

Quantum Phase Estimation - Report

Exact Eigenvalue Estimation of Hermitian Matrices

Let H be a Hermitian matrix. Let $|\lambda\rangle$ be an eigenvector of H with eigenvalue $\lambda \implies e^{iHt}|\lambda\rangle = e^{i\lambda t}|\lambda\rangle$

Of course, the measurements of QPE will *not* give us a direct approximation of λ , but rather of some "phase" θ such that:

$$e^{i\lambda t} = e^{i\theta} \implies \lambda = \frac{\theta}{t}$$

Recall that if the phase θ happens to be of the form $\frac{2\pi k}{2^n}$ then we can measure the ancilla qubits and get k exactly.

We can "force" this to happen. Let us say we pre-compute all eigenvalues $\lambda_1, \dots, \lambda_n$ of H and want to estimate the eigenvalue λ_i . Well, we can simply choose t such that:

$$\lambda_i = \frac{2\pi k}{t2^n}$$

This is equivalent to choosing:

$$t = \frac{2\pi k}{\lambda_i 2^n}$$

Hamiltonian used for this simulation

Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

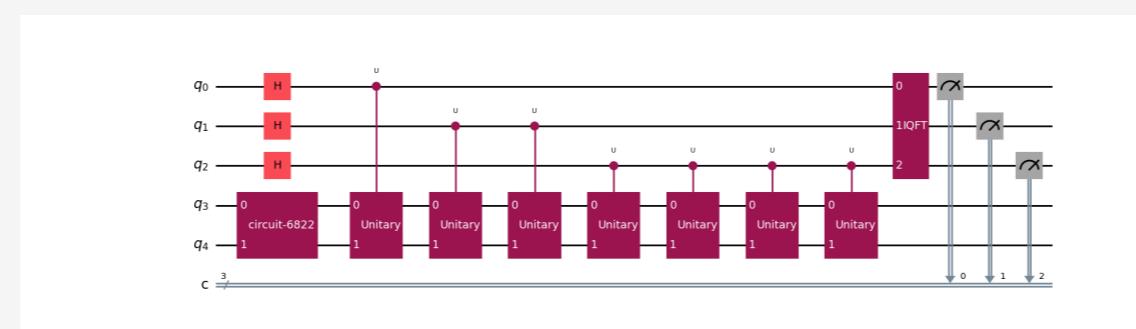
Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

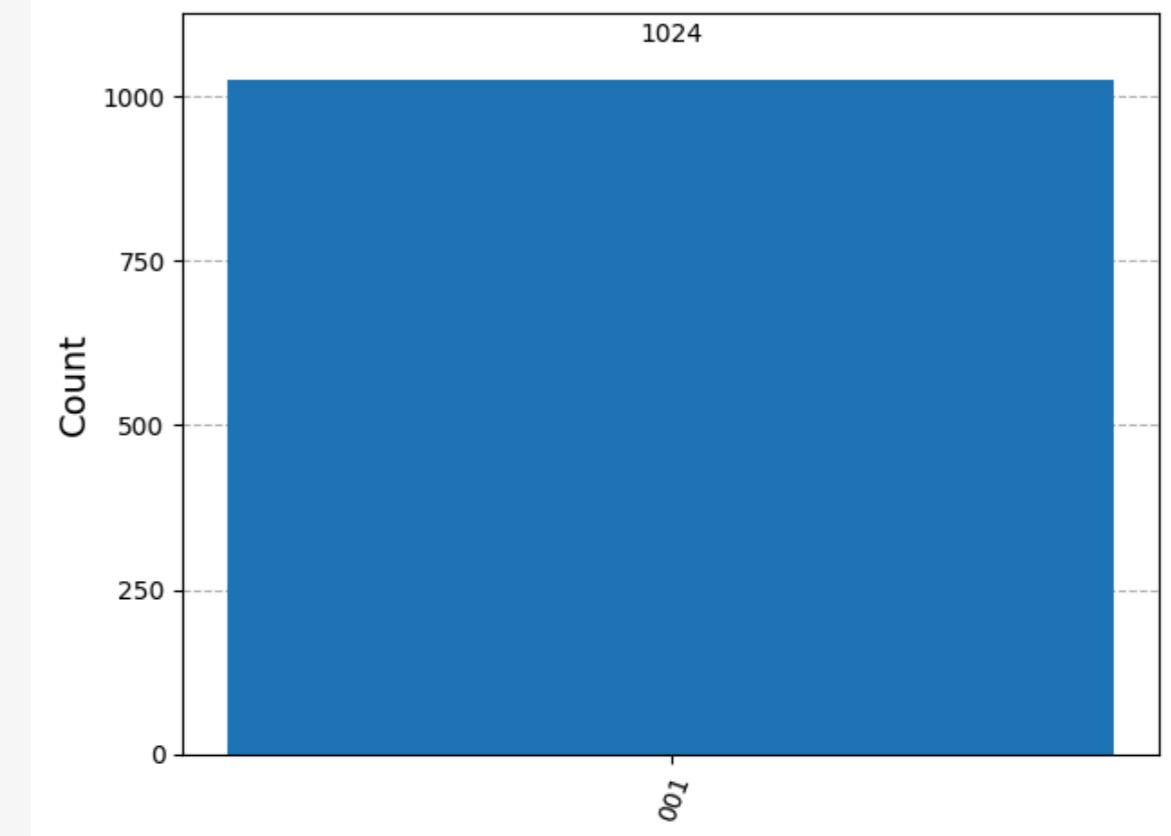
Time: -0.2513998258660903 | Ancilla: 3

Expected Binary: 001

Measured: 001



Quantum Circuit Diagram



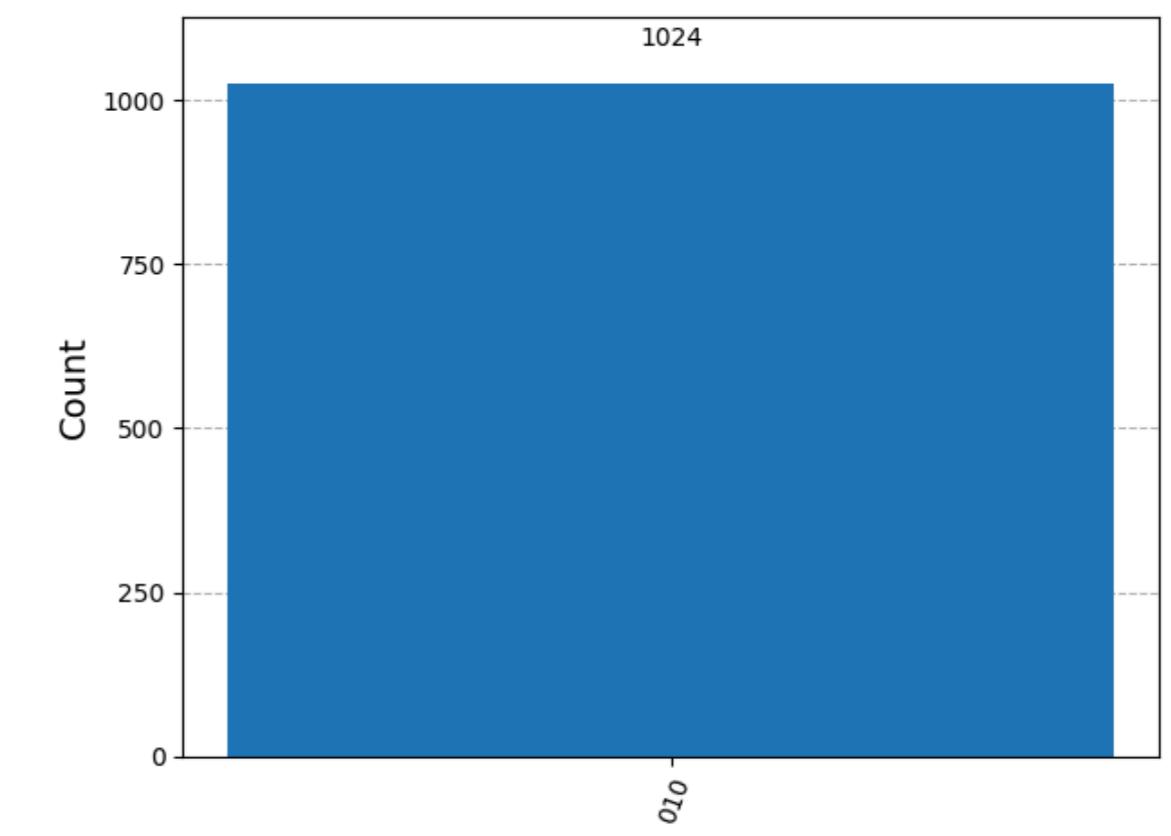
Time: -0.5027996517321806 | Ancilla: 3

Expected Binary: 010

Measured: 010



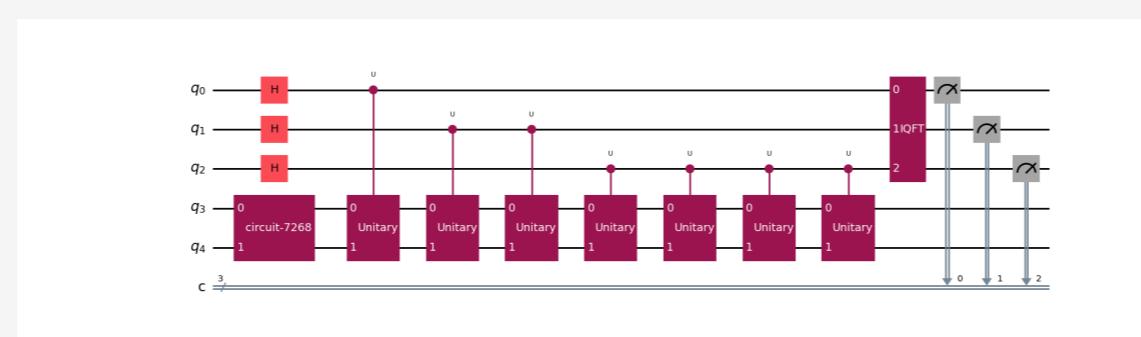
Quantum Circuit Diagram



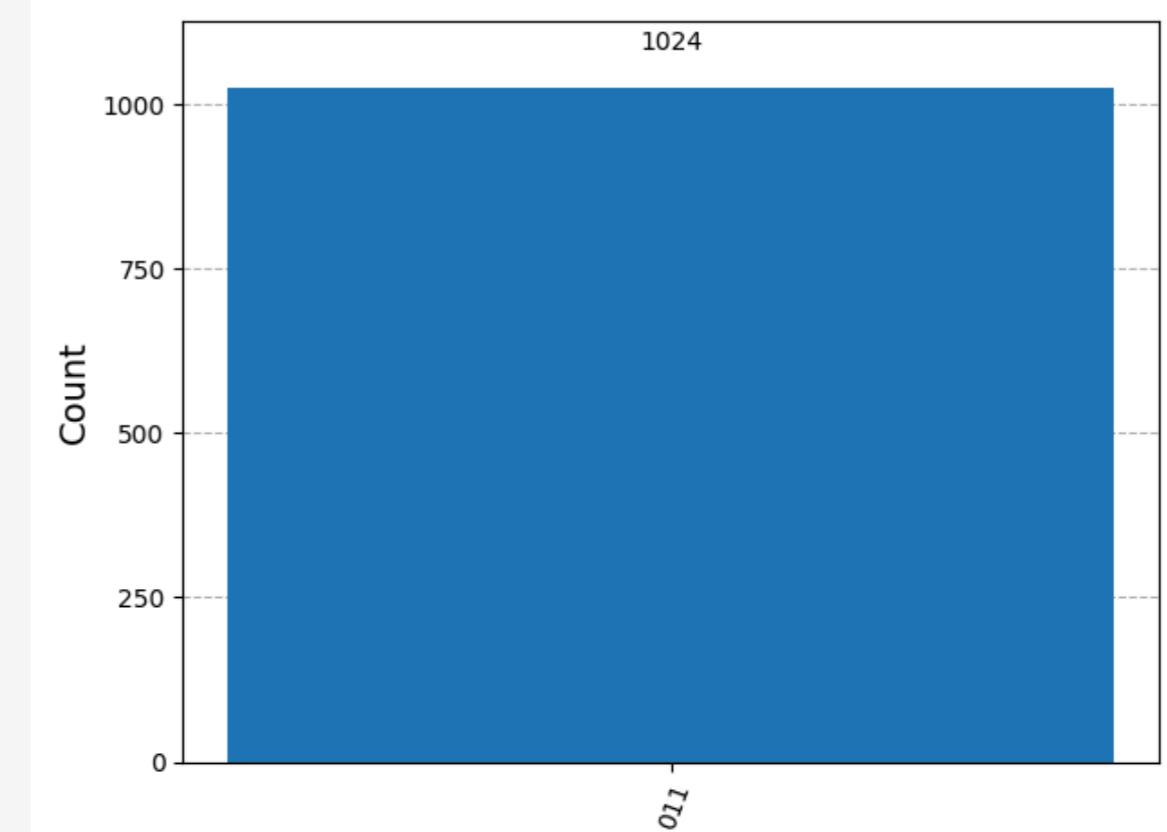
Time: -0.7541994775982708 | Ancilla: 3

Expected Binary: 011

Measured: 011



Quantum Circuit Diagram

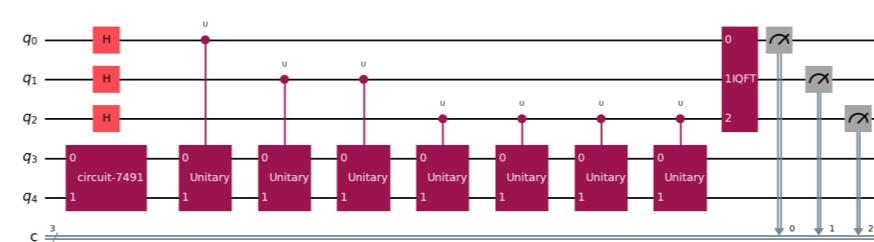


Results for Exact Phase Estimation

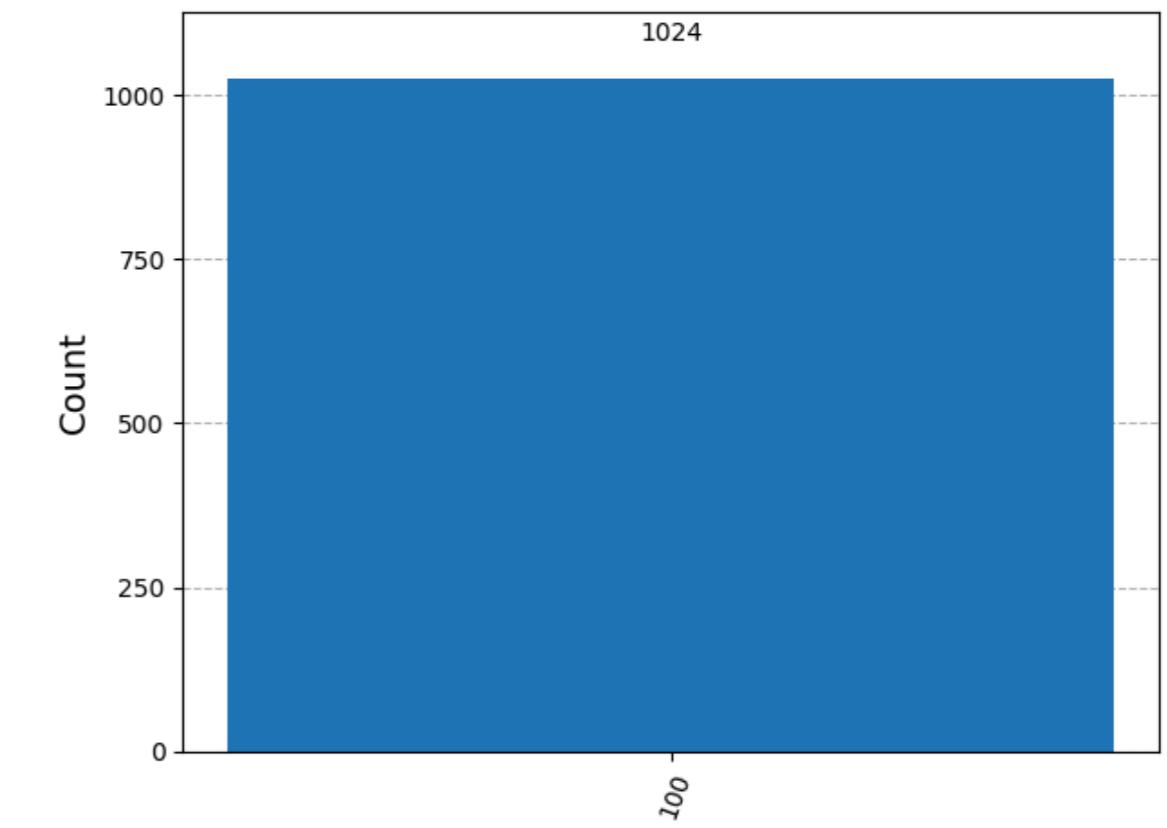
Time: -1.005599303464361 | Ancilla: 3

Expected Binary: 100

Measured: 100



Quantum Circuit Diagram



Results for Exact Phase Estimation

Approximate Eigenvalue Estimation of Hermitian Matrices

We aim to estimate the eigenvalues of a Hermitian matrix H using QPE.

$$e^{iHt}|\lambda\rangle = e^{2\pi i\varphi}|\lambda\rangle$$

$$\lambda = \frac{2\pi\varphi}{t}$$

Hamiltonian used for this simulation

Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

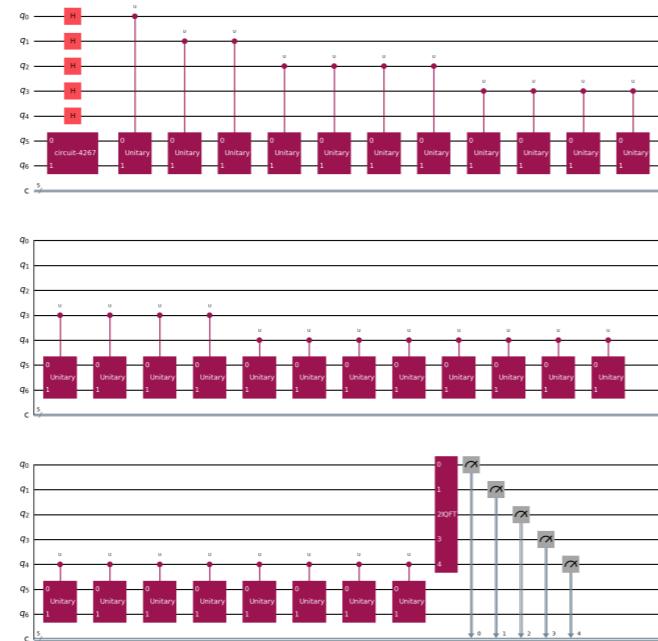
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.19099

Expected bitstring: (00110)

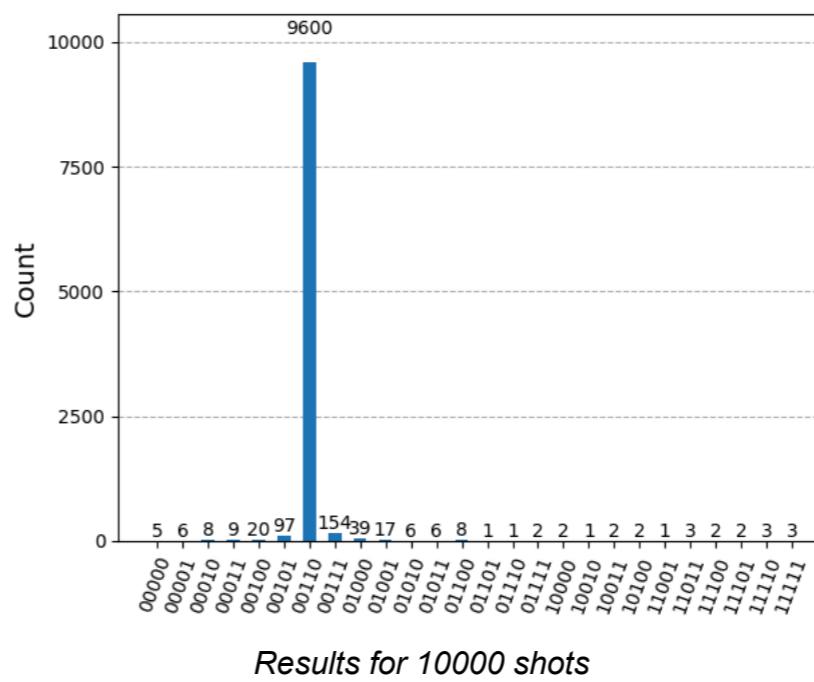
Most common measured bitstring (k): 00110

Phase ($\frac{k}{2^n}$): 0.1875

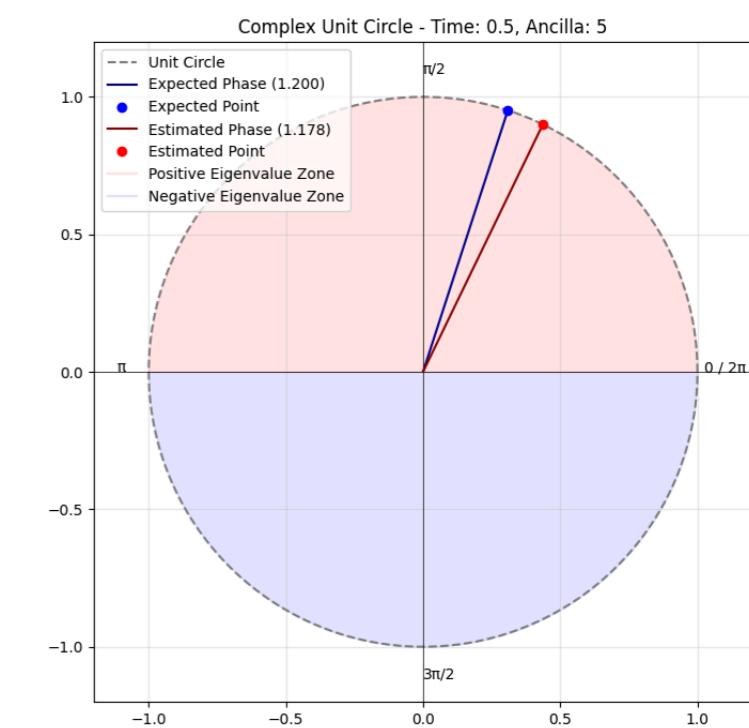
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 2.35619



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

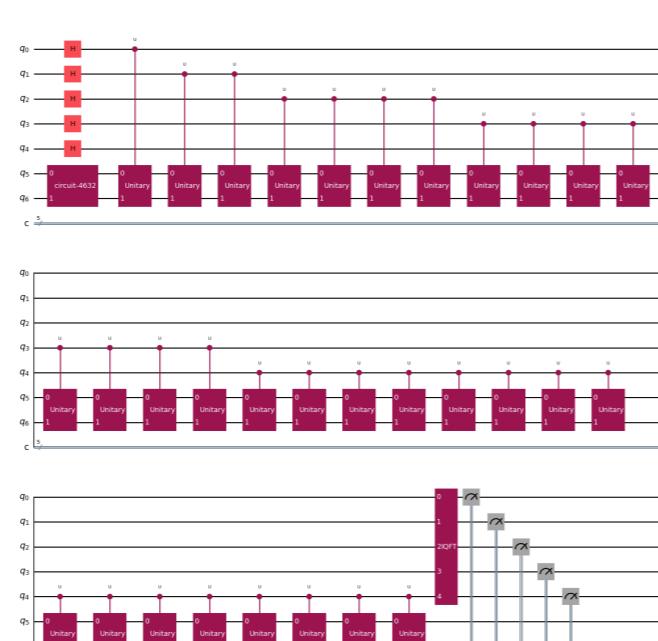
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.38197

Expected bitstring: (01100)

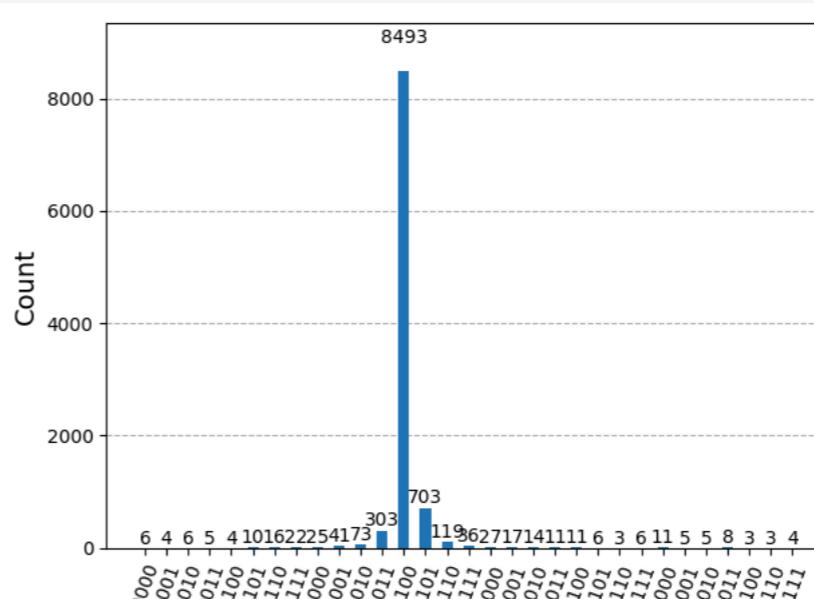
Most common measured bitstring (k): 01100

Phase ($\frac{k}{2^n}$): 0.375

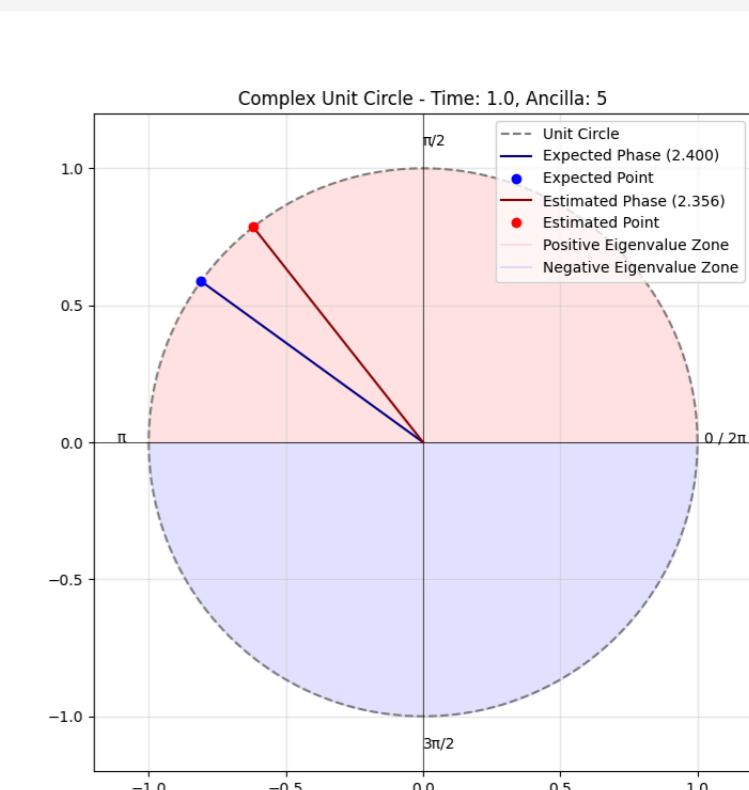
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 2.35619



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.01 | Shots: 10000 | Ancilla: 5

Exact energy (λ): -3.1241

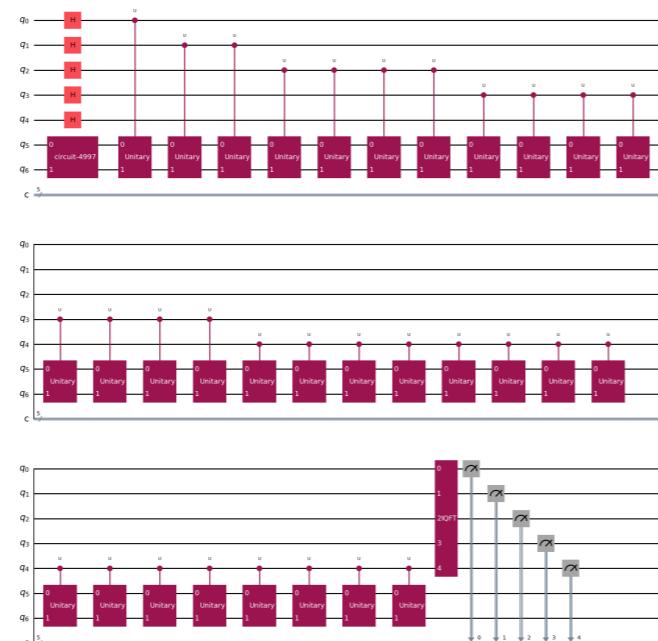
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.99503

Expected bitstring: (100000)

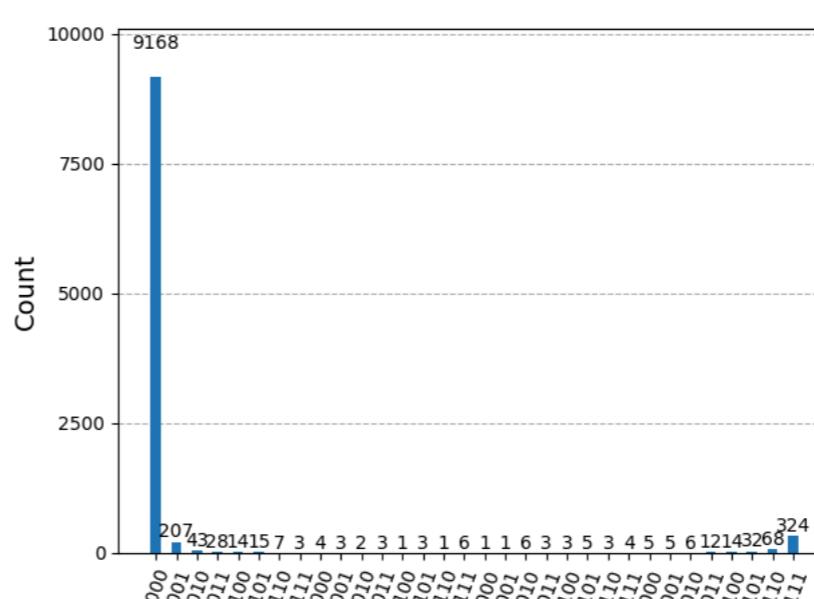
Most common measured bitstring (k): 00000

Phase ($\frac{k}{2^n}$): 0.0

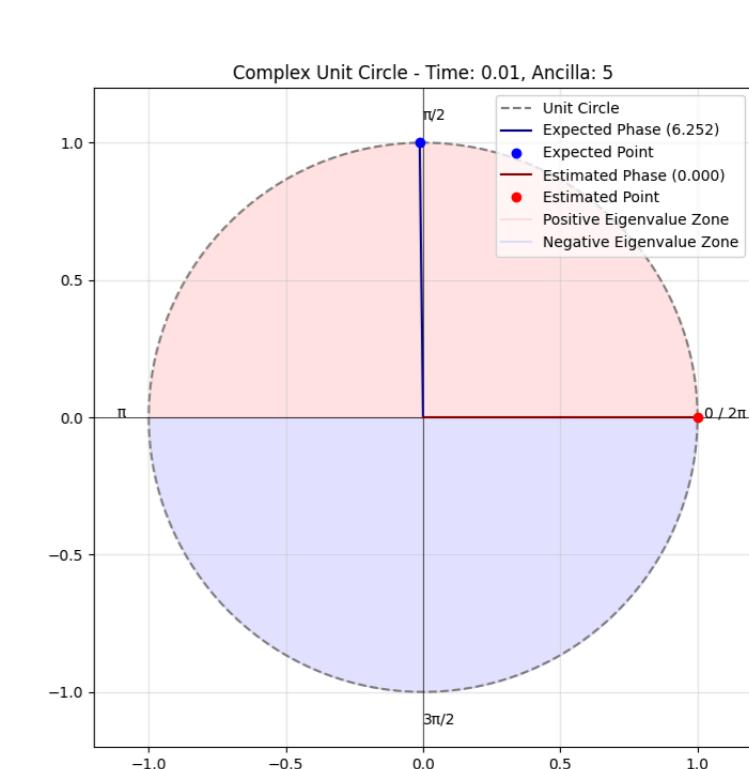
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 0.0



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.05 | Shots: 10000 | Ancilla: 5

Exact energy (λ): -3.1241

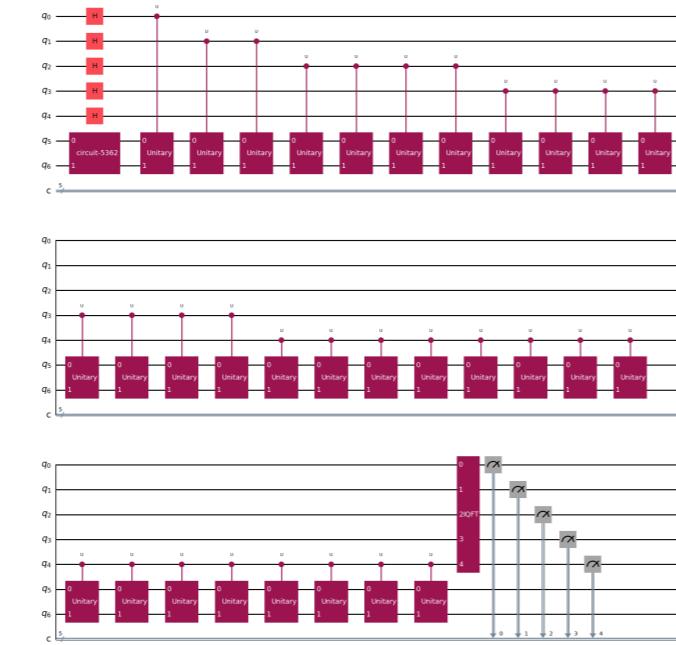
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.97514

Expected bitstring: (11111)

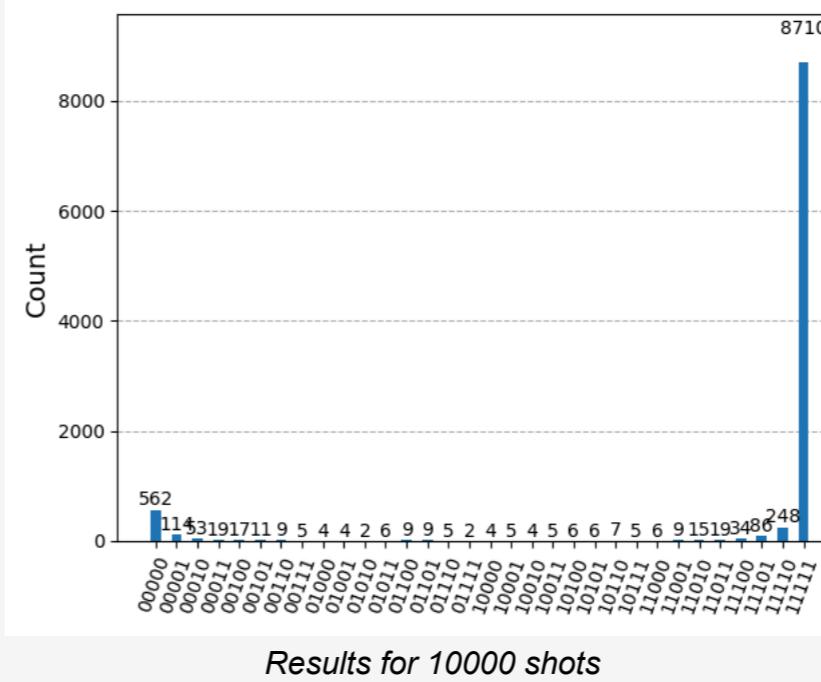
Most common measured bitstring (k): 11111

Phase ($\frac{k}{2^n}$): -0.03125

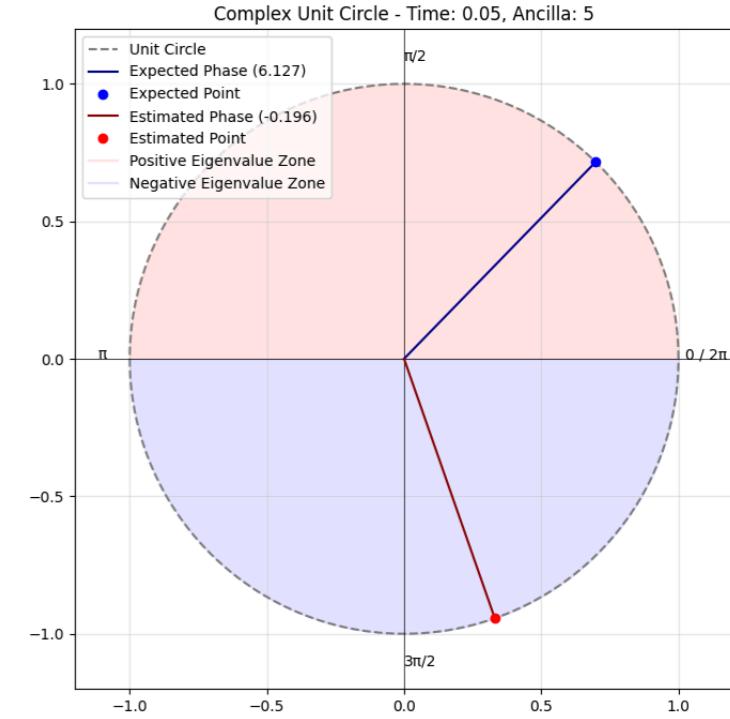
Estimated Energy ($\frac{2\pi\varphi}{t}$): -3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 5

Exact energy (λ): -3.1241

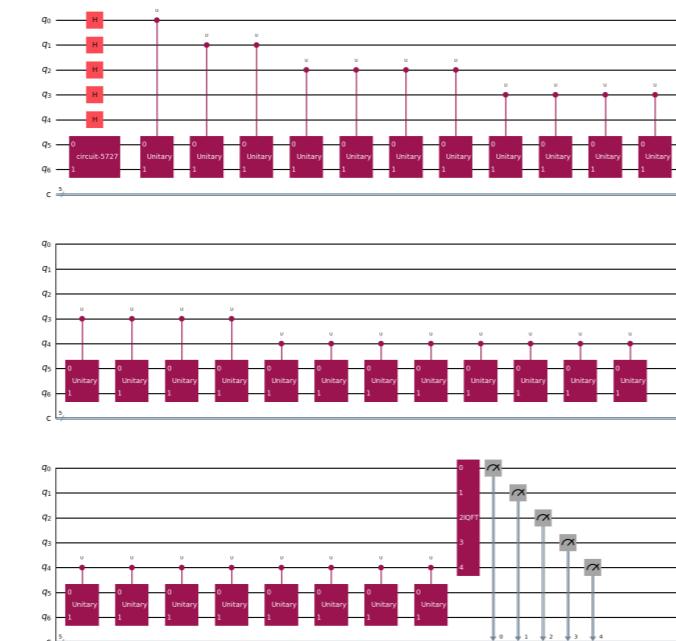
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.95028

Expected bitstring: (11110)

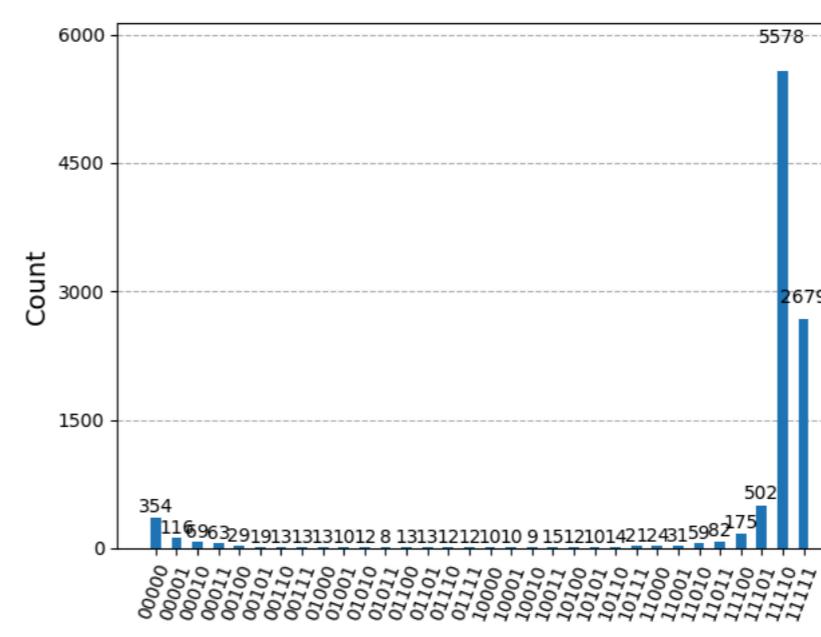
Most common measured bitstring (k): 11110

Phase ($\frac{k}{2^n}$): -0.0625

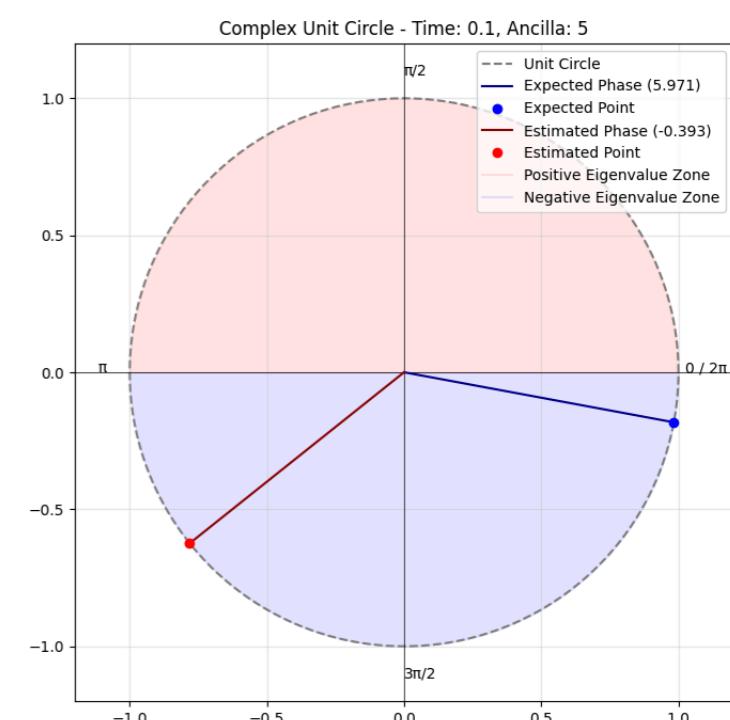
Estimated Energy ($\frac{2\pi\varphi}{t}$): -3.92699



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy (λ): -3.1241

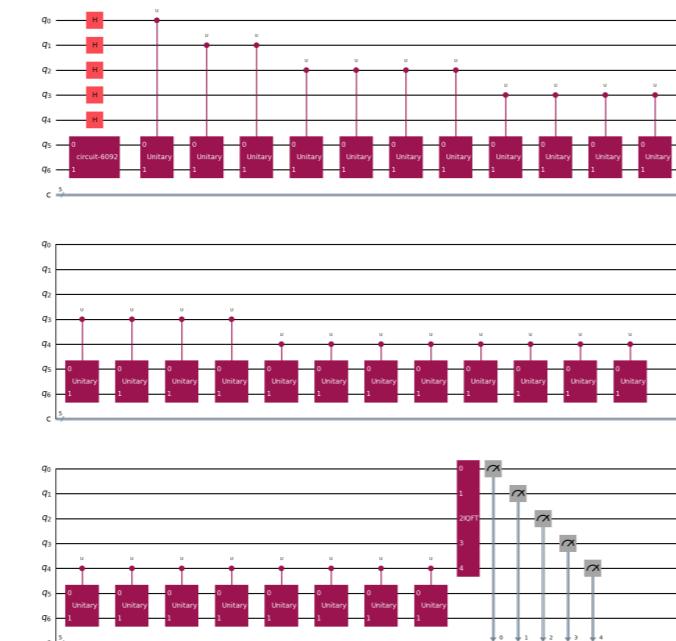
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.75139

Expected bitstring: (11000)

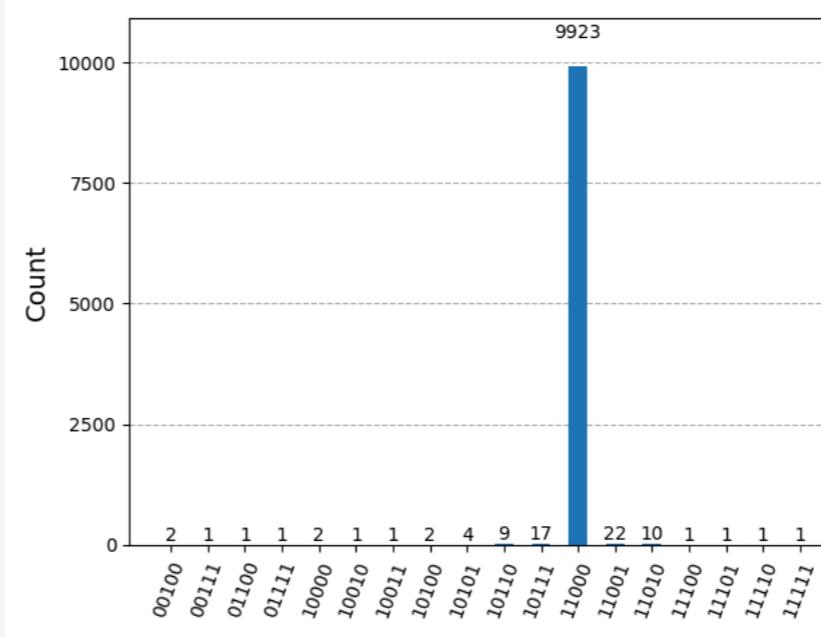
Most common measured bitstring (k): 11000

Phase ($\frac{k}{2^n}$): -0.25

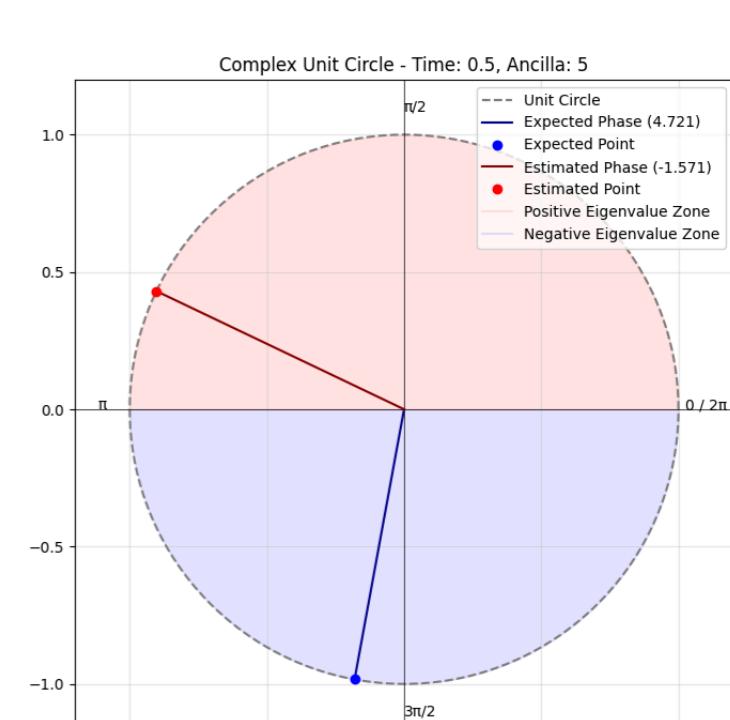
Estimated Energy ($\frac{2\pi\varphi}{t}$): -3.14159



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy (λ): -3.1241

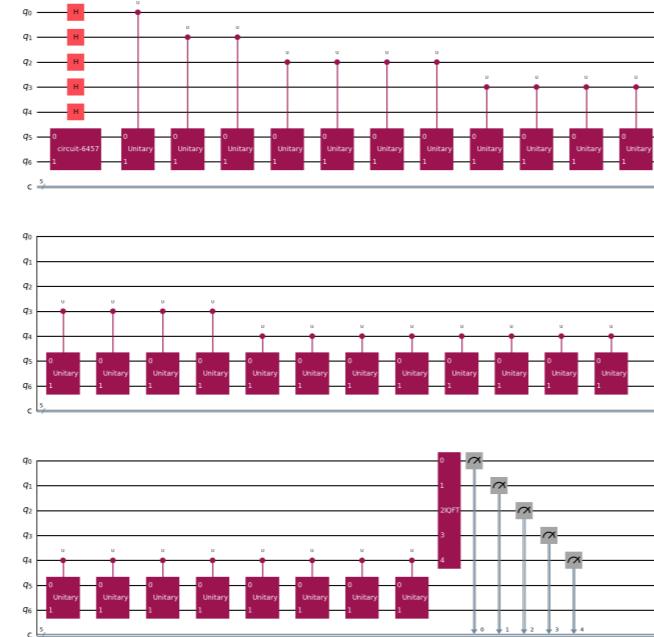
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.50278

Expected bitstring: (10000)

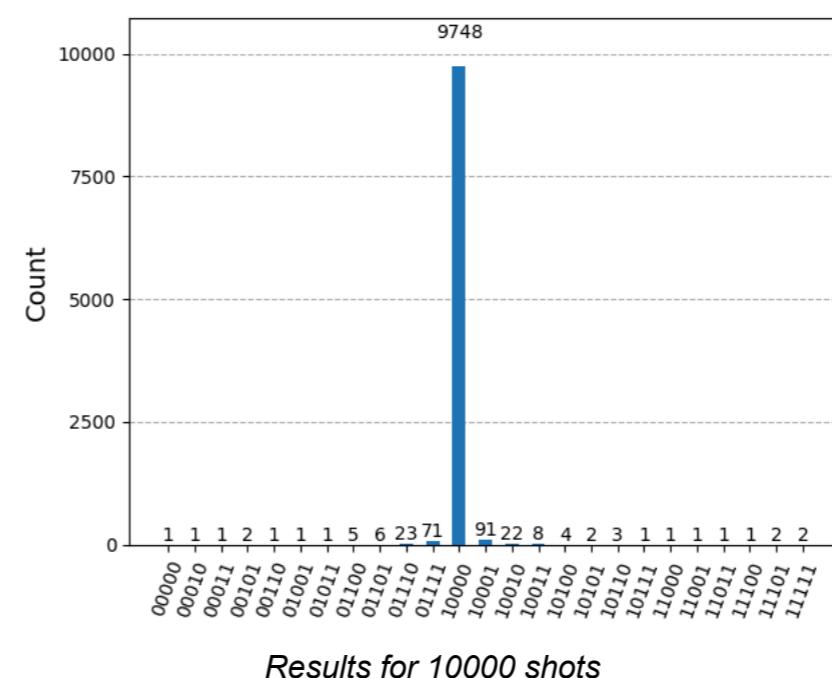
Most common measured bitstring (k): 10000

Phase ($\frac{k}{2^n}$): -0.5

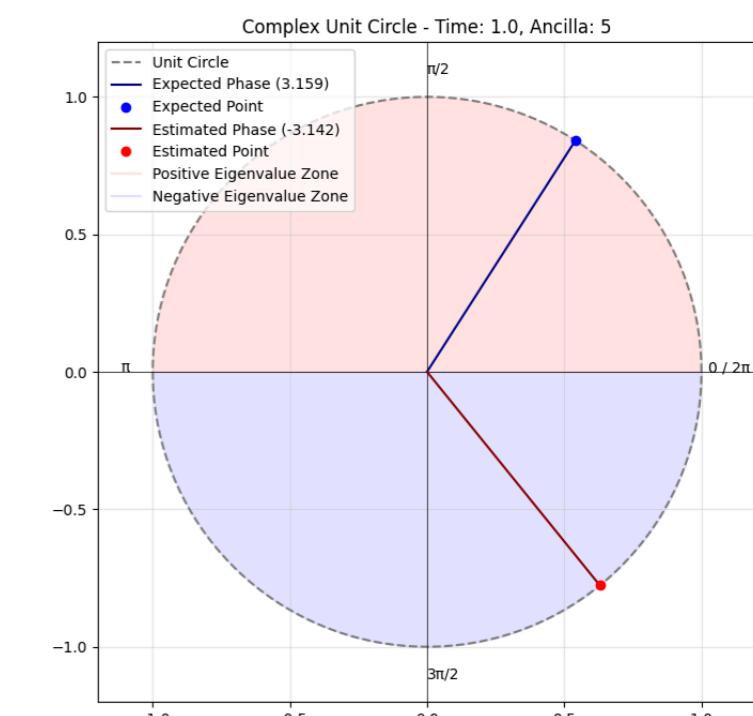
Estimated Energy ($\frac{2\pi\varphi}{t}$): -3.14159



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

qDRIFT Simulation

Hamiltonian used for this simulation

Tensor Product Form:

$$-1.2 \cdot Z \otimes Z + -1.2 \cdot Z \otimes Z + -1.0 \cdot X \otimes I + -1.0 \cdot I \otimes X$$

Matrix Form:

$$\begin{bmatrix} -2.4 & -1.0 & -1.0 & 0 \\ -1.0 & 2.4 & 0 & -1.0 \\ -1.0 & 0 & 2.4 & -1.0 \\ 0 & -1.0 & -1.0 & -2.4 \end{bmatrix}$$

Time: 0.01 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

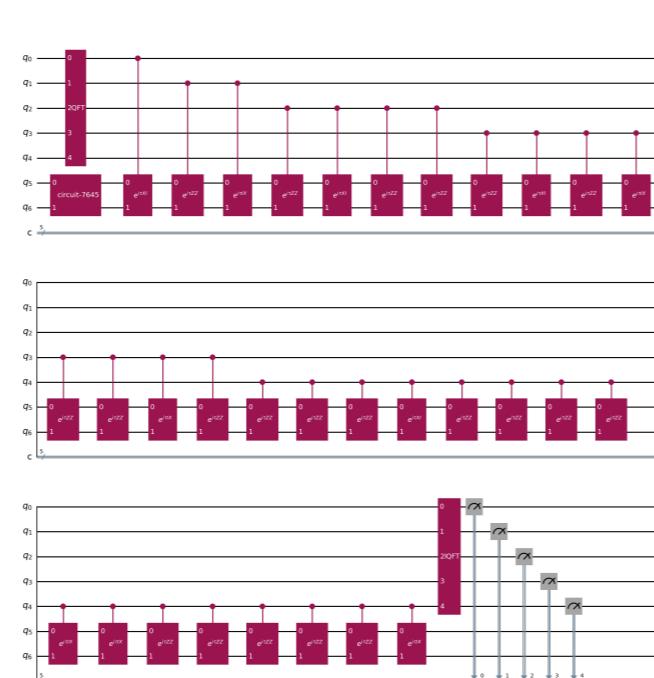
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.00382

Expected bitstring: (00000)

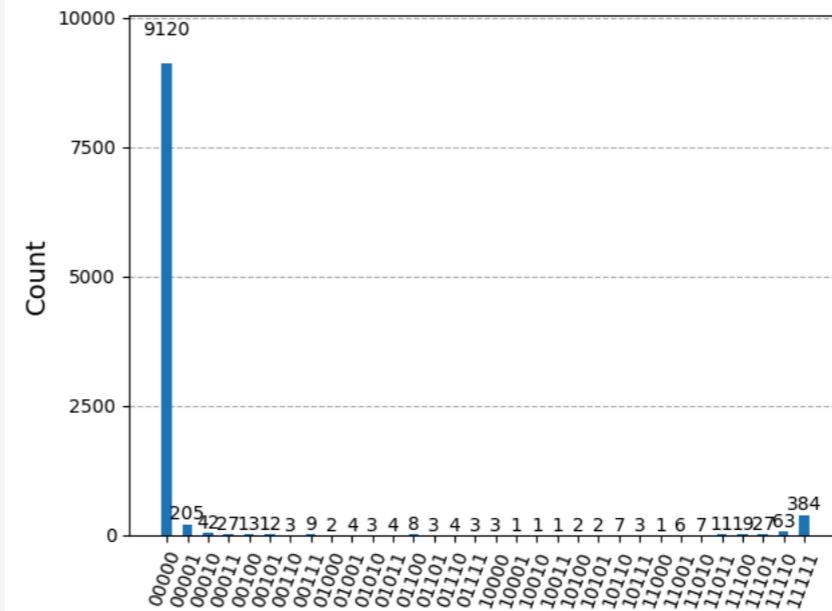
Most common measured bitstring (k): 00000

Phase ($\frac{k}{2^n}$): 0.0

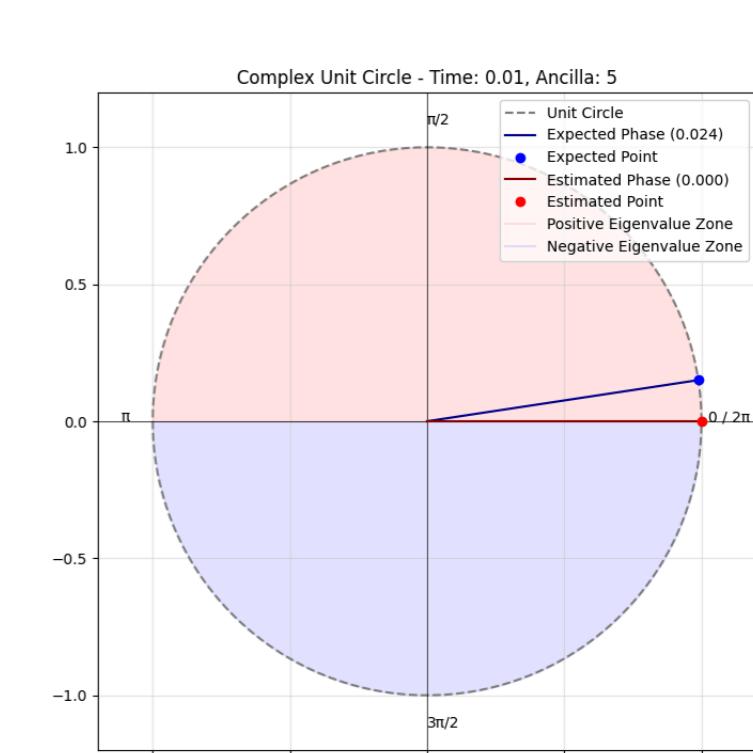
Estimated Energy ($\frac{2\pi\varphi}{t}$): 0.0



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.05 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

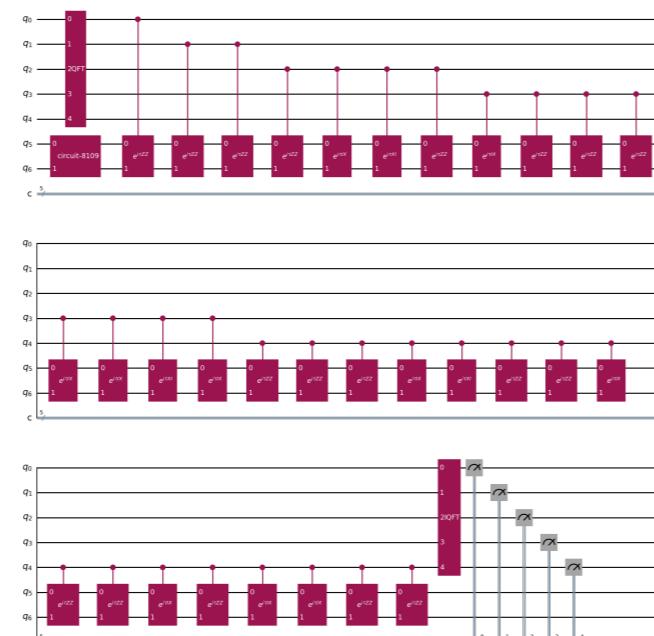
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.0191

Expected bitstring: (00001)

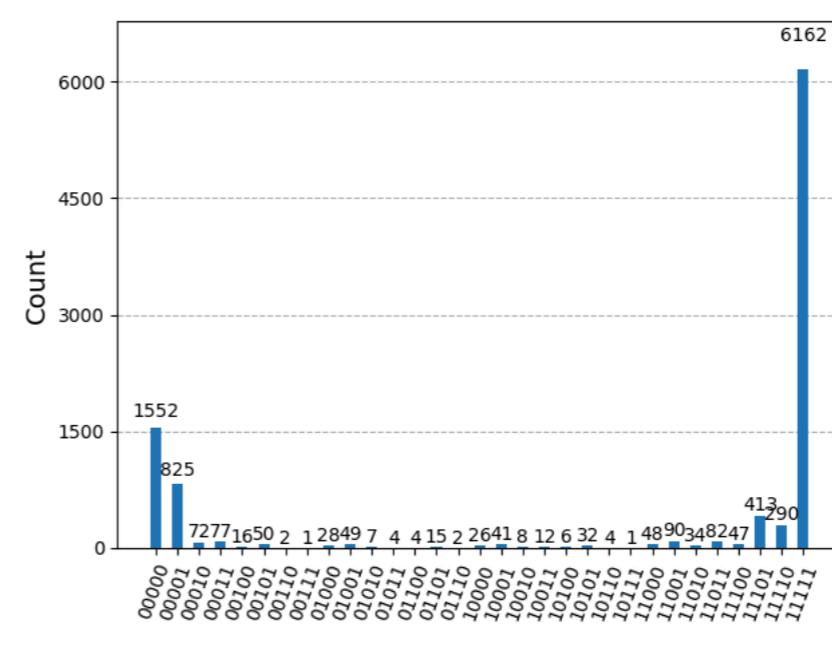
Most common measured bitstring (k): 11111

Phase ($\frac{k}{2^n}$): 0.96875

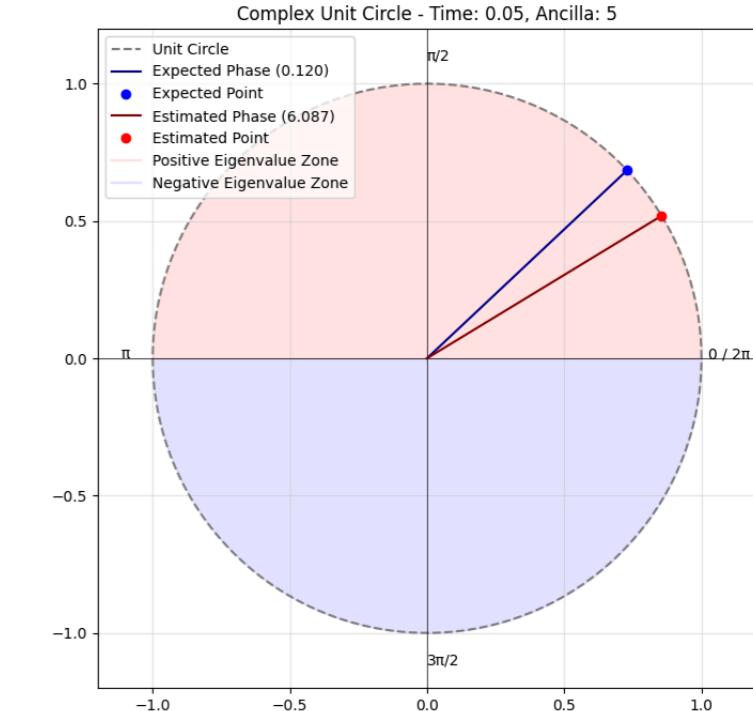
Estimated Energy ($\frac{2\pi\varphi}{t}$): 121.73672



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

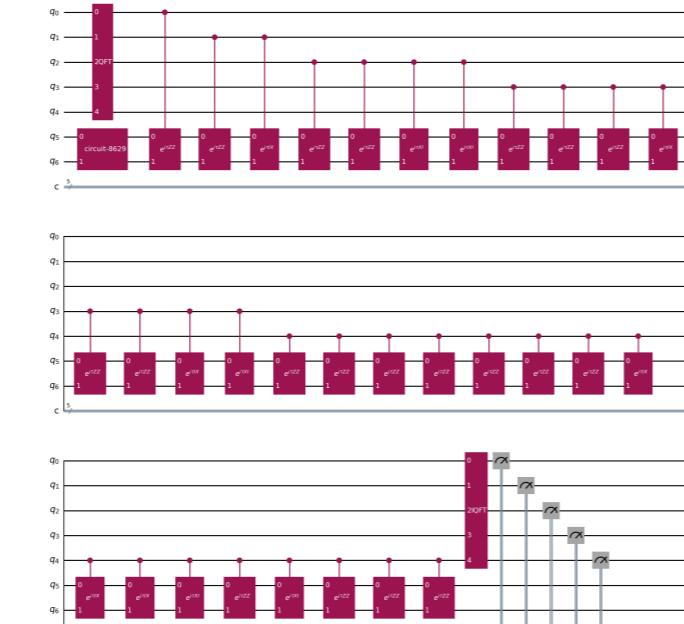
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.0382

Expected bitstring: (00001)

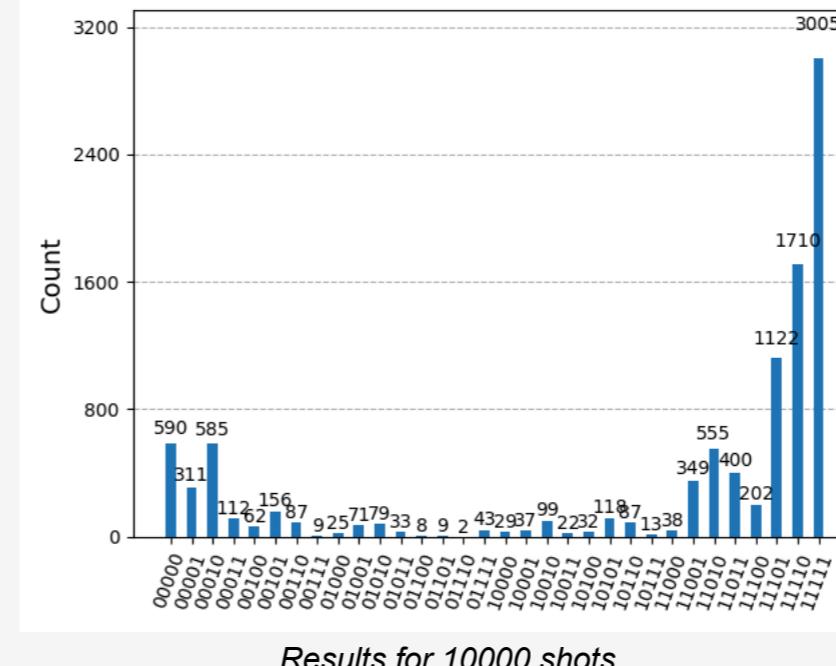
Most common measured bitstring (k): 11111

Phase ($\frac{k}{2^n}$): 0.96875

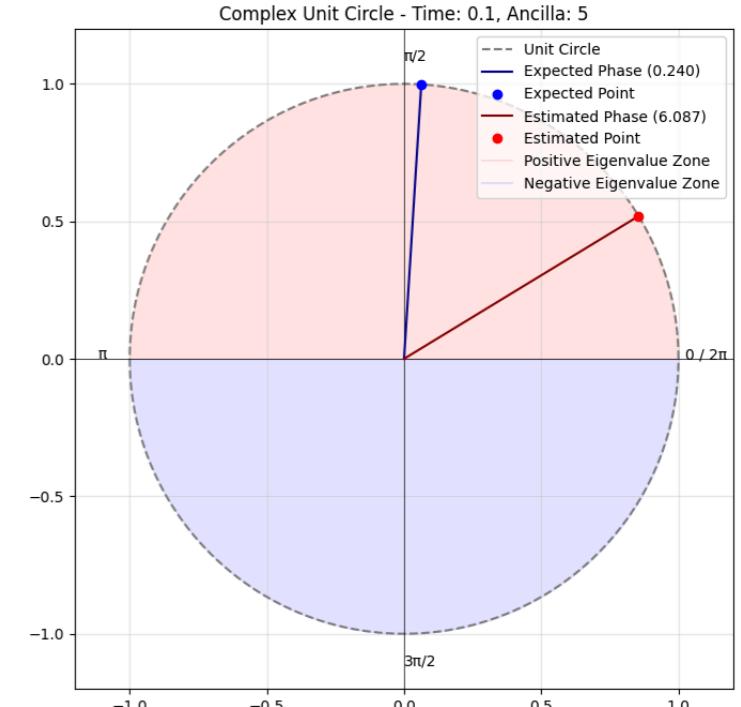
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 60.86836



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

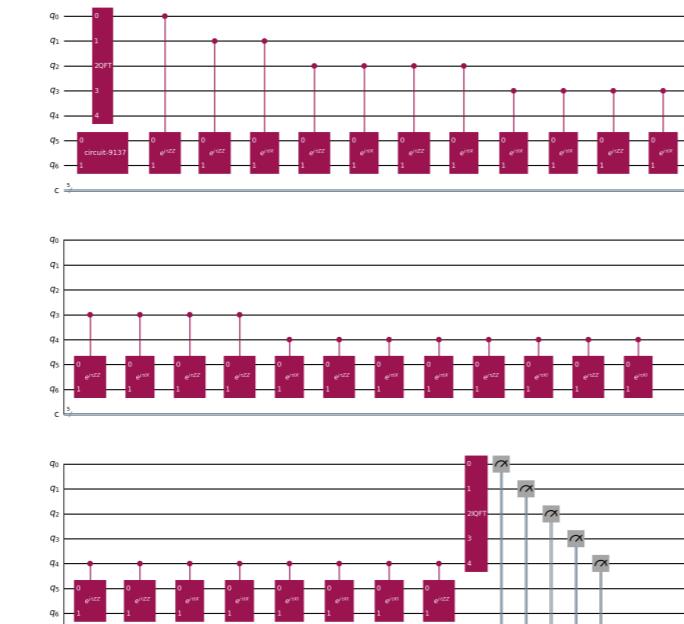
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.19099

Expected bitstring: (00110)

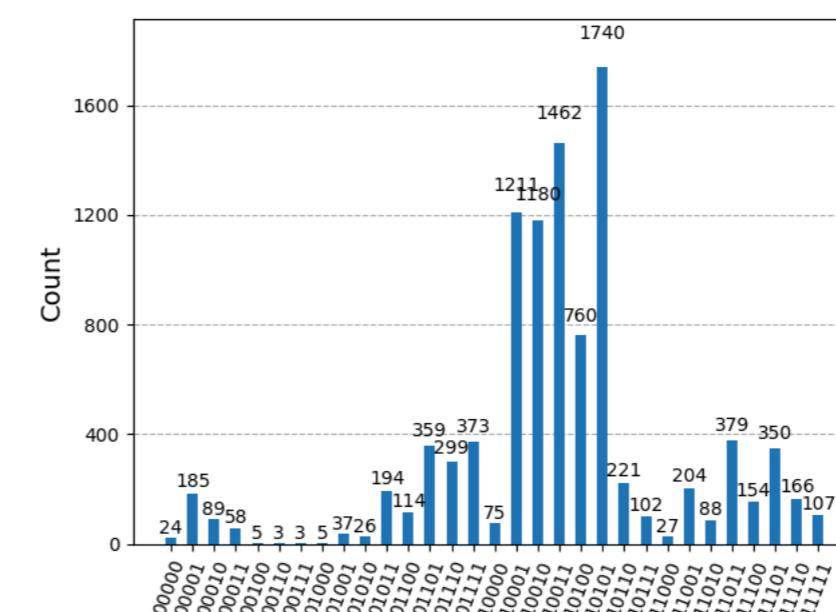
Most common measured bitstring (k): 10101

Phase ($\frac{k}{2^n}$): 0.65625

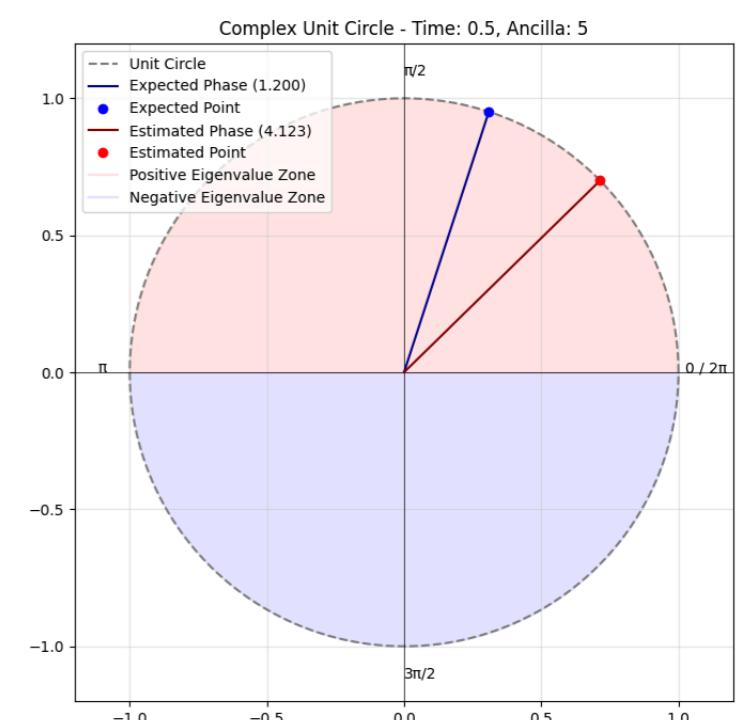
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 8.24668



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 5

Exact energy (λ): 2.4

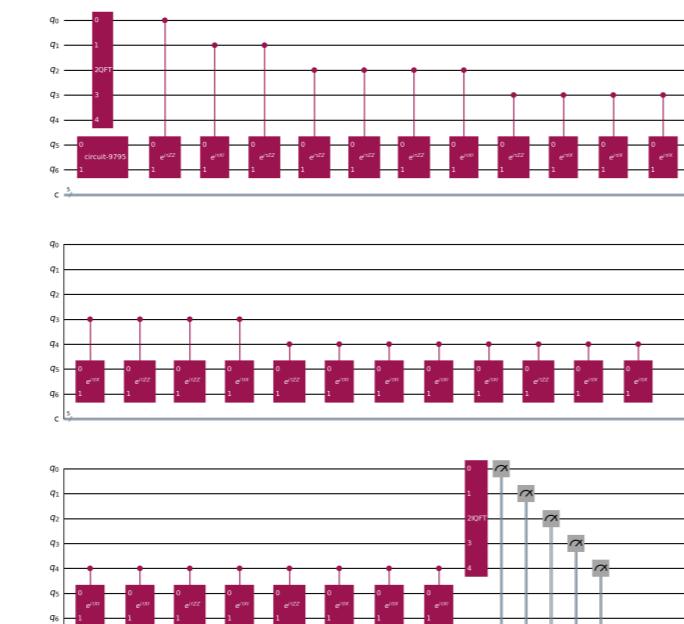
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.38197

Expected bitstring: (01100)

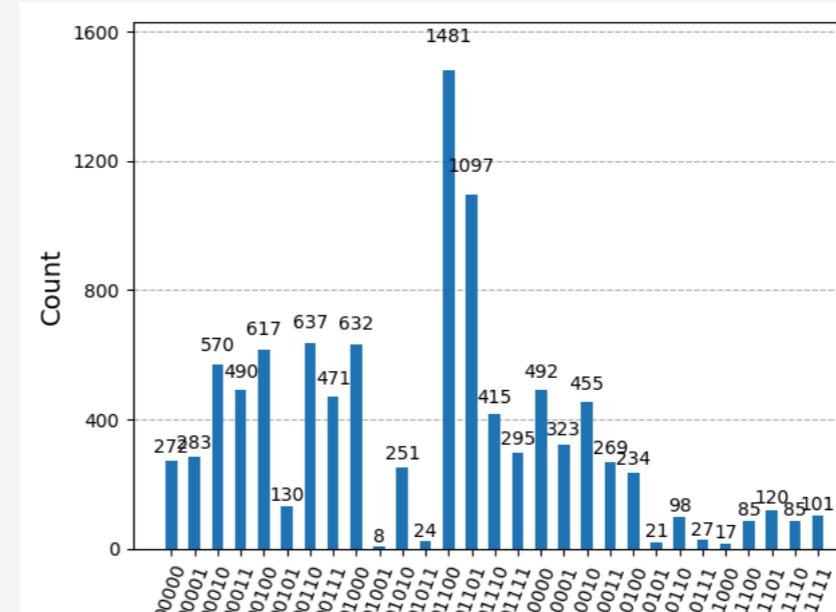
Most common measured bitstring (k): 01100

Phase ($\frac{k}{2^n}$): 0.375

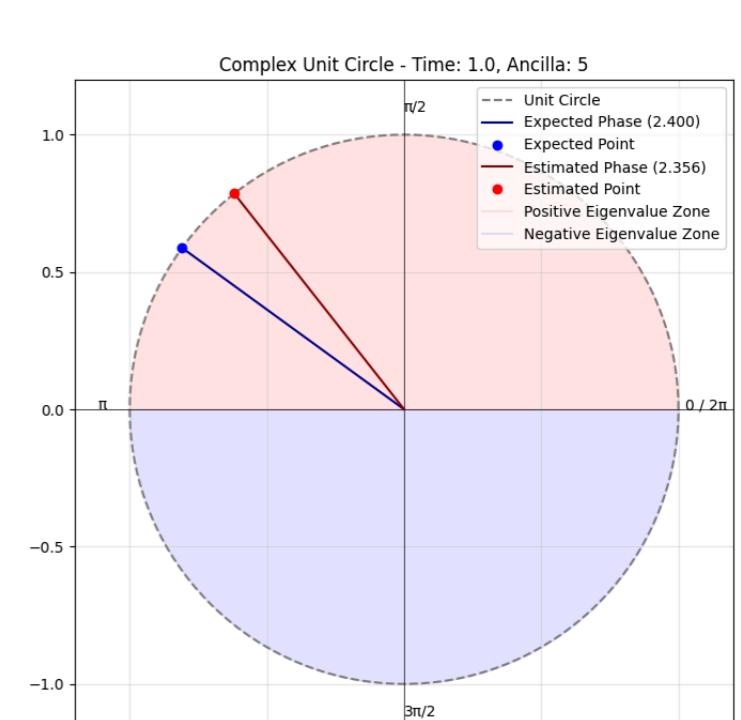
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 2.35619



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.01 | Shots: 10000 | Ancilla: 6

Exact energy (λ): 2.4

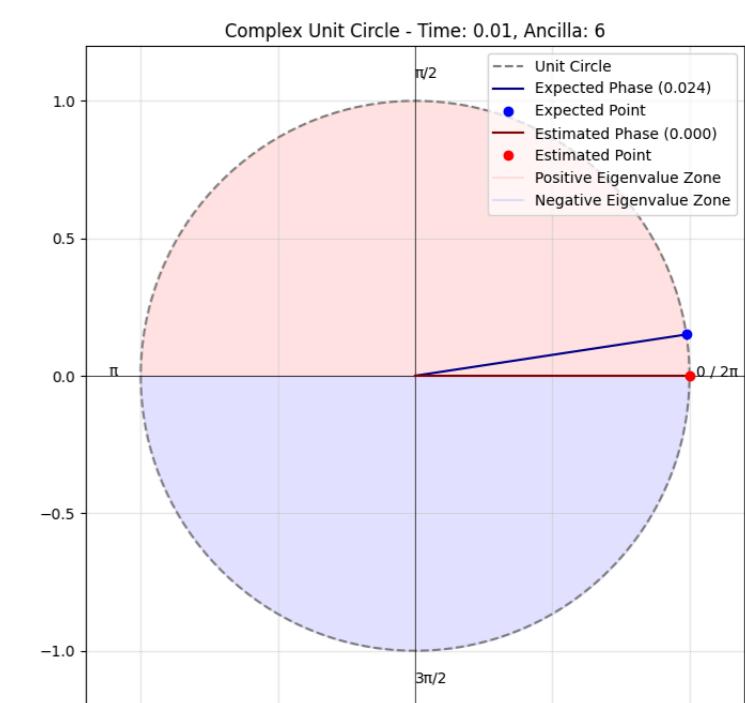
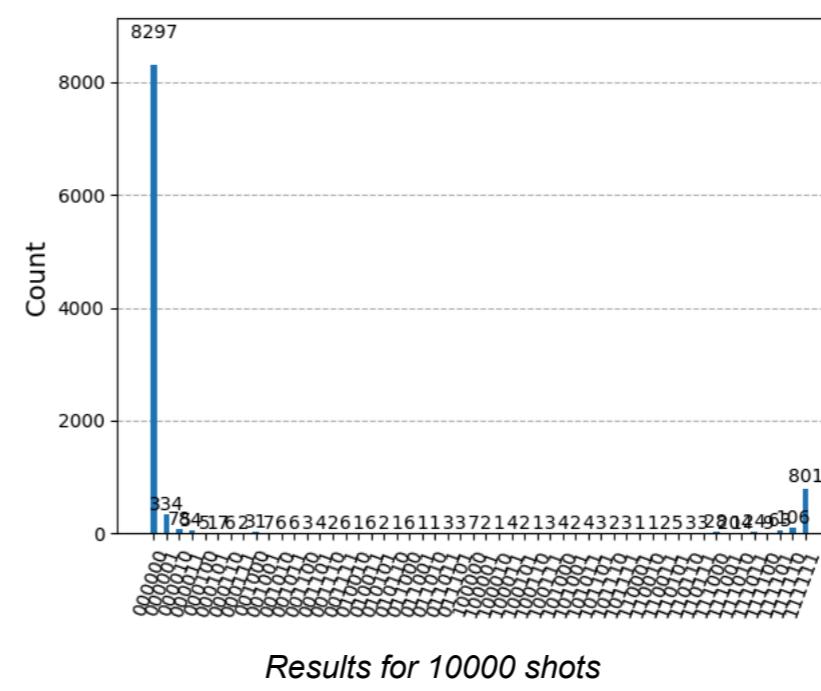
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.00382

Expected bitstring: (000000)

Most common measured bitstring (k): 000000

Phase ($\frac{k}{2^n}$): 0.0

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 0.0



Quantum Circuit Diagram

Time: 0.05 | Shots: 10000 | Ancilla: 6

Exact energy (λ): 2.4

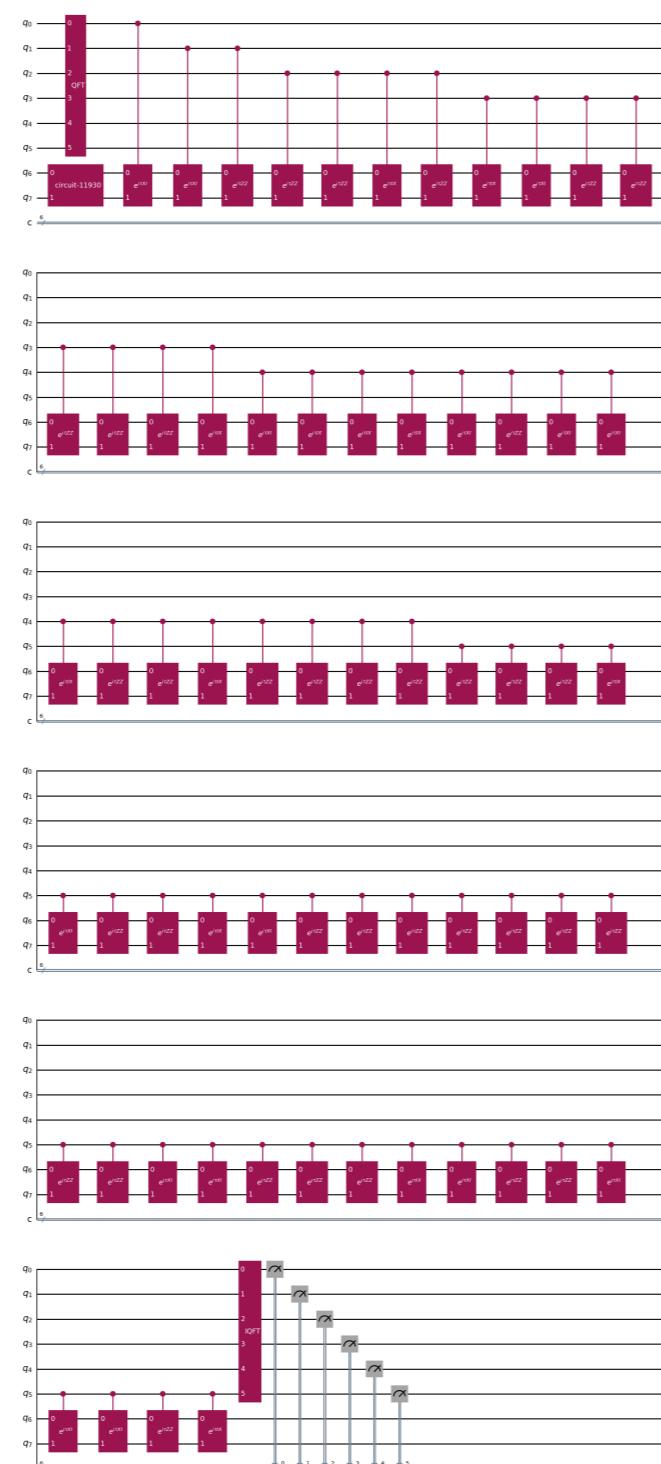
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.0191

Expected bitstring: (000001)

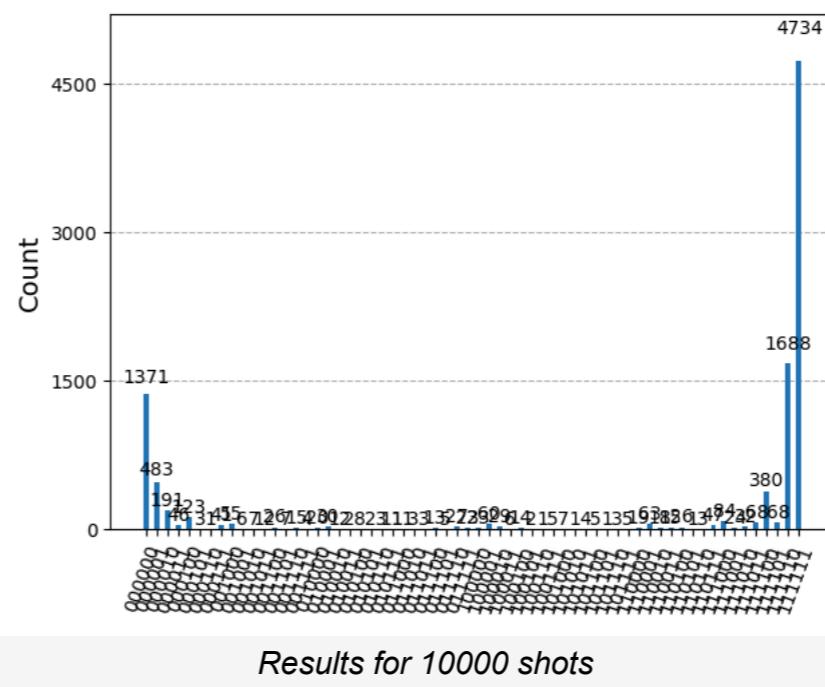
Most common measured bitstring (k): 111111

Phase ($\frac{k}{2^n}$): 0.98438

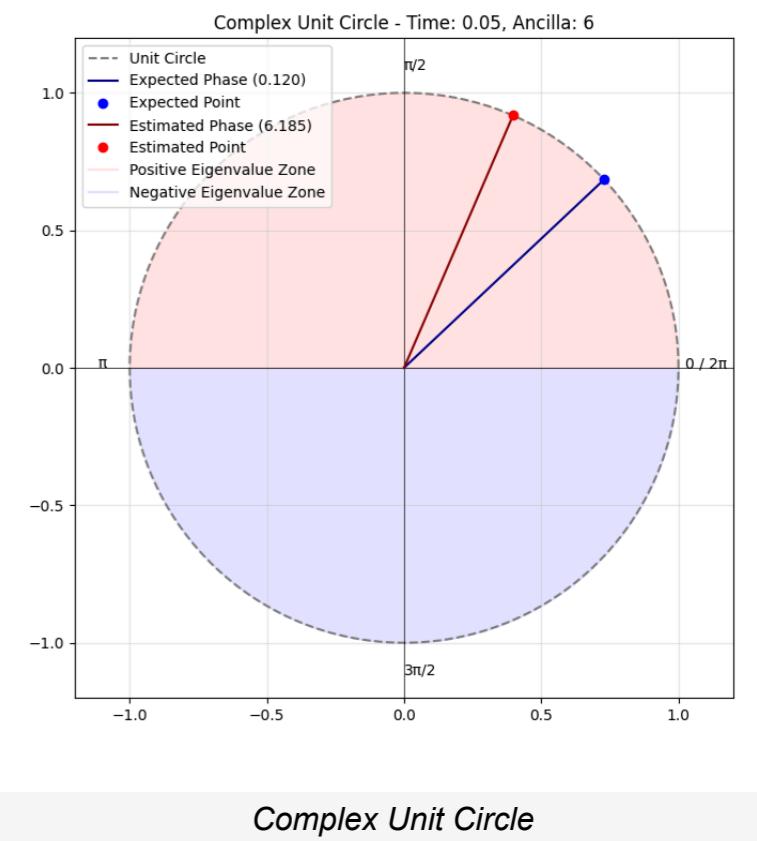
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 123.70021



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 6

Exact energy (λ): 2.4

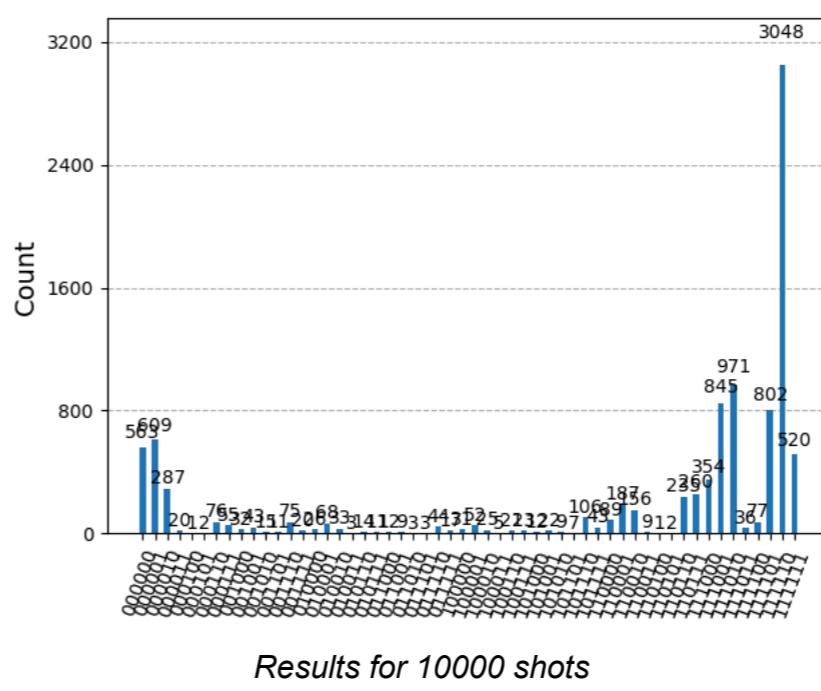
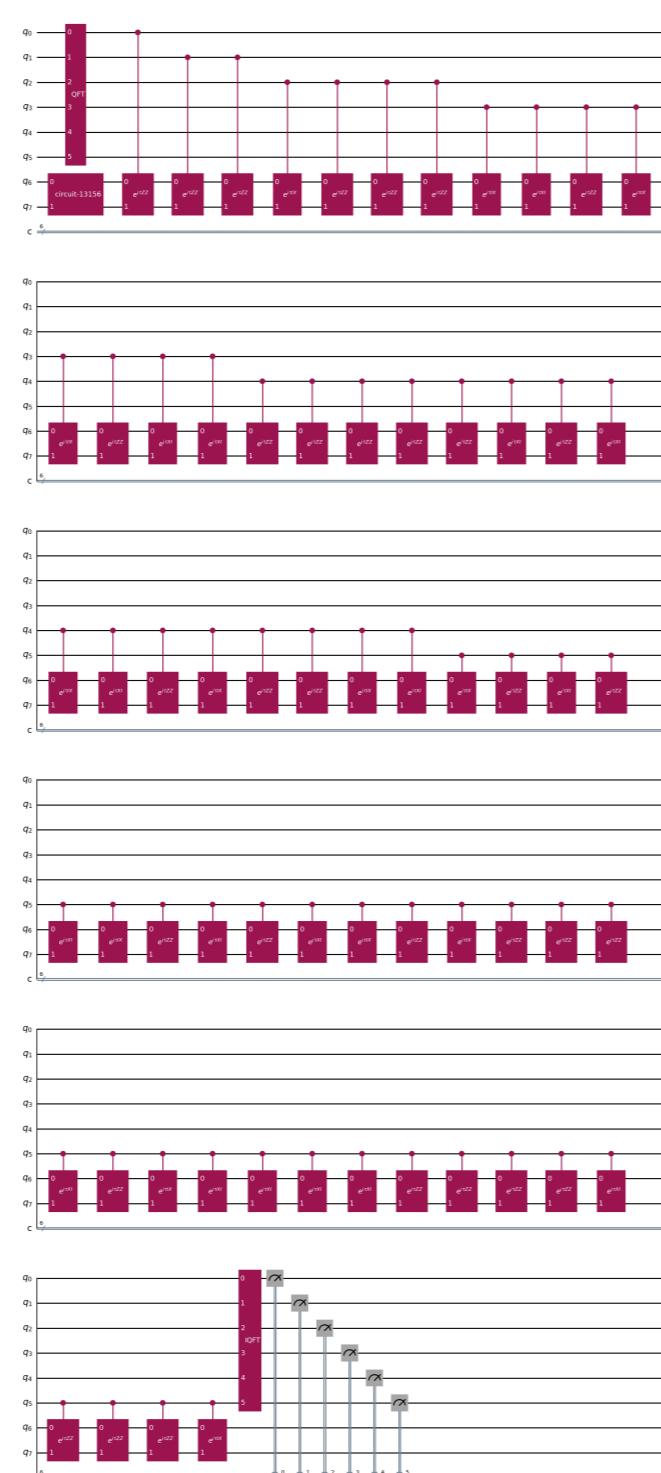
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.0382

Expected bitstring: (000010)

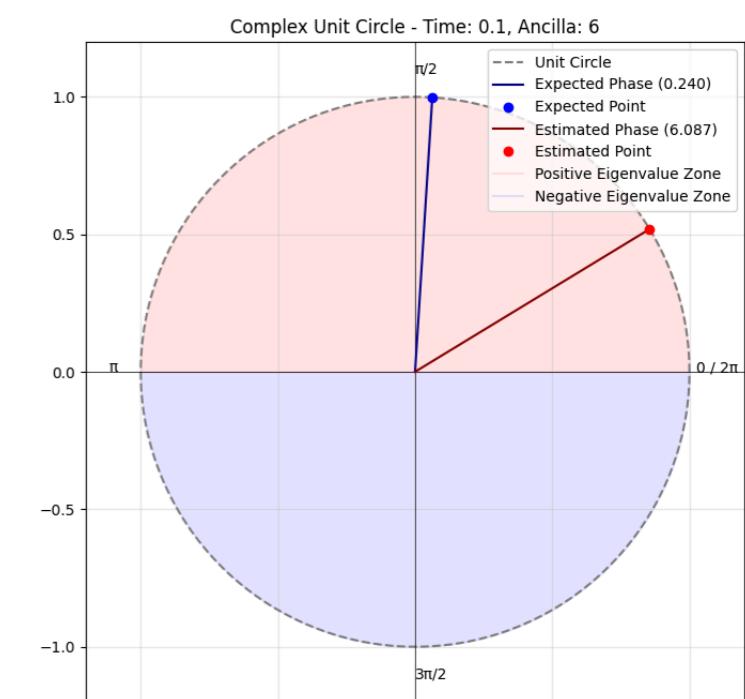
Most common measured bitstring (k): 111110

Phase ($\frac{k}{2^n}$): 0.96875

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 60.86836



Results for 10000 shots



Complex Unit Circle

Quantum Circuit Diagram

Time: 0.5 | Shots: 10000 | Ancilla: 6

Exact energy (λ): 2.4

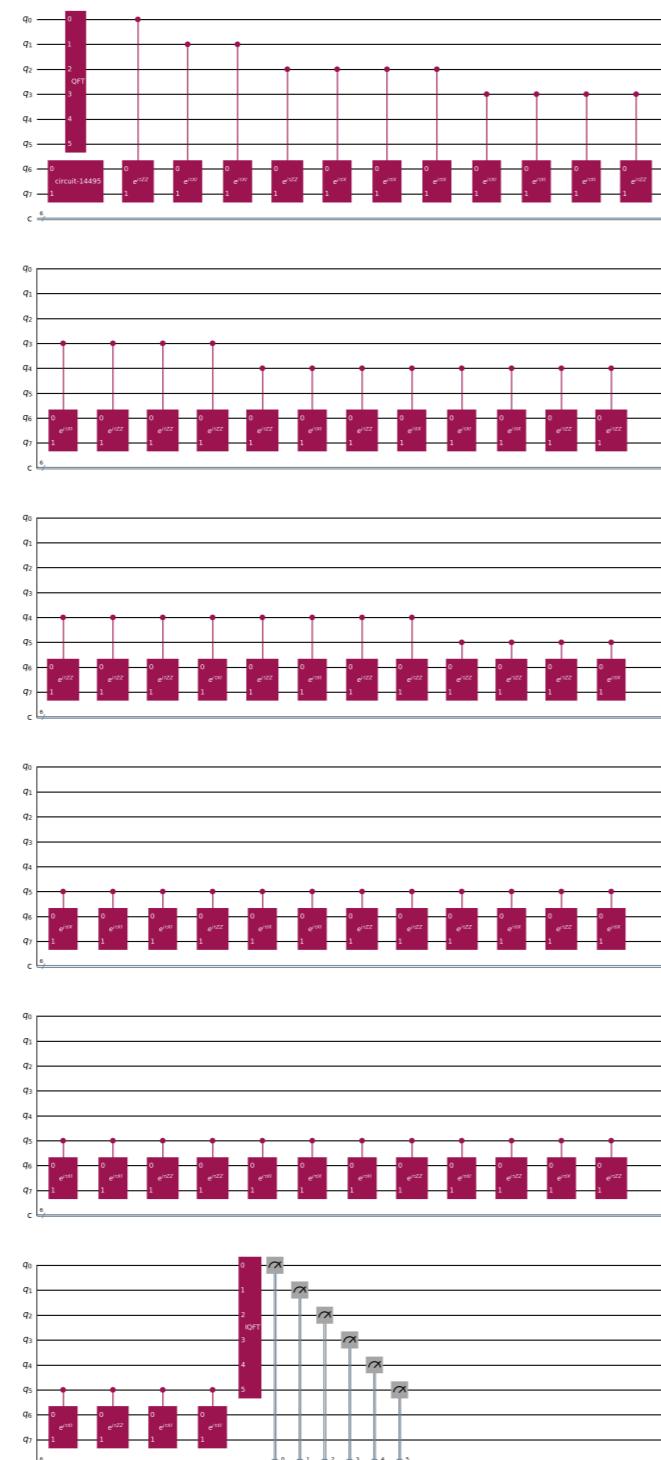
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.19099

Expected bitstring: (001100)

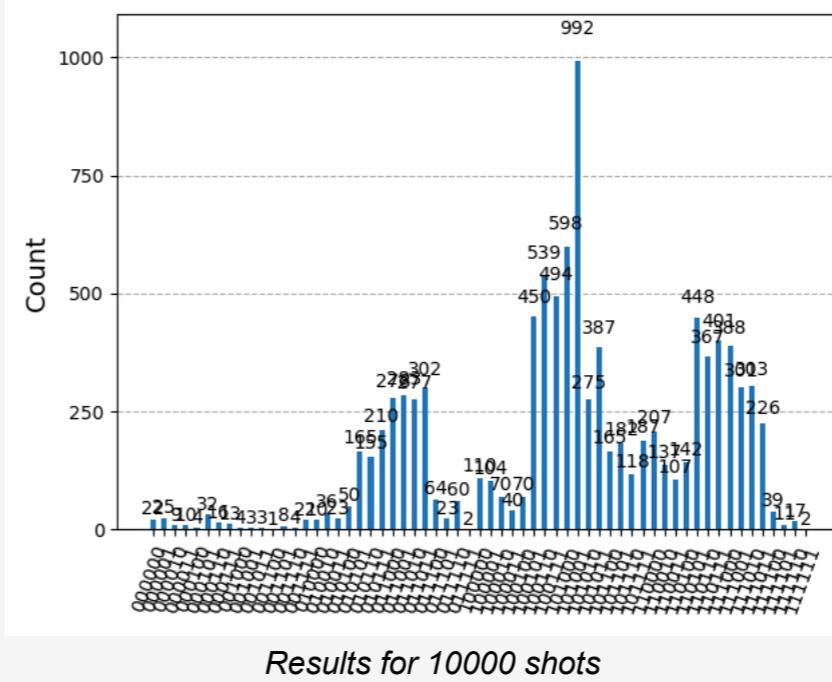
Most common measured bitstring (k): 101001

Phase ($\frac{k}{2^n}$): 0.64062

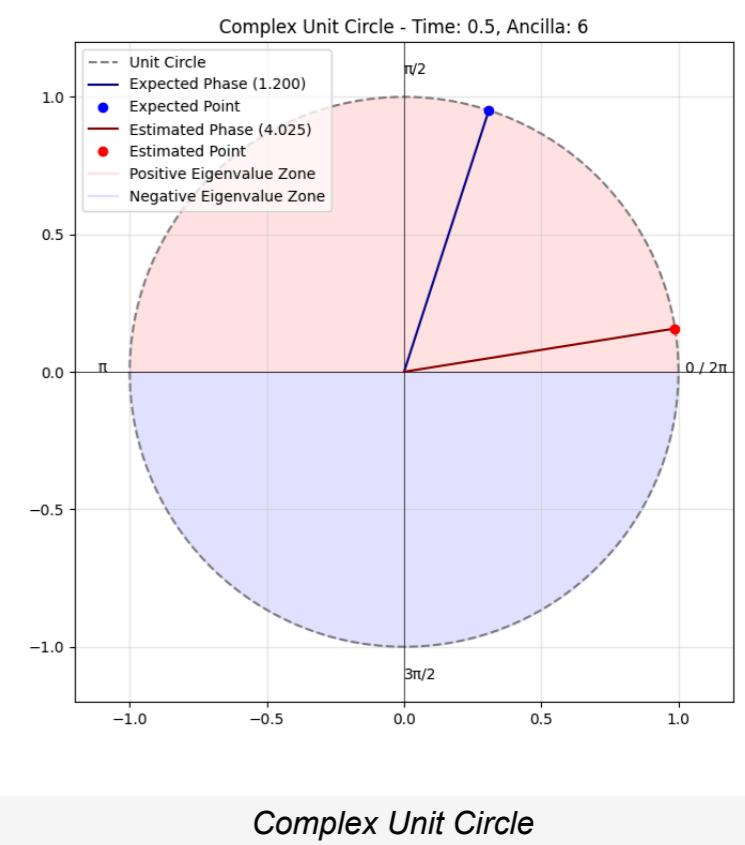
Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 8.05033



Quantum Circuit Diagram



Results for 10000 shots



Complex Unit Circle

Time: 1.0 | Shots: 10000 | Ancilla: 6

Exact energy (λ): 2.4

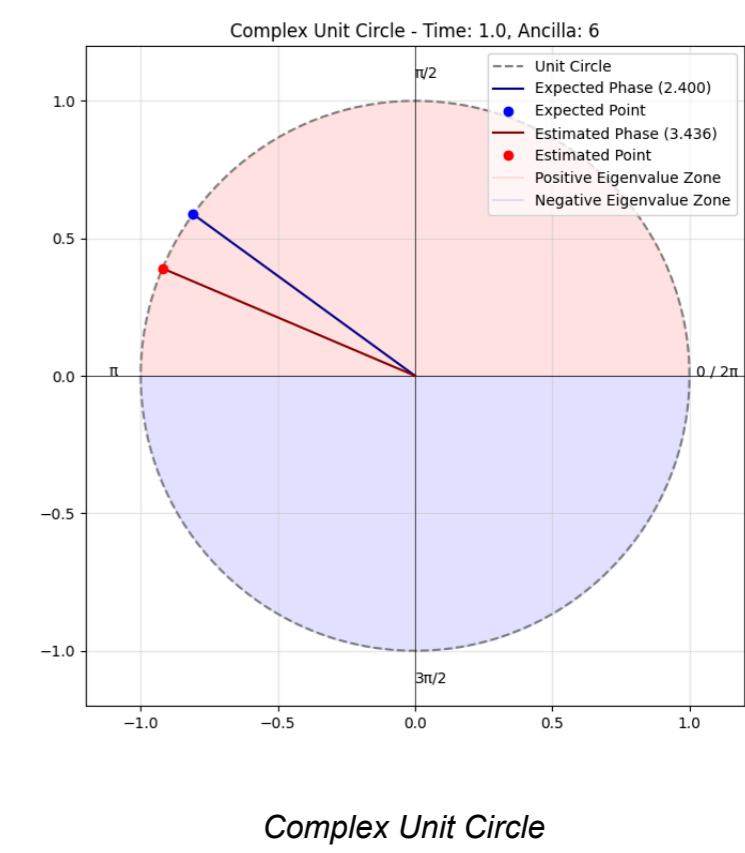
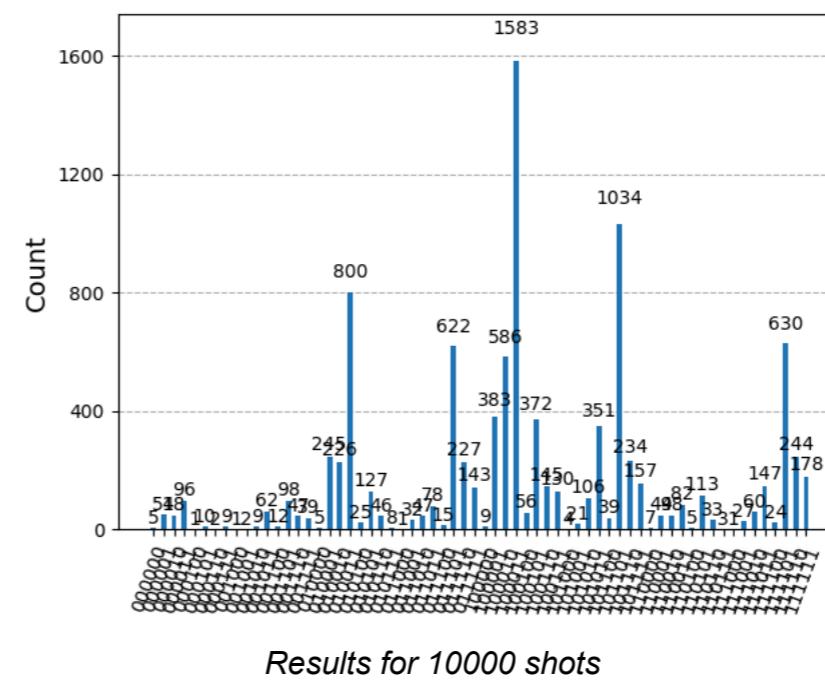
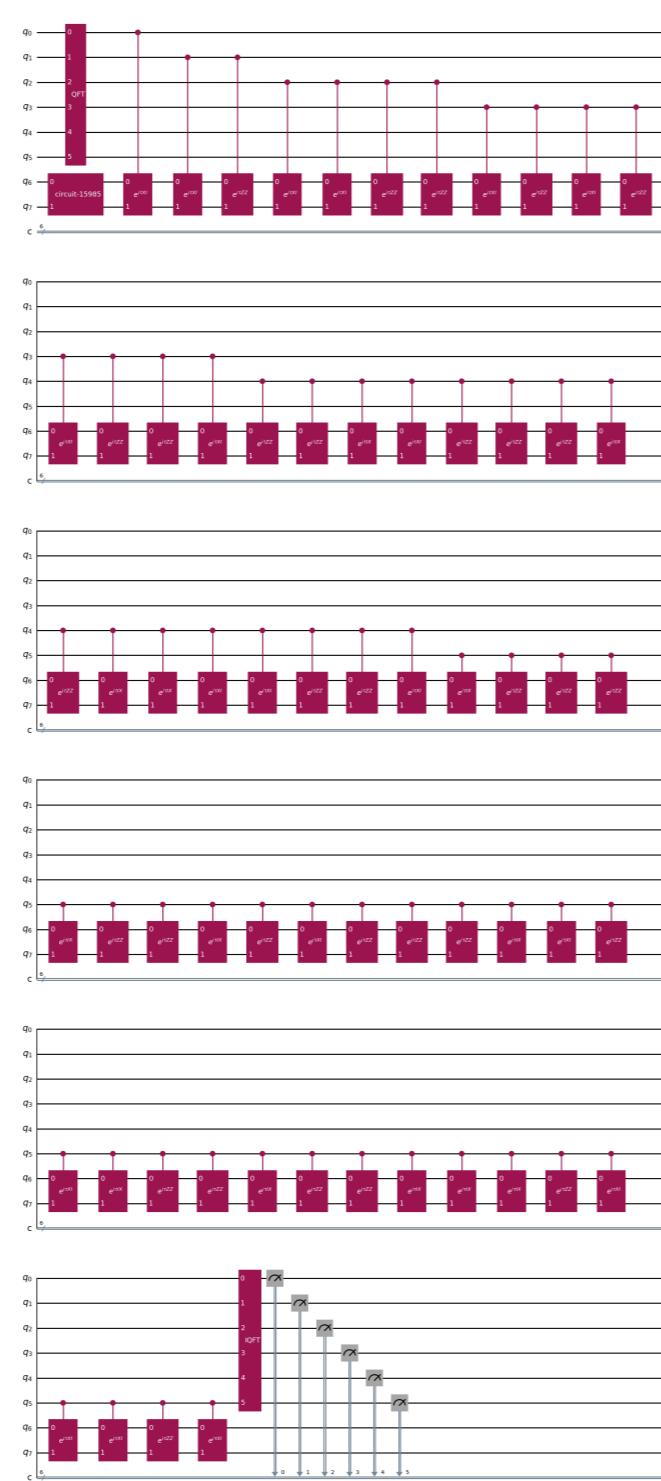
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.38197

Expected bitstring: (011000)

Most common measured bitstring (k): 100011

Phase ($\frac{k}{2^n}$): 0.54688

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 3.43612



Quantum Circuit Diagram

Time: 0.01 | Shots: 10000 | Ancilla: 7

Exact energy (λ): 2.4

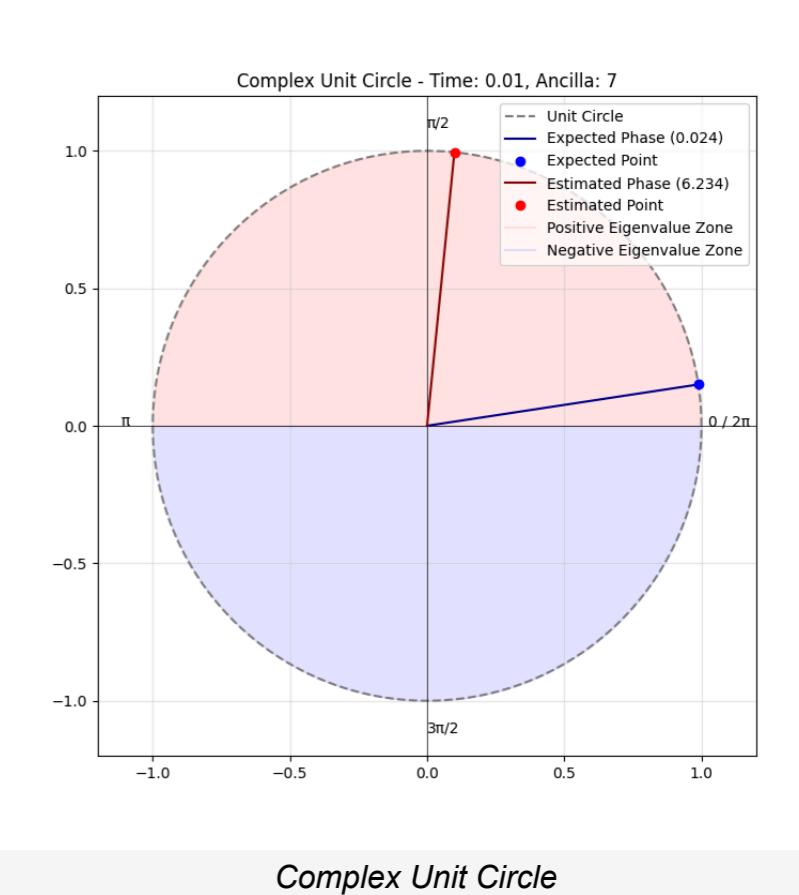
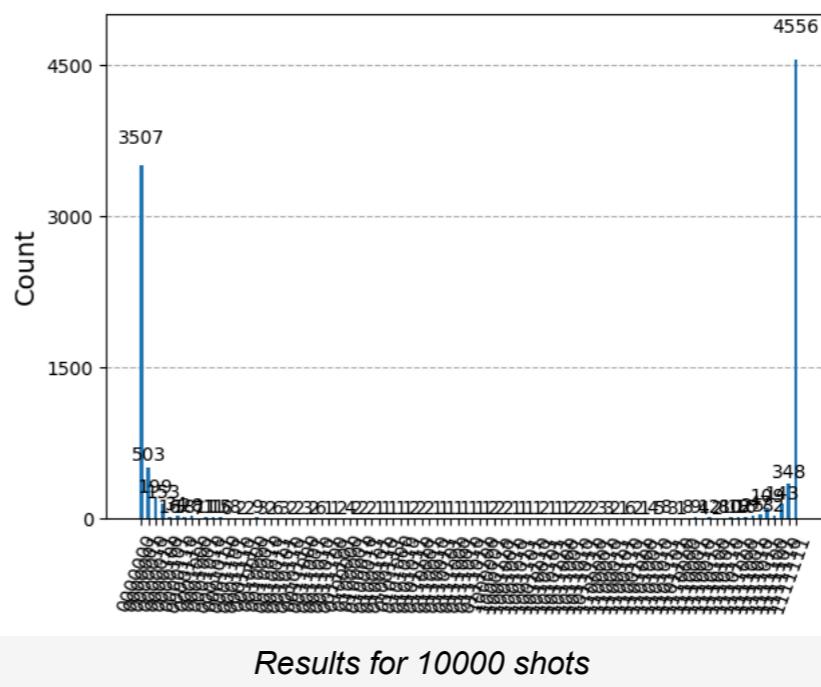
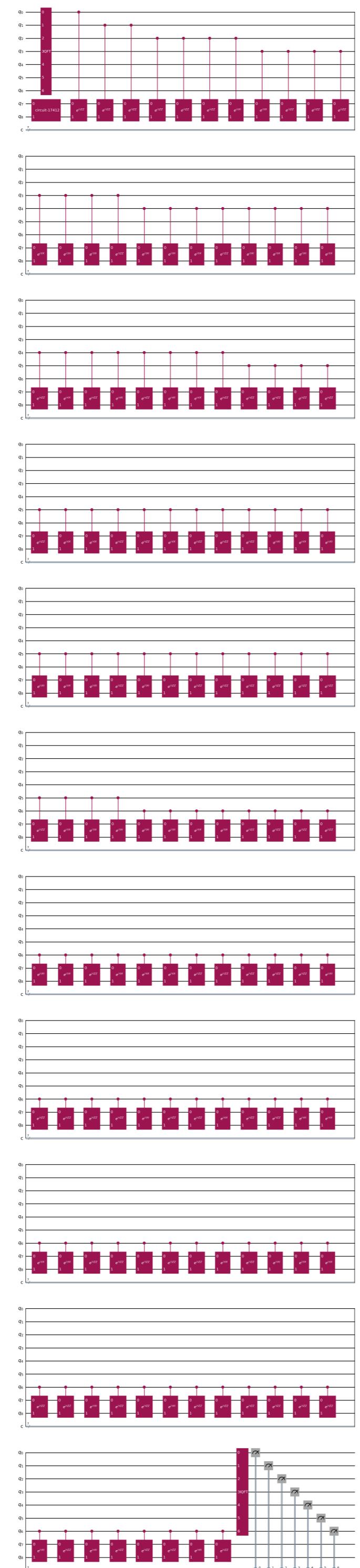
Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.00382

Expected bitstring: (0000000)

Most common measured bitstring (k): 1111111

Phase ($\frac{k}{2^n}$): 0.99219

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 623.40979



Quantum Circuit Diagram

Time: 0.05 | Shots: 10000 | Ancilla: 7

Exact energy (λ): 2.4

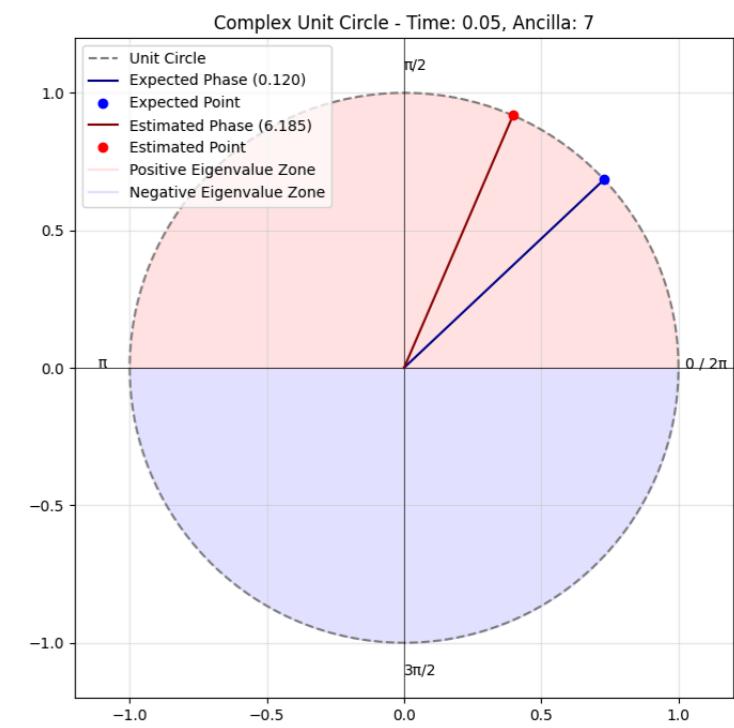
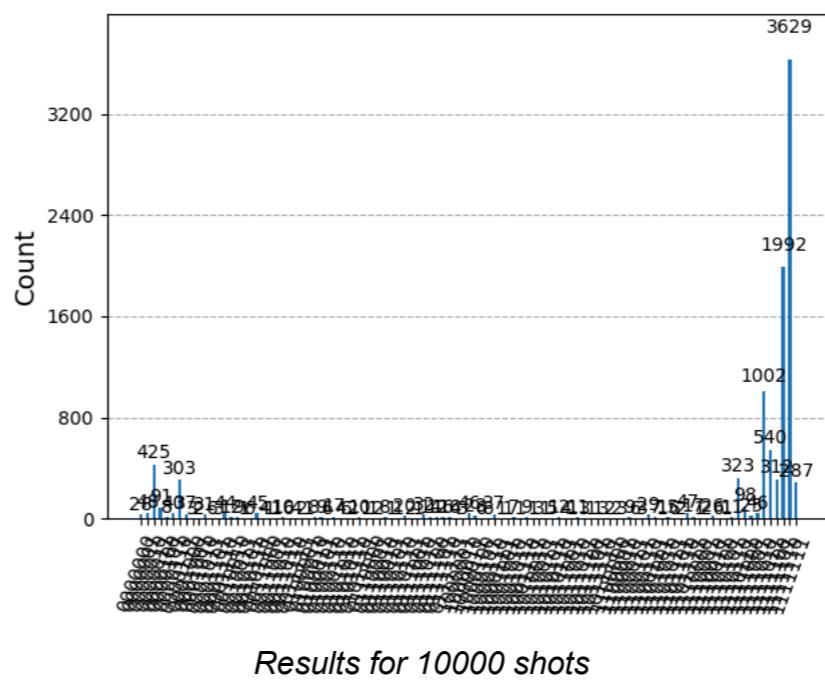
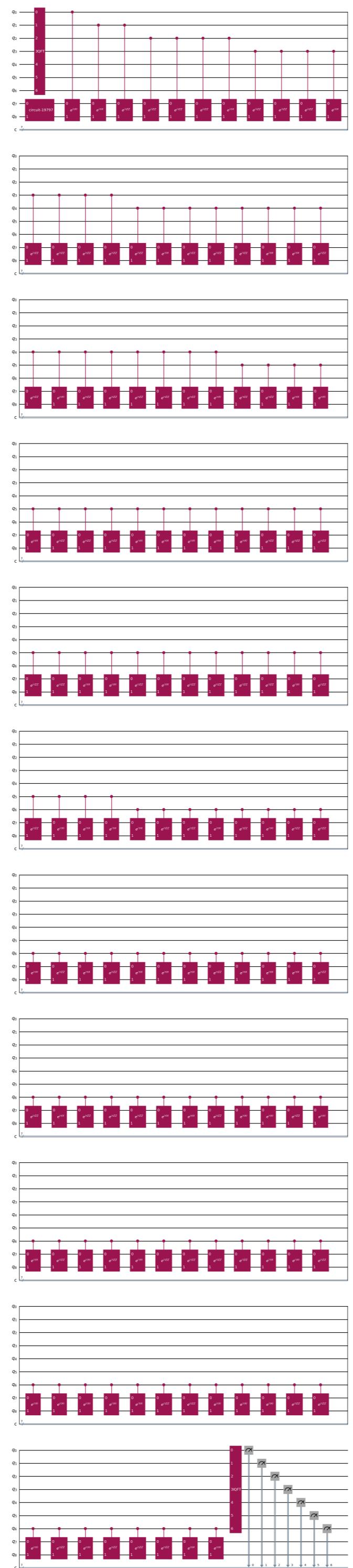
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.0191

Expected bitstring: (0000010)

Most common measured bitstring (k): 1111110

Phase ($\frac{k}{2^n}$): 0.98438

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 123.70021



Complex Unit Circle

Time: 0.1 | Shots: 10000 | Ancilla: 7

Exact energy (λ): 2.4

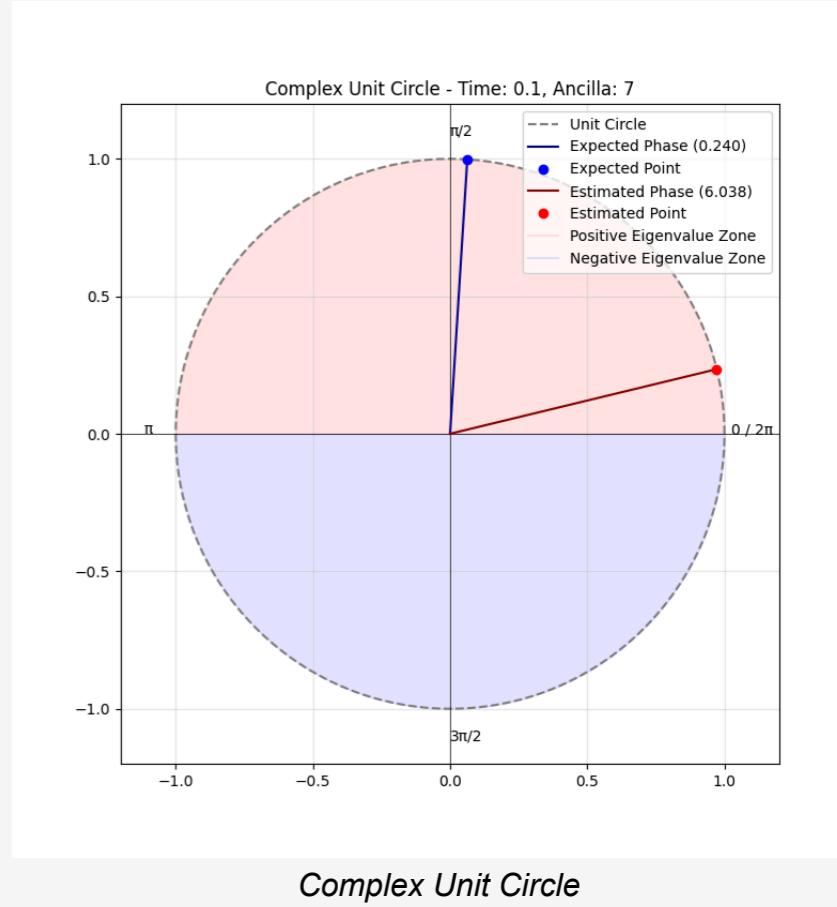
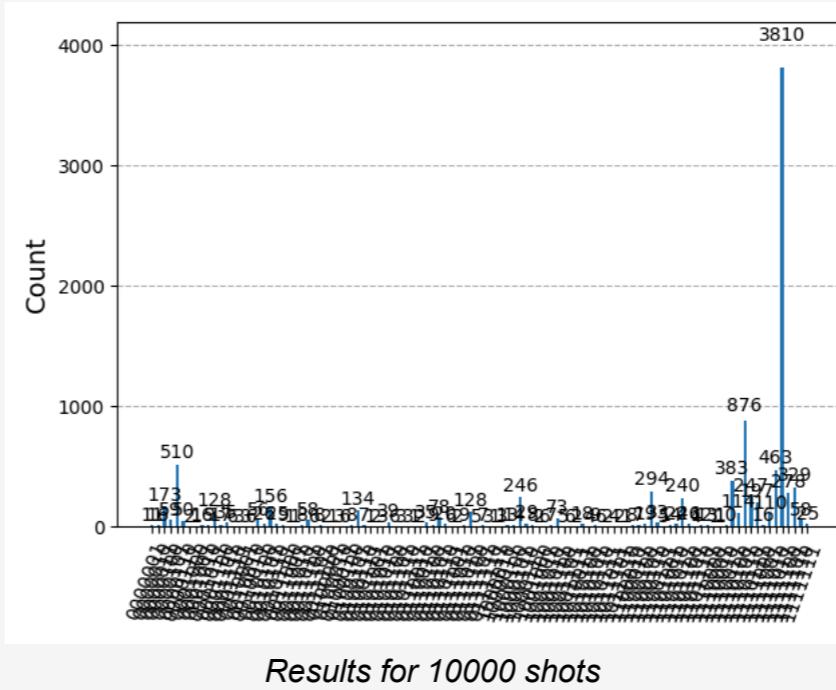
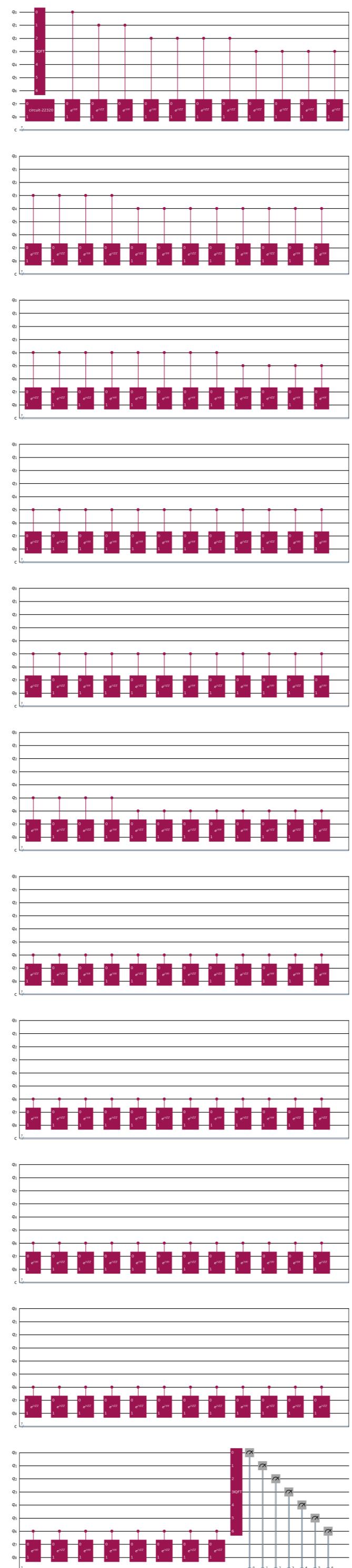
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.0382

Expected bitstring: (0000101)

Most common measured bitstring (k): 1111011

Phase ($\frac{k}{2^n}$): 0.96094

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 60.37748



Complex Unit Circle

Time: 0.5 | Shots: 10000 | Ancilla: 7

Exact energy (λ): 2.4

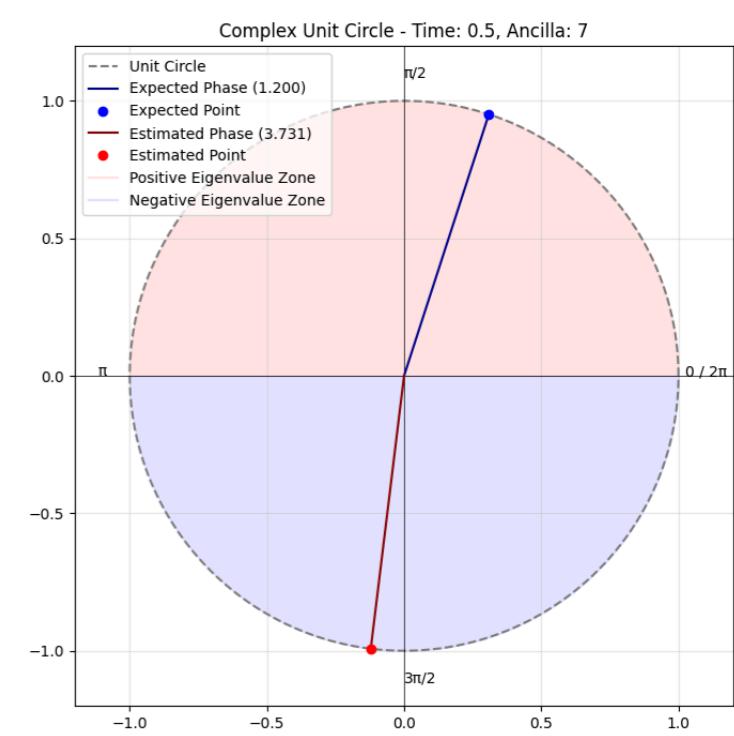
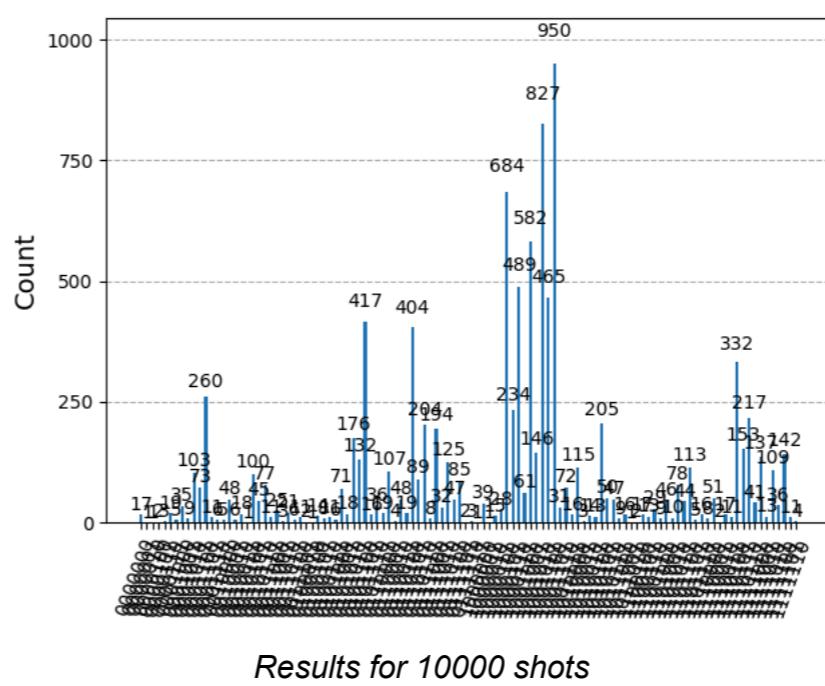
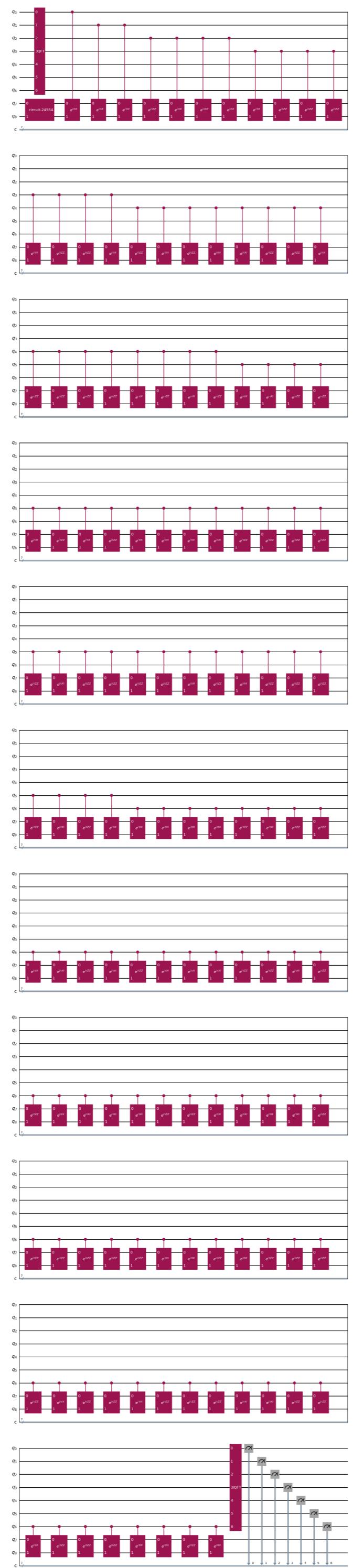
Exact expected phase ($\frac{\lambda \cdot t}{2 \cdot \pi} \bmod 1$): 0.19099

Expected bitstring: (0011000)

Most common measured bitstring (k): 1001100

Phase ($\frac{k}{2^n}$): 0.59375

Estimated Energy ($\frac{2 \cdot \pi \cdot \varphi}{t}$): 7.46128



Time: 1.0 | Shots: 10000 | Ancilla: 7

Exact energy (λ): 2.4

Exact expected phase ($\frac{\lambda \cdot t}{2\pi} \bmod 1$): 0.38197

Expected bitstring: (0110001)

Most common measured bitstring (k): 0011010

Phase ($\frac{k}{2^n}$): 0.20312

Estimated Energy ($\frac{2\pi \cdot \varphi}{t}$): 1.27627

