Channel Coding for Asynchronous Fiberoptic CDMA Communications

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***Abstract*—Several recent studies have explored the feasibility and systems performance aspects of a code division multiple access (CDMA) format for fiberoptic networks [1]–[13]. Previously discussed CDMA architectures would either have to tolerate a high bit error rate or be forced to use long code sequences in networks with even a moderate number of simultaneous users. The use of long sequences lowers the maximum achievable bit rate or places unrealistic requirements on the system hardware. This paper examines some of the possible improvements to system performance that could be realized by combining the CDMA format with external error correcting codes (ECC's) or a PPM format. It is determined that ECC's can be highly effective in lowering the BER, and/or increasing the achievable information bit rate and number of network users. The results are sufficiently encouraging to conclude that one should seriously consider including ECC's in any practical fiberoptic CDMA network.**

I. Introduction

CODE DIVISION multiple access (CDMA) has been proposed as a possible format for fiberoptic networks [1]–[13]. The baseline CDMA uses on–off keying (OOK) of binary data with a unique coded pulse sequence transmitted for each on-bit. Multiple accessing is achieved by having multiple sources, each with its own code sequence, superimpose their transmissions over a common fiber. The fibers can then be interconnected via star or other fiber systems to form the distribution network as shown in Fig. 1. Data bits are separated out at a receiving terminal by recognizing (correlating) the proper sequence of the desired source. Pulse code sequences can be passively generated from an initial OOK laser pulse by serial or parallel delay lines [11], [14], and sequence correlation can be achieved optically by corresponding matched delay lines. After correlating the desired sequence to a peak value, photodetection followed by threshold comparison can be used to detect the presence or absence of each bit. Minimal interference multiple accessing is achieved by using only sets of pulse code sequences that have low pairwise crosscorrelations. Optical CDMA has the advantage of permitting completely asynchronous transmitters, relatively simple, off-the-shelf laser sources (without need of wavelength control), standard photodetectors (without narrow optical filters) and higher peak power levels (relative to the average power) due to the laser pulsing. In addition, pulsed CDMA combines the higher speeds of optical signals with the more developed electronic processing to provide maximum performance efficiencies in converting digital data to optical transmission. Modified versions of coherent optical CDMA, such as spectral CDMA [9], [15] have been proposed, but are more complex and cannot be easily implemented with present day hardware.

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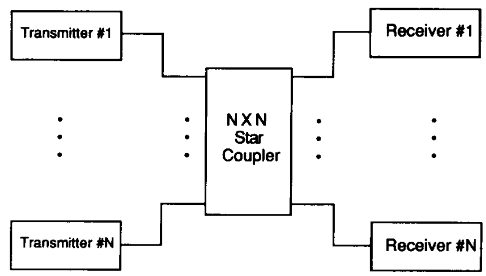

Fig. 1. CDMA network architecture.

A prime disadvantage with CDMA is the sacrifice in per-channel data rate (relative to the speeds available in the laser itself) that occurs in the insertion of code addressing. For typical laser sources, this generally limits the channel data rates to tens or hundreds of megahertz. Another important CDMA concern is the development of digital crosstalk between channels when multiplexing many simultaneous sources. This buildup will depend on the number of overlapping sources and on the specific correlation properties of the individual source codes. Channel crosstalk is the ultimate limit in link performance, and generally forces a restriction on the overall network capabilities.

In this paper we consider the use of external channel coding in the form of either forward error correction or modified waveform encoding, to aid in mitigating this crosstalk buildup and produce more efficient individual channel performance. While the advantages of channel coding are well known for the classical Gaussian noise channel, the application to the optical CDMA crosstalk channel is somewhat diverse, and care must be used inserting commonly accepted coding "gains".

II. Review of OOK-CDMA Fiber Systems

The performance parameters of an OOK–CDMA fiber link are directly related to the code sequence of the transmitters. The selection of candidate sets of pulse sequences is basically a problem in code design, and has now been rigorously

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formulated as such [16], [17]. Sets of optimal sequences (having both minimal levels of pairwise crosscorrelations and minimal levels of off-peak autocorrelations) have been designated as *pseudo-orthogonal* (PSO) codes, and their construction is now fairly well understood. Two different kinds of pseudo-orthogonality are defined, aperiodic and periodic. Aperiodic codes minimize the crosscorrelation and off-peak autocorrelation using conventional correlation. Periodic codes achieve this minimization when using periodic correlation, (or equivalently, correlating one code sequence with the periodic extension of the second code sequence). In a modulation format analogous to OOK, each source sends its own code sequence in order to represent a "1" in place of a pulse over the entire bit time. With this OOK format, the crosstalk is minimized if periodic codes are used. If a waveform modulation technique analogous to M-ary PPM is used, then the sources would transmit their code sequences in one of M possible slots. In this case, aperiodic codes can be used while still essentially maintaining the minimal crosstalk PSO properties. A set of N PSO sequences, each with w pulses (called the code weight), requires a specific code length (number of available pulse positions in each sequence). As a rough rule, the sequence length L for a PSO code set with N members requires [3]

$$L \cong Nw(w-1)/2 \qquad \text{(aperiodic)} \tag{1}$$

$$L \cong Nw(w-1) \qquad \text{(periodic)}. \tag{2}$$

Thus, code lengths must increase linearly with the number of PSO sequences in the set, for a specific code weight w. The weight w also determines the transmitter and correlator hardware complexity, and is the most important factor in determining the decoding performance. Since the code sequence of each source is sent to represent each OOK on-bit, the code length in (2) determines the bit rate of each CDMA channel. If the widths of each laser pulse (called the sequence chip time) is T_c, then the bit time is LT_c and the OOK channel bit rate is

$$R_b = \frac{1}{Nw(w-1)T_c} \qquad \text{b/s}. \tag{3}$$

Equation (3) shows that the number of multiple access channels N and the bit rate per channel R_b are always inversely related.

Bit decoding is based on OOK threshold testing after optical correlation and photodetection at each receiver. This threshold test is carried out each bit time in the presence of the receiver noise (shot noise, thermal noise, dark current, and photomultiplication noise) and the crosscorrelation noise (crosstalk) from overlapping channels. The resulting channel bit error probability PE is governed by two distinct asymptotic regions. The first is the low power region, where the detector noise dominates over the crosstalk, and the bit error probability behaves as

$$\mathrm{PE} \cong Q(\sqrt{\mathrm{SNR}}/2) \tag{4}$$

where $Q(x)$ is the Gaussian tail integral and

$$\mathrm{SNR} = \frac{(wK_s)^2}{wK_s + K_d}. \tag{5}$$

Here[1] is the received laser pulse photon count and K_d is the total detector noise photon count per chip time. In this region, increasing laser power directly increases K_s and lowers PE in (4).

As laser power further increases the system enters a transition region where the crosstalk begins to dominate, eventually producing an asymptotic PE "floor" governed by the crosstalk alone and completely independent of the laser power. This PE floor (PE_f) is simply the probability that interfering asynchronous code sequences of other channels happen to produce enough code chips that fall at the chip times of the desired sequence. This could produce an off-bit with an apparent high correlation value, leading to an erroneous decision. Assuming chip synchronous codes and independent uniformly distributed timing offsets between the asynchronous transmitters, the probability that a pair of PSO sequences of weight w and length L will have a pulse overlap is $w^2/2L$. If a threshold slightly below w is used, the probability that $N-1$ randomly located PSO sequences will produce w or more overlaps on a transmitter sending an off-bit is then

$$\mathrm{PE}_f = \frac{1}{2}\sum_{i=w}^{N-1}\binom{N-1}{i}\left(\frac{w^2}{2L}\right)^i\left(1-\frac{w^2}{2L}\right)^{N-1-i}. \tag{6}$$

Equation (6) is plotted in Fig. 2 as a function of w for various values of N using L in (2). The PE floor is continually reduced as the code weight w is increased. It is emphasized that PE_f is a finite discrete summation and care must be used in attempting to model this with any form of Gaussian statistics. Equations (3), (4), and (6) summarize the key design parameters tradeoffs for the standard OOK-CDMA fiber channel. Increasing the code weight always lowers the error probability floor PE_f, but will require longer sequences to support a given number of channels N and therefore lowers the individual channel bit rates. In addition, the hardware complexity of a CDMA system (delay line encoders and decoders) also increases at the higher code weights. This therefore raises the question of whether other techniques (such as channel coding) can also be used to improve the PE performance without suffering as much penalty in bit rate or code weight increases as the uncoded OOK–CDMA system.

III. Channel Coding

Channel coding is applied external to the CDMA link optics, as shown in Fig. 3. A channel coder is used at each transmitter to convert the data bits into binary channel symbols, which are then sent over the fiber link with the transmitter pulse code sequence. After optical CDMA detection at the receiver, channel decoding is used to convert the detected link symbols back to the data bits. The channel encoding can involve the simple insertion of error correction symbols, or can reformat the channel symbols. In either case, the objective is to obtain a lower decoded bit error rate than the link symbol error rate provided by the optics. It should be emphasized that the channel coding in Fig. 3 is done entirely in the electronic

[1] K_s itself potentially depends on w, due to the losses in the optical delay line generators and correlators. The actual functional dependence will depend on the manner in which the code generators are implemented.

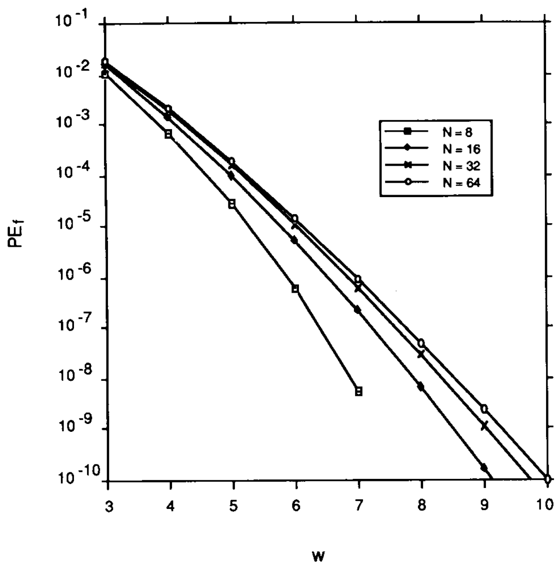

Fig. 2. Probability of bit error floor PE_f versus code weight w.

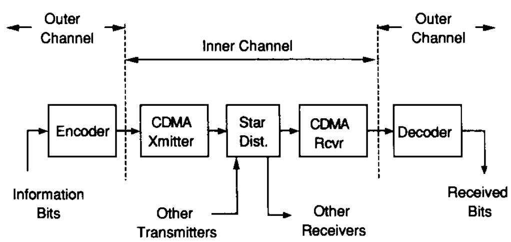

Fig. 3. Block diagram for coded CDMA system.

domain, and is applied completely external to the optics. Since channel coding hardware is fairly well developed, its insertion into a CDMA network should have little cost impact on the overall network implementation.

Two types of channel coding will be considered in this study. The first is the standard forward error correction coding using either Reed–Solomon (RS) block codes or convolutional codes. The transmitters of the optical network send the individual channel coded symbols as OOK–CDMA transmissions, using the pulse sequences to represent the on-bits. The second type of coding involves pulse position modulation (PPM) in which block of k data bits are channel encoded in one of $M = 2^k$ codewords, each of length M and containing a single one symbol (pulse) in a distinct word position (slot) to represent the data word. The CDMA transmitter then sends its pulse sequence to represent the PPM pulse at the proper slot position. Optical code correlation produces a correlation peak whose location relative to a PPM frame clock decodes the data block. PPM has the advantage of distributing the pulse times of the individual transmitters over a PPM frame, and thereby reducing the chance of multiple pulse overlaps during the decoding of any one transmitter. PPM–CDMA has been proposed and breadboarded as a viable format for transmitting multiplexed video over a common fiber channel [18].

In the following, the impact of channel coding on the PE floor is examined. The required conditions are that all systems have the same laser pulse width (chip time T_c), number of transmitters N, and channel data rate R_b. It will be assumed that all decoding errors due to crosstalk occur independently form bit to bit, which implies that the relative transmitter timing offsets are random and independent during any bit time. This avoids the problem of "lock-step" transmitters, where the same bit timing relation is maintained over long sequences of bits. The latter will not occur if there are slight frequency offsets in the bit clocks of each transmitter. It can also be avoided by purposely using slightly different sequence code lengths (appending different numbers of zeros to each sequence) so that the modified pulse sequences appear to slide past each other from bit to bit even though they initially were perfectly aligned. This offsetting disrupts the lock-step condition and makes the bit errors appear more independent. This adjustment, as well as other modifications (such as adaptive thresholding) have been studied elsewhere [19].

A. RS/OOK–CDMA Performance

In this channel coding approach, data bits are coded into Reed–Solomon (RS) codewords. A codeword of a (n, k) RS code contains n symbols. Each symbol is composed of m bits. The parameters n and m are related by

$$n \le 2^m - 1. \tag{7}$$

The code will correct any combination of t symbols errors where t is given by

$$t = (n - k)/2. \tag{8}$$

The probability of decoded bit error for a RS code is given by [20]

$$P_{bR} \le \left(\frac{2^{m-1}}{2^m - 1}\right) \sum_{j=t+1}^{n} \frac{j+t}{n} \binom{n}{j} p_s^i (1 - p_s)^{n-i} \tag{9}$$

where p_s is the probability of RS symbol error. With independent CDMA channel errors

$$p_s = 1 - (1 - \mathrm{PE})^m \tag{10}$$

where PE is the OOK–CDMA link error probability in Section II. For the purpose of comparison, the channel bit rate and chip time of the coded system will be constrained to be the same as that of the uncoded system. Since the insertion of RS coding reduces the channel bit rate relative to the link rate, the sequence length of the coded system must be reduced to maintain the same channel rate of the equivalent uncoded systems. For a fixed number of transmitters N, this implies the weight of the pulse sequence must be reduced. Let w_u be the uncoded CDMA code weight and w_{rs} be the RS code weight. Since w_{rs} must be reduced by some integer amount from w_u, w_{rs} can be written as

$$w_{rs} = w_u - q. \tag{11}$$

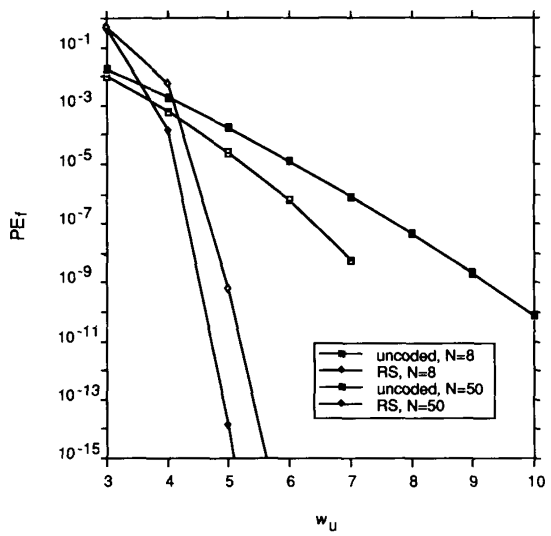

Fig. 4. Uncoded and RS coded bit error probability floor PE_f versus code weight w. $t=9$, $m=8$, $q=1$.

Equating the RS/OOK–CDMA coded data rate with the uncoded data rate in (3) with the same chip time requires

$$\left(\frac{n-2t}{n}\right)\frac{1}{Nw_{rs}(w_{rs}-1)}=\frac{1}{Nw_u(w_u-1)}. \qquad (12)$$

The parameter n in (7) can be varied in order to satisfy (12) with equality as closely as possible. Substituting (11) into (12) and solving for this value of n gives

$$n=\left\lceil\frac{2tw_u(w_u-1)}{2qw_u-q(q+1)}\right\rceil \qquad (13)$$

where $\lceil x\rceil$ is the nearest integer greater than or equal to x. The effective sequence length which equalizes the data bit rates is then set by

$$L_{rs}=\left\lfloor\left(\frac{n-2t}{n}\right)Nw_u(w_u-1)\right\rfloor \qquad (14)$$

where $\lfloor x\rfloor$ is the integer part of x. These equations must be evaluated to determine if RS coding has improved the PE performance at the same bit rate. The OOK–CDMA link PE_f is given by (6) with w_{rs} and L_{rs} inserted. Equations (9) and (10) can be evaluated to determine the RS coded PE_f. Fig. 4 plots the uncoded and RS coded PE_f versus w_u for the case $t=9$, $m=8$, and $q=1$. The figure shows that for $w_u>4$ the RS coding is indeed more effective in reducing PE_f than simply increasing the uncoded sequence code weight.

B. CC/OOK–CDMA Performance

In this system the channel coding uses convolutional coding (CC) to convert the data bit stream to a coded bit stream. Convolutional codes are characterized by their free distance d_f. Any path through the trellis which recombines with the correct path must differ from the correct path in d_f or more places. Let P_d be the probability of the event that the received vector's metric with an incorrect path which differs from the correct path in d places exceeds the metric of the received vector and the correct path. Let B_d be the total number of information bits which differ from the correct path on all incorrect paths which differ in d places from the correct path. Finally, define m_i as the number of input symbols of the convolutional code. The decoded probability of bit error is then given by [20]

$$P_{bC}<\frac{1}{m_i}\sum_{d=d_f}^{\infty}B_dP_d. \qquad (15)$$

Under the appropriate conditions, (15) can be approximated by considering only the first few terms. The parameters m_i, d_f, and B_d are a function of the structure of the given code. Examples of d_f and B_d for various convolutional codes, taken from [20] are listed in Table I. With values of m_i, d_f, and B_d known, only the quantity P_d remains to be evaluated. It can be shown, (Appendix) that for the OOK–CDMA optical channel

$$P_d=\binom{d}{d/2}\left(\frac{1}{2}\right)^{d+1}p_{01}^{d/2}+\left(\frac{1}{2}\right)^{d}\sum_{k=0}^{\frac{d}{2}-1}\binom{d}{k}p_{01}^{d-k}\quad d\text{ even}$$
$$=\left(\frac{1}{2}\right)^{d}\sum_{k=0}^{\frac{d-1}{2}}\binom{d}{k}p_{01}^{d-k}\quad d\text{ odd} \qquad (16)$$

where

$$p_{01}=\sum_{j=w_c}^{N-1}\binom{N-1}{j}\left(\frac{w_c^2}{2L_c}\right)^{j}\left(1-\frac{w_c^2}{2L_c}\right)^{N-1-j}. \qquad (17)$$

Parameters w_c and L_c are the convolutional coded network CDMA code weight and pulse sequence length, respectively. As with the Reed–Solomon code, it is desired to compare the uncoded and convolutional codes with an equal number of transmitters, an equal chip time, and with the convolutional coded channel bit rate equal to the uncoded channel bit rate. Similarly to (12), if R_c is the code rate of the convolutional code then w_u and w_c are related by

$$\frac{R_c}{Nw_c(w_c-1)}=\frac{1}{Nw_u(w_u-1)}. \qquad (18)$$

The code weight w_c can be written in terms of w_u as

$$w_c=\left\lfloor\frac{1}{2}+\frac{1}{2}\sqrt{1+4R_cw_u(w_u-1)}\right\rfloor. \qquad (19)$$

The pulse sequence length which equalizes the channel bit rates is

$$L_c=\lfloor R_cNw_u(w_u-1)\rfloor. \qquad (20)$$

Fig. 5 compares the uncoded PE_f versus w_u with that of various CC/CDMA systems, using the parameters from Table I. It is seen that these codes produce a significant reduction of the PE floor and are much better in regard than the RS block codes for values of w_u less than 6. This could be due to the fact that the CC system is better matched to the OOK–CDMA channel with its imbalance in decoding with on-bits and off-bits. This latter point is worthy of further study.

One can also use the CC advantages to increase the numbers of transmitters N while maintaining a fixed PE floor and data

TABLE I
PARAMETERS FOR SOME EXAMPLE CONVOLUTIONAL CODES (FROM [20]).

R_c	m_i	d_f	B_{d_f}	B_{d_f+1}	B_{d_f+2}	B_{d_f+3}	B_{d_f+4}
1/3	1	16	1	0	24	0	113
1/2	1	12	33	0	281	0	2189
2/3	2	8	97	0	2863	0	56633
3/4	3	6	12	342	1996	12296	78145

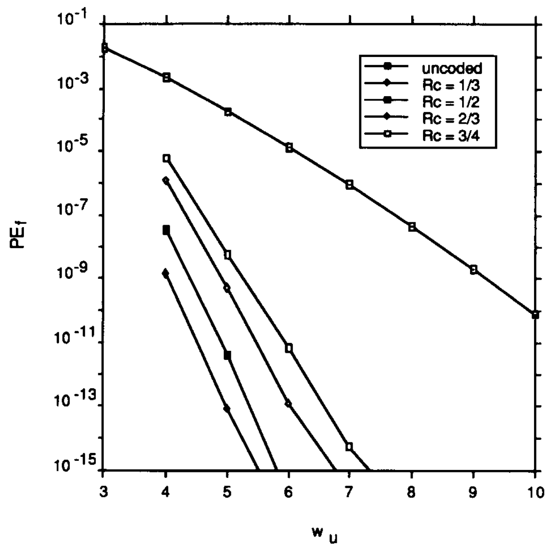

Fig. 5. PE_f versus uncoded w for various convolutional codes. $N = 50$.

rate. Fig. 6 plots the channel bit rate R_b versus the number N at $\mathrm{PE}_f = 10^{-9}$ for an uncoded system and for a CC system with a channel code rate of $R_c = 1/3$. It is seen, for example, that at a channel bit rate of 1.0 Mb/s, approximately 7 times more transmitters can be accommodated by the insertion of the coding.

IV. PPM–CDMA

Pulse position modulation is a form of channel coding in which blocks of bits are encoded into a code set composed of the M-length codewords [1000—], [01000—], [00100—], etc. When the ones represent on-symbols, these codewords represent PPM sequences. In PPM–CDMA, a transmitter sends its pulse sequence in one of the M time slots (symbol positions) to represent a block of $\log_2(M)$ data bits. The individual channel bit rate is

$$R_b = \frac{\log_2(M)}{\mathrm{MLT}_c} = \frac{2\log_2(M)}{MNw(w-1)T_c} \quad \text{[b/s]}. \tag{21}$$

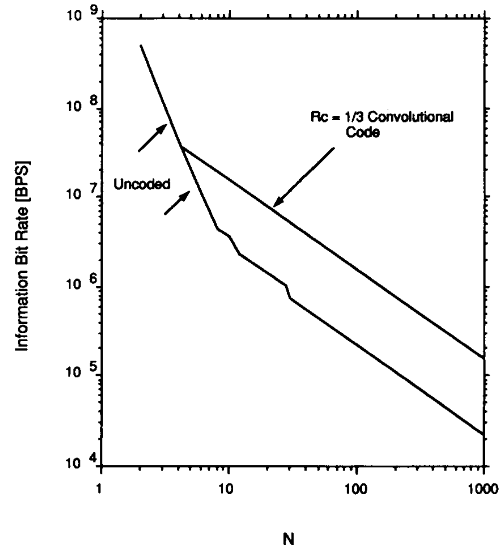

Fig. 6. Channel bit rate versus network size N for uncoded and convolutional coding. $\mathrm{PE}_f = 10^{-9}$. Chip time $Tc = 500$ ps.

The PPM bit rate is then $M/(2\log_2 M)$ times lower than the bit rate of an uncoded OOK–CDMA system using the same code weight w and sequence length L. Decoding is based on identifying the slot with the highest photon count. The bit error probability floor due to crosstalk buildup in an incorrect slot is approximately

$$\mathrm{PE}_f \le \frac{M}{2}\left[\frac{1}{2}\binom{N-1}{w}(p)^w(1-p)^{N-1-w} + \sum_{i=w+1}^{N-1}\binom{N-1}{i}p^i(1-p)^{N-i-1}\right] \tag{22}$$

where $p = w^2/(\mathrm{ML})$. This is obtained by using the fact with PSO sequences the probability that a given interfering transmitter will contribute a pulse of crosstalk to a peak slot correlation of the primary transmitter is w^2/ML. Since there can be as many as $N-1$ independent interfering transmitters, the quantity inside the bracket bounds the probability that any incorrect slot would be mistaken for the correct slot. The $M/2$ multiplier outside the bracket follows from the union bound and the conversion from PPM symbol error to bit error. An uncoded OOK–CDMA system operating at the same bit rate as (21) could use a sequence length

$$L_u = \left\lfloor \frac{\mathrm{ML}}{\log_2(M)} \right\rfloor \tag{23}$$

with a code weight

$$w_u = \left\lfloor \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\frac{L_u}{N}} \right\rfloor. \tag{24}$$

Fig. 7(a) and (b) show comparisons of the resulting PE_f for both PPM systems, and the equivalent uncoded OOK system with parameters in (23) and (24) for various values of M. In almost all the cases shown, the use of the longer sequence lengths and code weights makes the OOK performance slightly

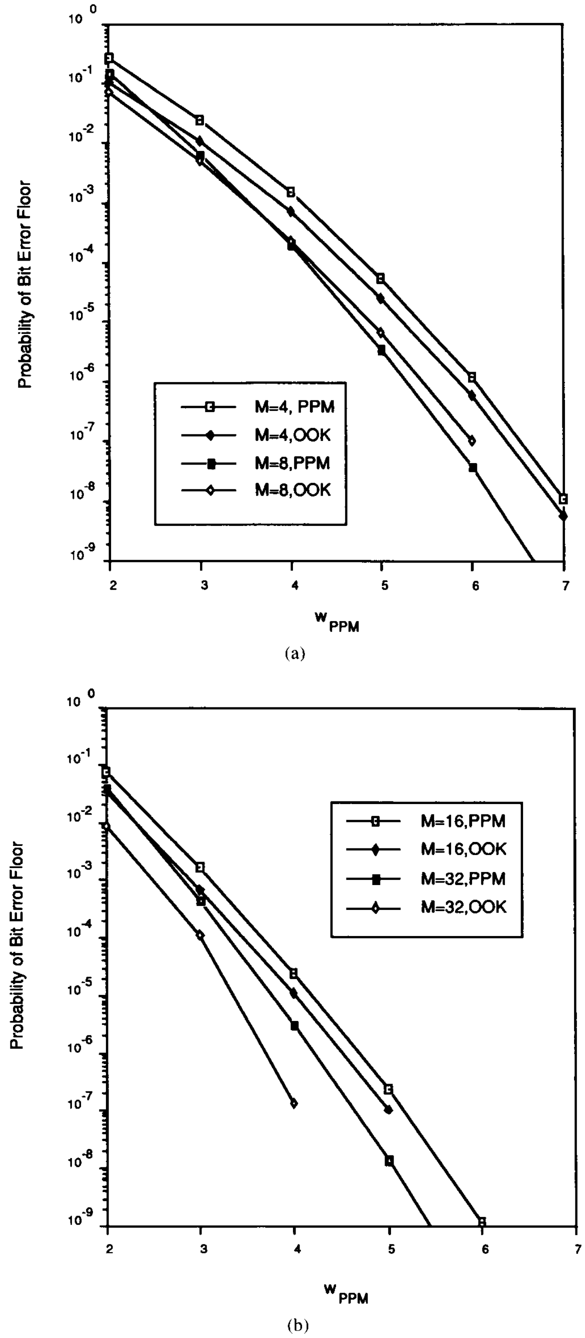

Fig. 7. PE_f versus code weight w for M-PPM and equivalent OOK. (a) $M = 4$ and 8, $N = 8$. (b) $M = 16$ and 32, $N = 8$.

better. Hence, the use of PPM is actually not advantageous for PE_f reduction in spite of the dispersing of the interfering pulses by the position encoding. Evidently the PPM sacrifice in channel bit rate in (20) is used to greater advantage in the equivalent OOK system.

It should be pointed out however that the advantage of PPM appears when average laser source power is considered. Since position modulation spreads its pulse power over longer frame times, its average power is much less than that of an OOK system that must pulse at the bit rate. (This fact does not directly effect the PE floor considered in this study). Conversely, with fixed average laser power, PPM develops a higher pulse power for combating detector noise. These power advantages were discussed in [18].

V. Conclusion

An asynchronous fiberoptic OOK–CDMA system will have its bit error probability ultimately limited by the crosstalk of other transmitters. This resulting PE floor can be reduced only by slowing the data rate with longer sequence lengths and higher weighted codes. This raises the question of whether external channel coding (RS coding, convolutional coding, etc.), can be more effective in reducing the PE performance of the network. We have shown that channel coding can indeed be more effective in reducing the PE floor. Both Reed–Solomon block codes and convolutional codes were considered, with both showing improved PE performance over the uncoded OOK–CDMA system at the same information bit rate. The convolutional codes tended to produce lower PE floors for the lower code weight values and hence would tend to have the highest network capacities. This result is important since the channel coding is applied with electronic hardware external to the optics. Hence, the channel coding should have limited impact on the overall system cost.

The use of PPM as a channel coding technique was also considered, since it also reduces the PE floor at the expense of data rate. It was shown that the PPM system was not as effective as the uncoded and error correction systems at the same data rate. Furthermore PPM requires a modification of the optical encoding and decoding processing from the standard OOK–CDMA format.

Appendix

In order to determine P_d in (16) for a CDMA optical system, consider a correct and incorrect set of bits which differ in d positions. For the correct and an incorrect path to differ, one path must have a 1 in a given position while the other has a 0 or vice versa. Given that an incorrect path through the trellis differs from the correct path in d positions, only these d bits of the received vector are relevant to which of the 2 paths will have the highest metric. Define the following parameters for these d bits:

k = The number of 1's in the d bits of the correct sequence. $(0 \leq k \leq d)$.

J_{10} = The number of $1 \rightarrow 0$ transitions from the correct to the received sequence. $(0 \leq J_{10} \leq k)$.

J_{01} = The number of $0 \rightarrow 1$ transitions from the correct to the received sequence. $(0 \leq J_{01} \leq d - k)$.

M_c = Metric of the received vector with the correct path.

M_i = Metric of the received vector with the incorrect path.

E_d = The event that the received vector's metric with an incorrect path which differs for the correct path in d places exceeds the metric of the received vector and the correct path.

In general, the probability of E_d can be written as

$$P_d = \sum_{k=0}^{d} \sum_{j_{01}=0}^{d-k} \sum_{j_{10}=0}^{k} \Pr\{E_d/K = k,\, J_{10} = j_{10},\, J_{01} = j_{01}\} \cdot \Pr\{J_{10} = j_{10},\, J_{01} = j_{01}/K = k\} \cdot \Pr\{K = k\}. \quad \text{(A1)}$$

Each term in (A1) can be evaluated for the general asymmetric channel. First, the probability of E_d given the number of $0 \to 1$ and $1 \to 0$ transitions depends on the values of the metrics M_c and M_i. Specifically,

$$\begin{aligned} \Pr\{E_d/K = k,\, J_{10} = j_{10},\, J_{01} = j_{01}\} &= 1 \quad \text{if} \quad M_i > M_c \\ &= 0.5 \quad \text{if} \quad M_i = M_c \\ &= 0 \quad \text{if} \quad M_i < M_c. \end{aligned} \quad \text{(A2)}$$

Since j_{01} $0 \to 1$ transitions and j_{10} $1 \to 0$ transitions are given to have occurred between the correct and received d bits of interest, the metric M_c is

$$M_c = j_{01} \log(p_{01}) + j_{10} \log(p_{10}). \quad \text{(A3)}$$

Under the condition of k 1's in the correct d bits, if there are j_{10} $1 \to 0$ transitions, the number of $0 \to 1$ transitions between the incorrect and received d bits is $k - j_{10}$. Likewise, given j_{01} $0 \to 1$ transitions between the correct and received d bits, there are $d - k - j_{01}$ $1 \to 0$ transitions between the incorrect and received sequences. This implies that

$$M_i = (k - j_{10}) \log(p_{01}) + (d - k - j_{01}) \log(p_{10}). \quad \text{(A4)}$$

By the use of (A3) and (A4) in (A2), $\Pr\{E_d/K = k,\, J_{10} = j_{10},\, J_{01} = j_{01}\}$ can be evaluated for each term in the summation.

Next $\Pr\{J_{10} = j_{10},\, J_{01} = j_{01}/K = k\}$ can be determined. With the chip synchronous interference assumption, the CDMA channel is memoryless. With k specified, the number of 1's and 0's in the d bits is fixed and the probabilities $0 \to 1$ and $1 \to 0$ transitions are independent. Hence,

$$\begin{aligned} &\Pr\{J_{10} = j_{10},\, J_{01} = j_{01}/K = k\} \\ &= \left[\binom{k}{j_{10}} p_{10}^{j_{10}} (1 - p_{10})^{k - j_{10}}\right] \cdot \left[\binom{d-k}{j_{01}} p_{01}^{j_{01}} (1 - p_{01})^{d-k-j_{01}}\right]. \end{aligned} \quad \text{(A5)}$$

Finally, with equally likely input bits,

$$\Pr\{K = k\} = \binom{d}{k} \left(\frac{1}{2}\right)^d. \quad \text{(A6)}$$

Combining the foregoing equations yields the following expression for P_d:

$$P_d = \sum_{k=0}^{d} \sum_{j_{01}=0}^{d-k} \sum_{j_{10}=0}^{k} \epsilon(j_{01}, j_{10}, k, d) \cdot \left[\binom{k}{j_{10}} p_{10}^{j_{10}} (1 - p_{10})^{k - j_{10}}\right] \cdot \left[\binom{d-k}{j_{01}} p_{01}^{j_{01}} (1 - p_{01})^{d-k-j_{01}}\right] \binom{d}{k} \left(\frac{1}{2}\right)^d \quad \text{(A7)}$$

where

$$\begin{aligned} \epsilon(j_{01}, j_{10}, k, d) &= 1 \quad M_i > M_c \\ &= 0.5 \quad M_i = M_c \\ &= 0 \quad M_i < M_c. \end{aligned} \quad \text{(A8)}$$

In the high power case where $N > w$, the CDMA channel becomes a Z channel. In this case (A7) can be simplified. First, since p_{10} is arbitrarily small only the $j_{10} = 0$ term in the summation contributes and P_d becomes

$$P_d = \sum_{k=0}^{d} \sum_{j_{01}=0}^{d-k} \epsilon(j_{01}, k, d) \cdot \left[\binom{d-k}{j_{01}} p_{01}^{j_{01}} (1 - p_{01})^{d-k-j_{01}}\right] \binom{d}{k} \left(\frac{1}{2}\right)^d \quad \text{(A9)}$$

now the metrics are

$$M_c = j_{01} \log(p_{01}) \quad \text{(A10)}$$

$$M_i = k \log(p_{01}) + (d - k - j_{01}) \log(p_{10}). \quad \text{(A11)}$$

Since p_{10} is arbitrarily small, the final term in (A11) is an arbitrarily large negative number. This means that M_i will always be less than M_c unless $j_{01} = d - k$. With d even, M_i is equal to M_c when $j_{01} = d$ if $k = d/2$. Otherwise, for M_i to be greater than M_c when $j_{01} = d - k$, k must be less than $d/2$. These observations lead to the final expression for P_d given by (16)

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