2,

(a) $\rho = \min\limits_{n=1,\dots N} y_n (w^T x_n + b)$

$$\begin{cases} \rho_1 = (-1)\cdot\left[1.2\times 0 + (-3.2)\times 0 + (-0.5)\right] = 0.5 \\ \rho_2 = (-1)\times\left[1.2\times 2 + (-3.2)\times 2 + (-0.5)\right] = 4.5 \\ \rho_3 = 1\times\left[1.2\times 2 + (-3.2)\times 0 + (-0.5)\right] = 1.9 \end{cases} \Rightarrow \rho = \min\{\rho_1, \rho_2, \rho_3\} = \underline{0.5} \ \#$$

(b)

new weights : $\frac{1}{\rho}(b, w) = \frac{1}{0.5}(b, w) \Rightarrow b = -1 \ ; \ w = \begin{bmatrix} 2.4 \\ -6.4 \end{bmatrix}$

$\Rightarrow y_n (w^T x_n + b)$:

$$(-1)\times\left[2.4\times 0 + (-6.4)\times 0 + (-1)\right] = 1$$
$$(-1)\times\left[2.4\times 2 + (-6.4)\times 2 + (-1)\right] = 9 \quad \Rightarrow \min\{1, 9, 3.8\} = \underline{1} \ \#$$
$$1\times\left[2.4\times 2 + (-6.4)\times 0 + (-1)\right] = 3.8$$

(c)

canonical representation :

solve $\underset{b, w}{\text{minimize}} \ \frac{1}{2} w^T w$ subject to $y_n (w^T x_n + b) \geq 1 \quad (n = 1, 2, 3)$

set $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$; b

$$\Rightarrow \begin{cases} (-1)\times(w_1\times 0 + w_2\times 0 + b) \geq 1 \\ (-1)\times(w_1\times 2 + w_2\times 2 + b) \geq 1 \\ 1\times(w_1\times 2 + w_2\times 0 + b) \geq 1 \end{cases} \Rightarrow \begin{cases} -b \geq 1 \\ -2w_1 - 2w_2 - b \geq 1 \\ 2w_1 + b \geq 1 \end{cases}$$

$$\Rightarrow \begin{cases} w_1 \geq 1 \\ w_2 \leq -1 \end{cases} \Rightarrow \begin{cases} w_1^2 \geq 1 \\ w_2^2 \geq 1 \end{cases}$$

$\Rightarrow \frac{1}{2} w^T w = \frac{1}{2}(w_1^2 + w_2^2) \geq \frac{1}{2}(1+1) = 1$ when $w_1 = 1$, $w_2 = -1$, $b = -1$

$\Rightarrow w^* = [w_1, w_2] = [1, -1]$

optimal hyperplane: $x_1 - x_2 - 1 = 0$

The line through blue points: $x_1 - x_2 = 0$

two line distance. $\dfrac{|1-0|}{\sqrt{1^2+(-1)^2}} = \dfrac{1}{\sqrt{2}}$ (margin)

$\dfrac{1}{\|w^*\|} = \dfrac{1}{\sqrt{1^2+(-1)^2}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow$ margin $= \dfrac{1}{\|w^*\|}$

Q.E.D.