Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		**2125753**

Air Support for Fighting Wildfires

Summary

The 2019-2020 bushfire season has seen significant damage in Australia.We helped the National Fire Service design the Integrated Bushfire Response System and calculate the budget accordingly.

To better deploy the UAVs, we developed **the Bushfire Model**.We collected data on the Australian bushfire last year, and we got the area and size of the forest fire in eastern Victoria. Considering that radio communication will be obstructed by the terrain, we developed **the Obstructed Radio Communication Model**. We simplified the calculation of the effect of terrain on radio communication distance based on electromagnetic theory and geospatial information.

We considered SSA UAVs and repeater UAVs separately. To evaluate and analyze the radio transmission capability of frontline forces and repeaters, we developed **the Repeater UAV Network Model**. Based on the method of VORONOI diagram in graph theory with the help of computer simulation, we made the basic planning on the partition of areas or regions. Combining the impact of the actual bushfire region and terrain on radio communication, we designed the optimal number of UAVs and deployment scheme. To evaluate the capability of UAVs to monitor fires, we developed **the SSA UAV Coverage Model**. Based on the characteristics of SSA UAVs, we turned the problem into a multi-UAV cooperative coverage route planning problem. We improved the Ant Colony Algorithm for path optimization and obtained the Rectangular-coverage Centralized Algorithm, which we used to obtain the optimal number of UAVs. Using the Hierarchical Analysis(AHP), we developed **the Integrated Bushfire Response Model**. Based on the Repeater UAV Network Model and the SSA UAV Coverage Model, we assigned appropriate weights to factors such as fire size, terrain, economy, and the capability of damage control to calculate the optimal number and combination of SSA and Repeater UAVs.

To estimate the likelihood of extreme bushfires in the next decade, we developed **the Bushfire Prediction Model**. We calculated the dynamic changes of bushfire regions in the next decade. We calculated the change in the number of UAVs, considering attrition costs. We finally calculated the incremental cost based on future bushfire predictions and attrition probabilities. To explore the impact of different terrain and bushfire sizes on repeater UAVs, we developed **the Repeater Location Model**. Using the Improved VORONOI Graph Algorithm, the distribution of the location of the displayed repeaters was simulated with the help of the computer.

We then performed a sensitivity analysis of the model and provided evidence to demonstrate the stability and reliability of the model. Finally we analyzed the strengths and weaknesses of the model. We also wrote an annotated budget request based on the proposed model to provide to the government for decision making.

Keywords: UAV, Voronoi Diagram, Ant Colony Algorithm, Route Planning, AHP

Contents

1 Introduction

In order to be able to respond quickly to major bushfires, we helped the National Fire Service design an integrated fire response system and calculate the budget accordingly. The system is capable of monitoring the situation on the fire ground without interruption using SSA drones. It also uses a network of repeater drones to ensure smooth radio communications.

The model can be devided into two major parts. One is to obtain a method for SSA drones to cover the whole region in bushfire, providing surveillance and support. The other is to obtain an ideal distribution pattern for repeater drones, with the smallest overlapping areas according to its functional range.

2 Problem Statement

The model provides an in-depth analysis of putting UAVs into mountain fire detection and control, and summarizes the following four core problems to be solved: the coverage path planning problem of UAV formations, the optimal solution of repeater distribution with minimum overlap area, the probability distribution model of annual loss of UAVs, and the multi-connected area form of the basic model.

- **Coverage path planning problem for UAV formations.** This model attempts to explore a model for giving an optimal combination of SSA UAVs and repeater UAVs.

  At the same time, in order to achieve the purpose of real-time transmission of signals, a more refined form of UAV formation is required, so the problem involves the combination of optimization problems in operations research.

- **Optimal solution of repeater distribution with minimum overlapping area.** The distribution of repeater UAVs can still be considered as an area coverage problem, and at the same time, the model needs to be solved to obtain a coverage solution with minimum overlapping area because of the overlap in the range of action of repeaters.

- **Probability distribution model of annual loss of UAVs.** The model also uses hierarchical analysis to assign weights to relevant factors such as terrain, fire range and frequency, and to synthesize various factors to give the final equipment configuration plan and detection action plan.

Ultimately, we will give the Victorian government a budget for the hill fire detection program. This budget is given by the previous question on the number of different types of drones required, and is supplemented by a consideration of the age of the drones in time. In the budget, for simplicity, the model uses the idealized concept of limiting the UAVs to two states during the inspection period, i.e., in service and obsolete, excluding maintenance, etc., to simplify the potential future value-added budget.

3 Assumptions and Notations

3.1 Assumptions

We make some general assumptions to simplify our model. These assumptions together with corresponding justification are listed below:

- **The area of interest in the model is in planar form.** In the case of small area of mountain fire occurrence area, it may be considered as a more concise flat form.
- **The detection range of the SSA UAV is circular.** Since there is a minimum pitch angle or maximum reconnaissance angle for the camera of the UAV, but it can rotate arbitrarily around the vertical axis, the detection range of the UAV at any moment is considered as circular in this model.
- **The area where the mountain fires occur is square.** When the scale of mountain fire is small, it is very concise and reasonable to approximate it by a square; when the scale of mountain fire is large, it is advisable to approximate it by a series of small squares.
- the position of impact along the bat.
- **Grid the area of fire occurrence.** Consider gridding the mountain fire occurrence area, under ideal conditions, it may be considered that the result of gridding, that is, the side length of the grid is two times the root of the SSA UAV detection range, or the side length of the mountain fire occurrence area is a positive integer multiple of two times the root of the SSA UAV detection range.
- **Ideal Drone Model** Mountain fire occurrence pattern is time-invariant within the examination period.Only two states of service and end-of-life exist for UAVs within the study area.The service life of UAVs obeys negative exponential distribution.Constant drone flight altitude.Constant energy consumption of the drone flying at a constant rate.The speed of the drone is transient.Reasonable distribution of personnel on the ground.Ideal control of mountain fires.

3.2 Notations

Symbol	Description
E	Field strength
E_0	Free space field strength
F	Free space field strength
L	Terrain loss
$\triangle\ \phi$	Phase difference of the reflected
P_k	Parent point
R_k	parent point
N_t	Time set
U_I	Drone set
w	Constraint distance
L_{LB}	Total distance lower bound
L_{RCA}	Total distance
TC	Target layer to criterion layer judgment matrix
CP	Target layer to criterion layer judgment matrix
M	Cost increase AUD
m	Drone unit price AUD
N	Total number of drones
Na	Drone loss

4 The Integrated Bushfire Response Model

4.1 The Bushfire Model

4.1.1 Background Analysis

In order to understand the extent and severity of fire distribution for the deployment of UAVs and search and rescue operations, we built a fire distribution model. We collected data [9] from this Australian forest fire to obtain the fire distribution and severity of the fires occurring in eastern Victoria. With the help of geographic analysis software, we estimated the major area of the fires. Using the model, we were able to determine the size of the area in which the UAV would work.

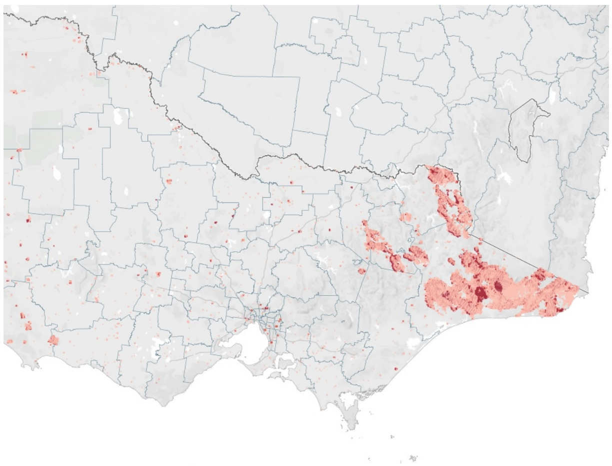

Figure 1: Fire Distribution Map

4.1.2 Model Construction

By retrieving information through the Internet, we succeeded in obtaining statistical data [8] on the occurrence of fires in Australia. Then, we used computer simulation and measurement to estimate the fire distribution map as shown in Figure 1. This map is mainly for the distribution of fires in Victoria, Australia. The map shows that fires in Australia are concentrated in the eastern part of Victoria and the southern border of New South Wales, with sporadic distribution in the southwest. The red areas marked on the map are fire areas. The shades of color indicate the extent of the fires that have occurred.

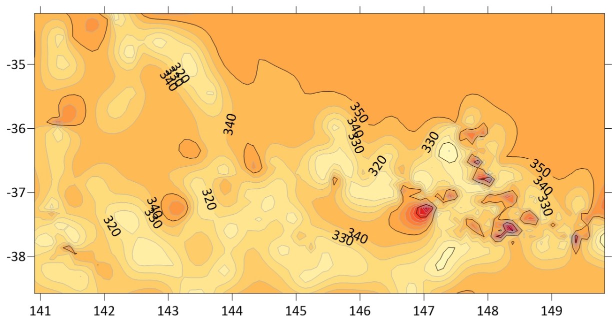

Figure 2: Fire Distribution Analysis Map

4.1.3 Conclusion

We obtained the main area in four parts as shown in Figure 2, namely 7034.5km^2, 2132.4km^2, 1803.1km^2, and 155.8km^2, through computer assistance, because the area of the rest of the fire area is too small, and the result is too ordinary in the process of using the gridding method, i.e., the number of repeaters does not exceed 1. Therefore, the above ordinary solution is not discussed and analyzed in depth.

Finally, by summing the area of the four parts, **we obtained the fire area of 11125.8 km^2 and the average value of the area of the four parts, which is 2781.45km^2**, to obtain the base value of the affected area for the model analysis. The square area is approximated and gridded to obtain a base figure that can be easily substituted into the model analysis.

4.2 The Obstructed Radio Communication Model

4.2.1 Background Analysis

The eastern part of Victoria, Australia, has a certain hilly topography as shown in Figure 3 and we mainly consider the topographic loss incurred when propagating along the hilly area. The topographic map of eastern Victoria shows the hilly conditions in the corresponding area.

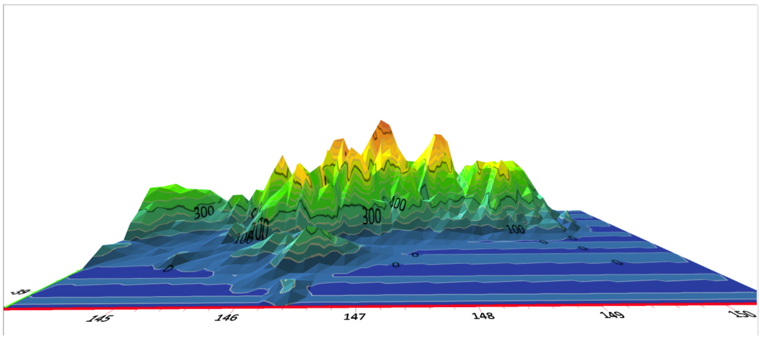

Figure 3: Topographical map of the fire area

4.2.2 Research Methodology

When the radio wave propagates along the hilly area, due to the unfavorable influence of the terrain, it has to produce extra loss, which is called "terrain loss"[3]. Many areas in the eastern part of Victoria, Australia are hilly terrain, and the influence of such loss must be considered when designing communication networks. Because of the undulating terrain in the hilly areas, the shape is very variable, so it is difficult to quantitatively describe the theoretical terrain loss, but there are still rules to follow.

According to the physical characteristics [3] of the propagation path, there are no more than two scenarios. a. When the wave propagates between two points, the hills between the two points do not block the direct path of the wave. b. The middle hill blocks the direct path of the waves, which can only go around the top to reach another point.

The hills in both cases have losses to the airwaves, and depending on the number of hills between two points, they can be classified as single-peak loss, double-peak loss and multi-peak loss [3]. We can establish the algorithm for the simple case first and deduce the algorithm for the complex case. The calculation of single-peak loss is discussed below.

4.2.3 Model Construction

In electromagnetic theory, the relative field strength $\frac{E}{E_0}$ of the electric wave crossing the obstacle and the terrain loss L can be expressed respectively as

$$\frac{E}{E_0} = Fe^{j\triangle\phi} \qquad (0 < \frac{E}{E_0} \leq 1) \tag{1}$$

$$L = 20logF \qquad (L \leq 0) \tag{2}$$

E_0 is the field strength of the electric wave propagating in free space, F is the bypassing system, and $\triangle\ \phi$ is the phase difference of the reflected or bypassing wave with respect to the direct path.

$$F = \frac{S + 0.5}{\sqrt{2}sin(\triangle\ \phi + \frac{\pi}{4})} \qquad (0 < F \leq 1) \tag{3}$$

$$\triangle\ \phi = tan^{-1}\left(\frac{S + 0.5}{C + 0.5}\right) - \frac{\pi}{4} \tag{4}$$

where C and S are Fresnel integrals.

$$C = \int_0^v cos(\frac{\pi}{2}X^2)dx \tag{5}$$

$$S = \int_0^v sin(\frac{\pi}{2}X^2)dx \tag{6}$$

V is a dimensionless covariate.

$$V = -h\sqrt{\frac{2}{\lambda}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)} \tag{7}$$

As shown in Figure 5,λ is the wavelength of the electric wave, and the meanings of h, γ_1 and γ_2 are the inter-peak distance. When the communication signal curves downward, h is negative and V is positive; when the communication signal curves upward, h is positive and V is negative.

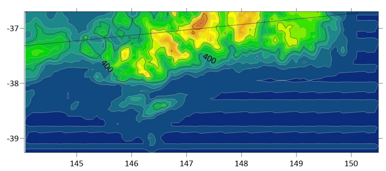

Figure 4: Contour topographic map

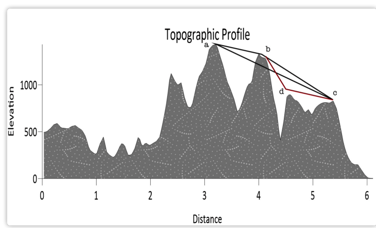

Figure 5: Communication blockage diagram

4.2.4 Conclusion

Based on the above mathematical analysis study, we found and applied the model to a topographic map of eastern Victoria, Australia. For the more typical topographic areas, we calculated the radio impact and simplified it to the following conclusions. The communication loss in the area where the fire occurred can be approximated as if it were the same area.

Considering a drop of 200 meters between the two peaks, 10 km each at the two endpoints of the convergence at the shelter, and the communication using meter wave communication, **we can get the maximum transmission distance of the signal attenuated to 8 km, with a mean value of 12 km**. We can use this data to complete the coverage of the repeater.

4.3 The Repeater UAV Network Model

4.3.1 Background Analysis

First analyze the role of the repeater drone, which carries HD and thermal imaging cameras and telemetry sensors that can be used to monitor the implementation of reporting frontline personnel as well as data from wearable devices. Only therefore the repeater UAV can help monitor the changing fire scene situation.

However, because of its limited wireless signal transmission, even with a transceiver that automatically redials high-powered signals, it still requires multiple UAVs to work together to ensure unobstructed signal propagation between the front line and the EOC. This requires a certain arrangement and networking of UAVs. Each hovering repeater UAV is a node, and the different nodes form a communication network.

In this network nodes communicate with each other through wireless chain transmission. But due to the limitation of transmitting frequency. Each repeater has only a certain coverage range, and when it wants to cover points outside its range, it needs a new repeater to cover the designated area with a series of coverage zones that overlap it completely. Each coverage area has one repeater. Frontline personnel in one coverage area can communicate with each other and the EOC. Frontline personnel in multiple coverage areas can communicate using several channels at the same time. Users between different coverage areas can be forwarded through intermediate nodes.

4.3.2 Research Methodology

We used Voronoi graph [4]and Delaunay triangle to solve the problem of site selection as shown in Figure 6 and Figure 7.

4.3.3 Voronoi graph and Delaunay triangle network

For the need of the construction of the model, we first introduce in a brief manner of the Voronoi graph [4] as shown in Figure 6 and Delaunay triangle network [5] as shown in Figure 7. They are reciprocal pairs of graphs, and are classical research problems in the field of computational geometry.

Voronoi graph:

Definition 2.1: A Voronoi diagram, also called a Tyson polygon or Dirichlet diagram [4], consists of a set of contiguous polygons formed by the perpendicular bisectors of lines connecting two neighboring points. n points that are distinct in the plane divide the plane according to

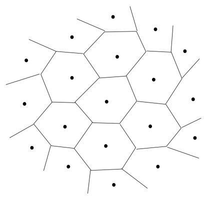

Figure 6: Voronoi diagram

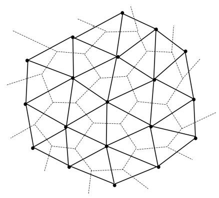

Figure 7: Delaunay triangle network

the nearest neighbor principle; each point is associated with its nearest neighboring region. A Delaunay triangle [5] is a triangle formed by connecting related points that share an edge with an adjacent Voronoi polygon. The center of the outer circle of a Delaunay triangle is a vertex of the Voronoi polygon associated with the triangle.

For a seed point $\{P_0, P_1, ..., P_n\}$ in a point set P_k , its Voronoi region P_k is defined as

$$R_k = \{x \in X | d(x, P_k) < d(x, P_j), j = \{0, 1, 2, \cdots, n\}, j \neq k\} \tag{8}$$

Theorem 4.1. *All points in a Voronoi polygon are closer to the parent point of the polygon than to any other parent point.*

Theorem 4.2. *Only those points that are closer to the parent of a Voronoi polygon can form an edge of that polygon.*

Theorem 4.3. *An edge of a polygon is a perpendicular bisector between its nearest two parent points.*

Definition 2.2: Voronoi polygons with common edges become adjacent Voronoi polygons. The triangular network formed by connecting the parent points of all adjacent Voronoi polygons becomes the Delaunay triangular network

Theorem 4.4. *Empty Outer Circle Property) In a Delaunay triangle network formed by a point set V, the outer circle of each triangle does not contain any other point in the point set V.*

Theorem 4.5. *Empty Outer Circle Property) In a Delaunay triangle network formed by a point set V, the outer circle of each triangle does not contain any other point in the point set V.*

4.3.4 Model Construction

We experimented with overlapping using different combinations, such as circles, and squares, and then explored the overlapping areas of external circles and compared them. The overlapping area can reflect the utilization of the resources of the UAV

After comparison, we found that when using the Voronoi map and the Delaunay triangle mesh, the Delaunay triangle is as close as possible to an equilateral triangle as shown in Figure 10, and its counterpart, the Voronoi map as shown in Figure 11, will be as close as possible to a regular hexagon [1].

When filling the region with a number of equal hexagons, when the centers of any three adjacent hexagons form a triangle, so that the sum of the overlapping areas of the external circles of each hexagon and the external circles of the six hexagons around it is minimized.

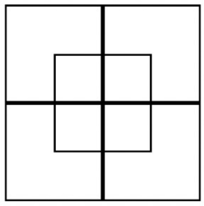

Figure 8: Square filling

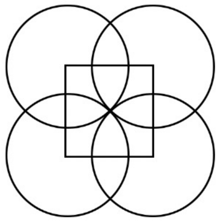

Figure 9: Round filling

Applied to our practical problem, this implies that the repeater UAV can provide communication coverage to the frontline personnel to the maximum extent.

This is a constrained nonlinear optimization problem [1]. We study the local configuration of this topology [1] so that the sides of the triangle formed by the center are of equal length. The two layers are parallel to each other. In this case, my use of UAV resources is minimal. Otherwise, the number of circles intersecting the external circles of the square hexagon needs to be increased and more overlapping area will be wasted.

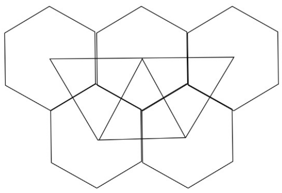

Figure 10: Hexagonal filling

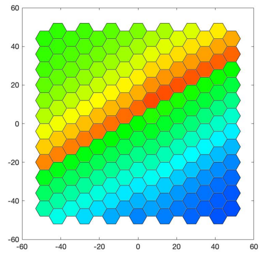

Figure 11: Hexagonal filling simulation

4.3.5 Conclusion

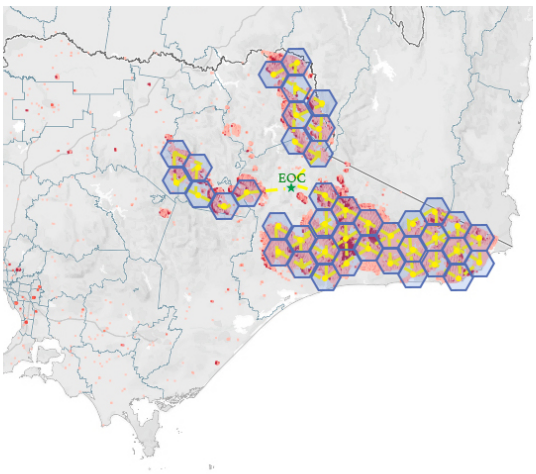

Figure 12: Repeater UAV distribution map

With the communication hindrance model, we obtained how the communication capability of the repeater UAV was affected by the terrain. Combining the data, we calculated the range of the radio in the event of a fire in eastern Victoria.

We then simulated with the corresponding model to obtain the optimal distribution as shown in Figure 12 and optimal number of repeater drones in a realistic geographical situation. The number of repeater drones required for this mission is 40.

4.4 The SSA UAV Coverage Model

4.4.1 Research Methodology

First, define the system model. The problem requires a solution to control a formation of UAVs to cover an area[2]. Define R as the rectangular area to be covered by the problem. The target area is gridded, and by the premise assumption, the root 2 times the UAV detection range of the side length of the grid is obtained, and it may be useful to note that the UAV detection range is r and the side length of the grid is c, i.e., $c = \sqrt{2}r$, as a way to normalize or unitize the system model. Now examine the vertices of the grid. If the UAV is able to traverse any vertex of any grid, also called a node [2], then full coverage of the area can be achieved. Without loss of generality, we may consider $c = 1$.

After defining the grid unit length c, we can partition the region R into grids. Without loss of generality, after the gridding process such that $c = 1$, the region R can be written as $R = \{(x, y)|0 < x \leq X, 0 \leq y \leq Y\}$, where X denotes the length of the rectangular region and Y denotes the width of the rectangular region.

Define the coordinates $p = (x, y)$ in the two-dimensional plane as the nodes of the grid, where $x, y \in N$, $p \in R$. $P = \{p_i = (x_i, y_i)|1 \leq i \leq n, p_i \in R\}$ denotes the set of all nodes, $n = XY$ denotes the region R of the number of nodes of the grid present in the interior. Define the distance between two points $a = (x_a.y_a)$, $b = (x_b, y_b,)$ in the two-dimensional plane, denoted as: $d(a, b) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$.

The inter-travelable distance is the individual grid length C. At time t, the position of the UAV u_j is seated and marked as $u_{j,t} = (x_{j,t}, y_{j,t})$,$t \in N_T$ where $N_T = \{0, 1, \cdots, T\}$, and T is the time when the final coverage task is completed. Next we define the three constraints that the UAV formation faces in completing the mission is.

Movement constraint: The UAV can only move along the grid lines toward the adjacent nodes and can move no more than 1.

$$d(u_{j,t}, u_{j,t+1}) \leq 1 \qquad \forall u_j \in U \qquad \forall t \in N_t \tag{9}$$

Connectivity constraint: The maximum communication distance w between UAVs indicates the minimum communication distance that can be guaranteed when two neighboring UAVs exchange information using communication devices.

$$d(u_{j,t}, u_{j+1,t}) \leq w \qquad \forall u_j \in U \qquad \forall t \in N_t \tag{10}$$

Coverage constraint: define the function $y_{i,j,t}$. to indicate that the UAV u_j scans the grid node p at moment t:

$$y_{i,j,t} = \begin{cases} 1 & u_{j,t} = P_i \\ 0 & \text{else} \end{cases} \tag{11}$$

Shows that for each node in the grid, there exists at least one UAV that has scanned it, and the coverage constraint can be expressed by the function $y_{i,j,t}$ in the following way.

$$\sum u_j \sum t y_{i,j,t} \geq 1 \qquad \forall P_i \in P \tag{12}$$

After the above constraints are defined, the optimization objective of the problem is studied. Since there are no individual differences in the UAVs in the UAV formation, all UAVs have the same flight radius. For the whole UAV formation, the member with the longest flight distance represents the movement distance or movement cost of the whole UAV formation.

4.4.2 Model Construction

To solve the above problem, the rectangular-region centralized algorithm, referred to as the RCA algorithm (Rectangular-coverage Centralized Algorithm), is hereby introduced.

First, the nature of the optimal solution is examined [2]. If the traversal is performed for all nodes in a non-repetitive scanning manner, the coverage task is constant after the gridding process. Therefore, a plausible reasoning is that the traversal task should be distributed equally to each UAV as much as possible, and at the same time, the number of repeated traversals should be minimized to obtain the optimal solution. Based on the analysis of the above properties, the present algorithm will be divided into two phases.

1. Deploying the UAVs to the corresponding positions on the x-axis.
2. Controlling all UAVs to complete the longitudinal coverage, i.e. traversing the y-axis.

Since different UAVs complete the longitudinal traversal at different points in time and there is a time interval between them, the UAV that completes the given task first needs to assist the remaining UAVs to complete part of the task in order to maximize the efficiency to reduce the task time.

The one member that flies the longest distance represents the distance or cost of movement of the entire formation [6]. By denoting the distance moved by the UAV u_j as L_j, we have $L_j = \sum_{t=0}^{\tau=1} d(u_{j,t}, u_{j,t+1})$, then the moving distance or moving cost of the UAV formation can be defined as $L_j least = max_{u_j}(L_J)$. Our goal is to minimize the distance travelled by the UAV formation. To summarize the above, the problem we study can be described in the following form.

$$min_{u_j \in U}(\sum_{t=0}^{\tau-1} d(u_{j,t}, u_{j,t+1})) \tag{13}$$

S.T.

$$d(u_{j,t}, u_{j,t+1}) \leq 1 \qquad \forall u_j \in U \qquad \forall t \in N_t \tag{14}$$

$$d(u_{j,t}, u_{j+1,t}) \leq w \qquad \forall u_j \in U \qquad \forall t \in N_t \tag{15}$$

$$\sum u_j \sum t y_{i,j,t} \geq 1 \qquad \forall P_i \in P \tag{16}$$

Using the area unidirectional scanning algorithm and the area chunking round-trip scanning algorithm [6], we obtain.

$$L_{LB} = XY + mY \tag{17}$$

Ultimately, according to the RCA algorithm, the total distance travelled by the UAV formation can be expressed in the following form.

$$L_{RCA} = L_{LB} + (m-1)(X+1) \tag{18}$$

The answer is easily obtained by applying a special form of rectangle, i.e., square. All of the above scenarios require that the traversed area be unicontiguous, i.e., that the flight altitude, or the height of the UAV from the ground, be constant. However, the actual situation is much more complicated than the one discussed above. The situation in the multiconected form should also be discussed.

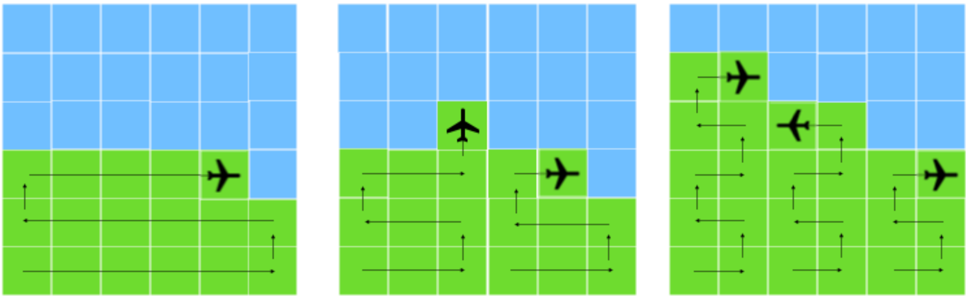

Figure 13: Aircraft flight path map

It is noted that, as shown in Figure13, a multi-connected region can be naturally divided into several rectangular regions, and therefore, applying the formula for the total distance travelled by the UAV formation derived by the RCA algorithm in the previous section, a similar formula for the multi-connected region is easily obtained based on the linearity of the distance superposition.

4.4.3 Conclusion

To save time, each drone was flown at the fastest speed. According to the parameters of the UAVs provided in the question, a flight at this speed can only last 25 minutes, plus 105 minutes for charging, for a total of 130 minutes and a flight distance of 20 km, which is a newly defined unit of distance measurement.

Following the flight pattern shown in Figure 13, we obtain the results in Figure 14. With a side length of 20 km, the UAV mission time: 520 minutes for one, 260 minutes for two or more. It can be seen that for the above square area, if too many UAVs are deployed at the same time, it will lead to overlapping of some UAVs' coverage tasks, so when the number of UAVs increases to a certain level, the mission time will no longer be shortened. Therefore, it is said that there is a limit value for the completion time of this traversal task.

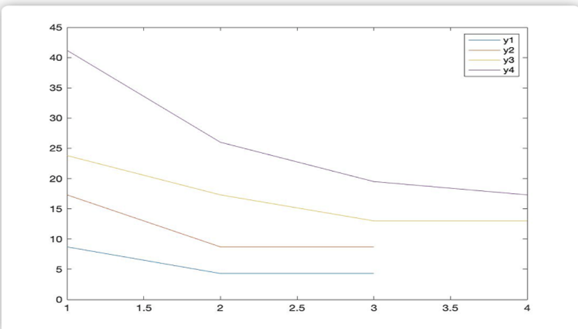

Figure 14: Search time with different number of aircraft and different basic range

Finally, we get two UAVs deployed per unit area, based on the calculation of the fire area of 40 units of area. **We can get the result that 80 SSA drones are deployed. These UAVs can achieve a complete detection of the fire area every 4.3 hours.**

4.5 The Integrated Bushfire Response Model

4.5.1 Background Analysis

The purpose of our integrated fire response model is to provide a weighted relationship between multiple influencing factors regarding the number and proportion of each of the two types of UAVs, so as to provide a more focused and rational approach to resource allocation and mobilization in practical applications, thus avoiding blind guesswork.

Our goal is to determine the number and proportion of UAVs needed to set up the fire risk in eastern Victoria, Australia, in the next ten years. To address these issues, we established the following four criteria, namely fire size, fire frequency, economic budget, and geographic topography, for a total of four factors. These four factors will ultimately determine the relationship between the number and proportion of two types of UAVs needed to respond to future fire hazards, which may be referred to as Type I UAVs and Type II UAVs, corresponding to SSA UAVs and repeater UAVs, respectively.

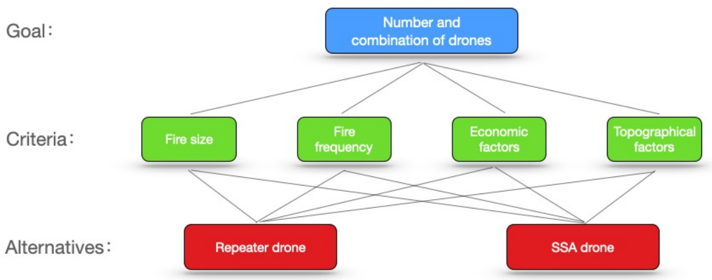

Figure 15: AHP structure

4.5.2 Research Methodology

There are generally four steps in using hierarchical analysis

- **Establish a hierarchical model.** The structure is devided into three levels, noted as the target level, the criterion level and the plan level.
- **Construct pairwise comparison array.** Starting from the 2nd level of the hierarchical model, for each factor of the same level that belongs to the previous level.
- **Calculate the weight vector and do the consistency test.** For each pairwise comparison array, the maximum feature root and the corresponding feature vector are calculated, and the consistency test is done by using the consistency index, random consistency index and consistency ratio.
- **Calculate the combination weight vector and do the combination consistency test.** Calculate the combined weight vector of the lowermost pair of targets and do the combined consistency test according to the formula.

4.5.3 Model Construction

By means of the following scale of scales, viz.

Scale of proportions factor i to factor j	Quantitative values
Equally important	1
Slightly important	3
Median important	5
Strongly important	7
Extremely important	9
Median value of two adjacent judgments	2,4,6,8

Table 1: Scale of proportions

We obtain the judgment matrix for the corresponding level, where for the ideal judgment matrix, the following equation holds constant.

$$a_{ij} = \frac{1}{a_{ji}} \tag{19}$$

By solving the eigenvalues and eigenvectors of the judgment matrix [7], calculated by Matlab, we obtain

$$TC = \begin{pmatrix} 1.0 & 1.0 & 1.0 & 2.0 \\ 1.0 & 1.0 & 1.0 & 2.0 \\ 1.0 & 1.0 & 1.0 & 2.0 \\ 0.5 & 0.5 & 0.5 & 1.0 \end{pmatrix} \tag{20}$$

	Fire size	Fire frequency	Economic factors	Topographical factors
Fire size	1.0	1.0	1.0	2.0
Fire frequency	1.0	1.0	1.0	2.0
Economic factors	1.0	1.0	1.0	2.0
Topographical factors	0.5	0.5	0.5	1.0

Table 2: Target layer for the criterion layer judgment matrix

$$TCX = \begin{pmatrix} -0.8944 & -0.5547 & -0.8944 & -0.7130 \\ 0.2981 & -0.5547 & 0.2981 & -0.3617 \\ 0.2981 & -0.5547 & 0.2981 & 0.5373 \\ 0.1491 & -0.2774 & 0.1491 & 0.2687 \end{pmatrix} \tag{21}$$

$$TCB = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

The feature vector corresponding to the maximum feature root of the judgment matrix is normalized and denoted as w [7]. The elements of w are the ranking weights of the same hierarchical factor with respect to the relative importance of a factor in the previous hierarchical factor. This process is called hierarchical single ranking.

After the consistency test, it is found that the weights are assigned more consistently, and the weights of size, frequency, economy, and topography are obtained as approximately. 0.2947, 0.2947, 0.2947, and 0.1467.

If a rough calculation is performed, the weights can be adjusted accordingly to. 0.3, 0.3, 0.3, and 0.1.

After that, the category I UAVs and category II UAVs under the influence of a single factor can be analyzed separately, and finally the above results are calculated as a weighted average according to the ratio of the weights determined by the hierarchical analysis method to obtain the final theoretical values. The above is the corresponding result of the criterion layer.

Similarly, the corresponding results of the program layer are

$$CP = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{23}$$

	Number of SSA drones	Number of repeater drones
Number of SSA drones	1.0	1.0
Number of repeater drones	1.0	1.0

Table 3: Judgment matrix of the criterion level for the program level.

$$CPX = \begin{pmatrix} -0.7.71 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix} \tag{24}$$

$$CPB = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \tag{25}$$

Similarly, the weights of quantity and proportion [**?**]are obtained as 0.6667 and 0.3333.

If a rough calculation is performed, the weights [7] can be adjusted accordingly to 0.7, 0.3, or 2/3, 1/3.

The subsequent operations are similar.

4.5.4 Conclusion

From the above analysis, we know that the ratio between the number of SSA UAVs and the number of repeater UAVs should be 2:1, which is consistent with the results given by our correlation quantitative model. **Based on the correlation quantitative model, we get that the number of SSA drones is 80 and the number of repeater drones is 40.**

5 The Bushfire Prediction Model

5.1 The Bushfire Prediction Model

5.1.1 Background Analysis

In order to assess the impact of extreme mountain fire events on the model over the next decade, it is first necessary to forecast the trend of mountain fires. The development of fires is a continuation of past fire trends, provided that external conditions remain unchanged. Therefore, by analyzing the historical statistics of mountain fires in Australia to build a fire prediction model, future fire trends can be obtained.

5.1.2 Research Methodology

We combined the above model with the fire situation in Australia in recent years for statistical analysis. Then we obtained the corresponding prediction results and derived the corresponding trend equations. The result is an exponential equation with an increasing trend of fire occurrence, indicating an increasing trend of fire occurrence in the region. We address the topography and derive the mean values of the number and size of possible fires

5.1.3 Model Construction

- **We performed the data collection and screening process.** On the official report of the Australian government's Department of Agriculture, we found the annual mountain fire area area of Victoria forest from 1980-2018 [10], consisting of unplanned fire and planned fire. As a mountain fire emergency department, obviously unplanned fire is the data we need to analyze, as the following figure

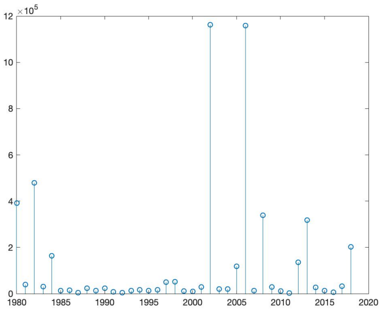

Figure 16: Fire area (hectares) 1980-2018

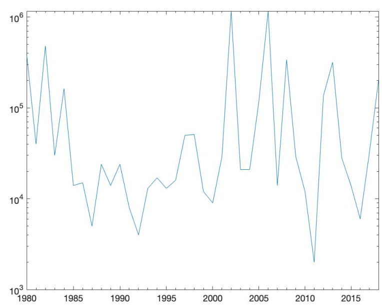

Figure 17: Average fire area

- **We fit the data exponentially.** It can be seen that the area of unplanned fire area has a tendency to increase over time. However, from the fitted curve, it is easy to see that the area of the area corresponding to the curve is not predictive of the extremes, and the area corresponding to the extremes is much higher than the fitted curve.

  In the Figure 15 below it can be seen more obviously that the area of the area of extreme mountain fire years and can account for about 70%, so the treatment of the area of extreme mountain fires becomes an issue that cannot be ignored.

- **We observe the year and number of extreme mountain fires.** In terms of year distribution, although the number of extreme mountain fire situations since 2000 accounts for a relatively large number, the pattern is not obvious. In 2002 and 2006, the two most serious mountain fires in 28 years occurred in just four years, while the area of the mountain fire area has been in a relatively stable state during the 16 years from 1985 to 2001.

  Thus, we can see that this extreme situation has considerable uncertainty, the exact year of occurrence and scale cannot be determined. However, an examination of the number of occurrences of extreme mountain fires can be obtained.

- **By examining various factors we conclude** There were 8 extreme mountain fires in 28 years, and it can be calculated that there are 2.85 extreme mountain fires per decade. On

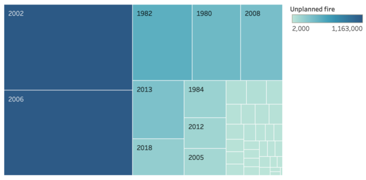

Figure 18: Annual fire area share chart

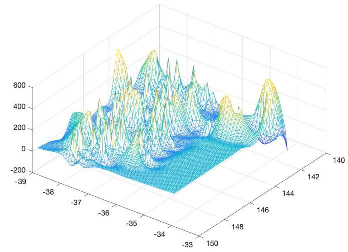

Figure 19: Fire frequency map of the mountain region

average, extreme mountain fires occur 2-3 times in a decade, which becomes one of the bases for our model improvement.

- **We examined the areas where extreme mountain fires are likely to occur.** A gridded analysis of the 2019-2020 Victorian wildfire distribution data shows that northeastern Victoria has a high frequency and size of fires, with areas of fire along and on both sides of the mountain range. This area is densely forested and has a high incidence of fires, and will remain a priority area for extreme mountain fire conditions over the next decade.

  The Figure 19 below shows the frequency of fires in the region in question, with satellite monitoring of the fires used to predict the size.

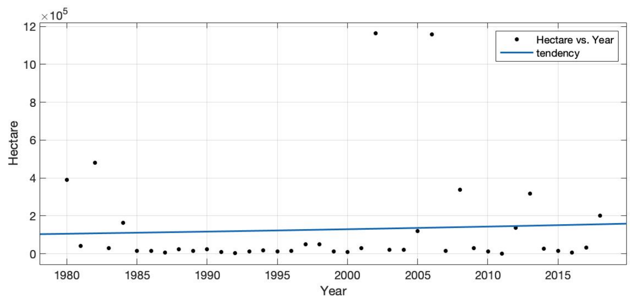

Figure 20: Trend of fire area

5.1.4 Conclusion

We have analyzed historical fires in Australia by collecting data, curve fitting, and extreme case processing. We obtained the future scenario of fires in the region. The results show that the occurrence of fires in the region is on the rise and fits certain mathematical quantitative relationships. **We conclude that extreme hill fires occur on average 2-3 times in a decade.**

5.2 Additional Fee Calculation Model

5.2.1 Background Analysis

Any electromechanical system requires appropriate safeguards and support equipment to support its proper operation. Safeguard and support equipment that can be configured for different levels.

For UAS used in fires, these machines need to be able to be operational on the front line at all times. Therefore we need to consider that in the future, in order to ensure the maintenance and repair of the UAS, we need to increase certain costs. This includes the repair of broken and damaged drones. For such expenses, we plan to build a mathematical model to calculate the attrition rate and the corresponding costs.

We also calculate some daily consumption costs, such as periodic replacement of batteries and parts, as well as lubricant, cleaning oil and storage and transportation costs.

5.2.2 Research Methodology

We have built a mathematical model about the wear and tear of the UAV. Through the corresponding mathematical analysis, we obtained the wear and tear rate about the UAV. Then we combined the query data to get the corresponding cost expenditures.

5.2.3 Model Construction

Referring to the statistical literature, it can be obtained that the service life of UAVs approximately obeys the exponential distribution. And the exponential distribution with good quality memorylessness will facilitate the calculation of annual obsolescence.

Denote x as the service life of the UAV. The time is discretely timed in units of one year.

Therefore, its distribution function is

$$F(x) = \begin{cases} 1 - \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases} \tag{26}$$

Denoting the scrapping rate of the UAV in one year as ρ, we have

$$\rho = 1 - \lambda e^{-\lambda} \tag{27}$$

Denoting the total number of drones as N and the annual scrapping of drones as N_a, we have $N_a = N\rho$

By reviewing relevant information, we understand that λ is approximately equal to 1/6.5, which is about 0.1538, so $e^{-\lambda}$ is about 0.1426. The total number of UAVs is known to be 120, so the annual scrapping of UAVs is about 17.

5.2.4 Conclusion

Therefore, the new investment in the next ten years is about $170,000 per year for the phase-out and replenishment of UAVs.

6 The Repeater Location Model

6.1 Background Analysis

The optimization was carried out for the repeater UAV network model. The model can show its superiority under more ideal geographical conditions. However, in more complex terrain changes, the radio range of the repeater UAVs will be affected, and if a single standard is still used, it will cause a loss of efficiency. We also need to develop programs to deal with the effects of different fire sizes.

6.2 Model Construction

The positive hexagon remains the most efficient in filling, and we continued the filling model using the positive hexagon. Then for different terrains, we calculated different side lengths of the hexagon. Instead of repeaters, different sizes of regular hexagons were used for different terrains.We collected the data [8] of this fire for simulation

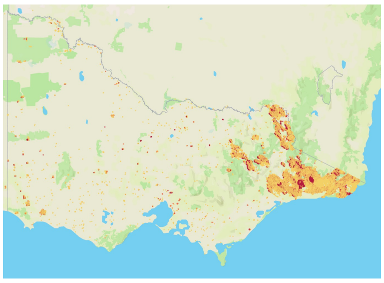

Figure 21: Fire distribution map

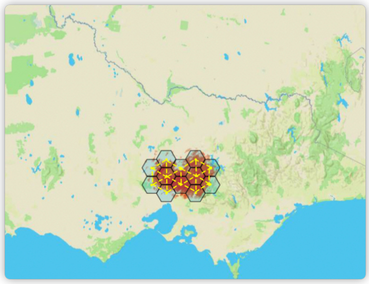

Figure 22: Small-scale fires in plain terrain

We discussed the location optimization of the repeater on different terrains for the same range as shown in Figure 22 and 23. In plain areas the repeaters have a wider radio range and therefore the simulation uses longer side lengths of the hexagon. In contrast, in mountainous jungle areas, where communication is blocked, we can use a smaller, more dense network of repeaters with smaller side lengths.

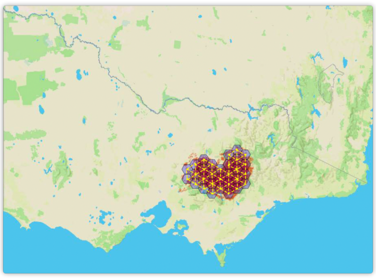

Figure 23: Large-scale fires in mountainous terrain

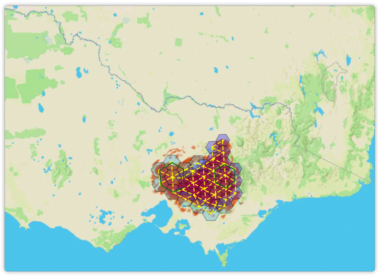

Figure 24: Large-scale fires in plain and mountainous terrain

We also discuss the effect of different sizes of fires on the repeater locations as shown in Figure 22 and 24. In small fires, repeaters can work at the same kind of range. In this case, the resource utilization is the highest. However, in large scale fires, the fire area includes plains and jungle mountains. If a single standard is still used for deployment, it will cause some waste of resources. In the simulation we divided the different terrains and deployed with repeaters of different ranges, and it can be seen that this combination is more efficient.

6.3 Conclusion

Our improved algorithm developed an optimized repeater drone position model for fires of different sizes occurring in different terrains, and computational simulations were performed and represented in conjunction with the eastern terrain of Victoria, Australia **As a result we have the optimal repeater UAV position model as shown in Figure 25.**

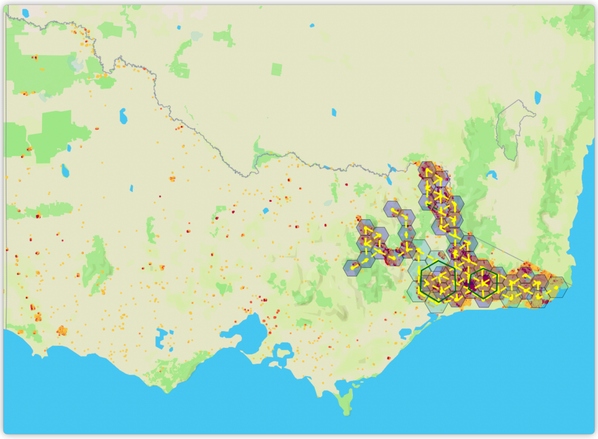

Figure 25: Optimized UAV Location

7 Sensitivity Analysis

For several sub-models of the mathematical model we developed, most of the data were relatively easy to obtain. However, in the prediction of cost increase in the next 10 years, we found that the end-of-life index λ of UAVs is difficult to measure, and several reviews of relevant information and data failed to obtain more accurate and consistent results. Therefore, the estimation of λ is not particularly accurate. λ has considerable room for fluctuation. Therefore, it is necessary to examine the sensitivity of the final cost increase to the UAV obsolescence index λ.

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$$Na = N(1 - e^{-\lambda}) \tag{28}$$

$$M = mN(1 - e^{-\lambda}) \tag{29}$$

$$M(\lambda, m) = mN(1 - e^{-\lambda}) \tag{30}$$

Since M is related to λ, the change $\triangle M$ due to $\triangle\lambda$ is examined and the proportion of its relative change is calculated, i.e. $\frac{\triangle M}{M} \cdot \frac{\lambda}{\triangle\lambda}$

$$\frac{\mathrm{d}M}{\mathrm{d}\lambda} = mNe^{-\lambda} \tag{31}$$

Defining the above equation as the sensitivity of M with respect to λ, or the sensitivity function.

$$S(M, \lambda) = \frac{\mathrm{d}M}{\mathrm{d}\lambda} \cdot \frac{\lambda}{M} = \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}} \tag{32}$$

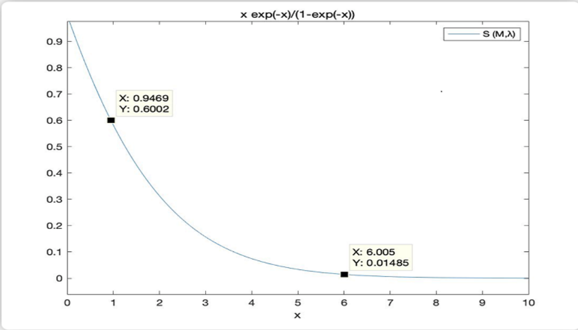

Figure 26: Sensitivity analysis

We can tell from the plot in Figure 26 that M is not sensitive anymore when λ is large enough, which means if the quality of drones is good enough, we can forget about the small fluctuation of λ.

8 Conclusion

8.1 Strengths

Our model is based on numerous theoretical foundations. After a careful search of relevant information and literature, we have selected some key parameters for building the model after several deliberations. The final result is a more complete model with a simple basic model as the backbone, which can analyze many practical and complex situations. Thus, our model is ideally close to the real situation.

In order to build a simple and elegant mathematical model, we have consciously made reasonable simplifications to build a model based on graph theory, coverage theory and radio communication theory to solve the severe problem of frequent and widespread hill fires in Victoria, Australia. Our model gives results that are consistent with reality and have a good degree of confidence.

We tested the applicability of our model by taking into account real-world scenarios and by considering the actual geographic location and topographic features of eastern Victoria, Australia. As a result, we were able to obtain a more generalized mathematical model. At the same time, our mathematical model is highly operational and feasible for all actual data results.

Since the tree species in the forests of eastern Victoria, Australia, differ significantly from those in other regions, future researchers can continue to explore the micro and macro effects of this biological factor in order to extend our findings in eastern Victoria, Australia, to the world. In the meantime, future researchers can simulate and demonstrate our existing results based on the meta-automata model, and further promote other research and presentation of related results.

8.2 Weaknesses

In order to simplify the mathematical model, we ignore a considerable number of disturbing factors, such as the wind factor. Similar omission may have a profound impact on the generalizability of the mathematical model. Therefore, for solving this problem, future researchers may need to analyze the above problem in depth from the perspective of spherical vector fields.

For other problems, future researchers also need to be cautious and take a steady and thoughtful step by step approach.

At the same time, it should be noted that some of the assumptions are not necessary. Just as the parallel axiom is not necessary for geometry, it is only one of the criteria for categorical discussions. Removing some of the assumptions and replacing them with other new ideas can lead to some interesting and useful results. For some of the controversial assumptions, we will have to wait for time and experimental tests.

8.3 Further Discussion

We have tried to construct a mathematical model that is applicable to the shape of various geographical regions. However, limited by our poor mathematical skills and less than proficient computer simulation techniques, we make many presuppositions for the validity of the existing mathematical model. For example.

We consider the small-scale spherical geometry problem as a more concise and simple planar geometry problem.

We assume that the detection area of the SSA UAV is ideally circular. We use a standard square to approximate the units of the fire occurrence area and a more desirable gridding treatment for mathematical analysis.

We may reduce some of the assumptions in the future and propose a general mathematical model.

9 Budget Request

To: Victoria State Government
From: Team 2125753
Date: February 8, 2021
Subject: Budget Request for CFA new department

Background

Drones are used to fight fires and can help firefighters quickly access information on the ground, while serving as signal relay stations to extend the range of personnel and have a significant role in hill fires.

In order to form CFA's new division–the Fire Emergency Response Division, new UAVs and their associated equipment needed to be acquired. We calculated the exact amount of drones and related equipment that needed to be purchased, as well as the supporting facilities and personnel expenditures. The data is presented in the following table.

Justification

Based on The Bushfire Prediction Model, we have derived extreme fire scenarios that are likely to occur in the next decade. Since the time of occurrence cannot be determined, we have to maintain the ability to respond to extreme mountain fires at all times.

Besides, after further analysis with The SSA UAV Coverage Model, The Repeater UAV Network Model and The Obstructed Radio Communication Model, we derived a specific number of drones with a total standing capacity of 120, of which the number of SSA drones is 80 and the number of repeater drones is 40. In addition, based on Additional Fee Calculation

	EXPENSES DESCRIPTION	BUDGET
1	SSA drones	$800,000
2	Repeater drones	$400,000
3	Replenished drones	$170,000
4	Radio Equipment	$4900
5	Fire Hose	$1000
6	Fire Vehicle Maintenance	$5700
7	Shop Supply/ Tools/ Equip	$3800
8	Clothing	$9000
9	Gen Medical / Fire Supply	$3200
10	Office	$900
11	Training Expense	$700
12	Salaries	$37000
13	Others	$18000
	Total	$1454200

Model, we calculated the annual attrition rate of UAVs to be 14.26%, so the number of UAVs to be replenished is 17.11.

Given the need to form a new department, we investigated other expenses such as Fire Vehicle Maintenance, Gen Medical / Fire Suppl, Fire Hose and that sort of things.Then, by reviewing the annual budget sheets of several fire departments, we got their specific amounts.

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