第十讲：函数

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2017 年 12 月 6 日

请独立完成作业，不得抄袭。 若参考了其它资料，请给出引用。 鼓励讨论，但需独立书写解题过程。

第一部分　作业

题目 (UD:13.3)

Which of the following are functions? Give reasons for your answers.

(a) Define f on $\mathbb{R}$ by f ={(x,y):x^2+y^2=4}.

(b) Define f :$\mathbb{R} \to \mathbb{R}$ by $f(x) = 1/(x+1)$.

(c) Define f :$\mathbb{R}^2 \to \mathbb{R}$ by $f(x,y) = x + y$.

(d) The domain of f is the set of all closed intervals of real numbers of the form [a,b], where a,b∈$\mathbb{R}$, a≤b, and f is defined by $f([a,b]) = a$.

(e) Define f : $\mathbb{N}\times\mathbb{N} \to \mathbb{R}$ by $f(n,m) = m$.

(f) Define f : $\mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & if \quad x \geq 0 \\ x & if \quad x \leq 0 \end{cases}$$

(g) Define f : $\mathbb{Q} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x+1 & if \quad x \in 2\mathbb{N} \\ x-1 & if \quad x \in 3\mathbb{N} \\ 2 & otherwise \end{cases}$$

(h) The domain of f is the set of all circles in the plane $\mathbb{N}^2$ and, if c is such a circle, define f by $f(c)$= the circumference of c.

(i) (*For students with a background in calculus.*) The domain of f is the set of all polynomials with real coefficients, and f is defined by $f(p) = p'$.(Here p$'$ is the derivative of p.)

(j) (*For students with a background in calculus.*) The domain of f is the set of all polynomials and f is defined by $f(p) = \int_0^1 p(x)dx$. (Here $\int_0^1 p(x)dx$ is the definite integral of p.)

解答:

(a) This is not a function, because it doesn't satisfy (i) or (ii). For example for x=0, y can be 2 or -2.

(b) This is not a function, because it doesn't satisfy (i), x=-1$\in$ dom(f) but this x makes the expression meaningless, we cannot find a y related to it.

(c) This is a function, because it satisfies (i) and (ii)

(d) This is a function, because it satisfies (i) and (ii)

(e) This is a function, because it satisfies (i) and (ii)

(f) This is a function, because it satisfies (i) and (ii)

(g) This is not a function, because it doesn't satisfy (ii). For example, for x=6, we have y=7 or 5.

(h) This is a function, because it satisfies (i) and (ii)

(i) This is a function, because it satisfies (i) and (ii)

(j) This is a function, because it satisfies (i) and (ii)

题目 (UD:13.4)

Let $f : P(R) \to Z$ be defined by

$$f(A) = \begin{cases} min(A \bigcap N) & if \quad A \bigcap N \neq \phi \\ -1 \quad if \quad A \bigcap N = \phi \end{cases} \tag{1}$$

Prove that f above is a well-defined function.

解答:

(i) for all A$\in$ $\mathbb{P}(\mathbb{R})$, whether A$\cap$ $\mathbb{N} = \emptyset$ has two cases.

and the two cases are both defined in f

(ii) For every A that satisfies A$\cap$ $\mathbb{N}$ =$\emptyset$, y is unique, namely -1.

For every A that satisfies A$\cap$ $\mathbb{N} \neq \emptyset$, there is only one A$\cap$ $\mathbb{N}$, and the minmum of A$\cap$ $\mathbb{N}$ is unique , so y is unique.

$\therefore$ for every A$\in$ $\mathbb{P}(\mathbb{R})$, the y it relates to is unique.

$\therefore$ f satisfies (i) and (ii)

$\therefore$ f is a well-defined function.

题目 (UD:13.5)

Let x be a nonempty set and let A be a subset of X. We define the Characteristic function of the set A by

$$X_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \in X \backslash A \end{cases} \tag{2}$$

(a)Since this is called the characteristic function, it probably is a function, but check this carefully anyway.

(b)Determine the domain and range of this function. Make sure you look at all possibilities for A and X.

解答:

The domain of is X, the range is {0,1}

(i) Since A is a subset of X, for every x∈X, there are two cases: x∈A or x∈X\A

The two cases are defined in the X_A

(ii) Since A is a subset of X, for every x∈X, there are two cases: x∈A or x∈X\A

(1) x∈A , the y is unique, namely 1

(2) x∈X\A, the y is also unique, namely 0

∴ for every x∈X, the y it relates to is unique.

∴ X_A satisfies (i) and (ii)

∴ X_A is a well-defined function.

题目 (UD:13.11)

Suppose that f is a function from a set A to a set B. Thus, we know that f is a subset of $A \times B$. Is the relation $\{(y, x) : (x, y) \in f\}$ necessarily a function from B to A? Why or why not? (Say as much as is possible to say with the given information.)

解答:

No,let us consider an example: f_1: (1,2) (2,2)

then according to the relation, we have f_2: (2,1) (2,2)

∴ for x=2, we have two y's: y=1 or y=2, it doesn't satisfy (ii)

题目 (UD:13.13)

Let X be a nonempty set. Find all relations on X that are both equivalence relations and functions.

解答:

x∼y:x=y

题目 (UD:14.8)

For each of the functions below, determine whether or not the function is one-to-one and whether or not the function is onto. If the function is not one-to-one, give an explicit example to show what goes wrong. If it is not onto, determine the range.

(a) Define $f : R \to R$ by $f(x) = 1/(x^2 + 1)$.

(b) Define $f : R \to R$ by $f(x) = sin(x)$. (Assume familiar facts about the sine function.)

(c) Define $f : Z \times Z \to Z$ by $f(n, m) = nm$.

(d) Define $f : R^2 \times R^2 \to R$ by $f((x, y),(u, v)) = xu + yv$. (Do you recognize this function?)

(e) Define $f : R^2 \times R^2 \to R$ by $f((x, y),(u, v)) = \sqrt{(x - u)^2 + (y - v)^2}$. (Do you recognize this function?)

(f) Let A and B be nonempty sets and let $b \in B$. Define $f : A \to A \times B$ by $f(a) = (a, b)$.

(g) Let X be a nonempty set, and P(x) the power set of X. Define $f : P(x) \to P(x)$ by $f(A) = X \backslash A$.

(h)Let B be a fixed proper subset of a nonempty set X. Define a function $f : P(X) \to P(X)$ by $f(A) = A \bigcap B$.

(i) Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2 - x & if \quad x < 1 \\ 1/x & otherwise \end{cases} \tag{3}$$

解答:

(a)

(1) This function isn't one-to-one, for y=1/2, we have x^2=1, then we have x=1 or x=-1

(2) This function isn't onto, the range of this function is (0,1]

(b)

(1) This function isn't one-to-one, for y=0, we have x=kπ k∈$\mathbb{Z}$

(2) This function isn't onto, the range of this function is [-1,1]

(c)

(1) This function isn't one-to-one, for y=2, we have nm=2, then we have (n=1 and m=2)or (n=2 and m=1)

(2) This function is onto.

(d)

(1) This function isn't one-to-one, for y=2, we can have (x=1, u=2,y=0,v=0) or (x=2,u=1,y=v=0) and so on

(2) This function is onto.

(e)

(1) This function isn't one-to-one, for y=2, we can have (x=2, u=0,y=0,v=0) or (x=0,u=2,y=v=0) and so on

(2) This function isn't onto, the range of this function is [0,+∞)

(f)

(1) This function is one-to-one.

(2) This function isn't onto, the range of this function is {(x,y):x∈ $A, y = b$}

(g)

(1) This function is one-to-one.

(2) This function is onto.

(h)

(1) This function isn't one-to-one, for y=B, we can have A=B or A=X and so on

(2) This function isn't onto, the range of this function is $\mathcal{P}$(B)

(i)

(1) This function is one-to-one.

(2) This function isn't onto, the range of this function is (0,+∞)

题目 (UD:14.12)

Let a, b, c, and d be real numbers with $a < b$ and $c < d$. Define a bijection from the closed interval $[a, b]$ onto the closed interval $[c, d]$ and prove that your function is a bijection.

解答:

f(x)=$\frac{d-c}{b-a}$(x-a) +c

Prove:

(1)if x1,x2$\in$[a,b] and f(x_1)=f(x_2), then $\frac{d-c}{b-a}$(x_1-a) +c=$\frac{d-c}{b-a}$(x_2-a) +c

$\therefore$ $\frac{d-c}{b-a}$(x_1-a)=$\frac{d-c}{b-a}$(x_2-a)

$\because$ a<b and c<d

$\therefore$ a-b<0, c-d<0

$\therefore$ x_1-a=x_2-a

$\therefore$ x_1=x_2

$\therefore$ this function is one-to-one

(2)$\because$ the range of x is [a,b]

$\therefore$ the range of (x-a) is [0,b-a]

$\therefore$ the range of $\frac{x-a}{b-a}$ is [0,1]

$\therefore$ the range of $\frac{d-c}{b-a}$(x-a) is [0,d-c]

$\therefore$ the range of $\frac{d-c}{b-a}$(x-a) +c is [c,d]

$\therefore$ this function is onto.

Based on (1) and (2), this function is bijective.

题目 (UD:14.13)

Let $F([0, 1])$ denote the set of all real-valued functions defined on the closed interval $[0, 1]$. Define a new function $\phi : F([0, 1]) \to R$ by $\phi(f) = f(0)$. Is ϕ a function from $F([0, 1])$ to R? Is it one-to-one? Is it onto? Remember to prove all claims, and to provide examples where appropriate.

解答:

(1) ϕ is a function from F([0,1]) to R. For every f defined on [0,1], we can have a unique f(0), thus a unique ϕ(f),and f(0)∈R. ∴ ϕ is a function from F([0,1]) to R.

(2) it is not one-to-one. for ϕf=f(0). we can have f(x)=x+f(0) or f(x)=2x+f(0) and so on

(3) it is onto, for every f(0)∈R, we can let f(x)=x+f(0) ∈ F([0,1])

题目 (UD:14.15)

Let f be a function, $f : R \to R$. Define a new function $f \cdot f$ by

$$(f \cdot f)(x) = f(x) \cdot f(x).$$

Prove that $f \cdot f$ is a function. Then do the remaining parts of the problem. (You may wish to work Problem 14.14, if you havem't already done so.)

(a) Does there exists a function f for which $f \cdot f$ is one-to-one? If not, why not? If there is, what is an example?

(b) Does there exist a function f for which $f \cdot f$ maps onto R? If not, what is $ran(f \cdot f)$? Your answer will be terms of $ran(f)$.

解答:

(i) ∵ f(x) is a function

∴ for every x∈ R, there is a f(x)∈R that it relates to.

∴ for every x∈ R, there is a f(x)*f(x)∈R that it relates to.

(ii) ∵ f(x) is a function

∴ for every x∈ R, there is an unique f(x)∈R that it relates to.

∴ for every x∈ R, there is an unique f(x)*f(x)∈R that it relates to.

Based on (i) and (ii), we have (f · f)(x) is a function

(a) Yes, let f(x)=2^x

(b)No, there doesn't exist a function f for which f · f maps onto R

(1) if ran(f)=(a,b) where a≤0≤b, then ran(f · f)=[0,$(\max(|a|,|b|))_2$). (for ran(f)=[a,b] or (a,b] or [a,b) is the similar)

(2) if ran(f)=(a,b) where a<b<0 or b>a>0, then ran(f · f)=($(\min(|a|,|b|))^2$. $(\max(|a|,|b|))^2$) (for ran(f)=[a,b] or (a,b] or [a,b) is the similar)

(3) if ran(f)=(a,+∞),where a≥0, then ran(f · f)=(a^2,+∞), (for ran(f)=[a,+∞) is the similar)

(4) if ran(f)=(a,+∞),where a<0, then ran(f · f)=[0,+∞). (for ran(f)=[a,+∞) is the similar)

(5) if ran(f)=(-∞,b),where b≥0, then ran(f · f)=[0,+∞). (for ran(f)=(-∞,b] is the similar)

(6) if ran(f)=(-∞,b),where b<0, then ran(f · f)=(b_2,+∞). (for ran(f)=(-∞,b] is the similar)

题目 (UD:15.1)

Find the compositions $f \circ g$ and $g \circ f$ assuming the domain of each is the largest set of real numbers for which the functions $f, g, f \circ g$, and $g \circ f$ make sense. In your solution to each of the following, give the compositions and the corresponding domain and range:

(a) $f(x) = 1/(1+x), g(x) = x^2$;

(b) $f(x) = x^2, G(X) = \sqrt{x}$ (simplify this one);

(c) $f(x) = 1/x, g(x) = x^2 + 1$;

(d) $f(x) = |x|, g(x) = f(x)$.

解答:

(a)

f∘g=f(g(x))=f(x^2)=$\frac{1}{1+x^2}$ dom(f∘g)=R ran(f∘g)=(0,1]

g∘f=g(f(x))=g($\frac{1}{1+x}$)=$\frac{1}{(1+x)^2}$ dom(g∘f)=R\{-1} ran(g∘f)=(0,+∞)

(b)

f∘g=f(g(x))=f($\sqrt{x}$)=x dom(f∘g)=[0,+∞) ran(f∘g)=[0,+∞)

g∘f=g(f(x))=g(x^2)=|x| dom(g∘f)=R ran(g∘f)=[0,+∞)

(c)

f∘g=f(g(x))=f(x^2+1)=$\frac{1}{1+x^2}$ dom(f∘g)=R ran(f∘g)=(0,1]

g∘f=g(f(x))=g($\frac{1}{x}$)=$\frac{1}{x^2}$+1 dom(g∘f)=R\{0} ran(g∘f)=(1,+∞)

(d)

f∘g=f(g(x))=f(f(x))=f(|x|)=|x| dom(f∘g)=R ran(f∘g)=[0,+∞)

g∘f=g(f(x))=g(|x|)=f(|x|)=|x| dom(g∘f)=R ran(g∘f)=[0,+∞)

题目 (UD:15.6)

The functions $f : R\backslash\{-2\} \to R\backslash\{1\}$ and $g : R\backslash\{1\} \to R\backslash\{-2\}$ defined by

$$f(x) = \frac{x-3}{x+2} \quad \textit{and} \quad g(x) = \frac{3+2x}{1-x}$$

are well-defined functions (you need not check this).

(a) Calculate $f \circ g$ and $g \circ f$.

(b) What can you conclude about f and g from your result in part (a)? If you use a theorem, give a reference.

解答:

(a)

f∘g=f(g(x))=f($\frac{3+2x}{1-x}$)=$\frac{\frac{3+2x}{1-x}-3}{\frac{3+2x}{1-x}+2}$=$\frac{3+2x-3(1-x)}{3+2x+2(1-x)}$=$\frac{5x}{5}$=x g∘f=g(f(x))=g($\frac{x-3}{x+2}$)=$\frac{3+\frac{2(x-3)}{x+2}}{1-\frac{x-3}{x+2}}$=$\frac{3(x+2)+2x-3}{x+2-(x-3)}$=$\frac{5x}{5}$=x

(b)

let A=R\{-2} B=R\{1}

And according to (a) we have f∘g=i$_B$ and g∘f=i$_A$

∴ according to Theorem 15.8, we have g=f^{-1}.

题目 (UD:15.7)

(a)If possible, find examples of functions $f : A \to B$ and $g : B \to A$ such that $f \circ g = i_b$ when:

(i) $A = \{1, 2, 3\}, B = \{4, 5\}$;

(ii) $A = \{1, 2\}, B = \{4, 5\}$;

(iii) $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$.

Draw diagrams of A and B in each case above.

(b) Give an example pf sets A and B, and functions $f : A \to B$ and $g : B \to A$ such that $f \circ g = i_B$, but $g \circ f \neq i_A$. (Thus the existence of a function g such that $f \circ g = i_B$ is not enough to conclude that f has an inverse!) Why doesn't this contradict Theorem 15.4, part(iv)?

(c) Give an example of sets A and B, anf functions $f : A \to B$ and $g : B \to A$ such that $g \circ f = i_A$, but $f \circ g \neq i_B$. (Thus the existence of a function g such that $g \circ f = i_A$ is not enough to conclude that f has an inverse!) Why doesn't this contradict Theorem 15.4, part(iv)?

(d) Let A and B be two sets, and let $f : A \to B$ be a function. Assume further that there exists a function $g : B \to A$ such that $f \circ g = i_B$. Must f be one-to-one? onto?

(e) Looking over your work above, what should be your strategy in solving a question like (d) above? Whatever you decide, use it to solve the following: Let f and g be as above and suppose $g \circ f = i_A$. Must f be one-to-one? onto?

解答:

(a)

(i) let:

$$f(x) = \begin{cases} 4 & if \quad x = 1 \\ x + 2 & if \quad x = 2, 3 \end{cases}$$

g(x)=x-2

(ii) let f(x)=x+3 and g(x)=x-3

(iii) this is not possible

(b)

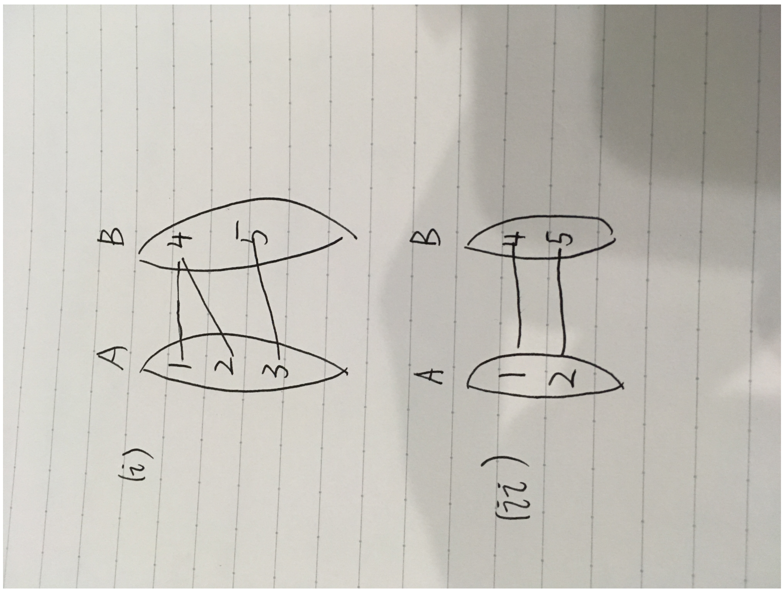

图 1: diagram of A and B

A={1,2,3},B={4,5} let:

$$f(x) = \begin{cases} 4 & if \quad x = 1 \\ x + 2 & if \quad x = 2, 3 \end{cases}$$

g(x)=x-2

This doesn't contradict Theorem 15.4,part(iv) beacuse f here cannot be a bijective function.

(c)

A={4,5},B={1,2,3} let:

$$g(x) = \begin{cases} 4 & if \quad x = 1 \\ x + 2 & if \quad x = 2, 3 \end{cases}$$

f(x)=x-2

This doesn't contradict Theorem 15.4,part(iv) beacuse g here cannot be a bijective function.

(d)

A={1,2,3},B={4,5} let:

$$f(x) = \begin{cases} 4 & if \quad x = 1 \\ x + 2 & if \quad x = 2, 3 \end{cases}$$

g(x)=x-2

As we cans see, f doesn't need to be one-to-one but it has to be onto.

(e)

f must be one-to-one but it doesn't have to be onto.

题目 (UD:15.11)

Suppose that $f : A \to B$ and g_1 and g_2 are functions from B to A such that $f \circ g_1 = f \circ g_2$. Show that if f is bijective, then $g_1 = g_2$. If $g_1 \circ f = g_2 \circ f$ and f is bijective, must $g_1 = g_2$?

解答:

(a)

$\because$ f is bijective

$\therefore$ we have f^{-1}

$\therefore$ $f^{-1} \circ f \circ g_1 = f^{-1} \circ f \circ g_2$

$\therefore$ $i_A \circ g_1 = i_A \circ g_2$

$\because$ $i_A \circ g_1 = g_1$ and $i_A \circ g_2 = g_2$

$\therefore$ $g_1 = g_2$

(b)

$\because$ f is bijective

$\therefore$ we have f^{-1}

$\therefore$ $g_1 \circ f \circ f^{-1} = g_2 \circ f \circ f^{-1}$

$\therefore$ $g_1 \circ i_B = g_2 \circ i_B$

$\because$ $g_1 \circ i_B = g_1$ and $g_2 \circ i_B = g_2$

$\therefore$ $g_1 = g_2$

题目 (UD:15.12)

Let $f : A \to A$ be a function. Define a relation on A by $a \sim b$ if and only if $f(a) = f(b)$. Is this an equivalence? If f is one-to-one, what is the equivalence class of a point $a \in A$?

解答:

Yes, this is an equivalence.

$E_a = \{a\}$

题目 (UD:15.13)

Let $f : A \to A$ be a function. Define a relation on A by $a \sim b$ if and only if $f(a) = b$. Is this an equivalence relation for an arbitrary function f? If not, is there a function for which it is an equivalence relation?

解答:

No. An example: f(x)=x-1 doesn't satisify the symmetric

Yes, f(x)=x

题目 (UD:15.14)

Let A, B, C, and D be nonempty sets. Let $f : A \to B$ and $g : C \to D$ be functions.

(a) Prove that if f and g are one-to-one, then $H : A \times C \to B \times D$ defined by

$$H(a, b) = (f(a), g(c))$$

is a one-to-one function. (Check that it is one-to-one and a function.)

(b) Prove that if f and g are onto, then H is also onto.

解答:

(a)

∵ f and g are one-to-one functions,

∴ for every a∈A and c∈C, we have there exist a u∈B that satisfies u=f(a) and a w∈B that satisfies w=g(c).

and for every u∈ran(f) and every w∈ ran(g), we have there exist a unique a∈A that satisfies u=f(a) and a unique c∈C that satisfies w=g(c).

∴ H is a one-to=one function

(b)

∵ f and g are onto functions,

∴ for every a∈A and c∈C, we have there exist a u∈B that satisfies u=f(a) and a w∈B that satisfies w=g(c).

and for every u∈B and w∈D, we have there exist a u∈A that satisfies u=f(a) and a w∈B that satisfies w=g(c).

∴ H is an onto function

题目 (UD:15.15)

Let A, B, C, and D be nonempty sets. Let $f : A \to B$ and $g : C \to D$ be functions. Consider H defined on $A \bigcup C$ by

$$H(x) = \begin{cases} f(x) & if \quad x \in A \\ g(x) & if \quad x \in C \end{cases} \tag{4}$$

Show that there exist sets A, B, C, and D for which H is not a function, But there also exist such sets for which H is a function. What conditions can we place on A and C that assume us that H is a function?

解答:

(a) Let A={1,2,3} B={3,4,5} C={0} D={1} this H is not a function

Let A={1,2} B={3,4} C={0} D={1} this H is a function

(b) for x∈A∩C, we let f(x)=g(x)

题目 (UD:15.20)

In this problem, we look at a function called the restriction function, which we now define.

If $f : A \to B$ is a function, and $A_1 \subset A$, we define another function $F : A_1 \to B$ by $F(a) = f(a)$ for all $a \in A_1$. This function F is called the restriction of f to A_1 and is usually denoted $f|_{A_1}$, We now turn to the problem:

(a) Prove that if f is one-to-one, then $f|_{A_1}$ is one-to-one.

(b) Prove that if $f|_{A_1}$ is onto, the f is onto.

解答:

(a)

∵ we have $\mathrm{A_1 \Subset A}$

∴ we can easily note that $\mathrm{ran(f|_{A1}) \subseteq ran(f)}$

∵ f is one-to-one

∴ for every $\mathrm{y \in ran(f)}$, we have there exists a unique x that satisfies y=f(x)

∵ $\mathrm{ran(f|_{A1}) \subseteq ran(f)}$

∴ for every $\mathrm{y \in ran(f|_{A1})}$, we have there exists a unique x that satisfies y=f(x)

∴ $\mathrm{f|_{A1}}$ is one-to-one.

(b)

∵ $\mathrm{f|_{A1}}$ is one-to-one.

∴ $\mathrm{ran(f|_{A1}) = B}$

∵ we have $\mathrm{A_1 \Subset A}$

∴ we can easily note that $\mathrm{ran(f|_{A1}) \subseteq ran(f)}$

∴ we have $\mathrm{B \subseteq ran(f)}$

∵ $\mathrm{ran(f) \subseteq B}$

∴ we have ran(f)=B

∴ f is onto

题目 (UD:16.19)

Let $f : A \to B$ be a function. Prove that if f is onto, then $\{f^{-1}(\{b\}) : b \in B\}$ partitions the set A.

解答:

Let $\mathrm{I}_b = \{\mathrm{f}^{-1}(\{\mathrm{b}\}) : \mathrm{b} \in \mathrm{B}\}$

∵ f is a function

∴ for every $\mathrm{y_1}$ and $\mathrm{y_2} \in \mathrm{B}$, if $\mathrm{f(x)=y_1}$ and $\mathrm{f(x)=y_2}$, we have $\mathrm{y_1=y_2}$

∴ $\mathrm{I}_{y_1} \cap \mathrm{I}_{y_2} = \emptyset$ for every $\mathrm{y_1}$ and $\mathrm{y_2} \in \mathrm{B}$

$\bigcup_{b \in B} \mathrm{I}_b = \bigcup_{b \in B} \mathrm{f}^{-1}(\{\mathrm{b}\}) = \mathrm{f}^{-1}(\bigcup_{b \in B}\{\mathrm{b}\}) = \mathrm{f}^{-1}(\mathrm{B}) = \mathrm{A}$

∴ $\{\mathrm{f}^{-1}(\{\mathrm{b}\}) : \mathrm{b} \in \mathrm{B}\}$ partitions the set A.

题目 (UD:16.20)

Suppose that $f : X \to Y$ is a function, and let A_1 and A_2 be subsets of X.

(a) If $f(A_1) = f(A_2)$, must $A_1 = A_2$?

(b) Let f be a bijective function. Show that if $f(A_1) = f(A_2)$, then $A_1 = A_2$. Indicate clearly where you use one-to-one or onto. Did you use both?

解答:

(a) it doesn't need to

(b)

$\because$ f(A_1)=f(A_2)

$\therefore$ {f(a):a$\in A_1$}={f(b):b$\in A_2$}

Let S={f(a):a$\in A_1$} T={f(b):b$\in A_2$}

$\therefore$ S $\subseteq$ T and T$\subseteq$S

(i)S $\subseteq$ T:

$\therefore$ for every s$\in$S ,we have s$\in$T

$\therefore$ for every f(a)$\in$S, we have f(c)$\in$T that satisfies f(c)=f(a)

$\because$ f is one-to-one

$\therefore$ if f(a)=f(c), then a=c

$\therefore$ for every a$\in A_1$, we have a $\in A_2$

$\therefore$ $A_1 \subseteq A_2$

(ii)T$\subseteq$S:

Similarly, we have $A_2 \subseteq A_1$

$\therefore$ A_1=A_2

And I only use one-to-one.

题目 (UD:16.21)

Suppose that $f : X \to Y$ is a function, and let B_1 and B_2 be subsets of Y.

(a) If $f^{-1}(B_1) = f^{-1}(B_2)$, must $B_1 = B_2$?

(b) Let f be a bijective function. Show that if $f^{-1}(B_1) = f^{-1}(B_2)$, then $B_1 = B_2$. Indicate clearly where you use one-to-one or onto. Did you use both?

解答:

(a) No,cause Y is not neccessarily the range.

(b)

$\because$ f is bijective

$\therefore$ f^{-1} is bijective

$\therefore$ according to 16.20(b), we have B_1=B_2 And I use both one-to-one and onto.

题目 (UD:16.22)

Let X be a nonempty set and let A_1 and A_2 be subsets of X. Recall the notation for characteristic function, x_A, defined in Problem 13.5.

(a) If $X_{A_1} = X_{A_2}$, must $A_1 = A_2$?

(b) Show that the product $X_{A_1} \cdot X_{A_2}$, which is defined pointwise on X by $(X_{A_1} \cdot X_{A_2})(x) = X_{A_1}(x) \cdot X_{A_2}(x)$, satisfies $X_{A_1} \cdot X_{A_2} = X_{A_1 \bigcap A_2}$.

(c) Show that $X_{A_1}(x) + X_{A_2}(x) - X_{A_1 \bigcap A_2} = X_{A_1 \bigcup A_2}$. (In other words, for each $x \in X$, we have $X_{A_1}(x) + X_{A_2}(x) - X_{A_1 \bigcap A_2} = X_{A_1 \bigcup A_2}$.)

(d) Can you find a similar result for $X_{X \setminus A_1}$?

解答:

(a) Yes.

(b)

for x$\in$ X there are four cases:

(i)if x$\in$A$_1$ and x$\in$A$_2$,

we have X_{A_1}(x)=1 and X_{A_2}(x)=1,

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=1 and $\mathrm{X}_{A_1\cap A_2}$(x)=1

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=$\mathrm{X}_{A_1\cap A_2}$(x)

(ii)if x$\in$A$_1$ and x$\notin$A$_2$,

we have X_{A_1}(x)=1 and X_{A_2}(x)=0,

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=0 and $\mathrm{X}_{A_1\cap A_2}$(x)=0

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=$\mathrm{X}_{A_1\cap A_2}$(x)

(iii)if x$\notin$A$_1$ and x$\in$A$_2$,

we have X_{A_1}(x)=0 and X_{A_2}(x)=1,

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=0 and $\mathrm{X}_{A_1\cap A_2}$(x)=0

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=$\mathrm{X}_{A_1\cap A_2}$(x)

(iv)if x$\notin$A$_1$ and x$\notin$A$_2$,

we have X_{A_1}(x)=0 and X_{A_2}(x)=0,

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=0 and $\mathrm{X}_{A_1\cap A_2}$(x)=0

$\therefore$ ($\mathrm{X}_{A_1} \cdot \mathrm{X}_{A_2}$)(x)=$\mathrm{X}_{A_1\cap A_2}$(x)

(c)

for x$\in$ X there are four cases:

(i)if x$\in$A$_1$ and x$\in$A$_2$,

we have X_{A_1}(x)=1 and X_{A_2}(x)=1,

$\therefore$ $\mathrm{X}_{A_1\cap A_2}$(x)=1

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$\mathrm{X}_{A_1\cap A_2}$(x)=1 and $\mathrm{X}_{A_1\cup A_2}$(x)=1

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$\mathrm{X}_{A_1\cap A_2}$(x)=$\mathrm{X}_{A_1\cup A_2}$(x)

(ii)if x$\in$A$_1$ and x$\notin$A$_2$,

we have X_{A_1}(x)=1 and X_{A_2}(x)=0,

$\therefore$ $\mathrm{X}_{A_1\cap A_2}$(x)=0

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$\mathrm{X}_{A_1\cap A_2}$(x)=1 and $\mathrm{X}_{A_1\cup A_2}$(x)=1

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$\mathrm{X}_{A_1\cap A_2}$(x)=$\mathrm{X}_{A_1\cup A_2}$(x)

(iii)if x$\notin$A$_1$ and x$\in$A$_2$,

we have X_{A_1}(x)=0 and X_{A_2}(x)=1,

$\therefore$ $X_{A_1\cap A_2}$(x)=0

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$X_{A_1\cap A_2}$(x)=1 and $X_{A_1\cup A_2}$(x)=1

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$X_{A_1\cap A_2}$(x)=$X_{A_1\cup A_2}$(x)

(iv)if x$\notin A_1$ and x$\notin A_2$,

we have X_{A_1}(x)=0 and X_{A_2}(x)=0,

$\therefore$ $X_{A_1\cap A_2}$(x)=0

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$X_{A_1\cap A_2}$(x)=0 and $X_{A_1\cup A_2}$(x)=0

$\therefore$ X_{A_1}(x)+X_{A_2}(x)-$X_{A_1\cap A_2}$(x)=$X_{A_1\cup A_2}$(x)

(d)

$X_{X\setminus A_1}$=X_X -X_{A_1}

第二部分　订正

题目 (题号)

题目。

错因分析： 简述错误原因（可选）。

订正：

正确解答。

第三部分　反馈

你可以写：

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- 等