**1, (3 points) Compute the eigenvalues and eigenvectors of matrix 𝐶1 = 𝐶 T 𝐶 ,**

First, we get the matrix:

Then calculate the matrix:

Determine the matrix:

Then we can get eigenvalues and

If the eigenvalue , calculate the equation:

Then the eigenvector is:

If the eigenvalue , calculate the equation:

Then the eigenvector is:

**2, (3 points) Compute LU Decomposition on matrix 𝐶2**

The calculate process of L

The calculate process of U

Define the row with R1, R2 and R3, in the first process, calculate 2R1 + R2, so the value in the L should be -2. Then calculate -3R1 + R3, so the value in the value should be 3, and we are done with LU decomposition:

**3, Please compute the CUR approximation. Here we assume the random selection of rows is 3 and 5, and random selection of columns is 1 and 3.**

Since the random selection of rows and columns are 3 and 5, 1 and 3.

Then we can create sample matrices C and R.

Calculate:

Therefore:

Then:

As we know the r=2, then we can get:

Similarly, with the same method, we can get:

Then, with above calculation, we can get:

We can get a CUR like:

Then we need construct U

First, we Create r x r matrix X as intersection of C and R:

Apply SVD on W:

From SVD definition, we can know:

Then start to calculate:

For U, the calculation is pretty same as the question1:

Similar with V, then we get:

Then we can get SVD:

Then we calculate the Pseudo-Inverse of :

Finally, we can get U:

Finally, we get CUR: