

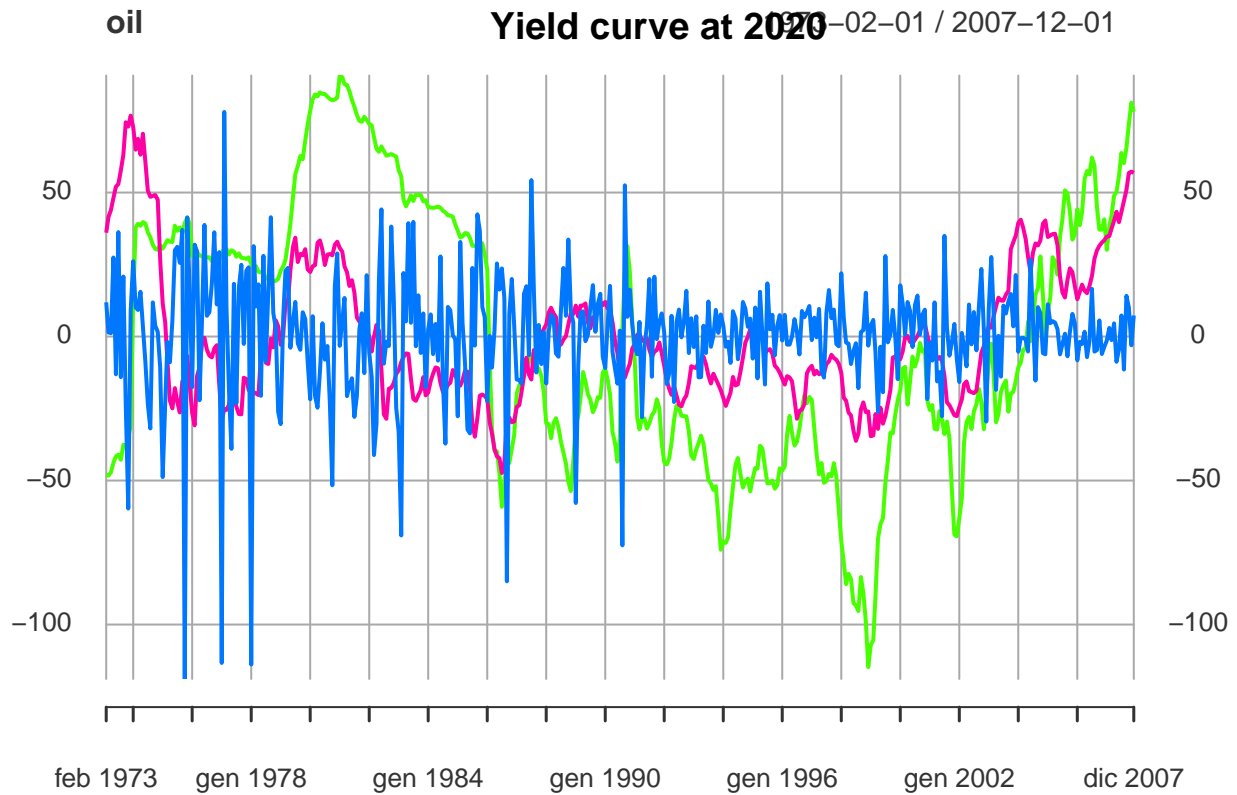
Assignement

Point 1

The time series below represents the monthly time series of:

1. % change in global crude oil production
2. the real price of oil
3. the real economy activity

from 1973:1 to 2007:12.



As we can see from acf its clear signaling the presence of an autocorrelation process.

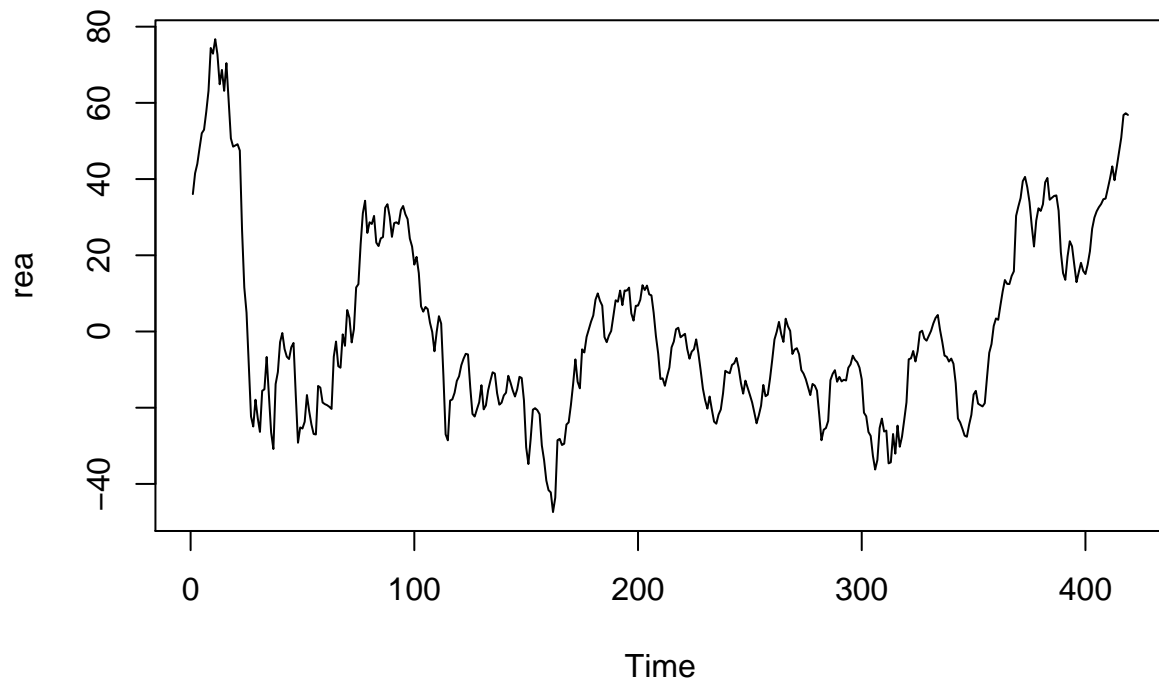
In order to test if the *rea* is an $I(1)$, we will use an ADF test with a minimum lag =1. We will perform the test specifying four different type of the process:

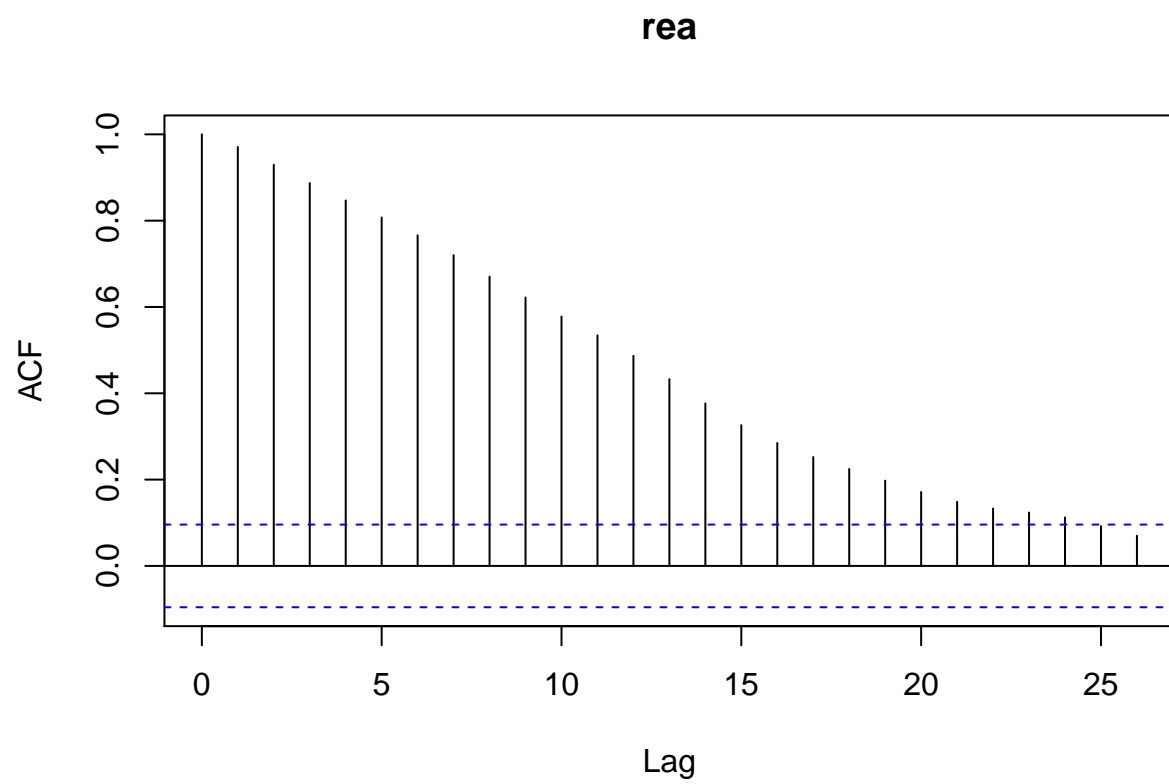
1. No constant, no trend
2. Constant
3. Constant with trend

First, we print the first time series graph. We perform the different types of the test with a maximum lag order of 12:

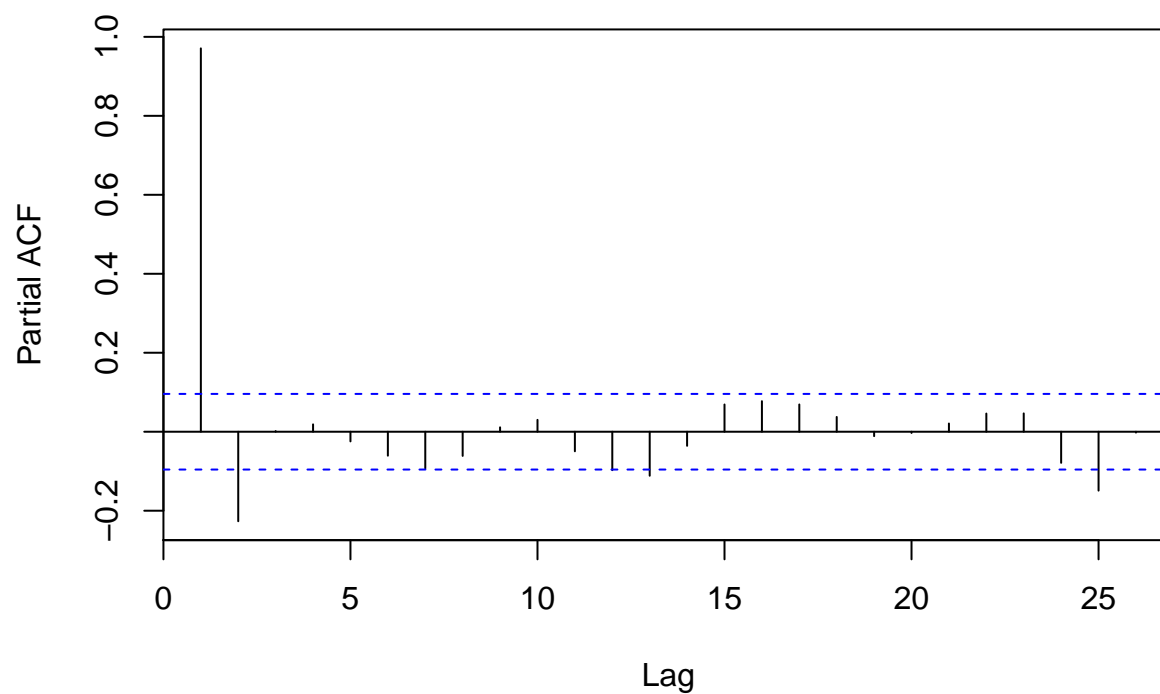
$$rea_t = \alpha + \sigma_1 rea_{t-1} + \dots + \sigma_{12} rea_{t-12}$$

The criteria for selection of the lag order is the one which has lower BIC:





Series timeseries



```
## [1] "Without constant and without time trend"
```

```
##
## === Test statistics =====
##          tau1
## statistic -3.056092
##
## === Test critical values ====
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
##
## === Combined output =====
## [1] "-3.06 [1]***"
```

```
## [1] "With constant and without time trend"
```

```
##
## === Test statistics =====
##          tau2      phi1
## statistic -3.064165 4.695391
##
## === Test critical values ====
##      1pct  5pct 10pct
## tau2 -3.44 -2.87 -2.57
## phi1  6.47  4.61  3.79
```

```
##
## === Combined output =====
## [1] "-3.06 [1]**"

## [1] "With constant and with time trend"

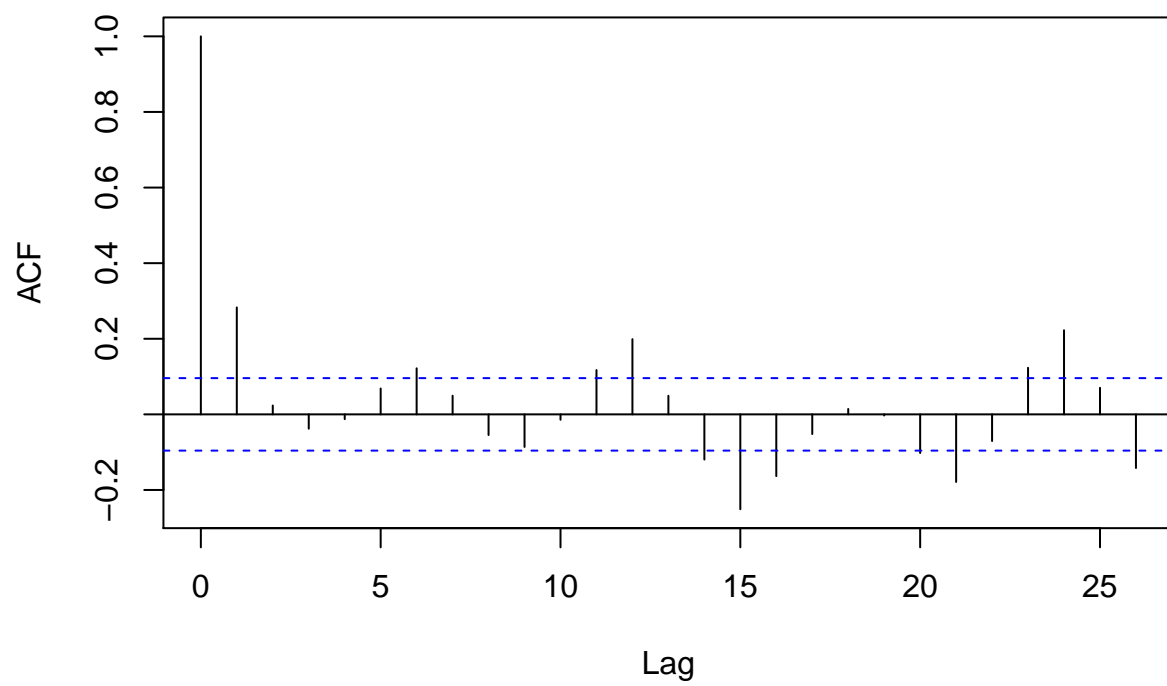
##
## === Test statistics =====
##          tau3      phi2      phi3
## statistic -3.283632 4.530211 6.794472
##
## === Test critical values ====
##      1pct  5pct 10pct
## tau3 -3.98 -3.42 -3.13
## phi2  6.15  4.71  4.05
## phi3  8.34  6.30  5.36
##
## === Combined output =====
## [1] "-3.28 [1]*"
```

The results of the ADF tests show the process is not stationary with a $\alpha < 10$, so there are no sufficient empirical evidence to reject the null. Thus the *rea* time series is not a covariance-stationary process with a minimum lag of order 1.

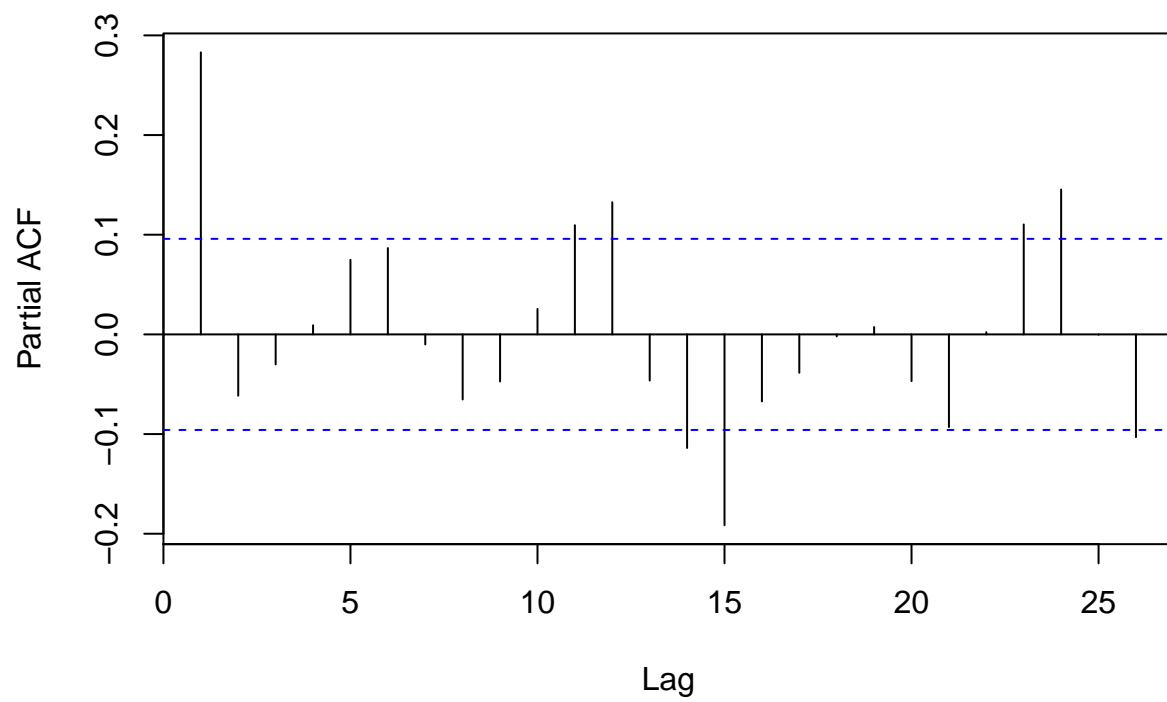
Point 2

We take the first difference of the timeseries *rea* and check if it is stationary with an adf test. Before that we print the time series of the first differences, the acf, and the pacf to understand the correct specification for the ADF test.

Series timeseries

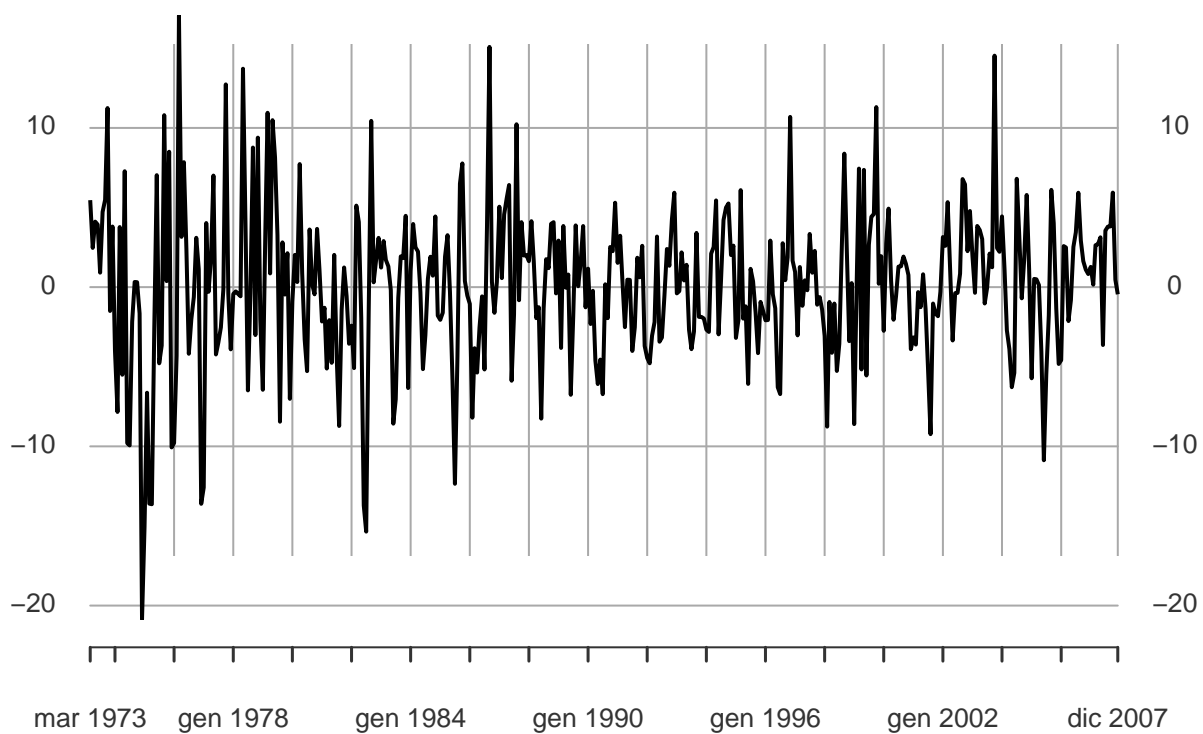


Series timeseries

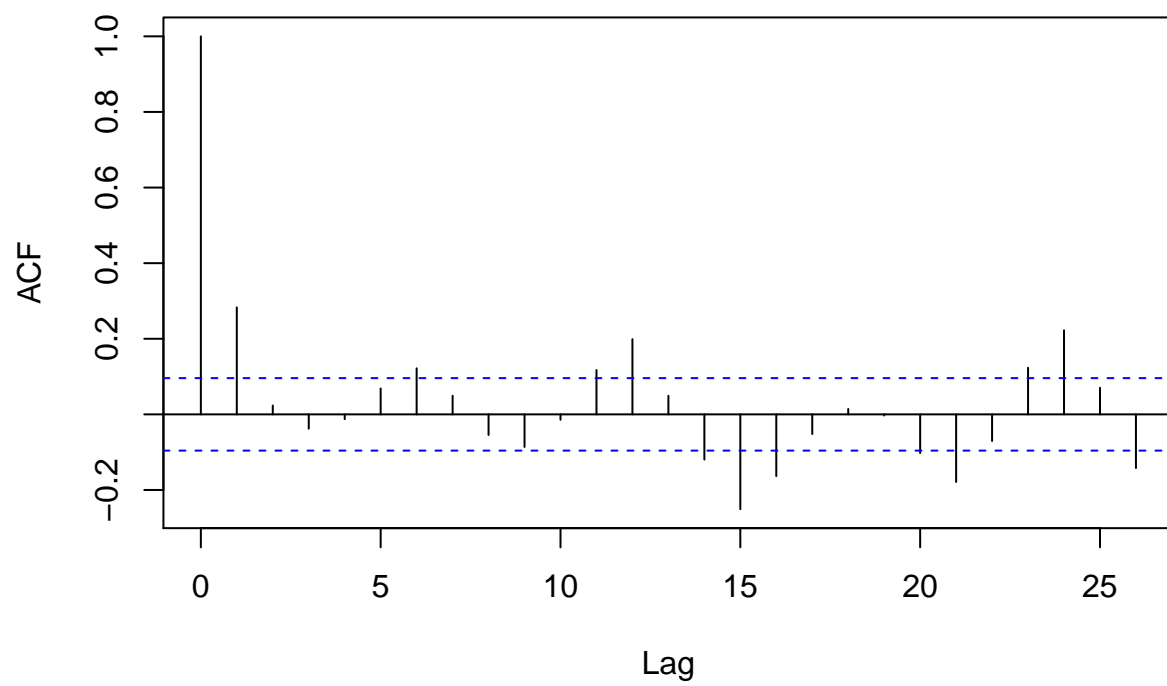


timeseries

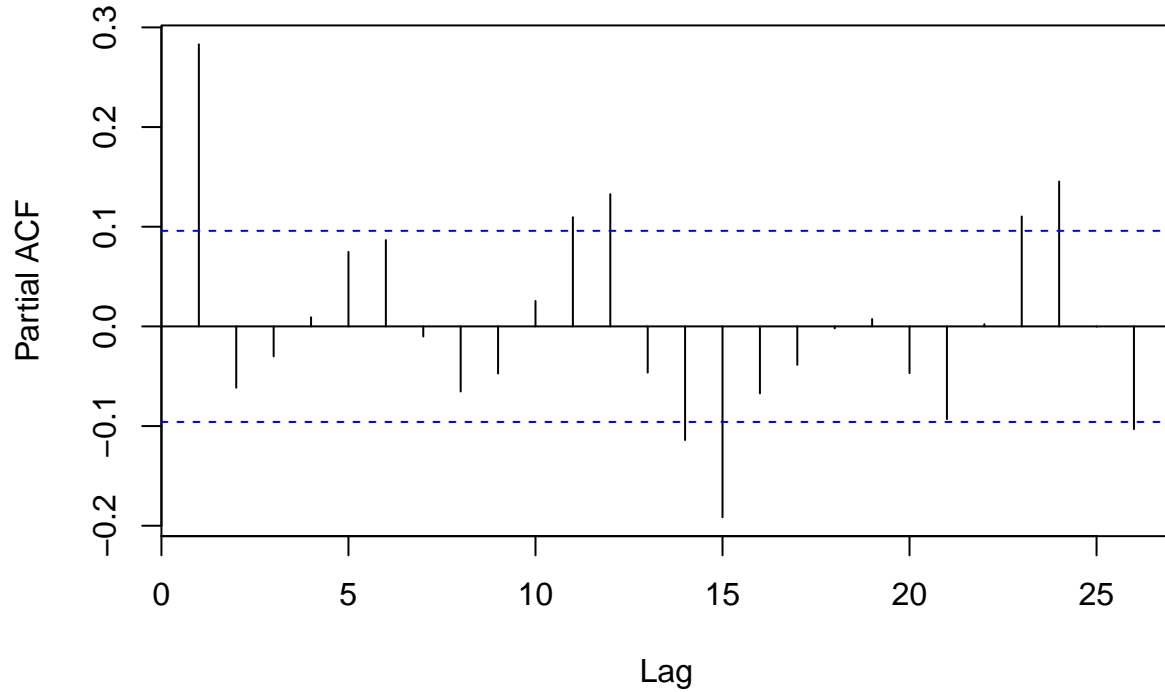
1973-03-01 / 2007-12-01



Series timeseries



Series timeseries



The above graphs clearly underline the stationarity of the process, indeed the acf for the $lag > 2$ the partial autocorrelation is not statistically different from 0. As for the partial autocorrelation that is statistically different only for some $lag > 10$. From the plot of the time series we can see a mean reverting process, and so I will opt for the specifications with constant and time trend, because it is less restrictive. So the test will have the following specifications:

$$\delta rea_t = \sigma_1 \delta rea_{t-1} + \dots + \sigma_{12} \delta rea_{t-12}$$

$$\delta rea_t = \alpha + \sigma_1 \delta rea_{t-1} + \dots + \sigma_{12} \delta rea_{t-12}$$

$$\delta rea_t = \alpha + \beta * t + \sigma_1 \delta rea_{t-1} + \dots + \sigma_{12} \delta rea_{t-12}$$

The test will be performed with all possible four specifications, and will be selected the specification with lower adf value.

```
## [1] "Without constant and without time trend"
```

```
##
## === Test statistics =====
##          tau1
## statistic -12.92825
##
## === Test critical values ===
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
##
## === Combined output =====
## [1] "-12.93 [1]***"
```

```
## [1] "With constant and without time trend"
```

```
##  
## === Test statistics =====  
##           tau2      phi1  
## statistic -12.91292 83.37187  
##  
## === Test critical values ====  
##      1pct  5pct 10pct  
## tau2 -3.44 -2.87 -2.57  
## phi1  6.47  4.61  3.79  
##  
## === Combined output =====  
## [1] "-12.91 [1]***"
```

```
## [1] "With constant and with time trend"
```

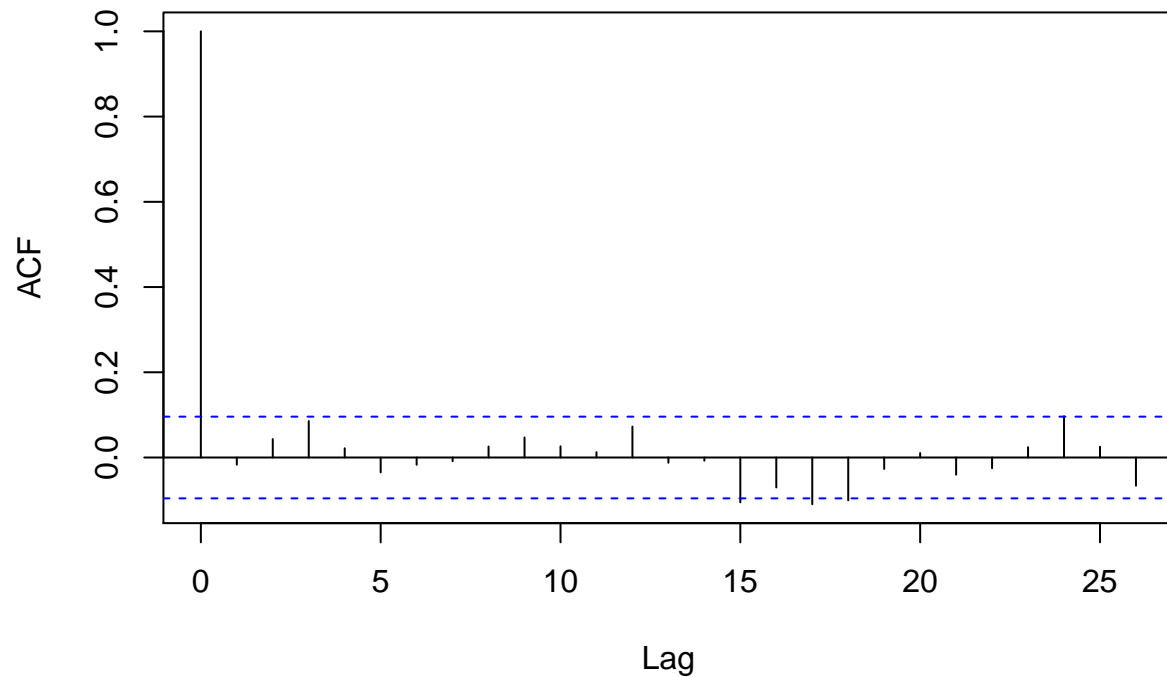
```
##  
## === Test statistics =====  
##           tau3      phi2      phi3  
## statistic -13.09473 57.15755 85.73616  
##  
## === Test critical values ====  
##      1pct  5pct 10pct  
## tau3 -3.98 -3.42 -3.13  
## phi2  6.15  4.71  4.05  
## phi3  8.34  6.30  5.36  
##  
## === Combined output =====  
## [1] "-13.09 [1]***"
```

The test above shows the stationarity of the process with an $\alpha \geq 1$, (indipendetemente dalla specificazione)
Thus, the order of integration of the *rea* is the second one, because the series is an $I(1)$.

Point 3

We select the best arma model setting the iper-paramenters (p,q), using the BIC criteria:

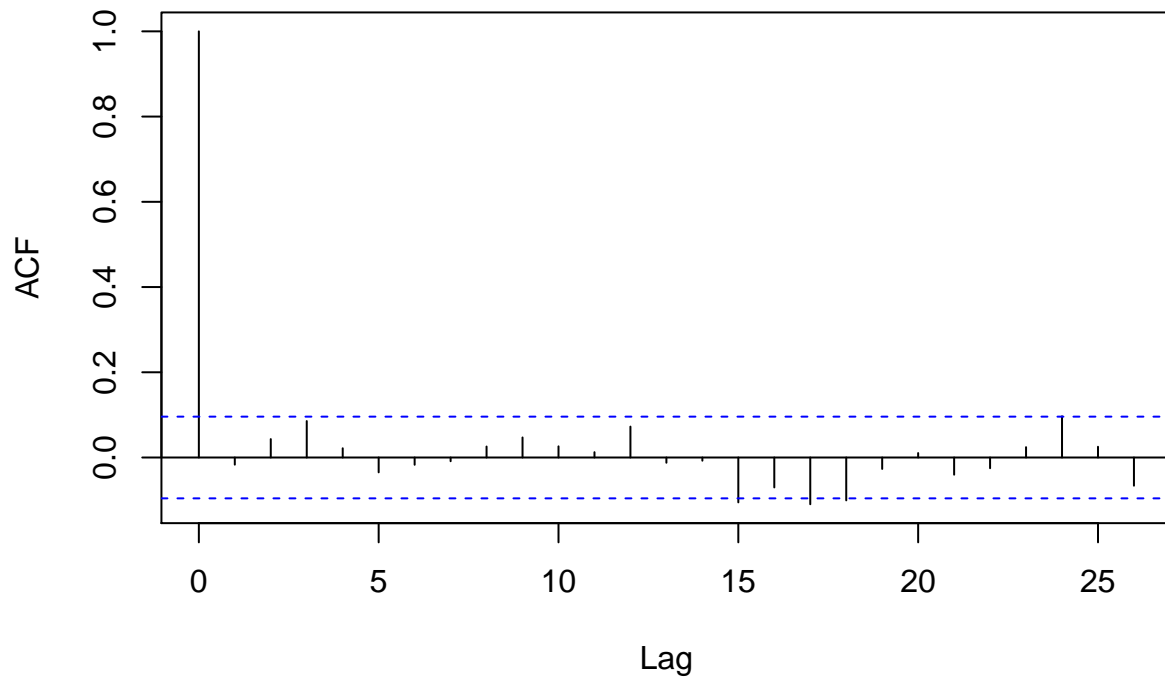
Series out\$residuals



```
## p    q
## 3 0 2

##
## Call:
## arima(x = timeseries, order = best_arima, method = "ML")
##
## Coefficients:
##          ar1      ar2      ar3      ma1      ma2  intercept
##          1.2744 -1.2749  0.2778 -1.0257  0.9999      0.0467
## s.e.    0.0481   0.0476  0.0479   0.0143  0.0206      0.2841
##
## sigma^2 estimated as 18.6:  log likelihood = -1207.93,  aic = 2429.85
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 0.004607344 4.312658 3.282594 65.46319 204.704 0.7697161 -0.016815
```

Series arma\$residuals



The autocorrelation function of the residuals it is not statistically different from 0, it looks like white noise. So the arma model adopted is one the fit perfectly the time series:

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \epsilon_t$$

The issue regarding this model is an overfitting one, since all the point in the timeseries has been used to fit the model, as opposite to the usual practice. But the aim of this model is not to provide a prediction for the series, but instead the understading of the process in the specific time span of the series.

Point 4

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,]  0 22.3   0.99
## [2,]  1 32.7   0.99
## [3,]  2 41.8   0.99
## [4,]  3 50.3   0.99
## [5,]  4 59.7   0.99
## [6,]  5 66.9   0.99
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]  0 22.3   0.99
```

```

## [2,] 1 32.7 0.99
## [3,] 2 41.9 0.99
## [4,] 3 50.5 0.99
## [5,] 4 60.1 0.99
## [6,] 5 67.4 0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,] 0 22.3 0.99
## [2,] 1 32.7 0.99
## [3,] 2 41.9 0.99
## [4,] 3 50.5 0.99
## [5,] 4 60.1 0.99
## [6,] 5 67.5 0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

```

```

## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,] 0 2.47 0.990
## [2,] 1 1.42 0.960
## [3,] 2 1.60 0.973
## [4,] 3 1.63 0.975
## [5,] 4 1.56 0.970
## [6,] 5 1.35 0.955
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,] 0 2.46 0.99
## [2,] 1 1.41 0.99
## [3,] 2 1.60 0.99
## [4,] 3 1.63 0.99
## [5,] 4 1.56 0.99
## [6,] 5 1.35 0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,] 0 2.47 0.99
## [2,] 1 1.42 0.99
## [3,] 2 1.61 0.99
## [4,] 3 1.64 0.99
## [5,] 4 1.57 0.99
## [6,] 5 1.36 0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

```

```

## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,] 0 1.852 0.984
## [2,] 1 0.579 0.811
## [3,] 2 0.886 0.899

```

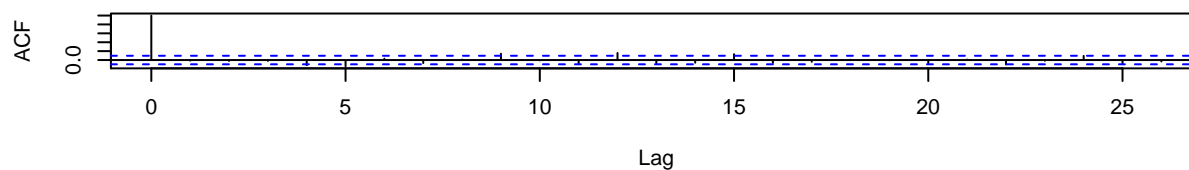
```

## [4,] 3 0.933 0.906
## [5,] 4 1.072 0.923
## [6,] 5 1.081 0.924
## Type 2: with drift no trend
## lag ADF p.value
## [1,] 0 1.847 0.990
## [2,] 1 0.579 0.989
## [3,] 2 0.886 0.990
## [4,] 3 0.933 0.990
## [5,] 4 1.071 0.990
## [6,] 5 1.081 0.990
## Type 3: with drift and trend
## lag ADF p.value
## [1,] 0 2.137 0.99
## [2,] 1 0.714 0.99
## [3,] 2 1.070 0.99
## [4,] 3 1.145 0.99
## [5,] 4 1.313 0.99
## [6,] 5 1.332 0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01

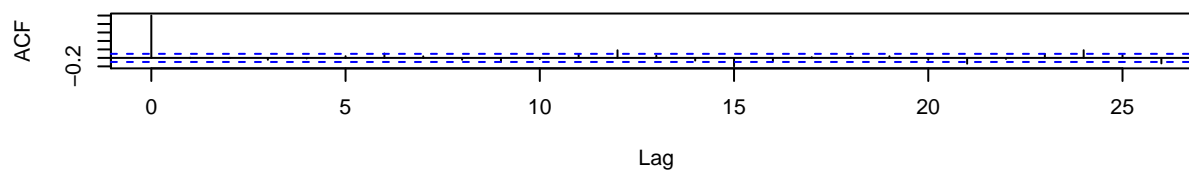
## AIC(n)
## 3

```

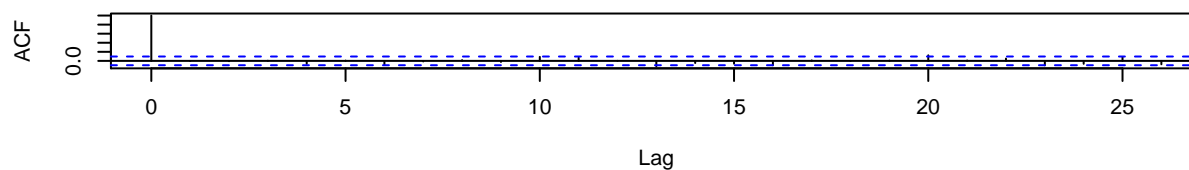
Series res[, 1]

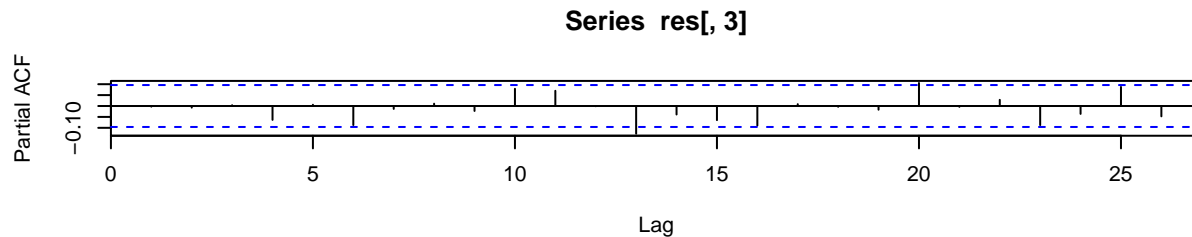
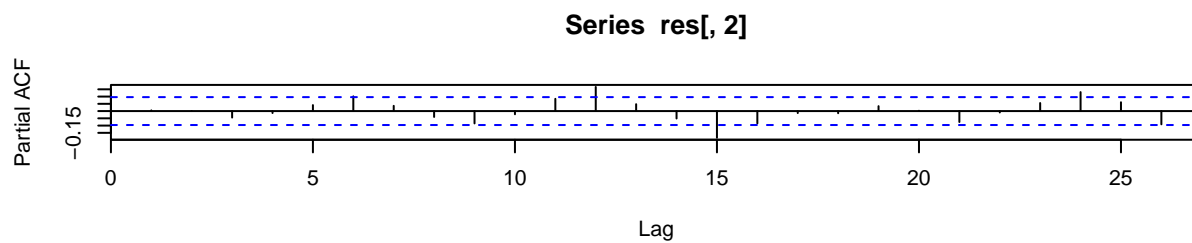
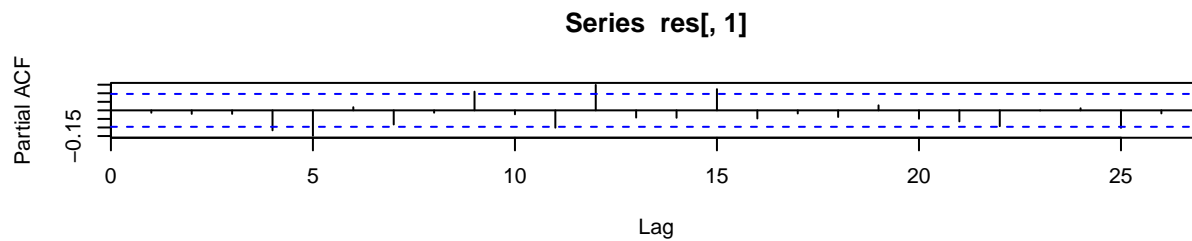


Series res[, 2]



Series res[, 3]



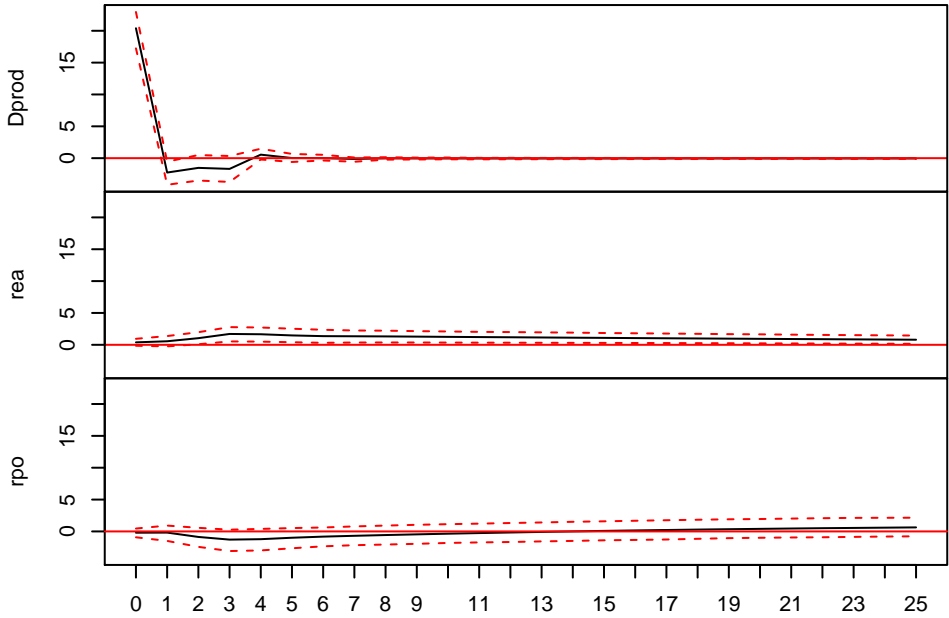


```
##           Dprod      rea      rpo
## Dprod 416.145308  7.824951 -4.099590
## rea    7.824951 20.483391  1.765876
## rpo   -4.099590  1.765876 38.132342
```

```
##           Dprod      rea      rpo
## Dprod  1.00000000 0.08475361 -0.03254402
## rea    0.08475361 1.00000000  0.06318480
## rpo   -0.03254402 0.06318480  1.00000000
```

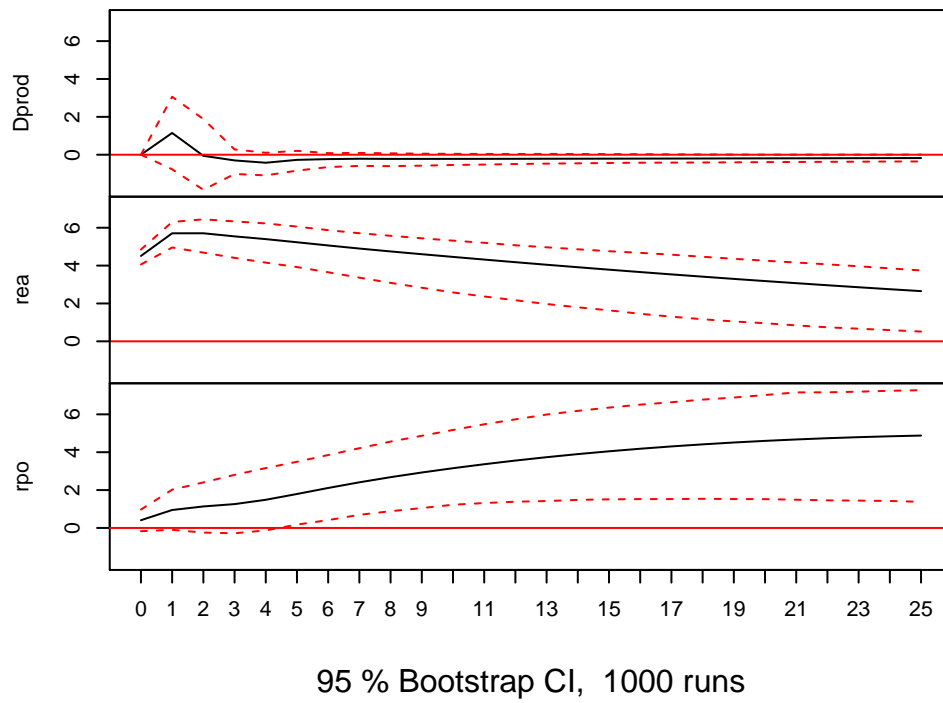
```
## [1] 0.9701644 0.9701644 0.4696721 0.4634054 0.4634054 0.4593787 0.4593787
## [8] 0.2924893 0.2924893
```


Orthogonal Impulse Response from Dprod

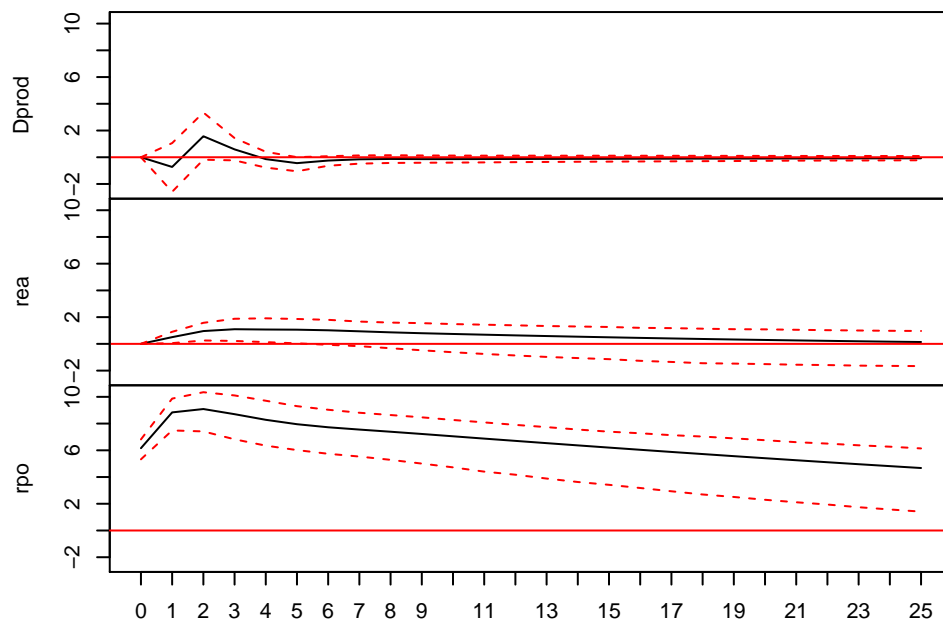


95 % Bootstrap CI, 1000 runs

Orthogonal Impulse Response from rea



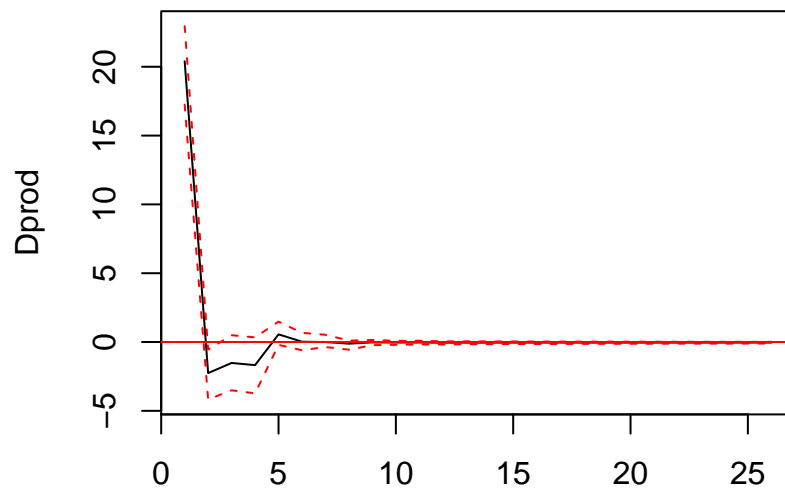
Orthogonal Impulse Response from rpo



95 % Bootstrap CI, 1000 runs

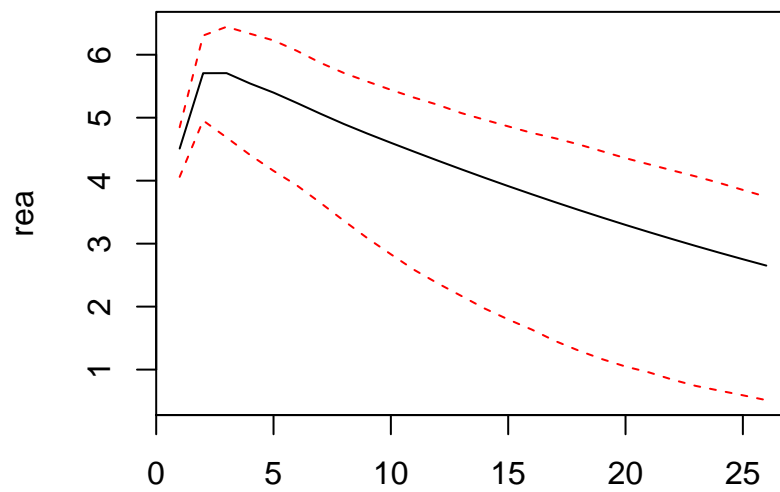
Point 5

Orthogonal Impulse Response from Dprod



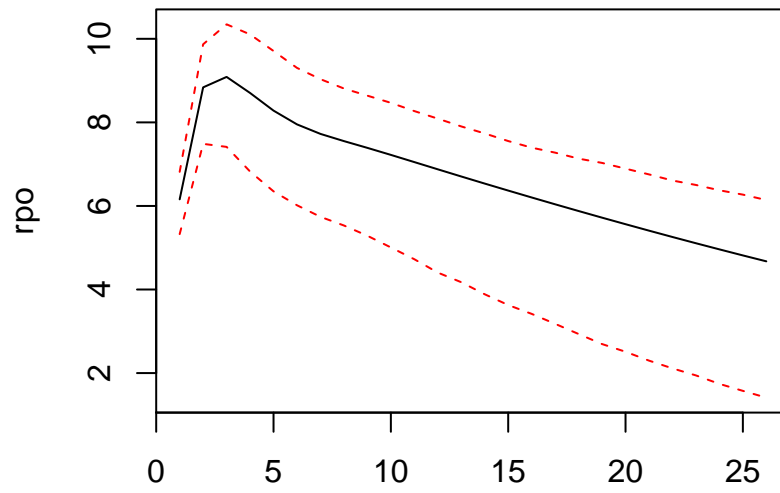
95 % Bootstrap CI, 1000 runs

Orthogonal Impulse Response from rea



95 % Bootstrap CI, 1000 runs

Orthogonal Impulse Response from rpo



95 % Bootstrap CI, 1000 runs