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Determination of an appropriate mother wavelet for de-noising of weak GPS correlation signals based on similarity measurements

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ABSTRACT

The Wavelet Transform (WT) is one of the most widely used tools for de-noising of Global Positioning System (GPS) signals which is a method for enhancing the sensitivity of GPS receivers in the acquisition of weak signals. The appropriate selection of mother wavelet which results in concentration of the major part of the power of a signal on a few numbers of wavelet coefficients is one of the most determinative factors in the efficiency of the de-noising process. Considering the importance of this issue, for the first time, in this paper, we introduce a quantitative method for selecting an appropriate mother wavelet for decomposing weak GPS correlation signals. Our proposed method uses the similarity measurements to choose the fittest mother wavelet. We use P, Q, and Magnitude-Shape (MS) criteria which are three individual remodeling factors, to evaluate the capability of different mother wavelets in the proper decomposition of weak GPS correlation signals. Eventually, the mother wavelet is selected based on our introduced Figure of Merit (FOM) which increases the reliability of the method. Our suggested method improves the sensitivity of GPS receivers in the acquisition of weak signals by enhancing the quality of the de-noising process using WT.

1. Introduction

De-noising by the means of the Wavelet Transform (WT) is one of the most efficient methods which are employed to improve the sensitivity of Global Positioning System (GPS) receivers in the acquisition of weak signals. According to this point, one of the taken approaches in the sensitivity enhancement of the acquisition stage in a GPS receiver is the quality improvement of the de-noising process. Identification and improvement of the effective factors in the efficiency of the de-noising process is a necessary prerequisite for achieving quality enhancement. In an efficient de-noising process using the WT, appropriate selection of the mother wavelet, optimum determination of the number of decomposition levels, efficient specification of the threshold, and using a proper thresholding method are the determinative factors. Considering this point, in this paper, we increase the quality of the de-noising process by suggesting a reliable quantitative method for proper determination of the mother wavelet. So, we can improve the sensitivity of a GPS receiver in the acquisition of weak signals by quality enhancement of the de-noising process.

Appropriate selection of the mother wavelet has a key role in concentrating the major part of the power of a signal in a few numbers of wavelet coefficients. Concentrating the power of the signal in a few numbers of wavelet coefficients makes it possible to easily separate signal and noise components by the means of thresholding. The more similarity between a mother wavelet and a specific signal results in a better decomposition of the signal's components into the wavelet coefficients. It is mentioned in [1] that there is no standard or general method to select a mother wavelet, and selection of the mother wavelet depends on the properties of the mother wavelet or the similarity between the mother wavelet and the signal. The research done in [1], classifies the employed methods for choosing the mother wavelet into two categories of the qualitative approaches and the quantitative approaches. In the qualitative approaches [2–6], some properties of the mother wavelets such as symmetry, regularity, vanishing moment, the degree of shift variance, orthogonality, compact support, and explicit expression are considered as a criterion to choose the mother wavelet, and the appropriate mother wavelet is determined by evaluating these properties. In some other researches [7–10] with the qualitative point of view, the similarity between the signal and the mother wavelet is evaluated by shape matching using visual inspection.

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In the quantitative approaches, the mother wavelet is selected based on some accurate mathematical methods. In the research done in [11], the measure of Minimum Description Length (MDL) is evaluated for appropriate mother wavelet determination. Based on the method proposed in [11], the MDL criterion selects the best mother wavelet by making the best compromise between fidelity of the estimation result to the data and the efficiency of the representation of the signal. The maximum cross-correlation coefficient is another employed criterion in [12–14] for choosing suitable mother wavelets. In [15], the similarity between mother wavelet and transient in a signal is evaluated based on Symmetric Distance Coefficient (SDC). This method is based on a fact that wavelet coefficients of a transient that has a similar shape to a mother wavelet are symmetric. Computation of maximum Compression Ratio (CR) is another technique which is suggested by [16] for selecting the most suited mother wavelet for decomposition of Vertical Ground Reaction Force (VGRF) signals. In another case, the research done in [17] introduces the Information Quality Ratio (IQR) as a new metric for mother wavelet selection. The suggestion of this metric is based on the idea that reconstructed signals have to maintain the essential information from original signals. Another research carried out in [18] chooses the mother wavelet using a combination of the Peak Signal to Noise Ratio (PSNR), the Mean Squared Error (MSE), and the max error criteria by assigning weights to each metric based on Analytic Hierarchy Process (AHP). In another method suggested by [19], remodeling factors are used for the determination of the mother wavelet. This research introduced P reconstruction criterion to measure the capability of various mother wavelets in appropriate decomposition of a signal into wavelet components. Getting along the path suggested by [19], in this paper, we employed three individual remodeling factors P, Q, and Magnitude-Shape (MS) in order to determine the most appropriate mother wavelet for decomposing GPS signals.

P is a criterion which assesses the similarity between the GPS signals and the de-noised reconstructed versions of them by the means of time domain parameters of the two groups of the signals. As mentioned in [19], evaluating the reconstruction capacity of different wavelet bases by the means of remodeling factors provides a measure of the capability of each basis in reconstructing the most similar signal to the original signal from its corresponding noisy signal. In addition, following the same guideline suggested in [19], we used two other frequency domain parameters assessment measures in order to evaluate the reconstructing capacity of different wavelet bases in reconstructing GPS signals. Q and MS are two individual similarity evaluation criteria which are suggested in [20,21], and they use frequency domain parameters of two signals for assessing their similarity. It is for the first time that these two measures are employed for choosing the appropriate mother wavelet for a specific signal.

Increasing the reliability of our proposed method is the reason for the employment of three individual criteria with three individual techniques of similarity assessment for mother wavelet selection. Eventually, for concluding the obtained results from individually employed criteria, we defined a Figure of Merit (FOM) to have a more accurate and reliable conclusion.

Our proposed method is the first quantitative method suggested for selecting an appropriate mother wavelet for decomposing GPS correlation signals.

The rest of this paper is organized as follows. In Section 2, the application of the WT for de-noising of weak GPS correlation signals is explained. Then, Section 3 provides our proposed method for determining the appropriate mother wavelet for de-noising GPS correlation signals. In Section 4, our proposed experimental framework is explained and obtained experimental results are presented. Finally, the conclusion of what has been done is provided in Section 5.

2. Application of the WT for de-noising of weak GPS correlation signals

Acquisition of weak GPS signals is a challenging issue in the positioning process. The goal of the acquisition stage is recognition of visible satellites and providing an appropriate estimation of Doppler shift and code phase. Estimating Doppler shift and code phase of a weak GPS signal is difficult for a typical receiver. So, in order to improve the capability of GPS receivers in acquiring weak signals, many methods have been proposed. De-noising using WT is one of the most efficient techniques suggested for this purpose. Various architectures are suggested for incorporating the de-noising block into the acquisition process. One of the most widely used architectures is a combination of the differential acquisition method and the wavelet de-noising block [22,23]. In this structure, the output of differential acquisition block which is known as GPS correlation signal, before being used in Doppler shift and code phase estimation is applied to the wavelet de-noising block for signal and noise isolation. The block diagram of this architecture is shown in Fig. 1.

As shown in Fig. 1, in this structure, the differential acquisition block is used for measuring the correlation of received GPS signals with the local carrier and C/A code. The differential acquisition is an efficient and widely used method for acquisition of GPS signals and complete explanations of this method are proposed in [24–29]. Based on the proposed instruction for differential acquisition method, GPS baseband signals which are specified by Eq. (1) after being multiplied by the local carrier for carrier demodulation are circularly correlated with C/A code in the frequency domain. This correlation is calculated by transferring the signals to the frequency domain using the Fast Fourier Transform (FFT). The resulting signal of this process is presented in Eq. (2).

$$S(t) = AC(t-\tau)D(t-\tau^{D})\cos(2\pi(f_{IF}+f_{Doppler})t+\varphi) \tag{1}$$

$$\begin{aligned} Z(k) &= FFT\left\{\sum_{m=0}^{N-1} c(-m)y(m-n)\right\} \\ &= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} c(-m)y(m-n)e^{-j2\pi kn/N} \\ &= \frac{1}{N}\sum_{m=0}^{N-1} c(m)e^{j2\pi km/N}\sum_{n=0}^{N-1} y(m+n)e^{-j2\pi k(m+n)/N} \\ &= \frac{1}{N}C^{*}(k)Y(k) \end{aligned} \tag{2}$$

In Eq. (1), A represents the signal amplitude, C is C/A spreading code, τ is code delay, D represents navigation data, τ^{D} is navigation data delay, and $f_{Doppler}$ is Doppler frequency shift.

In Eq. (2), c is C/A spreading code and y is the resulting signal of the multiplication of GPS baseband signal with the local carrier. Also, N represents the number of samples of each signal. As Eq. (2) is demonstrating the process of calculating the circular correlation of GPS baseband signals with local carrier and code, $Z(k)$ is the GPS correlation signal in the frequency domain. Applying inverse Fourier transform to $Z(k)$ and differentially integrating this signal in at least 2 consecutive time intervals using Eq. (3) results in GPS correlation signal.

$$Z_{sum}(n) = \sum_{i=2}^{M} |z_{i-1}(n).z_{i}(n)|, n = 0, 1, 2, ..., L-1 \tag{3}$$

In Eq. (3), $Z_{sum}(n)$ is representing GPS correlation signal and M is the number of consecutive time intervals in which differential integration is conducted. Also, in this equation L is the number of samples of the correlation signal.

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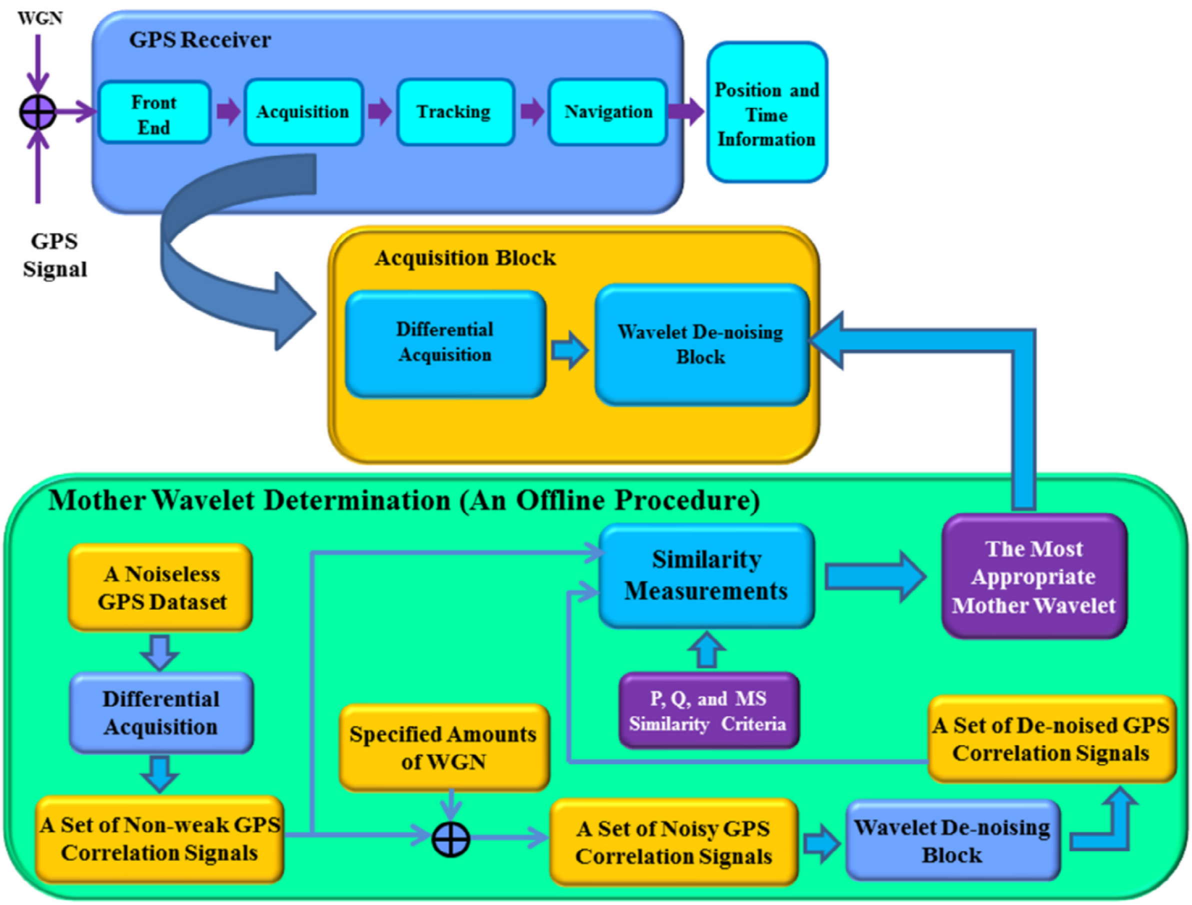

Fig. 1. The block diagram of the employed architecture for the acquisition process and the proposed mother wavelet determination method.

The wavelet de-noising block is an option which is joined to differential acquisition method to increase the overall efficiency. So, in this architecture, GPS correlation signals exported from differential acquisition block are our target signals for de-noising.

GPS received signals are a summation of original GPS signals and some other interference signals including White Gaussian Noise (WGN). In this paper, we consider the WGN as the only interference signal which influenced the received GPS signals.

As shown in Fig. 1, GPS signals which are summations of original GPS signals and Gaussian noise pass through differential acquisition block before gotten de-noised by the means of wavelet de-noising block. This transition may result in the change of Probability Density Function (PDF) of Gaussian noise. This point calls for more attention. Since most of the threshold determination methods are proposed to separate additive Gaussian noise from the signal, PDF of the input signal of the wavelet de-noising block is a determinative factor in the efficiency of the de-noising process. In [22], an investigation is done on the probability distribution of output signals of the differential acquisition block. Based on the results provided by [22], the mean value of the correlation power of the output signals of the differential acquisition method is zero and their probability distribution is similar to the distribution of Gaussian noise. It is mentioned in [22] that the probability density of the output noise of the differential acquisition block is not exactly fit the Gaussian distribution, but based on the mean value and the similarity in shape of both distribution functions, it can be assumed a Gaussian distribution. Therefore, the WT can be used as a de-noising tool for output signals of the differential acquisition method.

3. Determination of the appropriate mother wavelet for De-noising GPS correlation signals using reconstruction factors

Our proposed method in this paper for mother wavelet determination is based on reconstruction capacity of different wavelet bases. In this method, by adding Gaussian noise to GPS correlation signals, noisy GPS correlation signals are produced. These noisy signals are de-noised by the means of the WT. The method is repeated for all employed signals using different mother wavelets. In the next step, the similarity of the de-noised signals with the original GPS correlation signals is evaluated using our suggested similarity criteria. The maximum amount of similarity obtained by applying different mother wavelets determines the most appropriate mother wavelet for decomposing GPS correlation signals. Adding Gaussian noise to GPS correlation signals which are the outputs of the differential acquisition method is based on the obtained result of [22], in which the distribution of the output noise of the differential acquisition method can be assumed as Gaussian distribution. So, considering this result, our proposed method and obtained results are valid for output signals of the differential acquisition method. The block diagram of our proposed mother wavelet determination method is demonstrated in Fig. 1. In the following subsections, employed similarity criteria for implementation of our proposed mother wavelet determination method are introduced.

3.1. P Similarity criterion

P is a similarity criterion which is introduced in [19] and uses time domain parameters and properties of two signals to evaluate

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the similarity between them. This criterion, by measuring the local and global deviation of two signals from each other, assesses the amount of similarity between them. In order to achieve a comprehensive criterion based on time domain parameters, a weighted combination of the global and local deviations of two signals is provided and called P reconstruction factor. Based on the guideline proposed in [19], if we name the original signal S and the de-noised signal S_n, their local and global deviations can be calculated using Eqs. (4) and (5), respectively.

$$Local\_Deviation = \max(|S - S_n|) \tag{4}$$

$$Global\_Deviation = \sqrt{\frac{\sum_{i=1}^{L}(S_i - S_{n_i})^2}{L}} \tag{5}$$

In Eq. (5), L is representing the length of the signal. In order to achieve an overall measure of the deviation between two signals, a weighted combination of the global and local deviations is proposed in Eq. (6).

$$Overall\_Deviation = \frac{\left[rd_1 \times \sqrt{\frac{\sum_{i=1}^{L}(S_i - S_{n_i})^2}{L}} + rd_2 \times \max(|S - S_n|)\right]}{\sqrt{\frac{\sum_{i=1}^{L}(S_i - S_{n_i})^2}{L}}} \tag{6}$$

In Eq. (6), rd_1 and rd_2 are overall and local deviation factors used for weighting the global and local deviations of two signals. These coefficients must satisfy the conditions specified by Eq. (7).

$$\begin{cases} rd_1 \geqslant 0 \\ rd_2 \geqslant 0 \\ rd_1 + rd_2 = 1 \end{cases} \tag{7}$$

Eventually, the P similarity criteria can be represented as Eq. (8).

$$P = \frac{1}{Overall\_Deviation} \tag{8}$$

3.2. Q Similarity criterion

Another employed criterion for evaluating the similarity between two signals is introduced in [20]. This criterion which is called normalized error or Q similarity measure employs the frequency domain parameters of two signals to assess the amount of similarity between them. This criterion uses the concept of the norm in Sobolov space to propose a measure of similarity between two signals. For a function $f(t)$, the norm in Sobolov space is defined as Eq. (9).

$$\| f(.) \|^2 = \int_{-\infty}^{+\infty} |F(\omega)|^2 (1 + |\omega|^2)^k d\omega \tag{9}$$

In Eq. (9), $F(\omega)$ is the Fourier transform of $f(t)$, and the phrase $(1 + |\omega|^2)^k$ is a weighting factor for weighting higher frequencies. This factor depends on k. Since in the topic of similarity measurement there is no difference among different frequencies, the parameter k is set to 0 in order to have the same weights for different frequencies. So, Eq. (9) changes into Eq. (10) as follows.

$$\| f(.) \|^2 = \int_{-\infty}^{+\infty} |F(\omega)|^2 d\omega \tag{10}$$

It is mentioned in [20] that by the means of the concept of the norm in Sobolov space, the deviation between two signals can be specified using Eq. (11).

$$(Error_1)^2 = \| f_1(.) - f_2(.) \|^2 = \int |F_1(\omega) - F_2(\omega)|^2 d\omega \tag{11}$$

According to Eq. (11), the deviation of the two signals $f_1(t)$ and $f_2(t)$ from each other can be measured using their Fourier transforms $F_1(\omega)$ and $F_2(\omega)$. Since Eq. (11) represents the deviation between two signals, it can be used as a similarity criterion between them. In order to use the deviation specified by Eq. (11) as a similarity criterion, it should be defined in a manner that its provided amount of similarity does not change by scaling signals. So, by normalizing the deviation represented in Eq. (11), the normalized error or Q criterion of similarity is defined which it is represented in Eq. (12).

$$Q(f_1, f_2) = \frac{\| f_1(.) - f_2(.) \|}{\| f_1(.) \| + \| f_2(.) \|} = \frac{\left(\int |F_1(\omega) - F_2(\omega)|^2 d\omega\right)^{1/2}}{\left(\int |F_1(\omega)|^2 d\omega\right)^{1/2} + \left(\int |F_2(\omega)|^2 d\omega\right)^{1/2}} \tag{12}$$

According to Eq. (12), the maximum amount of similarity between the two signals is represented by 0 and the minimum amount is specified by 1.

3.3. MS similarity criterion

MS similarity measure is a criterion which is suggested by [21], and it uses frequency domain parameters of two signals to measure the similarity between them. Based on the method proposed in [21], the degree of similarity between two signals is evaluated by assuming a time-invariant systematic relationship between them. According to this method, if we assume the two signals which are the subjects of similarity analysis as input and output of a fictitious linear time-invariant system, the similarity between them can be evaluated by analyzing the magnitude and phase of the frequency response of this fictitious system. If $x(t)$ and $y(t)$ are the two deterministic signals which their similarity is being evaluated, and their Fourier transform representations are specified by Eqs. (13) and (14), the magnitude and phase of the frequency response of the fictitious system relating them to each other can be specified by Eqs. (15) and (16).

$$X(f) = |X(f)|e^{j\phi_X} \tag{13}$$

$$Y(f) = |Y(f)|e^{j\phi_Y} \tag{14}$$

$$|H(f)| = \left|\frac{Y(f)}{X(f)}\right| \tag{15}$$

$$\arg H(f) = \phi_y(f) - \phi_x(f) \tag{16}$$

The magnitude and phase functions specified by Eqs. (15) and (16) can be employed for introducing the Magnitude and Shape similarity functions.

For defining the Magnitude similarity function based on this method, the magnitude of the frequency response of the fictitious system is considered in the dB scale and it is represented by Eq. (17).

$$(|H(f)|)_{dB} = 20\log_{10}(|H(f)|) \tag{17}$$

According to Eq. (17), when the magnitudes of two signals in the frequency domain are equal, their degree of similarity obtained from Eq. (17) is 0 and this value increases as the difference in the magnitudes of the frequency responses of the two signals increases. Since for many of the suggested similarity measures, the degree of similarity between two signals is specified by a value

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which belongs to the range $[0, 1]$, a hyperbolic function which is represented in Eq. (18) is employed to map the proposed range of Eq. (17) into the range $[0, 1]$.

$$M_\alpha(f) = 1 - \tanh\left(\frac{\ln 3}{2} \cdot \frac{|20\log_{10}|H(f)||}{\alpha}\right) \tag{18}$$

In Eq. (18), α is a constant value which specifies the range of linear mapping. According to Eq. (18), the obtained Magnitude similarity measure $M_\alpha(f)$ is a function of frequency. The magnitude similarity can be specified with a singular value by averaging Eq. (18) over a specified frequency interval using Eq. (19).

$$M_i ndex = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} M_\alpha(f) df \tag{19}$$

In Eq. (19), f_1 and f_2 determine the range of intended frequency interval.

According to the method proposed in [21], the Shape similarity function specification is done using the phase function of the fictitious system. In this application, the group delay of the fictitious system is employed as a criterion to determine the amount of shape similarity between two signals. This parameter is represented in Eq. (20).

$$\tau(f) = -\frac{1}{2\pi} \frac{d\arg H(f)}{df} \tag{20}$$

According to Eq. (20), once the two signals exactly fit in shape, the value of the group delay function gets 0. Also, this value change into a constant non-zero amount in a case that there is a constant delay between two signals. It can be concluded from Eq. (20) that variant non-constant group delays are representatives of decrement in shape similarity of the two signals.

Following the same guideline as the Magnitude similarity function, for mapping the proposed range of the group delay function into the range $[0, 1]$, Eq. (21) is employed.

$$S_\beta(f) = 1 - \tanh\left(\frac{\ln 3}{2} \cdot \frac{|\tau(f)|}{\beta}\right) \tag{21}$$

In Eq. (21), β is a constant value which determines the linear mapping range. The Shape similarity function which is specified by Eq. (21) is a function of frequency. In order to obtain a singular value for the shape similarity in a specified range of frequency, same instruction as the one proceeded for the Magnitude similarity function in Eq. (19) can be followed. By averaging over a specified range of frequency using Eq. (22), a singular value as a measure of the shape similarity of the two signals can be obtained.

$$S_i ndex = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} S_\beta(f) df \tag{22}$$

To have an overall measure of the degree of similarity between two signals based on the magnitude and shape similarity of them, a linear combination of the two criteria introduced by Eqs. (19) and (22) can be presented by Eq. (23).

$$MS_i ndex = \omega_m.(M_i ndex) + \omega_s.(S_i ndex) \tag{23}$$

The coefficients ω_m and ω_s in Eq. (23) are weighting coefficients which are employed to specify the effectiveness of each of the magnitude and shape similarity criteria.

A summary of the characteristics of the employed similarity criteria is presented in Table 1.

4. Experimental framework and results

In this paper, we used the introduced similarity criteria in Sections 3.1 to 3.3 in order to measure the similarity between GPS correlation signals and their de-noised versions to choose the most appropriate mother wavelet for decomposing GPS correlation signals. In this way, we used the differential acquisition method to produce GPS correlation signals. In order to obtain accurate results and choose the most appropriate mother wavelet for decomposing GPS correlation signals, the collected correlation signals should not be weak and these signals should have as little interference as possible. Since the mother wavelet for GPS correlation signals is chosen based on the degree of similarity between original correlation signals and their de-noised versions, the precondition of non-weakness of employed correlation signals makes it possible that the chosen mother wavelet best fits the properties of the GPS correlation signals.

Table 1
A summary of the characteristics of the employed similarity criteria.

Similarity Criterion	P	Q	MS
Characteristics	Time Domain Analysis	Frequency Domain Analysis	Frequency Domain Analysis
	Local and Global Similarity Analysis	Similarity Analysis Based on the Concept of Norm in Sobolov Space	Similarity Analysis Based on the Group Delay and Magnitude Function of the Frequency Response of a Fictious System which is assumed between signals
	Maximum Similarity $\to \infty$	Maximum Similarity $\to 0$ Minimum Similarity $\to 1$	Maximum Similarity $\to 1$ Minimum Similarity $\to 0$

According to this point, in this paper, we used a non-weak GPS dataset in order to produce a set of non-weak GPS correlation signals by applying this dataset to the differential acquisition block. Intermediate Frequency (IF) of our used dataset is 3.02 MHz and it is sampled by the sampling frequency of 10 MHz. Also, the number of quantization bits in the employed dataset is 2 which resulted in 4 quantization levels. A part of this dataset is shown in Fig. 2.

Applying this dataset to the differential acquisition block results in providing GPS correlation signals of visible and invisible satellites. In this test, we used correlation signals of visible satellites. In Fig. 3, the result of the acquisition of 1 ms of the employed dataset is shown.

Fig. 3 is indicating the result of the acquisition of 1 ms of the employed dataset. In this figure, the vertical axis is representative of the acquisition metric which is the ratio of the first peak to the second peak in the corresponding correlation signal and the horizontal axis is specifying the PRN number of each satellite. As shown in Fig. 3, satellites number 27, 22, 21, 19, 18, and 14 are acquired by applying the differential acquisition method to 1 ms of the employed dataset. By applying the differential acquisition method to our dataset, we collected 7146 ms of correlation signals which belong to satellites number 27, 22, 21, 19, 18 and 14. A sample signal of the gathered collection which belongs to the satellite number 27 is shown in Fig. 4. The collected signals are GPS correlation signals which are entered to the de-noising block as inputs.

The next step is producing noisy signals by adding Gaussian noise to the collected GPS correlation signals. So, by adding Gaussian noise to our collection, we produced a set of noisy correlation signals. The reason for synthetically producing noisy signals is the vital need to preserve and have the corresponding original and noiseless signal of each noisy signal in order to be compared with de-noised versions of these noisy signals in similarity measurements of our proposed method. The added noise power was in an amount that the minimum SNR of the produced noisy signals decreased to 14 dB.

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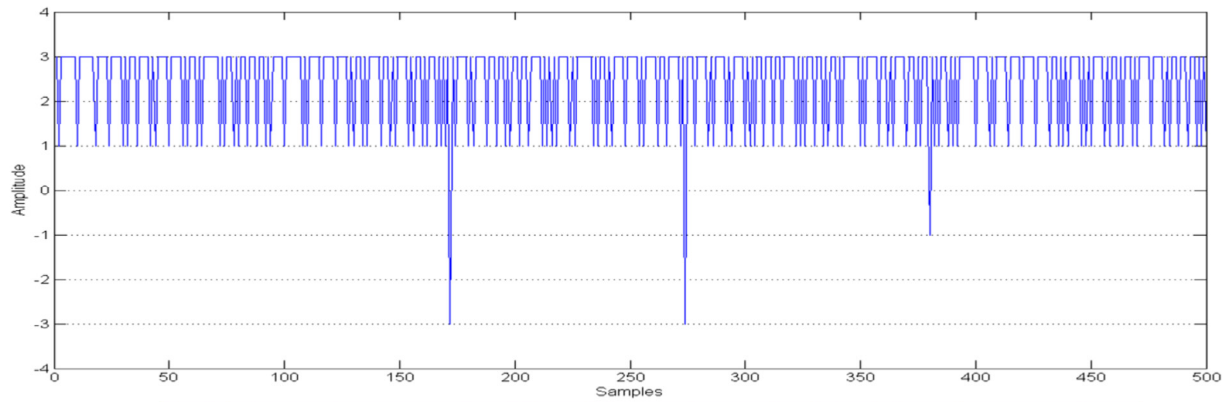

Fig. 2. A part of the non-weak employed dataset which is entered to differential acquisition block in order to produce GPS correlation signals.

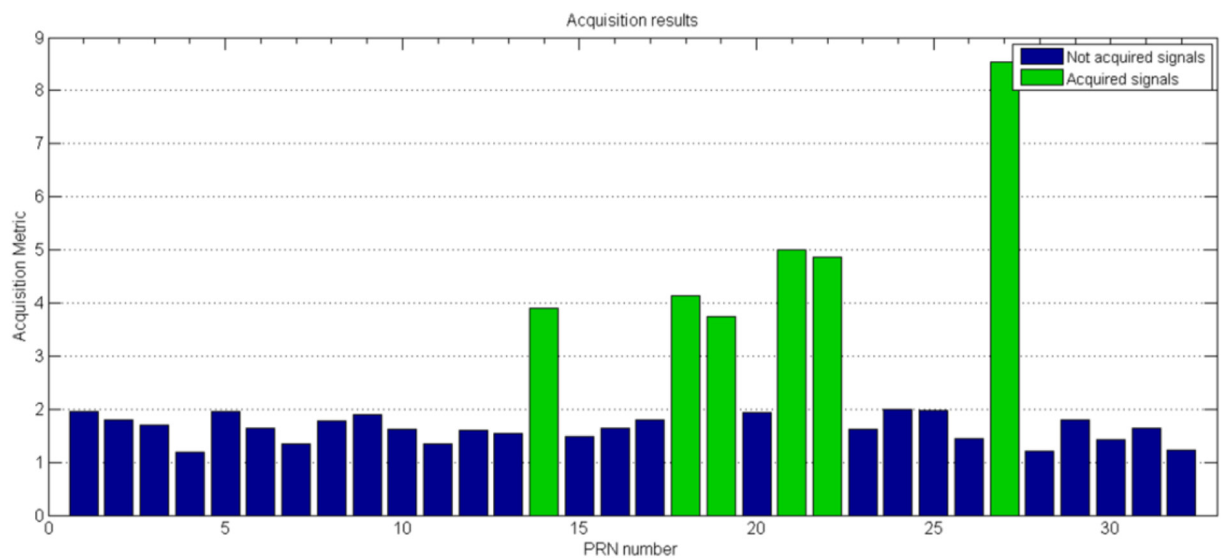

Fig. 3. The result of the acquisition of 1 ms of the employed dataset.

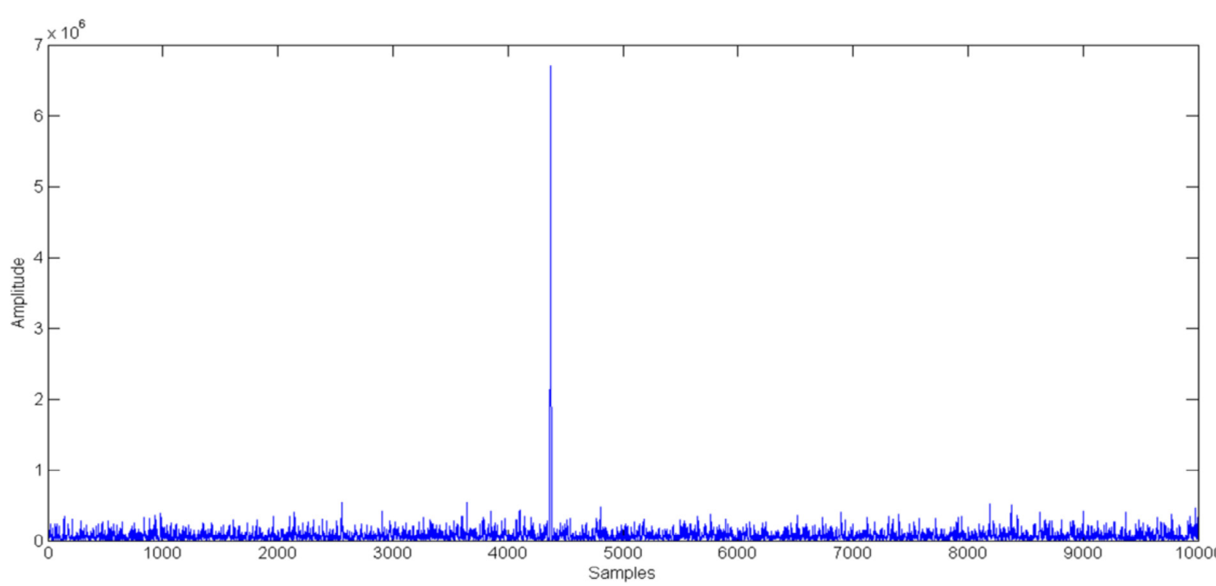

Fig. 4. A sample signal of the gathered collection of GPS correlation signals which belongs to the satellite number 27 and is used in the de-noising process.

Table 2
The best and worst obtained results of the employed similarity criteria and the calculated FOMs for mother wavelets from different families.

Number	Wavelet Family	Mother Wavelet	Averaged P	Averaged Q	Averaged MS	FOM
1	Daubechies	db1	0.30812	0.231359	0.909322	1.211019
45	Daubechies	db45	0.220675	0.274998	0.907838	0.728503
46	Symlet	sym2	0.287694	0.243771	0.909038	1.07283
63	Symlet	sym19	0.274987	0.244314	0.908596	1.02267
65	Coiflet	coif1	0.276798	0.246015	0.909157	1.022918
66	Coiflet	coif2	0.282745	0.243606	0.908921	1.054952
70	Reverse Biorthogonal	rbio1.1	0.30812	0.231359	0.909322	1.211019
77	Reverse Biorthogonal	rbio3.1	0.064698	0.833107	0.90667	0.070411
85	Biorthogonal	bior1.1	0.30812	0.231359	0.909322	1.211019
92	Biorthogonal	bior3.1	0.388512	0.566497	0.909544	0.623779

After producing noisy signals, the noisy produced signals were de-noised by the means of the WT. The de-noising process was done by employing different mother wavelets, but all other effective parameters in the quality of the de-noising process such as the number of decomposition levels, threshold selection method, and thresholding technique were kept unchanged in all iterations. So, the only effective parameter in the quality of the de-noising process was mother wavelet and its fitness to the properties of GPS correlation signals. In this way, we proposed a framework in which we set the number of decomposition levels to the maximum possible number in accordance to the length of the signal which it is obtained using Eq. (24), and we used the Sqtwolog well-known threshold determination method which is completely explained in [30,31] for threshold selection, and also we employed hard thresholding method to de-noise the noisy GPS correlation signals.

$$K_{\max} = \lfloor \log_2 N \rfloor \tag{24}$$

In Eq. (24), $K_{\max}$ determines the maximum number of decomposition levels and N is representing the length of the signal. According to the sampling frequency of the employed dataset, the number of samples of the collected correlation signals in 1 ms is 10000. Substituting the value of N in Eq. (24) results in the number of decomposition levels become 13.

In our proposed experimental framework, mother wavelets from various wavelet families have participated. Various wavelet families provide different trade-offs between the degree of compactness in the localization of basis functions in space and their smoothness [32]. Mother wavelets from various families differ in features such as the length of support of the mother wavelet, the speed of decaying of coefficients, symmetry, orthogonality, and biorthogonality [33]. More information on different wavelet families can be obtained in [33]. We used 99 different mother wavelets from 5 different families including Daubechies, Symlet, Coiflet, Biorthogonal and Reverse Biorthogonal for de-noising 7146 ms noisy GPS correlation signals.

In the next step of the test, the resulted de-noised signals obtained from applying each mother wavelet to 7146 ms noisy signals were evaluated by the means of employed P, Q, and MS similarity criteria, and the similarity of these de-noised signals with their corresponding original correlation signals was measured. Following these steps resulted in 7146 individual values for each similarity criterion and each experimented mother wavelet. To achieve a singular value for each similarity criterion which was measured for each mother wavelet, the average of the obtained results were calculated. As our employed similarity measures are three individual criteria which concentrate on individual properties of the signal, a FOM is defined to have an overall evaluation of the fitness of different mother wavelets. The defined FOM is represented in Eq. (25).

$$FOM = \frac{P \times MS}{Q} \tag{25}$$

Employing the FOM defined by Eq. (25), the FOM was calculated for each mother wavelet using the results obtained from the averaging process. The best and worst obtained results for mother wavelets from each family are gathered in Table 2.

The best and worst obtained results for mother wavelets from different families are presented in Table 2 which the best overall results have appeared in rows number 1, 70, and 85. Based on the presented results in Table 2, it can be concluded that db1, rbio1.1, and boir1.1 which are three equivalent mother wavelets with identical wavelet and scaling functions and actually represent Haar wavelet resulted in best outcomes in our experimental framework, and according to the time and frequency domain similarity evaluations done by the means of our employed similarity criteria, these mother wavelets are the most appropriate wavelet bases for decomposing GPS correlation signals.

Since our employed criteria evaluate local and global similarity in the time domain and magnitude and phase similarity in the frequency domain, it seems that our experimental framework provides a comprehensive evaluation of similarity between two signals, and the determined mother wavelets are selected based on a reliable quantitative method. Although achieving the highest accuracy was our goal in the proposed method, the obtained results show that in our case the selected mother wavelet was the simplest one too, which it was a further advantage for our application.

It seems to be necessary to emphasize that serious SNR deviations from the reported range of this research, may cause changes in results, but the suggested method is still valid. Therefore, in order to have a reasonable action, it is better to have approximate preset information about the approximate range of the SNR of the received GPS signals.

5. Conclusion

In this paper, we suggested an accurate quantitative method for selecting the most appropriate mother wavelet to be employed in the de-noising process of weak GPS correlation signals by the means of the WT. Our proposed method was a quantitative method which was based on similarity measurements of the original GPS correlation signals and their de-noised versions. In this way, we employed P, Q and MS individual similarity criteria which use different parameters from time and frequency domains to evaluate the degree of similarity between two signals. These criteria conducted a comprehensive evaluation of the similarity between two signals and provided a reliable framework for choosing the most appropriate mother wavelet for decomposing weak GPS correlation signals. In the proposed method, a FOM was defined that concluded the results obtained from individual similarity criteria. Our experimental results showed that db1, rbio1.1, and bior1.1 which actually represent the Haar wavelet were the best options for being employed in de-noising of weak GPS correlation signals using WT for signals with minimum SNR of 14 dB.

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