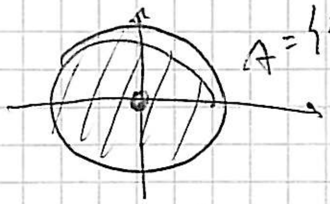
(c)



$A = \{x^2+y^2 = \frac{1}{9}\}$

l'unico pto critico interno è $(0,0)$

Al bordo $f(x,y) = (x^2+y^2)^2 - \frac{1}{4}(x^2+y^2) - \frac{3}{4}x^2 = \left(\frac{1}{9}\right)^2 - \frac{1}{4}\left(\frac{1}{9}\right) - \frac{3}{4}x^2$

$= \frac{1}{81} - \frac{1}{4}\left(\frac{1}{9}\right) - \frac{3}{4}x^2 = \frac{1}{4}\left(\frac{4}{81} - \frac{1}{9} - \frac{3}{4}x^2\right) = \frac{1}{4}\left(-\frac{5}{81} - \frac{3}{4}x^2\right)$

sul bordo

Max se $x=0$ $y = \pm\frac{1}{3}$ $f(0, \pm\frac{1}{3}) = \frac{1}{4}\left(-\frac{5}{81}\right)$

Min se $x = \pm\frac{1}{3}$ $y=0$ $f(\pm\frac{1}{3}, 0) = \frac{1}{4}\left(-\frac{5}{81} - \frac{3}{4}\right)$

$f(0,0) = 0 \Rightarrow \max_A f = 0$ in $(0,0)$

$\min_A f = -\frac{34}{324}$ in $(\pm\frac{1}{3}, 0)$

d) $\sup_{\mathbb{R}^2} f$ $\quad f(x,0) = x^4 - x^2 \xrightarrow{x\to+\infty} +\infty$

$\Rightarrow \sup_{\mathbb{R}^2} f = +\infty$

e) $\lim_{(x^2+y^2)\to\infty} f(x,y) \to \lim_{\rho\to\infty} (\rho^2)^2 - \rho^2 + \frac{3}{4}\rho^2\sin^2\theta =$

$= \lim_{\rho\to\infty} \rho^4 - \rho^2 + \frac{3}{4}\rho^2\sin^2\theta = +\infty$

f è cont e $f \xrightarrow{x^2+y^2\to\infty} +\infty$ $\Rightarrow$ f ha un min assoluto

$\Rightarrow$ è limitata dal basso