

ANALYSIS OF ALGORITHMS

COMP 2230-02 | WINTER 2026

In Previous Class



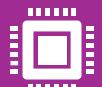
Data is any raw or unstructured piece of information that can be processed to get some information.



Data Structures is about how data can be stored in different structures.



Algorithms is about how to solve different problems, often by searching through and manipulating data structures.



Learning about **Data Structures and Algorithms (DSA)** will help us to use large amounts of data to solve problems efficiently.



Learning Objectives

1. Who do we decide the problem is an "ALGORITHM"?
 2. Properties of Algorithm
 3. Mapping Examples to Properties
 4. Why do we need to ANALYZE an ALGORITHM?
 5. What to ANALYZE?
 6. Algorithm Efficiency , Problem Size & Growth Function
 7. Asymptotic Notations/Complexity
 8. Big-Oh Notation & How to calculate it?

What is an ALGORITHM?

An algorithm is a set of step-by-step instructions to solve a given problem or achieve a specific goal.

Algorithm are ideas behind computer program.

Examples of an ALGORITHM

- 1. Directions to somebody's house**
(Step-by-step instructions with a clear start and end)
- 2. A recipe for cooking a cake**
(Ordered steps, inputs = ingredients, output = cake)
- 3. Morning routine**
Wake up → brush teeth → shower → get dressed → leave
- 4. ATM cash withdrawal**
Insert card → enter PIN → choose amount → collect cash
- 5. Traffic light system**
Green → Yellow → Red → repeat (with timing rules)
- 6. Logging into an email account**
Enter username → enter password → click login



Examples of an ALGORITHM

Sorting	Sorting books on a shelf : Compare titles → arrange alphabetically
Making	Making a phone call : Unlock phone → dial number → press call
Selecting	Online food ordering : Select restaurant → choose items → add to cart → pay
Washing	Washing clothes in a washing machine : Load clothes → add detergent → select mode → start
Crossing	Crossing a road safely : Look left → look right → look left again → cross
Evaluating	Exam paper evaluation : Read answer → compare with solution → assign marks
Parking	Parking a car : Check space → reverse slowly → adjust → stop



What is the deciding criteria to check
if it is an **ALGORITHM**?

Properties of ALGORITHMS



INPUT – what the algorithm takes in as input.



OUTPUT – what is the goal.



DEFINITENESS – the steps are defined.



CORRECTNESS - should produce the correct output.



Finiteness - the steps required should be finite.



EFFECTIVENESS - each step must be able to be performed in a finite amount of time.

Mapping Examples to Properties

Example	Input	Output	Definiteness	Correctness	Finiteness	Effectiveness
Directions to somebody's house	Starting location, destination	Arrival at the house	Step-by-step route clearly defined	Following steps gets you to the house	Finite number of steps	Each step (turn, go straight) can be performed
Recipe for cooking a cake	Ingredients, utensils	Cooked cake	Steps for mixing, baking, cooling are clearly defined	Following steps produces cake	Finite steps (mix → bake → cool)	Each step can be performed in real time
Morning routine	Wake-up time	Ready to leave for day	Steps like brush teeth, shower, get dressed	Following steps prepares a person	Finite steps	Each step executable in finite time
ATM cash withdrawal	Bank card, PIN, withdrawal amount	Cash dispensed	Steps: insert card → enter PIN → choose amount → collect cash	Correct cash is given	Finite steps	Each step can be done in real time
Traffic light system	Current light status	Next light in cycle	Green → Yellow → Red → repeat	Changes follow correct sequence	Steps repeat in cycles, but each transition is finite	Each light change happens in finite, timed duration

Mapping Examples to Properties

Example	Why It's Not a Proper Algorithm	Which Component Fails
“Do whatever you feel like today”	No clear steps; outcome unpredictable	Definiteness, Correctness
“Keep searching for a lost pen forever”		
“Think of a number and guess it”		
“Write an essay whenever inspiration strikes”		
“Make the best cake instantly without a recipe”		
“Solve world hunger in one step”		

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“Make the best cake instantly without a recipe”	Steps impossible to perform; vague	Effectiveness, Definiteness
“Solve world hunger in one step”	Not executable in finite time; outcome unclear	Effectiveness, Finiteness, Correctness

Hands-on Activity 1

Look at each task. Decide if it meets all the algorithm components. Discuss in pairs why it is or isn't an algorithm.

GROUP 1 : Playing a song on a music app

GROUP 2 : Waiting for inspiration to start a painting

GROUP 3 : Washing Hands

GROUP 4 : Deciding to wear something

GROUP 5 : Following a GPS to reach a destination

GROUP 6 : Guessing lottery numbers

Possible Solutions

Group	Task	Algorithm?	Explanation
Group 1	Playing a song on a music app	<input checked="" type="checkbox"/> Yes	Steps are finite, definite, and executable: open app → select song → play. Input = song choice; output = song plays.
Group 2	Waiting for inspiration to start a painting		
Group 3	Washing hands		
Group 4	Deciding to wear something		
Group 5	Following a GPS to reach a destination		
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Group 3	Washing hands		
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Group 6	Guessing lottery numbers	<input type="checkbox"/> No	Steps are not definite, not correct, not effective; output cannot be guaranteed; no finite procedure ensures success.

If a task has a defined **input**, produces a correct **output**, has **finite**, **unambiguous steps**, and is **effective**, then it can be implemented as a computer program



Why we need ALGORITHM ANALYSIS?

- Writing a working program is not good enough.
- The program may be inefficient!
- If the program is executed on BIG Data / Large Datasets, then the running time becomes an issue.

Why we need ALGORITHM ANALYSIS?

Algorithm analysis is the process of evaluating an algorithm in terms of efficiency, correctness, and resource usage before implementing it.

BETTER algorithm save **TIME, SPACE & COST**, making programs **FASTER** and more **SCALABLE**.

What to ANALYZE?

CORRECTNESS – does the input/output relation matches the algorithm requirement

AMOUNT OF WORK DONE – basic operations to do task finite amount of time

AMOUNT OF SPACE USED – memory used

SIMPLICITY & CLARITY – verification & implementation

OPTIMALITY – is it impossible to do better?

What to ANALYZE?

TIME COMPLEXITY : Amount of computer time it needs to execute the program to get the intended result.

SPACE COMPLEXITY : Memory requirements based on the problem size.

ALGORITHM EFFICIENCY

Definition: The **efficiency of an algorithm** measures how effectively it uses **resources**, usually **CPU time**.

Algorithm analysis helps categorize algorithms based on efficiency.

ALGORITHM EFFICIENCY



Everyday Example: Washing Dishes



Washing a dish → 30 seconds



Drying a dish → 30 seconds



Total per dish = 60 seconds = 1 minute



n dishes → n minutes

ALGORITHM EFFICIENCY

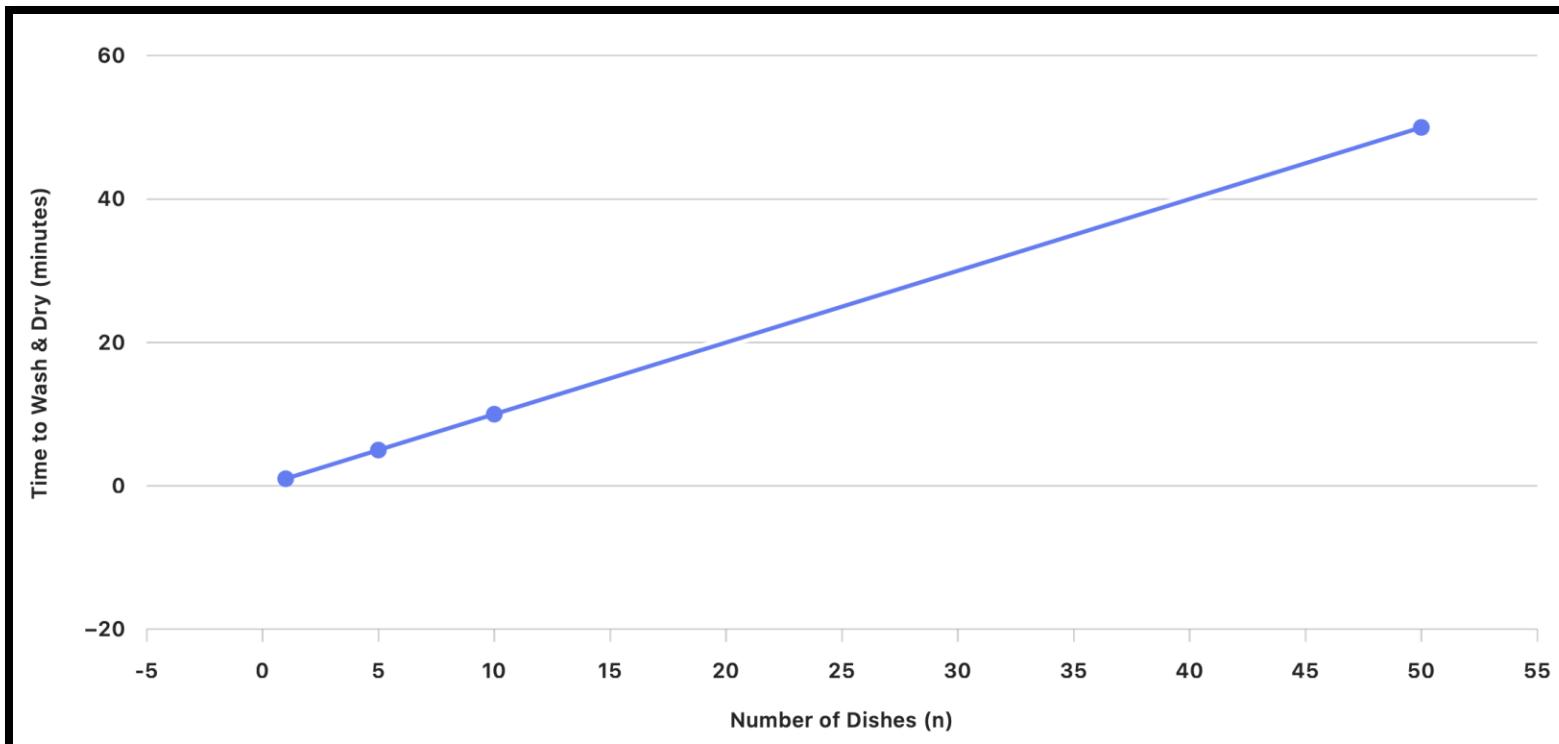


"EFFICIENCY TELLS US HOW RESOURCES SCALE WITH INPUT SIZE"

Dishes (n)	Time to wash & dry (minutes)
1	1
5	5
10	10
50	50

ALGORITHM EFFICIENCY

Efficiency grows **linearly** with input in this example
→ **O(n)** time complexity.



$$T(n)=k \cdot n$$

Where:

- $T(n)$ = Time to wash & dry
- n = Number of dishes
- k = Constant time per dish (in this example, $k = 1$ minute)

Hands-On Activity 2

TASK : Finding Your Favorite Snack

Scenario:

- There's a small snack cupboard with 10 different snacks.
- You want to find your favorite snack.



Checking one by one

There are **10 snacks**. It takes **15 seconds** to check each snack. If you find your favorite snack on the **k-th try**, the time taken is?



Checking one by one

There are **10 snacks**. It takes **15 seconds** to check each snack. If you find your favorite snack on the **k-th try**, the time taken is?

Position (k)	Time (seconds)	Time (minutes)
1	15	0.25

$$T(k) = 15 * k \text{ seconds}$$

Or

$$T(k) = (15 * k) / 60 \text{ minutes}$$

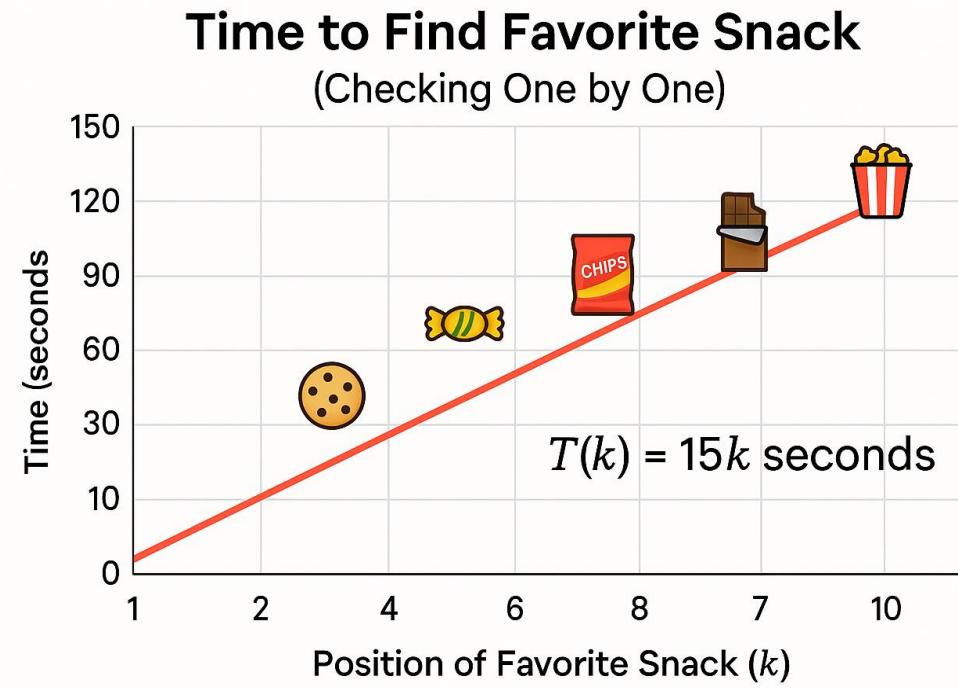
Checking one by one

There are **10 snacks**. It takes **15 seconds** to check each snack. If you find your favorite snack on the **k-th try**, the time taken is?

Position (k)	Time (seconds)	Time (minutes)
1	15	0.25
2	30	0.5
3	45	0.75
4	60	1
5	75	1.25
6	90	1.5
7	105	1.75
8	120	2
9	135	2.25
10	150	2.5

ALGORITHM EFFICIENCY

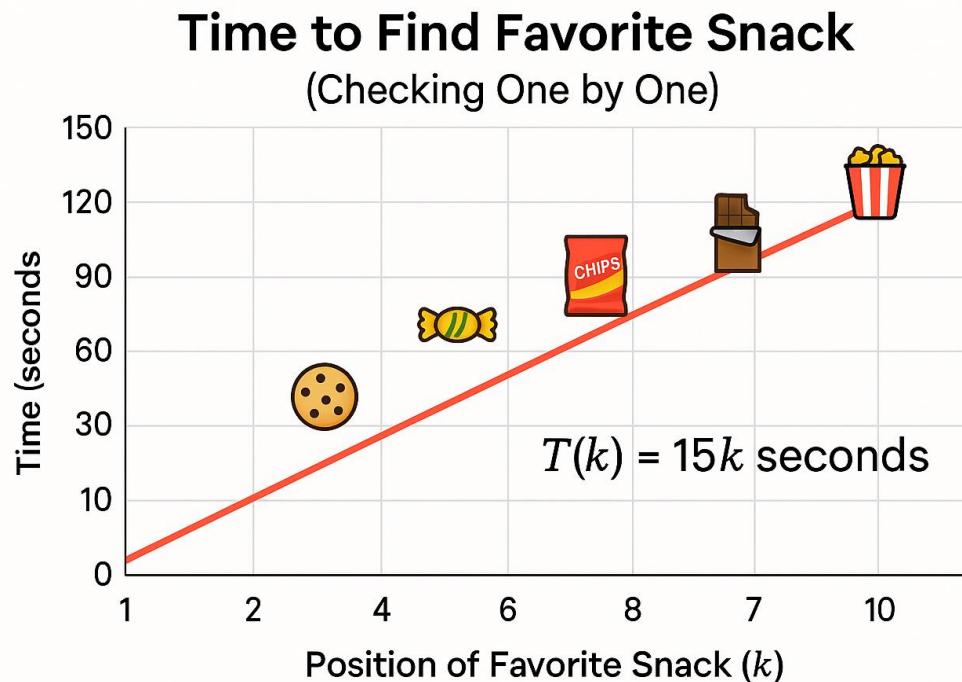
Now what would be the WORST-CASE and the BEST-CASE scenarios ?



ALGORITHM EFFICIENCY



Now what would be the WORST-CASE and the BEST-CASE scenarios ?



You find your favorite snack **on the first try**.

$T(1) = 15$ seconds / 0.25 mins

This is the **minimum time** possible.

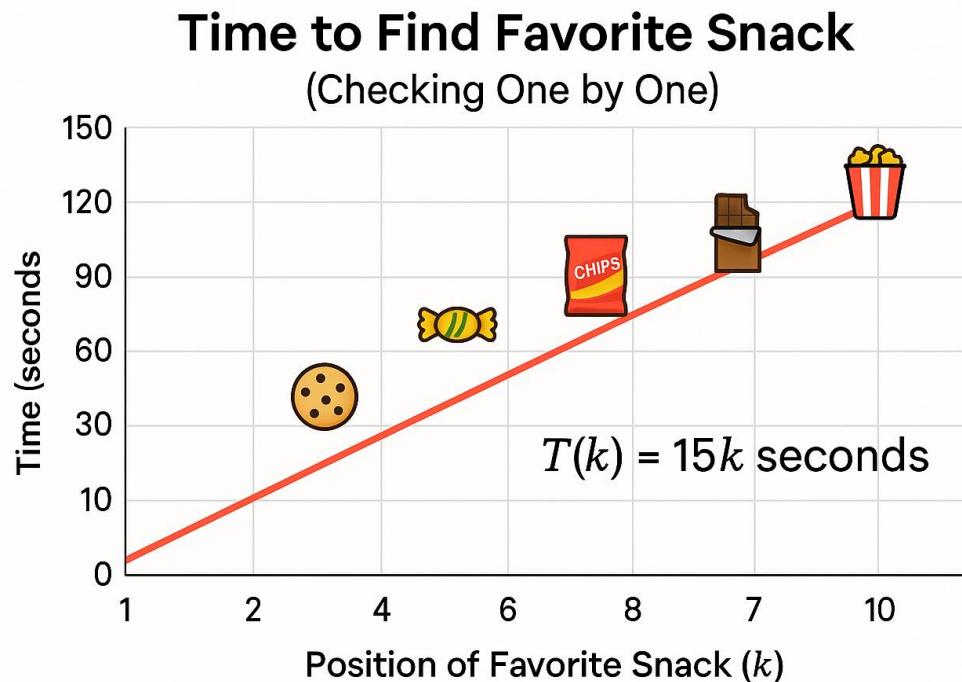
In algorithm terms:

Best case = $O(1)$ (constant time) because you only check one snack.

ALGORITHM EFFICIENCY



Now what would be the WORST-CASE and the BEST-CASE scenarios ?



Your favorite snack is **last in the cupboard (10th position)**.

$$T(10)=15 \times 10=150 \text{ seconds / 2.5 mins}$$

This is the **maximum time** possible.

In algorithm terms:

Worst case = $O(n)$ because you check all snacks.

PROBLEM SIZE



For every algorithm we want to analyze, we need to define the size of the problem



The dishwashing problem has a size n – number of dishes to be washed & dried



The finding snacks problem has a size n – number of snacks in the cupboard.

PROBLEM SIZE

For a search algorithm, the size of the problem is the size of the search pool

For a sorting algorithm, the size of the program is the number of elements to be sorted

PROBLEM SIZE



For every algorithm we analyze, we define the **size of the problem**:



Dishwashing Problem:

Size = n → number of dishes to be washed/dried.



Search Problem:

Size = n → number of items in the search pool.



Sorting Problem:

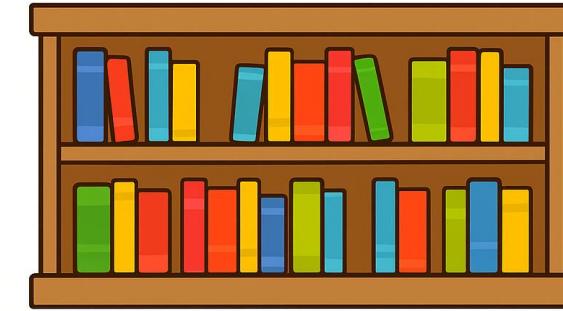
Size = n → number of elements to be sorted.

What is the search size if you looking for your favorite snack?



10 snacks

What is the search size if you looking for your favorite book?



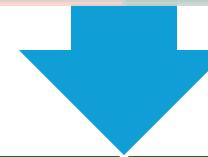
20 books

GROWTH FUNCTIONS

We must also decide what we are trying to efficiently optimize

time complexity – CPU time

space complexity – memory space



CPU time is generally the focus



A growth function shows the relationship between the size of the problem (n) and the value optimized (time)

ASYMPTOTIC NOTATION

Asymptotic notations are the terminology that enables meaningful statements about **time & space complexity**.

To calculate running time of an ALGORITHM

ASYMPTOTIC NOTATION

The time required by the given algorithm falls under **THREE** types:

1. **Best-case time or the minimum time** required in executing a program.
2. **Average-case time or the average time** required in executing a program.
3. **Worst-case time or the maximum time** required in executing program

ASYMPTOTIC NOTATION

Best Case: Denoted by Ω (Omega)

→ Represents the minimum time an algorithm will take.

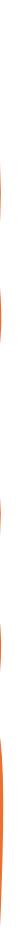
Average Case: Denoted by Θ (Theta)

→ Represents the average time for an algorithm.

Worst Case: Denoted by O (Big O)

→ Represents the maximum time an algorithm will take.

Big-Oh Notation



In computing science, Big-Oh Notation denoted by O is used to classify algorithm according to how their run time and space requirement grows as the input size grows.

Big-Oh Notation is really concerned with " WHAT HAPPENS WHEN WE SCALE IN TERMS OF DATA"

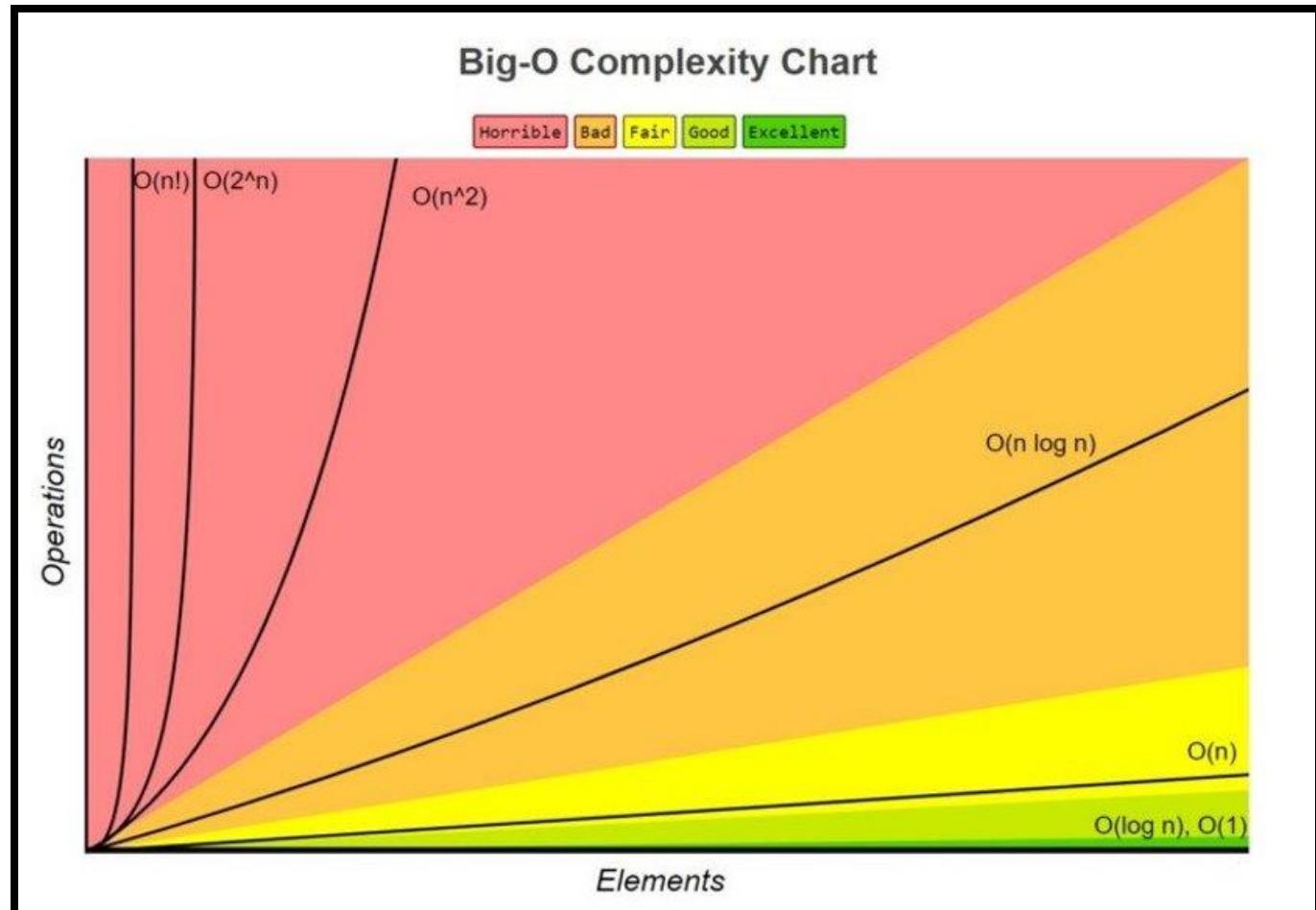


Why is it important for us to understand Big-Oh Notation?

Big-Oh Notation Definition

- Given two functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $f(n) \leq c*g(n)$ for all $n \geq n_0$.
- $f(n)$ is $O(g(n))$ if $f(n)$ grows no faster than $c*g(n)$ for all $n \geq n_0$, where c and n_0 are constants.

$$F(n) = O(g(n))$$



Example 1 : Adding numbers

```
int total = 0;

for (int i = 0; i < n; i++) {
    total += 1;          // same as total = total + 1
    System.out.println(i);
}
```

How do we calculate Big-Oh Notation?



First, we will get the count of operations an algorithm is going to take.

Statement/Operation	Frequency
total = 0	
for(i=0,i<n, i++)	
total+=1	print(i)

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How do we calculate Big-Oh Notation?



First, we will get the count of operations an algorithm is going to take.

Statement/Operation	Frequency
total = 0	1
for(i=0,i<n, i++)	n
total+=1	1+1=2
print(i)	

How do we calculate Big-Oh Notation?

Second, we put together the steps count into $f(n)$ function

Statement/Operation	Frequency	
total = 0	1	$F(n) = O(g(n))$
for(i=0,i<n, i++)	n	$F(n) = 1 + 2 * n = 2n + 1$
	1+1 = 2	
total+=1	print(i)	

How do we calculate Big-Oh Notation?

Third, we ignore the lower order terms and consider only highest order term.

Statement/Operation	Frequency
total = 0	1
for(i=0,i<n, i++)	n
	1+1 = 2
total+=1	print(i)

$$F(n) = 1 + 2 * n = 2n + 1$$

2n

How do we calculate Big-Oh Notation?

Fourth, we ignore the constant associated with the highest order term.

Statement/Operation	Frequency
total = 0	1
for(i=0,i<n, i++)	n
	1+1 = 2
total+=1	print(i)

$$F(n) = 1 + 2 * n = 2n + 1$$

n

How do we calculate Big-Oh Notation?

The final step, the value remaining after the 4 step is = $O(f(n))$.

Statement/Operation	Frequency
total = 0	1
for(i=0,i<n, i++)	n
	1+1 = 2
total+=1	print(i)

$$F(n) = 1 + 2 * n = 2n + 1$$

Therefore, the Big Oh value $O(n)$

Example 2

$$f(n) = 3n^2 + 2n + 1000\log n + 5000$$

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After ignoring lower order terms, we get the highest order term as $3n^2$

After ignoring the constant 3, we get n^2

Therefore the Big O value of this expression is $O(n^2)$

Example 3

$$f(n) = 3n^3 + 2n^2 + 5n + 1$$

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After ignoring lower order terms, we get the highest order term as $3n^3$

After ignoring the constant 3, we get n^3

Therefore the Big O value of this expression is $O(n^3)$

Dominant Term in Asymptotic Notation

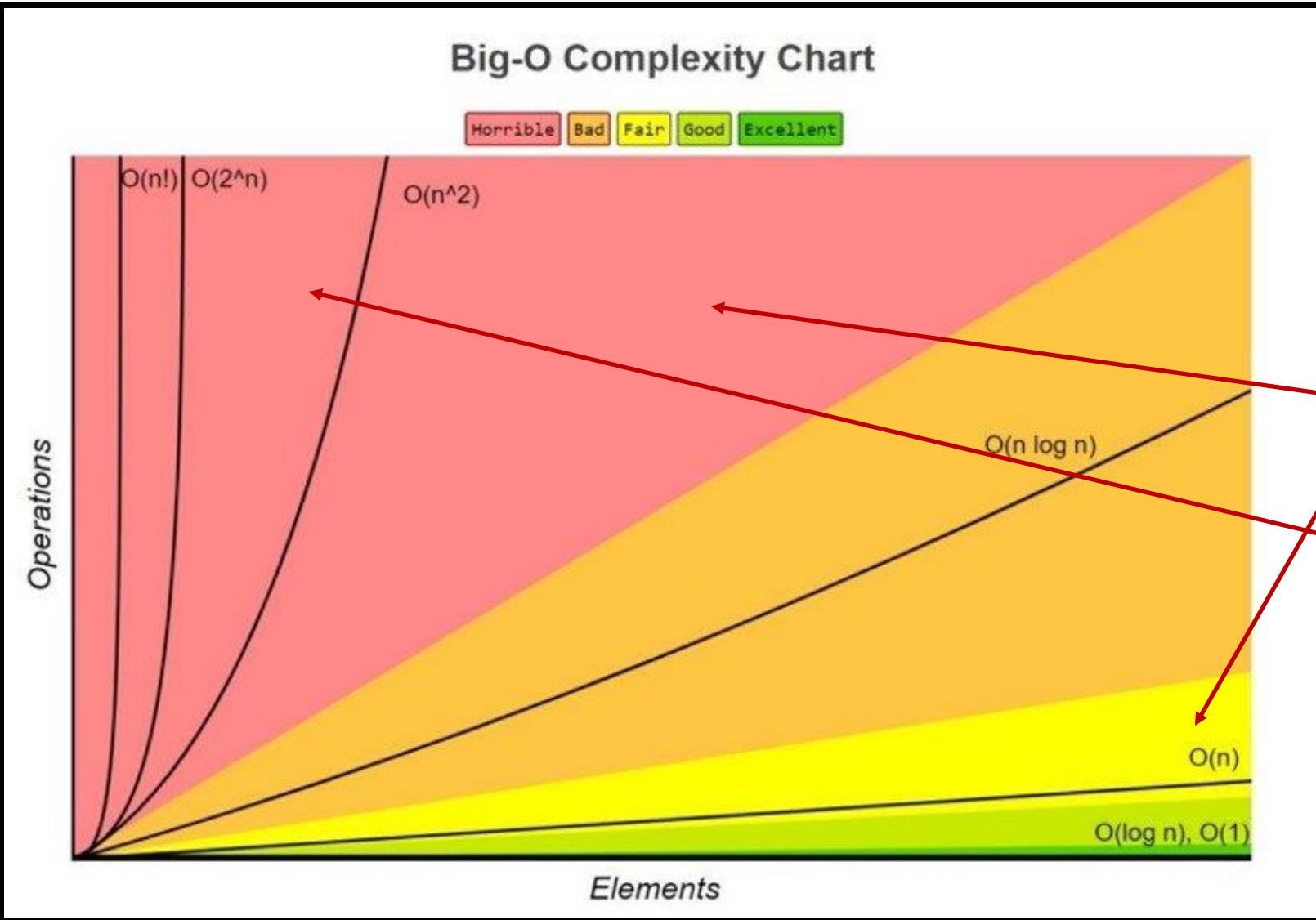
Asymptotic complexity is based on the ***dominant term*** of the growth function – the term that increases most quickly as n increases.

- In Example 1, the highest order term was $-n$ that is our ***Dominant Term***
- In Example 2, the highest order term was $-n^2$ that is our ***Dominant Term***
- In Example 3, the highest order term was $-n^3$ that is our ***Dominant Term***

Order of Growth in Asymptotic Complexity

Order of growth describes how fast an algorithm's running time increases as input size (n) grows, based on the dominant term of its function.

Function $f(n)$	Dominant Term	Order of Growth	Big-O Notation
$2n + 1$	n	Linear	$O(n)$
$3n^2 + 2n + 1$	n^2	Quadratic	$O(n^2)$
$1000 \log n + 5000$			
$3n^3 + 2n^2 + 5n + 1$	n^3	Cubic	$O(n^3)$



Example 1 : $O(n)$

Example 2 : $O(n^2)$

Example 3 : $O(n^3)$

Example : If Else Statement

```
if (condition) {  
    Statement1;  
} else {  
    Statement2;  
}
```

Either Stmt 1 or Stmt 2 will execute.

Solution

Because only one statement will be executed in if else, we will consider

$$\begin{aligned} \text{Max}(\text{time(st1)}, \text{time(st2)}) &\sim \\ \text{Max(O(n), 1)} \end{aligned}$$

Statement

```
if(condition)
{
    stmt 1;
}
else
{
    stmt2;
}
```

Either Stmt1 or Stmt2 will execute.
So, Worst case is

Max(time(stmt1, stmt2))
Max (O(n), O(1)) → O(n)

Example :

Nested For Statement

```
for (condition) {  
    for (condition) {  
        Statements;  
    }  
}
```

Note: For all the "for loop" statement multiply by the product

Solution

```
for(i=0; i <n; i++)  
{  
    for(i=0; i <n; i++)  
    {  
        Stmt;  
    }  
}
```

Total : n *n → O(n²)

In the Next Class?

Implement	Hands-on Lab Work : Implement algorithms discussed in theory (Use lab computers for coding and testing.)
Run	Run Programs : Execute sample algorithms with varying input sizes. Observe actual time and space usage.
Analyze	Analyze Complexity : Compare empirical results with theoretical Big-O. Plot graphs for performance trends.
Share	Discussion & Q&A : Share findings and clarify doubts in next session.