Publicly Verifiable Zero-Knowledge and Post-Quantum Signatures From VOLE-in-the-Head

Crypto'23 Submission 482

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Motivations

- Efficient VOLE-based DVZK
- How to transform DVZK to (NI)ZK?

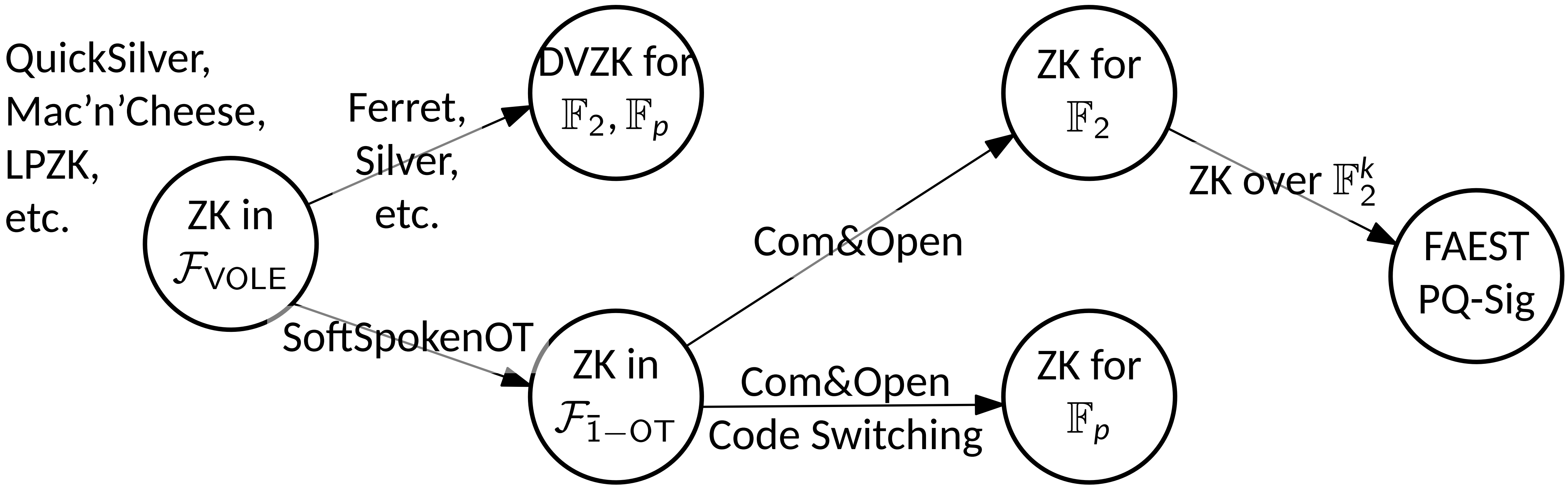
- P.S. Landscape of Efficient Zero Knowledge

	zk-SNARK, GKR, etc.	GCZK	DVZK
Prover Computation	$\Omega(\lvert C \rvert)$	$O(\lvert C \rvert)$	$O(\lvert C \rvert)$
Prover Memory	$\Omega(\lvert C \rvert)$	$O(1)$	$O(1)$
Proof Size	$O(\log(\lvert C \rvert))$	$O(\kappa \cdot \lvert C \rvert)$	$O(\lvert C \rvert)$ or $O(\lvert w \rvert + d)$
Verifier Type	Universal	Designated	Designated
Advantage	Low-Bandwidth Small Circuit	High-Bandwidth Large Circuit	High-Bandwidth Large Circuit Polynomials

Main techniques (of DVZK):

- Random (subfield) VOLE
- Low-Degree Test

Contributions



- Observation 1: In DVZK, Verifier is **public coin** and VOLE output can be delayed to the very end after all communications
- Observation 2: Subspace VOLE (SoftSpokenOT) allows reduction to OT
- Observation 3: OT can be replaced with com-and-open

Performance of the ZK Compilers

Table 1. Comparison of linear-size zero-knowledge proof systems

Protocol	Field*	Model	Comm./gate†	Assumption
VOLE-ZK [YSWW21]‡	$\mathbb{F}_2$	deg-d constraints	1	LPN
VOLE-ZK [DIO21, YSWW21]‡	$\mathbb{F}_p$	deg-d constraints	1	LPN
Limbo [dOT21]	$\mathbb{F}_2$	Circuits (free XOR)	42 (11)	Hash
Limbo [dOT21]	$\mathbb{F}_p$	Circuits (free add)	40 (11)	Hash
VOLE-in-the-head (§E.3)	$\mathbb{F}_2$	deg-d constraints	16 (5)	Hash
VOLE-in-the-head (§5.1)	$\mathbb{F}_p$	deg-d constraints	3 (2)	Hash

* $p \approx 2^{64}$

† Soundness error at most 2^{-128} (2^{-40}). Cost is average number of field elements sent per AND/mult. gate, for a circuit with 2^{20} such gates.

‡ Designated-verifier only

Preliminary: VOLE as IT-MAC (Linear Commitment)

Receiver/$\mathcal{V}$

$\Delta \in \mathbb{F}_{p^r}$
(global key)
$\mathbf{v} \in \mathbb{F}_{p^r}^n$
(MAC Key)

$\mathcal{F}_{\text{rVOLE}}$

Sender/$\mathcal{P}$

$\mathbf{a} \in \mathbb{F}_{p^r}^n$ (message)
$\mathbf{b} \in \mathbb{F}_{p^r}^n$ (MAC Tag)

Sender commits to $\mathbf{x}$ by sending $\delta_x := \mathbf{x} - \mathbf{a}$

IT-MAC $[\mathbf{x}] := (\mathbf{x}, \mathbf{v}, \mathbf{b})$ subject to $\mathbf{v} = \mathbf{b} + \mathbf{x} \cdot \Delta$

- Linear Homomorphism: $[x] + [y] \mapsto [x + y]$
- Open($[x]$): $\mathcal{P} \to \mathcal{V} : (x, b)$, $\mathcal{V}$ checks $v = b + x \cdot \Delta$
- Batched Open:

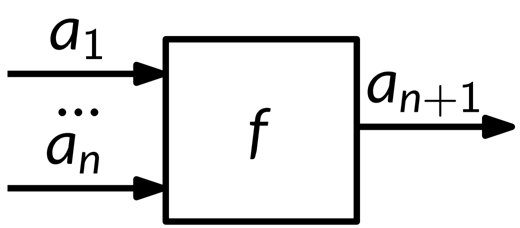
Receiver/$\mathcal{V}$ Sender/$\mathcal{P}$

$\{\chi_i\}$

$\mathbf{x}$, Open($[\sum_i \chi_i \cdot \mathbf{x}_i]$)

- $\mathcal{P}$ opens a different value $\to \mathcal{P}$ guesses Δ
- Soundness error $= \frac{1}{p^r}$

Starting Point: Designated Verifier Zero Knowledge

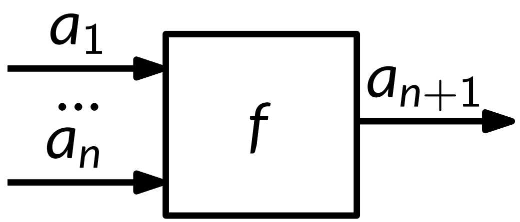


$$\underbrace{\mathbf{v} = \Delta}_{\mathcal{V}} \cdot \underbrace{\mathbf{a} + \mathbf{b}}_{\mathcal{P}}$$

$$f(\mathbf{a}) = f_d(\mathbf{a}) + f_{d-1}(\mathbf{a}) ... + f_0$$

$$\begin{aligned} f(\mathbf{v}) &= f_d(\mathbf{v}) + f_{d-1}(\mathbf{v}) + ... + f_0 \\ &= f_d(\mathbf{a})\Delta^d + f_{d-1}(\mathbf{a})\Delta^{d-1} + ... + f_0 + f_r(\mathbf{a}, \mathbf{b}) \end{aligned}$$

Starting Point: Designated Verifier Zero Knowledge

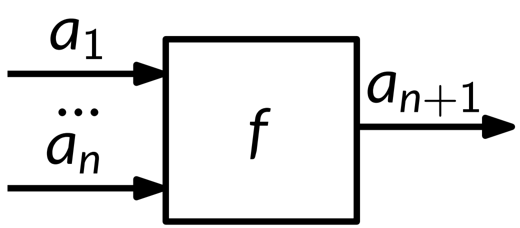


$$\underbrace{\mathbf{v} = \Delta}_{\mathcal{V}} \cdot \underbrace{\mathbf{a} + \mathbf{b}}_{\mathcal{P}}$$

$$\begin{aligned} f(\mathbf{a}) &= f_d(\mathbf{a}) + f_{d-1}(\mathbf{a}) ... + f_0 \\ f(\mathbf{v}) &= f_d(\mathbf{v}) + f_{d-1}(\mathbf{v}) + ... + f_0 \\ &= f_d(\mathbf{a})\Delta^d + f_{d-1}(\mathbf{a})\Delta^{d-1} + ... + f_0 + f_r(\mathbf{a}, \mathbf{b}) \end{aligned}$$

$$\begin{aligned} g(\mathbf{v}) &:= f_d(\mathbf{v}) + \Delta f_{d-1}(\mathbf{v}) + ... + \Delta^{d-1} f_1(\mathbf{v}) + \Delta^d f_0 - \Delta^{d-1} v_{n+1} \\ &= (f_d(\mathbf{a}) + ... + f_0 - a_{n+1})\Delta^d + \underbrace{f'_{r,\mathbf{a},\mathbf{b}}(\Delta)}_{\deg(\Delta) < d} \end{aligned}$$

Starting Point: Designated Verifier Zero Knowledge



$$\underbrace{\mathbf{v} = \Delta}_{\mathcal{V}} \cdot \underbrace{\mathbf{a} + \mathbf{b}}_{\mathcal{P}}$$

$$f(\mathbf{a}) = f_d(\mathbf{a}) + f_{d-1}(\mathbf{a})... + f_0$$

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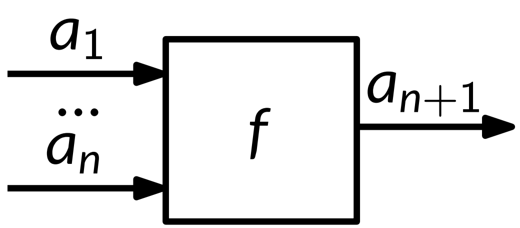
Π^{d-1}_{gen}

$$\left.\begin{aligned} v_1 &= a_1\Delta + b_1 \\ v_2\Delta &= a_2\Delta^2 + b_2\Delta \\ &\vdots \\ v_{d-1}\Delta^{d-2} &= a_{d-1}\Delta^{d-1} + b_{d-1}\Delta^{d-2} \end{aligned}\right\} + \Rightarrow g^*(\Delta)$$

Starting Point: Designated Verifier Zero Knowledge



$$\underbrace{\mathbf{v} = \Delta}_{\mathcal{V}} \cdot \underbrace{\mathbf{a} + \mathbf{b}}_{\mathcal{P}}$$

$$f(\mathbf{a}) = f_d(\mathbf{a}) + f_{d-1}(\mathbf{a}) ... + f_0$$

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$$\begin{aligned} g(\mathbf{v}) &:= f_d(\mathbf{v}) + \Delta f_{d-1}(\mathbf{v}) + ... + \Delta^{d-1} f_1(\mathbf{v}) + \Delta^d f_0 - \Delta^{d-1} v_{n+1} \\ &= (f_d(\mathbf{a}) + ... + f_0 - a_{n+1})\Delta^d + \underbrace{f'_{r,\mathbf{a},\mathbf{b}}(\Delta)}_{\deg(\Delta) < d} \end{aligned}$$

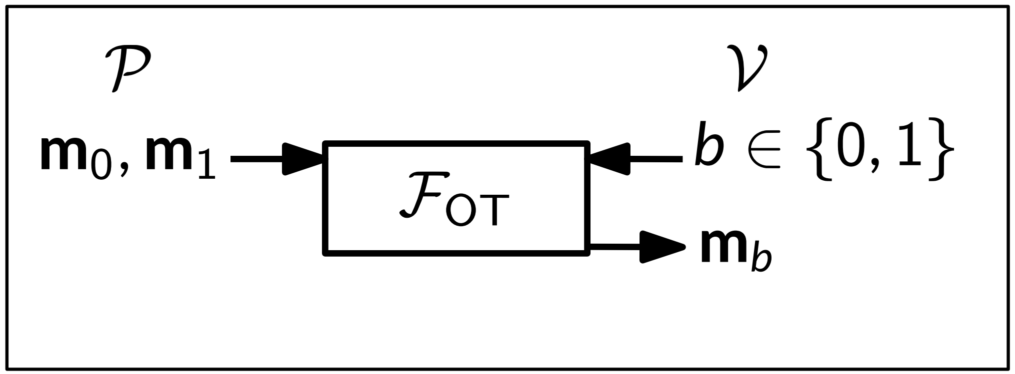
Π_{gen}^{d-1}

$$\left.\begin{aligned} v_1 &= a_1\Delta + b_1 \\ v_2\Delta &= a_2\Delta^2 + b_2\Delta \\ &\vdots \\ v_{d-1}\Delta^{d-2} &= a_{d-1}\Delta^{d-1} + b_{d-1}\Delta^{d-2} \end{aligned}\right\} + \Rightarrow g^*(\Delta)$$

- Sends collapsed, masked coeff. of $g(\mathbf{v})$
- Soundness: $\frac{d}{p}$

Starting Point: Public Coin $\mathcal{F}_{\mathrm{OT}}$ by Com&Open

- For public-coin $\mathcal{V}$, we have public-coin $\binom{2}{1}$-OT



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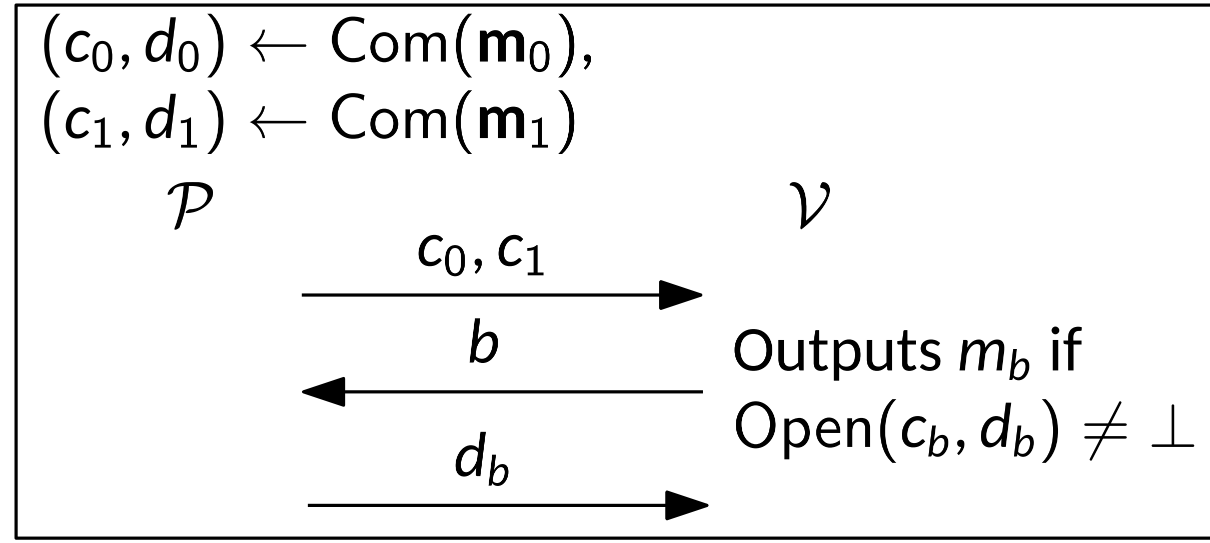
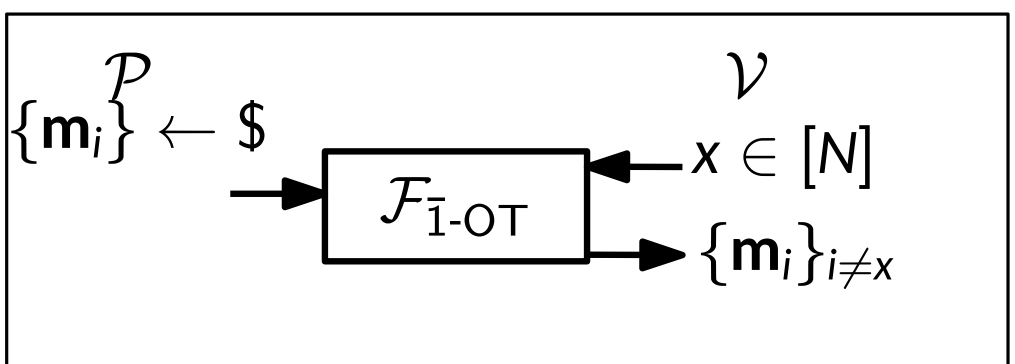


- In particular, we have public-coin **random** $\binom{N}{N-1}$-OT with $O(\log N)$ comm.



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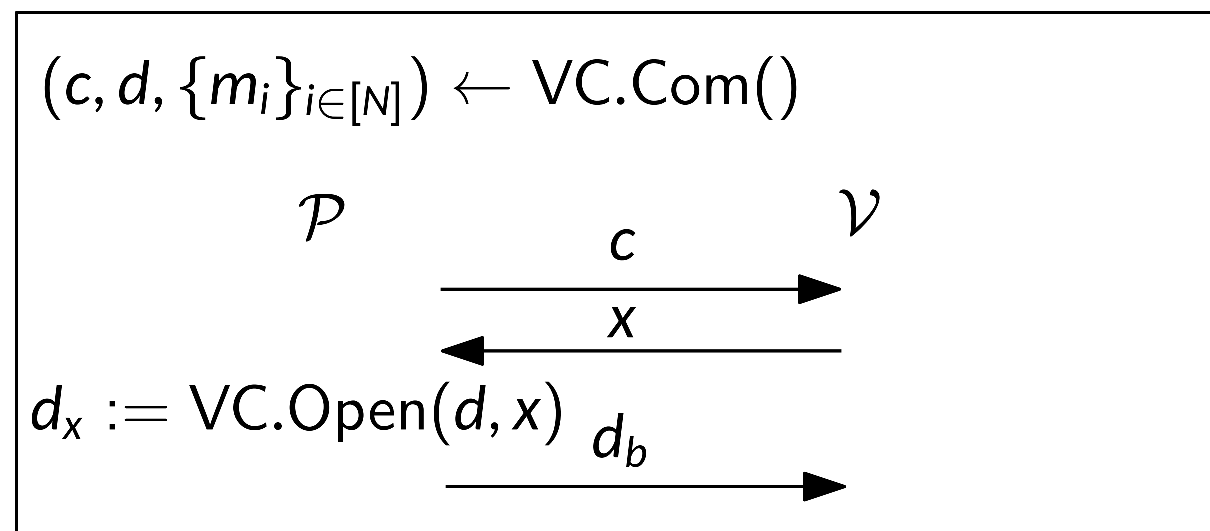
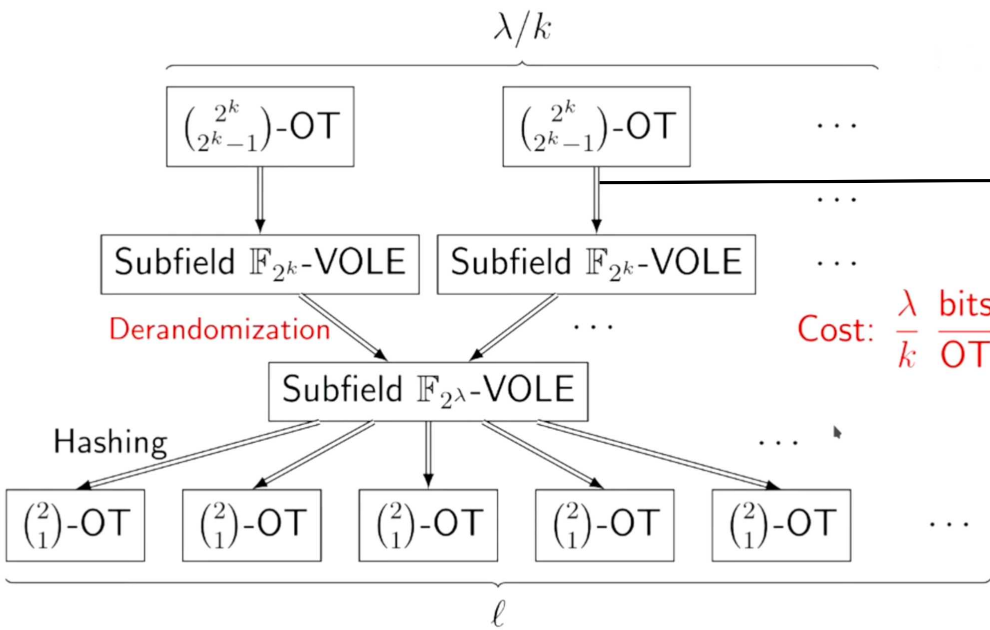


Next Step: From $\mathcal{F}_{\bar{1}\text{-OT}}$ to Subspace $\mathcal{F}_{\text{VOLE}}$ (SoftSpokenOT)



$\mathcal{P}$		$\mathcal{V}$
$\{m_i\}_{i\in[2^k]}$	$\leftarrow \boxed{\mathcal{F}_{\bar{1}\text{-OT}}} \rightarrow$	$\{m_i\}_{i\neq x}$
$\mathbf{a} := \sum_i m_i$		$\Delta := x$
$\mathbf{b} := \sum_i -i \cdot m_i$		$\mathbf{v} := \sum_i (\Delta - i)\cdot m_i$

$$\mathbf{b} = \mathbf{v} - \mathbf{a} \times \Delta \quad \in \mathbb{F}_{2^k}$$

Repeat $\ell = \lceil \kappa/k \rceil$ times

$$B = V' - A' \times \begin{pmatrix} \Delta_1 & & \\ & \ddots & \\ & & \Delta_\ell \end{pmatrix}$$

κ-bit entropy

From $\mathcal{F}_{\bar{1}\text{-OT}}$ to Subspace $\mathcal{F}_{\mathsf{VOLE}}$ (SoftSpokenOT), Continued

- Goal: q^{-d}-sound IT-MAC

$$\ell \quad \underbrace{\boxed{B}}_{n_{\mathcal{C}}} = \underbrace{\boxed{V}}_{n_{\mathcal{C}}} - \underbrace{\boxed{A}}_{k_{\mathcal{C}}} \times \underbrace{\boxed{G_{\mathcal{C}}}}_{n_{\mathcal{C}}} \times \begin{bmatrix}\Delta_1 & & \\ & \ddots & \\ & & \Delta_{n_{\mathcal{C}}}\end{bmatrix}$$

$\mathbb{F}_{2^\kappa} \equiv \mathbb{F}_2^\kappa$

$$\Delta \in \mathbb{F}_{2^\kappa} \equiv \boxed{Rep(\kappa)} \times \begin{bmatrix}\Delta_1 & & \\ & \ddots & \\ & & \Delta_{\kappa}\end{bmatrix}$$

$$A' \cdot \begin{bmatrix}G_{\mathcal{C}} \\ H_{\mathcal{C}}\end{bmatrix}^{-1} = [A \| C]$$

- Send Syndrome

$$\boxed{B} = \boxed{V'} - \boxed{A'} \times \begin{bmatrix}\Delta_1 & & \\ & \ddots & \\ & & \Delta_{n_{\mathcal{C}}}\end{bmatrix}$$

$$\boxed{A} \times \boxed{G_{\mathcal{C}}} + \boxed{C} \times \boxed{H_{\mathcal{C}}}$$

From $\mathcal{F}_{\bar{1}\text{-OT}}$ to Subspace $\mathcal{F}_{\text{VOLE}}$ (SoftSpokenOT), Continued

- $\mathcal{V}$ locally sets $V = V' - C \cdot H_{\mathcal{C}}$

$$\boxed{B} = \boxed{V'} - \left(\boxed{A} \times \boxed{G_{\mathcal{C}}} + \boxed{C} \times \boxed{H_{\mathcal{C}}} \right) \times \begin{bmatrix} \Delta_1 & & \\ & \ddots & \\ & & \Delta_{n_{\mathcal{C}}} \end{bmatrix}$$

$$= \boxed{V} - \boxed{A} \times \boxed{G_{\mathcal{C}}} \times \begin{bmatrix} \Delta_1 & & \\ & \ddots & \\ & & \Delta_{n_{\mathcal{C}}} \end{bmatrix}$$

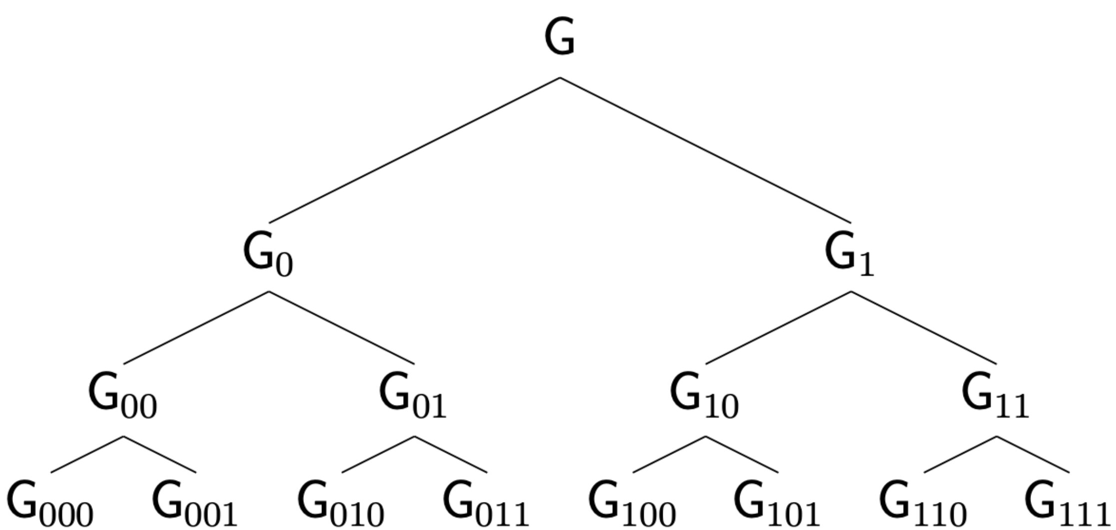
- Consistency Check: Use Linear-UHF to hash and reveal some rows to check $\mathcal{C}$-Δ-relations

Theorem 2. Protocol Π_{sVOLE} securely realizes $\mathcal{F}_{\text{sVOLE}}$ with distinguishing advantage $\binom{n_{\mathcal{C}}}{k_{\mathcal{C}}+1} \cdot \varepsilon$

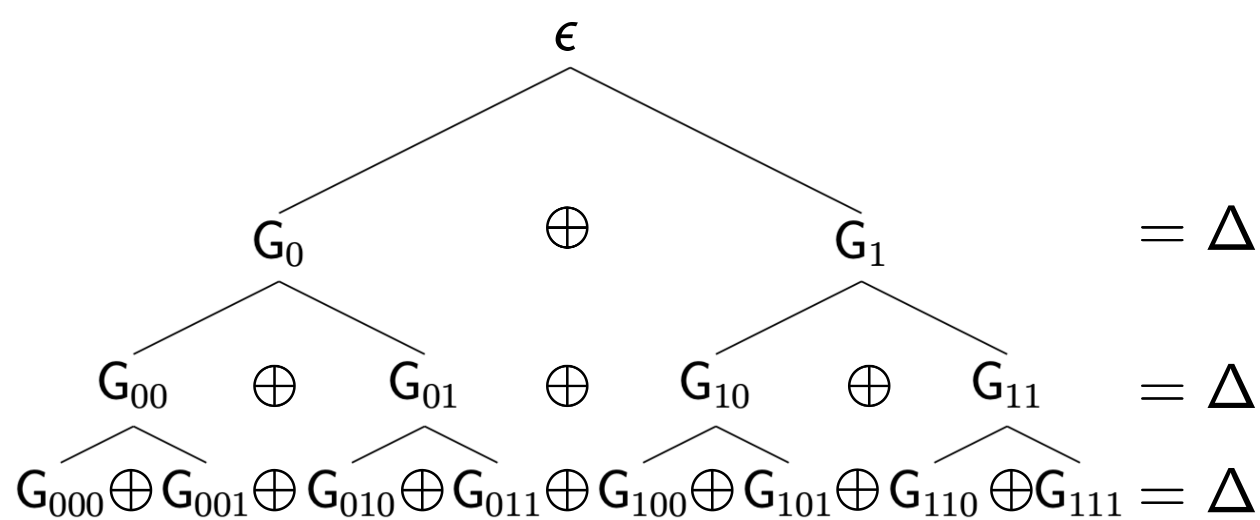
Half-tree Optimization

- Save computation/communication by introducing correlation at each level

GGM Tree



Correlated GGM Tree



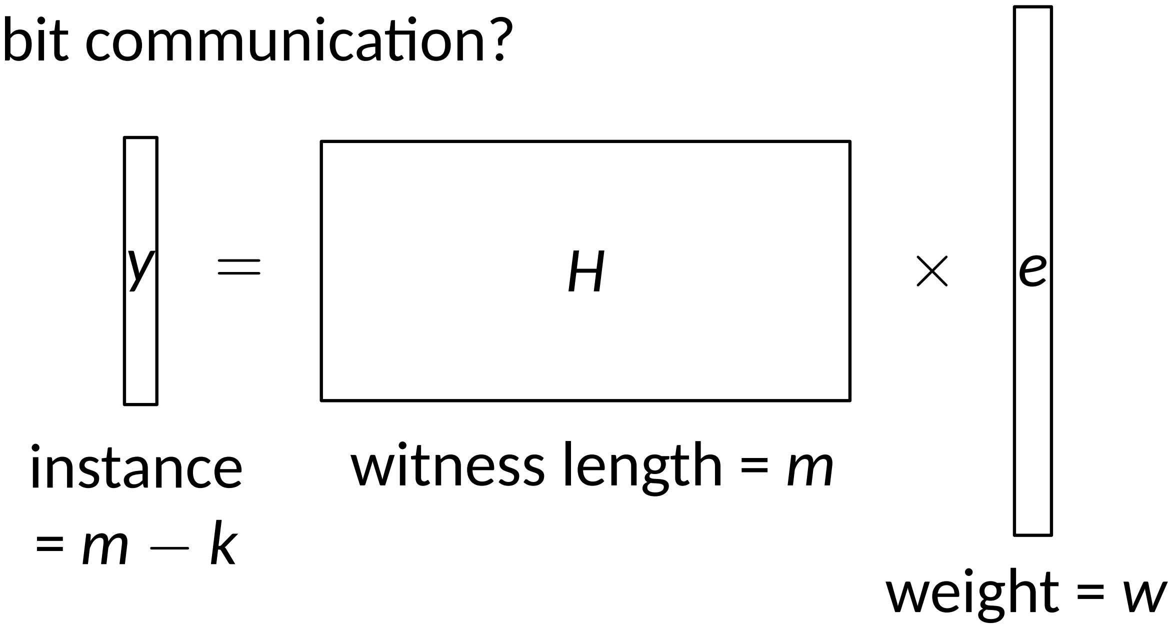
- Expansion: $G_{00} || G_{01} = \mathsf{PRG}(G_0)$ $\quad$ $G_{00} = \mathsf{H}(G_0), G_{01} = G_0 \oplus G_{00}$
- Costs: $N\times$ RO or $2N\times$ RP $\quad$ $N\times$ RP
- Initial Setup: $G \leftarrow \mathbb{F}_2^\kappa$ $\quad$ $G_0 = k \leftarrow \mathbb{F}_2^\kappa \quad G_1 = \Delta - k$

Optimization?

- We need $\ell := m + 2\kappa$ random bits for QuickSilver
- Half-tree gives κ bits
- How to expand it into ℓ bits with less than ℓ bit communication?

Scheme	SD Parameters					MPC Parameters			
	q	m	k	w	d	$\lvert\mathbb{F}_{\text{poly}}\rvert$	$\lvert\mathbb{F}_{\text{points}}\rvert$	t	p
Variant 1	2	1280	640	132	1	2^{11}	2^{22}	6	$\approx 2^{-69}$
Variant 2	2	1536	888	120	6	2^{8}	2^{24}	5	$\approx 2^{-79}$
Variant 3	2^8	256	128	80	1	2^{8}	2^{24}	5	$\approx 2^{-78}$

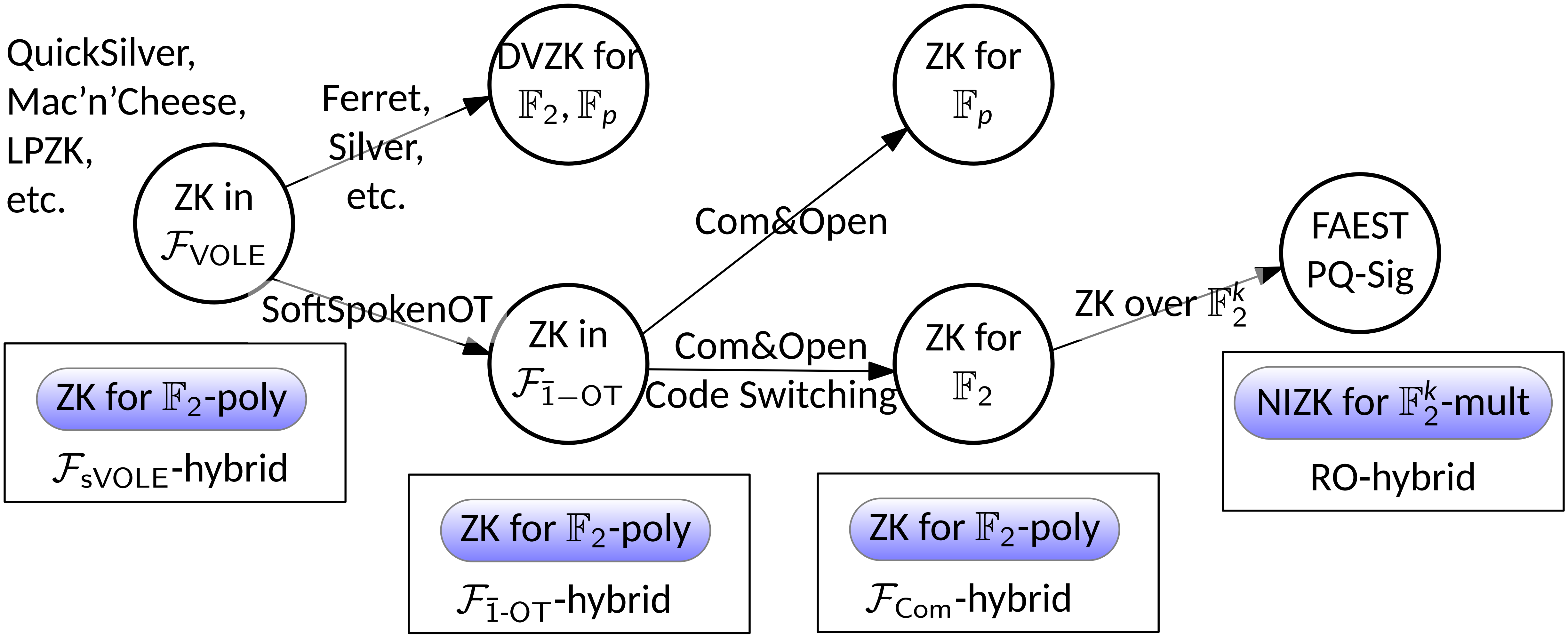
Table 3: SD and MPC parameters.



n	k	h	Best [34]	d_{conj} plain	(f,u)	d_{conj}	XL hybrid Sec. 4.2
2^{22}	64770	2735	104	2	$(0,0)$	2	<u>103</u>
2^{20}	32771	1419	99	3	$(1159,2)$	2	<u>98</u>
2^{18}	15336	760	95	3	$(657,7)$	2	104
2^{16}	7391	389	91	4	$(373,10)$	2	108
2^{14}	3482	198	86	6	$(197,11)$	2	106
2^{12}	1589	98	83	8	$(88,13)$	2	103
2^{10}	652	57	94	12	$(54,9)$	2	101

Table 2. Hybrid approach of Section 4.2 over $\mathbb{F}_2$ (Modeling 2).

ZK for Polynomial Constraints Over **Small** Fields



The 3-Round Protocol

Protocol $\Pi^t_{\mathsf{2D\text{-}Rep}}$

PARAMETERS: Code $\mathcal{C}_{\mathsf{Rep}} = [\tau, 1, \tau]_p$ with $\mathbf{G}_{\mathcal{C}} = (1 \ldots 1) \in \mathbb{F}_p^{1\times\tau}$. VOLE size $q = p^r$.
INPUTS: Polynomials $f_i \in \mathbb{F}_{p^k}[X_1, \ldots, X_\ell]_{\leq 2}$, $i \in [t]$. The prover $\mathcal{P}$ also holds a witness $\mathbf{w} \in \mathbb{F}_p^\ell$ such that $f_i(\mathbf{w}) = 0$ for all $i \in [t]$.

Round 1. $\mathcal{P}$ does the following:

1. Call the functionality $\mathcal{F}_{\mathsf{sVOLE}}^{p,q,S_\Delta,\mathcal{C}_{\mathsf{Rep}},\ell+r\tau,\mathcal{L}}$ and receive $\mathbf{u} \in \mathbb{F}_p^{\ell+r\tau}, \mathbf{V} \in \mathbb{F}_q^{(\ell+r\tau)\times\tau}$.
   $\mathcal{V}$ receives done.
2. Compute $\mathbf{d} = \mathbf{w} - \mathbf{u}_{[1..\ell]} \in \mathbb{F}_p^\ell$ and send $\mathbf{d}$ to $\mathcal{V}$.
3. For $i \in [\ell + 1..\ell + r\tau]$, embed $u_i \hookrightarrow \mathbb{F}_{q^\tau}$.
   For $i \in [\ell + r\tau]$, lift $\mathbf{v}_i \in \mathbb{F}_q^\tau$ into $v_i \in \mathbb{F}_{q^\tau}$.
   For $i \in [\ell]$, also embed $w_i \hookrightarrow \mathbb{F}_{q^\tau}$.

Round 2. $\mathcal{V}$ sends challenges $\chi_i \in \mathbb{F}_{q^\tau}$, $i \in [t]$.

Round 3. $\mathcal{P}$ does the following:

1. For each $i \in [t]$, compute $A_{i,0}, A_{i,1} \in \mathbb{F}_{q^\tau}$ such that
$$c_i(Y) = \bar{f}_i(w_1, \ldots, w_n) \cdot Y^2 + A_{i,1} \cdot Y + A_{i,0}.$$
2. Compute
$$u^* = \sum_{i\in[r\tau]} u_i X^{i-1} \qquad v^* = \sum_{i\in[r\tau]} v_i X^{i-1},$$
   where $\mathbb{F}_{q^\tau} \simeq \mathbb{F}_p[X]/F(X)$.
3. Compute $\tilde{b} = \sum_{i\in[t]} \chi_i \cdot A_{i,0} + v^* \in \mathbb{F}_{q^\tau}$ and $\tilde{a} = \sum_{i\in[t]} \chi_i \cdot A_{i,1} + u^* \in \mathbb{F}_{q^\tau}$ and send $(\tilde{a}, \tilde{b})$ to $\mathcal{V}$.

Verification. $\mathcal{V}$ runs the following check:

1. Call $\mathcal{F}_{\mathsf{sVOLE}}^{p,q,S_\Delta,\mathcal{C}_{\mathsf{Rep}},\ell+r\tau,\mathcal{L}}$ on input (get) and obtain $\boldsymbol{\Delta} \in \mathbb{F}_q^\tau$, $\mathbf{Q} \in \mathbb{F}_q^{(\ell+r\tau)\times\tau}$ such that $\mathbf{Q} = \mathbf{V} + \mathbf{u}^T\mathbf{G}_{\mathcal{C}}\mathsf{diag}(\boldsymbol{\Delta})$.
2. Compute $\mathbf{Q}' = \mathbf{Q}_{[1..\ell]} + \mathbf{d}^T\mathbf{G}_{\mathcal{C}}\mathsf{diag}(\boldsymbol{\Delta}) = \mathbf{V}_{[1..\ell]} + \mathbf{w}^T\mathbf{G}_{\mathcal{C}}\mathsf{diag}(\boldsymbol{\Delta})$.
3. Lift $\boldsymbol{\Delta}, \mathbf{q}'_1, \ldots, \mathbf{q}'_\ell, \mathbf{q}_{\ell+1}, \ldots, \mathbf{q}_{\ell+r\tau} \in \mathbb{F}_q^\tau$ into $\Delta, q'_1, \ldots, q'_\ell, q_{\ell+1}, \ldots, q_{\ell+r\tau} \in \mathbb{F}_{q^\tau}$.
4. For each $i \in [t]$, compute
$$c_i(\Delta) = \sum_{h\in[0,2]} \bar{f}_{i,h}(q'_1, \ldots, q'_\ell) \cdot \Delta^{2-h}$$
5. Compute $q^* = \sum_{i\in[r\tau]} q_{\ell+i} \cdot X^{i-1}$ such that $q^* = v^* + u^*\Delta$.
6. Compute $\tilde{c} = \sum_{i\in[t]} \chi_i \cdot c_i(\Delta) + q^*$.
7. Check that $\tilde{c} \stackrel{?}{=} \tilde{a} \cdot \Delta + \tilde{b}$.

Theorem 4. The Protocol $\Pi^t_{\text{2D-Rep}}$ is a ZKPoK with soundness error $\frac{3}{p^{r\tau}}$.

How to Handle Arbitrary $\mathcal{C}$?

- For subspace VOLE with general code $[n_{\mathcal{C}}, k_{\mathcal{C}}, d_{\mathcal{C}}]$ and witness $\mathbf{w} = \mathbb{F}_p^{\ell \times k_{\mathcal{C}}}$
- The committed witness is as follows

$$\ell \; \underbrace{\boxed{\quad B \quad}}_{n_{\mathcal{C}}} = \boxed{\quad V \quad} - \underbrace{\boxed{\; A \;}}_{k_{\mathcal{C}}} \times \underbrace{\boxed{\; G_{\mathcal{C}} \;}}_{n_{\mathcal{C}}} \times \begin{bmatrix} \Delta_1 & & \\ & \ddots & \\ & & \Delta_{n_{\mathcal{C}}} \end{bmatrix}$$

Problem: Only row-wise linearity

In $Rep(\kappa)$, $k_{\mathcal{C}} = 1$

- Solution: Simulate VOLE in $\mathcal{P}$'s head once again

$\mathcal{P}$		$\mathcal{V}$
Prepares $(2\ell + 2)$ rows in A'		
$A' = \begin{bmatrix} A \\ B \end{bmatrix}$	$\xrightarrow{\text{DVZK } \pi \text{ as if } \Delta' \text{ is the key}}$	
	$\xleftarrow{\Delta'}$	
	$\xrightarrow{V = B + \Delta' \cdot A}$	

$\mathcal{P}$ and $\mathcal{V}$ continue the subspace VOLE simulation

$\mathcal{V}$ accepts if

- π is valid under Δ'
- The opening of V is correct under $\mathrm{diag}(\vec{\Delta})$

The Code-Switching Technique

Protocol $\Pi^t_{\mathsf{2D-LC}}$

The protocol is parameterized by an $[n_\mathcal{C}, k_\mathcal{C}, d_\mathcal{C}]_p$ linear code $\mathcal{C}$, set $S_\Delta \subset \mathbb{F}_p^{n_\mathcal{C}}$ and a leakage space $\mathcal{L}$ (used in $\mathcal{F}_{\mathsf{sVOLE}}$).

INPUTS: Both parties hold a set of polynomials $f_i \in \mathbb{F}_p[X_1, \ldots, X_\ell]_{\leq 2}$, $i \in [t]$. $\mathcal{P}$ also holds a witness $\mathbf{w} \in \mathbb{F}_p^{k_\mathcal{C}\ell}$ such that $f_i(\mathbf{w}) = 0$, for all $i \in [t]$.

Round 1. $\mathcal{P}$ does as follows:

1. $\mathcal{P}$ and $\mathcal{V}$ call $\mathcal{F}_{\mathsf{sVOLE}}^{p,p,S_\Delta,\mathcal{C},2\ell+1,\mathcal{L}}$, $\mathcal{P}$ receives $\mathbf{U} \in \mathbb{F}_p^{(2\ell+2)\times k_\mathcal{C}}$, $\mathbf{V} \in \mathbb{F}_p^{(2\ell+2)\times n_\mathcal{C}}$, while $\mathcal{V}$ gets the message done.
2. $\mathcal{P}$ sets $\mathbf{V}_1 = \mathbf{V}_{[1..\ell+1]}$, $\mathbf{V}_2 = \mathbf{V}_{[\ell+2..2\ell+2]}$ and $\mathbf{R} = \mathbf{U}_{[\ell+2..2\ell+2]}$
3. $\mathcal{P}$ commits to its witness by sending $\mathbf{D} = \mathbf{W} - \mathbf{U}_{[1..\ell]}$.

Round 2. $\mathcal{V}$ samples $\boldsymbol{\chi} \leftarrow \mathbb{F}_p^t$ and sends it to $\mathcal{P}$.

Round 3. $\mathcal{P}$ proceeds as follows.

1. For each $i \in [t]$, compute

$$g_i(Y) := \sum\nolimits_{h\in[0,2]} f_{i,h}(\mathbf{r}_1 + \mathbf{w}_1 \cdot Y, \ldots, \mathbf{r}_\ell + \mathbf{w}_\ell \cdot Y) \cdot Y^{2-h}$$
$$= \sum\nolimits_{h\in[0,1]} A_{i,h} \cdot Y^h$$

2. Compute $\widetilde{\mathbf{b}} = \sum_{i\in[t]} \chi_i \cdot A_{i,0} + \mathbf{r}_{\ell+1}$ and $\widetilde{\mathbf{a}} = \sum_{i\in[t]} \chi_i \cdot A_{i,1} + \mathbf{u}_{1,\ell+1}$, where $\mathbf{u}_{1,i}$ is the ith row of $\mathbf{U}$.
3. Send $(\widetilde{\mathbf{b}}, \widetilde{\mathbf{a}})$ to $\mathcal{V}$.

Round 4. $\mathcal{V}$ samples $\Delta' \leftarrow \mathbb{F}_p$ and sends it to the prover.

Round 5. $\mathcal{P}$ sends $\mathbf{S} = \mathbf{R} + \mathbf{U}_{[1..\ell+1]} \cdot \Delta' \in \mathbb{F}_p^{(\ell+1)\times n_\mathcal{C}}$ to $\mathcal{V}$

Round 6. $\mathcal{V}$ samples $\boldsymbol{\eta} \leftarrow \mathbb{F}_p^{\ell+1}$ and sends it to $\mathcal{P}$

Round 7. $\mathcal{P}$ computes $\widetilde{\mathbf{v}} = \boldsymbol{\eta}^\top(\mathbf{V}_2 + \mathbf{V}_1 \cdot \Delta')$ and sends it to $\mathcal{V}$.

Verification. $\mathcal{V}$ runs the following checks.

1. *Check the constraints:*
   - Compute $\mathbf{S}' = \mathbf{S} + \begin{bmatrix}\mathbf{D}\\ 0\end{bmatrix} \cdot \Delta' = \mathbf{R} + \begin{bmatrix}\mathbf{W}\\ \mathbf{u}_{\ell+1}\end{bmatrix} \cdot \Delta'$.
   - For each $i \in [t]$, compute
   $$\mathbf{c}_i(Y) = \sum_{h\in[0,2]} f_{i,h}(\mathbf{s}'_1, \ldots, \mathbf{s}'_\ell) \cdot Y^{2-h}.$$
   - Let $\widetilde{\mathbf{s}} = \sum_{i\in[t]} \chi_i \cdot \mathbf{c}_i(\Delta') + \mathbf{s}'_{\ell+1}$.
   - Check that $\widetilde{\mathbf{s}} = \widetilde{\mathbf{b}} + \widetilde{\mathbf{a}} \cdot \Delta'$.
2. *Check the opening of* $\mathbf{S}$*:*
   - Call $\mathcal{F}_{\mathsf{sVOLE}}^{p,p,S_\Delta,\mathcal{C},2\ell+1,\mathcal{L}}$ on input (get) and obtain $\boldsymbol{\Delta} \in \mathbb{F}_p^{n_\mathcal{C}}$ and $\mathbf{Q} \in \mathbb{F}_p^{(2\ell+2)\times n_\mathcal{C}}$ such that $\mathbf{Q} = \mathbf{V} + \mathcal{C}(\mathbf{U}) \cdot \mathsf{diag}(\boldsymbol{\Delta})$
   - Set $\mathbf{Q}_1 = \mathbf{Q}_{[1..\ell+1]}$ and $\mathbf{Q}_2 = \mathbf{Q}_{[\ell+2..2\ell+2]}$.
   - Check that
   $$\boldsymbol{\eta}^\top(\mathbf{Q}_2 + \mathbf{Q}_1 \cdot \Delta') = \widetilde{\mathbf{v}} + \boldsymbol{\eta}^\top \cdot \mathcal{C}(\mathbf{S}) \cdot \mathsf{diag}(\boldsymbol{\Delta})$$

Theorem 3. The protocol $\Pi^t_{\text{2D-LC}}$ is a SHVZKPoK with soundness error $\frac{3}{p} + 2|S_\Delta|^{-d_\mathcal{C}}$ in the $\mathcal{F}_{\mathsf{sVOLE}}^{p,S_\Delta,\mathcal{C},2(\ell+2),\mathcal{L}}$-hybrid model

The Problem with LPN-based State-of-the-Art

Sender/$\mathcal{P}$ Receiver/$\mathcal{V}$

(k_0^1, k_1^1) $\Delta_B[1]$

... $\rightarrow \boxed{\mathcal{F}_{\mathsf{OT}}} \leftarrow$ ...

(k_0^ℓ, k_1^ℓ) $\Delta_B[\ell]$

The Problem with LPN-based State-of-the-Art

Sender/$\mathcal{P}$ Receiver/$\mathcal{V}$

$$\begin{array}{ccc} (k_0^1, k_1^1) & & \Delta_B[1] \\ \dots & \rightarrow \boxed{\mathcal{F}_{\mathsf{OT}}} \leftarrow & \dots \\ (k_0^\ell, k_1^\ell) & & \Delta_B[\ell] \end{array}$$

Extend (u)

$$m_1 := \mathsf{PRF}(k_0^1, j) + \mathsf{PRF}(k_1^1, j) + u$$

$$\dots$$

$$m_\ell := \mathsf{PRF}(k_0^\ell, j) + \mathsf{PRF}(k_1^\ell, j) + u$$

$$\longrightarrow$$

$$u\Delta_B[i] =$$

$$\underbrace{\mathsf{PRF}(k_0^i, j)}_{\text{Sender}} + \underbrace{\mathsf{PRF}(k_{\Delta_B[i]}^i, j) + m_i\Delta_B[i]}_{\text{Receiver}}$$

$\boxed{\text{Use LHL to remove selective failure leackage on } \Delta}$

The Problem with LPN-based State-of-the-Art

Sender/$\mathcal{P}$ Receiver/$\mathcal{V}$

$(k_0^1, k_1^1) \rightarrow \boxed{\mathcal{F}_{\mathsf{OT}}} \leftarrow \Delta_B[1]$

$\ldots \qquad\qquad \ldots$

$(k_0^\ell, k_1^\ell) \qquad\qquad \Delta_B[\ell]$

Extend (u)

$$m_1 := \mathsf{PRF}(k_0^1, j) + \mathsf{PRF}(k_1^1, j) + u$$

$$\ldots$$

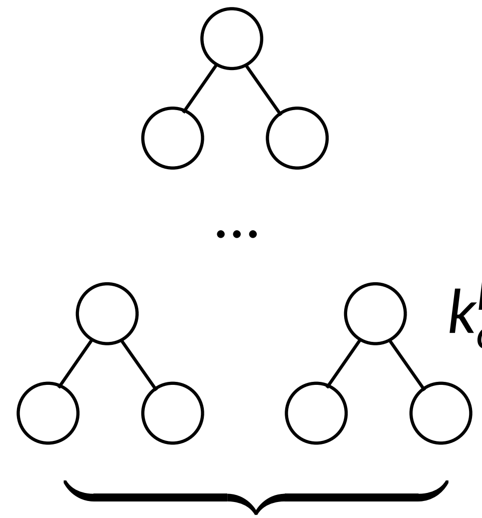
$$m_\ell := \mathsf{PRF}(k_0^\ell, j) + \mathsf{PRF}(k_1^\ell, j) + u$$

$$\longrightarrow$$

$$u\Delta_B[i] =$$

$$\underbrace{\mathsf{PRF}(k_0^i, j)}_{\text{Sender}} + \underbrace{\mathsf{PRF}(k_{\Delta_B[i]}^i, j) + m_i\Delta_B[i]}_{\text{Receiver}}$$

Receiver/$\mathcal{V}$ Sender/$\mathcal{P}$



$\mathbf{v}$

$$e \in \mathbb{F}_p^{2^h} \text{ s.t. } \begin{cases} e_\alpha = u \\ e_{\bar{\alpha}} = 0 \end{cases}$$

k_o^1, k_e^1

k_o^{h-1}, k_e^{h-1} $\boxed{\mathcal{F}_{\mathsf{OT}}}$ $\alpha_1, \ldots, \alpha_h$

k_o^h, k_e^h

$$m = \sum \mathbf{v}_i + v_u$$

$$\longrightarrow$$

Use LHL to remove selective failure leackage on Δ

The Problem with LPN-based State-of-the-Art

Sender/$\mathcal{P}$ Receiver/$\mathcal{V}$

$$(k_0^1, k_1^1), \dots, (k_0^\ell, k_1^\ell) \rightarrow \boxed{\mathcal{F}_{\mathsf{OT}}} \leftarrow \Delta_B[1], \dots, \Delta_B[\ell]$$

Extend (u)

$$m_1 := \mathsf{PRF}(k_0^1, j) + \mathsf{PRF}(k_1^1, j) + u$$

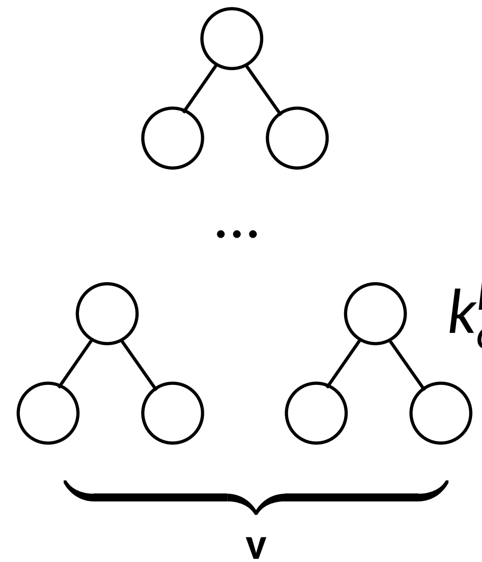
$$\dots$$

$$m_\ell := \mathsf{PRF}(k_0^\ell, j) + \mathsf{PRF}(k_1^\ell, j) + u$$

$$\longrightarrow$$

$$u\Delta_B[i] = \underbrace{\mathsf{PRF}(k_0^i, j)}_{\text{Sender}} + \underbrace{\mathsf{PRF}(k^i_{\Delta_B[i]}, j) + m_i\Delta_B[i]}_{\text{Receiver}}$$

Receiver/$\mathcal{V}$ Sender/$\mathcal{P}$



k_o^1, k_e^1

k_o^{h-1}, k_e^{h-1}

k_o^h, k_e^h

$$e \in \mathbb{F}_p^{2^h} \text{ s.t. } \begin{cases} e_\alpha = u \\ e_{\bar{\alpha}} = 0 \end{cases}$$

$$\boxed{\mathcal{F}_{\mathsf{OT}}} \; \alpha_1, \dots, \alpha_h$$

$$m = \sum \mathbf{v}_i + v_u \longrightarrow$$

Recover $\mathbf{b}_i, i \neq \alpha$

Recover $\mathbf{b}_\alpha$

Use LHL to remove selective failure leackage on Δ

The Problem with LPN-based State-of-the-Art

Sender/$\mathcal{P}$ Receiver/$\mathcal{V}$

(k_0^1, k_1^1) ... (k_0^ℓ, k_1^ℓ) $\rightarrow$ $\boxed{\mathcal{F}_{\mathrm{OT}}}$ $\leftarrow$ $\Delta_B[1]$ ... $\Delta_B[\ell]$

Extend (u)

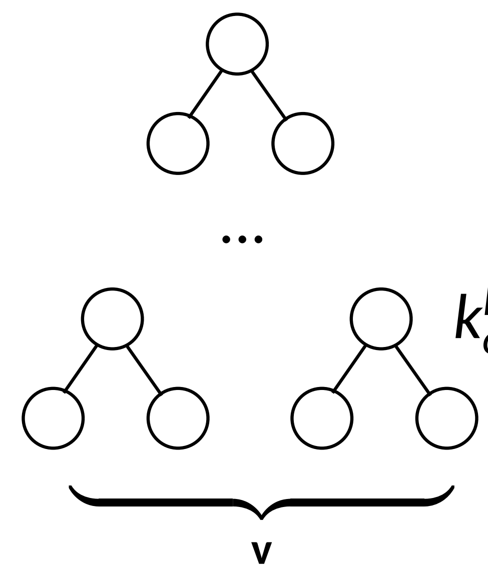
$$m_1 := \mathrm{PRF}(k_0^1, j) + \mathrm{PRF}(k_1^1, j) + u$$

...

$$m_\ell := \mathrm{PRF}(k_0^\ell, j) + \mathrm{PRF}(k_1^\ell, j) + u$$

$$u\Delta_B[i] = \underbrace{\mathrm{PRF}(k_0^i, j)}_{\text{Sender}} + \underbrace{\mathrm{PRF}(k^i_{\Delta_B[i]}, j) + m_i\Delta_B[i]}_{\text{Receiver}}$$

Receiver/$\mathcal{V}$ Sender/$\mathcal{P}$



k_o^1, k_e^1 ... k_o^{h-1}, k_e^{h-1} k_o^h, k_e^h

$$e \in \mathbb{F}_p^{2^h} \text{ s.t. } \begin{cases} e_\alpha = u \\ e_{\bar{\alpha}} = 0 \end{cases}$$

$\boxed{\mathcal{F}_{\mathrm{OT}}}$ $\alpha_1, \dots, \alpha_h$

$$m = \sum \mathbf{v}_i + v_u$$

Recover $\mathbf{b}_i, i \neq \alpha$

Recover $\mathbf{b}_\alpha$

- Use Multiple $\mathcal{F}_{\mathrm{spVOLE}}$ to get sparse **e**
- Use LPN* to expand to pseudorandom **u**

Use LHL to remove selective failure leackage on Δ

Com&Open doesn't work when $\mathcal{P}$ is OT receiver

FAEST Signature

- Apply FS transform to $\Pi^t_{\text{2D-LC}}$ scheme
- $\text{pk} = x, y \in \mathbb{F}_2^{128}$, $\text{sk} = k \in \mathbb{F}_2^{128}$
- Relation: $y = \text{Enc}_k(x)$
- For AES128, S-box is $\mathbb{F}_{2^8}$ inversion, so we can use 2D polynomial to express it

Theorem 5. *The Π_{FAEST} protocol, defined as*

$$\Pi_{\mathsf{FAEST}} = \mathsf{FS}^{H_{\mathsf{FS}}}[\mathsf{O2C}^{H_{\mathsf{O2C}}}[\Pi_{\textit{2D-Rep-OT}}]],$$

is a zero-knowledge non-interactive proof system in the CRS+RO model with knowledge error

$$2 \cdot (Q_{\mathsf{FS}} + Q_{\mathsf{Verify}}) \cdot \frac{2}{p^{r\tau}} + M \cdot (Q_{\mathsf{FS}} + Q_{\mathsf{Verify}}) \cdot \mathsf{AdvEB}^{\mathsf{VC}}_{\mathcal{A}'}[Q_{H_{\mathsf{O2C}}}] + \mathsf{AdvDist}^{\mathsf{VC.Setup},\mathsf{VC.TSetup}}_{\mathcal{D}},$$

where M is an upper bound on the number of VC *commitments sent during a run of* $\mathsf{O2C}[\Pi_{\textit{2D-Rep-OT}}]$.

Claimed Performance of FAEST

Scheme	$t_{\mathcal{P}}$ (ms)	$t_{\mathcal{V}}$ (ms)	|sign| (B)	Assumption
SDitH [FJR22b] (fast)	13.40	12.70	17 866	SD $\mathbb{F}_2$
SDitH [FJR22b] (short)	64.20	60.70	12 102	SD $\mathbb{F}_2$
SDitH [FJR22b] (fast)	6.40	5.90	12 115	SD $\mathbb{F}_{256}$
SDitH [FJR22b] (short)	29.50	27.10	8 481	SD $\mathbb{F}_{256}$
Rainier_3 [DKR+22]	2.96	2.92	6 176	RAIN_3
Rainier_4 [DKR+22]	3.47	3.42	6 816	RAIN_4
Limbo [dOT21] (fast)	2.61	2.25	23 264	Hash
Limbo [dOT21] (short)	24.51	21.82	13 316	Hash
SPHINCS+-SHA2 [HBD+22] (fast)	4.40	0.40	17 088	Hash
SPHINCS+-SHA2 [HBD+22] (short)	88.21	0.15	7 856	Hash
Falcon-512 [PFH+22]	0.11	0.02	666	Lattice
Dilithium2 [LDK+22]	0.07	0.03	2 420	Lattice
FAEST (**this work**, fast, $q = 2^8$)	2.28	2.11	6 583	Hash
FAEST (**this work**, short, $q = 2^{11}$)	11.05	10.18	5 559	Hash

Linear Combination Opening

- We can save the C-matrix communication if verifier only need to get a linear combination of the matrix B
- First P and V run Com/OT to get A, B', U'
- For a linear combination $\mathbf{r}$, P simply sends $\hat{c} := \mathbf{r}^T \cdot C \in \mathbb{F}_{2^\kappa}^{\tau}$ to the verifier
- Now the two parties can compute

$$\mathbf{r}^T \cdot B = \mathbf{r}^T \cdot A' + [0 \| \hat{c}] \cdot \mathrm{diag}(\Delta) + u \cdot [1\,1 \ldots 1] \cdot \mathrm{diag}(\Delta)$$

- Perform consistency check as usual after sending $\hat{c}$

SD-in-the-Head

- An alternative approach towards Hamming weight checking
- Let S encodes the noise $S(\gamma_i) = \phi(e_i)$ for $i \in [m]$
- Let Q encodes the non-zero positions