Assignemnt 2: Term structure and expectations

Exercise 1:

The exogenous endowment shock is a two states Markov process, defined as:

$$\frac{\Omega_t}{\Omega_{t-1}} = \omega_t \quad with\ \omega_t \epsilon (h, l)$$

With the following symmetric transition matrix:

$$\Pi = \begin{bmatrix} \phi & (1-\phi) \\ (1-\phi) & \phi \end{bmatrix}$$

Let $\beta = 0.96$, $\gamma = 2$, $h = 1.05$, $l = 0.97$ and $\phi = 0.3$.

Knowing this, we cannot use the same 4-cases framework The probability of finding ourselves in state h or state l now depends on the state of the economy in the previous period.

Thus, we will have two different yield curves, conditional on the state of the Markov process we are starting with at time t.

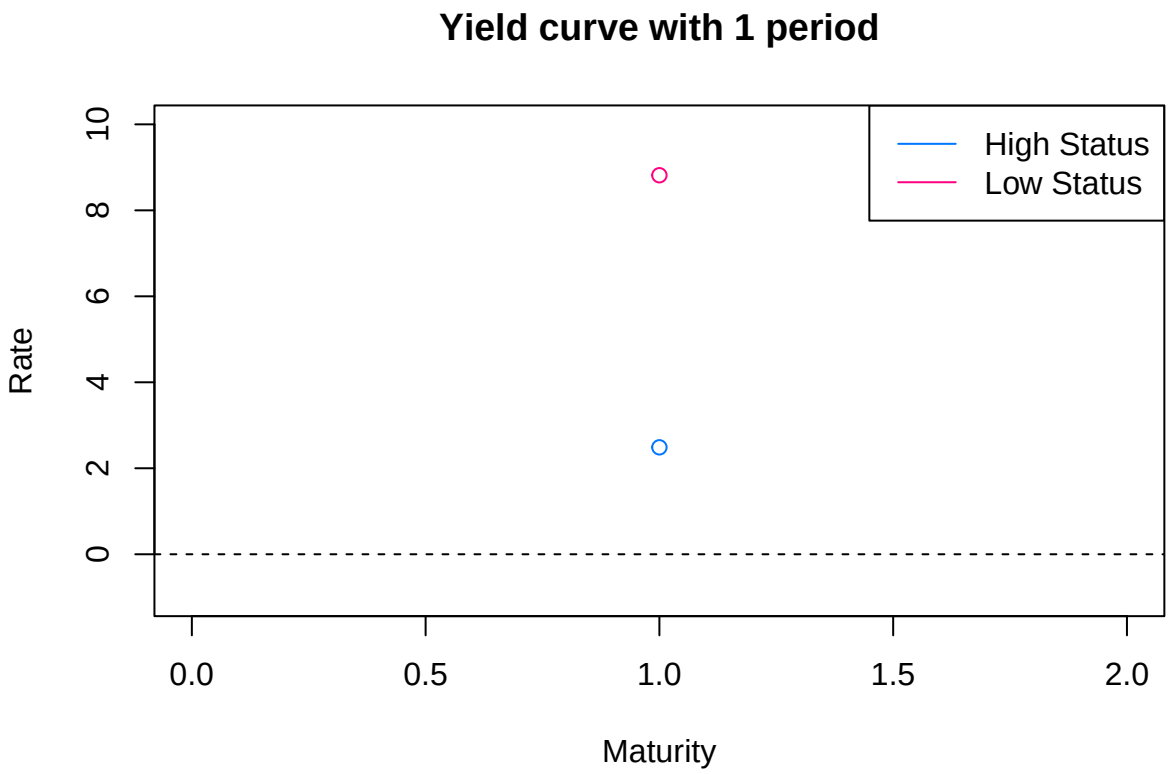
We will generate a function that takes as inputs the two status of the markow chain $(high, low)$, the transition matrix Π and the number of step (n) to itereate the Markov process so the algorithm is scalable. The algorithm follow these steps:

1. iteration count <- 1
2. $$States = \Pi * \begin{bmatrix} high \\ low \end{bmatrix}$$
3. $$yieds = -\ln(\beta) - \frac{1}{count} \ln(States)$$
4. adding yields to output
5. $$\Pi = \Pi * \Pi$$
6. $$state = state * state^T$$
7. restart at point 1 until $count = n$

Iteration for 1 period

We wanted also to have a measure of the short-term interest rate in period t. In the real world, this is the overnight interest rate, set by the Central Bank. Let's look now at the output:

	t=1
h	0.024872482618492
l	0.088160702553818

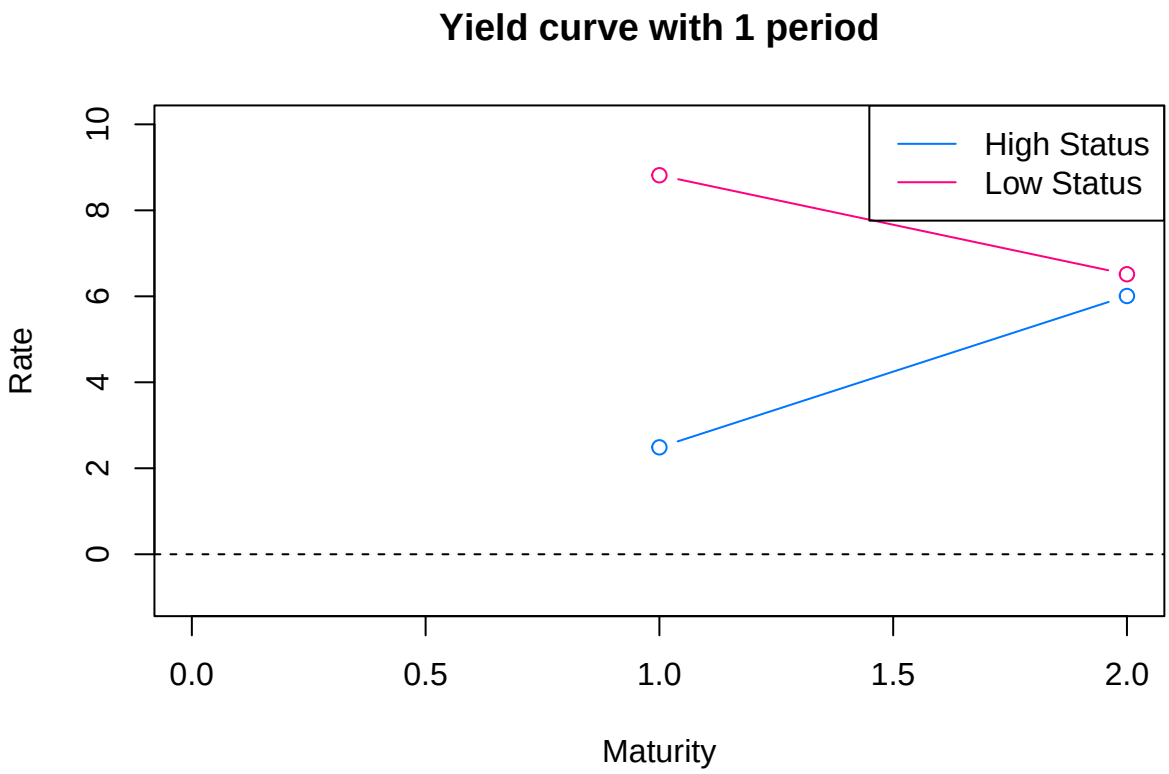


At time 1 the economy can be at state high or a state low, if it is in high status the yield curve starts at 2.49% otherwise the economy is in low state then the yield curve starts at 8.88%.

We observe that even if the curve predicts a recovery in the long term, does not return to the first-period yield level. We believe this is because in the starting period the state h is certain, while in the second it is only slightly more likely than a recession, thus the yield is lower.

Introducing two time (n = 2)

	t=1	t=2
h	0.024872482618492	0.060070817604076
l	0.088160702553818	0.065132229132245

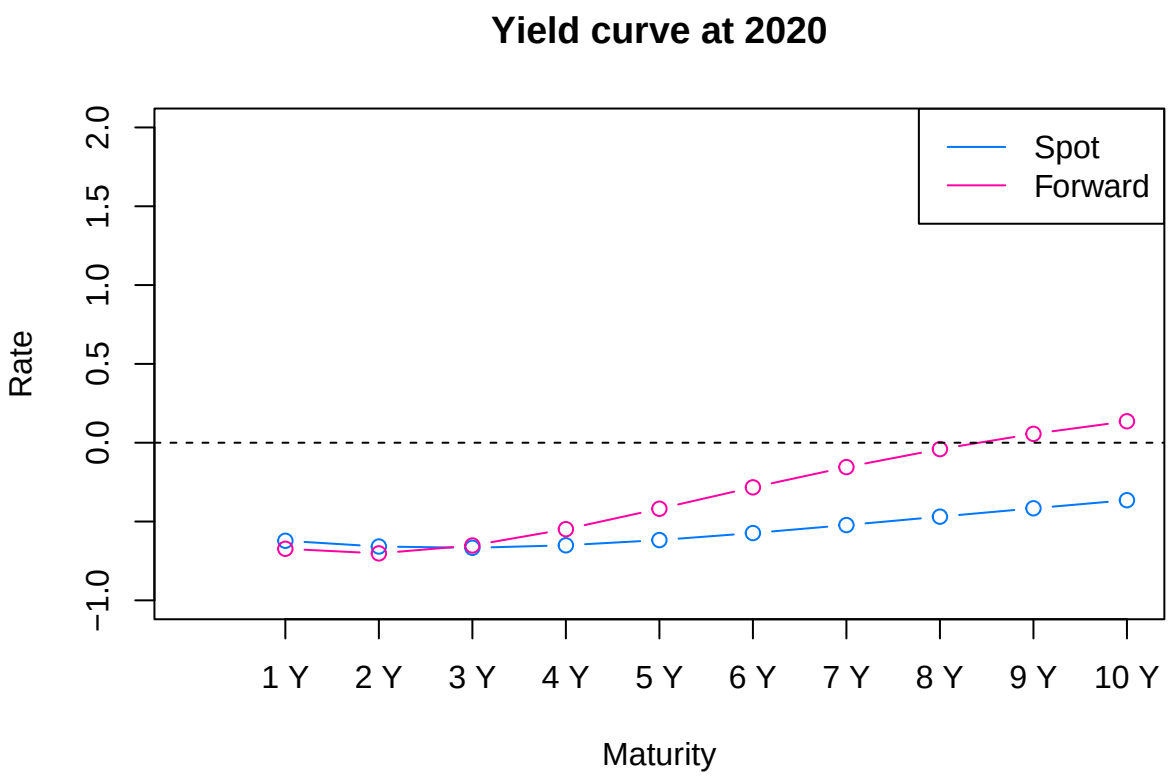


In this case, the yield curve starts as in the previous case at time t=1, in which the economy can be in a good state or in a bad state as mentioned above. Conditioning to state of the economy at t= 1 and itereting the markov process to generate new transition probability ($n = 2$), we get the yields curve at time t=2.

According to the red line rappresting the low state of the economy, the slope is negative indicating a recession. While the high status yield it has a positive slope coeficient. Both yield converges around 6% this is probably due to the convergence of the transition probability as the number of iteration becames greater.

Exercise 2

Plot yield curves 2020



The plot above represents the spot and forward yield curve in July 2020. The spot yield is entirely in the negative area of interest rate due to the APP of the ECB. Even the forward curve is negative until the 8Y maturity. Indeed, In addition, the ECB enlarged the APP program to face the pandemic in the EU countries, lowering the yield curve in the short maturity.

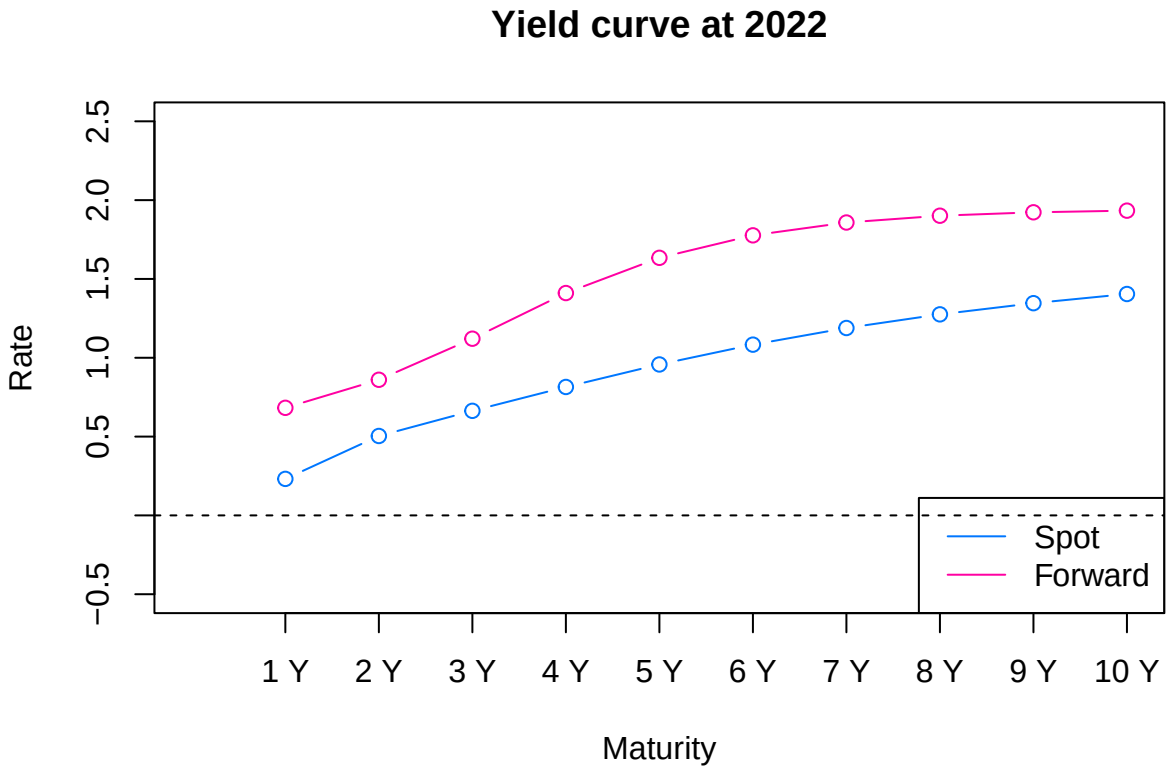
The curves are upward-sloping after 3Y, and the forward rate premium becomes higher as maturity increases. Suggesting that agents had been expecting the recovery of consumption at the pre-covid level, a tightening in the monetary policy, and probably higher inflation rates.

Regarding the maturity before 3Y, the forward rate is above the spot rate, and both have negative slopes. Leading to the belief that the agents had incorporated a recession in the 3 years ahead. As Engstrom and Sharpe (2018) find that a near-term negative spread may only predict recessions because it reflects the market's expectation that a contracting economy will induce the central bank to lower its policy rate. Indeed, the nominal interest rate is driven by the following:

- $E(Inflationrates)$ -> inflation risk premium
- $E(Monetarypolicy)$ -> real rate risk premium

Hence, if investors see higher odds of a recession, the long-term inflation risk premium in Treasury bonds will fall. In contrast, an increase in the recession probability would increase the real rate risk premium asked by agents. One interpretation can be that if investors see a greater risk of recession, they will attribute a higher value to short-term assets that they can quickly liquidate to finance spending on goods and services. It seems reasonable because the covid had already broken out in July, and the prices incorporated the expectation of future covid measures that hinder consumption.

Plot yield curves 2022



The plot above represents the forward and spot yield curves for different maturity quoted in July 2022. Compared to the previous plot, the forward yield curve is steeper. Thus the forward risk premium is higher for each maturity than the last plot. A possible interpretation is that the agents had expected higher inflation, an increase in monetary tightening, asking for a higher risk premium. Regarding the slopes of the two curves, the forward rate slope is positive, greater in absolute values compared to one year before ar each maturity. While the slope of the curves are both positive positive. The slopes and the forward risk premium are positive, this hints that agents believe in a positive state of the economy in the long run.