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(a) Before split $-\left(\left(\frac{4}{9}\right) * \log_2 \frac{4}{9} + \left(\frac{5}{9}\right) * \log_2 \frac{5}{9}\right) \cong 0.991$

(b) Split by a_1

$$\frac{4 * -\left(\left(\frac{3}{4}\right) * \log_2 \frac{3}{4} + \left(\frac{1}{4}\right) * \log_2 \frac{1}{4}\right) + 5 * -\left(\left(\frac{1}{5}\right) * \log_2 \frac{1}{5} + \left(\frac{4}{5}\right) * \log_2 \frac{4}{5}\right)}{9} \cong 0.761$$

Information gains: $0.991 - 0.761 = 0.23$

Split by a_2

$$\frac{5 * -\left(\left(\frac{2}{5}\right) * \log_2 \frac{2}{5} + \left(\frac{3}{5}\right) * \log_2 \frac{3}{5}\right) + 4 * -\left(\left(\frac{2}{4}\right) * \log_2 \frac{2}{4} + \left(\frac{2}{4}\right) * \log_2 \frac{2}{4}\right)}{9} \cong 0.984$$

Information gains: $0.991 - 0.984 = 0.007$

According to the entropy index, split by a_1 is better.

(c) Before split: $\left(1 - \left(\frac{4}{9}\right)^2 - \left(\frac{5}{9}\right)^2\right) \cong 0.494$

Split by a_1 : $\frac{4}{9} * \left(1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right) + \frac{5}{9} * \left(1 - \left(\frac{1}{5}\right)^2 - \left(\frac{4}{5}\right)^2\right) \cong 0.3444$

Split by a_2 : $\frac{5}{9} * \left(1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2\right) + \frac{4}{9} * \left(1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2\right) \cong 0.4888$

According to the Gini Index, split by a_1 is better.