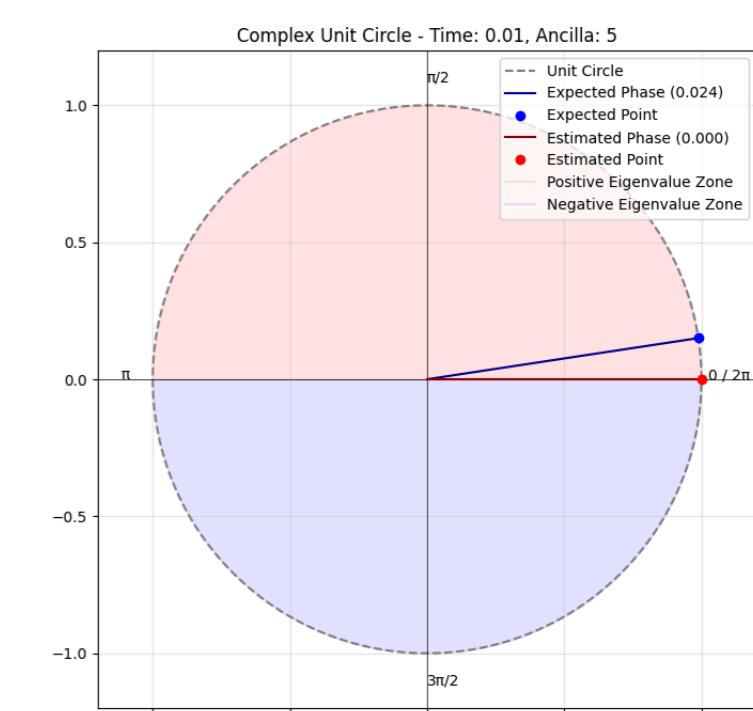
Estimated Energy ( $\frac{2\pi\varphi}{t}$ ): 0.0



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.05 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

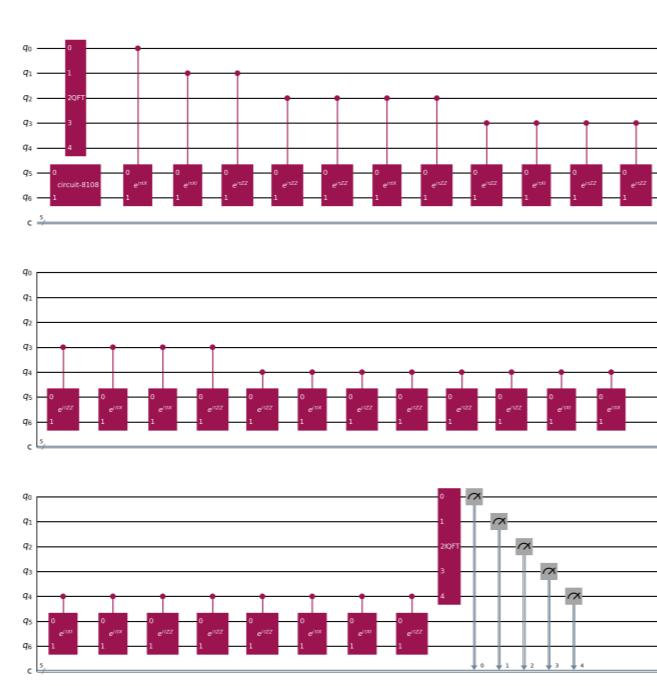
Exact expected phase ( $\frac{\lambda \cdot t}{2\pi} \bmod 1$ ): 0.0191

Expected bitstring: (00001)

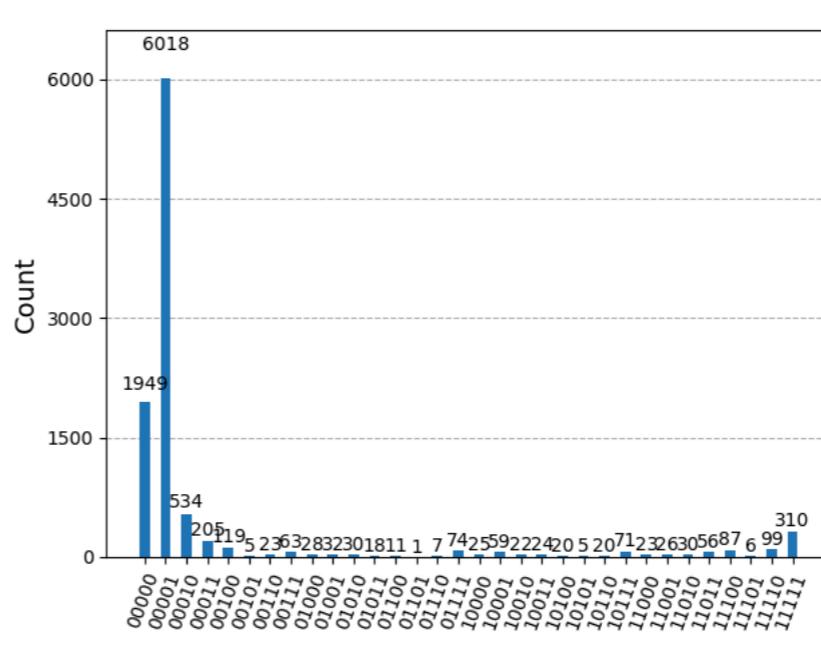
Most common measured bitstring ( $k$ ): 00001

Phase ( $\frac{k}{2^n}$ ): 0.03125

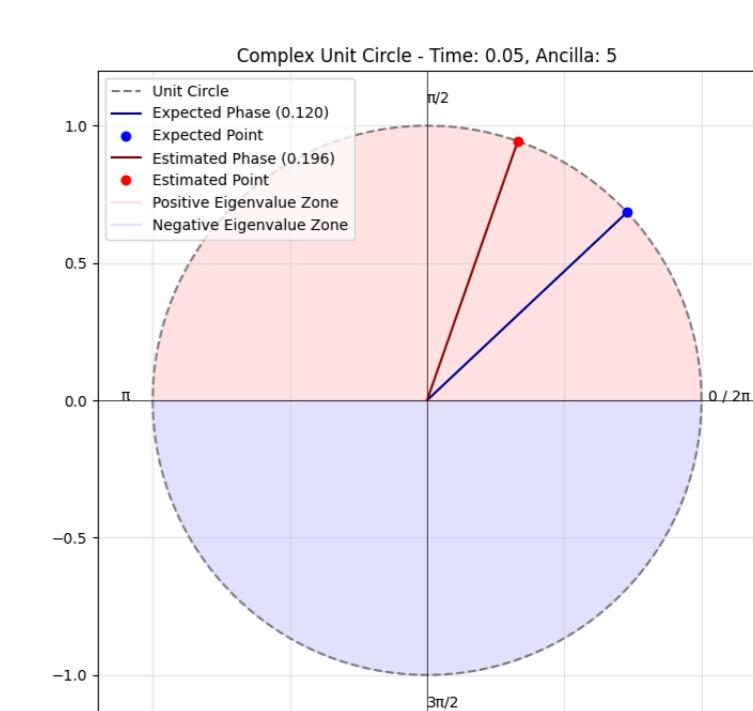
Estimated Energy ( $\frac{2\pi\varphi}{t}$ ): 3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

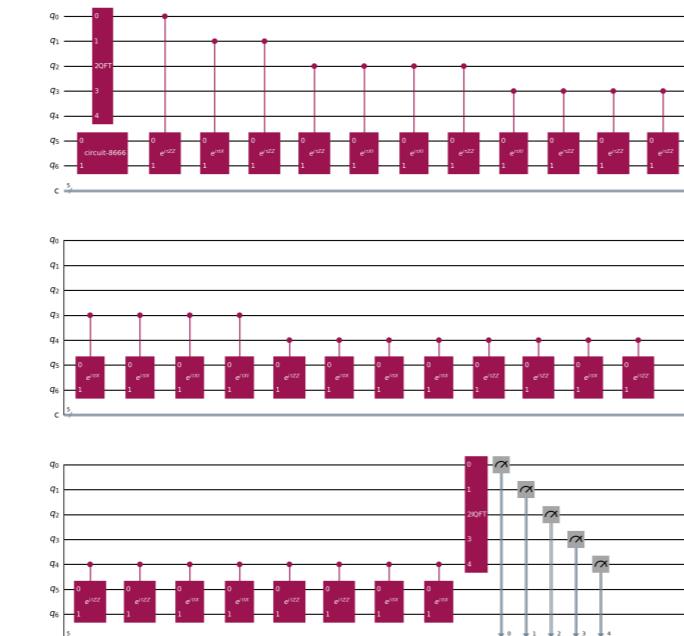
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0382

Expected bitstring: (00001)

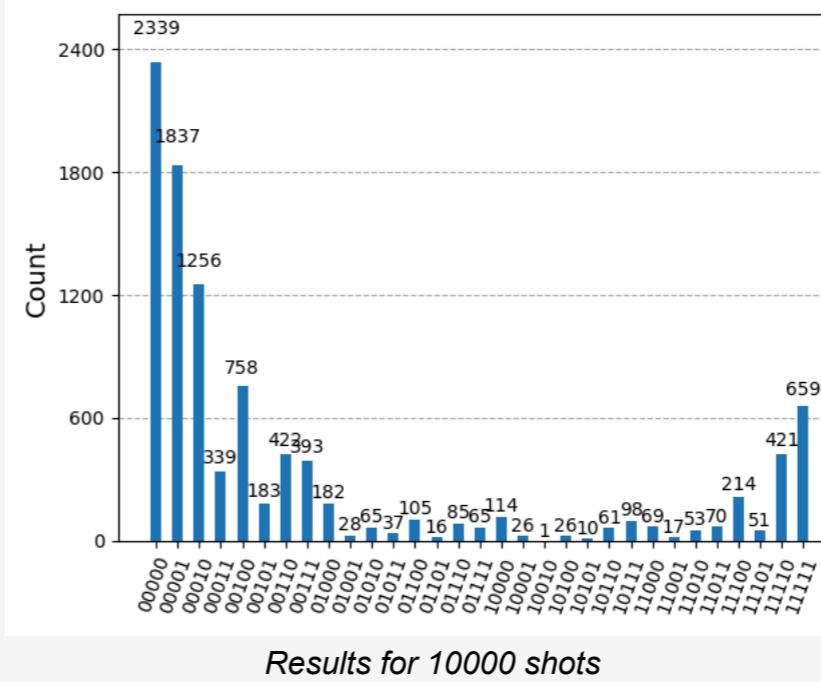
Most common measured bitstring ( $k$ ): 00000

Phase ( $\frac{k}{2^n}$ ): 0.0

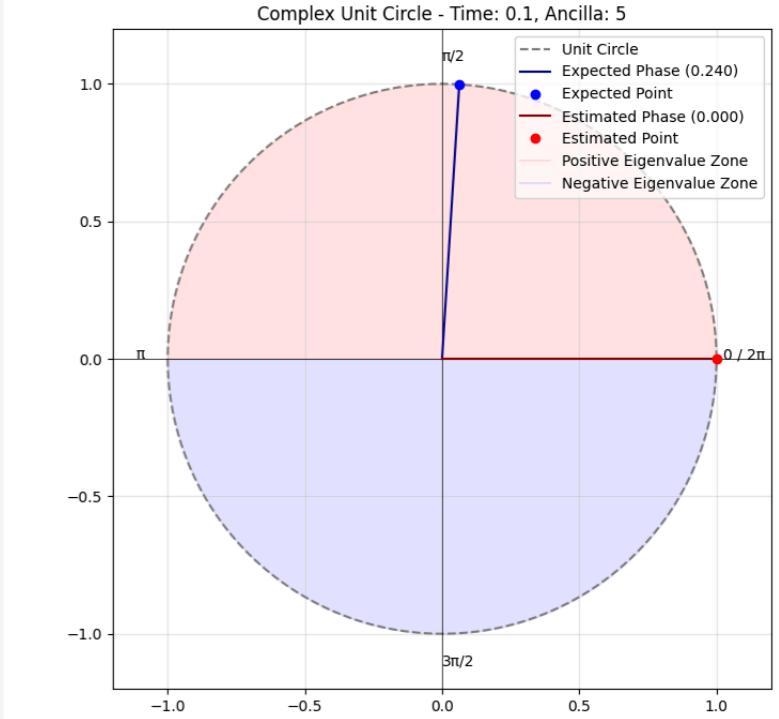
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 0.0



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

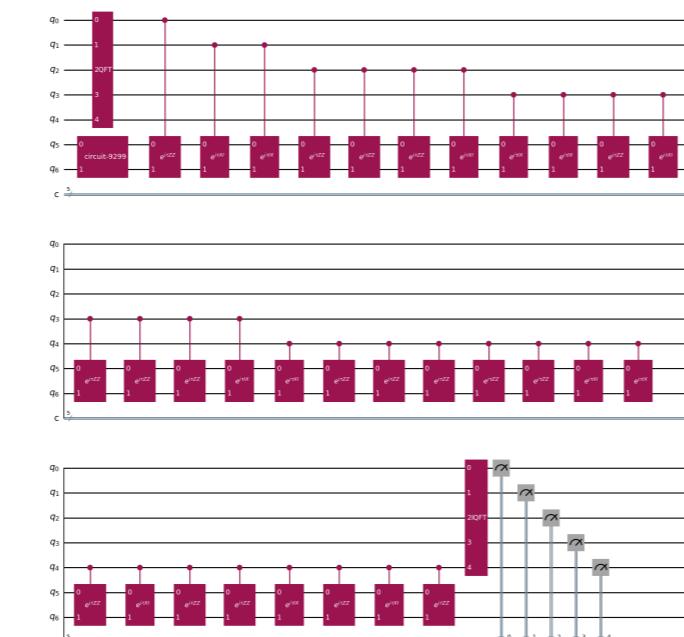
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.19099

Expected bitstring: (00110)

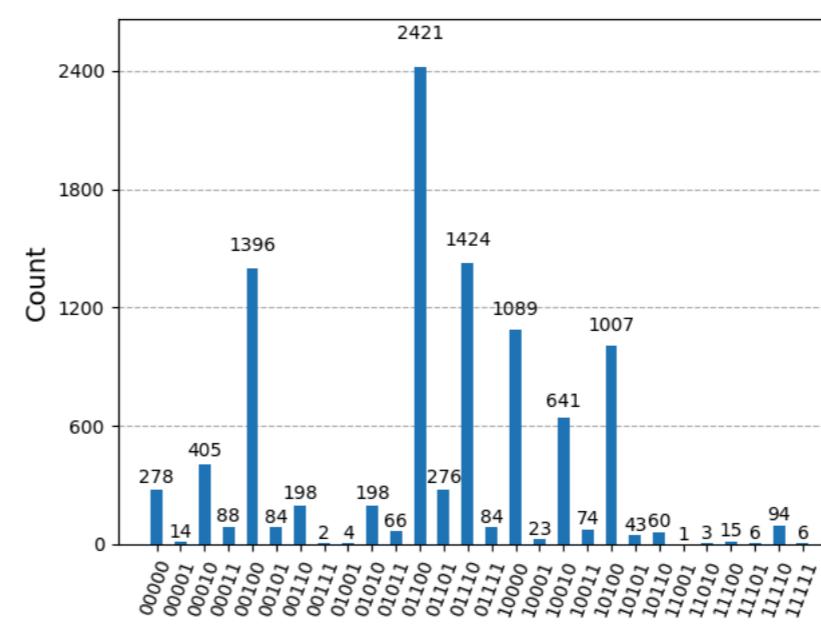
Most common measured bitstring ( $k$ ): 01100

Phase ( $\frac{k}{2^n}$ ): 0.375

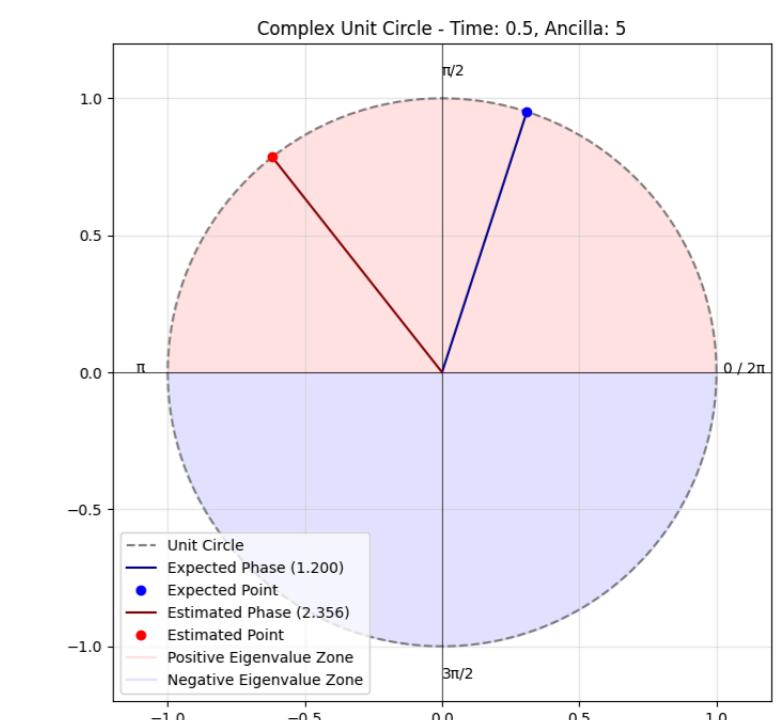
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 4.71239



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy ( $\lambda$ ): 2.4

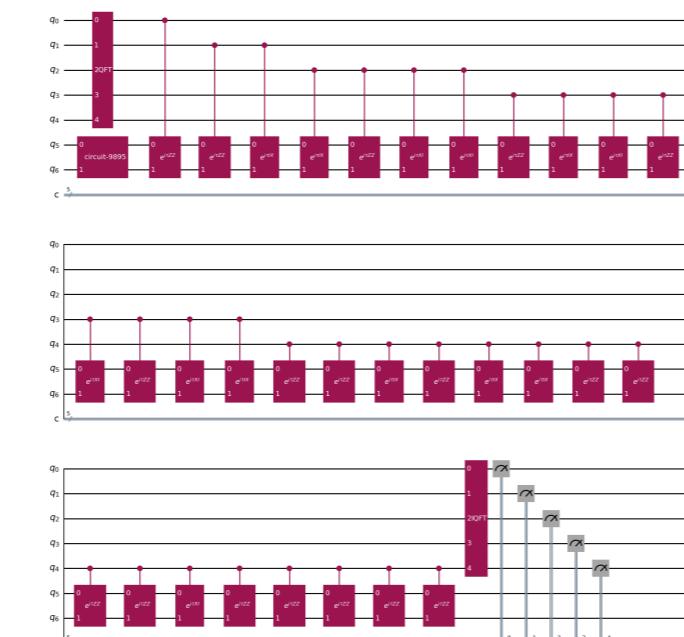
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.38197

Expected bitstring: (01100)

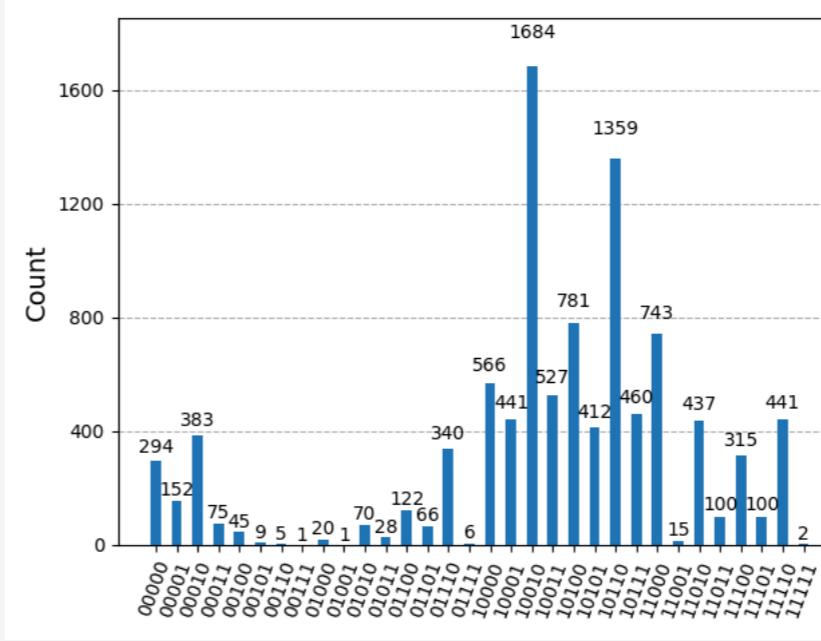
Most common measured bitstring ( $k$ ): 10010

Phase ( $\frac{k}{2^n}$ ): 0.5625

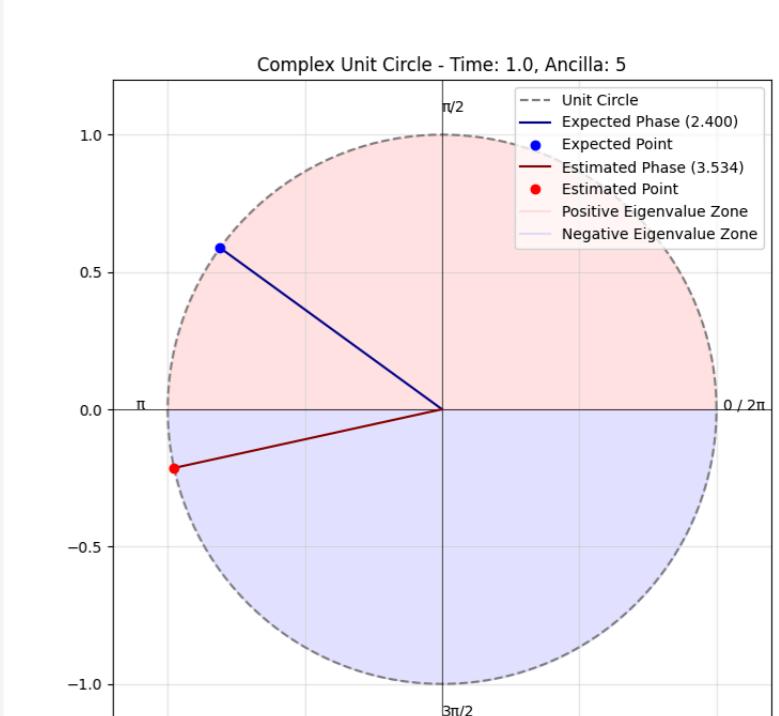
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 3.53429



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.01 | Shots: 10000 | Ancilla: 6

Exact energy ( $\lambda$ ): 2.4

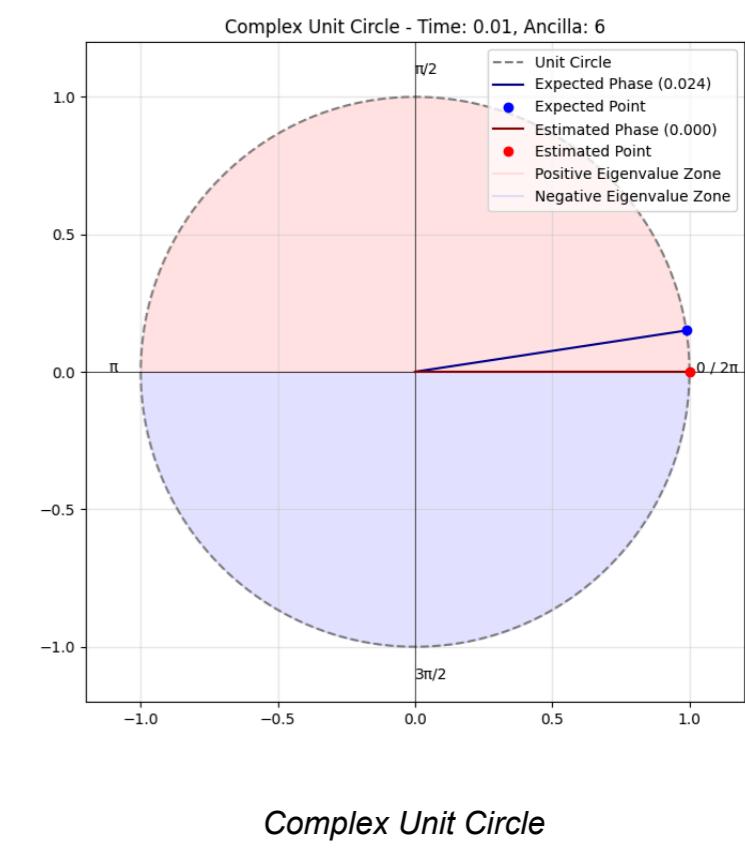
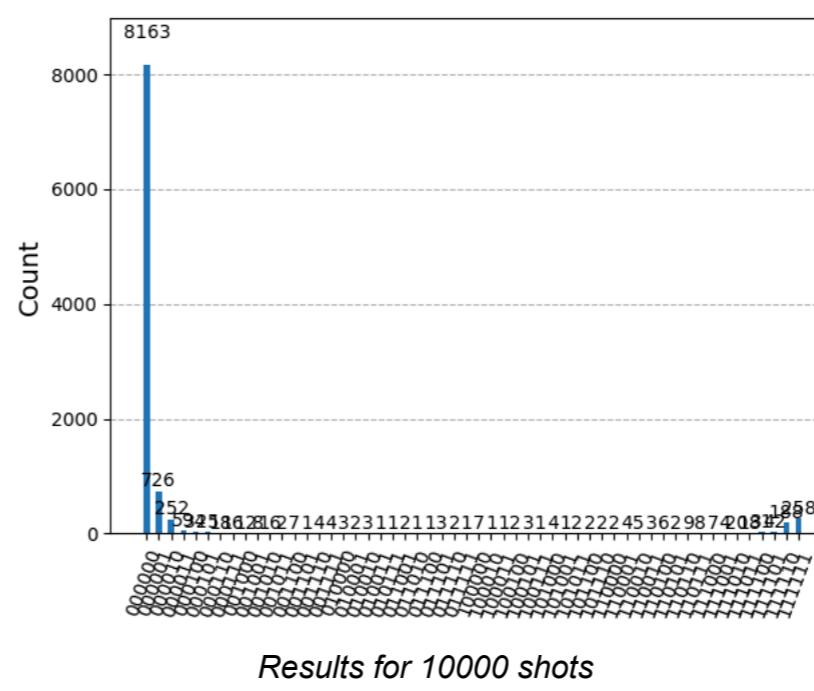
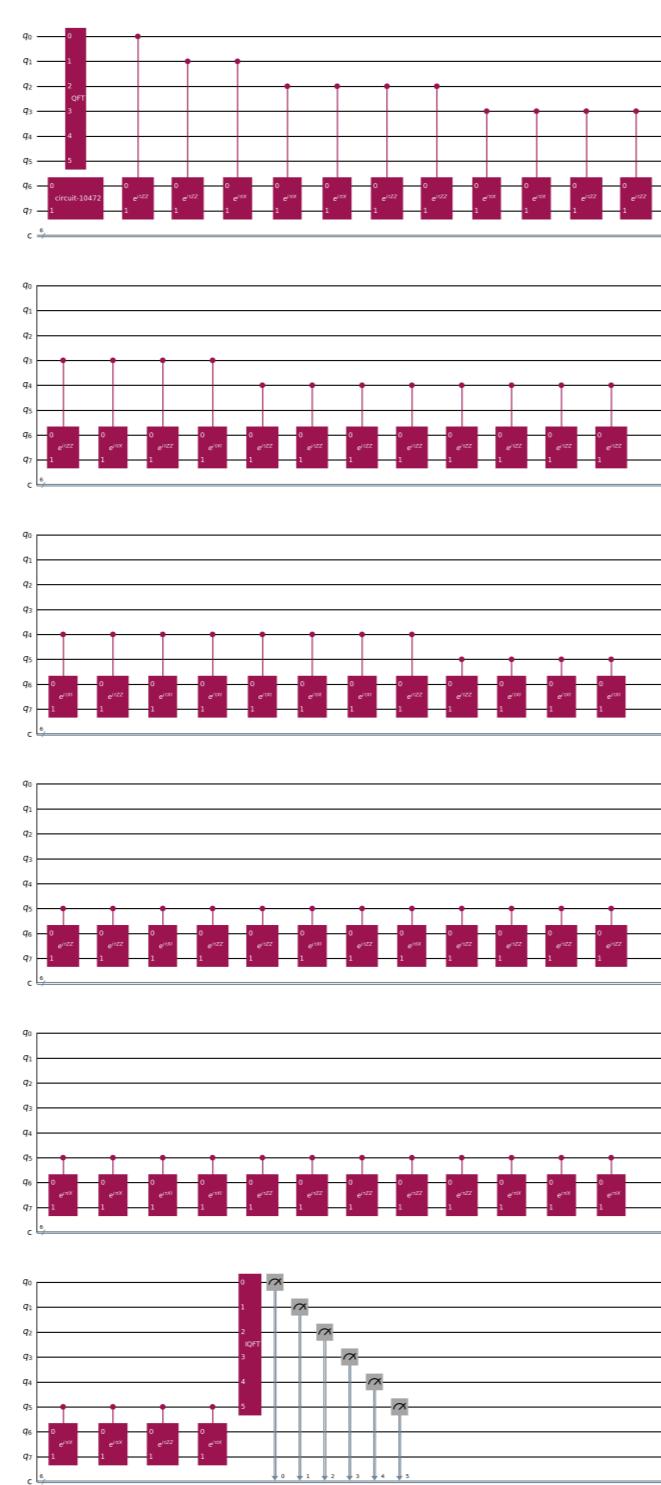
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.00382

Expected bitstring: (000000)

Most common measured bitstring ( $k$ ): 000000

Phase ( $\frac{k}{2^n}$ ): 0.0

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 0.0



Quantum Circuit Diagram

Time: 0.05 | Shots: 10000 | Ancilla: 6

Exact energy ( $\lambda$ ): 2.4

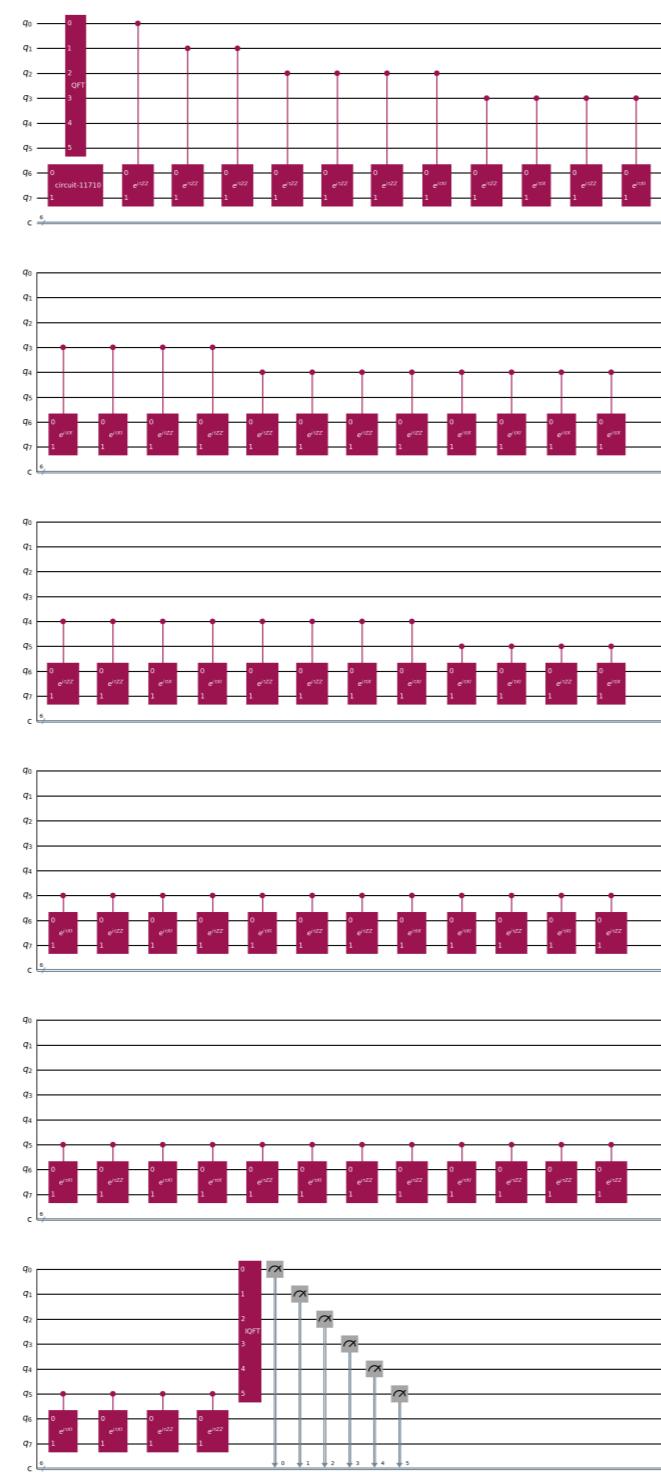
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0191

Expected bitstring: (000001)

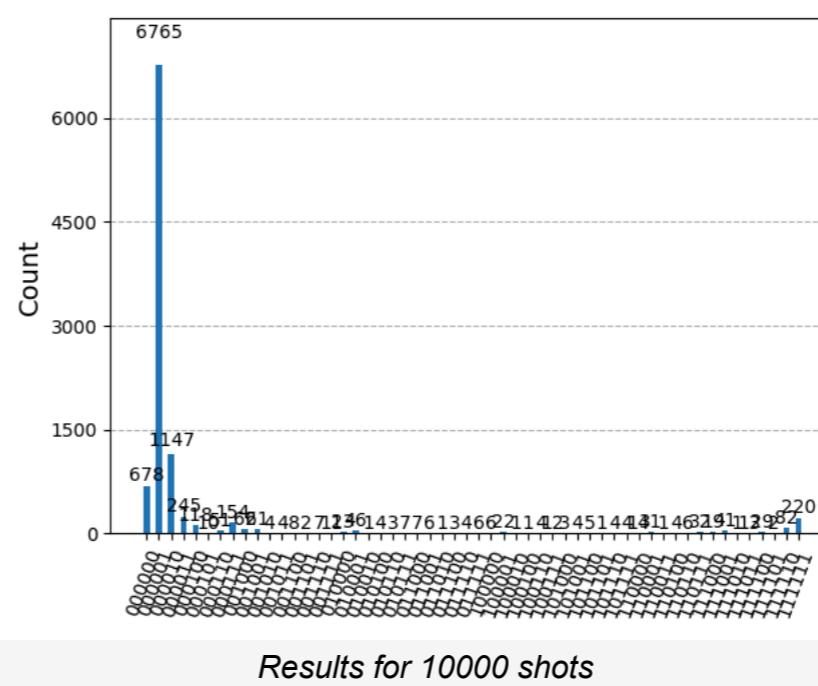
Most common measured bitstring ( $k$ ): 000001

Phase ( $\frac{k}{2^n}$ ): 0.01562

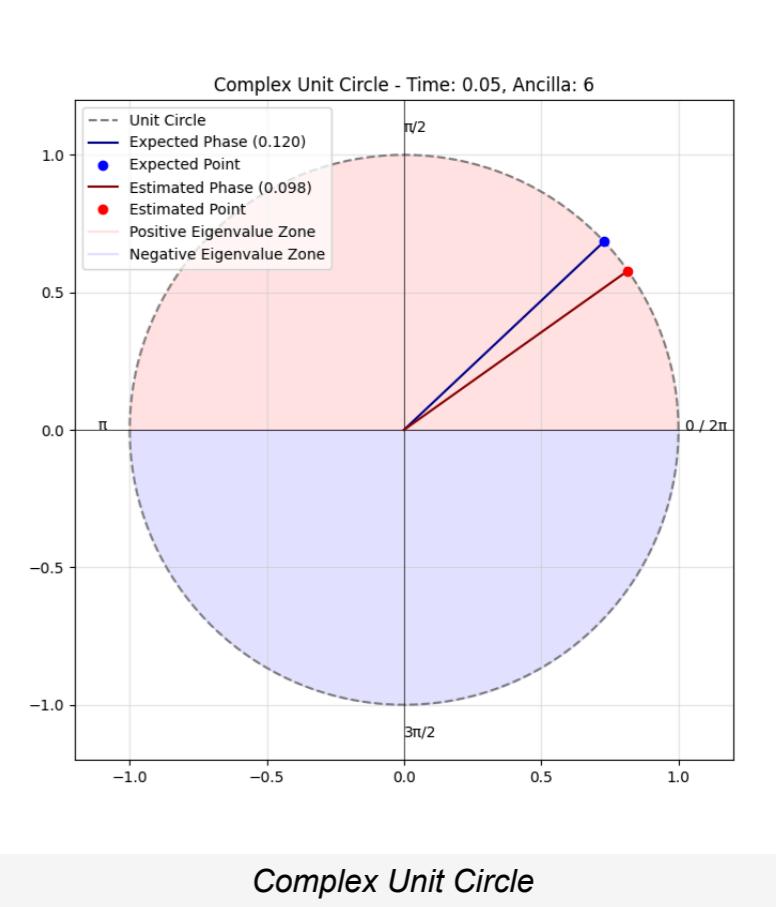
Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 1.9635



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

**Time: 0.1 | Shots: 10000 | Ancilla: 6**

Exact energy ( $\lambda$ ): 2.4

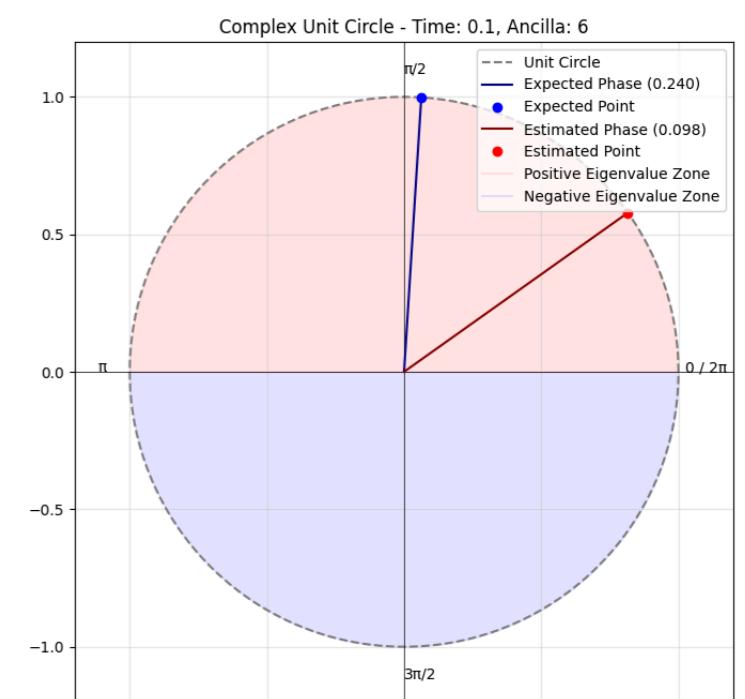
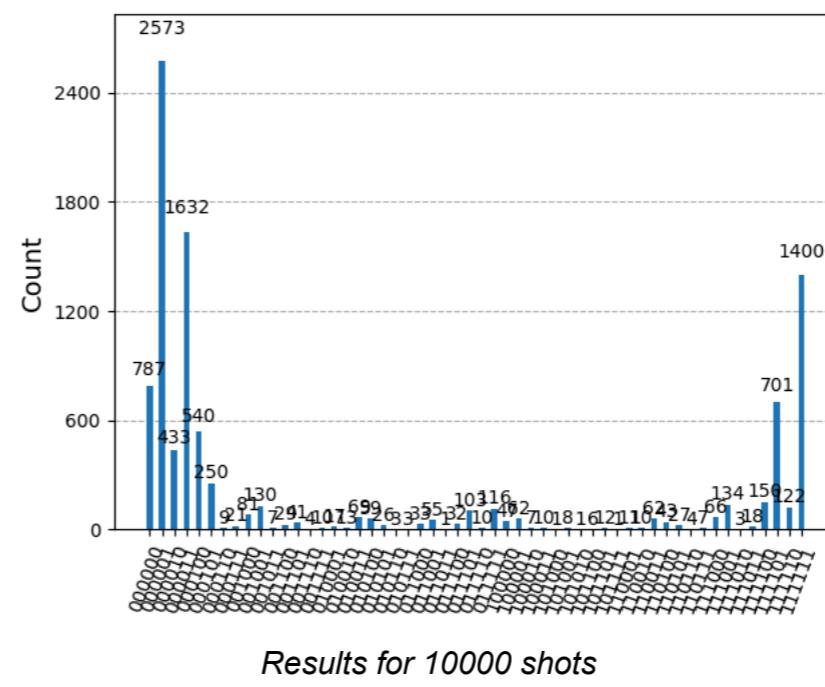
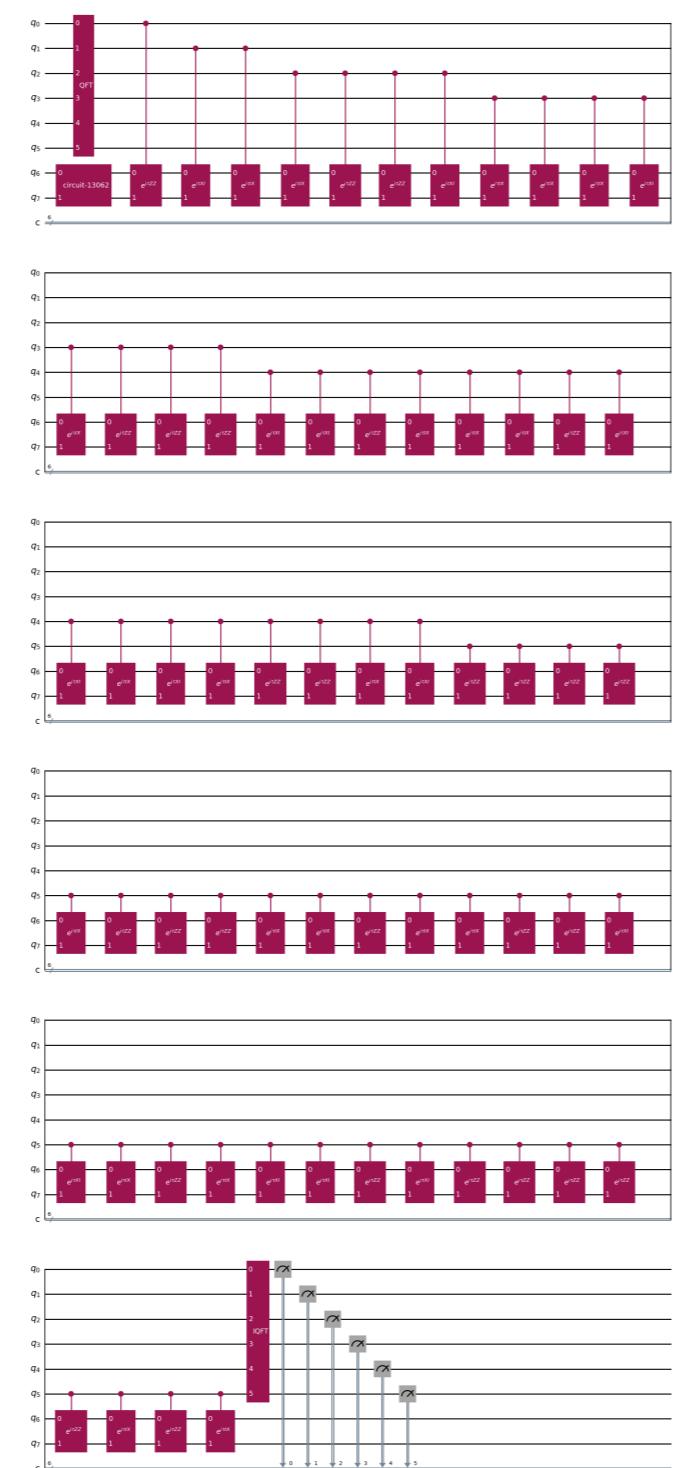
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0382

Expected bitstring: (000010)

Most common measured bitstring ( $k$ ): 000001

Phase ( $\frac{k}{\gamma^n}$ ): 0.01562

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 0.98175



## *Quantum Circuit Diagram*

Time: 0.5 | Shots: 10000 | Ancilla: 6

Exact energy ( $\lambda$ ): 2.4

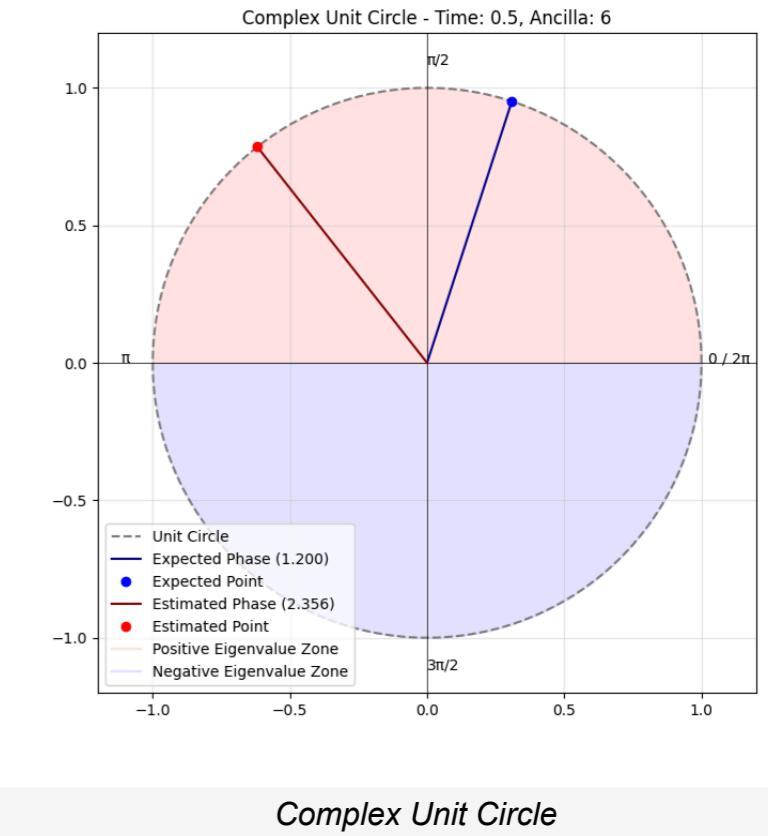
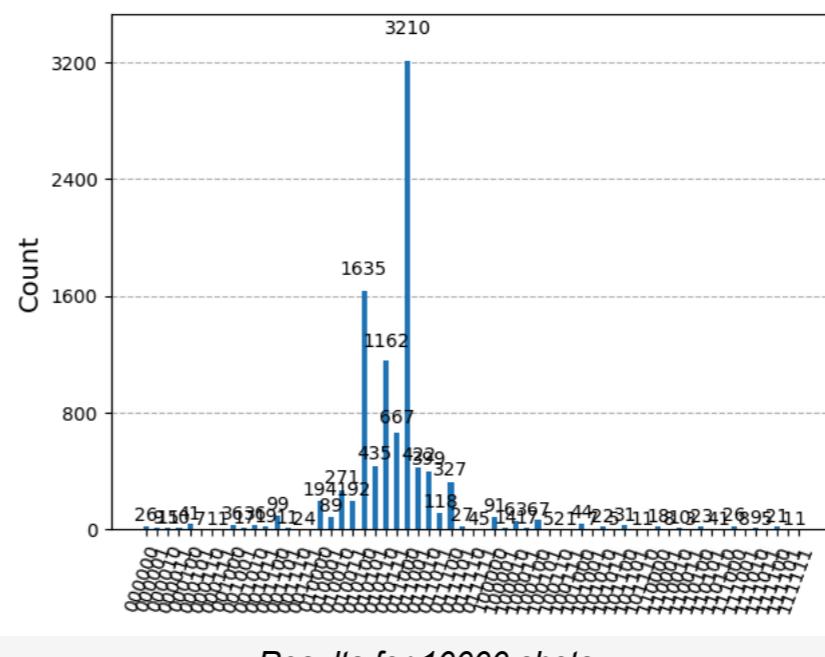
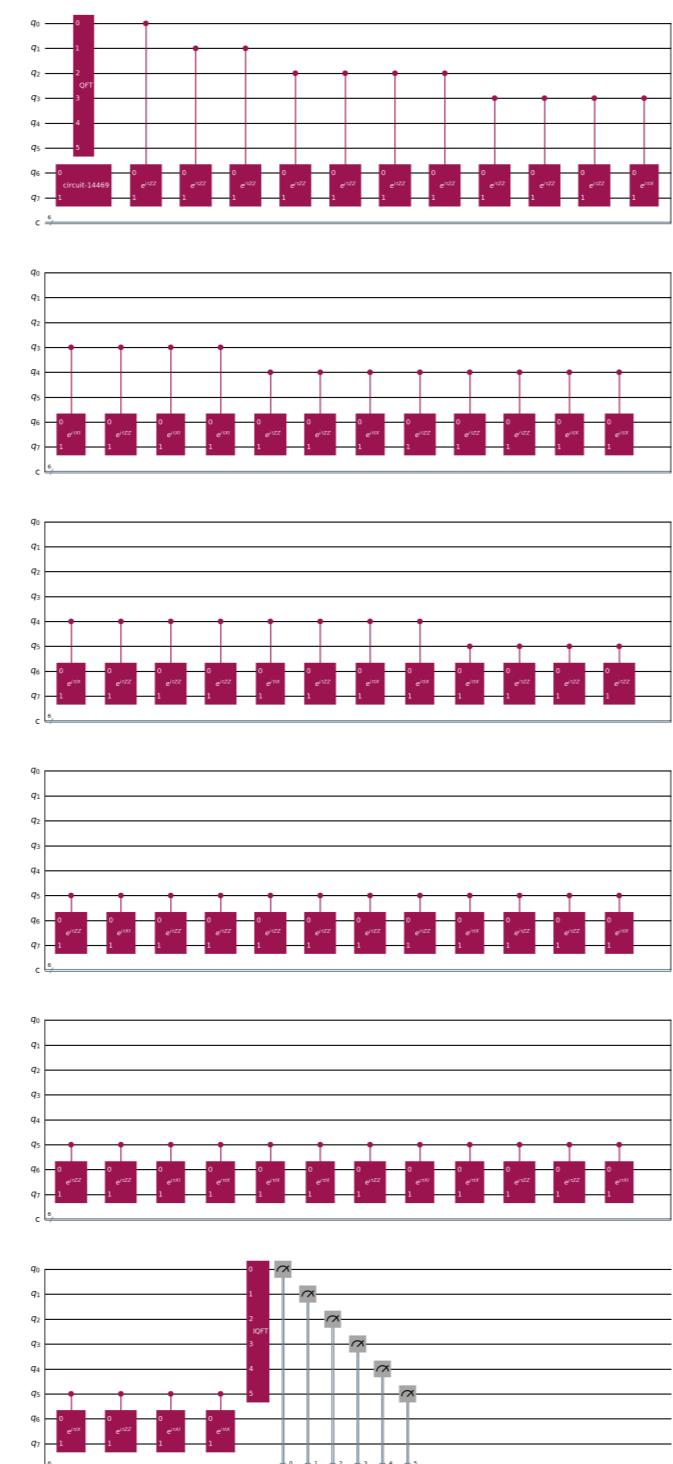
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.19099

Expected bitstring: (001100)

Most common measured bitstring ( $k$ ): 011000

Phase ( $\frac{k}{2^n}$ ): 0.375

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 4.71239



Time: 1.0 | Shots: 10000 | Ancilla: 6

Exact energy ( $\lambda$ ): 2.4

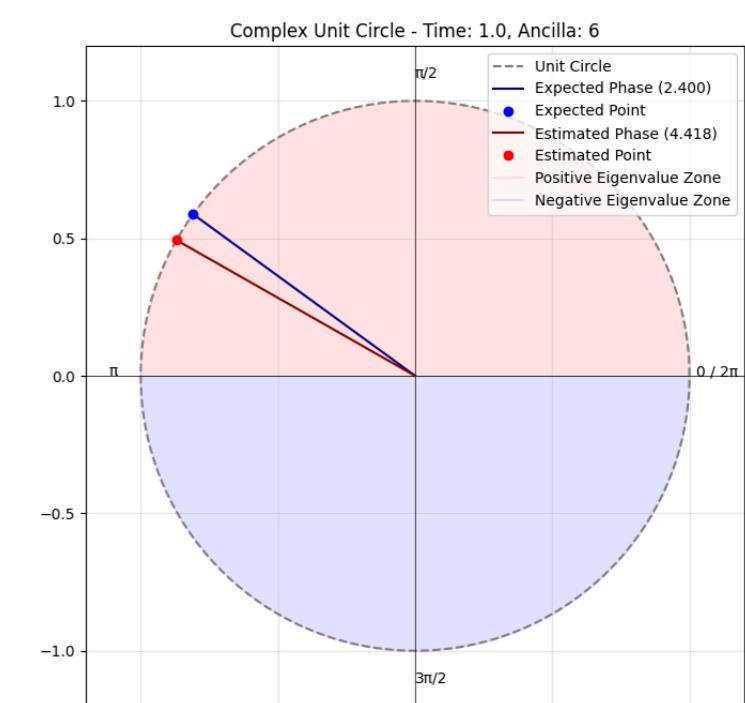
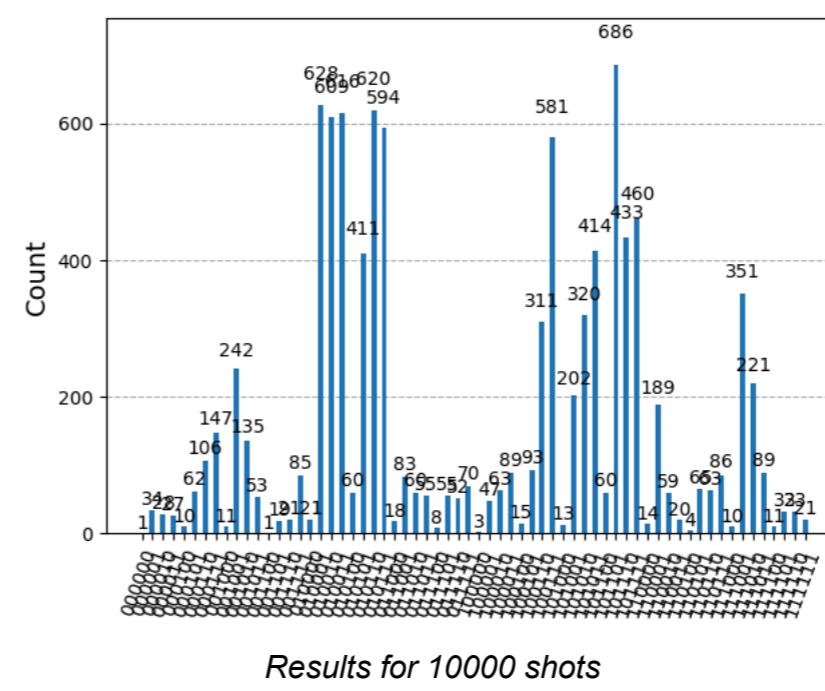
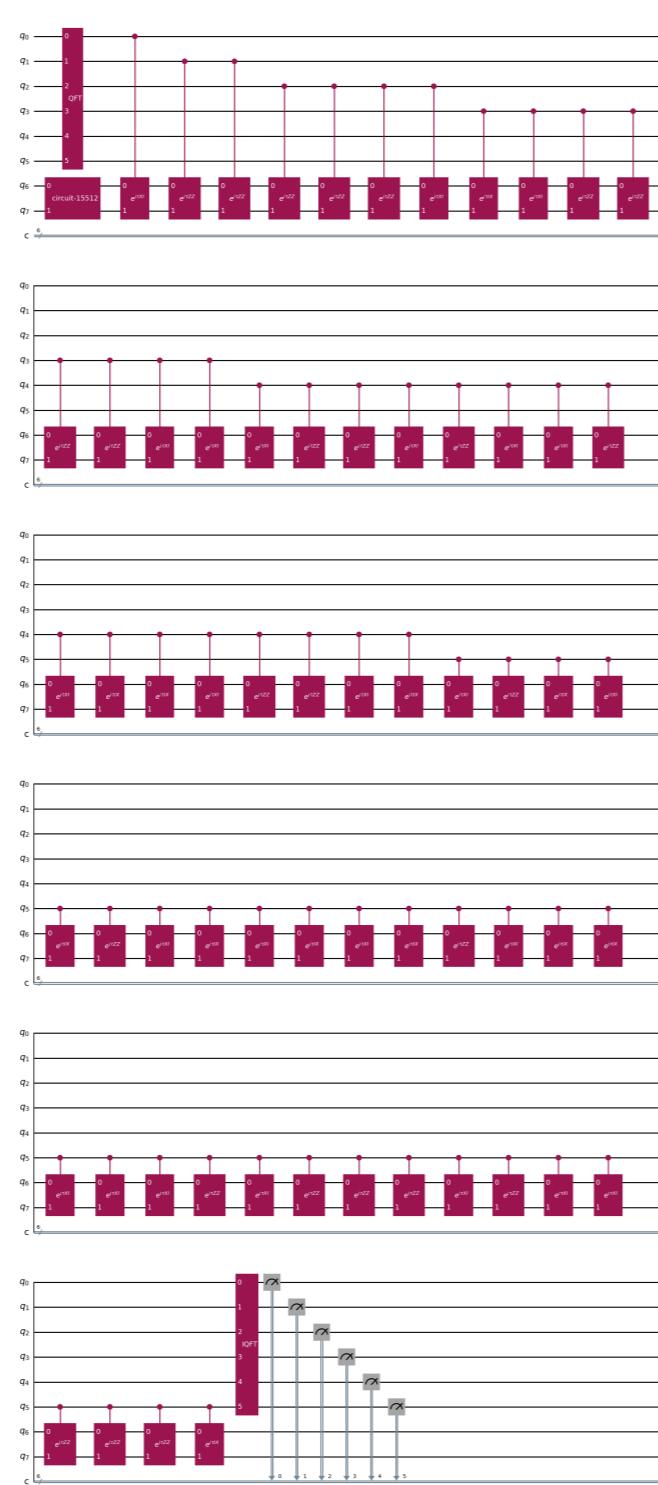
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.38197

Expected bitstring: (011000)

Most common measured bitstring ( $k$ ): 101101

Phase ( $\frac{k}{2^n}$ ): 0.70312

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 4.41786



Quantum Circuit Diagram

Time: 0.01 | Shots: 10000 | Ancilla: 7

Exact energy ( $\lambda$ ): 2.4

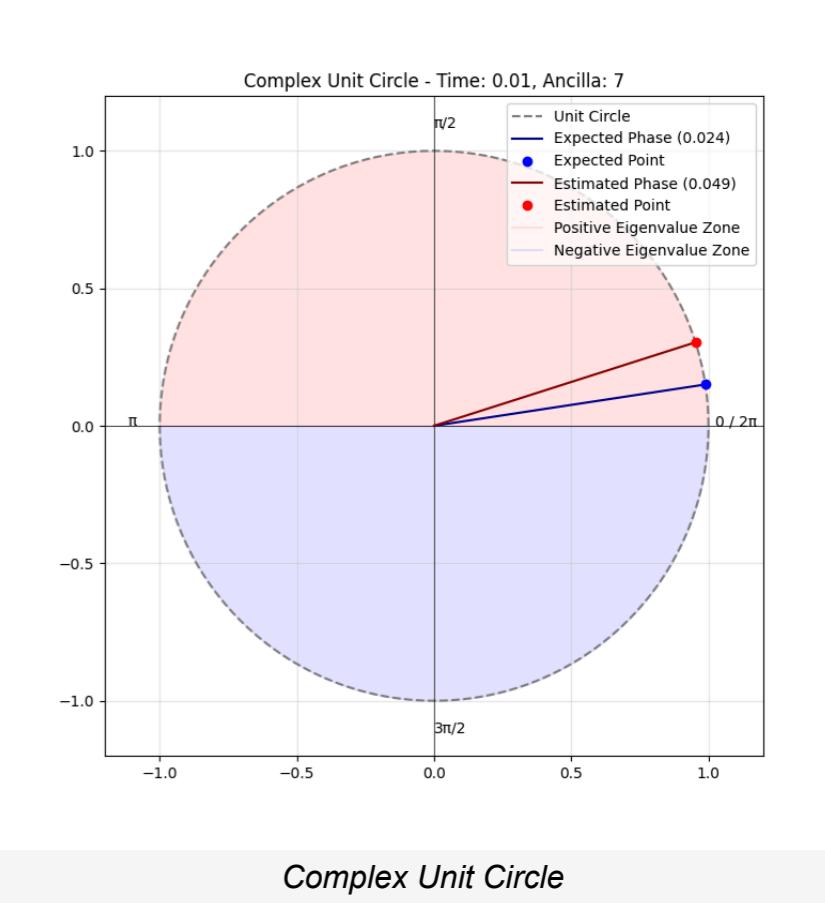
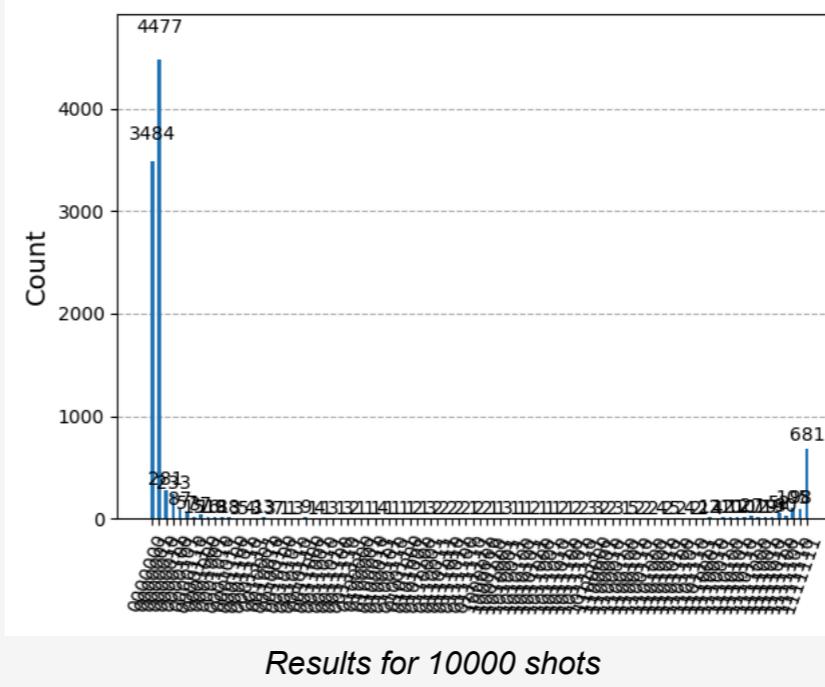
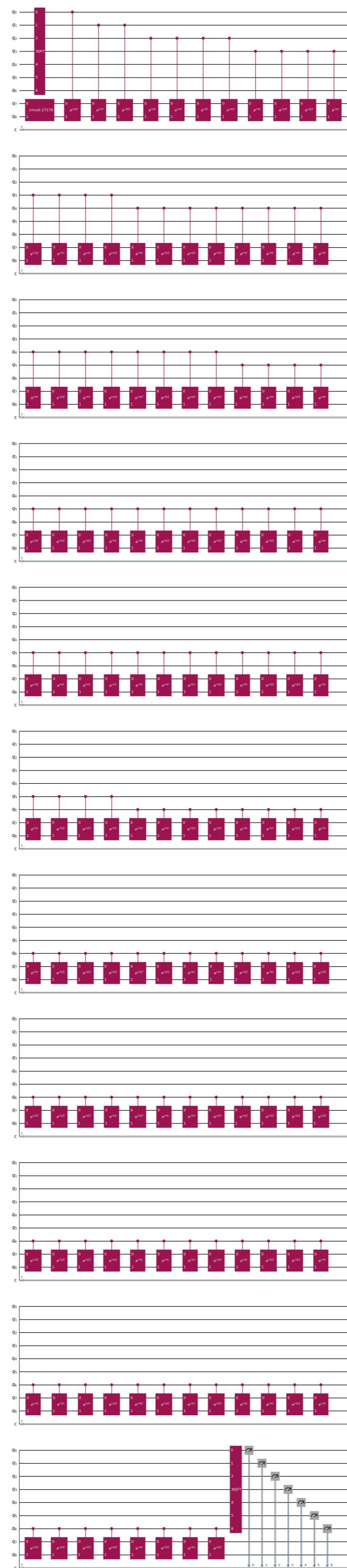
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.00382

Expected bitstring: (0000000)

Most common measured bitstring ( $k$ ): 0000001

Phase ( $\frac{k}{2^n}$ ): 0.00781

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 4.90874





Time: 0.05 | Shots: 10000 | Ancilla: 7

Exact energy ( $\lambda$ ): 2.4

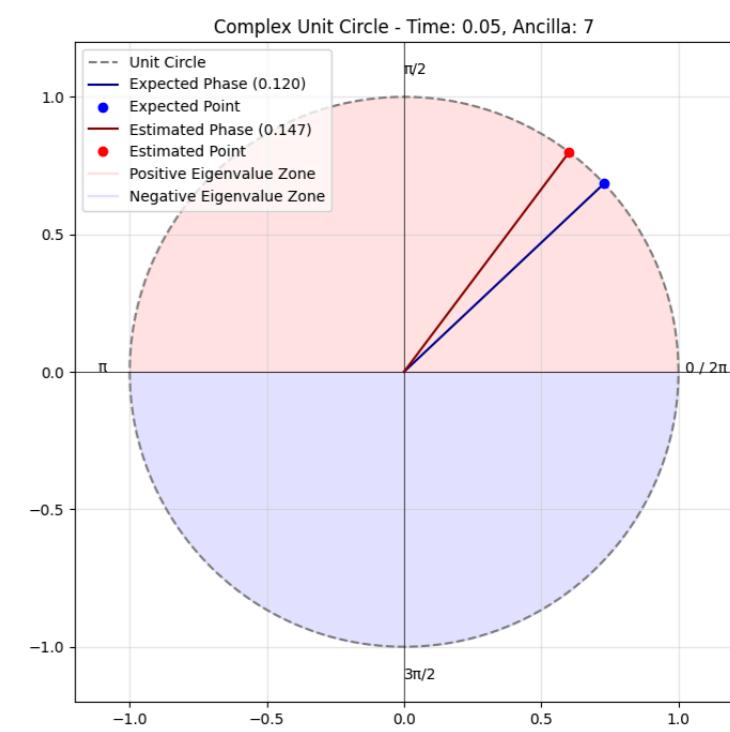
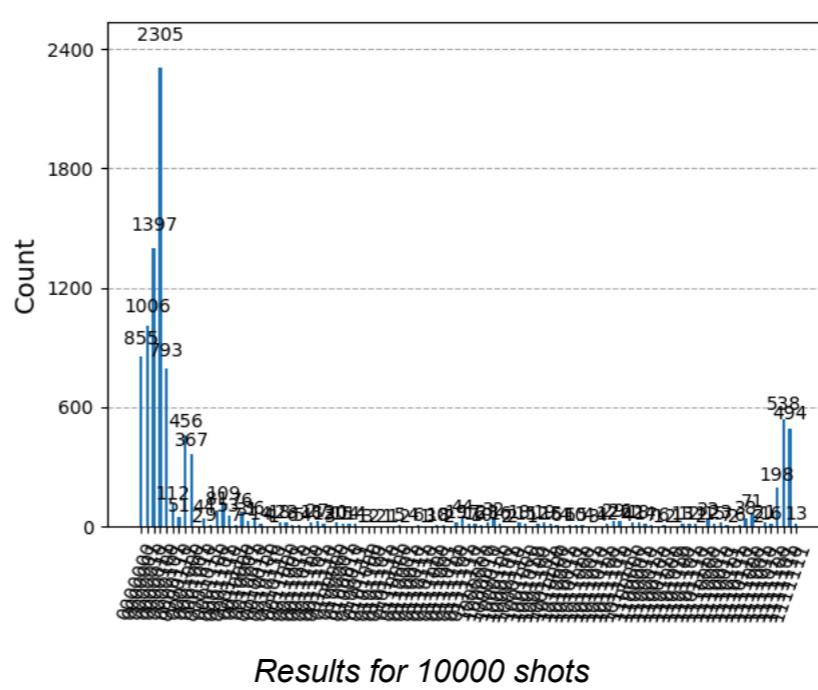
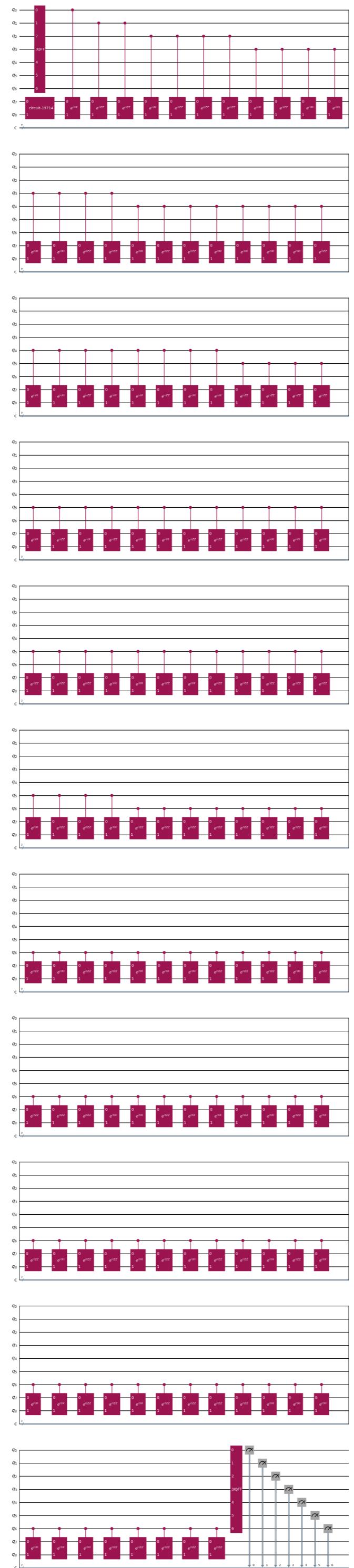
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0191

Expected bitstring: (0000010)

Most common measured bitstring ( $k$ ): 0000011

Phase ( $\frac{k}{2^n}$ ): 0.02344

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 2.94524



Complex Unit Circle



Time: 0.1 | Shots: 10000 | Ancilla: 7

Exact energy ( $\lambda$ ): 2.4

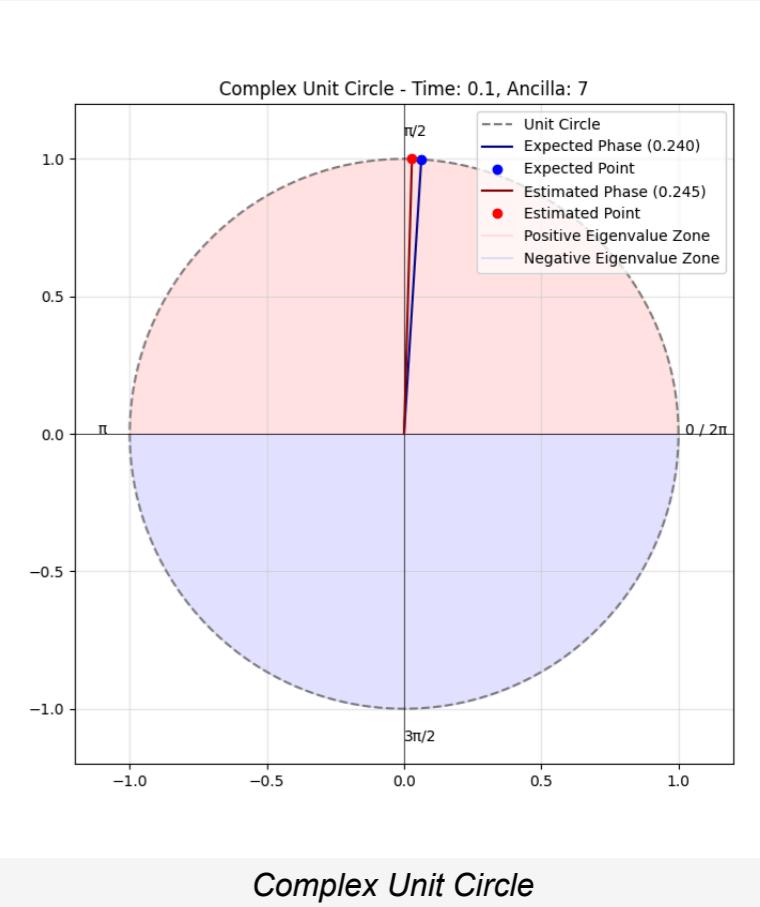
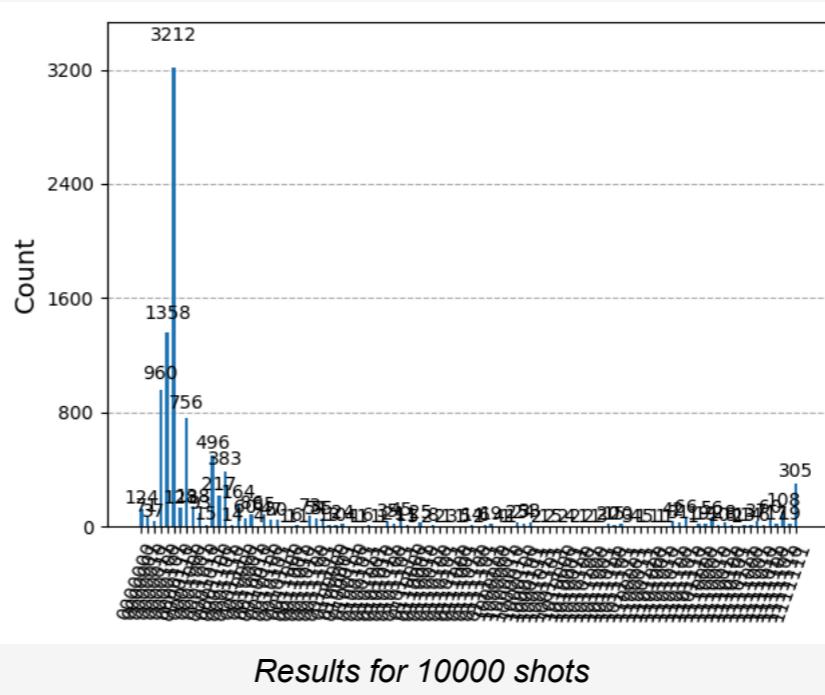
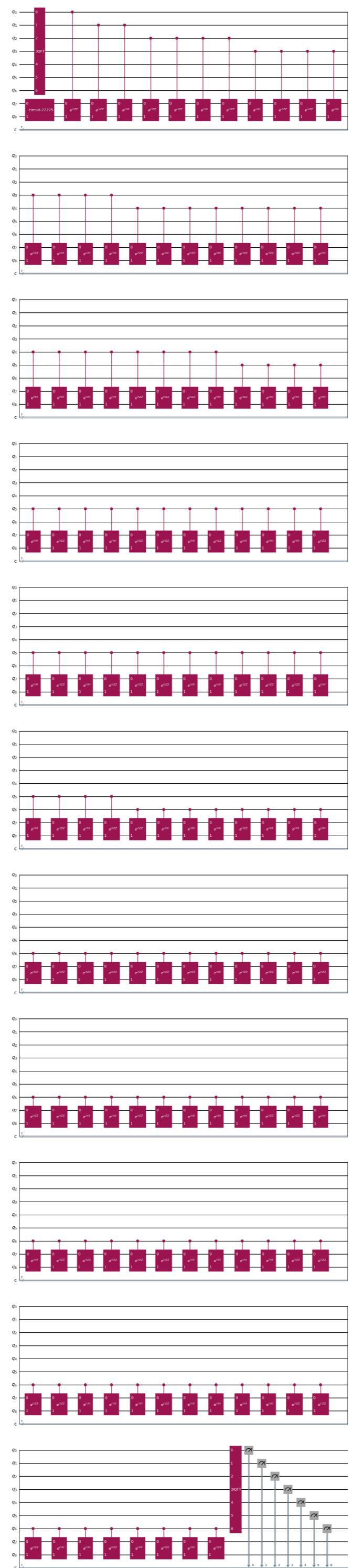
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.0382

Expected bitstring: (0000101)

Most common measured bitstring ( $k$ ): 0000101

Phase ( $\frac{k}{2^n}$ ): 0.03906

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 2.45437





Time: 0.5 | Shots: 10000 | Ancilla: 7

Exact energy ( $\lambda$ ): 2.4

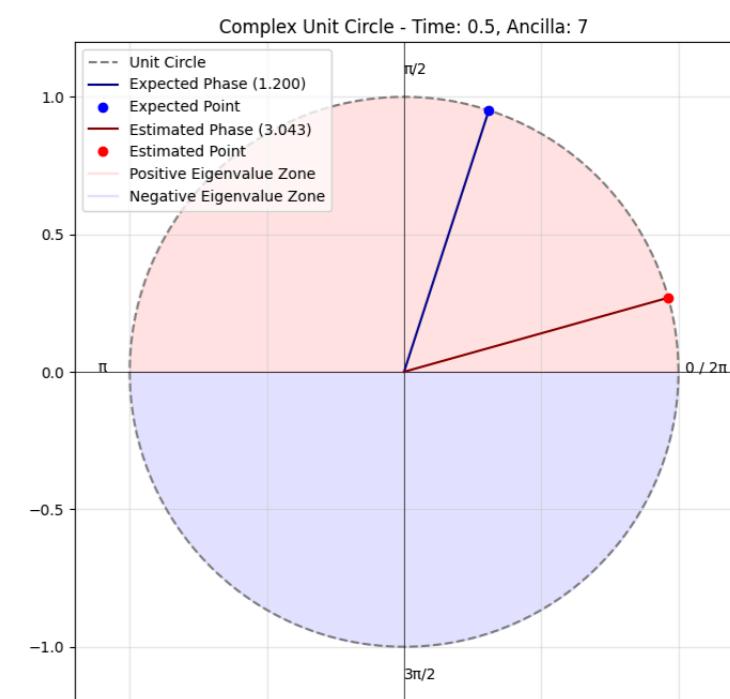
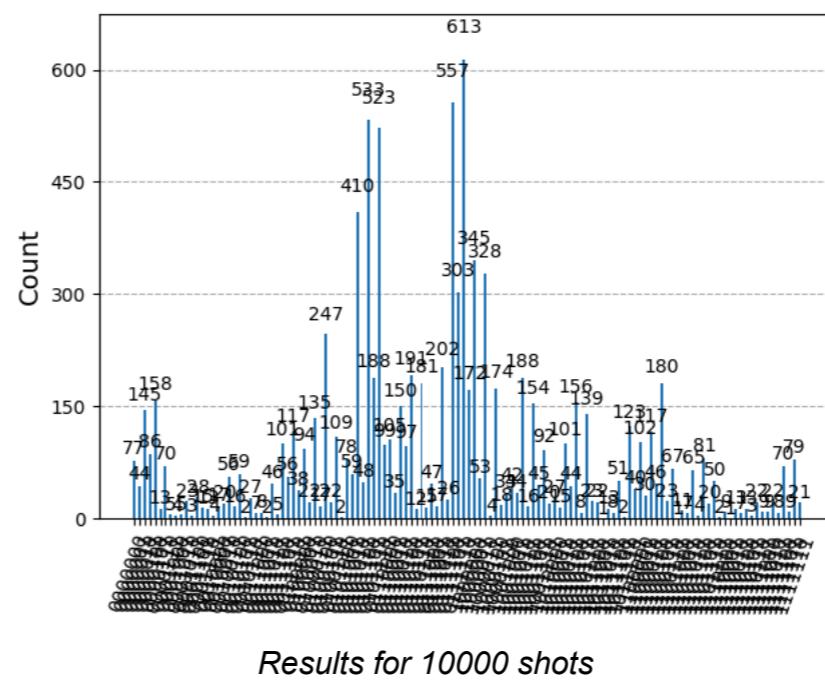
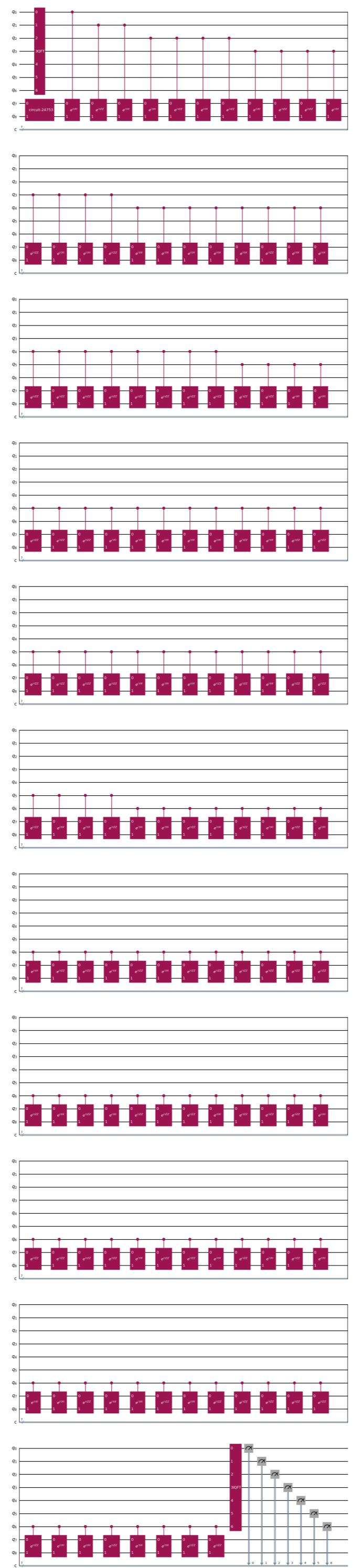
Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.19099

Expected bitstring: (0011000)

Most common measured bitstring ( $k$ ): 0111110

Phase ( $\frac{k}{2^n}$ ): 0.48438

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 6.08684



Complex Unit Circle



Time: 1.0 | Shots: 10000 | Ancilla: 7

Exact energy ( $\lambda$ ): 2.4

Exact expected phase ( $\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$ ): 0.38197

Expected bitstring: (0110001)

Most common measured bitstring ( $k$ ): 1011110

Phase ( $\frac{k}{2^n}$ ): 0.73438

Estimated Energy ( $\frac{2 \cdot \pi \cdot \varphi}{t}$ ): 4.61421

