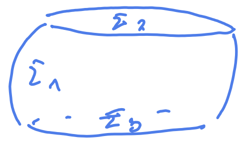
______________________ (Cognome) ______________________ (Nome) ____________ (Matricola)

Per passare è necessario realizzare almeno 8 punti sui 16 disponibili nei primi 2 esercizi e almeno 18 in totale

Esercizio 1. Si considerino il campo $F = (2xz, e^z + 4x^3, z + 2)$ e il solido $V = \{x^2 + y^2 + z^2 \leq 9, -2 \leq z \leq 2\}$

(a). [4p]Si calcoli il flusso totale di F uscente da V



$\operatorname{div} F = 2z + 1$

$$\Phi(F, V) = \iiint_V \operatorname{div} F \underset{\text{per simmetria}}{=} \iiint_V 1 \, dx\, dy\, dz = \int_{-2}^{2} \iint_{x^2+y^2 \leq 9 - z^2} dx\, dy \quad dz =$$

$$= \int_{-2}^{2} \pi (9 - z^2) = \pi \left[9 - z^2 \right]_{-2}^{2} = \pi \left(36 - \frac{16}{3} \right) = \frac{92}{3} \pi$$

(b). [4p]Si calcoli il flusso su ogni faccia di V

$$\Phi(F, \Sigma_2) = \iint_{\substack{x^2+y^2 \leq 5 \\ z=2}} F \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} d\sigma = \iint_{\substack{x^2+y^2 \leq 5 \\ z=2}} z + 2 \, d\sigma = \iint_{x^2+y^2 \leq 5} 4 \, dx\, dy = 20\pi$$

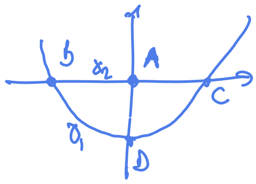
$$\Phi(F, \Sigma_3) = \iint_{\substack{x^2+y^2 \leq 5 \\ z=-5}} F \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -\iint_{\substack{x^2+y^2 \leq 5 \\ z=-2}} z + 2 = \iint_{x^2+y^2 \leq 5} 0 = 0$$

$$\Phi(F, \Sigma_1) = \pi \left(36 - \frac{16}{3} \right) - 20\pi = \pi \left(16 - \frac{16}{3} \right) = \frac{32}{3} \pi$$

Esercizio 2. Dato $D = \left\{ \frac{x^2}{8} - 8 \le y \le 0 \right\}$

(a). [5p]Trovare, se esistono, massimi e minimi di $f(x, y) = 4 - 2y + \frac{x^2}{3}$ su D

$\exists$ max e min per Weierstrass



$\nabla f = \begin{pmatrix} \frac{2}{3}x \\ -2 \end{pmatrix} \neq 0 \Rightarrow$ max e min $\in \partial D$

Su γ_2 $\quad f = 4 + \frac{x^2}{3}$ ha minimo in $x = 0$ e max per $x = \pm 8$

(da prov. anche pti singolari)

Su γ_1 $\quad f = 4 - \frac{x^2}{4} + 20 - \frac{x^2}{3} = 20 - \frac{7x^2}{12}$ $\quad$ max in $x = 0$, min in $x = \pm 8$

MIN $\rightarrow$ $f(A) = 4$ $\qquad f(D) = 20 \leftarrow$ MAX

$f(B) = f(C) = 4 + \frac{16}{3} = \frac{28}{3}$

(b). [3p]Calcolare $\iint_D y dx dy$

$$\iint_D y \, dx \, dy = \int_{-8}^{8} \int_{\frac{x^2}{8}-8}^{0} y \, dy \, dx = \int_{-8}^{8} \frac{1}{2} \left[\frac{x^2}{8} - 8 \right]^2 dx$$

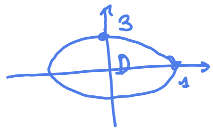
$$= \frac{1}{2} \int_{-8}^{8} \left(\frac{x^4}{64} - 2x^2 + 64 \right) dx = \frac{1}{2} \left[\frac{x^5}{5 \cdot 64} + \frac{2x^3}{3} + 64x \right]_{-8}^{8} = \frac{8^5}{5 \cdot 64} + \frac{2 \cdot 8^3}{3} + 64 \cdot 8 =$$

$$= 8^3 \left[\frac{1}{5} + \frac{2}{3} + 1 \right] = 8^3 \frac{3 + 10 + 15}{15} = \frac{8^3 \cdot 28}{15} = \frac{14336}{15}$$

Esercizio 3. Si consideri il campo vettoriale $F = \left(\overset{P}{e^{x^2} - y}, \overset{Q}{\frac{x^2}{4} + e^y}\right)$

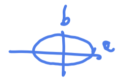
(a). [4p]Si usi il teorema di Gauss Green per determinare la circuitazione del campo lungo il bordo di $D = \{9x^2 + y^2 \leq 9\}$ percorso in senso antiorario

$$\int_{\partial^+ D} P\,dx + Q\,dy = \iint_D -P_y + Q_x \, dx\,dy = \iint_D -1 + \frac{x}{2}\, dx\,dy =$$



per simmetria

$$= \iint_D -1 \, dx\,dy = -\iint_D 1 \, dx\,dy = -3\pi$$

(Area ellisse $= \pi a b$ 

)

(b). [3p]Si calcoli il lavoro del campo lungo la curva $\gamma = (1, t)$, con $0 \leq t \leq 2\pi$.

$\dot\gamma = (0, 1)$

$$\int_\gamma F \cdot ds = \int_0^{2\pi} (e - t)\cdot 0 + \left(\frac{1}{4} + e^t\right) 1 \, dt = \int_0^{2\pi} \frac{1}{4} + e^t \, dt =$$

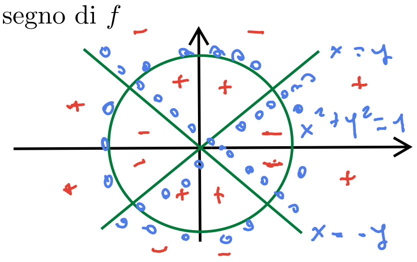
$$= \frac{\pi}{2} + \left[e^t\right]_0^{2\pi} = \frac{\pi}{2} + \left(e^{2\pi} - 1\right)$$

$(x-y)(x+y)(x^2+y^2-1)$

$\shortparallel$

Esercizio 4. Si consideri la funzione $f(x,y) = (x^2-y^2)(x^2+y^2-1)$

[2p]Si studi il segno di f



[5p]Si studino e si classifichino i punti critici di f

$f = x^4 - x^2 - y^4 + y^2$ $\qquad \nabla f = \begin{pmatrix} 4x^3 - 2x \\ 2y - 4y^3 \end{pmatrix} = 0 \iff \begin{cases} 2x(2x^2-1) = 0 \\ 2y(1-2y^2) = 0 \end{cases}$

pt. critici : $(0,0)$, $\left(0, \pm\frac{\sqrt{2}}{2}\right)$, $\left(\pm\frac{\sqrt{2}}{2}, 0\right)$, $\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$

$H_f = \begin{pmatrix} 12x^2-2 & 0 \\ 0 & 2-12y^2 \end{pmatrix}$ $\qquad (0,0)$ sella (dal segno)

$H_f\left(0, \pm\frac{\sqrt{2}}{2}\right) = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix}$ max $\qquad H_f\left(\pm\frac{\sqrt{2}}{2}, 0\right) = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$ min

$H_f\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right) = \begin{pmatrix} 6 & 0 \\ & -4 \end{pmatrix}$ sella

[2p]Si scriva l'equazione del piano tangente al grafico di f nel punto $(1,0)$

$\nabla f(1,0) = (2,0)$

Piano tg $\qquad z = f_x(0,1)\cdot(x-1) + f_y(0,1)(y-0) = 2(x-1) + 0$

$\boxed{z = 2x-2}$