Theoretical framework

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Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

1 Theoretical framework

The economy consists of firms that are risk-neutral and have a constant discount rate represented by the parameter $0 < \beta < 1$. These firms exhibit heterogeneity in terms of their productivity and net worth. They utilize a production technology that takes capital (or production units) as the sole input, with diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as c to initiate production. Following production, firms make decisions regarding the allocation of profits for the next period. The remaining portion of profits is invested in a risk-free asset. Firms face a choice: they can either remain in the market and reinvest their profits or exit the market entirely, investing their entire net worth, denoted as e, in the risk-free asset.

When firms choose to exit the market, they forgo any future profits, but they also free themselves from the financial burden of the fixed cost represented by c. Consequently, firms opt to exit the market when the expected profits fail to outweigh the fixed cost, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

This value, obtained from investing in the risk-free asset, is equal to $q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r)-1] e_{t+s+1}$. Notably, when the condition $\beta(1+r) \leq 1$ holds true, this value simplifies to q. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors.

The agents in this economy are the firms themselves, and they aim to maximize their value over time by selecting an optimal level of capital denoted as k. The production function, after accounting for the fixed cost c, can be expressed as follows: $Y = Z(\theta + \epsilon)k^\alpha$.

- Z: is the stochastic aggregate productivity common across firms
- θ: is a persistent firm-specific productivity shock (model as a Markov Chain)
- ϵ: is the firm-specific productivity shock $\epsilon \sim \mathcal{N}(0, \sigma)$
- k^α: capital or production units as in (Caballero Hammour, AER)

The timeline of the events is the following

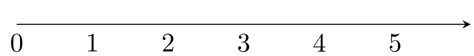

Figure 1: Timeline of Events

1. firm knows Z, θ, k^α, e where e is its endowment[1]
2. firm compute the k need to maximizes the expected value of the firm range can varies from $[0, +\infty]$ if $k = 0$ it means that the firm decide to exit.

[1] which is different from k since the firm can borrow money: $d = c + k - e$ where d stays for debt

3. at the end of the period the firm observe ϵ_t and the aggregate shock

4. repays its debt and the fixed operating cost $c+k-e$, the firm its left with the end of period net worth q

5. decide the amount the dividend to distribute $q-e_{t+1}$ observe the productivity shock θ_{t+1}, Z_{t+1} and the step restart from 1

1.1 Frictionless economy

In a frictionless economy, firms have the option to borrow an amount denoted as $c+k-e$ at the risk-free interest rate $r=\frac{1}{\beta}-1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_k E \int \max[q, \max_{e_{t+1}}(q - e_{t+1} + \beta V_{FL}(e_{t+1}, \theta_{t+1}, Z_{t+1}))]\, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta+\epsilon)k^\alpha + (1-\delta)k - (1+r)(c+k-e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_k E \int [Z(\theta+\epsilon)k^\alpha - (1+r)c\, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta_{t+1}, Z_{t+1})]$$

In the absence of market frictions, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta_{t+1} < \underline{\theta}_{FL}(Z_{t+1})$, where $\underline{\theta}_{FL}(Z_{t+1})$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z_{t+1}) = 0$.