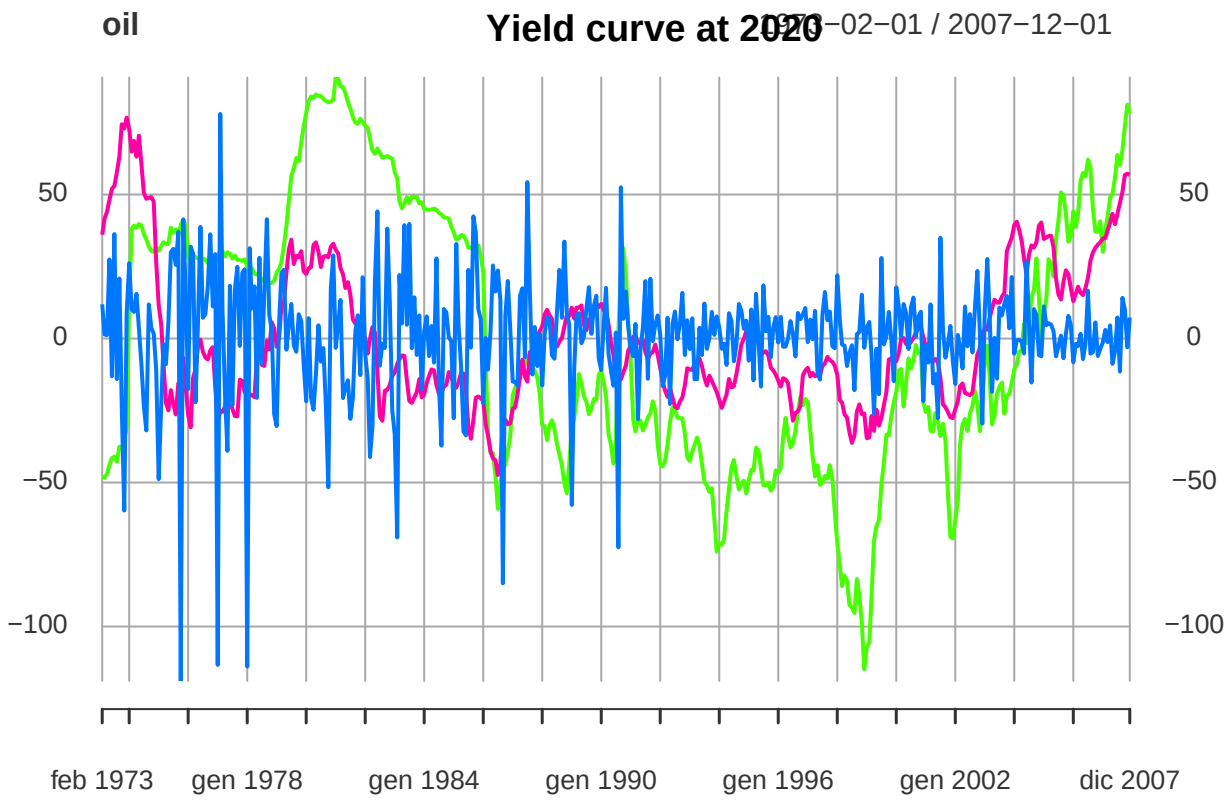
Assignement

Point 1

The time series below represents the monthly time series of:

1. % change in global crude oil production
2. the real price of oil
3. the real economy activity

from 1973:1 to 2007:12.



As we can see from acf its clear signaling the presence of an autocorrelation process. From the partial autocorrelation function we can infer that it's probably first-order autocorrelation since the only significant column is the first one (also the second one, but it has a negative sign).

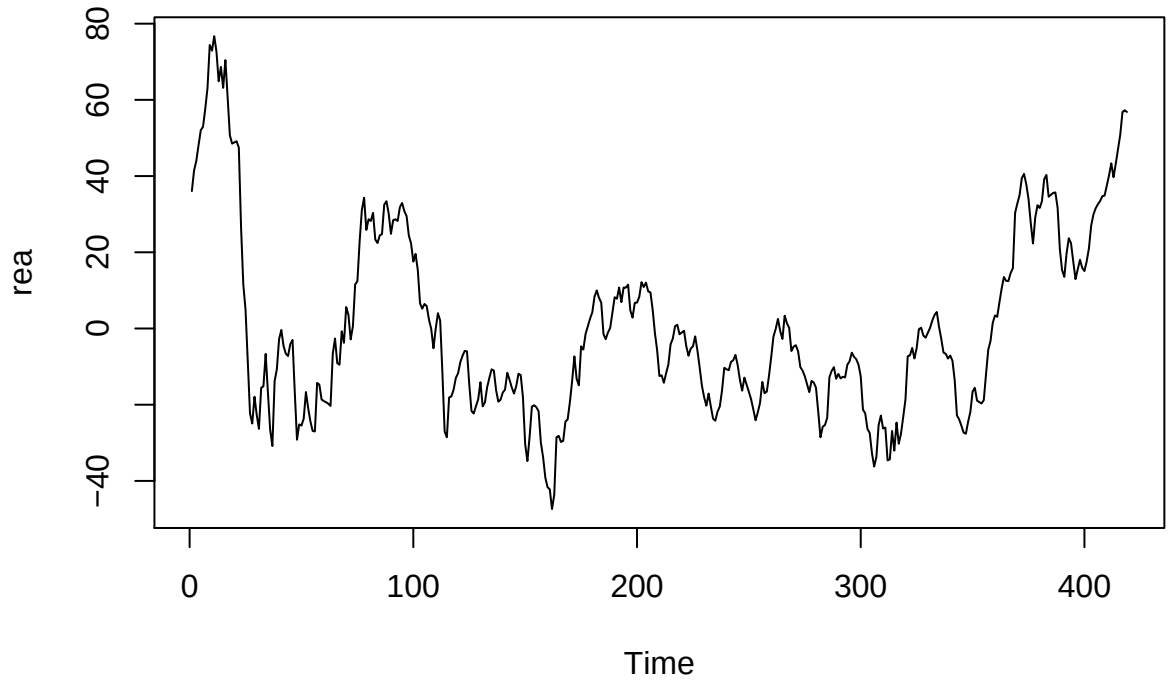
In order to test if the *rea* is an $I(1)$, we will use an ADF test with a minimun lag =1. We will perform the test specifing four different type of the process:

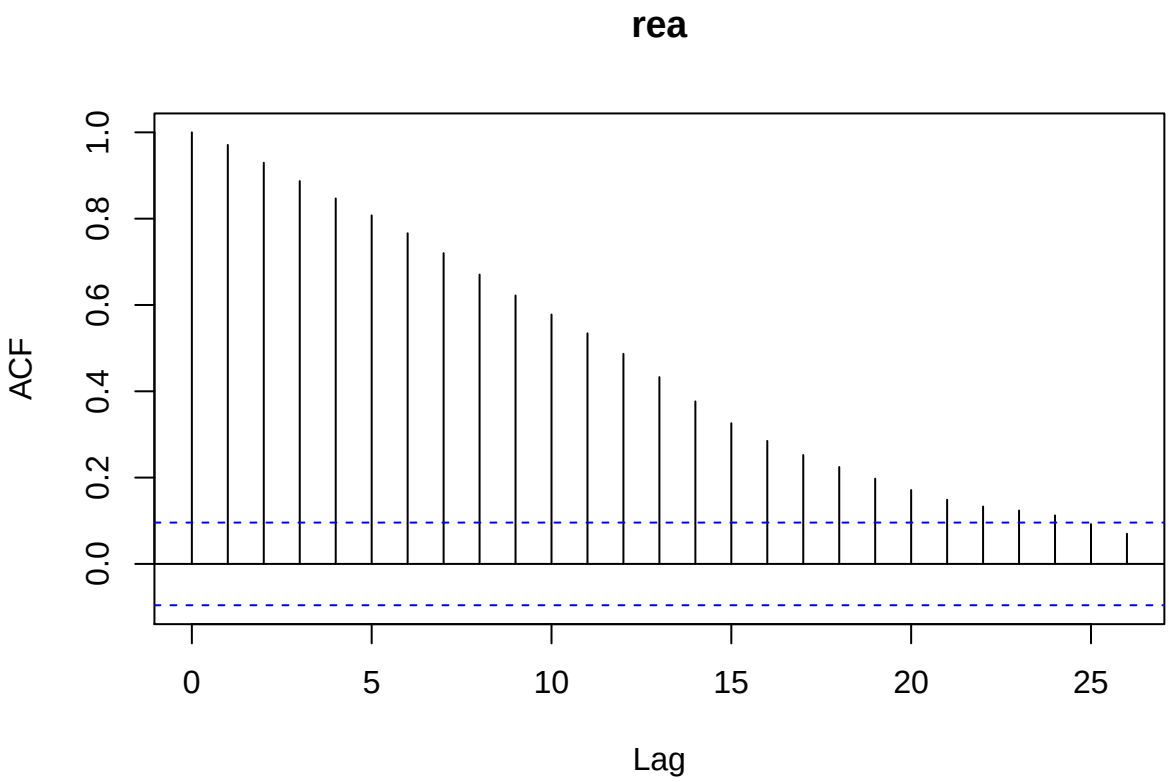
1. No constant, no trend
2. Constant
3. Constant with trend

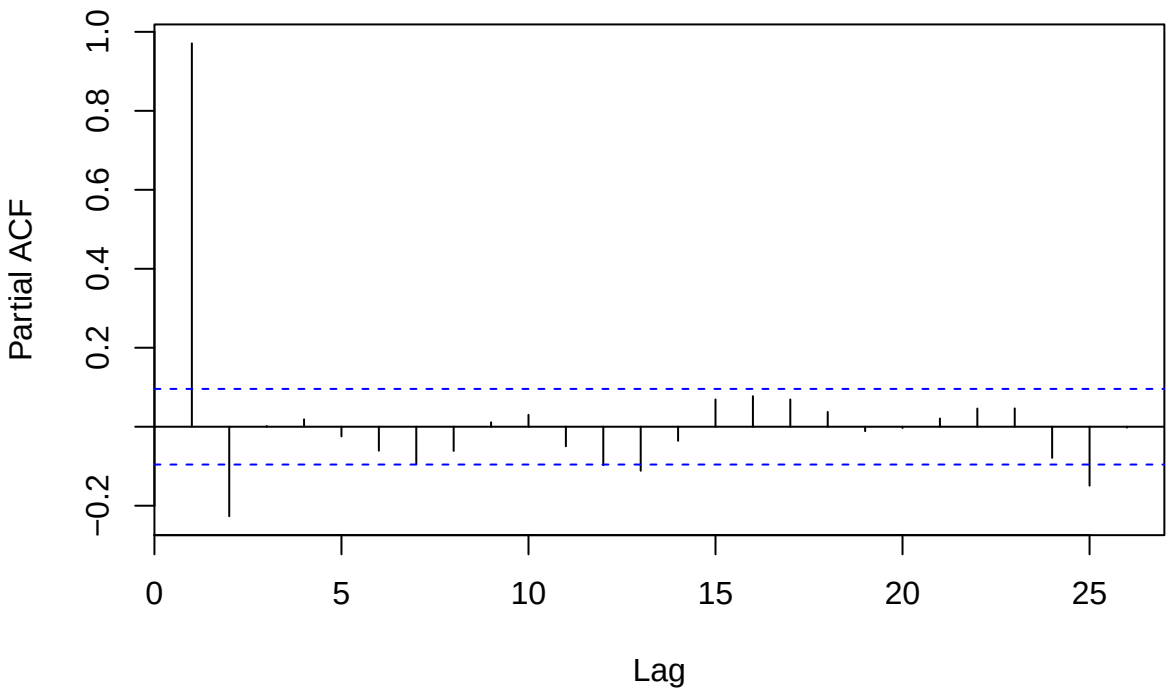
First, we print the first time series graph. We perform the different types of the test with a maximum lag order of 12:

$$rea_t = \alpha + \sigma_1 reat_{t-1} + ... + \sigma_1 2\delta reat_{t-13}$$

The criteria for selection of the lag order is the one which has lower BIC:




rea
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
Lag



	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	1	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	1	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	1	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	2	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	2	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	2	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	2	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	2	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	2	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	3	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	3	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	3	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	3	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	3	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	3	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	3	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	3	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	3	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	4	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	4	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	4	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	4	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	4	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.

	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	4	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	4	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	4	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	4	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	4	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	4	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	4	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	5	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	5	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	5	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	5	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	5	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	5	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	5	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	5	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	5	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	5	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	5	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	5	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	5	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	5	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	5	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	6	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	6	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	6	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	6	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	6	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	6	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	6	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	6	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	6	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	6	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	6	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	6	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	6	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	6	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	6	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	6	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	6	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	6	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	7	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	7	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	7	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	7	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	7	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	7	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	7	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	7	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	7	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	7	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	7	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.

	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	7	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	7	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	7	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	7	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	7	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	7	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	7	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	7	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	7	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	7	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	8	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	8	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	8	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	8	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	8	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	8	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	8	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	8	no drift no trend	8	-3.3318	0.01	Stat	Stat
rw	8	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	8	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	8	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	8	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	8	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	8	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	8	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	8	with drift no trend	8	-3.3321	0.0154	Stat	Stat
rw	8	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	8	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	8	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	8	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	8	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	8	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	8	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	8	with drift and trend	8	-3.3119	0.0686	No Stat.	Stat
rw	9	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	9	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	9	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	9	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	9	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	9	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	9	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	9	no drift no trend	8	-3.3318	0.01	Stat	Stat
rw	9	no drift no trend	9	-3.4479	0.01	Stat	Stat
rw	9	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	9	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	9	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	9	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	9	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	9	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	9	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	9	with drift no trend	8	-3.3321	0.0154	Stat	Stat

	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	9	with drift no trend	9	-3.454	0.01	Stat	Stat
rw	9	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	9	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	9	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	9	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	9	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	9	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	9	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	9	with drift and trend	8	-3.3119	0.0686	No Stat.	Stat
rw	9	with drift and trend	9	-3.4427	0.0478	Stat	Stat
rw	10	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	10	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	10	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	10	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	10	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	10	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	10	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	10	no drift no trend	8	-3.3318	0.01	Stat	Stat
rw	10	no drift no trend	9	-3.4479	0.01	Stat	Stat
rw	10	no drift no trend	10	-3.2258	0.01	Stat	Stat
rw	10	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	10	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	10	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	10	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	10	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	10	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	10	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	10	with drift no trend	8	-3.3321	0.0154	Stat	Stat
rw	10	with drift no trend	9	-3.454	0.01	Stat	Stat
rw	10	with drift no trend	10	-3.2318	0.0202	Stat	Stat
rw	10	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	10	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	10	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	10	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	10	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	10	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	10	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	10	with drift and trend	8	-3.3119	0.0686	No Stat.	Stat
rw	10	with drift and trend	9	-3.4427	0.0478	Stat	Stat
rw	10	with drift and trend	10	-3.2348	0.0819	No Stat.	Stat
rw	11	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	11	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	11	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	11	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	11	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	11	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	11	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	11	no drift no trend	8	-3.3318	0.01	Stat	Stat
rw	11	no drift no trend	9	-3.4479	0.01	Stat	Stat
rw	11	no drift no trend	10	-3.2258	0.01	Stat	Stat
rw	11	no drift no trend	11	-3.5172	0.01	Stat	Stat

	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	11	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	11	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	11	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	11	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	11	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	11	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	11	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	11	with drift no trend	8	-3.3321	0.0154	Stat	Stat
rw	11	with drift no trend	9	-3.454	0.01	Stat	Stat
rw	11	with drift no trend	10	-3.2318	0.0202	Stat	Stat
rw	11	with drift no trend	11	-3.529	0.01	Stat	Stat
rw	11	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	11	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	11	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	11	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	11	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.
rw	11	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	11	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	11	with drift and trend	8	-3.3119	0.0686	No Stat.	Stat
rw	11	with drift and trend	9	-3.4427	0.0478	Stat	Stat
rw	11	with drift and trend	10	-3.2348	0.0819	No Stat.	Stat
rw	11	with drift and trend	11	-3.5484	0.0375	Stat	Stat
rw	12	no drift no trend	1	-1.6457	0.0961	No Stat.	Stat
rw	12	no drift no trend	2	-2.5702	0.0104	Stat	Stat
rw	12	no drift no trend	3	-2.4271	0.0166	Stat	Stat
rw	12	no drift no trend	4	-2.4375	0.0162	Stat	Stat
rw	12	no drift no trend	5	-2.5514	0.0112	Stat	Stat
rw	12	no drift no trend	6	-2.801	0.01	Stat	Stat
rw	12	no drift no trend	7	-3.2152	0.01	Stat	Stat
rw	12	no drift no trend	8	-3.3318	0.01	Stat	Stat
rw	12	no drift no trend	9	-3.4479	0.01	Stat	Stat
rw	12	no drift no trend	10	-3.2258	0.01	Stat	Stat
rw	12	no drift no trend	11	-3.5172	0.01	Stat	Stat
rw	12	no drift no trend	12	-3.8179	0.01	Stat	Stat
rw	12	with drift no trend	1	-1.6443	0.4692	No Stat.	No Stat.
rw	12	with drift no trend	2	-2.5669	0.1025	No Stat.	No Stat.
rw	12	with drift no trend	3	-2.4235	0.1595	No Stat.	No Stat.
rw	12	with drift no trend	4	-2.4339	0.1554	No Stat.	No Stat.
rw	12	with drift no trend	5	-2.5482	0.11	No Stat.	No Stat.
rw	12	with drift no trend	6	-2.7977	0.0626	No Stat.	Stat
rw	12	with drift no trend	7	-3.2132	0.0211	Stat	Stat
rw	12	with drift no trend	8	-3.3321	0.0154	Stat	Stat
rw	12	with drift no trend	9	-3.454	0.01	Stat	Stat
rw	12	with drift no trend	10	-3.2318	0.0202	Stat	Stat
rw	12	with drift no trend	11	-3.529	0.01	Stat	Stat
rw	12	with drift no trend	12	-3.8297	0.01	Stat	Stat
rw	12	with drift and trend	1	-1.6235	0.736	No Stat.	No Stat.
rw	12	with drift and trend	2	-2.5446	0.3465	No Stat.	No Stat.
rw	12	with drift and trend	3	-2.3969	0.4087	No Stat.	No Stat.
rw	12	with drift and trend	4	-2.4043	0.4055	No Stat.	No Stat.
rw	12	with drift and trend	5	-2.5178	0.3578	No Stat.	No Stat.

	N of lags	Type	lag	ADF	p.value	Stationary at 5%	Stationary at 10%
rw	12	with drift and trend	6	-2.7702	0.2515	No Stat.	No Stat.
rw	12	with drift and trend	7	-3.1891	0.0898	No Stat.	Stat
rw	12	with drift and trend	8	-3.3119	0.0686	No Stat.	Stat
rw	12	with drift and trend	9	-3.4427	0.0478	Stat	Stat
rw	12	with drift and trend	10	-3.2348	0.0819	No Stat.	Stat
rw	12	with drift and trend	11	-3.5484	0.0375	Stat	Stat
rw	12	with drift and trend	12	-3.863	0.0156	Stat	Stat

```
## [1] "Without constant and without time trend"

##
## === Test statistics =========
##                tau1
## statistic -3.056092
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
##
## === Combined output =========
## [1] "-3.06 [1]***"

## [1] "With constant and without time trend"

##
## === Test statistics =========
##                tau2     phi1
## statistic -3.064165 4.695391
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau2 -3.44 -2.87 -2.57
## phi1  6.47  4.61  3.79
##
## === Combined output =========
## [1] "-3.06 [1]**"

## [1] "With constant and with time trend"

##
## === Test statistics =========
##                tau3     phi2     phi3
## statistic -3.283632 4.530211 6.794472
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau3 -3.98 -3.42 -3.13
## phi2  6.15  4.71  4.05
## phi3  8.34  6.30  5.36
```

```
##
## === Combined output =========
## [1] "-3.28 [1]*"
```

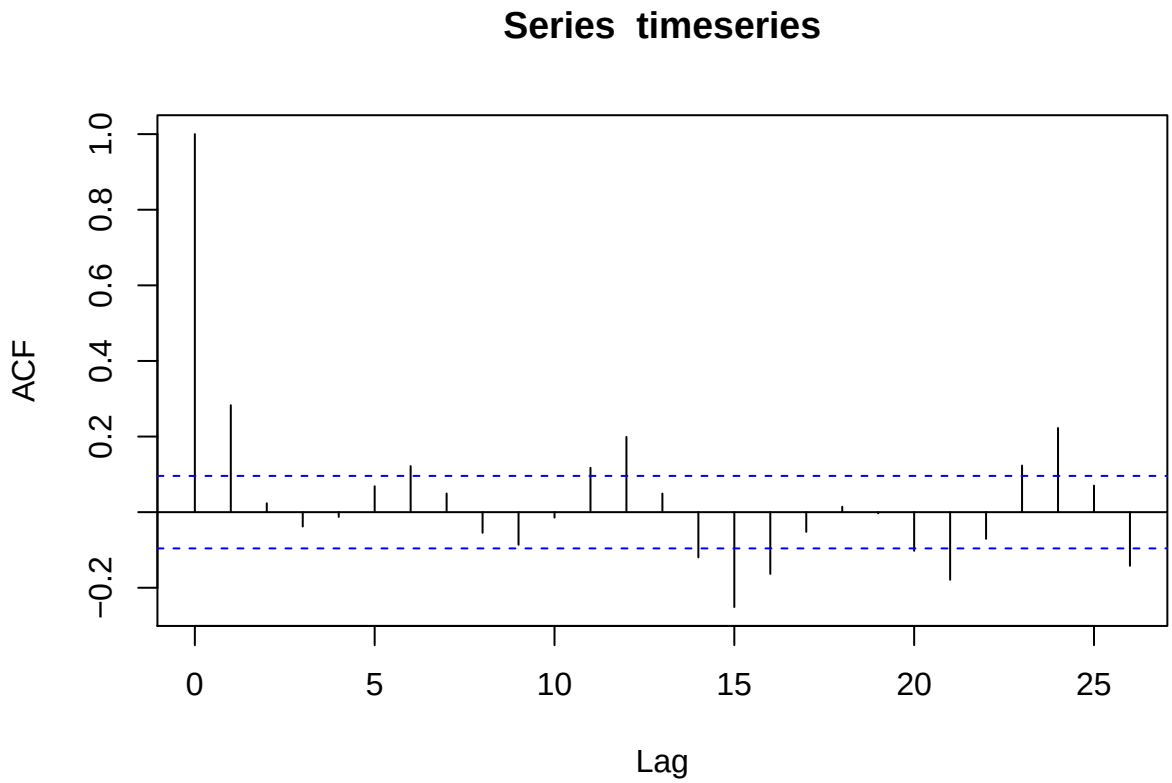
The results of the ADF tests shows that the process is stationary with the simplest specification (without constant and time trend), up to the third significance level (over 1%). However, the other possible specification, which add a constant and then also a time trend present higher p-values, thus the specification we are going to select is the first one. This is consistent with what we should expect, since rea_t is computed as a percentage deviation from the mean (it's basically an indicator of the business cycle).

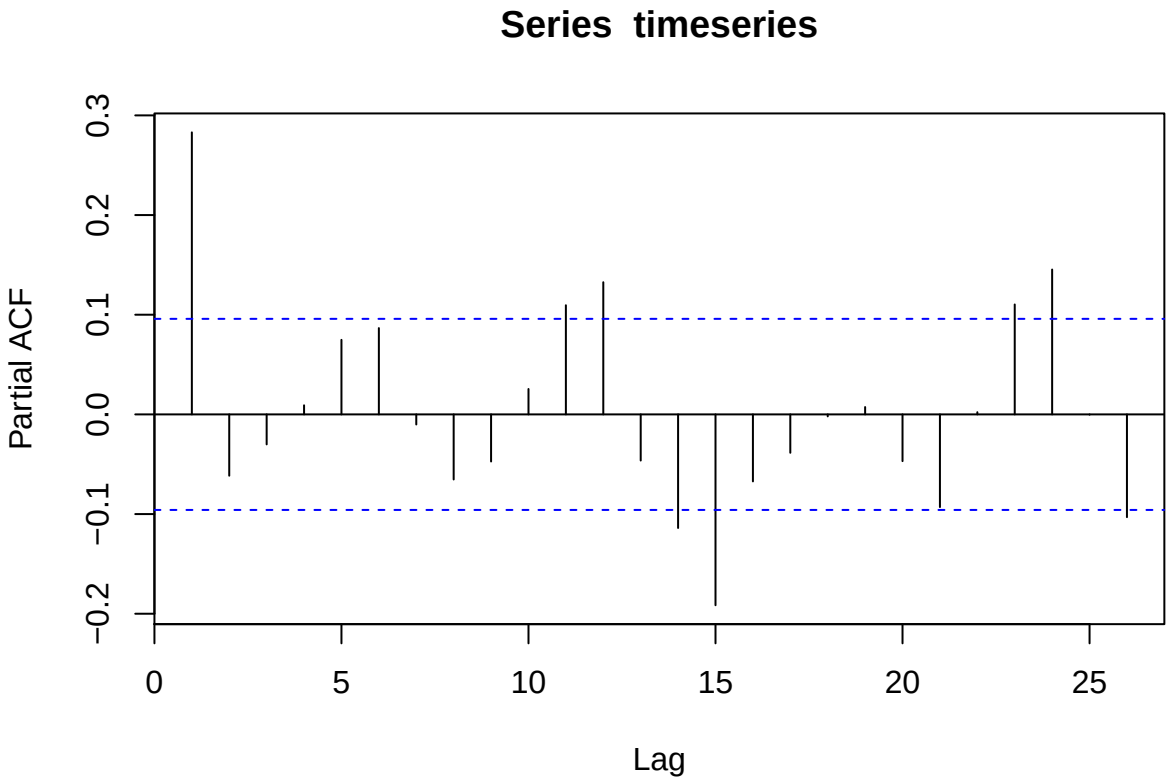
Thus, the specification we select in the end is:

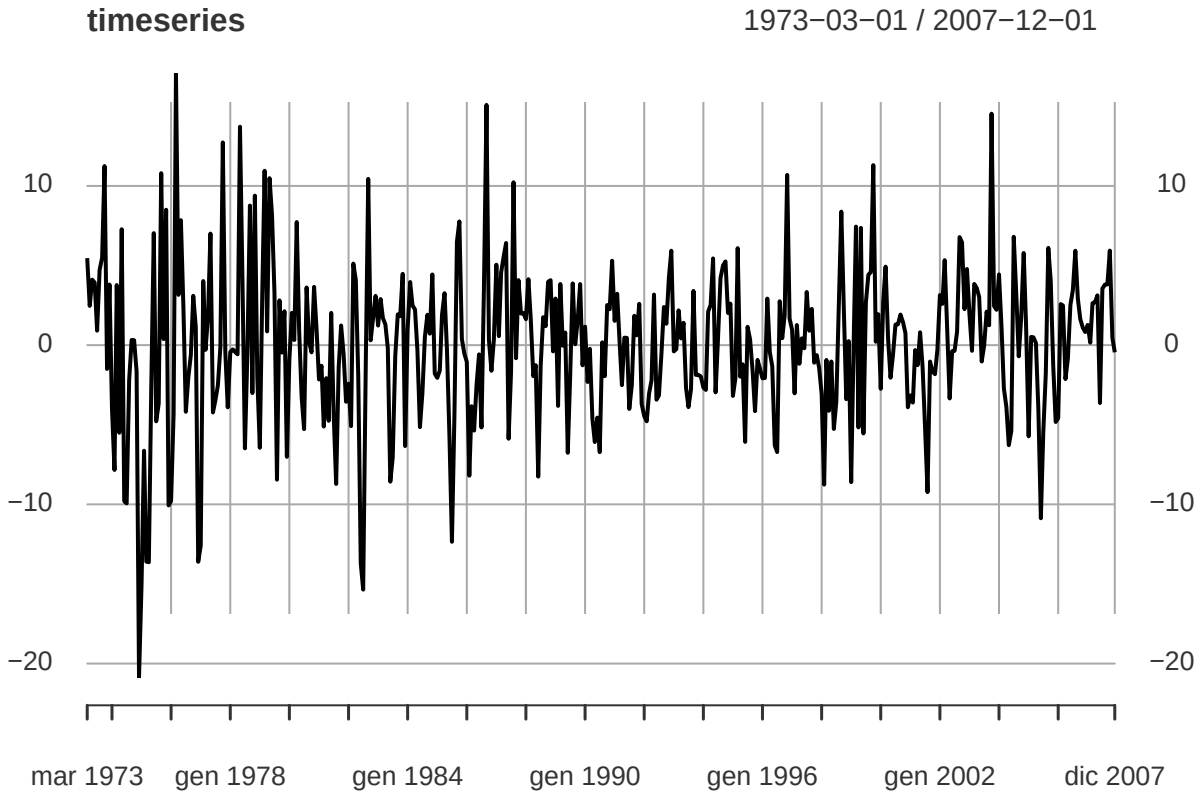
$$\delta rea_t = \sigma_1 \delta reat_{t-1} + ... + \sigma_1 2 \delta reat_{t-13}$$

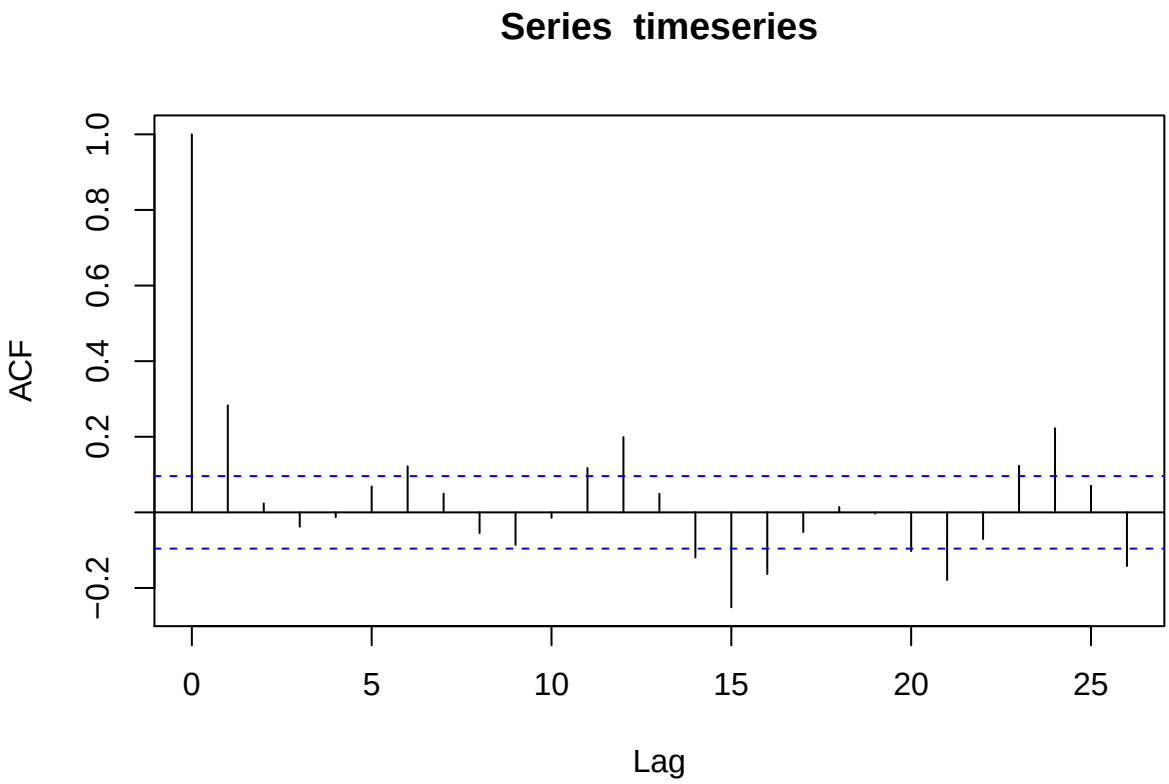
Point 2

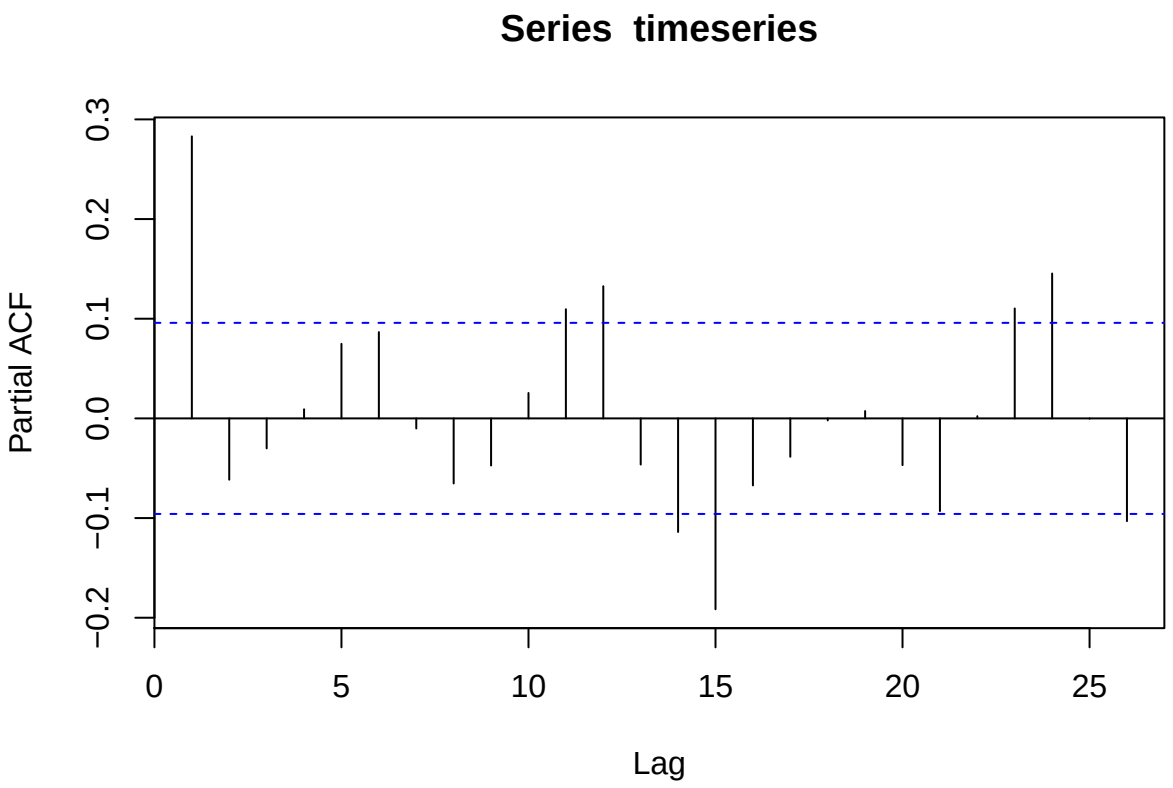
We take the first difference of the timeseries *rea* and check if it is stationary with an adf test. Before that we print the time series of the first differences, the acf, and the pacf to understand the correct specification for the ADF test.




Series timeseries
Partial ACF
0.3
0.2
0.1
0.0
−0.1
−0.2
0
5
10
15
20
25
Lag


timeseries
1973−03−01 / 2007−12−01
10
0
−10
−20
mar 1973
gen 1978
gen 1984
gen 1990
gen 1996
gen 2002
dic 2007


Series timeseries
ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0
5
10
15
20
25
Lag



The above graphs clearly underline the stationarity of the process, indeed the acf for the $lag > 2$ the partial autcorrelation is not statisticaly different from 0. As for the partial autcorrelation that is statistically different only for some lag>10. From the plot of the time series we can see a mean reverting process, and so I will opt for the specifications with constant and time trend, becouse it is less restricitve. So the test will have the following specifications:

$$\delta rea_t = \sigma_1 \delta reat_{t-1} + ... + \sigma_1 2 \delta reat_{t-13}$$

$$\delta rea_t = \alpha + \sigma_1 \delta reat_{t-1} + ... + \sigma_1 2 \delta reat_{t-13}$$

$$\delta rea_t = \alpha + \beta * t + \sigma_1 \delta reat_{t-1} + ... + \sigma_1 2 \delta reat_{t-13}$$

The test will be performed with all passible four specification, and will be selected the specification with lower adf value.

```
## [1] "Without constant and without time trend"

##
## === Test statistics =========
##                 tau1
## statistic -12.92825
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
##
## === Combined output =========
## [1] "-12.93 [1]***"
```

```
## [1] "With constant and without time trend"

##
## === Test statistics =========
##                tau2     phi1
## statistic -12.91292 83.37187
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau2 -3.44 -2.87 -2.57
## phi1  6.47  4.61  3.79
##
## === Combined output =========
## [1] "-12.91 [1]***"

## [1] "With constant and with time trend"

##
## === Test statistics =========
##                tau3     phi2     phi3
## statistic -13.09473 57.15755 85.73616
##
## === Test critical values ====
##       1pct  5pct 10pct
## tau3 -3.98 -3.42 -3.13
## phi2  6.15  4.71  4.05
## phi3  8.34  6.30  5.36
##
## === Combined output =========
## [1] "-13.09 [1]***"
```

The test above shows the stationarity of the process with an $\alpha >= 1$, (indipendetemente dalla specificazione) Thus, the order of integration of the *rea* is the second one, because the series is an $I(1)$.

Point 3

We select the best arma model setting the iper-paramenters (p,q), using the BIC criteria:

The autcorrelation function of the residuals it is not statistically different from 0, it looks like white noise. So the arma model adopted is one the fit perfectly the time series:

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \epsilon_t$$

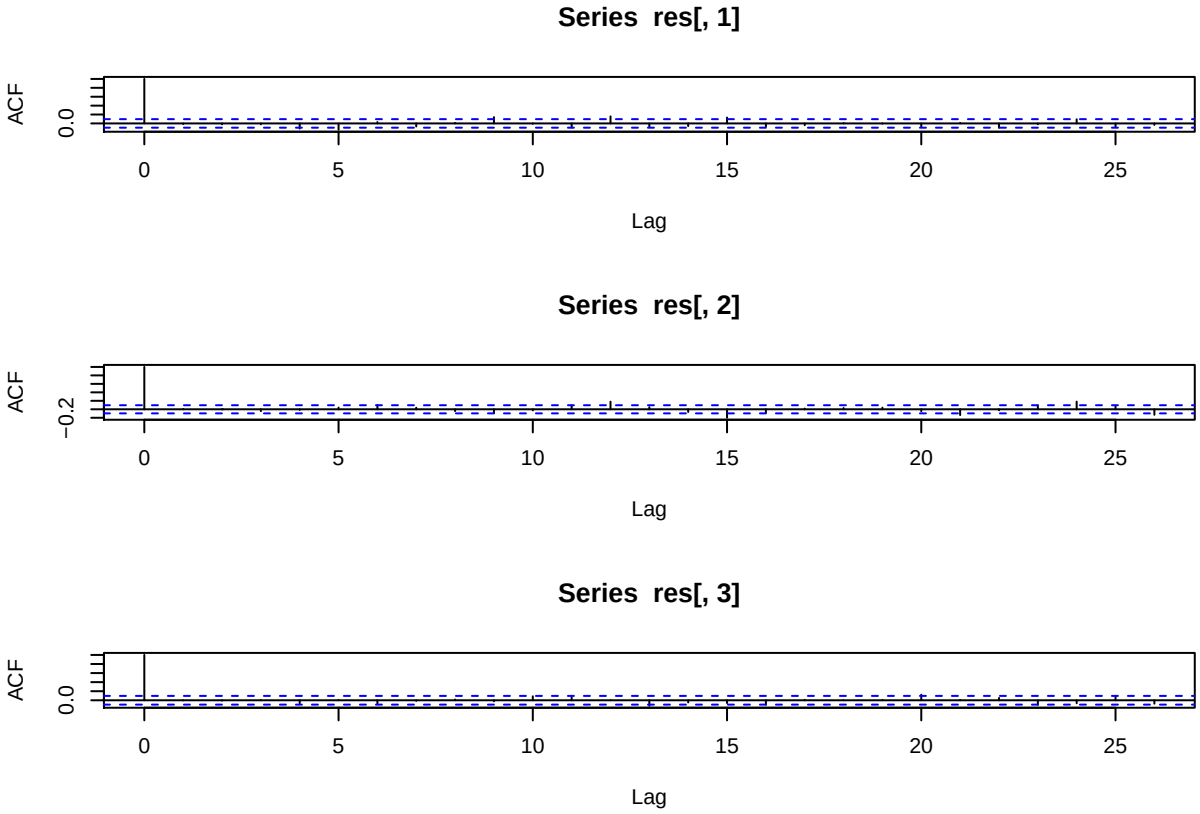
The issue regarding this model is an overfitting one, since all the point in the timeseries has been used to fit the model, as opposite to the usual practice. But the aim of this model is not to provide a prediction for the series, but instead the understading of the process in the specific time span of the series.

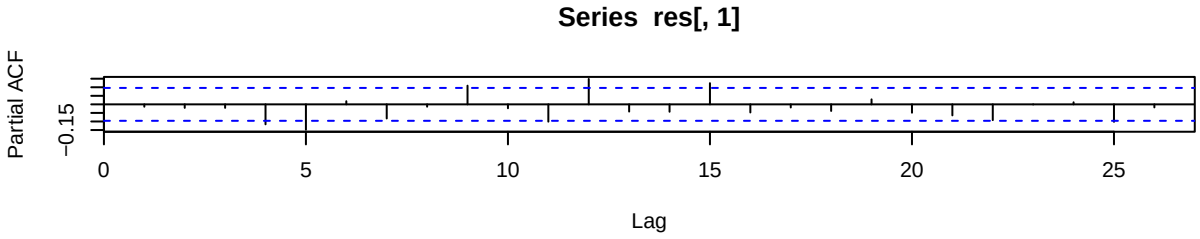
Point 4

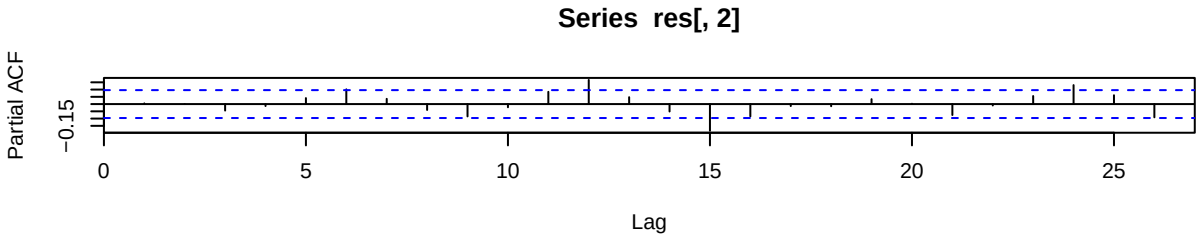
```
## Augmented Dickey-Fuller Test
## alternative: stationary
```

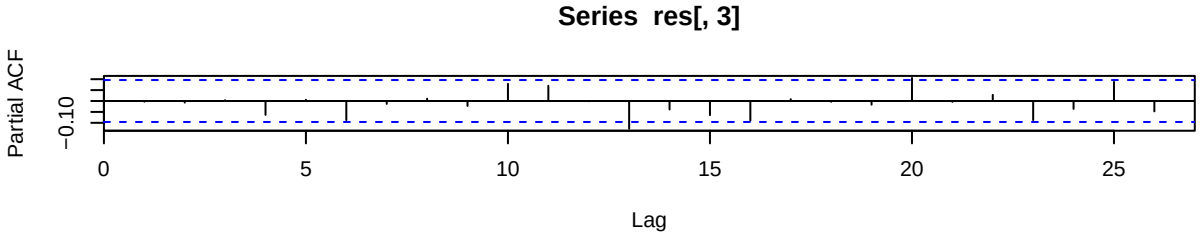
```
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,]   0 22.3    0.99
## [2,]   1 32.7    0.99
## [3,]   2 41.8    0.99
## [4,]   3 50.3    0.99
## [5,]   4 59.7    0.99
## [6,]   5 66.9    0.99
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 22.3    0.99
## [2,]   1 32.7    0.99
## [3,]   2 41.9    0.99
## [4,]   3 50.5    0.99
## [5,]   4 60.1    0.99
## [6,]   5 67.4    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 22.3    0.99
## [2,]   1 32.7    0.99
## [3,]   2 41.9    0.99
## [4,]   3 50.5    0.99
## [5,]   4 60.1    0.99
## [6,]   5 67.5    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,]   0 2.47   0.990
## [2,]   1 1.42   0.960
## [3,]   2 1.60   0.973
## [4,]   3 1.63   0.975
## [5,]   4 1.56   0.970
## [6,]   5 1.35   0.955
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 2.46    0.99
## [2,]   1 1.41    0.99
## [3,]   2 1.60    0.99
## [4,]   3 1.63    0.99
## [5,]   4 1.56    0.99
## [6,]   5 1.35    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 2.47    0.99
## [2,]   1 1.42    0.99
## [3,]   2 1.61    0.99
## [4,]   3 1.64    0.99
## [5,]   4 1.57    0.99
```

```
## [6,]   5 1.36    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag   ADF p.value
## [1,]   0 1.852   0.984
## [2,]   1 0.579   0.811
## [3,]   2 0.886   0.899
## [4,]   3 0.933   0.906
## [5,]   4 1.072   0.923
## [6,]   5 1.081   0.924
## Type 2: with drift no trend
##      lag   ADF p.value
## [1,]   0 1.847   0.990
## [2,]   1 0.579   0.989
## [3,]   2 0.886   0.990
## [4,]   3 0.933   0.990
## [5,]   4 1.071   0.990
## [6,]   5 1.081   0.990
## Type 3: with drift and trend
##      lag   ADF p.value
## [1,]   0 2.137    0.99
## [2,]   1 0.714    0.99
## [3,]   2 1.070    0.99
## [4,]   3 1.145    0.99
## [5,]   4 1.313    0.99
## [6,]   5 1.332    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

## AIC(n)
##      3
```


Series res[, 1]
ACF
0.0
0
5
10
15
20
25
Lag
Series res[, 2]
ACF
−0.2
0
5
10
15
20
25
Lag
Series res[, 3]
ACF
0.0
0
5
10
15
20
25
Lag



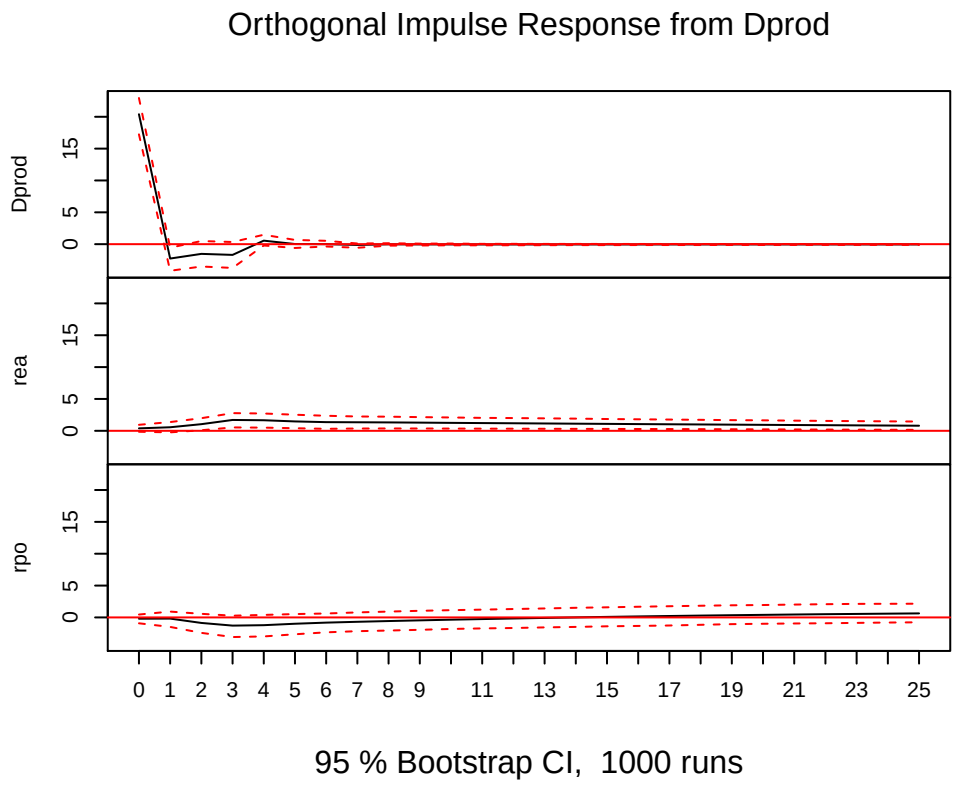


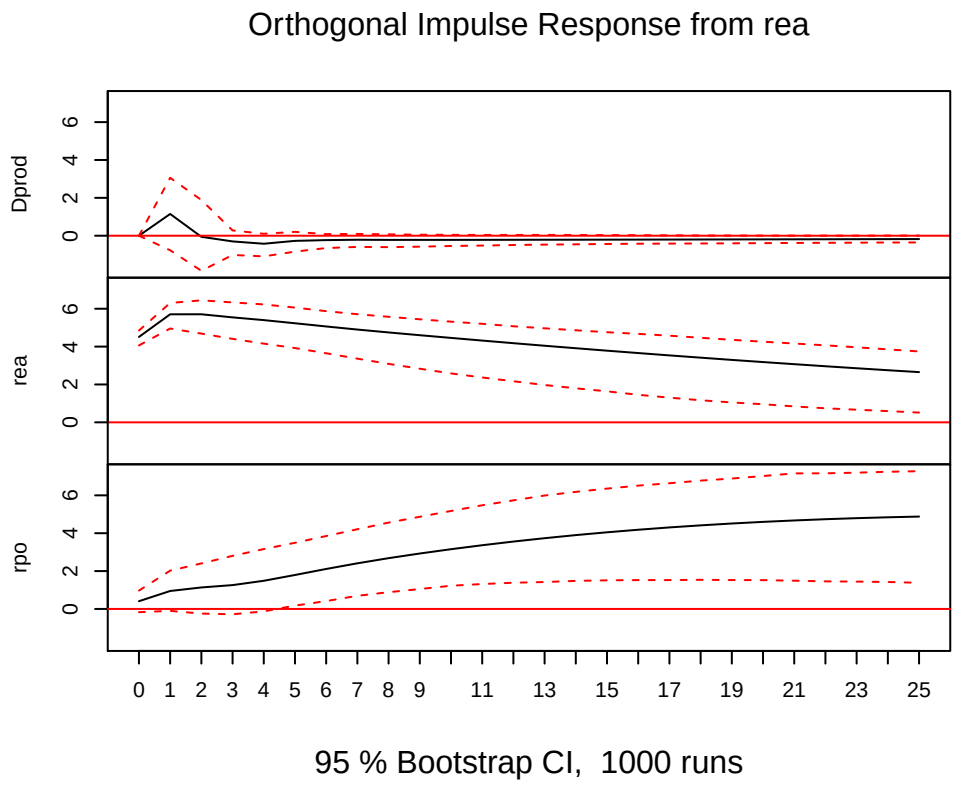


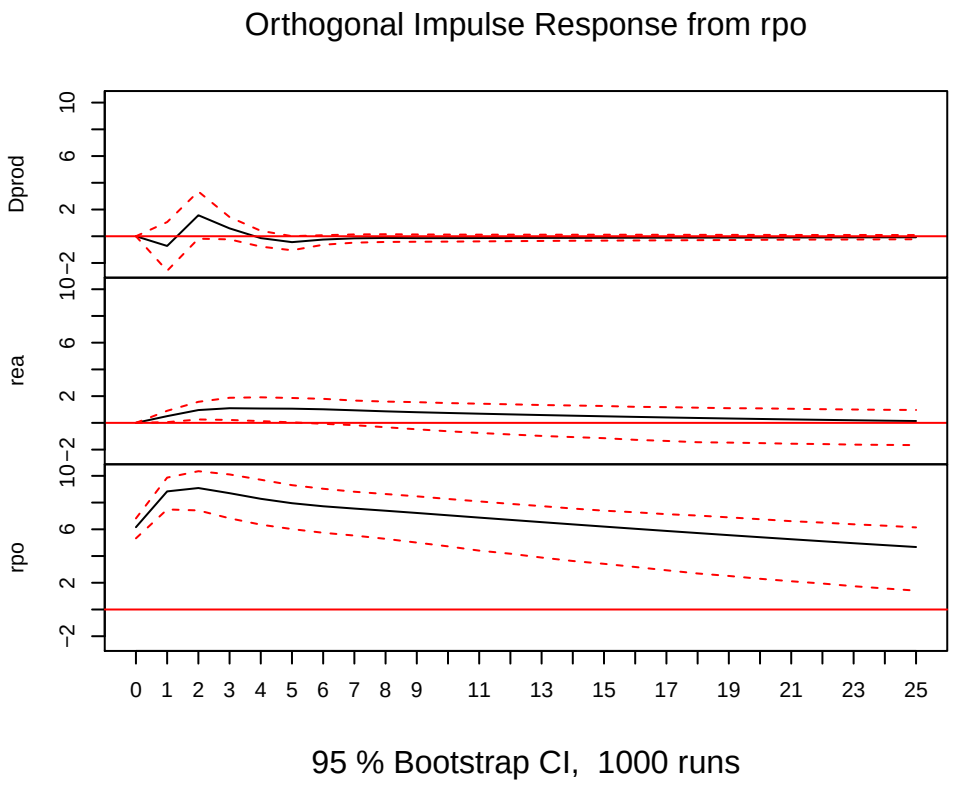

```
##            Dprod       rea       rpo
## Dprod 416.145308  7.824951 -4.099590
## rea     7.824951 20.483391  1.765876
## rpo    -4.099590  1.765876 38.132342
```

```
##             Dprod        rea         rpo
## Dprod  1.00000000 0.08475361 -0.03254402
## rea    0.08475361 1.00000000  0.06318480
## rpo   -0.03254402 0.06318480  1.00000000
```

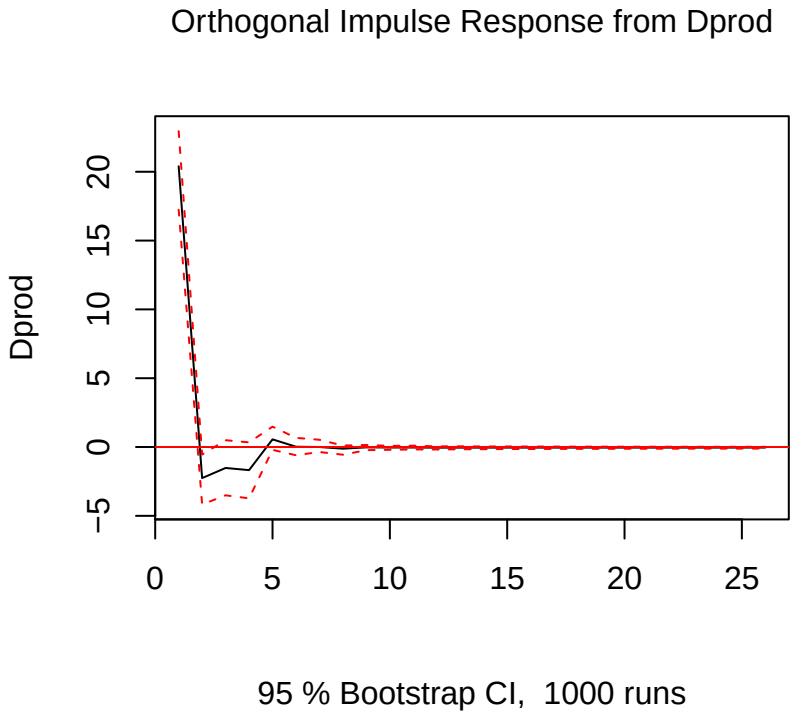
```
## [1] 0.9701644 0.9701644 0.4696721 0.4634054 0.4634054 0.4593787 0.4593787
## [8] 0.2924893 0.2924893
```

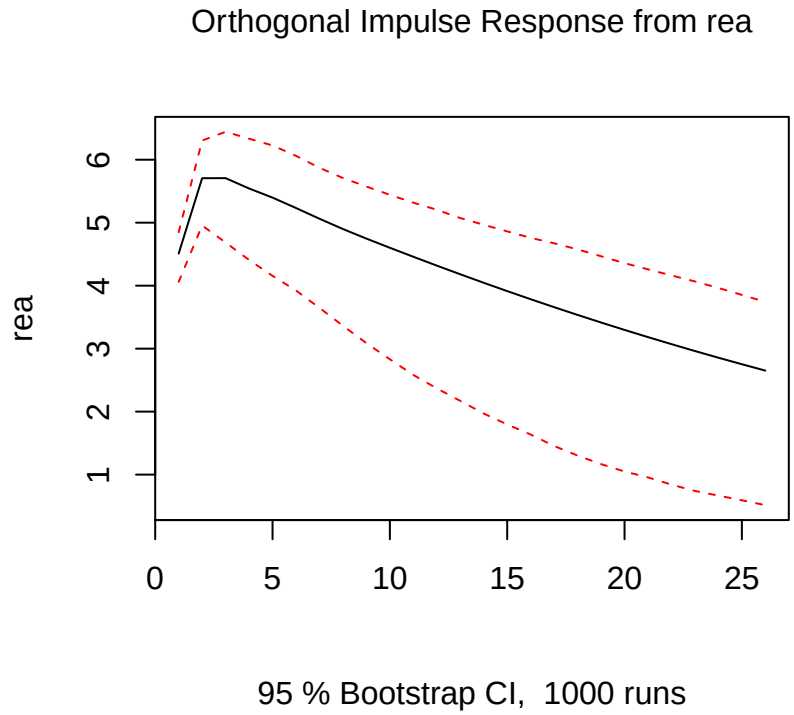

Orthogonal Impulse Response from Dprod
Dprod
rea
rpo
0 5 15
0 1 2 3 4 5 6 7 8 9 11 13 15 17 19 21 23 25
95 % Bootstrap CI, 1000 runs

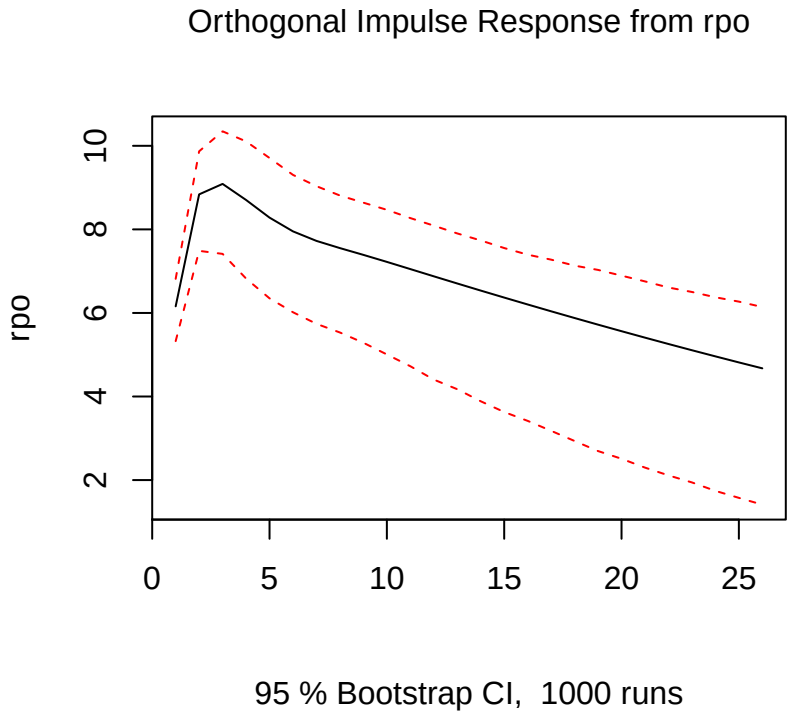

Orthogonal Impulse Response from rea
Dprod
rea
rpo
0 2 4 6
0 1 2 3 4 5 6 7 8 9 11 13 15 17 19 21 23 25
95 % Bootstrap CI, 1000 runs


Orthogonal Impulse Response from rpo
Dprod
rea
rpo
10
6
2
10−2
6
2
10−2
6
2
−2
0 1 2 3 4 5 6 7 8 9 11 13 15 17 19 21 23 25
95 % Bootstrap CI, 1000 runs

Point 5


Orthogonal Impulse Response from Dprod
Dprod
−5
0
5
10
15
20
0
5
10
15
20
25
95 % Bootstrap CI, 1000 runs


Orthogonal Impulse Response from rea
rea
1
2
3
4
5
6
0
5
10
15
20
25
95 % Bootstrap CI, 1000 runs


Orthogonal Impulse Response from rpo
rpo
2
4
6
8
10
0
5
10
15
20
25
95 % Bootstrap CI, 1000 runs