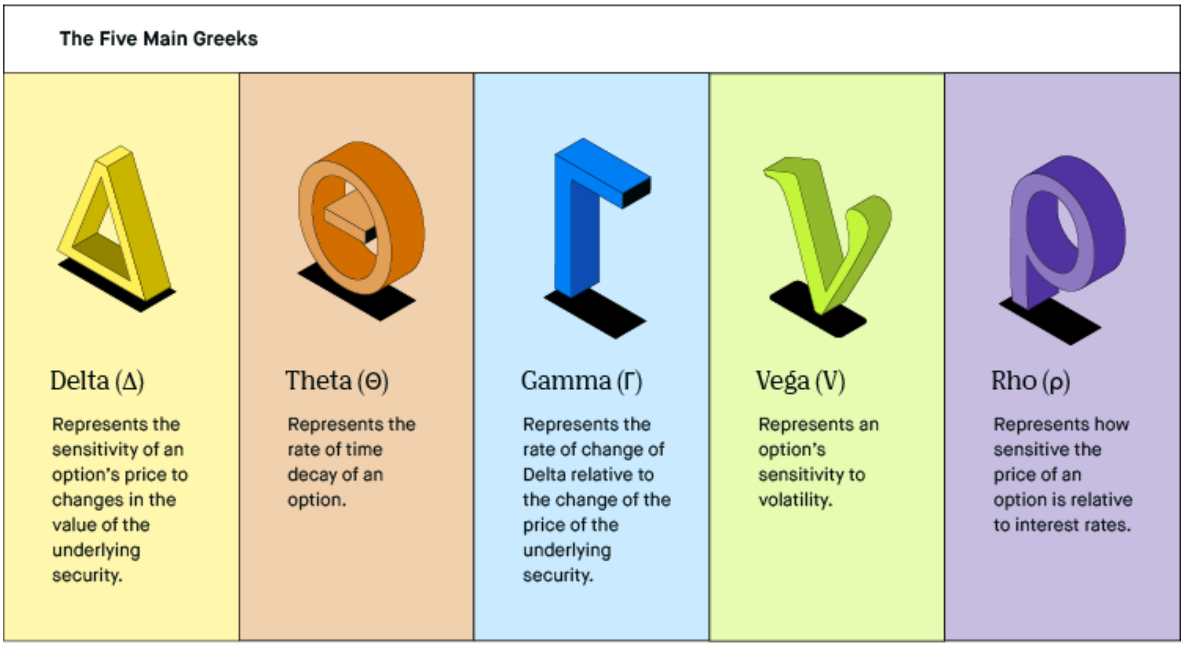
OPTIONS GREEKS - DELTA



HEMANT THAPA

Option Greeks are advanced financial metrics employed by traders to assess the sensitivity of option contracts to various factors associated with the underlying asset. These factors typically include the price of the underlying asset, its volatility, and the passage of time. The Greeks serve as a valuable tool for implementing effective hedging strategies and mitigating risks within a position. Additionally, they facilitate the optimization of options positions at any given moment.

Among the various types of option Greeks, Delta stands out as a prominent metric utilized to calculate the rate of change in the price of an option contract in response to fluctuations in the price of the underlying asset. Delta is symbolized by the Greek letter Δ and plays a crucial role in assessing the price dynamics of options.m

$$\Delta = \frac{\partial V}{\partial S}$$

Where V is the value of the option and S the price of the underlying asset.

Option on Microsft stock has a delta of 0.3, it essentially means that a 1 dollars change in the price of the underlying asset i.e., Microsoft stock, will lead to a change of 0.3 dollar in the price of the option contract.

Delta is commonly used to assess the risk exposure of derivatives positions and to manage portfolio risk. It is expressed as a numerical value between -1 and +1, representing the degree of price movement correlation between the derivative and the underlying asset.

Delta hedging refers to the strategic approach undertaken by traders when establishing positions based on the delta sensitivity of option contracts. The objective is to attain a portfolio that is delta-neutral, thereby effectively mitigating the risks associated with fluctuations in the underlying asset prices. Delta hedging, owing to its intricate nature, is predominantly employed by seasoned professionals within substantial financial institutions.

In the realm of options, the delta value of a call option is invariably positive, indicating its responsiveness to changes in the underlying asset's price. Conversely, a put option possesses a consistently negative delta, signifying its inverse relationship to fluctuations in the underlying asset's value.

LIBRARIES

In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
import seaborn as sns
import datetime
import math
import warnings
warnings.filterwarnings("ignore")
```

1. Right to Buy vs. Right to Sell: A call option gives the holder the right to buy an underlying asset at the strike price, while a put option gives the holder the right to sell the underlying asset at the strike price.

2. Market Expectation: Investors typically buy call options when they expect the price of the underlying asset to rise, as they can profit from buying the asset at a lower strike price and selling it at a higher market price. On the other hand, investors purchase put options when they anticipate the price of the underlying asset to fall, as they can profit from selling the asset at a higher strike price and buying it back at a lower market price.

3. Payoff Profile: The payoff profile of a call option is asymmetric. If the price of the underlying asset rises above the strike price, the call option holder can benefit from the price increase. However, if the price falls or remains below the strike price, the maximum loss is limited to the premium paid for the option. For put options, the payoff profile is also asymmetric. If the price of the underlying asset falls below the strike price, the put option holder can profit. If the price rises or remains above the strike price, the maximum loss is limited to the premium paid for the option.

1. Risk and Reward: With call options, the potential for profit is unlimited if the price of the underlying asset rises significantly. However, the maximum loss is limited to the premium paid for the option. For put options, the potential for profit is limited to the difference between the strike price and the market price of the underlying asset, while the maximum loss is limited to the premium paid for the option.

1. BLACK-SCHOLES-MERTON MODEL

The Black-Scholes-Merton model is a widely used mathematical model in finance, specifically in the field of options pricing and analysis. It was developed by economists Fischer Black, Myron Scholes, and Robert Merton in the early 1970s. This groundbreaking model provides a theoretical framework for valuing European-style options on stocks, indices, currencies, and other underlying assets.

The Black-Scholes-Merton model assumes that the price of the underlying asset follows a geometric Brownian motion, meaning it moves randomly but with a constant volatility. It also assumes that there are no transaction costs, no dividends paid on the underlying asset, and that markets are efficient.

The model calculates the fair value of an option by taking into account several factors, including the current price of the underlying asset, the strike price of the option, the time to expiration, the risk-free interest rate, and the volatility of the underlying asset. The model provides a formula, known as the Black-Scholes-Merton formula, which quantifies the relationship between these variables and determines the theoretical price of the option.

The Black-Scholes-Merton model is valuable not only for pricing options but also for understanding the factors that influence option prices. It introduced the concept of option Greeks, such as delta, gamma, theta, vega, and rho, which measure the sensitivity of an option's price to changes in the underlying asset price, volatility, time decay, and other factors.

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = call option price

N = CDF of the normal distribution

S_t = spot price of an asset

K = strike price

r = risk-free interest rate

t = time to maturity

σ = volatility of the asset

CALL OPTION

Black-Scholes-Merton model, the formula for calculating the delta for a European-style option on a non-dividend paying stock is given by:

$$\Delta_{Call} = N(d_1)$$

PUT OPTION

Black-Scholes-Merton model, the formula for calculating the delta for a European-style option on a non-dividend paying stock is given by:

$$\Delta_{Put} = N(d_1) - 1$$

BENEFITS OF THE BLACK-SCHOLES MODEL

1. Provides a Framework: The Black-Scholes model provides a theoretical framework for pricing options. This allows investors and traders to determine the fair price of an option using a structured, defined methodology that has been tried and tested.
2. Allows for Risk Management: By knowing the theoretical value of an option, investors can use the Black-Scholes model to manage their risk exposure to different assets. The Black-Scholes model is therefore useful to investors not only in evaluating potential returns but understanding portfolio weakness and deficient investment areas.
3. Allows for Portfolio Optimization: The Black-Scholes model can be used to optimize portfolios by providing a measure of the expected returns and risks associated with different options. This allows investors to make smarter choices better aligned with their risk tolerance and pursuit of profit.
4. Enhances Market Efficiency: The Black-Scholes model has led to greater market efficiency and transparency as traders and investors are better able to price and trade options. This simplifies the pricing process as there is greater implicit understanding of how prices are derived.
5. Streamlines Pricing: On a similar note, the Black-Scholes model is widely accepted and used by practitioners in the financial industry. This allows for greater consistency and comparability across different markets and jurisdictions.

LIMITATIONS OF THE BLACK-SCHOLES MODEL

1. Limits Usefulness: As stated previously, the Black-Scholes model is only used to price European options and does not take into account that U.S. options could be exercised before the expiration date.
2. Lacks Cashflow Flexibility: The model assumes dividends and risk-free rates are constant, but this may not be true in reality. Therefore, the Black-Scholes model may lack the ability to truly reflect the accurate future cashflow of an investment due to model rigidity.
3. Assumes Constant Volatility: The model also assumes volatility remains constant over the option's life. In reality, this is often not the case because volatility fluctuates with the level of supply and demand.
4. Misleads Other Assumptions: The Black-Scholes model also leverages other assumptions. These assumptions include that there are no transaction costs or taxes, the risk-free interest rate is constant for all maturities, short selling of securities with use of proceeds is permitted, and there are no risk-less arbitrage opportunities. Each of these assumptions can lead to prices that deviate from actual results.

		Call	Put
Delta	$\frac{\partial V}{\partial S}$	$N(d_1)$	$-N(-d_1) = N(d_1) - 1$
Gamma	$\frac{\partial^2 V}{\partial S^2}$	$\frac{N'(d_1)}{S\sigma\sqrt{T-t}}$	
Vega	$\frac{\partial V}{\partial \sigma}$	$SN'(d_1)\sqrt{T-t}$	
Theta	$\frac{\partial V}{\partial t}$	$-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$	$-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$
Rho	$\frac{\partial V}{\partial r}$	$K(T-t)e^{-r(T-t)}N(d_2)$	$-K(T-t)e^{-r(T-t)}N(-d_2)$

2. OPTION

A call or put option is a type of financial contract that gives the holder (buyer) the right, but not the obligation, to buy an underlying asset at a predetermined price (strike price) within a specific period (expiration date). The underlying asset can be a stock, commodity, index, or other financial instruments.

When an investor buys a call or put option, they are speculating that the price of the underlying asset will rise before the option's expiration. By owning a call or put option, the investor has the opportunity to profit from the potential price increase of the underlying asset while limiting their downside risk.

KEY COMPONENTS OF A OPTION INCLUDE

1. Underlying Asset: The asset that the option is based on, such as a stock or an index.
2. Strike Price: The predetermined price at which the underlying asset can be bought (in the case of a call option) upon exercising the option. In case of put option, the strike price is the predetermined price at which the underlying asset can be sold.
3. Expiration Date: The date on which the option contract expires. After this date, the option is no longer valid.
4. Premium: The price paid by the option buyer to acquire the call option. It represents the cost of purchasing the right to buy the underlying asset. The premium is influenced by factors such as the current price of the underlying asset, the strike price, the time to expiration, and market volatility.
5. Option Holder: The buyer of the call or put option who has the right to exercise the option.
6. Option Writer: The seller of the call option who is obligated to deliver the underlying asset if the option is exercised by the buyer.

3. DELTA

Delta is a commonly used options Greek that measures the sensitivity of an option's price to changes in the price of the underlying asset. In the case of a call option, delta represents the change in the option price relative to a 1 dollar change in the underlying asset's price.

Delta ranges from 0 to 1 for call options. A delta of 0 means that the option price will not change at all with a 1 dollar change in the underlying asset's price.

A delta of 1 means that the option price will move in lockstep with the underlying asset, gaining or losing $1 for every 1 dollar change in the underlying asset's price. Delta values between 0 and 1 indicate a partial price movement relative to the underlying asset.

The delta of a call option is given by: $\Delta = N(d_1)$

where $d_1 = \dfrac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ and $N(x) = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt$

S: underlying
K: strike
T: maturity
σ: implied volatility (at the strike)
r: interest rates

Since gamma is the rate of change of delta with respect to the underlying: $\Gamma = \dfrac{\partial \Delta}{\partial S}$, differentiating $N(d_1)$ w.r.t. S gives

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}} \quad \text{where } N'(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

Since the delta of a put is given by $\Delta = N(d_1) - 1$, differentiating has the same effect. Therefore the gamma of a put is the same as the gamma of call.

Note that as spot S, moves away from the strike *K*, $N'(d_1)$ will become small and gamma will decrease very rapidly. Then, since $N'(d_1)$ is independent of the monetary units of the underlying, gamma will be inversely proportional to the value of the underlying.

Call Option Delta: The delta of a call option represents the change in the option price for a 1 dollar change in the underlying asset's price. A call option with a delta of 0.5 would be expected to increase by 0.50 dollars if the underlying asset price rises by 1 dollar.

Put Option Delta: The delta of a put option represents the change in the option price for a 1 dollars change in the underlying asset's price. A put option with a delta of -0.5 would be expected to decrease by 0.50 dollars if the underlying asset price rises by 1 dollars.

In [2]:
```
def calculate_call_delta(S, K, r, t, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * t) / (sigma * np.sqrt(t)
    delta = norm.cdf(d1)
    return delta
```

EXAMPLE 1

3.1 TIME TO EXPIRATION

```
In [3]: t = 0.5  #6 months, where 1 represent year
```

In the context of options, when you mention "0.5" as a time value, it usually represents 0.5 years or 6 months. This is because time is typically measured in terms of the time to expiration of the option, which is often expressed in years or fractions of a year.

3.2 CURRENT PRICE OF THE UNDERLYING ASSET

```
In [4]: S = 100
```

In this specific case, the code is setting the value of S to 100. This could mean that the underlying asset, such as a stock, is currently priced at 100 dollars.

The variable S is often used as a placeholder to represent the current price of the underlying asset in option pricing models or when performing calculations involving options.

3.3 STRIKE PRICE OF THE CALL OPTION

```
In [5]: K = 105
```

In options trading, the strike price, also known as the exercise price or simply the "K" in the given 105, refers to the predetermined price at which the underlying asset can be bought or sold when exercising an option.

For call options, the strike price is the price at which the holder of the call option can buy the underlying asset. If the market price of the underlying asset is higher than the strike price at expiration, the call option may be exercised, allowing the holder to purchase the asset at the predetermined strike price.

the variable K is assigned a value of 105, indicating that the strike price for the call option is 105 dollars. This means that the option holder has the right to buy the underlying asset at a price of 105 dollars per share if they choose to exercise the call option.

3.4 RISK FREE INTEREST RATE

```
In [6]: r = 0.05
```

The variable "r = 0.05" in the given code snippet represents the risk-free interest rate. In options pricing models, such as the Black-Scholes model, the risk-free interest rate is used to discount future cash flows and calculate the present value of the option.

3.5 VOLATILITY OF THE UNDERLYING ASSET

In [7]:
```
sigma = 0.2
```

Sigma (σ) represents the volatility of the underlying asset. Volatility is a measure of the price fluctuation or variability of the underlying asset's returns over a specific period.

Volatility is a crucial input in option pricing models, such as the Black-Scholes model, as it quantifies the uncertainty or risk associated with the underlying asset. Higher volatility implies a greater potential for price movements, leading to higher option prices.

Sigma is typically expressed as an annualized standard deviation of the underlying asset's returns. It is commonly calculated using historical price data or derived from implied volatility, which is derived from the current option prices in the market.

Sigma = 0.2 is an example value for sigma. It indicates a volatility of 0.2 or 20% per year for the underlying asset.

In [8]:
```
call_delta = calculate_call_delta(S, K, r, t, sigma)
print(f"The delta of the call option is {call_delta:.4f}")
```

```
The delta of the call option is 0.4612
```

SIMULATION OF CALL OPTION

In [44]:
```
K = 105  # Strike price of the call option
r = 0.05  # Risk-free interest rate
t = 0.5  # Time to expiration in years
sigma = 0.2  # Volatility of the underlying asset
```

In [45]:
```
print("Current price of underlying assets:", S,"dollars")
```

```
Current price of underlying assets: 100 dollars
```

In [46]:
```
def calculate_option_delta(S, K, r, t, sigma, option_type):
    d1 = (math.log(S / K) + (r + 0.5 * sigma**2) * t) / (sigma * math.sqr
    if option_type == "call":
        delta = norm.cdf(d1)
    elif option_type == "put":
        delta = norm.cdf(-d1) - 1  # Use -d1 and subtract 1 for put optio
    else:
        raise ValueError("Invalid option type. Please specify 'call' or '
    return delta
```

In [47]:
```
#Random price
prices = [x for x in range(1, 201)]
print(prices)
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 2
1, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39
, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57,
58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 7
6, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94
, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 1
10, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124,
125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139
, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 1
54, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168,
169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183
, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 1
98, 199, 200]
```

In [48]:

```python
call_deltas = []
for price in prices:
    call_delta = calculate_call_delta(price, K, r, t, sigma)
    call_deltas.append(call_delta)
    print(f"The delta of the call option with price {price} is {call_delt
```

```
The delta of the call option with price 1 is 0.0000
The delta of the call option with price 2 is 0.0000
The delta of the call option with price 3 is 0.0000
The delta of the call option with price 4 is 0.0000
The delta of the call option with price 5 is 0.0000
The delta of the call option with price 6 is 0.0000
The delta of the call option with price 7 is 0.0000
The delta of the call option with price 8 is 0.0000
The delta of the call option with price 9 is 0.0000
The delta of the call option with price 10 is 0.0000
The delta of the call option with price 11 is 0.0000
The delta of the call option with price 12 is 0.0000
The delta of the call option with price 13 is 0.0000
The delta of the call option with price 14 is 0.0000
The delta of the call option with price 15 is 0.0000
The delta of the call option with price 16 is 0.0000
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The delta of the call option with price 20 is 0.0000
The delta of the call option with price 21 is 0.0000
The delta of the call option with price 22 is 0.0000
The delta of the call option with price 23 is 0.0000
The delta of the call option with price 24 is 0.0000
The delta of the call option with price 25 is 0.0000
The delta of the call option with price 26 is 0.0000
The delta of the call option with price 27 is 0.0000
The delta of the call option with price 28 is 0.0000
The delta of the call option with price 29 is 0.0000
The delta of the call option with price 30 is 0.0000
The delta of the call option with price 31 is 0.0000
The delta of the call option with price 32 is 0.0000
The delta of the call option with price 33 is 0.0000
The delta of the call option with price 34 is 0.0000
The delta of the call option with price 35 is 0.0000
The delta of the call option with price 36 is 0.0000
The delta of the call option with price 37 is 0.0000
The delta of the call option with price 38 is 0.0000
The delta of the call option with price 39 is 0.0000
The delta of the call option with price 40 is 0.0000
The delta of the call option with price 41 is 0.0000
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The delta of the call option with price 42 is 0.0000
The delta of the call option with price 43 is 0.0000
The delta of the call option with price 44 is 0.0000
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The delta of the call option with price 47 is 0.0000
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The delta of the call option with price 49 is 0.0000
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The delta of the call option with price 52 is 0.0000
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The delta of the call option with price 56 is 0.0000
The delta of the call option with price 57 is 0.0000
The delta of the call option with price 58 is 0.0000
The delta of the call option with price 59 is 0.0001
The delta of the call option with price 60 is 0.0001
The delta of the call option with price 61 is 0.0002
The delta of the call option with price 62 is 0.0003
The delta of the call option with price 63 is 0.0004
The delta of the call option with price 64 is 0.0006
The delta of the call option with price 65 is 0.0008
The delta of the call option with price 66 is 0.0012
The delta of the call option with price 67 is 0.0017
The delta of the call option with price 68 is 0.0024
The delta of the call option with price 69 is 0.0033
The delta of the call option with price 70 is 0.0044
The delta of the call option with price 71 is 0.0059
The delta of the call option with price 72 is 0.0078
The delta of the call option with price 73 is 0.0101
The delta of the call option with price 74 is 0.0130
The delta of the call option with price 75 is 0.0165
The delta of the call option with price 76 is 0.0208
The delta of the call option with price 77 is 0.0258
The delta of the call option with price 78 is 0.0318
The delta of the call option with price 79 is 0.0388
The delta of the call option with price 80 is 0.0469
The delta of the call option with price 81 is 0.0562
The delta of the call option with price 82 is 0.0667
The delta of the call option with price 83 is 0.0785
The delta of the call option with price 84 is 0.0917
The delta of the call option with price 85 is 0.1063
The delta of the call option with price 86 is 0.1222
The delta of the call option with price 87 is 0.1396
The delta of the call option with price 88 is 0.1583
The delta of the call option with price 89 is 0.1784
The delta of the call option with price 90 is 0.1997
The delta of the call option with price 91 is 0.2223
The delta of the call option with price 92 is 0.2460
The delta of the call option with price 93 is 0.2707
The delta of the call option with price 94 is 0.2963
The delta of the call option with price 95 is 0.3227
The delta of the call option with price 96 is 0.3497
The delta of the call option with price 97 is 0.3772
The delta of the call option with price 98 is 0.4050
The delta of the call option with price 99 is 0.4331
The delta of the call option with price 100 is 0.4612
The delta of the call option with price 101 is 0.4892
The delta of the call option with price 102 is 0.5170
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```
The delta of the call option with price 103 is 0.5444
The delta of the call option with price 104 is 0.5714
The delta of the call option with price 105 is 0.5977
The delta of the call option with price 106 is 0.6234
The delta of the call option with price 107 is 0.6484
The delta of the call option with price 108 is 0.6724
The delta of the call option with price 109 is 0.6956
The delta of the call option with price 110 is 0.7178
The delta of the call option with price 111 is 0.7391
The delta of the call option with price 112 is 0.7592
The delta of the call option with price 113 is 0.7784
The delta of the call option with price 114 is 0.7964
The delta of the call option with price 115 is 0.8135
The delta of the call option with price 116 is 0.8294
The delta of the call option with price 117 is 0.8444
The delta of the call option with price 118 is 0.8583
The delta of the call option with price 119 is 0.8713
The delta of the call option with price 120 is 0.8833
The delta of the call option with price 121 is 0.8944
The delta of the call option with price 122 is 0.9047
The delta of the call option with price 123 is 0.9141
The delta of the call option with price 124 is 0.9227
The delta of the call option with price 125 is 0.9306
The delta of the call option with price 126 is 0.9378
The delta of the call option with price 127 is 0.9444
The delta of the call option with price 128 is 0.9503
The delta of the call option with price 129 is 0.9557
The delta of the call option with price 130 is 0.9606
The delta of the call option with price 131 is 0.9650
The delta of the call option with price 132 is 0.9690
The delta of the call option with price 133 is 0.9725
The delta of the call option with price 134 is 0.9757
The delta of the call option with price 135 is 0.9785
The delta of the call option with price 136 is 0.9811
The delta of the call option with price 137 is 0.9834
The delta of the call option with price 138 is 0.9854
The delta of the call option with price 139 is 0.9872
The delta of the call option with price 140 is 0.9887
The delta of the call option with price 141 is 0.9902
The delta of the call option with price 142 is 0.9914
The delta of the call option with price 143 is 0.9925
The delta of the call option with price 144 is 0.9934
The delta of the call option with price 145 is 0.9943
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The delta of the call option with price 154 is 0.9984
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The delta of the call option with price 198 is 1.0000
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The delta of the call option with price 200 is 1.0000
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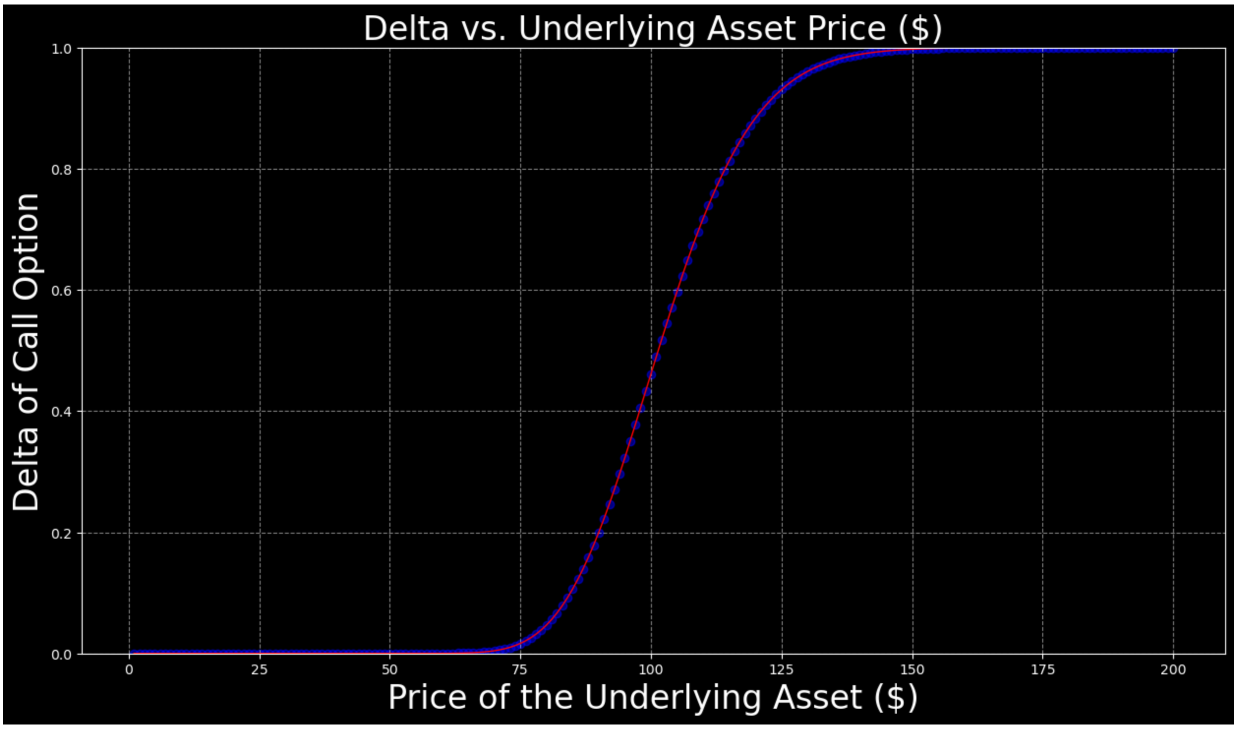
In [49]:

```
plt.style.use('dark_background')
plt.figure(figsize=(15,8))
plt.plot(prices, call_deltas, color="red", linewidth=1)
plt.scatter(prices, call_deltas, color="blue",alpha=0.5)
plt.xlabel("Price of the Underlying Asset ($)", fontsize=24)
plt.ylabel("Delta of Call Option", fontsize=24)
plt.title("Delta vs. Underlying Asset Price ($)", fontsize=24)
plt.grid(True, linestyle="--", alpha=0.5)
plt.ylim(0, 1)  # Set the y-axis limits from -1 to 0
plt.show()
```



EXAMPLE 2

SIMULATION OF PUT OPTION

In [50]:
```
K = 105  # Strike price of the put option
r = 0.05  # Risk-free interest rate
t = 0.5  # Time to expiration in years
sigma = 0.2  # Volatility of the underlying asset
```

In [54]:
```
def calculate_option_delta(S, K, r, t, sigma, option_type):
    d1 = (math.log(S / K) + (r + 0.5 * sigma**2) * t) / (sigma * math.sqr
    if option_type == "call":
        delta = norm.cdf(d1)
    elif option_type == "put":
        delta = norm.cdf(-d1) - 1  # Use -d1 and subtract 1 for put optio
    else:
        raise ValueError("Invalid option type. Please specify 'call' or '
    return delta
```

In [55]:
```
prices = [x for x in range(1, 201)]
```

In [56]:
```
put_deltas = []
for price in prices:
    put_delta = calculate_option_delta(price, K, r, t, sigma, "put")
    put_deltas.append(put_delta)
    print(f"The delta of the put option with price {price} is {put_delta:
```

```
The delta of the put option with price 1 is 0.0000
The delta of the put option with price 2 is 0.0000
The delta of the put option with price 3 is 0.0000
The delta of the put option with price 4 is 0.0000
The delta of the put option with price 5 is 0.0000
The delta of the put option with price 6 is 0.0000
The delta of the put option with price 7 is 0.0000
```

```
The delta of the put option with price 8 is 0.0000
The delta of the put option with price 9 is 0.0000
The delta of the put option with price 10 is 0.0000
The delta of the put option with price 11 is 0.0000
The delta of the put option with price 12 is 0.0000
The delta of the put option with price 13 is 0.0000
The delta of the put option with price 14 is 0.0000
The delta of the put option with price 15 is 0.0000
The delta of the put option with price 16 is 0.0000
The delta of the put option with price 17 is 0.0000
The delta of the put option with price 18 is 0.0000
The delta of the put option with price 19 is 0.0000
The delta of the put option with price 20 is 0.0000
The delta of the put option with price 21 is 0.0000
The delta of the put option with price 22 is 0.0000
The delta of the put option with price 23 is 0.0000
The delta of the put option with price 24 is 0.0000
The delta of the put option with price 25 is 0.0000
The delta of the put option with price 26 is 0.0000
The delta of the put option with price 27 is 0.0000
The delta of the put option with price 28 is 0.0000
The delta of the put option with price 29 is 0.0000
The delta of the put option with price 30 is 0.0000
The delta of the put option with price 31 is 0.0000
The delta of the put option with price 32 is -0.0000
The delta of the put option with price 33 is -0.0000
The delta of the put option with price 34 is -0.0000
The delta of the put option with price 35 is -0.0000
The delta of the put option with price 36 is -0.0000
The delta of the put option with price 37 is -0.0000
The delta of the put option with price 38 is -0.0000
The delta of the put option with price 39 is -0.0000
The delta of the put option with price 40 is -0.0000
The delta of the put option with price 41 is -0.0000
The delta of the put option with price 42 is -0.0000
The delta of the put option with price 43 is -0.0000
The delta of the put option with price 44 is -0.0000
The delta of the put option with price 45 is -0.0000
The delta of the put option with price 46 is -0.0000
The delta of the put option with price 47 is -0.0000
The delta of the put option with price 48 is -0.0000
The delta of the put option with price 49 is -0.0000
The delta of the put option with price 50 is -0.0000
The delta of the put option with price 51 is -0.0000
The delta of the put option with price 52 is -0.0000
The delta of the put option with price 53 is -0.0000
The delta of the put option with price 54 is -0.0000
The delta of the put option with price 55 is -0.0000
The delta of the put option with price 56 is -0.0000
The delta of the put option with price 57 is -0.0000
The delta of the put option with price 58 is -0.0000
The delta of the put option with price 59 is -0.0001
The delta of the put option with price 60 is -0.0001
The delta of the put option with price 61 is -0.0002
The delta of the put option with price 62 is -0.0003
The delta of the put option with price 63 is -0.0004
The delta of the put option with price 64 is -0.0006
The delta of the put option with price 65 is -0.0008
The delta of the put option with price 66 is -0.0012
The delta of the put option with price 67 is -0.0017
The delta of the put option with price 68 is -0.0024
```

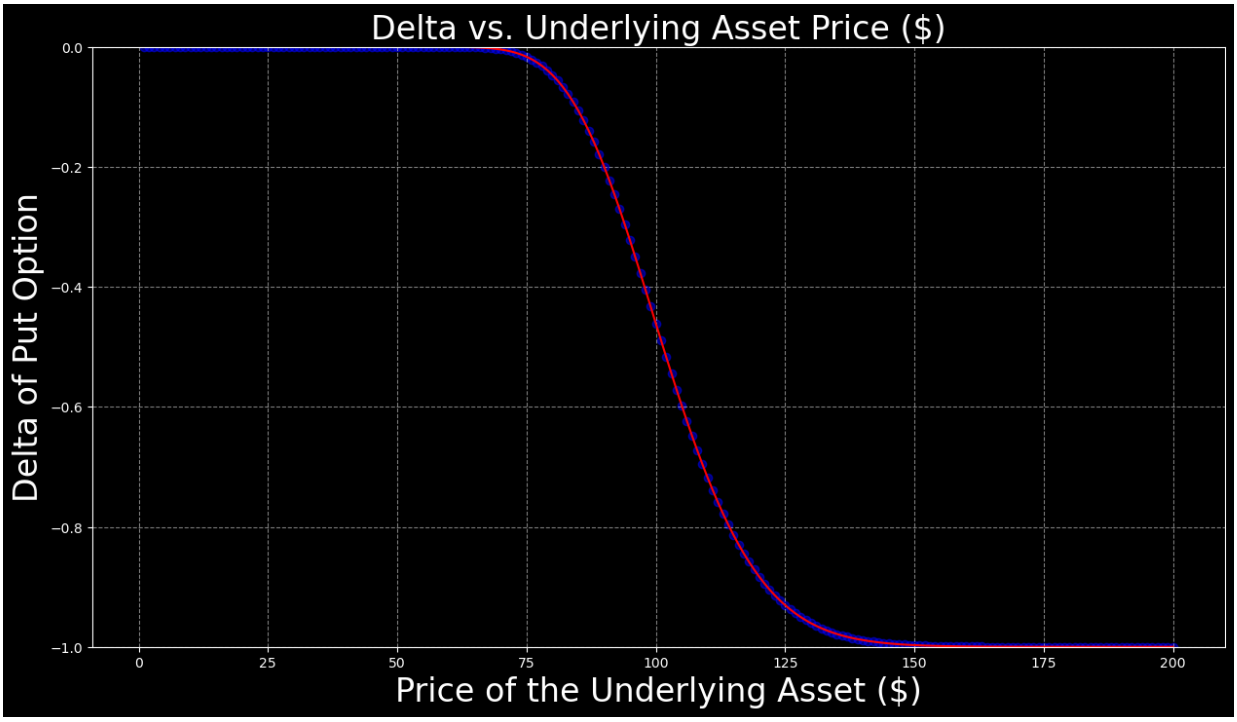
```
The delta of the put option with price 69 is -0.0033
The delta of the put option with price 70 is -0.0044
The delta of the put option with price 71 is -0.0059
The delta of the put option with price 72 is -0.0078
The delta of the put option with price 73 is -0.0101
The delta of the put option with price 74 is -0.0130
The delta of the put option with price 75 is -0.0165
The delta of the put option with price 76 is -0.0208
The delta of the put option with price 77 is -0.0258
The delta of the put option with price 78 is -0.0318
The delta of the put option with price 79 is -0.0388
The delta of the put option with price 80 is -0.0469
The delta of the put option with price 81 is -0.0562
The delta of the put option with price 82 is -0.0667
The delta of the put option with price 83 is -0.0785
The delta of the put option with price 84 is -0.0917
The delta of the put option with price 85 is -0.1063
The delta of the put option with price 86 is -0.1222
The delta of the put option with price 87 is -0.1396
The delta of the put option with price 88 is -0.1583
The delta of the put option with price 89 is -0.1784
The delta of the put option with price 90 is -0.1997
The delta of the put option with price 91 is -0.2223
The delta of the put option with price 92 is -0.2460
The delta of the put option with price 93 is -0.2707
The delta of the put option with price 94 is -0.2963
The delta of the put option with price 95 is -0.3227
The delta of the put option with price 96 is -0.3497
The delta of the put option with price 97 is -0.3772
The delta of the put option with price 98 is -0.4050
The delta of the put option with price 99 is -0.4331
The delta of the put option with price 100 is -0.4612
The delta of the put option with price 101 is -0.4892
The delta of the put option with price 102 is -0.5170
The delta of the put option with price 103 is -0.5444
The delta of the put option with price 104 is -0.5714
The delta of the put option with price 105 is -0.5977
The delta of the put option with price 106 is -0.6234
The delta of the put option with price 107 is -0.6484
The delta of the put option with price 108 is -0.6724
The delta of the put option with price 109 is -0.6956
The delta of the put option with price 110 is -0.7178
The delta of the put option with price 111 is -0.7391
The delta of the put option with price 112 is -0.7592
The delta of the put option with price 113 is -0.7784
The delta of the put option with price 114 is -0.7964
The delta of the put option with price 115 is -0.8135
The delta of the put option with price 116 is -0.8294
The delta of the put option with price 117 is -0.8444
The delta of the put option with price 118 is -0.8583
The delta of the put option with price 119 is -0.8713
The delta of the put option with price 120 is -0.8833
The delta of the put option with price 121 is -0.8944
The delta of the put option with price 122 is -0.9047
The delta of the put option with price 123 is -0.9141
The delta of the put option with price 124 is -0.9227
The delta of the put option with price 125 is -0.9306
The delta of the put option with price 126 is -0.9378
The delta of the put option with price 127 is -0.9444
The delta of the put option with price 128 is -0.9503
The delta of the put option with price 129 is -0.9557
```

```
The delta of the put option with price 130 is -0.9606
The delta of the put option with price 131 is -0.9650
The delta of the put option with price 132 is -0.9690
The delta of the put option with price 133 is -0.9725
The delta of the put option with price 134 is -0.9757
The delta of the put option with price 135 is -0.9785
The delta of the put option with price 136 is -0.9811
The delta of the put option with price 137 is -0.9834
The delta of the put option with price 138 is -0.9854
The delta of the put option with price 139 is -0.9872
The delta of the put option with price 140 is -0.9887
The delta of the put option with price 141 is -0.9902
The delta of the put option with price 142 is -0.9914
The delta of the put option with price 143 is -0.9925
The delta of the put option with price 144 is -0.9934
The delta of the put option with price 145 is -0.9943
The delta of the put option with price 146 is -0.9950
The delta of the put option with price 147 is -0.9957
The delta of the put option with price 148 is -0.9963
The delta of the put option with price 149 is -0.9968
The delta of the put option with price 150 is -0.9972
The delta of the put option with price 151 is -0.9976
The delta of the put option with price 152 is -0.9979
The delta of the put option with price 153 is -0.9982
The delta of the put option with price 154 is -0.9984
The delta of the put option with price 155 is -0.9987
The delta of the put option with price 156 is -0.9988
The delta of the put option with price 157 is -0.9990
The delta of the put option with price 158 is -0.9991
The delta of the put option with price 159 is -0.9993
The delta of the put option with price 160 is -0.9994
The delta of the put option with price 161 is -0.9995
The delta of the put option with price 162 is -0.9995
The delta of the put option with price 163 is -0.9996
The delta of the put option with price 164 is -0.9997
The delta of the put option with price 165 is -0.9997
The delta of the put option with price 166 is -0.9998
The delta of the put option with price 167 is -0.9998
The delta of the put option with price 168 is -0.9998
The delta of the put option with price 169 is -0.9998
The delta of the put option with price 170 is -0.9999
The delta of the put option with price 171 is -0.9999
The delta of the put option with price 172 is -0.9999
The delta of the put option with price 173 is -0.9999
The delta of the put option with price 174 is -0.9999
The delta of the put option with price 175 is -0.9999
The delta of the put option with price 176 is -1.0000
The delta of the put option with price 177 is -1.0000
The delta of the put option with price 178 is -1.0000
The delta of the put option with price 179 is -1.0000
The delta of the put option with price 180 is -1.0000
The delta of the put option with price 181 is -1.0000
The delta of the put option with price 182 is -1.0000
The delta of the put option with price 183 is -1.0000
The delta of the put option with price 184 is -1.0000
The delta of the put option with price 185 is -1.0000
The delta of the put option with price 186 is -1.0000
The delta of the put option with price 187 is -1.0000
The delta of the put option with price 188 is -1.0000
The delta of the put option with price 189 is -1.0000
The delta of the put option with price 190 is -1.0000
```

```
The delta of the put option with price 191 is -1.0000
The delta of the put option with price 192 is -1.0000
The delta of the put option with price 193 is -1.0000
The delta of the put option with price 194 is -1.0000
The delta of the put option with price 195 is -1.0000
The delta of the put option with price 196 is -1.0000
The delta of the put option with price 197 is -1.0000
The delta of the put option with price 198 is -1.0000
The delta of the put option with price 199 is -1.0000
The delta of the put option with price 200 is -1.0000
```

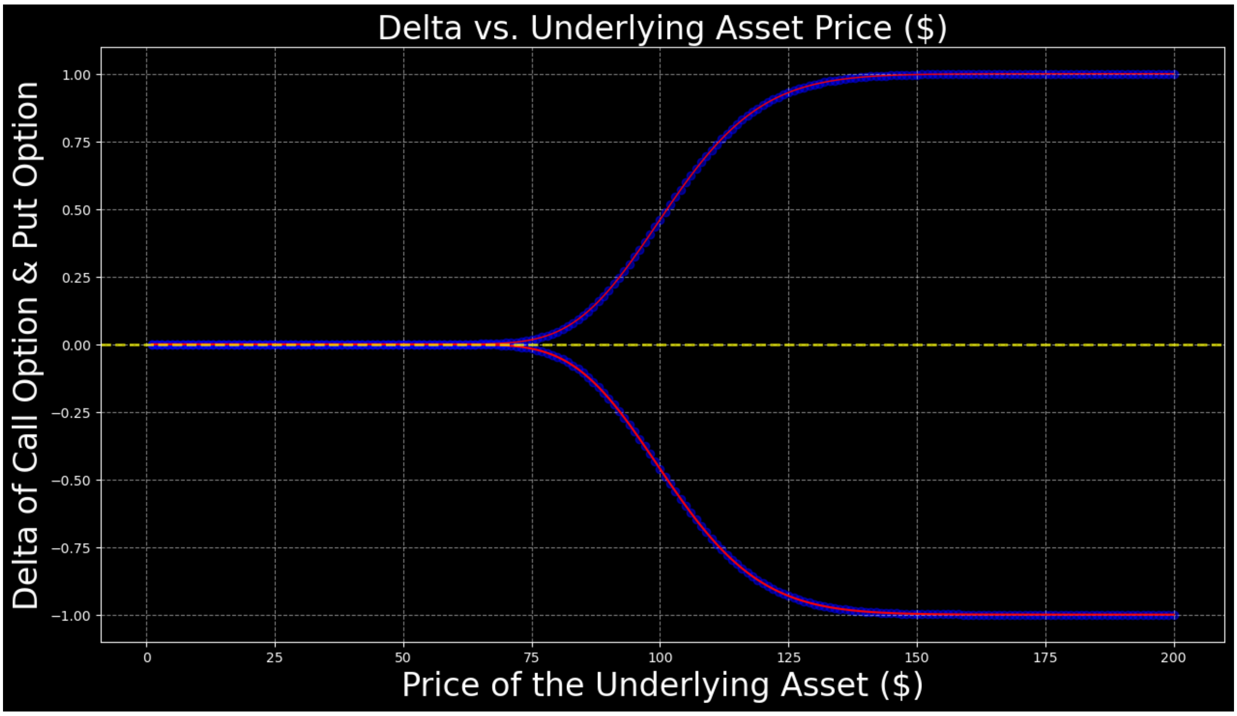
In [57]:

```python
plt.style.use('dark_background')
plt.figure(figsize=(15,8))
plt.plot(prices, put_deltas, color="red")
plt.scatter(prices, put_deltas, color="blue", alpha=0.5)
plt.xlabel("Price of the Underlying Asset ($)", fontsize=24)
plt.ylabel("Delta of Put Option", fontsize=24)
plt.title("Delta vs. Underlying Asset Price ($)", fontsize=24)
plt.grid(True, linestyle="--", alpha=0.5)
plt.ylim(-1, 0)  # Set the y-axis limits from -1 to 0
plt.show()
```



In [60]:

```python
plt.style.use('dark_background')
plt.figure(figsize=(15,8))
plt.plot(prices, call_deltas, color="red", linewidth=1)
plt.scatter(prices, call_deltas, color="blue",alpha=0.5)
plt.plot(prices, put_deltas, color="red")
plt.scatter(prices, put_deltas, color="blue", alpha=0.5)
plt.xlabel("Price of the Underlying Asset ($)", fontsize=24)
plt.ylabel("Delta of Call Option & Put Option", fontsize=24)
plt.title("Delta vs. Underlying Asset Price ($)", fontsize=24)
plt.grid(True, linestyle="--", alpha=0.5)
plt.axhline(0, color="yellow", linestyle="--", linewidth=2, alpha=0.7)
plt.show()
```



EXAMPLE 3

In [342…

```
strike_price = 100
premium_call = 3.5
premium_put = 2.8
expiration_price = np.linspace(70, 130, 100)
```

In [343…

```
payoff_call = np.maximum(expiration_price - strike_price, 0) - premium_ca
payoff_put = np.maximum(strike_price - expiration_price, 0) - premium_put
payoff_total = payoff_call + payoff_put
```

4. DELTA HEDGING

Option traders employ delta hedging as a strategic measure to mitigate the risks associated with price fluctuations in the underlying asset. By continuously executing buy and sell transactions on the underlying asset, traders aim to attain delta neutrality. This practice finds prominence within financial institutions as it allows them to effectively manage their option book, where delta is computed at the individual option level and subsequently aggregated at the book level. Additionally, delta hedging enables these institutions to generate margins through their option writing activities.

Maintaining delta neutrality necessitates constant monitoring of market conditions and promptly executing trades to align the portfolio's delta with the desired neutrality level. This proactive approach, referred to as dynamic hedging in the realm of options, ensures that the delta of the option contract remains balanced amidst fluctuations in both the underlying asset's price and the option contract itself. By diligently undertaking dynamic hedging strategies, option traders can effectively manage and control their exposure to market movements, thereby optimizing risk management and potential returns.

$$C_{t+\delta t} - C_t - \Delta(S_{t+\delta t} - S_t) = r \cdot \delta t$$

LIMITATIONS OF DELTA HEDGING

TRANSACTION COST

The implementation of delta hedging necessitates frequent buying and selling of the underlying asset, resulting in substantial transaction costs. These costs add to the expenses associated with utilizing delta hedging as a means to optimize portfolio performance and mitigate price-related risks. Consequently, it is essential for traders to judiciously assess and factor in these transaction costs when considering the viability and effectiveness of delta hedging strategies. Furthermore, prudent traders may periodically adjust their option positions to optimize their portfolios and minimize transaction costs.

ILLIQUID MARKETS

In instances where a particular asset market exhibits low liquidity, the practice of delta hedging becomes challenging. In such circumstances, traders encounter difficulties in continuously buying or selling the underlying asset to neutralize the price impact and maintain delta neutrality. The limited availability of counterparties and reduced trading volumes in illiquid markets impede the seamless execution of delta hedging strategies, thereby hindering traders from effectively managing their risk exposures. As a result, traders operating in illiquid markets must adapt their approach, potentially employing alternative risk management techniques to address the limitations imposed by illiquidity.

EXAMPLE 4

Let us consider a call option contract with the following characteristics: the underlying asset is an Microsoft stock, the option strike price (K) is equal to 300 dollars and the time to maturity (T) is of one month (i.e., 0.08333 years).

At the time of valuation, the price of the Apple stock (S) is 300 dollars, the volatility (σ) of Microsoft stock is 30% and the risk-free rate (r) is 3% (market data).

The delta of a call option is approximately equal to 0.53.

Using the above value, we can say that due to a 1 dollar change in the price of the underlying asset, the price of the option will change by 0.53 dollar.

In [345...
```
def calculate_option_delta(S, K, r, t, sigma, option_type):
    d1 = (math.log(S / K) + (r + 0.5 * sigma**2) * t) / (sigma * math.sqr
    if option_type == "call":
        delta = norm.cdf(d1)
    elif option_type == "put":
        delta = norm.cdf(-d1) - 1  # Use -d1 and subtract 1 for put optio
    else:
        raise ValueError("Invalid option type. Please specify 'call' or '
    return delta
```

In [346...
```
time_to_maturity = 1/12 #t
print("One month: ",round(time_to_maturity,5))
```

```
One month:  0.08333
```

In [347...
```
underlying_price = 300 #S
strike_price = 300 #K
risk_free_rate = 0.03 #r
volatility = 0.3 #sigma
option_type = "call"
```

In [348...
```
delta = calculate_option_delta(underlying_price, strike_price, time_to_ma
print("Delta:", delta)
```

Delta: 0.5295319206345476