第八讲：集合及其运算

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2017 年 11 月 19 日

请独立完成作业，不得抄袭。
若参考了其它资料，请给出引用。
鼓励讨论，但需独立书写解题过程。

第一部分　作业

题目 (UD: 6.7)

Find an expression for each of the shaded sets in the Venn diagrams of Figure 6.5

解答：

from left to right, from up to down

(a) $B \setminus (A \cap B)$

(b) $(A \cup B) \setminus (A \cap B)$

(c) $A \cap B \cap C$

(d) $(B \cap C) \setminus A$

(e) $((A\cap B)\cup(A\cap C)\cup(B\cap C)) \setminus (A \cap B \cap C)$

题目 (UD: 6.16)

In each part of this problem, two sets, A and B, are defined. Prove that $A \Subset B$ in each of the following:

(a) $A=\{x^2 : x\in Z\}$ and $B=Z$;

(b) $A=R$ and $B=\{2x : x\in R\}$;

(c) $A=\{(x,y)\in R^2 : y=(5-3x)/2\}$ and $B=\{(x,y)\in R^2 : 2y+3x=5\}$.

解答：

(a)

$\because x\in Z$

$\therefore x=2k$ or $2k+1$ $(k\in Z)$

(1) if $x=2k$, then $x^2=4k^2$, $\therefore 4k^2 \in Z$, $\therefore x \in Z$

(2) if $x=2k+1$ then $x^2=4k^2+4k+1$, $\therefore 4k^2+4k+1 \in Z$, $\therefore x\in Z$

Based on (1) and (2) we can conclude that for all x $\in$ Z ,we have $x^2 \in$ Z, namely B
$\therefore$ A$\Subset$ B

(b)
for every y $\in$ R,namely A, we have y/2 $\in$ R
we can let y=2x
$\therefore$ for every 2x $\in$ R,namely A, we have x $\in$ R
$\therefore$ that property satisfies the defination of B
$\therefore$ for every 2x $\in$ R,namely A, we have 2x $\in$ B
$\therefore$ A$\Subset$ B

(c)
for every (x,y) $\in$ A, we have y=(5-3x)/2
$\therefore$ for every (x,y) $\in$ A, we have 2y+3x=5
$\therefore$ for every (x,y) $\in$ A, we have (x,y)$\in$ B
$\therefore$ A$\Subset$B

题目 (UD: 6.17)
Prove that one set is a proper subset of the other one in each of the following:
(a) A={(x,y) $\in R^2$: xy>0 } and B={(x,y) $\in R^2$:$x^2 + y^2 > 0$ };
(b) A=$\emptyset$ and B={(x,y)$\in R^2$: $x^2 + y^2 \leq 0$ }.

解答:
(a)
(1) for every (x,y) $\in$ A,we have xy>0
$\therefore$ xy$\neq$ 0
$\therefore$ $x^2 + y^2 > 0$
$\therefore$ that property satisfies the defination of B
$\therefore$ for every (x,y) $\in$ A,we have (x,y) $\in$ B
$\therefore$ A$\Subset$B
(2) (2,-5)$\in$ B but (2,-5)$\notin$ A
$\therefore$ A $\neq$ B
Based on (1) and (2), A$\subsetneqq$ B

(b)
(1) we need to prove for every x, if x$\in$ A,then x$\in$ B
Since there are no elements in the empty set, the antecedent is always false. Therefore the implication is always true.
$\therefore$ A $\Subset$ B
(2) (0,0) $\in$ B, but (0,0) $\notin$ A

$\therefore$ A$\neq$B

Based on (1) and (2), A$\subsetneq$B

题目 (UD:7.1)

In this problem we refer to statements of Theorem 7.4.

(a) Prove statement 2.

$(A^{C})^{C}$=A.

(b) Prove statement 14.

A $\cap$(B $\cup$ C) = (A $\cap B$)$\cup$(A $\cap$ C). (Distributive property)

(c) Prove statement 16.

X $\setminus$(A $\cap$ B)=(X $\setminus$ A)$\cup$ (X $\setminus$ B).(Demorgan's law)

(When X is the universe we also write $(A \cap B)^{C} = A^{C} \cup B^{C}$

(d) Prove statement 18.

A$\Subset$B if and only if (X $\setminus$ B) $\Subset$ (X $\setminus$ A).

(When X is the universe we also write A$\Subset$B if and only if $B^{C} \Subset A^{C}$)

(e) Prove statement 20.

A $\cap$ B=B if and only if B$\Subset$A.

解答:

(a)

(1) for every x$\in$ $(A^{C})^{C}$,we have x$\notin$ (A^{C})

$\therefore$ we have x$\in$ A

$\therefore$ $(A^{C})^{C} \Subset A$

(2) for every x$\in$ A,we have x$\notin$ (A^{C})

$\therefore$ we have x$\in$ $(A^{C})^{C}$

$\therefore$ $A \Subset (A^{C})^{C}$

Based on (1) and (2),we have $(A^{C})^{C}$=A.

(b)

(1) for every x $\in$ A $\cap$(B $\cup$ C),x$\in$ A and x$\in$ (B $\cup$ C)

$\therefore$ there are two cases:

(i) x$\in$ A and x$\in$ B :

$\therefore$ x$\in$(A $\cap$ B)

$\therefore$ x$\in$(A $\cap B$)$\cup$(A $\cap$ C)

(ii) x$\in$ A and x$\in$ C

$\therefore$ x$\in$(A $\cap$ C)

$\therefore$ x$\in$(A $\cap B$)$\cup$(A $\cap$ C)

according to (i)and(ii),we can conclude that A $\cap$(B $\cup$ C) $\Subset$ (A $\cap B$)$\cup$(A $\cap$ C)

(2) for every x $\in$ (A $\cap B$)$\cup$(A $\cap$ C), x$\in$ (A$\cap$B) or x$\in$ (A$\cap$C)

$\therefore$ there are two cases:

(i) x$\in$ (A$\cap$B)

x$\in$ A and x$\in$ B

x$\in$ A and x$\in$ (B $\cup$ C)

x $\in$ A $\cap$(B $\cup$ C)

(ii) x$\in$ (A$\cap$C)

x$\in$ A and x$\in$ C

x$\in$ A and x$\in$ (B $\cup$ C)

x $\in$ A $\cap$(B $\cup$ C)

according to (i) and (ii),we can conclude that (A $\cap B$)$\cup$(A $\cap$ C) $\Subset$ A $\cap$(B $\cup$ C)

Based on (1) and (2) ,we can conclude that A $\cap$(B $\cup$ C) = (A $\cap B$)$\cup$(A $\cap$ C)

(c)

(1) for every x $\in$ X $\backslash$(A $\cap$ B),we have x$\in$ X and x$\notin$ (A $\cap$ B)

$\therefore$ x $\in$ X and x$\in$ (A$\cap$ B)^{C}

$\therefore$ x$\in$ X and x$\in$((A$\backslash$(A$\cap$ B)) $\cup$ (B$\backslash$(A$\cap$ B)) $\cup$ (A$\cup$B)^{C})

$\therefore$ there are three cases:

(i) x$\in$X and x$\in$ (A$\backslash$(A$\cap$ B))

$\therefore$ x$\in$ X and x$\notin$ B

$\therefore$ x$\in$ (X$\backslash$ B)

$\therefore$ x$\in$ (X $\backslash$ A)$\cup$ (X $\backslash$ B)

(ii) x$\in$X and x$\in$ (B$\backslash$(A$\cap$ B))

$\therefore$ x$\in$ X and x$\notin$ A

$\therefore$ x$\in$ (X$\backslash$ A)

$\therefore$ x$\in$ (X $\backslash$ A)$\cup$ (X $\backslash$ B)

(iii) x$\in$ X and x$\in$ (A$\cup$B)^{C}

$\therefore$ x$\in$ X and x$\notin$ A and x$\notin$ B

$\therefore$ x$\in$ (X $\backslash$ A)$\cap$ (X $\backslash$ B)

$\therefore$ x$\in$ (X $\backslash$ A)$\cup$ (X $\backslash$ B)

according to (i), (ii) and (iii), we can conclude that X $\backslash$(A $\cap$ B) $\Subset$ (X $\backslash$ A)$\cup$ (X $\backslash$ B)

(2)for every x $\in$ (X $\backslash$ A)$\cup$ (X $\backslash$ B),we have x$\in$ X and x$\in$((A$\backslash$(A$\cap$ B)) $\cup$ (B$\backslash$(A$\cap$ B)) $\cup$ (A$\cup$B)^{C})

$\therefore$ there are three cases:

(i) x$\in$X and x$\in$ (A$\backslash$(A$\cap$ B))

$\therefore$ x$\in$ X and x$\in$((A$\backslash$(A$\cap$ B)) $\cup$ (B$\backslash$(A$\cap$ B)) $\cup$ (A$\cup$B)^{C})

$\therefore$ x $\in$ X and x$\in$ (A$\cap$ B)^{C}

$\therefore$ x $\in$ X and x$\notin$ (A$\cap$ B)

$\therefore$ x $\in$ X $\backslash$(A $\cap$ B)

(ii) $x \in X$ and $x \in (B\backslash(A\cap B))$

$\therefore x\in X$ and $x\in((A\backslash(A\cap B)) \cup (B\backslash(A\cap B)) \cup (A\cup B)^C)$

$\therefore x \in X$ and $x\in (A\cap B)^C$

$\therefore x \in X$ and $x\notin (A\cap B)$

$\therefore x \in X \backslash(A \cap B)$

(iii) $x\in X$ and $x\in (A\cup B)^C$

$\therefore x\in X$ and $x\in((A\backslash(A\cap B)) \cup (B\backslash(A\cap B)) \cup (A\cup B)^C)$

$\therefore x \in X$ and $x\in (A\cap B)^C$

$\therefore x \in X$ and $x\notin (A\cap B)$

$\therefore x \in X \backslash(A \cap B)$

according to (i), (ii) and (iii), we can conclude that $(X \setminus A)\cup (X \setminus B)$ $X \backslash(A \cap B) \Subset (X \backslash(A \cap B))$

according to (1) and (2),we can conclude that $X \backslash(A \cap B)=(X \setminus A)\cup (X \setminus B)$.

(d)

(1) Because $A\Subset B$, we have $B^C \Subset A^C$

for every $x\in(X\backslash B)$, we have $x\in (X \cap B^C)$

and $\because B^C \Subset A^C$

$\therefore x\in (X \cap A^C)$

$\therefore (X \setminus B) \Subset (X \setminus A)$

(2) $\because (X \setminus B) \Subset (X \setminus A)$

$\therefore (X \cap B^C) \Subset (X \cap A^C)$

$\therefore B^C \Subset A^C$

$\therefore A\Subset B$

Based on (1) and (2),we can conclude that $A\Subset B$ if and only if $(X \setminus B) \Subset$

(e)

(1) $\because A \cap B=B$

$\therefore$ for every $x\in B$, we have $x\in(A \cap B)$

$\therefore x\in A$

$\therefore B\Subset A$

(2) $\because B\Subset A$

$\therefore$ for every $x \in B$, we have $x \in A$

(i)

$\therefore$ for $x \in (A \cap B)$, we have $x \in B$

$\therefore A\cap B \Subset B$

(ii)

$\therefore$ for $x \in B$,we have $x\in A$

$\therefore$ x$\in$ A$\cap$B

$\therefore$ B $\Subset$ (A$\cap$B)

accoding to (i) and (ii) ,we can conclude that A$\cap$B=B

Based on (1) and (2),we can conclude that A $\cap$ B=B if and only if B$\Subset$A

题目 (UD:7.8)

Consider the following sets:

(i) (A$\cap$B) \ (A$\cap$B$\cap$C),

(ii) A$\cap$B \ (A$\cap$B$\cap$C),

(iii) A$\cap$ B $\cap$ $C^{\complement}$,

(iv) (A$\cap$B) \ C,and

(v) (A\C) $\cap$ (B \C).

(a) Which of the sets above are written ambiguously,if any?

(b) Of the sets above that make sense, which ones equal the set sketched in Figure 7.2?

(c) Prove that (A$\cap$B)\C=(A\C) $\cap$ (B\C).

解答:

(a) the second one is written ambiguously.

(b) all above except the second one equal the set sketched in Figure 7.2

(c)

(1) for every x $\in$ (A$\cap$B)\C,we have x$\in$ (A $\cap$ B) and x$\notin$ C

$\therefore$ x$\in$ (A$\cap$ B) and x$\in$ $C^{\complement}$

$\therefore$ x$\in$ (A $\cap$ B $\cap$ $C^{\complement}$)

$\therefore$ x$\in$ (A$\cap$$C^{\complement}$) $\cap$ (B$\cap$$C^{\complement}$)

$\therefore$ x$\in$ (A\C) $\cap$ (B\C)

$\therefore$ (A$\cap$B)\C $\Subset$ (A\C) $\cap$ (B\C)

(2) for every x $\in$ (A\C) $\cap$ (B\C),we have x$\in$ (A$\cap$$C^{\complement}$) $\cap$ (B$\cap$$C^{\complement}$)

$\therefore$ x$\in$ (A $\cap$ B $\cap$ $C^{\complement}$)

$\therefore$ x$\in$ (A$\cap$ B) and x$\in$ $C^{\complement}$

$\therefore$ x$\in$ (A $\cap$ B) and x$\notin$ C

$\therefore$ x$\in$ (A$\cap$B)\C

$\therefore$ (A\C) $\cap$ (B\C) $\Subset$ (A$\cap$B)\C

According to (1) and (2),we can conclude that (A$\cap$B)\C=(A\C) $\cap$ (B\C).

题目 (UD:7.9)

In this problem you will prove that the union of two sets can be rewritten as the union of two disjoint sets.

(a) Prove that the two sets A\B and B are disjoint.

(b) Prove that A∪=(A\B) ∪ B.

解答:

(a)

(1) for every $x \in A\backslash B$,we have $x\in A$ and $x\notin B$

$\therefore x\notin B$

(2) for every $x \in B$, we have $x\notin B^C$

$\therefore x\notin A\backslash B$

Based on (1) and (2),we can conclude that A\B and B are disjoint.

(b)

(1) for every $x \in A\cup B$, we have $x\in A$ or $x\in B$

for $x\in B$,we can show that $x\in (A \setminus B) \cup B$

for $x\in A$,there are two cases:

(i) $x\in A\cap B$

$\therefore x\in B$

$\therefore x\in (A \setminus B) \cup B$

(ii) $x\in A\setminus (A\cap B)$

$\therefore x\in A\setminus B$

$\therefore x\in (A \setminus B) \cup B$

$\therefore$ we can conclude that $A\cup B \Subset (A\backslash B)\cup B$.

(2) for every $x \in (A\backslash B)\cup B$,we have $x\in (A\backslash)$ or $x\in B$.

(i). $x\in (A\backslash)$

$\therefore x\in A$

$\therefore x\in A\cup B$

(ii).$x\in B$

$\therefore x\in A\cup B$

$\therefore$ we can conclude that $(A\backslash B)\cup B \Subset A\cup B$

Based on (1) and (2), $A\cup=(A\backslash B) \cup B$

题目 (UD:7.10)

Prove or disprove: If $A\cup B=A\cup C$, then $B=C$;

解答:

let $A=\{1,2,3\}$ $B=\{1\}$ $C=\{2\}$

then we have $A\cup B = A\cup C = \{1,2,3\}$,but $B\neq C$

So the statement is not true.

题目 (UD:7.11)

Prove or give a counterexample for the following statement.

Let X be the universe and A,B⋐X. If A∩Y=B∩Y for all Y⋐ X,then A=B.

解答:

(1) Let Y=A, then we have A∩A=B∩A

∴ A=B∩A

∴ A⋐B

(2) Let Y=B, then we have A∩B=B∩B

∴ A∩B=B

∴ B⋐A

Based on (1) and (2),we can conclude that A=B

So the statement is true.

题目 (UD:8.1)

Consider the intervals of real numbers given by A_n=[0,1/n),B_n=[0,1/n],and C_n=(0,1/n).

(a)Find $\bigcup_{n=1}^{\infty} A_n$,$\bigcup_{n=1}^{\infty} B_n$,and $\bigcup_{n=1}^{\infty} C_n$.

(b)Find $\bigcap_{n=1}^{\infty} A_n$,$\bigcap_{n=1}^{\infty} B_n$,and $\bigcap_{n=1}^{\infty} C_n$.

(c)Does $\bigcup_{n\in N} A_n$ make sense? Why or why not?

解答:

(a)

$\bigcup_{n=1}^{\infty} A_n$=[0,1)

$\bigcup_{n=1}^{\infty} B_n$=[0,1]

$\bigcup_{n=1}^{\infty} C_n$=(0,1)

(b)

$\bigcap_{n=1}^{\infty} A_n$={0}

$\bigcap_{n=1}^{\infty} B_n$={0}

$\bigcap_{n=1}^{\infty} C_n$=∅

(c)

It doesn't make sense.

∵ 0∈N, but 0 cannot be the denominator.

题目 (UD:8.4)

Prove or give a counterexample:Let {A_n:n∈Z^+} and {B_n:n∈Z^+} be two indexed families of set. If $A_n \subsetneq B_n$ for all n∈ Z^+, then

$\bigcap_{n=1}^{\infty} A_n \subsetneq \bigcap_{n=1}^{\infty} B_n$.

(Recall that A$\subsetneq$B means strict inclusion; that is, A$\Subset$B and A$\neq$B.)

解答:

This statement is not true.

We can let B_n={1,2}

when n is odd let A_n={1}

when n is even let A_n={2}

$\therefore$ for all n$\in$ Z^+, we have $A_n \subsetneq B_n$

but $\bigcap_{n=1}^{\infty} A_n = \bigcap_{n=1}^{\infty} B_n$ $\therefore$ the statement is not true.

题目 (UD:8.7)

Suppose that {A_α:$\alpha\in$I} is an indexed family of subsets of a set X, and that B is a subset of X.

(a) If A_α =$\emptyset$ for some $\alpha \in$ I,prove that $\bigcap_{\alpha\in I} A_\alpha$ =$\emptyset$.

(b) If A_α =X for some $\alpha \in$ I,prove that $\bigcup_{\alpha\in I} A_\alpha$ =X.

(c) If B$\Subset A_\alpha$ for every $\alpha \in$ I, prove that B$\Subset$ $\bigcap_{\alpha\in I} A_\alpha$.

解答:

(a) Suppose that $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$, we have there exists an x$\in$ $\bigcap_{\alpha\in I} A_\alpha$ =$\emptyset$

$\therefore$ we have there exists an x $\in A_\alpha$ for all $\alpha \in$ I

$\therefore$ $A_\alpha \neq \emptyset$ for all $\alpha \in$ I

but it contradicts with the condition that A_α =$\emptyset$ for some $\alpha \in$ I

$\therefore$ what we suppose is wrong

$\therefore$ $\bigcap_{\alpha\in I} A_\alpha$ =$\emptyset$

(b) Suppose that $\bigcup_{\alpha\in I} A_\alpha \neq$ X, we have there exists an x $\in$ X but $\notin \bigcup_{\alpha\in I} A_\alpha$

$\therefore$ there exists an x$\in$ X but $\notin A_\alpha$ for all $\alpha \in$ I

but it contradicts with the condition that A_α =X for some $\alpha \in$ I

$\therefore$ what we suppose is wrong

$\therefore$ $\bigcup_{\alpha\in I} A_\alpha$ =X

(c) Suppose that B$\nsubseteq$ $\bigcap_{\alpha\in I} A_\alpha$, we have there exists an x$\in$ B but $\notin \bigcap_{\alpha\in I} A_\alpha$.

$\therefore$ there exists an x$\in$ B but $\notin A_\alpha$ for some $\alpha \in$ I

but it contradicts with the condition that B$\Subset A_\alpha$ for every $\alpha \in$ I

$\therefore$ what we suppose is wrong

$\therefore$ B$\Subset$ $\bigcap_{\alpha\in I} A_\alpha$

题目 (UD:8.8)

Define

$$A=R\setminus \bigcap_{n\in Z^+} (R\setminus \{-n,-n+1,...,0,...,n-1,n\}).$$

The set A should be familiar to you. Guess what it is and then prove that your guess is correct.

解答:

A=Z

According to Exercise 8.9, we have $\bigcap_{n\in Z^+}$ (R\ {-n,-n+1,...,0,...,n-1,n})=R \ ($\bigcup_{n\in Z^+}$ {-n,-n+1,...,0,...,n-1,n}) $\therefore$ A=R\ (R \ Z)

$\therefore$ A=R\ $\complement_R^Z$

$\therefore$ A=Z

题目 (UD:8.9)

Guess a simpler way to express the set A defined as

$$A=Q\setminus \bigcap_{n\in Z}(R\setminus\{2n\}),$$

and then prove that your guess is correct.

解答:

A={2n|n∈Z}

According to Exercise 8.9, we have $\bigcap_{n\in Z}$(R\{2n})=R\ $\bigcup_{n\in Z}${2n}

$\therefore$ A= Q\ (R\ {2n|n∈Z})

$\therefore$ A= Q\ $\complement_R^{\{2n|n\in Z\}}$

$\therefore$ A={2n|n∈Z}

题目 (UD:8.11)

A collection of sets$\{A_\alpha:\alpha \in I\}$ is said to be a pairwise disjoint collection if the following is satisfied :For all $\alpha\beta \in$I, if $A_\alpha \cap A_\beta \neq \emptyset$, then $A_\alpha=A_\beta$. Suppose that each set A_α is nonempty.

(a) Give an example of pairwise disjoint sets A_1,A_2,A_3,....

(b) What is the contrapositive of "if $A_\alpha \cap A\beta \neq \emptyset$, then $A_\alpha=A_\beta$"?

(c) What is the converse of "if $A_\alpha \cap A\beta \neq \emptyset$, then $A_\alpha=A_\beta$"?

(d) If $\{A_\alpha : \alpha \in I\}$ is a pairwise disjoint collection, does the assertion you found in (b) hold for all α and β in I?

(e) If the assertion that you found in (b) holds for all α and β in I, is $\{A_\alpha : \alpha \in I\}$ a pairwise disjoint collection?

(f) If $\{A_\alpha : \alpha \in I\}$ is a pairwise disjoint collection of sets, does it follow that $\bigcap_{\alpha\in I}A_\alpha$ $=\emptyset$?

(g) If $\bigcap_{\alpha\in I}A_\alpha =\emptyset$, is $\{A_\alpha : \alpha \in I\}$ necessarily a pairwise disjoint collection of sets?

解答:

(a) $A_n=\{1\}$

(b) if $A_\alpha \neq A_\beta$, then $A_\alpha \cap A\beta = \emptyset$

(c) if $A_\alpha = A_\beta$, then $A_\alpha \cap A\beta \neq \emptyset$

(d) yes

(e) yes

(f) no

(g) no

题目 (UD:9.2)

(a) Show that $P(A) \cup P(B) \Subset P(A \cup B)$.

(b) Show that $P(A) \cup P(B) \neq P(A \cup B)$ by exhibiting two concrete sets, A and B, for which the aforementioned inequality holds.

解答:

(a)

for every $x \in P(A) \cup P(B)$, we have $x \Subset A$ or $x \Subset B$.

$\therefore x \Subset A \cup B$

$\therefore x \in P(A \cup B)$

$\therefore P(A) \cup P(B) \Subset P(A \cup B)$

(b)

Let $A=\{1\}$ $B=\{2\}$ $A \cup B=\{1,2\}$

$P(A)=\{\{1\},\emptyset\}$, $P(B)=\{\{2\},\emptyset\}$, $P(A \cup B)=\{\{1\},\{2\},\{1,2\},\emptyset\}$

$P(A) \cup P(B)=\{\{1\},\{2\},\emptyset\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

题目 (UD:9.4)

Show that $A \Subset B$ if and only if $P(A) \Subset P(B)$.

解答:

(1) for every $x \in P(A)$, we have $x \Subset A$

$\therefore x \Subset B$

$\therefore x \in P(B)$

$\therefore$ we can conclude that $P(A) \Subset P(B)$

(2) for every $x \Subset A$, we have $x \in P(A)$

$\therefore x \in P(B)$

$\therefore$ x$\Subset$B

$\therefore$ we can conclude that A$\Subset$B

Based on (1) and (2), A$\Subset$B if and only if P(A) $\Subset$ P(B)

题目 (UD:9.12)

(a) Prove the following:

Let A,B,C and D be nonempty sets. Then A $\times$ B=C $\times$ D if and only if A=C and B=D.

(b) Where did your proof use the fact that the sets were nonempty?

解答:

(a)

(1)

$\because$ A,B,C and D are nonempty sets

$\therefore$ A$\times$B=$\{$(x,y)$|$x$\in$A,y$\in$B$\}$ $\neq \emptyset$

C$\times$D=$\{$(z,w)$|$z$\in$C,w$\in$D$\}$ $\neq \emptyset$

$\because$ A $\times$ B=C $\times$ D

$\therefore$ for every (x,y) $\in$ A$\times$B ,we can find (x,y) $\in$ C$\times$D

for every (z,w) $\in$ C$\times$D ,we can find (z,w) $\in$ A$\times$B

$\therefore$ for every x $\in$A, we can find x$\in$C (this is also the same for y)

for every z$\in$C,we can find z$\in$ A(this is also the same for w)

$\therefore$ A$\Subset$C and C$\Subset$ A, B$\Subset$D and D$\Subset$ B

$\therefore$ A=C,B=D

(2)

$\because$ A,B,C and D are nonempty sets

$\therefore$ A$\times$B=$\{$(x,y)$|$x$\in$A,y$\in$B$\}$ $\neq \emptyset$

C$\times$D=$\{$(z,w)$|$z$\in$C,w$\in$D$\}$ $\neq \emptyset$

$\because$ A=C and B=D

$\therefore$ A$\Subset$C and C$\Subset$ A, B$\Subset$D and D$\Subset$ B

$\therefore$ for every x $\in$A, we can find x$\in$C (this is also the same for y)

for every z$\in$C,we can find z$\in$ A(this is also the same for w)

$\therefore$ for every (x,y) $\in$ A$\times$B ,we can find (x,y) $\in$ C$\times$D

for every (z,w) $\in$ C$\times$D ,we can find (z,w) $\in$ A$\times$B

$\therefore$ A $\times$ B$\Subset$C $\times$ D and C $\times$ D $\Subset$ A $\times$ B

$\therefore$ A $\times$ B=C $\times$ D

Based on (1) and (2),Let A,B,C and D be nonempty sets. Then A $\times$ B=C $\times$ D if and only if A=C and B=D.

(2)

when I want to illustrate that $A\times B \neq \emptyset$ and $C\times D \neq \emptyset$

题目 (UD:9.13)

Suppose A,B,C and D are four sets. If $A\times B \Subset C\times D$, must $A \Subset C$ and $B \Subset D$? Why or why not?

解答:

No, it doesn't need to.
We can let $C \Subset A$ and $A \neq C$ and $B=D=\emptyset$
We can still have $A\times B \Subset C\times D$

题目 (UD:9.14)

Let A,B and C be sets. If the statements below are true prove them.
If they are false,give a counterexample:
(a) $A\times(B\cup C) = (A\times B) \cup (A\times C)$;
(b) $A\times(B\cap C) = (A\times B) \cap (A\times C)$;

解答:

(a)
(1)for every $(x,y)\in A\times(B\cup C)$, we have $x\in A$ and $y\in (B\cup C)$
$\therefore$ there are two cases:
    (i) $x\in A$ and $y\in B$
        $\therefore (x,y)\in A\times B$
        $\therefore (x,y)\in (A\times B)\cup(A\times C)$
    (ii) $x\in A$ and $y\in C$
        $\therefore (x,y)\in A\times C$
        $\therefore (x,y)\in (A\times B)\cup(A\times C)$
$\therefore$ according to (i) and (ii),$A\times(B\cup C) \Subset (A\times B) \cup (A\times C)$
(2)for every $(x,y) \in (A\times B) \cup (A\times C)$, we have $(x,y)\in A\times B$ or$(x,y)\in A\times C$
$\therefore$ therefore there are two cases:
    (i)$(x,y)\in A\times B$
        $x\in A$ and $y\in B$
        $x\in A$ and $y\in (B\cup C)$
        $(x,y)\in A\times(B\cup C)$
    (ii)$(x,y)\in A\times C$
        $x\in A$ and $y\in C$
        $x\in A$ and $y\in (B\cup C)$
        $(x,y)\in A\times(B\cup C)$

$\therefore$ according to (i) and (ii),$(A\times B) \cup (A\times C) \Subset A\times(B\cup C)$

Based on (1) and (2),$A\times(B\cup C) = (A\times B) \cup (A\times C)$

(b)

(1)for every $(x,y)\in A\times(B\cap C)$, we have $x\in A$ and $y\in B\cap C$

$\therefore$ we have ($x\in A$ and $y\in B$) and ($x\in A$ and $y\in C$)

$\therefore$ we have $(x,y)\in(A\times B)$ and $(x,y)\in(A\times C)$

$\therefore$ we have $(x,y)\in (A\times B) \cap (A\times C)$

$\therefore A\times(B\cap C) \Subset (A\times B) \cap (A\times C)$

(2)for every $(x,y)\in A\times(B\cap C)$, we have $(x,y)\in(A\times B)$ and $(x,y)\in(A\times C)$

$\therefore$ we have ($x\in A$ and $y\in B$) and ($x\in A$ and $y\in C$)

$\therefore$ we have $x\in A$ and $y\in B\cap C$

$\therefore$ we have $(x,y)\in A\times(B\cap C)$

$\therefore (A\times B) \cap (A\times C) \Subset A\times(B\cap C)$

Based on (1) and (2),$A\times(B\cap C) = (A\times B) \cap (A\times C)$

题目 (UD:9.16)

This problem introduces rigorous definitions of an ordered pair and Cartesian product. Let A be a set and $a,b\in A$. We define the ordered pair of a and b with first coordinate a and second coordinate b as

$$(a,b)=\{\{a\},\{a,b\}\}.$$

Using this definition prove the following.

(a) If $(a,b)=(x,y)$, then $a=x$ and $b=y$.

(b) If $a\in A$ and $b\in B$, then $(a,b) \in P(P(A\cup B))$.

Now we are able to define the Cartesian product of the two sets A and B as the set $A\times B=\{x\in P(P(A\cup B))\ x=(a,b)$ for some $a\in A$ and some $b\in B\}$.

(c) Use the above definations to prove that if $A\Subset C$ and $B\Subset D$,then $A\times B \Subset C\times D$.

This is a pretty complicated defination. It is also not our idea, but rather an idea that was born from axioms. P.Halmos' book,[31],is an excellent reference for this subject.

解答:

(a)

$\because (a,b)=(x,y)$

$\therefore \{\{a\},\{a,b\}\}=\{\{x\},\{x,y\}\}$

$\therefore \{a\}==\{x\}$ and $\{a,b\}=\{x,y\}$

$\therefore a=x$ and $b=y$

(b)

$\because a\in A$ and $b\in B$

$\therefore a\in A\cup B$ and $b\in A\cup B$

$\therefore$ a$\in$ P(A$\cup$B) and a,b$\in$ P(A$\cup$B)

$\therefore$ {{a},{a,b}}$\in$P(P(A$\cup$B))

$\therefore$ (a,b) $\in$ P(P(A$\cup$B))

(c)

(1) Prove if a=x and b=y, then (a,b)=(x,y)

$\because$ a=x and b=y

$\therefore$ {a}=={x} and {a,b}={x,y}

$\therefore$ {{a},{a,b}}={{x},{x,y}}

$\therefore$ (a,b)=(x,y)

(2)$\because$ A$\Subset$C and B$\Subset$D

$\therefore$ for every x$\in$A, we have x$\in$C

for every y$\in$B, we have y$\in$D

$\therefore$ for every (x,y) $\in$ (A$\times$B),we have (x,y) $\in$ (C$\times$D)

$\therefore$ A$\times$B $\Subset$ C$\times$D

第二部分　订正

题目 (题号)

题目。

错因分析： 简述错误原因（可选）。

订正：

正确解答。

第三部分　反馈

你可以写：

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- 等