Homework 7

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Chapter 4: Exercises: 4.1-6, 4.9

4.1 (a.)Yes the DFA M accepts 0100.
(b.)No M does not accept 011.
(c.)No this input has only a single component and therefore is not in the correct form.
(d.)No the first component is not a regular expression and so the input is not of the correct form.
(e.)No M's language is not empty
(f.)Yes M accepts the same language as itself.

4.2 *Proof.* The language can be express as the set L=$\{\langle R, S\rangle$ | R is a DFA and is S is a regex where $L(R) = L(S)\}$.Let the Turning Machine T be the TM that decides L.
T="On input $L = \langle R, S\rangle$:
1.Convert S to a NFA N.
2.Convert N to a DFA D.
3. By using Theorem 4.5. Feed R and D into a TM M that decides whether or they are equal or not.
(a.) If M says R and D are equal by accepting then accept.
(b.) If M says R and D are not equal by rejecting then reject. " ■

4.3 *Proof.* Let the Turning Machine T be the TM that decides ALL_{DFA}.
T="On input $\langle A\rangle$, where A is a DFA.
1.Convert A to A^c, the DFA that accepts the complement of the language being accepted by A.
2.Using Theorem 4.4 as a subroutine, check and see if $L(A^c) = \emptyset$ or not.
3.If $L(A^c) = \emptyset$ accept otherwise reject." ■

4.4 *Proof.* Let the Turning Machine T be the TM that decides $A_{\varepsilon CFG}$.
T="On input $\langle G\rangle$ where G is a context free grammar.
1.Convert the grammar G to a CFG G' in CNF.
2.If G' contains the production $S \rightarrow \varepsilon$ accept, otherwise reject. " ■

4.5 *Proof.* Let $s_1, s_2, ...$ be a list of all strings in Σ^*. The following TM T recognizes $\overline{E_{TM}}$.
T="On input $\langle M\rangle$, where M is a TM:
1.Repeat the following for i=1,2,3...
2.Run M for i steps on each input,$s_1, s_2, ..., s_i$.
3.If M has accepted any of these, accept.Otherwise reject." ■

4.6 (a.)No f is not one-to-one because of f(1)=f(3).
(b.)No f is not onto because their is not a x∈X such that f(x)=10.
(c.)No f is not a correspondence because f is not one-to-one and onto.
(d.)Yes g is one-to-one.
(e.)Yes g is onto.
(f.)Yes g is a correspondence because g is one-to-one and onto.

4.9 *Proof.* A relation is known as a equivalence relation if it is reflexive, transitive, and symmetric. A same size relation is a equivalence relation if and only if it is reflexive, transitive, and symmetric.First, it is reflexive because the identity function $f(x) = x, \forall x \in A$ is a correspondence $f : A \to A$. Second it is symmetric because any correspondence has an inverse, which itself is a correspondence. Lastly, it is transitive because the function $f : A \to B$ is a bijective function from A to B and the function $g : B \to C$ is a bijective function from B to C. Therefore the composition of two bijective functions f and g is also a bijective function from A to C. Hence the same size relation is reflexive,symmetric, and transitive so the same size relation is a equivalence relation. ∎