

ES 1

$$f(x, y) = (x^2 + y^2)^2 - x^2 - \frac{1}{2} y^2$$

$$a) \nabla f = \begin{pmatrix} 2x(x^2 + y^2) - 2x \\ 4y(x^2 + y^2) - \frac{1}{2}y \end{pmatrix} = \begin{pmatrix} 4x^3 + 4xy^2 - 2x \\ 4yx^2 + 4y^3 - \frac{1}{2}y \end{pmatrix}$$

$$\nabla f \in \mathbb{C}' \Rightarrow \text{d}f_{(x,y)} \left[ \begin{pmatrix} h \\ k \end{pmatrix} \right] = (4x^3 + 4xy^2 - 2x)h + (4x^2y + 4y^3 - \frac{1}{2}y)k \in \text{d}f_{(x,y)} \in \mathbb{R}^2$$

$$b) \nabla f = \begin{cases} 2x(2x^2 + 2y^2 - 1) = 0 \\ y(4x^2 + 4y^2 - \frac{1}{2}) = 0 \end{cases} \quad \textcircled{J} \quad \begin{cases} x = 0 \\ y(\frac{4}{2} - \frac{1}{2}) = 0 \end{cases}$$

$$A \begin{cases} x = 0 \\ y = 0 \end{cases} \quad B \begin{cases} x = 0 \\ y = \pm \frac{\sqrt{2}}{2} \end{cases} \quad C \begin{cases} x^2 + y^2 = \frac{1}{2} \\ y(4 \cdot \frac{1}{2} - \frac{1}{2}) = 0 \end{cases} \quad D, E \begin{cases} x = \pm \frac{\sqrt{2}}{2} \\ y = 0 \end{cases}$$

$$A = (0, 0) \quad B = (0, \frac{\sqrt{2}}{2}) \quad C = (0, -\frac{\sqrt{2}}{2}) \quad D = (\frac{\sqrt{2}}{2}, 0) \quad E = (-\frac{\sqrt{2}}{2}, 0)$$

$$H_f = \begin{pmatrix} 12x^2 + 4y^2 - 2 & 8xy \\ 8xy & 4x^2 + 12y^2 - \frac{1}{2} \end{pmatrix}$$

$$|H_f(0,0)| = 1 \quad f_{xx}(0,0) = -2 \Rightarrow (0,0) \text{ max}$$

$$|H_f(0, \pm \frac{\sqrt{2}}{2})| = \begin{vmatrix} \frac{1}{2} - 2 & 0 \\ 0 & \frac{3}{2} - \frac{1}{2} \end{vmatrix} = \begin{vmatrix} -\frac{3}{2} & 0 \\ 0 & 1 \end{vmatrix} \quad \text{alle}$$

$$|H_f(\pm \frac{\sqrt{2}}{2}, 0)| = \begin{vmatrix} 6 - 2 & 0 \\ 0 & 2 - \frac{1}{2} \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 0 & \frac{3}{2} \end{vmatrix} \quad \text{min}$$