Problem Chosen	2022	Team Control Number
B	MCM/ICM Summary Sheet	2226802

Smart Distribution of Water and Hydroelectric Power

Summary

People construct dams on rivers to store water and build reservoirs to manage water resources. The availability of water for general usage and electricity generation has been an outstanding feature of modern life. We built models to address water allocation plans in five US states (Arizona, California, Wyoming, New Mexico, and Colorado).

We have established a **Water and Hydropower Distribution Model**, based on supply and demand conditions, to make allocation plans to satisfy individual state water and hydropower needs. First of all, we used **water and hydroelectric balance equation** to calculate the optimal allocation between the two dams. It shows that the overall water and hydroelectric efficiency is highest when the Loren Canyon Dam delivers $1 \times 10^9 m^3$ of water to the Hoover Dam. By collecting data, we have made detailed statistics on the distance from dams, terrain complexity, economic development, etc. in each state, and then determined the evaluation indicators. Next, we used a **Multi-Objective Optimization** method to determine the optimal water allocation plan. The model can create more electricity while meeting general water needs and reduce supply costs. According to the results of this model, we calculated that it takes 26.5 days to meet the demand when one month is the specified cycle through the relationship between the scheduling flow and the flow rate. Over time, an additional $6 \times 10^9 m^3$ of water needs to be replenished each month to ensure that demand is met.

We used the **Analytic Hierarchy Process(AHP)** to build a model to make decisions about the **conflict** between general usage and power generation of water. Thinking over resource scarcity, ecological environment, etc., we determined the hierarchical structure. By establishing judgment matrix, we finally got the weight of 3 schemes: water supply priority, power supply priority, simultaneous water and electricity supply which is [0.560, 0.212, 0.228]. Thus, the optimal scheme is to give priority to water supply. In the case of **insufficient water resources**, we applied GDP, water scarcity and water demand as 3 measurements to build a priority of five states' water supply through the **TOPSIS algorithm**. With the help of the fusion assignment method, we calculated the result which shows AZ, CA, WY, NM, CO as [0.3164, 0.7715, 0.0362, 0.3035, 0.3173] to maximize efficiency. Additionally, we set up a plan that guarantees Mexico's rights to water claims and calculated that $1.692 \times 10^9 m^3$ of water would flow from the Colorado River into the Gulf of California after the allocation was implemented.

Then we performed a sensitivity analysis of the model and provided evidence. We separately considered the effects of changes in demand, increased renewable energy, and economical policy to demonstrate the stability and reliability of the model. Finally, we analyzed the advantages and disadvantages of the model. We also wrote a two-page article based on the proposed model for managers to make decisions.

Keywords: Multi-Objective Optimization, Analytic Hierarchy Process(AHP), TOPSIS

Contents

1 Introduction

A dam is a barrier that stops or restricts the flow of surface water or underground streams. Reservoirs created by dams not only suppress floods but also provide water for activities such as irrigation, human consumption, industrial use, aquaculture, and navigability. Hydropower is often used in conjunction with dams to generate electricity.

Arizona, California, Wyoming, New Mexico, and Colorado need to determine the best way to manage water use and electricity production at Glen Canyon and Hoover Dam. We are responsible for providing a practical and efficient solution to this problem.

2 Problem Statement

Our model was developed to develop resource allocation scenarios for water and electricity from the Glen Canyon Dam and Hoover Dam to five states. We need to complete the following four questions.

- **Distribution of water and electricity.** We established a water and hydropower allocation model to meet individual state water and hydropower needs and equitable allocation. The water and electricity resources of the two dams should be allocated to the five states for use.

  The model needs to account for the link between the two dams, remaining water and Mexico's water debt.

- **Conflict between water for general usage and electricity production.** We need to resolve the question of who has the higher priority in the event of a conflict. We need to comprehensively consider various factors to measure the social benefits of both to make decisions.

- **Water shortage problem.** If there is not enough water to meet the demand, we need to consider two aspects. On the one hand, it is to find other alternative resources, and on the other hand, it is to reduce the supply of water sources and set supply priorities. The second aspect is the problem we should be solving in terms of the model.

- **The problem of changing needs.** Over time, water needs and indicators change in different regions.What may happen is the growth and decline of population, agriculture and industry, the increase in the proportion of renewable energy, the implementation of additional water and electricity saving measures, etc. We need to judge the impact of these changes in demand.

3 Assumptions and Notations

3.1 Assumptions

We make some general assumptions to simplify our model. These assumptions together with corresponding justification are listed below:

- **We assume fixed water and hydropower needs across the five states.** Using fixed requirements facilitates more intuitive description and modeling of resource allocation. Our data is based on the actual situation through statistical website survey, which can guarantee the scientific accuracy of our results.

- **We assume separate demand analysis and supply analysis in water resources and hydropower.** Demand is processed according to statistics, with total supply allocated to five states.Supply is divided into water and hydroelectric water. Water is water drawn from reservoirs for general use. Hydroelectric water is the amount of water that is drawn downstream.

  Glen's water for power generation is included in Hoover's total. For each reservoir, First, the water volume is calculated from the water level, and the extracted water volume is divided into water for general use and water for hydropower generation.

- **The model runs for a period of one month.** Demand is measured by water and electricity usage in five states over the period. The measure of supply is derived from factors such as actual water levels and ecological water levels during the cycle.

  After comprehensive consideration, we set the model period to be one month. The model assumes no additional water supply, so it is reasonable to estimate demand and supply on a monthly basis considering the actual situation.

- **Simplified indexes for supply costs.** We approximate the distance from the dam from the geometric center of each state as the average transport distance. This both accounts for differences in reality and simplifies the model.

3.2 Notations

Symbol	Description
Qg_i	Monthly average water supply of the dams
V_i	Initial storage capacity of the dams before water delivery
Vs_i	Storage capacity of the dams
V_i'	Final capacity of the dams after delivery and warehousing
QR_i	Monthly average inflow of the dams
QC_i	Average monthly outflow of the dams
Q_i	Dams power generation flow
N_{min}	Lower water levels of the dams that can generate electricity
$x_{i,j}$	Water quantity transfer from dams to j state
$L_{i,j}$	Distance from the dams to j state
$H_{i,j}$	altitude difference from the dams to j state
SC_j	Scarcity degree of water resources of j state
v_i	Transferable water quantity of the dams
Wat_j	Water demand of j state
E_i	Entropy
w_j	Weight of indicators
S	Euclidean distance in TOPSIS

The rest of the variables will be marked in the specific question.

4 Distribution of Water and Electricity Model

4.1 Background

We have established a hydropower distribution model to satisfy the hydropower demand and equitable distribution of various states. The water and electricity of the two dams are supposed to be allocated to five states. The model analyzes not only the level of demand for water and electricity in five states, but the supply quantity of two dams. The core of the model measures the supply cost by reduction dimension. Then we uesd the Multi-Objective Optimization method to minimize the total supply cost and alleviate water scarcity in 5 states, so as to obtain the allocation scheme.

4.2 Demand Analysis

To determine the demand for reservoir water and dam hydropower in five states, we looked for agricultural, industrial, and residential water and electricity usage in five states. Considering that each state's water and electricity consumption is not entirely dependent on reservoir water resources and dam hydropower, we further searched for information on the state's water sources and the share of hydropower in total electricity consumption.

We found that states are highly dependent on reservoir water resources for water sources. However, in terms of electricity consumption, the proportion of hydropower generation is very small, which is determined by the capacity of hydropower generation. Limited capacity means it cannot be used as a primary energy source.

Depending on the urgency of demand, we will focus on the allocation of water resources in the following analysis. This is reflected in the fact that we will try to meet the needs of the states in terms of water consumption as much as possible. In terms of electricity consumption, we maximize the value of hydroelectric power generation under the premise of meeting the water consumption of each state, and then allocate it according to the proportional value of each state's electricity consumption.

State-by-state water demand and electricity usage data are shown in the table below

Table 1: Monthly water consumption table [9](KWh)

State \ Indicators	Residential water consumption	Industrial water consumption	Agricultural water consumption	Monthly water consumption	Proportion
Arizona	1.1×10^8	6.9×10^5	4.2×10^8	5.3×10^8	0.15
California	3.8×10^8	4.5×10^7	1.7×10^9	2.1×10^9	0.61
Wyoming	1.0×10^7	9.1×10^5	2.4×10^8	2.6×10^8	0.07
New Mexico	1.9×10^7	3.9×10^5	1.6×10^8	1.8×10^8	0.05
Colorado	7.6×10^7	9.5×10^6	3.9×10^8	3.9×10^8	0.11

In order to facilitate representation and calculation, we calculated the proportional value of the data. The advantage of the proportional value is that the magnitude and unit of the original data can be ignored, which is convenient to substitute into the model as a weight for calculation. This

Table 2: Monthly electricity consumption table [10](KWh)

State \ Indicators	Monthly electricity consumption	Proportion
Arizona	1.1×10^{10}	0.25
California	1.9×10^{10}	0.45
Wyoming	4.2×10^{9}	0.1
New Mexico	3.4×10^{9}	0.08
Colorado	5.4×10^{9}	0.12

technique is also mostly used in subsequent data processing.

In addition, we set the statistical period to be one month, that is, the state's demand for water and electricity in one month, which is also the update frequency of our model.

4.3 Supply Analysis

From the demand analysis, the focus of the supply and demand analysis is the water supply of the reservoir.

By consulting relevant information, we obtained information such as the surface area and total water volume of the two reservoirs, but this cannot obtain the water supply volume of the reservoirs. Because the reservoir must have sufficient water storage to maintain the dynamic balance of the water level. Therefore, the total water volume cannot be used as the upper limit for pumping.

With this in mind, we looked at the ecological water levels, the lower limit of homeostasis, of Lake Powell [5] and Lake Mead [6], which are 1077 meters and 366 meters above sea level respectively.

After obtaining the ecological water level, we can calculate the water supply of the two reservoirs separately.

Table 3: Information about the lakes

Lake \ Indicators	Surface area(m^2)	Surface elevation(m)	Average depth(m)	Water volume(m^3)
Lake Mead	6.4×10^{8}	372	69	3.2×10^{10}
Lake Powell	6.5×10^{8}	1080	40	3×10^{10}

However, we also need to consider the linkage between the two reservoirs. Lake Powell is upstream from Lake Mead, which means that the amount of water released by Lake Powell will add to Lake Mead's water volume. According to this condition, we divide the water volume into two uses, namely general water and water for power generation. General water is directly drawn from the reservoir, and water for power generation is the amount of water released from the dam, which will generate electricity resources.

Under these conditions, the amount of water released by Lake Powell could not only increase the water volume of Lake Mead, but also be used for hydroelectric power generation. Considering only this condition, the more water that Lake Powell should release, the better. But we need to add the limit of ecological water level, that is, the water level cannot be lower than 1077m.

Figure 1: Glen Canyon Dam

Figure 2: Hoover Dam

In order to obtain the specific value of the water release from Lake Powell under multiple conditions, we adopted the method of multi-objective optimization to set up the water transfer model.

4.3.1 Water Transfer Model

The Glen Canyon dam [7] is located at the upstream of the Hoover [8] dam. Therefore, it can provide some water for the Hoover dam, and the water storage capacity of Lake Powell is much larger than that of Lake Mead. When establishing the water transfer simulation model [1], based on the principle of the overall layout of the project, without considering the water distribution between the five states, the water transfer quantity is simulated by using the water balance equation and different constraints. Finally the optimal solution of the water transfer from Glen Canyon dam to Hoover dam is obtained.

According to the water balance equation, the model is established as follows:

Target function:

$$maxW = \sum Qg_i \tag{1}$$

Where: W represents the total water supply of the two dams in the dispatching period (one month) (c), Qg_i represents the monthly average water supply of the dam(m^3), and I represents the number of dams, i represents for dam number, $i = 1$ stands for the Glen Canyon dam, $i = 2$ stands for the Hoover dam.

Constraint Condition:

Water balance constraints:

$$V_i' = V_i + QR_i - QC_i \tag{2}$$

V_i' is the final capacity (m^3) of the dam after delivery and warehousing, V_i is the initial storage capacity of the dam before water delivery (m^3), QR_i is the monthly average inflow of the ith dam (m^3 / month), QC_i is the average monthly outflow of the ith dam (m^3 / month). i is the same as above.

Storage capacity constraint:

$$minV_i \leq Vs_i \leq maxV_i \tag{3}$$

$minV_i$ is the lowest limit of the storage capacity of the ith dam, usually corresponding to the dead water level and the ecological water level(m^3), $maxV_i$ is the upper limit of the storage capacity of i dam, which usually stands for the flood control level (m^3), and Vs_i is the storage capacity of the ith dam.

Electricity production constraint:

$$minN_i \leq K_iQ_i \leq maxN_i \tag{4}$$

$maxN_i$, $minN_i$ represent upper and lower water levels of the ith dams that can generate electricity(kW), K_i is the power generation coefficient per unit volume of the ith dam, Q_i is the ith dams power generation flow(m^3/s).

Solving Method:

Apply the self iterative simulation algorithm to solve the model [2], the steps are shown as below:

- **Step1** Set the long term demand quantity.
- **Step2** Water transfer from the Glen Canyon dam.

  At the beginning, the algorithm analyze the relationship between water level P and the ecological water level of Powell. After considering the ecological water level, the water volume of the Glen Canyon dam is the maximum water supply quantity, and the remaining water is used for power generation and stored in the Hoover dam (Lake Mead).
- **Step3** Lake Mead for water storage and transfer.

  As the additional guidance remarks, the Glen Canyon dam can store water to the Hoover dam. The algorithm judges whether the residual water exceeds the upper limit of the Hoover dam and whether the storage space of the Hoover dam meets the residual water volume in that month. If the storage space is greater than 0 and the residual water, all the residual water will be replenished into the Hoover dam. If the storage space is greater than 0 and less than the residual water, the amount of water replenished into the Hoover dam = residual water - storage space, then return to the modified Powell water transferred quantity. If the storage space is less than 0, no water will be stored and the Hoover dam will discharge.
- **Step4** After the completion of water transfer and storage, the optimal water supplement from the Glen Canyon dam to the Hoover dam is calculated.

Through this model, it is finally obtained that the monthly water release of the Glen Canyon Dam is $1 \times 10^9 m^3$. The efficiency of power generation is the highest.

After the allocated water is distributed and recalculated, the water and electricity that can be provided by the two dams can be obtained.

The Glen Canyon Dam can provide $1 \times 10^9 m^3$ water. The Hoover Dam can provide $3.2 \times 10^9 m^3$ water.

4.4 Model Construction

After obtaining the situation of supply and demand, we will establish a model to allocate the five states. In the process of distribution, we take the supply costs of the five states into account, such as distance and terrain factors and so on. By assessing the cost and benefit of supply, we decide to take the method of MOP to make decision, so then we can obtain the distribution under the specified demand and supply. We also considered the time needed to meet the demand, the amount of residual water and Mexico's claim for water resources.

4.4.1 Concept of MOP

In the case of single objective optimization, there is only one target function. Any two solutions can be compared to get an undisputed optimal solution, which is opposite to MOP. The concept of MOP is that when multiple objectives need to be achieved in a certain situation, due to the internal conflict between objectives, the optimization of objectives is at the cost of the deterioration of other objectives. Therefore, it is difficult to find the unique solution. Instead, coordination and compromise are made among them to make the overall objective as optimal as possible.

4.4.2 Analysis Process

Based on the water supply and cost of two dams and the scarcity of water resources in five states, an MOP model is constructed, which is a complex distribution method with multiple constraints and meet the need to coordinate multiple interests.

Target Function:

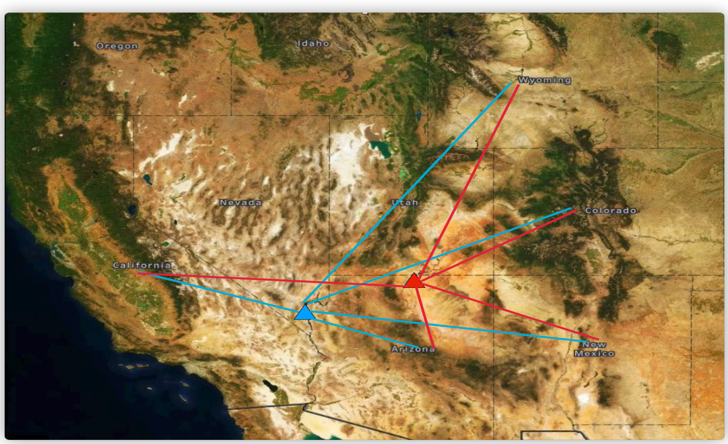


Figure 3: Distance between states and dams

- **The shortest total transportation distance.** The cost of water resources supply largely depends on the distance between the state and the dam including the cost of pipeline construction and actual transportation. In practice, the maintenance cost of pipeline also needs to be considered. The shorter the distance, the cheaper for the prime cost. Therefore, the transportation distance should be investigated and the closer dam should be selected for priority transportation.

$$min \sum L_{i,j} \sum x_{i,j} \tag{5}$$

$x_{i,j}$ is the water quantity transfer from ith dams to j state(m^3) , $L_{i,j}$ is the distance from the ith dams to j statekm), i represents for dam number, $i = 1$ stands for the Glen Canyon dam, $i = 2$ stands for the Hoover dam, j is the state number.

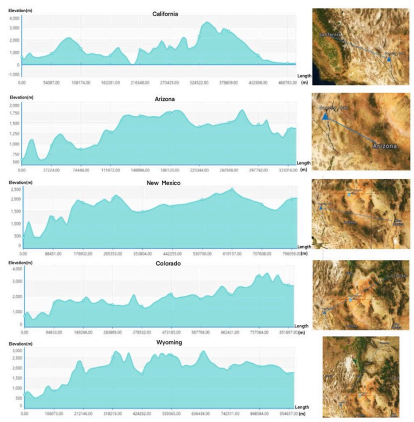


Figure 4: Elevation conditions-Hoover

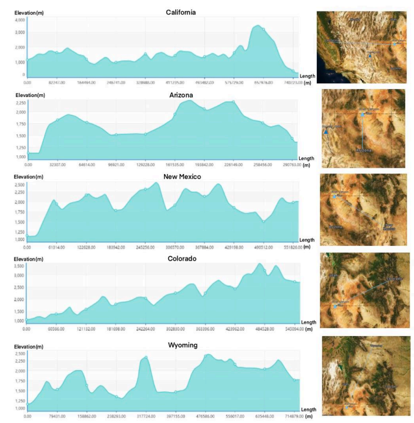


Figure 5: Elevation conditions-Glen Canyon

- **Minimum altitude difference.** The transportation of water resources also needs to consider the terrain and altitude factors. The greater the altitude difference from the dam, the greater the difficulty of transportation and the higher the technical requirements. It includes the configuration and use of pumps. Therefore, the altitude should be statistics and the dam with smaller altitude difference should be selected for priority transportation.

$$min \sum H_{i,j} \sum x_{i,j} \tag{6}$$

$H_{i,j}$ is the altitude difference from the ith dams to j state(km). $x_{i,j}$, i,j are same as above.

- **Optimal allocation of water scarcity.** States with scarce water resources should give priority to be supplied by nearby dams, which can meet the state's water demand as priority and obtain the highest overall benefits. The scarcity of water resources should be obtained by comparing the distribution of water sources in the state and comprehensively thinking overăthe water demand.

$$max \sum SC_{i,j} \sum x_{i,j} \tag{7}$$

$SC_{i,j}$ is scarcity degree of water resources of j state, $x_{i,j}$, i,j are same as above.

Constraint Condition:

$$s.t. = \begin{cases} minV_i < \sum x_{i,j} \leq a_i & i = 1,2 \\ \sum x_{i,j} = b_j & j = 1,2,...,5 \\ x_{i,j} \geq 0 \end{cases} \tag{8}$$

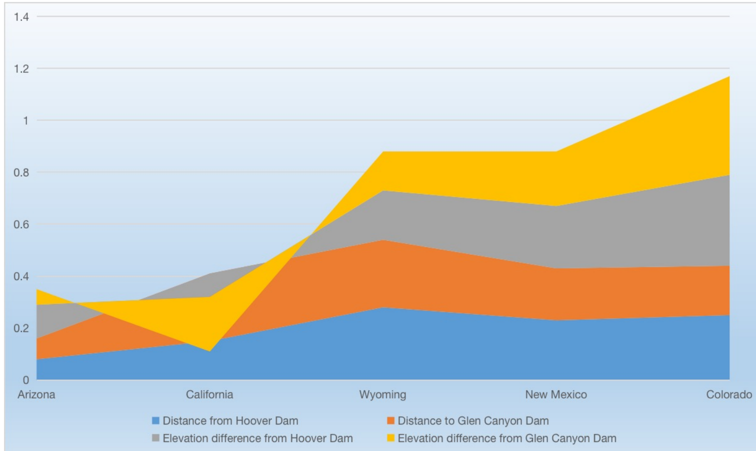


Figure 6: Cost of water transportation from dams

V_i is the transferable water quantity of the ith dams(m^3), $minV_i$ is the lowest limit of the storage capacity of the ith dam, usually corresponding to the dead water level and the ecological water level(m^3), Wat_j represents the water demand of j state(m^3). $x_{i,j}$, i,j are same as above.

Above is the mathematical formula part of the model, and the best allocation method can be obtained by substituting real data.

By querying authoritative data, we get the original data of various indicators in various states. In order to facilitate the calculation on the model, we calculate the proportion value of each data based on the original data, as shown in the table below:

Table 4: Water scarcity and transportation difficulty factors

State \ Indicators	Water scarcity	Distance-Hoover	Distance-Glen Canyon	Elevation difference-Hoover	Elevation difference-Glen Canyon
Arizona	0.23	0.08	0.08	0.13	0.06
California	0.16	0.15	0.26	-0.09	-0.21
Wyoming	0.12	0.28	0.26	0.19	0.15
New Mexico	0.25	0.23	0.20	0.24	0.21
Colorado	0.24	0.25	0.19	0.35	0.38

4.4.3 Conclusion

Substitute the proportional data into the model, and after calculation, we get the specific water resources allocation, as shown in the following table. Looking at the data, we can see that our calculations have a high degree of confidence.

First, the water needs of all five states were met.

Second, taking California's water sources as an example, all water resources are drawn from

Table 5: Water resource allocation plan and hydroelectric power allocation ratio

State \ Indicators	Water from Hoover dam(m^3)	Water from Glen Canyon dam(m^3)	Hydroelectric power distribution ratio
Arizona	3.9×10^8	2.5×10^8	0.25
California	2.6×10^9	0	0.45
Wyoming	6×10^7	2.5×10^8	0.1
New Mexico	1.2×10^8	1.0×10^8	0.08
Colorado	7×10^7	4.0×10^8	0.12

the Hoover Reservoir. This is because the distance and elevation difference between California and Hoover Dam is lower than that of Glen Canyon Dam.

Third, taking the water source of Wyoming as an example, even though the distance and elevation difference between Wyoming and Hoover Dam are very far, a certain amount of water is still drawn from Hoover Dam. This is because Lake Powell, where the Glen Canyon Dam is located, needs to maintain its ecological water level and cannot provide enough water to all surrounding states.

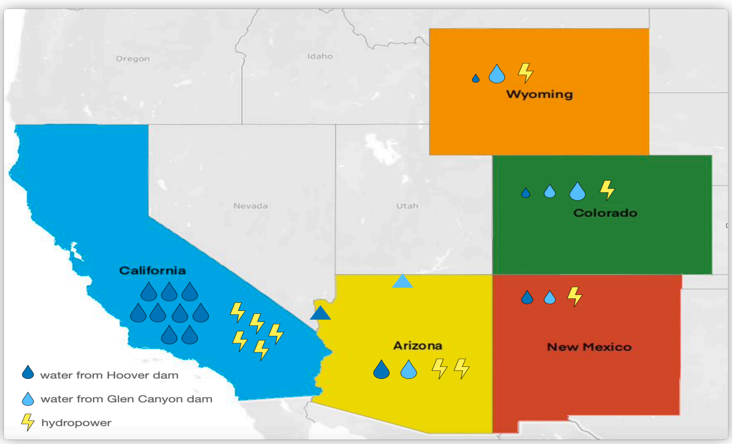


Figure 7: Electricity and water distribution of states

As stated in the supply and demand analysis, once the water needs of the states are met, we maximize the amount of hydroelectric power.

Specifically reflected in the data, maximize the sum of the water released by the Glen Canyon Dam and the Hoover Dam. This part has been calculated in the water transfer model, which are $1 \times 10^9 m^3$ and $1.8 \times 10^9 m^3$ respectively. Adding them together gives a maximum value of $2.8 \times 10^9 m^3$.

Finally, it can be allocated proportionally according to the electricity demand of each state.

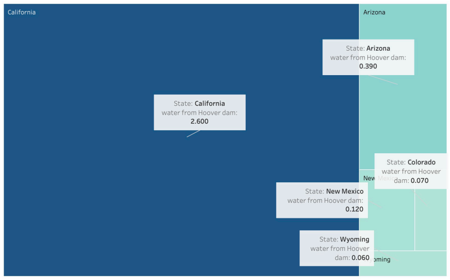


Figure 8: Water distribution of Hoover

Figure 9: Water distribution of Glen Canyon

4.5 Time Allowance

Without additional water supply and assume the demand is fixed, we calculate the time required to satisfy the demand through the model. The allocation and scheduling time of water sources mainly depends on the scheduling time between the two dams and the time required for transportation to the states.

We investigated the length of the Colorado River between the two dams and calculated the allocation time of water resources with a fixed time period of one month. We get a total flow of $1 \times 10^9 m^3$, The average velocity of the Colorado River from the Glen Canyon Dam to the Hoover dam is 410 cubic meters per second, and the evaporation along the way is $6 \times 10^7 m^3$. Through mathematical calculation, the time is about 26.5 days.

$$1 \times 1000000000 \div 410 \div 24 \div 3600 \times 0.94 = 26.5 days \tag{9}$$

According to the statistical results of transportation in various states, combined with the speed of pipeline water supply, time of water supply though pipeline to users can be ignored compared with the dispatching time between dams. Therefore, the time to meet the demand should be 26.5 days.

4.6 Residual Water

If the ecological water level is met, the remaining water resources after the allocation of water resources in the five states will be released through the Hoover Dam. The specific embodiment in the data is the amount of water released by the Hoover Dam, which is $1.8 \times 10^9 m^3$.

This part of the water resources will supplement downstream water sources, flow through Mexico, and eventually from the Colorado River into the Gulf of California.

However, we also need to consider evaporation. According to the data, evaporation in the Colorado River Basin accounts for 6% of the total water. Subtracting evaporation from the amount of water released by the Hoover Dam calculates that the Hoover Dam can also provide 1.692×10^9 cubic meters of water downstream.

4.7 Mexico Water Debt

Our model scientifically calculates distribution scheme of water and electricity used by five states, and calculates the remaining water, which should be input to the downstream to ensure ecological stability and resources available to other countries.

If five states use more water than their share, Mexico can file a claim in the form of water debt or monetary compensation. According to the needs of water debt, more water will be released upstream. The price of 1.50 US dollars per cubic meter of water can be compensated with reference to the data of the US Environmental Protection Agency.

5 Conflict between Water and Electricity Model

5.1 Background

In fact, there are often conflicts between the distribution of general usage and power generation of water. Therefore, we will use the analytic hierarchy process [3] to determine which resource to be supplied first with the help of the demand measurement standard of states in the distribution of water and electricity model.

5.2 Concept of AHP

Analytic hierarchy process (AHP) is a hierarchical weight decision analysis method proposed by American operations research scientist Sati in the early 1970s when studying the topic of "power distribution according to the contribution of various industrial departments to national welfare" for the U.S. Department of Defense, using network system theory and multi-objective comprehensive evaluation method.

5.3 Model Construction

5.3.1 Indicator Selection

Five indicators are selected to measure the priority of water and electricity.

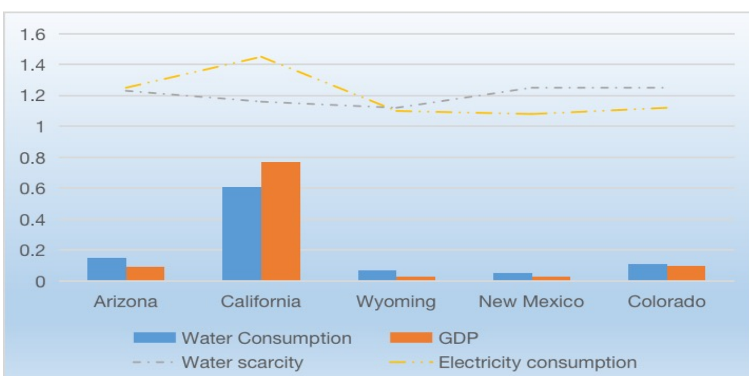


Figure 10: Indicators of priority measurement

- **Resource Scarcity.** It refers to the degree of dependence on the resource for each states. For instance, if a state receives up to 80% of its water resources from dams and only 10% of its electricity, so the supply of water is more dependent on dams. We should give priority to meeting the supply of resources that depends more on dams.

- **Economic Benefits.** It refers to the benefits obtained by the state using the resources. For example, the economic benefits generated by using water in a state are far greater than those obtained by power generation, so priority should be given to satisfying the demand for water rather than power generation.

- **Utilization Efficiency.** It refers to the utilization efficiency of the state for the resource. If the utilization efficiency is high, it can be preferred and obtain greater benefits. If the utilization efficiency is low, it is supposed be compensated and replaced by other ways.

- **Ecological Environment.** It refers to the impact of the resource on the ecological environment. For water and electricity, water is more important for agriculture. Thus, it has a greater impact on the ecological environment. For ecological reasons, priority should be given to ensuring water supply.

- **Social impact.** It refers to the importance of the resource to maintain social stability. Electricity is more important for the normal operation of society and plays a more obvious supporting role for industry and housing compared with water. Therefore, the social impact weight of electricity is higher.

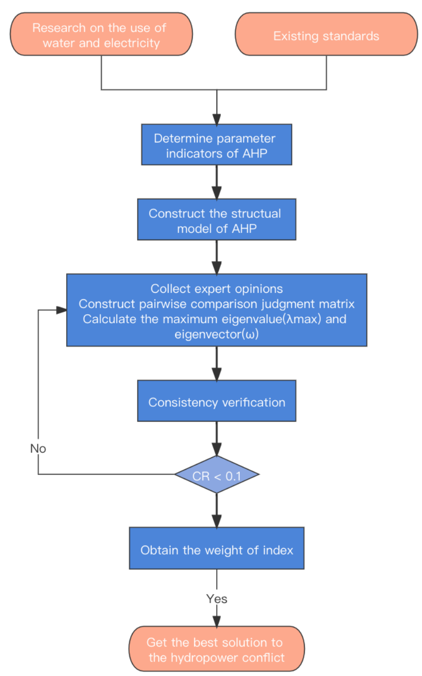
5.3.2 Algorithm process

- **Step1** For the overall goal of solving conflict between water and electricity, divide the target into multiple single objectives.

- **Step2** Determine the elements and indicators level of different objectives in the specific division. Invite the committee to give corresponding scores to build a judgment matrix.

- **Step3** Extract the effective information from the judgment matrix and use the effective information to calculate the weight vector.

- **Step4** Based on the weight vector, check the consistency of the matrix to judge whether it is consistent.



The specific algorithm process is as follows:

Through the data collecte from the investigation, clarify the specific objectives of solving water and hydropower conflicts and determine the indicators parameters.

- **Objective:** Reasonably solve conflicts between water and electricity.
- **Criteria:** Comprehensively consider the impact of water and electricity on economy, society and environment. Then select the following five criteria indicators as follows. A1 Resource Scarcity, A2 Ecological Environment, A3 Utilization Efficiency, A4 Economic Benefits, A5 Social Impact.
- **Scheme** B1: Water supply priority, B2: Power supply priority, B3: Simultaneous water and electricity supply.

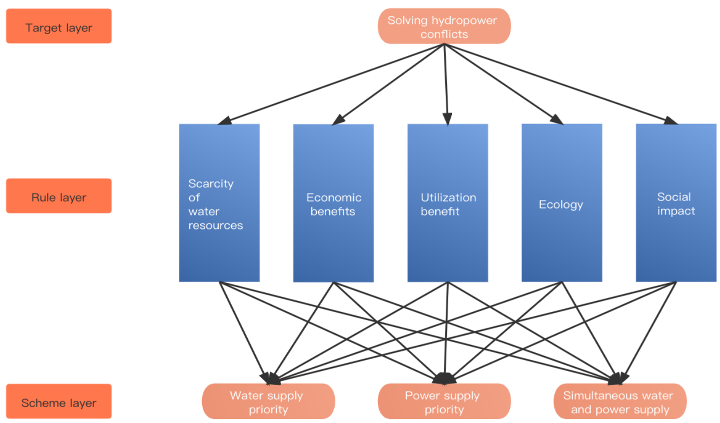


Figure 11: Hierarchical structure of evaluation indicators

Construct judgment matrix: Compare the importance of each element in the same level with respect to a certain criterion in the previous level, and construct a judgment matrix. The value reflects people's understanding of the relative importance of each evaluation factor. Generally, the scale method of 1 to 9 and its reciprocal is adopted, as shown in the table below.

Table 6: Scale of proportions

Scale of proportions factor i to factor j	Quantitative values
Equally important	1
Slightly important	3
Median important	5
Strongly important	7
Extremely important	9
Median value of two adjacent judgments	2,4,6,8

The actual judgmental comparison is done by the evaluation committee. The comparison of commercial aspects can be done by experts in finance, accounting and economic management; technical aspects can be compared by technical experts; ecological aspects can be compared by environmental experts and so on.

The results of the experts' comparisons are expressed numerically in a judgment matrix by introducing appropriate scales. The judgment matrix is as follows.

$$A = \begin{pmatrix} 1.00 & 0.50 & 3.00 & 5.00 & 5.00 \\ 2.00 & 1.00 & 7.00 & 5.00 & 5.00 \\ 0.33 & 0.14 & 1.00 & 3.00 & 0.50 \\ 0.20 & 0.20 & 0.33 & 1.00 & 1.00 \\ 0.20 & 0.20 & 2.00 & 1.00 & 1.00 \end{pmatrix} \tag{10}$$

$$B1 = \begin{pmatrix} 1.00 & 5.00 & 3.00 \\ 0.20 & 1.00 & 0.33 \\ 0.33 & 3.00 & 1.00 \end{pmatrix} \tag{11}$$

$$B2 = \begin{pmatrix} 1.00 & 7.00 & 5.00 \\ 0.14 & 1.00 & 0.33 \\ 0.20 & 3.00 & 1.00 \end{pmatrix} \tag{12}$$

$$B3 = \begin{pmatrix} 1.00 & 0.33 & 0.50 \\ 3.00 & 1.00 & 3.00 \\ 2.00 & 0.33 & 1.00 \end{pmatrix} \tag{13}$$

$$B4 = \begin{pmatrix} 1.00 & 0.20 & 0.33 \\ 5.00 & 1.00 & 3.00 \\ 3.00 & 0.33 & 1.00 \end{pmatrix} \tag{14}$$

$$B5 = \begin{pmatrix} 1.00 & 0.33 & 0.50 \\ 3.00 & 1.00 & 2.00 \\ 2.00 & 0.50 & 1.00 \end{pmatrix} \tag{15}$$

A is the judgment matrix of the target layer on the factors of the criterion layer, B_j is the judgment matrix of the criterion A_j on the scheme layer, j is the notation of factors in the criterion layer.

Perform consistency test

The characteristic root method is used to derive the ranking weights from the two-by-two comparison between elements of the judgment matrix, and the maximum characteristic root of the judgment matrix is calculated as follows.

$$AW = max\lambda \times W \tag{16}$$

$$B_jW = max\lambda_j \times W \tag{17}$$

where $max\lambda$, $max\lambda_j$ are the maximum characteristic roots of the judgment matrix and the eigenvector W it corresponds to is the estimate of the ranking weight vector after normalization. A, B_j, j are the same as above elements in this segment.

Calculating the consistency index CI

$$CI = \frac{\lambda - n}{n - 1} \tag{18}$$

Obviously, the larger the value of CI, the more the judgment matrix deviates from full consistency. The higher the order n of the judgment matrix (i.e., the more indicator factors involved in the comparison), the larger the deviation caused by human beings.

Calculation of consistency ratio CR

$$CR = \frac{CI}{RI} \tag{19}$$

When $CR < 0.1$, it is considered that the judgment matrix has satisfactory consistency, which can be accepted. When $CR > 0.1$, appropriate corrections should be made to the judgment matrix.

Hierarchical total ranking and consistency test Weights are synthesized from the highest level to the lowest level layer by layer, and the total judgment consistency test is performed. The calculation formula and matrix are as follows.

The specific values are brought in for calculation to get the hierarchical total ranking weights.

Table 7: Hierarchical total ranking weights

	A1	A2	A3	A4	A5
a	0.294	0.461	0.091	0.063	0.091
B1	0.638	0.731	0.157	0.105	0.163
B2	0.105	0.081	0.594	0.638	0.540
B3	0.258	0.188	0.249	0.258	0.297

Then we check the total consistency ratio:

$$CR = \frac{a_1 CI_1 + a_2 CI_2 + ... + a_m CI_m}{a_1 RI_1 + a_2 RI_2 + ... + a_m RI_m} \tag{20}$$

The total hierarchical ranking was calculated to pass the consistency test. Finally, the weights are shown as follows. B1 = 0.560, B2 = 0.212, B3 = 0.228

5.4 Conclusion

By using AHP to evaluate comprehensive indicators consisting of resource scarcity, ecological environment, energy efficiency, economic and social factors, we obtained the result that the supply of water resources is more important, so that when conflicts arise between water and electricity, priority should be given to water resources.

6 Insufficient Water Allocation

6.1 Background

The water supply may not be able to satisfy the demand due to the depletion of lakes and the decrease of water levels caused by the drought.

We use the demand measures of sates from the allocation of water and electricity model to calculate the demand indicators for each state with the help of TOPSIS [4].

6.2 Concept of TOPSIS

TOPSIS is an important comprehensive evaluation algorithm, which determines the closeness between the unit to be evaluated and the optimal solution by calculating the Euclidean distance between the indicators and the ideal solutions. It is an effective algorithm for comparing, ranking and selecting multiple design solutions under multi-attribute conditions.

6.3 Algorithm Process

The allocation of water to the five states in the case of insufficient dam water requires multiple factors, which is a multi-attribute decision problem. The priority of allocation should fully consider the economic development status of the five states, the degree of demand for water resources and other factors.

Therefore, this study constructs a multi-attribute decision system that includes three indicators: GDP, water scarcity and water demand of each state. Based on the actual situation, they are all efficiency-based indicators.

Normalized processing

$$b_{i,j} = \frac{a_{i,j}}{\sqrt{\sum a_{i,j}^2}} \qquad i = 1, 2, ..., m \qquad j = 1, 2, ..., n \tag{21}$$

A is the decision matrix for the multi-attribute decision problem, B is the normalized decision matrix, m is the total number of states(i=1 is AZ, i=2 is CA, i=3 is WY, i=4 is NM, i = 5 is CO), n is the number of indicators.

Construct the weighted norm matrix Z

The fusion assignment method is used for weight calculation, which integrates both the subjective AHP and the objective entropy weight method. The final weight coefficients are obtained by taking the average value. This can relatively avoid the situation that the result is too objective or too subjective attributed to the single choice of assignment method, and improve the reliability of the assignment.

AHP has been introduced in the resolution of water and hydropower conflict model.The entropy weight method is an objective method and its calculation is displayed as follows.

Standardize the original data matrix, and for the profitability indicators with the largest and the smallest as the best:

$$r_{i,j} = \frac{max\{x_{i,j}\} - x_{i,j}}{max\{x_{i,j}\} - min\{x_{i,j}\}} \tag{22}$$

$$r_{i,j} = \frac{x_{i,j} - max\{x_{i,j}\}}{max\{x_{i,j}\} - min\{x_{i,j}\}} \tag{23}$$

The definition of entropy:

In the problem of n indicators and m evaluation objects, entropy of the jth indicator can be defined as:

$$E_i = -k \sum f_{i,j} ln f_{i,j} \qquad i = 1, 2, ..., m \tag{24}$$

The definition of entropy weight:

The entropy-weight of the ith indicator can be calculated as:

$$w_j = \frac{1 - E_j}{n - \sum E_j} \qquad 0 \le w_j \le 1 \tag{25}$$

The weights of the indicators are obtained by fusion weighting $w = [0.223, 0.498, 0.279]$

$$z_{i,j} = w_{i,j} \times b_{i,j} \qquad i = 1, 2, ..., m \qquad j = 1, 2, ..., n \tag{26}$$

Determine the ideal positive solution M* and the ideal negative solution M0

For the positive ideal solution $M * (m_j *)$, the value of each benefit-based indicator must be the maximum value of the corresponding indicator among all the solutions, and the value of each cost-based indicator must be the minimum value of the corresponding indicator among all the solutions. Conversely, the negative ideal solution $M \circ (m_j \circ)$ can be found.

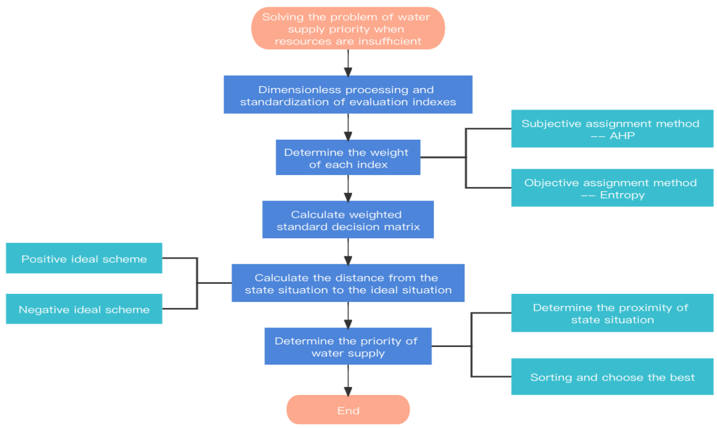


Figure 12: Flow chart of TOPSIS

Calculate the distance of each solution from the positive and negative ideal solution

The distance from alternative di to the ideal positive solution:

$$S_i * = \sqrt{\sum (z_{i,j} - m_{i,j} *)^2} \qquad i = 1, 2, ..., m \tag{27}$$

The distance from alternative di to the ideal negative solution:

$$S_i \circ = \sqrt{\sum (z_{i,j} - m_{i,j} \circ)^2} \qquad i = 1, 2, ..., m \tag{28}$$

Calculate the combined queuing metrics for each program

$$h_i = \frac{S_i \circ}{S_i \circ + S_i *} \tag{29}$$

The result of AZ, CA, WY, NM, CO is shown as follows in the order: $h = [0.31640.77150.03620.30350.3173]$

6.4 Conclusion

Through calculation and comparison, we obtained the degree of dependence for the five states on water supply and electricity supplied by the dam. When there is a water shortage, combined with the Conflict Model, the water supply is at the top of the list.

Among the five states, priority should be given to California, Colorado and Arizona, followed by New Mexico, and finally Wyoming.

7 Sensitivity Analysis

In our model, we have considered only the present demand and calculated it as a fixed value. Over time, the demand for water and electricity changes dynamically, which can affect the operation of the model, including the judge metrics and allocation scenarios, so we need to focus on the sensitivity of water and electricity demand and analyze trends.

Table 8: Changes in the weight of water allocation by state

	Arizona	California	Wyoming	New Mexico	Colorado
Arizona+5%	2.46%	0%	0%	0%	0%
Arizona+10%	2.52%	0%	-2.76%	0%	0%
California+5%	-2.08%	0.23%	-8.84%	-0.73%	-1.38%
Colorado+5%	1.23%	0%	0%	0%	-1.38%

Sensitivity coefficient is an index reflecting the sensitivity of project benefits to factors. The higher the coefficient, the higher the sensitivity. The calculation formula is:

$$E = \frac{\Delta A}{\Delta F} \tag{30}$$

E is the sensitivity coefficient of evaluation indicator A to factor F, ΔF is the change rate of uncertain factor F(%), and the ΔA is the change rate of A when the change of uncertain factor F is the ΔF.

Through the research, we found that the change amount of uncertainty in sensitivity analysis is usually set at 5%, 10%, 15%, 20%, etc. When the water demand of a state changes to a certain extent, the weights of water allocation changes in the five states are shown in the following table.

7.1 Changes in General Usage

Demand for water and electricity changes over time. In the water and hydropower allocation model, an increase in demand will result in a larger indicator weight for the state's water demand, which will allow more hydropower resources to be allocated. On the contrary, it will allocate less water resources.

For water distribution, when water demand decreases, more water can be transferred from the Glen Canyon dam to the Hoover dam, which will increase power production and generate higher efficiency.

7.2 Renewable Energy

An increase in the proportion of renewable energy technologies will change the model's metrics and the state's resource scarcity leading to a decrease in these weights.

When allocations are made, states which product based on renewable energy sources receive a reduced share of the water and electricity, allowing resources to be optimally distributed and effectively improving the utilization of hydropower.

7.3 Saving Water and Electricity

The government may develop policies to limit the usage of water and hydropower. This means that there will be less demand in the state, which will cause a smaller indicator weighting of this. so that the allocation of water and hydropower resources in this sate will be lower, which could improve the overall utilization of water resources.

Besides, it will be more environmentally friendly and achieve the sustainable economic and social development.

8 Summary

8.1 Advantages

Our model is based on numerous theoretical foundations. After a careful search of relevant information and literature, we picked out some key parameters for building the model after several deliberations. In the end, a relatively complete model with a simple basic model as the backbone has been established, which can analyze various time-complicated situations. Therefore, our model is consistent with the reality.

We consciously made reasonable simplifications in order to build simple and elegant mathematical models. Demand and supply analysis is performed by using iterative simulation algorithms, MOP algorithms, and AHP to solve allocation problems.

We thought over the actual situation, including the data of the dams, the demand situation of each state and the determination of measurement indicators, which we found and estimated from the websites of professional statistical offices to ensure the accuracy and scientific validity of our model. As a result, we were able to obtain a more generalized mathematical model.

Although we are not able to predict the changing conditions precisely, our model is flexible, fault-tolerant and adaptable enough to cope with the basic assignment problem and adapt its characteristics to obtain satisfactory results. For the existing model, we can easily modify some of the parameters and introduce new variables for more in-depth studies.

8.2 Disadvantages

Limited by the investigation data, we were not able to obtain more accurate results. If a higher standard of results are needed in the future, greater improvements to the existing model will be required. Such improvements are also multifaceted, so it requires substantial additions of theoretical knowledge from multiple fields and levels, such as more scientific metrics and decision models.

For the requirement of simplifying the mathematical model, we ignore a considerable number of confounding factors. For example, we set no external water replenishment and a period of one month with fixed values of supply and demand. Similar omissions may have profound effects on the generalizability of the mathematical model. In the case, for solving this problem, future researchers may need to deeply dissect the above problem from a dynamical change perspective. For other problems, future researchers also need to be cautious, so as to take a steady and thoughtful approach.

9 Article

Make Dams Efficient again

As we all know that a dam is a barrier that stops or restricts the flow of surface water or underground streams. But it can do more than that, a dam means a reservoir, which means hydroelectric power. With the linkage of the two reservoirs, the dams can do more.That's what happened with the Hoover Dam and Glen Canyon Dam. To make dams effective again, we have proposed a new allocation of water resources and hydroelectric power. This is the result of quantitative analysis of several models based on a large amount of data from five states, with efficiency as the first criterion.

Maximum efficiency in linkage

There is a relationship between the Glen Canyon Dam and the Hoover Dam where the amount of water released by the Glen Canyon Dam can increase the water resources of the Hoover Dam. This relationship has very significant value. First, the amount of water released from Glen Canyon Dam can be used directly for hydroelectric power generation. Secondly, this is a cost-free long-distance water transportation method, and effective use can save a lot of costs. Under this premise, we constructed a water transfer model and a hydropower distribution model by combining the distances and elevation differences between states and the dam. It is calculated that the most efficient water transfer method is the Glen Canyon Dam with a monthly water release of $1 \times 10^9 m^3$, and the lowest cost hydropower allocation results are shown in the table below.

Indicators / State	Water from Hoover dam(m^3)	Water from Glen Canyon dam(m^3)	Hydroelectric power distribution ratio
Arizona	3.9×10^8	2.5×10^8	0.25
California	2.6×10^9	0	0.45
Wyoming	6×10^7	2.5×10^8	0.1
New Mexico	1.2×10^8	1.0×10^8	0.08
Colorado	7×10^7	4.0×10^8	0.12

Water allocation is the first priority

The priority of resource allocation needs to be clarified first. If there is a resource mismatch, the efficiency must be low. The importance of a resource depends on its irreplaceability in people's lives. To determine the demand for reservoir water and dam hydropower in the five states, we looked for information on the five states' water use and electricity use, as well as information on the state's water sources and hydroelectricity's share of total electricity use. We found that states are highly dependent on reservoir water resources for water sources. However, in terms of electricity consumption, the proportion of hydropower generation is very small, which is determined by the capacity of hydropower generation.Limited capacity means it cannot be used as a primary energy source. In addition, we also comprehensively considered factors such as ecological environment, energy efficiency, economy and society, and constructed a hydropower conflict model using AHP. It is calculated that the general water use priority: hydropower generation priority: the weights of the same priority are 0.560:0.212:0.228. It is concluded that the general water use priority is the highest. This is exactly the choice we should take in the hydropower conflict.

Maximize the value of unit water resources

Allocating resources is to maximize value. But how to maximize value? The most direct idea is to create greater value per unit of water resources. The specific embodiment is the GDP of each state. So the idea is to give states with higher GDP a greater distributional weight. This is respect for economic value.In addition, we considered water scarcity and water usage by state. Combining these factors, we use the TOPSIS method to build a water shortage model, calculate the weight of water resources allocation in the case of insufficient water resources, and sort to get its priority.

Weight h = (0.3164(AZ), 0.7715(CA), 0.0362(WY), 0.3035(NM), 0.3173(CO))

Priority: California > Colorado > Arizona > New Mexico > Wyoming

Summarize

In today's increasingly severe climate change, the possibility of drought conditions is greatly increased. Therefore, improving efficiency is what we must pay attention to when allocating water. At the same time, with the development of technology. Mathematical tools are becoming more and more capable of quantifying the real world, and the results of quantification are becoming more and more credible. Referencing the results of mathematical models to solve efficiency problems is gradually becoming the choice of more and more people.

Finally, we hope that this article sheds some light on the problems that our readers face, which is the greatest affirmation of our work.

References

[1] J. Wu. Study on mathematical model of urban water supply and distribution system [J] Journal of Xiangtan Normal University (Natural Science Edition). 2005(02).

[2] B. Kong. Study on multi-objective optimal allocation of pump station-reservoir power station group [D] Xian: Engineering Science and Technology II (Water Conservancy and Hydropower Project).2021.

[3] F. Cheng. Evaluation of water resources carrying capacity in Taiyuan based on AHP [J] China Management Informatization. 2021,24(01).

[4] H. Wang, S. Zhong. Multi-attribute decision-making for dam type comparison and selection of hydropower projects based on entropy weight-TOPSIS [D] Yellow River. 2021,43(S1).

[5] https://en.wikipedia.org/wiki/Lake_Mead

[6] https://en.wikipedia.org/wiki/Lake_Powell

[7] https://en.wikipedia.org/wiki/Hoover_Dam

[8] https://en.wikipedia.org/wiki/Glen_Canyon_Dam

[9] https://www.sciencebase.gov/catalog/item/get/5af3311be4b0da30c1b245d8

[10] https://www.eia.gov/electricity/state/