Theoretical framework

Riccardo Dal Cero

October 4, 2023

Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

1 Theoretical framework

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.
In each period, firms incur a fixed production cost denoted as c to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as e, in the risk-free asset.
Firms opt to exit the market when expected profits no longer outweigh the fixed cost c, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset. The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s[\beta(1+r)-1]e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to q. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors.
In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as k. The production function, accounting for the fixed cost c, is expressed as follows: $Y = Z(\theta + \epsilon)k^\alpha$.
Key variables include:

- Z: Stochastic aggregate productivity common across firms.
- θ: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- ϵ: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \sigma)$.
- k^α: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

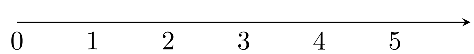

Figure 1: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of Z, θ, k^α, e (where e represents its endowment, different from k since the firm can borrow money with $d = c + k - e$).

2. The firm computes the optimal k to maximize the expected value of the firm, with k ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.

3. At the end of the period, the firm observes ϵ and the aggregate shock.

4. The firm repays its debt and the fixed operating cost $(c + k - e)$, resulting in an end-of-period net worth q.

5. The firm decides on the amount of dividends to distribute $(q - e')$, observes the productivity shock θ', Z', and the process restarts from step 1.

1.1 Frictionless economy

In a frictionless economy, firms have the option to borrow an amount denoted as $c + k - e$ at the risk-free interest rate $r = \frac{1}{\beta} - 1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_k E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - (1 + r)(c + k - e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_k E \int [Z(\theta + \epsilon)k^{\alpha} - (1 + r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market frictions, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

1.2 Economy with Credit Market Frictions

After production, the firm privately observes the temporary shock ϵ, while financial intermediaries can only observe it at a cost of μk^{α}. For one-period debt contracts, financial intermediaries observe ϵ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth e, current productivity θ, and aggregate productivity value Z, all observable by both the financial intermediary and the firm at no additional cost.

HP1 (Hypothesis 1): The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\bar{\epsilon}$ is determined by the equation:

$$Z(\theta + \bar{\epsilon})k^{\alpha} + (1 - \delta)k = (1 + \widetilde{r})(c + k + e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component θ.

The financial intermediary lends $(c + k - e)$ to the firm only if the expected income from the loan equals the opportunity cost of the funds, as expressed by the inequality:

$$(1 + \widetilde{r})(k + c + e)(1 - \Phi(\bar{\epsilon})) + \int_{-\infty}^{\bar{\epsilon}} [Z(\theta + \bar{\epsilon})k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}] \, d\Phi(\epsilon) \geq (1 + r)(c + k + e)$$

The financial contract is characterized by $(k, \bar{\epsilon})$. Given Z, θ, e, the participation constraint indicates the default threshold $\bar{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied.

In fact, given θ, Z, there is a unique threshold $e_b(\theta, Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta + G(\overline{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\overline{\epsilon}_b) = (1+r)(k_b + c - \underline{e_b})$$

where $\overline{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta + \overline{\epsilon})k^\alpha + (1-\delta)k - (1+\widetilde{r})(k + c - e) & \text{if } \epsilon \geq \overline{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^\alpha(\epsilon - \overline{\epsilon}); 0]$$

1.3 The firm's problem

Define V as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e, \theta, Z) = \max_{(k, \overline{\epsilon})} E\left\{\int I(q)q + (1 - I(q)) \max[q, \max_{e'} q - e' + \beta V(e', \theta', \zeta')]\, d\Phi(\epsilon)\right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1. $$Z[\theta + G(\overline{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\overline{\epsilon}_b) \geq (1+r)(k_b + c - \underline{e_b})$$

2. $$q = \max[Zk^\alpha(\epsilon - \overline{\epsilon}); 0]$$

3. $$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.