Monotonic Elastic-Plastic for Power-Law
w/ Isotropic Hardening

$$f = \begin{cases} F - F_{yo}(1 + a\zeta)^n \\ -F - F_{yo}(1 + a\zeta)^n \end{cases} \quad w/ \quad F = AE(\varepsilon - \varepsilon^p)$$

$$\dot{f} = \frac{\partial f}{\partial \zeta}\frac{d\zeta}{dt} + \frac{\partial f}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial t} + \frac{\partial f}{\partial \varepsilon^p}\frac{\partial \varepsilon^p}{\partial t}$$

$$\dot{f} = \begin{cases} AE(\dot{\varepsilon} - \dot{\varepsilon}^p) - F_{yo}\, n(1 + \zeta)^{n-1} a\dot{\zeta} \\ -AE(\dot{\varepsilon} - \dot{\varepsilon}^p) - F_{yo}\, n(1 + a\zeta)^{n-1} a\dot{\zeta} \end{cases}$$

Using $\dot{\zeta} = F\dot{\varepsilon}^p$

$$\dot{f} = \begin{cases} AE(\dot{\varepsilon} - \dot{\varepsilon}^p) - F_{yo}\, n(1 + a\zeta)^{n-1} aF\dot{\varepsilon}^p \\ -AE(\dot{\varepsilon} - \dot{\varepsilon}^p) - F_{yo}\, n(1 + a\zeta)^{n-1} aF\dot{\varepsilon}^p \end{cases}$$

Taking first case, for plastic flow $\dot{f} = 0$

$$\dot{f} = 0 = AE(\dot{\varepsilon} - \dot{\varepsilon}^p) - F_{yo}(1 + a\zeta)^{n-1} aF\dot{\varepsilon}^p$$

Using $\dot{\varepsilon}^p = \beta\dot{\varepsilon}$

$$0 = AE\dot{\varepsilon} - AE\beta\dot{\varepsilon} - F_{yo}n(1 + a\zeta)^{n-1} aF\beta\dot{\varepsilon}$$

$$0 = \dot{\varepsilon}\left[AE - AE\beta - F_{yo}n(1 + a\zeta)^{n-1} aF\beta\right]$$

$\dot{\varepsilon} \neq 0$, Thus $\uparrow$ has to be zero

$$AE\beta + F_{yo}n(1 + a\zeta)^{n-1} aF\beta = AE$$

Dividing through by AE, solving for β

$$\boxed{\beta = \frac{1}{1 + \varepsilon_{yo}na(1 + a\zeta)^{n-1}|F|}}$$