第十三讲：布尔代数

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请独立完成作业，不得抄袭。
若参考了其它资料，请给出引用。
鼓励讨论，但需独立书写解题过程。

第一部分　作业

题目 (第一题)

证明布尔代数是有界有补分配格，有界有补分配格是布尔代数

解答:

(1) prove:A Boolean algebra B is a bounded, distributive and complemented lattice.

By Theorem 15.2 and axiom [B1], every Boolean algebra B satisfies the associative, commutative, and absorption laws and hence is a lattice where + and ∗ are the join and meet operations, respectively. With respect to this lattice, a+1 = 1 implies a   1 and a ∗ 0 = 0 implies 0   a,for any element a∈ B. Thus B is a bounded lattice. Furthermore, axioms [B2] and [B4] show that B is also distributive and complemented.

(i) Commutive laws: ∵ a+b=b+a and a*b=b*a, ∴ a∨b=b∨a and a∧b=b∧a

(ii) Associative laws: ∵ (a+b)+c=a+(b+c) and (a*b)*c=a*(b*c), ∴ (a∨b)∨c=a∨(b∨c) and (a∧b)∧c=a∧(b∧c)

(iii) Absorption laws: ∵ a*(a+b)=a and a+(a*b)=a, ∴ a∧(a∨b)=a and a∨(a∧b)=a

(iv) bounded: a+1 = 1 implies a   1 and a ∗ 0 = 0 implies 0   a,for any element a∈ B. Thus B is a bounded lattice.

(v) According to [B2], we can conclude that a∨(b∧c)=(a∨b)∧(a∨c) and a∧(b∨c)=(a∧b)∨(a∧c)

(vi) According to [B4], we can conclude that a∨a'=1 and a∧a'=0

∴ A Boolean algebra B is a bounded, distributive and complemented lattice.

(2) prove:A bounded, distributive and complemented lattice B ia s Boolean algebra.

a lattice where + and ∗ are the join and meet operations

(i) communitive laws: ∵ a∨b=b∨a and a∧b=b∧a, ∴ a+b=b+a and a*b=b*a

(ii) Distributive laws: ∵ a∨(b∧c)=(a∨b)∧(a∨c) and a∧(b∨c)=(a∧b)∨(a∧c),∴ a+(b∗c)=(a+b)∗(a+c) and a∗(b+c)=(a ∗ b)+(a ∗ c)

(iii) Identity laws: ∵ 0 is the infimun of B and 1 is the supremum of B, ∴ a∨0=a and

$a \wedge 1 = a$, $\therefore$ a+0=a and a*1=a
(iv) Complement laws: $\because$ a' is the complement of a, $\therefore$ $a \vee a'=1$ and $a \wedge a'=0$,$\therefore$ a+a'=1 and a*a'=0
$\therefore$ A bounded, distributive and complemented lattice B ia s Boolean algebra.
Based on (1) and (2), 布尔代数是有界有补分配格，有界有补分配格是布尔代数.

题目 (第二题)
证明 SM 定理 15.6

解答:
$a=u_1+u_2+\cdots+u_r$ and $b=v_1+v_2+\cdots+v_s$ and $a'=w_1+w_2+\cdots+w_t$ and $\{u_1,u_2,dots,u_r\}\cap\{w_1,w_2,dots,w_t\}=\emptyset$
and $\{u_1,u_2,dots,u_r\}\cup\{w_1,w_2,dots,w_t\}=$ A
and $a+b=u_1+u_2+\cdots+u_r+v_1+v_2+\cdots+v_s$
and $a*b=x_1+x_2+\cdots+x_q$($x_1,x_2,\ldots,x_q$ all are the common elements of $\{u_1,u_2,dots,u_r\}$ and $\{v_1,v_2,dots,v_s\}$)
$\therefore$ $f(a)=\{u_1,u_2,dots,u_r\}$ and $f(b)=\{v_1,v_2,dots,v_s\}$ and $f(a')=\{w_1,w_2,dots,w_t\}$
and $f(a+b)=\{u_1,u_2,dots,u_r\}\cup\{v_1,v_2,dots,v_s\}$ and $f(a*b)=\{u_1,u_2,dots,u_r\}\cap\{v_1,v_2,dots,v_s\}$
$\therefore$ f(a+b)=f(a)+f(b), and f(a*b)=f(a)*f(b) and f(a')=f(a)'
$\therefore$ The above mapping f:$B\rightarrow P(A)$ is an isomorphism.

题目 (第三题)
证明等势的布尔代数均同构

解答:
We define Boolean Algebra A and B have the same number of elements.
$\therefore$ according to the problem solved above, we can note that A is isomorphic to P(C) with the mapping f where C is the set of atoms of A and B is isomorphic to P(D) with the mapping g where D is the set of atoms of B.
And according to Corollary 15.7, we note that C and D have the same number of elements.
$\therefore$ As proved before, we can define a bijective function h: $S\rightarrow T$ is a isomorphism.
$\therefore$ $g^{-1}\circ h\circ f$ if is a isomorphism.
$\therefore$ A and B are isomorphic.

第二部分　订正

题目 (题号)
题目。

错因分析： 简述错误原因（可选）。

订正：
正确解答。

第三部分　反馈

你可以写：

- SM 第十四章中，书上关于最大（maximal）最小（minimal）元的定义与 ppt 中关于他们的定义不一致，极大极小直的定义也产生了冲突，以哪一个为准?

- 想问一下，我们以后的课程（包括除问题求解外的课程）会学《深入理解计算机系统》这本书吗?