Finite Elements

Rick Koster
Ruben Termaat

February 7, 2018

Contents

Chapter 1

1D-case

1.1 Assignment 1

For the domain of $x = [0, 1]$ the following formulas are given:

$$\begin{aligned} -D\triangle u + \lambda u = f(x), \\ -D\frac{du}{dx}(0) = 0, \\ -D\frac{du}{dx}(1) = 0 \end{aligned} \tag{1.1}$$

Here $\triangle$ equals the $\nabla \cdot \nabla$ operator. In order to find the Weakform of the given equations of (1.1), we first multiply both sides by ϕ and integrate both sides over the domain ω.

$$\int_\Omega \phi(-D\triangle u + \lambda u)d\Omega = \int_\Omega \phi f(x)d\Omega \tag{1.2}$$

Now by rewriting and then using partial integration the following equation can be found:

$$\int_\Omega (-D\nabla \cdot (\phi\nabla u) + D\nabla\phi\nabla u + \phi\lambda u)d\Omega = \int_\Omega \phi f(x)d\Omega \tag{1.3}$$

Applying Gauss on the first term:

$$\int_\Omega -D\vec{n} \cdot (\phi\nabla u)d\tau + \int_\Omega (D\nabla\phi \cdot \nabla u + \phi\lambda u)d\Omega = \int_\Omega \phi f(x)d\Omega \tag{1.4}$$

Using the boundary conditions from formula (1) we find that the integral over the boundary equals to 0 and we find the following Weakform:

$$\int_\Omega (D\nabla\phi \cdot \nabla u + \phi\lambda u)d\omega = \int_\Omega \phi f(x)d\Omega \tag{1.5}$$

1.2 Assignment 2

The next step is to apply the Galerkin equations to the found weakform, where u is replaced by $\sum_{j=1}^{n} c_i \phi_j$ and $\phi(x) = \phi(x)_i with i = [1, .., n]$.

$$\sum_{j=1}^{n} c_i \int_0^1 (D\nabla\phi_i \cdot \nabla\phi_j + \lambda\phi_i\phi_j) d\Omega = \int_0^1 \phi_i f(x) d\Omega \tag{1.6}$$

Which is of the form of $S\vec{c} = \vec{f}$

1.3 Assignment 3

1.4 Assignment 4

1.5 Assignment 5

Now the found Galerkin equations can be used to compute S_{ij} the element matric, over a generic line element e_i.

$$S\vec{c} = \sum_{j=1}^{n} c_i \int_0^1 (D\nabla\phi_i \cdot \nabla\phi_j + \lambda\phi_i\phi_j) d\Omega \tag{1.7}$$

Now to solve S we solve the following equation, over the internal line element.

$$S_{ij}^{e_i} = -D \int_{e_k} \nabla\phi_i \cdot \nabla\phi_j d\Omega + \lambda \int_{e_k} \phi_i\phi_j dx \tag{1.8}$$

1.6 Assignment 6

1.7 assignment 7

1.8 Assignment 8

Again using the found Galerkin Equations(1.6) in order to compute the element vector f_i over a generic line-element.

$$f_i^{e_n} = \int_{e_n} \phi_i f dx \tag{1.9}$$

$$f_i^{e_n} = \frac{|x_k - x_{k-1}|}{(1+1+0)!} f(\vec{x}) = \frac{|x_k - x_{k-1}|}{2} \begin{bmatrix} f_{k-1}^{e_n} \\ f_k^{e_n} \end{bmatrix} \tag{1.10}$$

1.9 Assignment 9

1.10 Assignment 10