An agent based model

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Abstract

Our model aims at representing the voting mechanism of the main decision-making body of the Euro-system the governing council. Moreover, we design the discussion preceding the interest rate voting session relying on the following assumption: Among agents we could single out two subgroups hawks and doves, within the same group agents show homogenous preferences, thus we model the two groups as two agents.

1 Model specification

We defined hawks as the representatives of a high-debt country who are more inclined to support higher interest rates as means of controlling inflation. While, doves refer to individuals who are more accommodating in the monetary policy approach, favoring lower interest rates. Moreover, we have assumed that both agents act rationally, and that they know each other's proposals.

From now on we will refer to hawks as agents in group j and doves in group i.

The utility function of the council voters (agent) is:

$$U(x_i, x_j) = -\beta_{i1}(x_i - T)^2 - \beta_{i2}(x_i - N_i)^2 - ((1-\alpha)x_i - \alpha x_j)^2$$

where:

- x_i: is the changing rates proposed by each agent i
- T: is the taylor rule (Assumed to be common knowledge)

- N_i: is the agents national preferences about interest rate
- β_{i1}, β_{i2}: weights assigned to taylor rule and to national preferences
- $x \in [-50; 50] b.p.$
- *alpha*: the share of hawks on the total number of voters

The first term $\beta_{i1}(x_i - T)^2$ represents the disutility coming from deviations from the taylor rule prescription, meaning the further x_i is from T the higher is the disutility. The second term $\beta_{i2}(x_i - N_i)^2$ represents the disutility coming from deviations from the country's national directive, meaning the further is the agent i from the country's national directive, the lower the utility. The third term $(x_i - x_j)^2$ represents the disutility coming from deviations from the proposal of the others.

2 Solving the model

We solve the maximization problem of the agent i and j
FOC:

$$-2\beta_{i1}(x_i - T) - 2\beta_{i2}(x_i - N_i) - 2(1-\alpha)[(1-\alpha)x_i - \alpha x_j] = 0$$

$$\beta_{i1}x_i - \beta_{i1}T + \beta_{i2}x_i - \beta_{i2}N_i + x_i(1-\alpha)^2 - x_j(1-\alpha)\alpha = 0$$

$$x_i = \frac{\beta_{i1}T + \beta_{i2}N_i + x_j(1-\alpha)\alpha}{\beta_{i1} + \beta_{i2} + (1-\alpha)^2}$$

$$x_j = \frac{\beta_{j1}T + \beta_{j2}N_j + x_i(1-\alpha)\alpha}{\beta_{j1} + \beta_{j2} + \alpha^2}$$

Solving the system of equations we derived the Nash equilibrium:

$$x_i^N = \frac{q_i w_j + (1-\alpha)\cdot \alpha q_j}{w_i \cdot w_j - 1}$$

$$x_j^N = \frac{q_j w_i + (1-\alpha)\cdot \alpha q_j}{w_i \cdot w_j - 1}$$

- $q_j = \beta_{i1}T + \beta_{i2}N_i$; $q_i = \beta_{j1}T + \beta_{j2}N_j$
- $w_j = \beta_{1,j} + \beta_{2,j} + \alpha^2$; $w_i = \beta_{1,i} + \beta_{2,i} + (1-\alpha)^2$

The Nash equilibrium suggests that the interest rate proposals of the two agents depend on their own preferences ($\beta_{1,i}$ and $\beta_{2,i}$ for agent i and $\beta_{1,j}$ and $\beta_{2,j}$ for agent j), the Taylor rule (T), and the interest rate proposal of the other agent (x_j for agent i and x_i for agent j). The closed-form solutions for the interest rate proposals indicate that the interest rate proposals of the two agents are linear functions of their own preferences, the Taylor rule, and the interest rate proposal of the other agent. This suggests that the interest rate proposals of the two agents are interdependent and that they need to take into account the actions of the other agent in order to maximize their own utility.

2.1 Parametrization of the model

The parameters for the hawks representative agents are set as $\beta_{1,j} = 2$ and $\beta_{2,j} = 1$, indicating that they give more weight to the Taylor rule prescription compared to the national preferences. These hawks represent less indebted countries, as reflected by their preference for a higher interest rate target of $N_j = 0.5$. On the other hand, the doves representative agents are characterized by $\beta_{1,i} = 2$ and $\beta_{2,i} = 1.5$, indicating a greater weight given to national preferences over the Taylor rule. The doves represent more indebted countries, and their preference for a lower interest rate target is reflected by $N_i = 0$. The Taylor rule target is set to $T = 0.25$. Finally, we imposed the share of hawks α is equal to 60%. Based on the given parameter values, the best reply functions of the doves and hawks are plotted to visualize the Nash equilibrium in the graph1. The green line indicates the best reply function of the doves, while the black line represents the best reply function of the hawks. The nash equilibrium is in $(0.16, 0.3)$ meaning that the qualified majority will vote for an increase of the interest rate about 20 b.p.

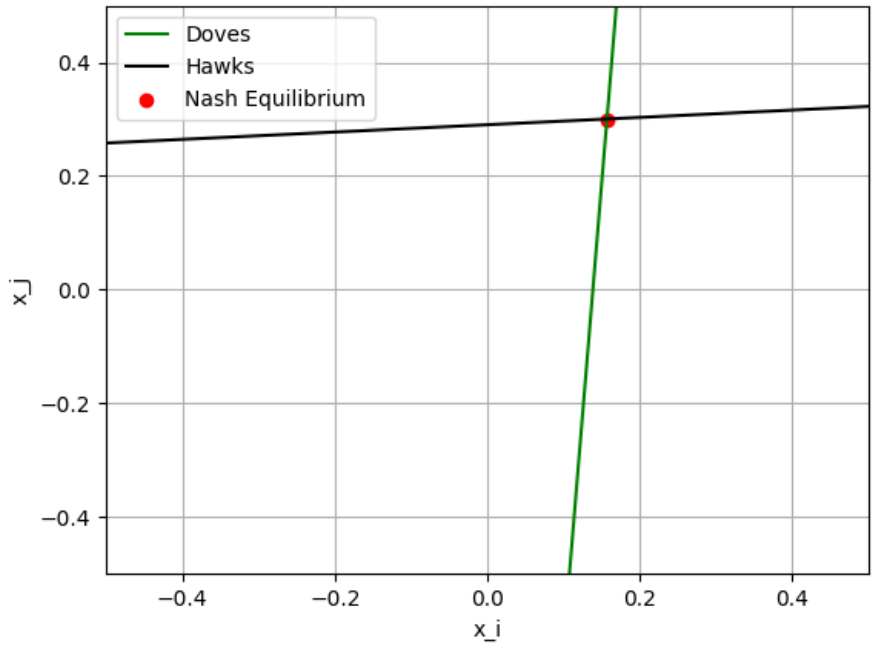

Figure 1: Nash equilibrium and best reply function

Table 1: Rationalizability Learning Steps

Step	**Doves**	**Hawks**
0	[-0.5, 0.5]	[-0.5, 0.5]
1	[0.10382514, 0.16939891]	[0.26190476, 0.33333333]
2	[0.1537861, 0.15846995]	[0.30503513, 0.30971897]
3	[0.15661433, 0.15692146]	[0.30860377, 0.30893833]
4	[0.15684833, 0.15687027]	[0.30880579, 0.30882772]
5	[0.15686158, 0.15686302]	[0.3088225, 0.30882407]
6	[0.15686268, 0.15686278]	[0.30882345, 0.30882355]
7	[0.15686274, 0.15686275]	[0.30882352, 0.30882353]
8	[0.15686274, 0.15686275]	[0.30882352, 0.30882353]