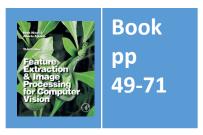
Lecture 3 Image Sampling

COMP3204 & COMP6223 Computer Vision

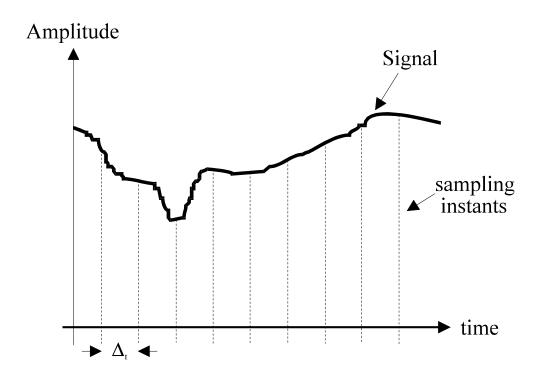
How is an image sampled and what does it imply?



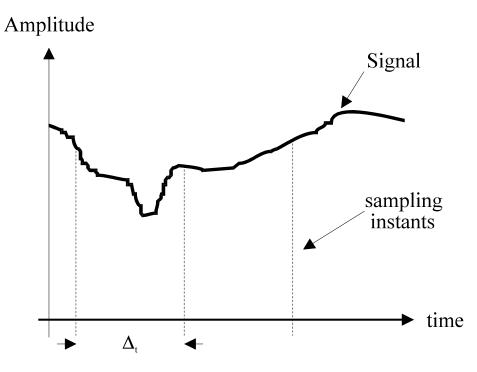
Department of Electronics and Computer Science



Sampling at Different Frequencies



(a) sampling at high frequency



(b) sampling at low frequency





Aliasing in Sampled Imagery





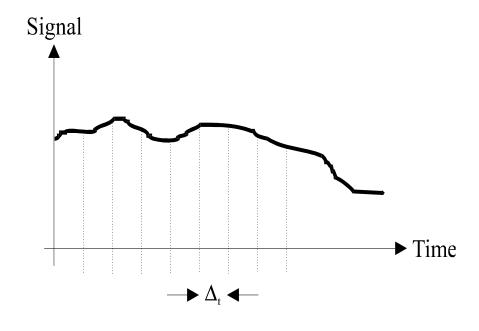






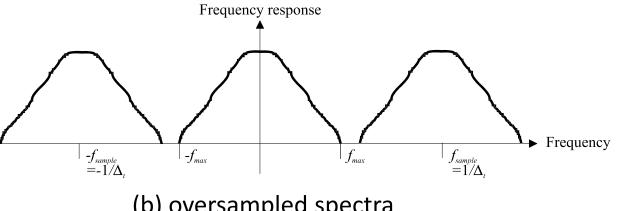
(c) low resolution – aliased

Sampled Spectra

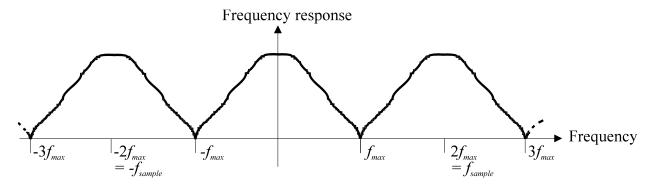


(a) sampled signal

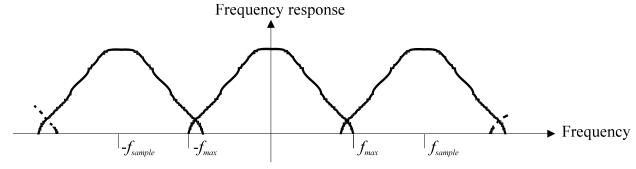






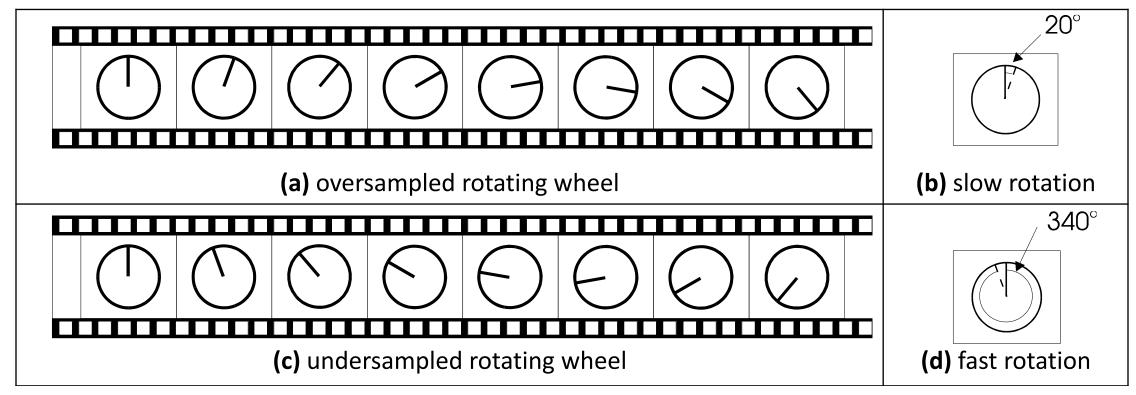


(c) sampling at the Nyquist rate



(d) undersampled, aliased, spectra

Correct and Incorrect Apparent Wheel Motion



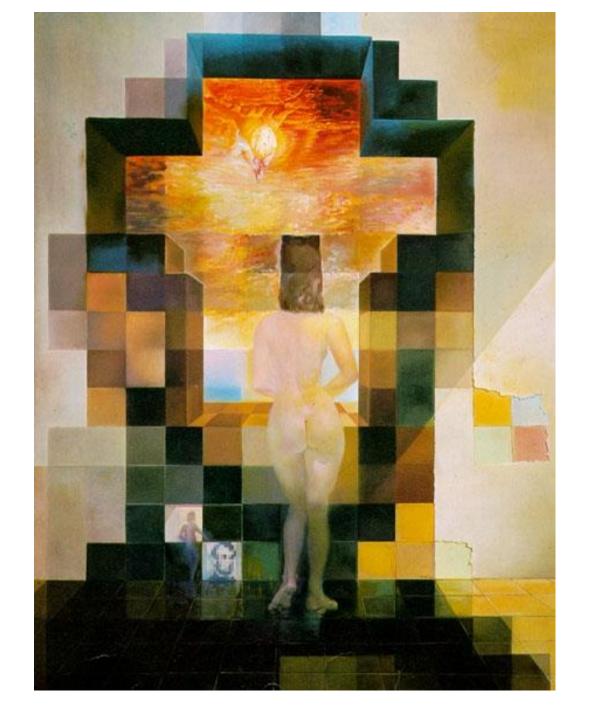




Sampling theory

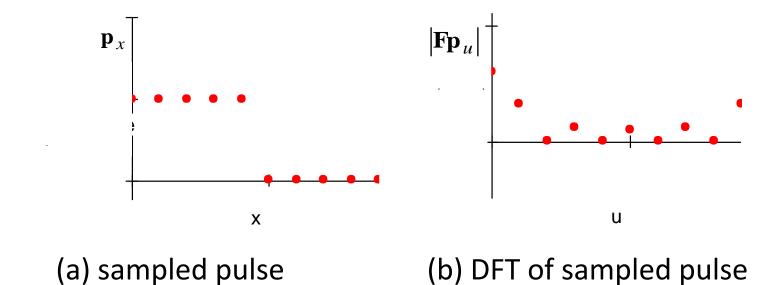
- Nyquist's sampling theorem is 1D
- E.g. speech 6kHz, sample at 12 kHz
- Video bandwidth (CCIR) is 5MHz
- Sampling at 10MHz gave 576×576 images
- Guideline: "two pixels for every pixel of interest"



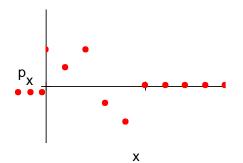


https://www.pinterest. com/pin/27542333343 1517864/

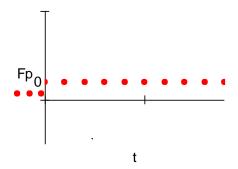
Transform Pair for Sampled Pulse



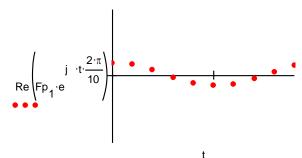


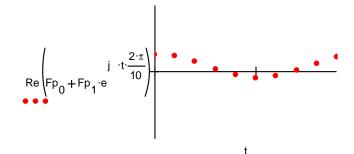


(a) original sampled signal

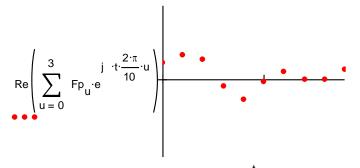


(c) second coefficient Fp₁

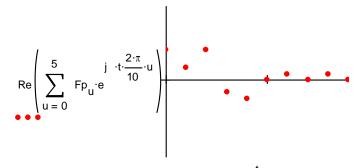




(b) first coefficient Fp₀



(d) adding Fp₁ and Fp₀



(f) adding all six frequency components

e) adding Fp₀, Fp₁, Fp₂ and Fp₃ (f) adding all six frequency compon Signal Reconstruction from Transform Components

2D Fourier transform

Forward transform
$$\mathbf{F}\mathbf{P}_{u,v} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \mathbf{P}_{x,y} e^{-j\left(\frac{2\pi}{N}\right)(ux+vy)}$$

Where two dimensions of space, x and y two dimensions of frequency, u and v image NxN pixels $\mathbf{P}_{x,v}$

Inverse transform

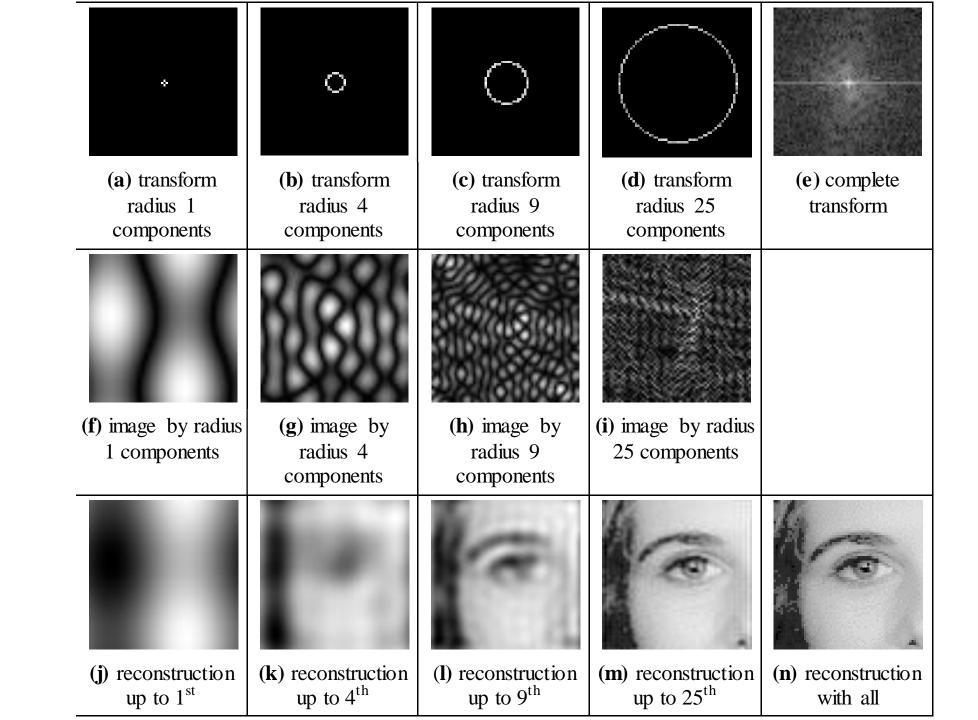
$$\mathbf{P}_{x,y} = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \mathbf{F} \mathbf{P}_{u,v} e^{j\left(\frac{2\pi}{N}\right)(ux+vy)}$$





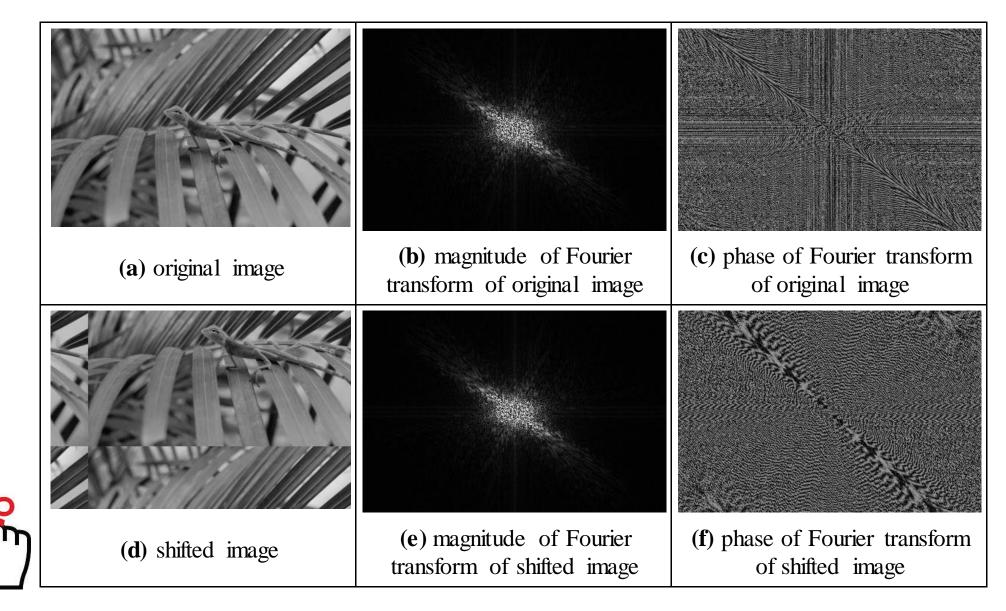
π??







Shift invariance



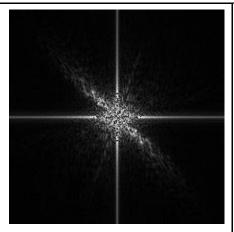
Rotation



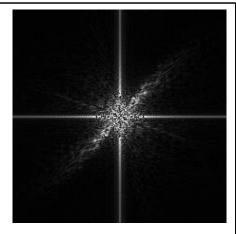
(a) original image



(b) rotated image



(c) transform of original image



(d) transform of rotated image

$$\mathbf{FP}_{u,v} = \frac{1}{N} \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} \mathbf{P}_{x,y} e^{-j\left(\frac{2\pi}{N}\right)(uy+vx)}$$

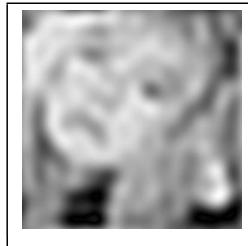


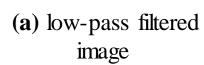


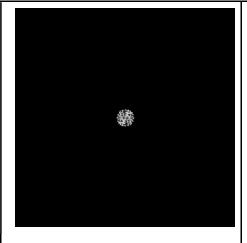
Filtering

Fourier gives access to frequency components





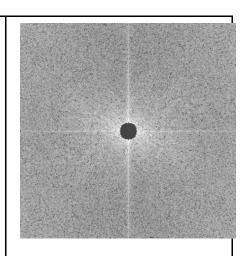




(b) low-pass filtered transform



(c) high-pass filtered image



(d) high-pass filtered transform

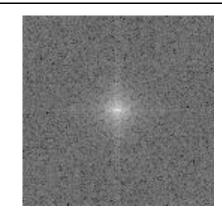




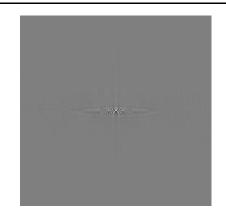
Other transforms

• For Lena

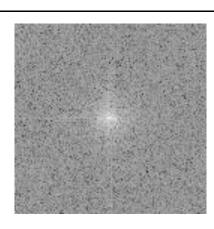








(b) discrete cosine transform



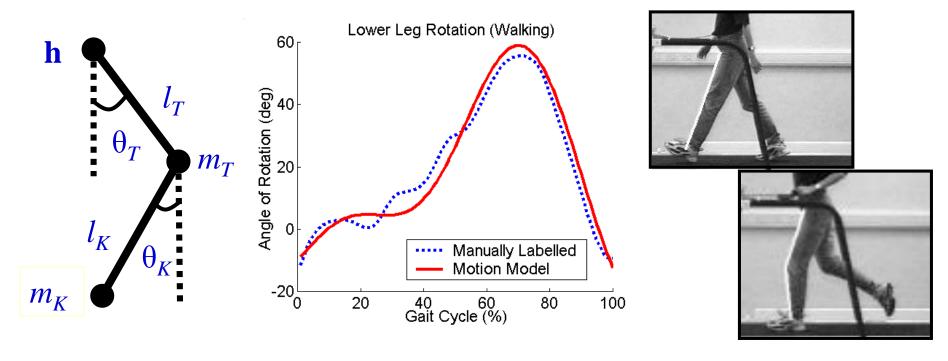
(c) Hartley transform



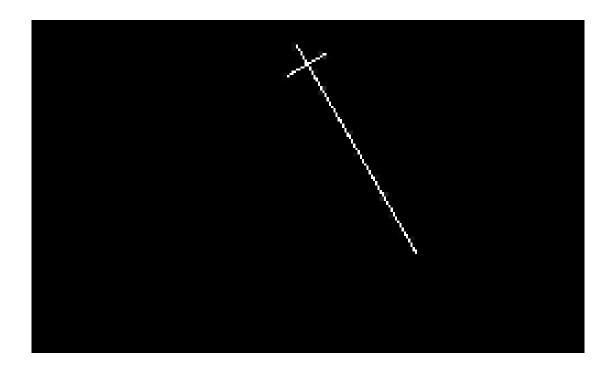


Modelling Gait(s)

- Extended pendular thigh-model, based on angles
- Uses forced oscillator/ bilateral symmetry/ phase coupling

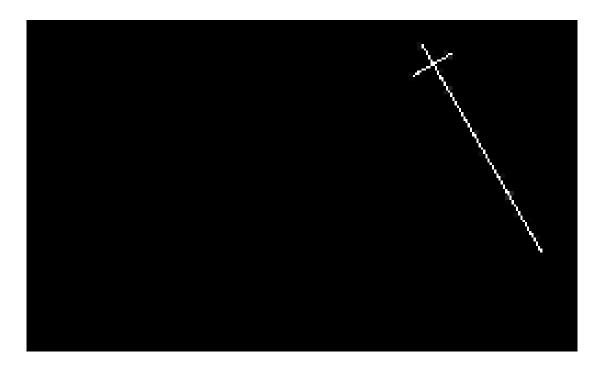


Modeling the Thigh's Motion 1



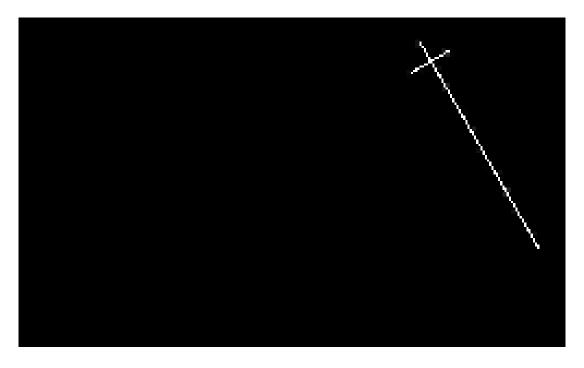
$$vs_x(t) = A\cos(\omega t + \phi)$$

Modeling the Thigh's Motion 2



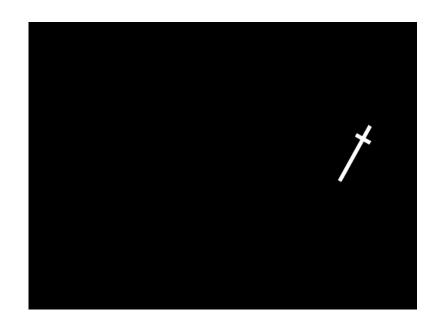
$$vh_x(t) = Vx + A\cos(\omega t + \phi)$$

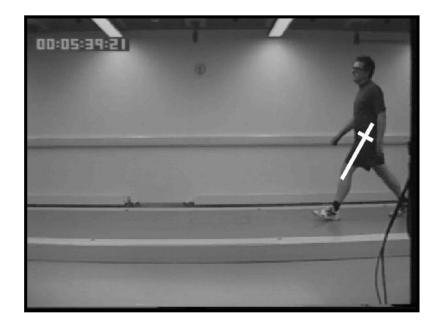
Modeling the Thigh's Motion 3



$$\phi(t) = a_0 + \sum_{k=1}^{N} \left[b_k \cos(k\omega_0 t + \psi) \right]$$

Validity?





Applications of 2D FT

- Understanding and analysis
- Speeding up algorithms
- Representation (invariance)
- Coding
- Recognition/ understanding (e.g. texture)

