第十二讲：偏序关系和格

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请独立完成作业，不得抄袭。 若参考了其它资料，请给出引用。 鼓励讨论，但需独立书写解题过程。

第一部分　作业

题目 (SM: 14.32)

Let $B = \{a, b, c, d, e, f\}$ be ordered as in Fig.14-17(b).

(a) Find all minimal and maximal elements of B.

(b) Does B have a first or last element?

(c) List two and find the number of consistent enumerations of B into the set $\{1, 2, 3, 4, 5, 6\}$.

解答:

(a) mimimal elements: d and f

maximal elements: a

(b) It doesn;t have a first element, but it has a last element.

(c)

f: f(a)=6, f(b)=5, f(c)=4, f(d)=3, f(e)=2, f(f)=1

f: f(a)=6, f(b)=4, f(c)=5, f(d)=3, f(e)=2, f(f)=1

The number of consistent enumerations of B is 11.

题目 (SM: 14.44)

Suppose the following are three consistent enumerations of an ordered set $A = \{a, b, c, d\}$:

$$[(a, 1), (b, 2), (c, 3), (d, 4)], [(a, 1), (b, 3), (c, 2), (d, 4)], [(a, 1), (b, 4), (c, 2), (d, 3)]$$

Assuming the Hasse diagram D of A is connected, draw D.

解答:

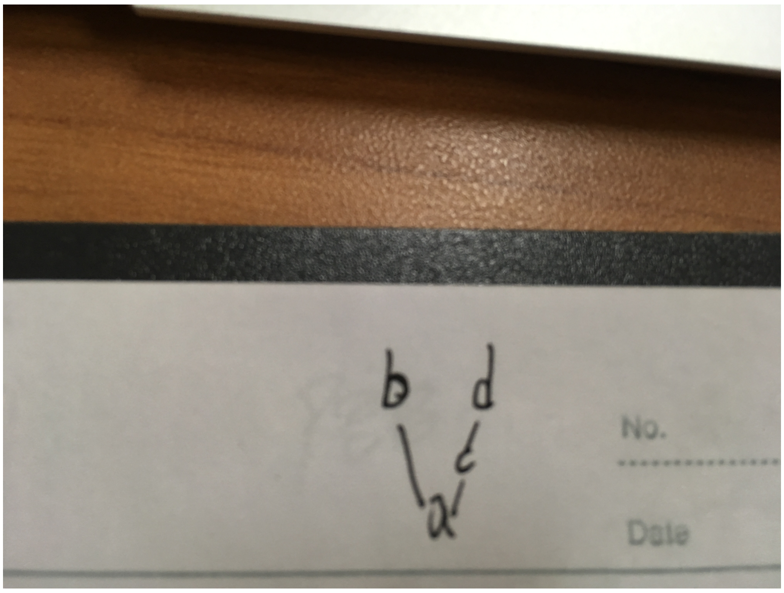


图 1: Hasse diagram

题目 (SM: 14.46)

Consider the English alphabet $A = \{a, b, c, ..., y, z\}$ with the usual (alphabetical) order. Recall A∗ consists of all words in A. Let L consist of the following list of elements in A∗:

gone, or, arm, go, an, about, gate, one, at, occur

(a) Sort L according to the short-lex order, i.e., first by length and then alphabetically.

(b) Sort L alphabetically.

解答:

(a) an, at, go, or, arm, one, about, gate, gone, occur.

(b) about, an, arm, at, gate, go, gone, occur, one, or.

题目 (SM: 14.58)

Show that the isomorphism relation $A \cong B$ for ordered sets in an equivalence relation, that is:

(a) $A \cong A$ for any ordered set A. (b) If $A \cong B$, then $B \cong A$. (c) If $A \cong B$ and $B \cong C$, then $A \cong C$.

解答:

(a) we can define f: A→A by f(a)=a, and we can easily note that this remains the order relations.

(b) For A≅B, we can find a bijective function f: A→B that satisfies (i) if a>b, then f(a)>f(b) (ii) if a||b then f(a)||b. Therefore there is a bijective function f^{-1}, which also satisfies (i) if u>v(f(a)>f(b)), then f^{-1}(u)>f^{-1}(v) (a>b) and (ii) if u||v(f(a)||f(b)), then f^{-1}(u)||f^{-1}(v) (a||b)

(c) for A≅B and B≅C, we can find f: A→B and g: B→C that satisfy the two properties. Therefore we can define F: A→C by F=g∘f

if a>b, then f(a)>f(b), then g(f(a))>g(f(b)), namely F(a)>F(b)

If a||b, then f(a)||f(b), then g(f(a))||g(f(b)), namely F(a)||F(b)

Therefore F also satisfies the two properties.

Therefore A≅C

题目 (SM: 14.62)

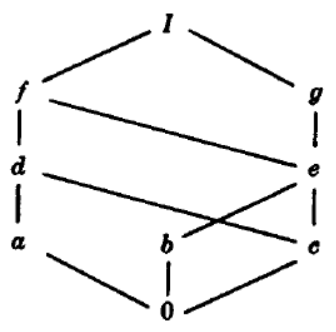
Suppose A and B are well-ordered isomorphic sets. Show that there is only one similarity mapping $f : A \to B$.

解答:

Since A and B are well-ordered isomorphic sets, the elements in A(or B) are all comparable and we can find a bijective function f: A→B

A and B are linearly ordered. Therefore we can form A into a chain and B into a chain. and the mapping from A to B should be parallel (as f has to perserve the order relation). If there is another similarity mapping g, g must be have lines crossed, and it breaks the order relation, therefore g is not a similarity mapping.

Therefore there is only one similarity mapping $f : A \to B$.



(b)

Consider the lattice M in Fig.14-19(b).

(a) Find all join-irreducible elements.

(b) Find the atoms.

(c) Find complements of a and b, if they exist.

(d) Express each x in M as the join of irredundant join-irreducible elements.

(e) Is M distributive? Complemented?

解答:

(a) a,b,c,g,0

(b) a,b,c

(c) a:g b:none;

(d) I=a∨g, f=a*vee*b, d=a*vee*c, a=a∨0, e=b∨c, g=b∨g, c=c∨0, b=b∨0, 0=0∨0

(e)it is not distributive or complemented.

题目 (SM: 14.70)

Consider the lattice $D_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$, the divisors of 60 ordered by divisibility.

(a) Draw the diagram of D_{60}.

(b) Which elements are join-irreducible and which are atoms?

(c) Find complements of 2 and 10, if they exist.

(d) Express each number x as the join of a minimum number of irredundant join irreducible elements.

解答:

(a)

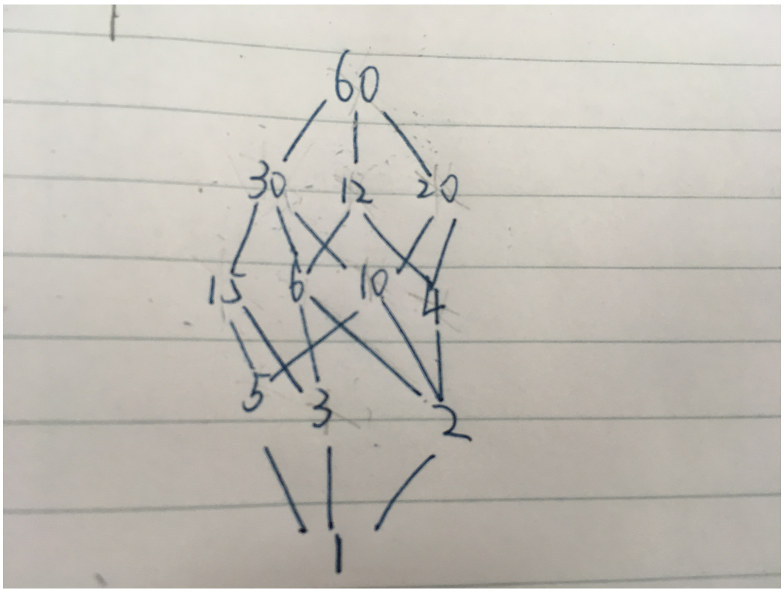


图 2: diagram

(b) join-irreducible: 1,2,3,4,5

atoms: 2,3,5

(c) complement of 2 doesn't exist.

complement of 10 doesn't exist.

(d) 60=5∨4∨3, 30=2∨3∨5, 20=4∨5, 15=3∨5, 12=3∨4, 10=2∨5, 6=2∨3.

题目 (SM: 14.75)

A lattice M is said to be modular if whenever $a \le c$ we have the law

$$a \vee (b \wedge c) = (a \vee b) \wedge c$$

(a) Prove that every distributive lattice is modular.

(b) Verify that the non-distributive lattice in Fig.14-7(b) is modular; hence the converse of (a) is not true.

(c) Show that the nondistributive lattice in Fig.14-7(a) is non-modular.(In fact, one can prove that every non-modular lattice contains a sublattice isomorphic to Fig.14-7(a).)

解答:

(a) because a≤c, then a∨c=c

a∨(b∨c)=(a∨b)∧(a∨c)=(a∨b)∧c.

Therefore every distributive lattice is modular.

(b) I≥a, a∨(b∧I)=I=(a∨b)∧I. For b and c, it is similiar.

(c) c≥a, a∨(b∧c)=a while (a∨b)∧c=c, therefore a∨(b∧c)≠(a∨b)∧c. Therefore the non-distributive lattice in Fig.14-7(b) is modular, hence the converse of (a) is not true.

第二部分　订正

题目 (UD:13.5)

Let x be a nonempty set and let A be a subset of X. We define the Characteristic function of the set A by

$$X_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \in X \backslash A \end{cases} \tag{1}$$

(a)Since this is called the characteristic function, it probably is a function, but check this carefully anyway.

(b)Determine the domain and range of this function. Make sure you look at all possibilities for A and X.

错因分析:　简述错误原因（可选）。

订正:

(a)

(i) Since A is a subset of X, for every x∈X, there are two cases: x∈A or x∈X\A

The two cases are defined in the X_A

(ii) Since A is a subset of X, for every x∈X, there are two cases: x∈A or x∈X\A

(1) x∈A , the y is unique, namely 1

(2) x∈X\A, the y is also unique, namely 0

∴ for every x∈X, the y it relates to is unique.

∴ X_A satisfies (i) and (ii)

∴ X_A is a well-defined function.

(b)

the domain of this function is X

(i) A=∅, ran(f)={0}

(ii) A=X, ran(f)={1}

(iii) A ≠X and A≠ ∅, ran(f)={0,1}

题目 (UD:14.12)

Let a, b, c, and d be real numbers with $a < b$ and $c < d$. Define a bijection from the closed interval $[a, b]$ onto the closed interval $[c, d]$ and prove that your function is a bijection.

订正：

f(x)=$\frac{d-c}{b-a}$(x-a) +c

Prove:

(1)if x1,x2∈[a,b] and f(x_1)=f(x_2), then $\frac{d-c}{b-a}$(x_1-a) +c=$\frac{d-c}{b-a}$(x_2-a) +c

∴ $\frac{d-c}{b-a}$(x_1-a)=$\frac{d-c}{b-a}$(x_2-a)

∵ a<b and c<d

∴ a-b<0, c-d<0

∴ x_1-a=x_2-a

∴ x_1=x_2

∴ this function is one-to-one

(2)for every y∈ [c,d], let f(x)=y

∴ $\frac{d-c}{b-a}$(x-a) +c=y

∴ x= $\frac{b-a}{d-c}$(y-c) +a

∵ y∈[c,d]

∴ x∈[a,b]

∴ for every y∈[c,d], we can find x∈[a,b] that satisfies y=f(x)

∴ this function is onto

Based on (1) and (2), this function is bijective.

题目 (UD:15.14)

Let A, B, C, and D be nonempty sets. Let $f : A \to B$ and $g : C \to D$ be functions.

(a) Prove that if f and g are one-to-one, then $H : A \times C \to B \times D$ defined by

$$H(a, b) = (f(a), g(c))$$

is a one-to-one function. (Check that it is one-to-one and a function.)

注： 一开始的部分先说明 H 是一个 well-defined 的 function

订正：

(a)

∵ f and g are one-to-one functions,

∴ for every a∈A and c∈C, we have there exist a u∈B that satisfies u=f(a) and a w∈B that satisfies w=g(c).

and if f(a)=f(b), then a=b, for g is the similarly

∴ H is a one-to=one function

题目 (UD:16.21)

Suppose that $f : X \to Y$ is a function, and let B_1 and B_2 be subsets of Y.

(b) Let f be a bijective function. Show that if $f^{-1}(B_1) = f^{-1}(B_2)$, then $B_1 = B_2$. Indicate clearly where you use one-to-one or onto. Did you use both?

订正：

(b)

∵$f^{-1}(B_1) = f^{-1}(B_2)$

Let a∈ $f^{-1}(B_1)$,b∈$f^{-1}(B_2)$， x=f(a)∈ B_1, y=f(b)∈ B_2

∵ f is onto,

∴we can always find x and y

for a=b, we have f(a)=f(b) ,then x=y

∴ we can easily conclude that B_1=B_2

And I use only onto

题目 (第六讲)

写出你现在用的 C++ 语言的算术表达式的完整严格的文法。

注： 参考了张天昀同学的关于 BNF 文法的作业

解答：

赋值语句:

<assignment>::=<variable> = <expression>

运算表达式:

<expression> ::= <element> | (<expression>) | <increment> | <decrement> | <unary-operators><expression> | <expression><multiplicative-operators><expression> | <expression><additive-operators><expression>

运算符

<increment> ::= ++<expression> | <expression>++

<decrement>::= –<expression> | <expression>–

<unary-operators> ::= + | - | empty

<multiplicative-operators> ::= * | /

<additive-operators> ::= + | -

元素

<element> ::= <value> | <variable>

<value> ::= <integer> | <non-integer>

<non-integer> ::= <integer>.<integer>

<integer> ::= <digit> | <integer> <digit>

<variable> ::= <nondigit> | <variable><character>

<character> ::= <digit> | <nondigit>

<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

<nondigit> ::= A | B | C | ... | Z | a | b | c | ... | z | _

<equation> ::= <expression> == <expression>

<non-equation> ::= <expression> != <expression>

<less-than> ::= <expression> < <expression>

<less-than-or-equal> ::= <expression> <= <expression>

<more-than> ::= <expression> > <expression>

<more-than-or-equal> ::= <expression> >= <expression>

第三部分　反馈

你可以写：

- 对课程及教师的建议与意见
- 教材中不理解的内容
- 希望深入了解的内容
- 等