

# PDE Final Overview

- Understand Dimension & coordinates (1D, 2D, 3D, spherical, etc)
- Separation of Variables

Based on equation (only care about variables in derivatives)

Ex:  $U_{xx} + U_{yy} + U_{zz} = 0$

$\Rightarrow U = X(x) Y(y) Z(z)$

$-\frac{h^2}{2m} \psi_{xx} + V \psi = E \psi$

$\Rightarrow \psi = X(x) T(t)$

- You obtain as many ODEs as variables in derivatives

$U_{xx} + U_{yy} + U_{zz} = 0$  3 variables, 3 ODEs

$X'' Y Z + X Y'' Z + X Y Z'' = 0$

$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0$

- ODEs obtained come from equation type & coordinates

Usually: Cylindrical/Polar  $\Rightarrow$  Cauchy-Euler ODE

Spherical  $\Rightarrow$  Bessel ODE

- Boundary conditions give: Eigenvalues & Eigen Functions

- Initial conditions give: Orthogonality Condition

In general, after I.C.

all things that  $\rightarrow F(x) = \sum_{n=1}^{\infty} H_n P_n$

Function that will be subject to orthogonality

don't have summation

Constant to be found

- Take dot product with  $P_m$

$\int_a^b F(x) P_m dx = \int_a^b \sum_{n=1}^{\infty} H_n P_n P_m w(x) dx$

$w(x)$  depends on coordinates