Theoretical framework

Riccardo Dal Cero

March 11, 2024

Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

Contents

Chapter 1

Introduction

In macroeconomic theory, the investigation of business cycles and long-term growth trajectories traditionally unfolds in distinct academic silos, drawing a parallel to the distinct realms of quantum mechanics and Einstein's theory of relativity in physics. This academic segregation, however, obscures a fundamental and profound question: How do business cycles influence long-term economic growth? The exploration of this question is more than an academic exercise; it underpins the practical understanding of short-term economic policies, such as automatic stabilizers, and their profound long-term impacts on the economy.

Embarking on this exploration, my research primarily dwells in the realm of theory, supplemented by rigorous simulation and calibration exercises. The intricate complexity of business cycles, particularly evident during periods of economic downturn and recovery, challenges empirical approaches due to the plethora of confounding variables. Thus, a theoretical lens, rather than a purely empirical one, is employed to dissect and understand these phenomena.

Central to this theoretical framework is an examination of the role of financial market frictions during economic recessions. A key inquiry here is the investigation of policy interventions, such as demand stabilizers, and their potential effect in attenuating the 'cleansing effect' of recessions. This exploration is pivotal in understanding whether such policies might inadvertently lead to a reduced economic baseline or steady state in the long term.

The conceptual foundation of this investigation is inspired by an ecological analogy the cyclical dynamics observed between predator and prey populations in nature. This natural cycle, when observed over extended periods, reveals not just self-contained oscillations but also underlying trends of population growth for both predators and prey. This observation leads to a compelling analogy for economic cycles: while they appear as short-term fluctuations, they might be underpinned by long-term growth trajectories.

In natural ecosystems, interventions aimed at stabilizing these cycles such as protecting prey during times of increased predation might seem beneficial in the short term. However, such interventions often neglect the critical and natural process of selection. This interference disrupts evolutionary mechanisms, potentially leading to unforeseen consequences over time, such as the propagation of traits detrimental to the species' survival and adaptability in changing environments.

My thesis extends this analogy to the economic sphere, positing a similar selective mechanism at play in economic systems. The primary focus is on the recession's cleansing effect, which might be analogous to natural selection in ecology. This effect could potentially 'weed out' less productive firms, leaving a market landscape dominated by more efficient players. The exploration aims to decipher whether such an economic 'natural selection' mechanism exists and, if so, how it shapes the fabric of productivity, innovation, and growth in the long term. Through this lens, the research endeavors to contribute a nuanced understanding of the intricate interplay between short-term economic fluctuations and long-term economic evolution, offering insights into the design and implications of economic policies. In the following sections, we will delve deeper into specific theories that bridge the gap between business cycles and long-term economic growth. However, it is beneficial first to embark on a brief historical journey through the evolution of thought regarding business cycles, to understand the context and development of these interconnected economic theories. This exploration will provide a foundation for appreciating the diversity of perspectives and the progression of ideas that have shaped our understanding of the intricate relationship between short-term economic fluctuations and long-term growth trajectories.

Chapter 2

Literature review

2.1 Business cycle history

2.1.1 Theories Connecting Business Cycles to Long-Term Growth

In the domain of economic theory, the relationship between business cycles (BC) and long-term growth is dissected into two principal schools of thought. The conventional viewpoint suggests that long-term growth is chiefly propelled by technological progress. Within this framework, technological advancements are often viewed as exogenous—arising outside the economic model's explanatory scope, as highlighted in the seminal contributions of Solow [1956] and Swan [1956]. This perspective treats technological progress as an independent variable that exerts influence on economic growth without being influenced by the economy's internal dynamics.

Contrastingly, an alternative strand of theoretical work aims to endogenize technological progress, weaving it into the fabric of the economic process. These models embed factors such as incentives for innovation, the value of education, and the accumulation of knowledge through economic activities, epitomizing the 'learning by doing' paradigm. A prominent example of this approach is found in Stadler [1990], which posits technological progress as an emergent property of economic behavior and incentives, rather than a mysterious external force.

A critical aspect of the 'learning by doing' model is its premise that technological frontiers are contingent upon the existing knowledge base, which expands primarily through practical experience. Consequently, periods of economic expansion witness a sharp increase in the knowledge stock, driven by higher employment levels, whereas recessions tend to stabilize or even diminish this stock due to reduced employment rates. This dynamic suggests that economies devoid of cyclical fluctuations might attain superior steady-state growth, as employment remains consistently high, fostering continuous technological advancement. From this perspective, the concept of a 'cleansing effect'—whereby economic downturns eliminate low-productivity jobs and ostensibly strengthen the economy—is challenged. The elimination of even low-productivity roles can erode the overall knowledge base.

Such theories reframe the discourse on stabilization policies, particularly fiscal interventions, by highlighting their role in sustaining employment and, by extension, supporting the technological frontier even in downturns.

To illustrate this theory's implications more vividly, consider a nuanced example: an antiquated factory with limited land resources discovers an innovative method to utilize an old tractor more efficiently. Despite the ingenuity of this breakthrough, if the broader economy has moved beyond the technology that the tractor represents, the innovation might not significantly contribute to the overall knowledge stock or push the technological frontier forward. This scenario prompts a fundamental inquiry: does innovation at the lower end of the skill spectrum or within outdated technological contexts meaningfully advance the technological frontier? Or, would it be more beneficial for economic growth to transition such small-scale innovations into larger entities equipped with modern technologies?

One significant critique concerns the disparity in learning opportunities across different sectors and among individuals. The model's premise of uniform learning opportunities does not always align with the reality that some industries, such as the technology sector, offer rapid innovation and learning environments compared to more traditional manufacturing industries, where the pace of learning and innovation may be inherently slower due to the nature of the work processes involved.

Furthermore, the model may not adequately address the issue of structural unemployment that can arise from technological advancements. As certain workers benefit from "learning by doing," leading to increased productivity, others may find their skills becoming obsolete due to automation and other technological changes. This dynamic is evident in the automation of routine manufacturing jobs, which, while fostering "learning by doing" in fields like robotics and software engineering, simultaneously leads to structural unemployment for workers displaced by these technologies.

Another point of contention is the potential for diminishing returns to learning. The assumption that "learning by doing" continuously fuels growth may not hold up against the reality that initial rapid gains in productivity tend to taper off as workers gain proficiency, suggesting that the benefits of learning may diminish over time.

The model also potentially overlooks the externalities and spillover effects associated with "learning by doing." Technological advancements in one firm or sector do not automatically translate into broader economic growth if these advancements remain isolated and do not benefit other sectors or industries. This is illustrated by a software company that develops cutting-edge algorithms, enhancing its productivity but failing to contribute to the wider economy if the knowledge remains proprietary.

This nuanced exploration challenges the simplistic notion of 'learning by doing' by questioning the value and impact of incremental innovations within the broader economic and technological ecosystem. It underscores the complexity of technological progress and its interplay with economic dynamics, inviting a deeper investigation into the mechanisms that drive long-term growth and the role of policy in nurturing an environment conducive to continuous innovation and knowledge expansion.

The contemporary perspective on technological advancement, when viewed as a product of incremental contributions from every market participant, appears overly simplistic. A more accurate depiction of technological progress recognizes it as predominantly driven by those at the forefront of research. The expansion of the technology frontier is essentially shaped by the knowledge and breakthroughs of these leading-edge innovators. Other entities in the economy adopt these innovations at varying paces, influenced by the associated adoption costs. While firms that are not at the innovation frontier may achieve marginal efficiency gains through adoption, the impact of such improvements is often minimal. These marginal innovations are frequently a reflection of the adopting firms' constraints, particularly their inability to invest in more advanced and expensive technologies. Consequently, these incremental innovations have limited potential for widespread diffusion, as they stem from a position of necessity rather than pioneering advancement.

Another theory describe a recession as a period in which the opportunity cost of investing in a productivity enhancing projects is lower since the workforce is not fully in demand to produce goods. Doing this would lead in theory to higher productivity in the period of expansion. The key role here is that the productivity-enhancing activity is costly and thus divert capital and labor force from production as Hayek Hayek [1933] explained. For this view to be valid two key aspects should be true at the same time: in the first place the expectations about the length of the recession should reinter in the short-term otherwise it is cheaper to destroy some production units (labor and capital) to accommodate the slow in demand. The last condition is that internal resources must be less costly than external ones, however, it would be cheaper to higher more skilled workers and fire the low-skill one. An additional remark is the this theory describes all those initiatives like worker formation that affect only marginally the productivity of a firm. It misses the main mechanics in which a firm can increase its productivity sharply: thorough technical innovation, and to do so you need a research program where the workforce is fully dedicated to it and not diverted from production.

Another theory posits that recessions offer a period in which the opportunity cost of investing in productivity-enhancing projects are lower, primarily because the workforce is not fully engaged in producing goods. Theoretically, this would lead to higher productivity during subsequent periods of expansion. A crucial element in this perspective is the acknowledgment that productivity-enhancing activities are expensive, thereby diverting capital and labor away from immediate production, a concept Hayek Hayek [1933] elucidated.

For this viewpoint to hold, two critical conditions must be concurrently satisfied: firstly, expectations regarding the duration of the recession must be short-term. If the recession is anticipated to be prolonged, it becomes economically viable to dismantle some production units (both labor and capital) to adjust to reduced demand. Secondly, the cost of utilizing internal resources for such productivity-enhancing ventures must be lower than the cost of acquiring external resources. Otherwise, it might be more economical to hire more skilled workers and lay off less skilled ones.

An additional observation about this theory is that it accounts for initiatives like worker training, which only marginally affect a firm's productivity. This overlooks the primary mechanism through which a firm can significantly boost its productivity: through technical innovation. To achieve substantial innovation, a dedicated research program is essential, where the workforce is fully committed to innovation efforts rather than being diverted to current production tasks. This highlights a gap in the theory, suggesting that while reallocating resources during recessions may offer some productivity benefits, the most dramatic productivity improvements are likely achieved through focused innovation and research activities, not merely through the opportunistic reallocation of resources during economic downturns.

An intricate theory that elaborates on the dynamics of economic recessions and the associated lower opportunity costs is grounded in the concept of labor hoarding, as discussed in the seminal work by Clark Clark [1973]. This theory posits that firms maintain employment levels higher than what immediate efficiency metrics might dictate. The rationale behind such a strategy is to prepare for a potential surge in demand, ensuring that the firm can quickly ramp up production without the delays associated with recruiting and training new employees. However, this strategic maneuver towards the internal possibility frontier—where firms optimize their readiness for future demand—does not manifest as observable changes in employment rates. Consequently, this leads to discrepancies or residuals in the statistical series of employment, which do not align with what might be considered the level of optimal employment, a phenomenon further analyzed in the work of Burnside et al. [1993].

This labor hoarding theory offers a partial explanation for the strong pro-cyclically of measured productivity. During economic upturns, firms can immediately respond to increased demand using their hoarded labor, thereby appearing more productive. Conversely, during downturns, the reluctance to shed this excess labor, due to the anticipation of future demand recovery, results in lower observable productivity levels. This behavior underscores a strategic depth in firm management, navigating through the cyclical economic waves by balancing between immediate efficiency and long-term responsiveness.

Expanding on this foundation, it becomes evident that the decision to engage in productivity-enhancing activities during recessions is not merely a reaction to lower opportunity costs but also a strategic consideration influenced by expectations of the recession's duration and the comparative costs of internal versus external resources. If firms anticipate a short-lived recession, the logic of hoarding labor and investing in internal productivity initiatives becomes compelling. However, this strategy hinges on the assumption that improving the skill set of the existing workforce or reallocating resources towards innovation is less costly than the alternative—acquiring new, possibly more skilled labor post-recession.

The opportunity cost (OC) approach closely aligns with the theory of labor hoarding, which seeks to elucidate the pronounced procyclicality of measured productivity. This observation implies that during economic downturns, firms appear to retreat towards the interior of their production possibility frontier, opting for a strategic reduction in operational efficiency rather than workforce downsizing. This strategy is partly attributed to the invisible nature of one crucial input—effort—to statisticians, and the economic rationale that, given the costs associated with employee turnover, it proves more economically viable for firms to dial back effort during slumps instead of terminating employment.

An intriguing alternative to diminishing effort is the redirection of employee tasks from immediate production roles to undertakings that bolster the firm's long-term productivity. At first glance, this strategy bears a resemblance to labor hoarding but carries the added outcome that these so-called shadow activities, embraced during recessions, eventually manifest as enhancements in total factor productivity.

The concept of adjustment costs does not singularly confine firms from adapting their production factors according to operational necessities. This opportunity cost mechanism could theoretically extend to a macroeconomic scale, influencing individual entities via price adjustments. During periods marked by diminished production value, the immediate returns from production activities (e.g., wages for workers) decline in comparison to alternative endeavors, notably human capital accumulation, whose benefits are pegged to future earnings. This economic mechanism could precipitate a resource reallocation towards these alternative activities. The empirical observation that education durations tend to extend during economic recessions lends credence to this argument. Nevertheless, with the exception of leisure, most sectors shadow the movements of aggregate GDP. Thus, if productivity-enhancing activities (PEAs) are to occur during recessions, the resource reallocation process must predominantly unfold within the firms themselves.

One notable deviation might be labor reallocation. As demonstrated by Davis and Haltiwanger [1992], job destruction exhibits a more countercyclical pattern compared to job creation. Viewing job reallocation through the lens of both destruction and creation suggests a countercyclical trend, positing job reallocation as an investment in cultivating superior firm-worker matches, thereby sowing the seeds for heightened productivity in the future. Davis and Haltiwanger [1992]further postulate, within the framework of a model featuring a representative agent, that economic recessions present an optimal window for labor reallocation, highlighting a strategic dimension to workforce management during downturns that might ultimately contribute to long-term productivity gains.

The ”lame ducks” theory, initially proposed by Caballero and Hammour [1994], offers an intriguing perspective on recessions as mechanisms that phase out less profitable, outdated capital. This theory delineates how the destruction of older units during downturns is more pronounced than the construction of new ones, characterizing recessions as periods marked by the systematic elimination of obsolete capital, hence the moniker ”lame ducks” theory. Notably, this framework sheds light on observations documented by Davis and Haltiwanger [1992], positioning it as a prominent theoretical approach that will be delved into more thoroughly in subsequent discussions.

Despite its insights, this model lacks consideration of the financial dimensions of firms, an aspect addressed by the theoretical work of Osotimehin and Pappadà [2017]. Their research weaves the financial decision-making process into the broader context of intertemporal capital decisions, highlighting how financial frictions influence the lender's participation constraint. The study reveals that, despite financial frictions, the cleansing effect of recessions on productivity persists, potentially leading to a more pronounced productivity surge during expansion phases. This analysis serves as a pivotal foundation for the new theoretical framework introduced in this thesis, marking a significant departure from traditional views and emphasizing the multifaceted impact of recessions on firm productivity and economic recovery.

In the forthcoming sections, / we will explore the theoretical underpinnings that form the basis of the new, streamlined theoretical framework introduced in this thesis. Our examination begins with the insights of Caballero and Hammour [1994], focusing on the interplay between the destruction and creation margins in economic cycles. Subsequently, we will delve into the work of Osotimehin and Pappadà [2017] (2017), which sheds light on the financial dimensions, particularly how capital lending frictions can precipitate misallocations. These studies collectively inform the development of our theoretical framework, setting the stage for a comprehensive analysis of economic dynamics and firm behavior during recessions.

2.2 Literature review of theoretical models

2.2.1 Introduction

This section of the literature review examines two influential theoretical models that analyze the cleansing effect of recessions on the economy. The first model, introduced by Caballero and Hammour [1994], utilizes a vintage model of creative destruction to investigate how industries adapt to cyclical demand variations. This model is pioneering in its approach, highlighting how recessions can facilitate the removal of outdated and less efficient production units, potentially leading to an overall increase in industry productivity. The model's insights are derived from a framework where production units of varying vintages coexist, and the rate of technological adoption plays a crucial role in determining industry responses to economic fluctuations.

In contrast, the second model explored in this review, by Osotimehin and Pappadà [2017], introduces financial frictions into the analysis of the cleansing effect of recessions. This addition brings a new layer of complexity to the understanding of how economic downturns affect firm dynamics, particularly by influencing the selection process through which firms enter and exit the market. The model underscores the role of credit constraints in mediating the impact of recessions, suggesting that financial frictions can dampen the potential productivity gains that might otherwise arise from the purging effects of a downturn.

Both models rely on numerical methods to solve their respective frameworks, acknowledging the intricate dynamics and non-linearities inherent in their analyses. By comparing these models, this review seeks to illuminate the diverse mechanisms through which recessions can influence economic outcomes, as well as the varying implications of introducing different types of market frictions into theoretical frameworks.

2.2.2 The cleansing effect by Caballero

In the first paper that rationalizes the cleansing effect of recessions, Caballero and Hammour [1994] and published in the American Economic Review in 1994, the primary aim was to investigate how industries respond to cyclical variations in demand. They did this by employing a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations in two ways: by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model they used incorporated heterogeneous firms, where production units embodied the most advanced technology at the time of their creation. The costs associated with creating new units slowed down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a reduction in demand can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour's research Blanchard et al. [1990] offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard and Diamond Blanchard et al. [1990], as well as Steven Davis and John Haltiwanger Davis and Haltiwanger [1992], in their respective works from 1990. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study Davis and Haltiwanger [1992], where they assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986, it is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry. The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

Theoretical model

The model in question is a vintage model that simulates an industry experiencing exogenous technological progress. Within this model, production units are constructed using a fixed proportion of labor and capital, and they are continually being created and phased out. Notice that only the creation of new production units incurs a cost. This simplification is plausible, particularly in the context of the United States, where the expense associated with hiring is typically higher than the cost of termination, as demonstrated by Abdulkadiroğlu and Kranton (2003) Abowd and Kramarz [2003].

In this model, when a production unit is created at a specific time t_0, it embodies the most advanced technology available at that moment and consistently generates a uniform output represented by $A(t_0)$ throughout its operational lifetime. The productivity of this technology denoted as $A(t)$, experiences continuous growth at an exogenously determined constant rate $\delta \geq 0$. This growth in technology can be interpreted in two ways: either as the adoption of new technology or as a product innovation. In the latter scenario, a continuum of perfectly substitutable products can yield varying levels of output.

$$[f(a,t) \qquad 0 \leq a \leq \overline{a}(t)]$$

The above function represents the cross-sectional density of the production units aged a at time t, where $\overline{a}(t)$ is the age of the oldest production unit at time t. The first assumption is that $f(a,t)$ and $\overline{a}(t)$ are continuous functions. The mass of production units at time t is given by:

$$N(t) = \int_{\overline{a}(t)}^{0} f(a,t)da$$

$N(t)$ is a measure of either the industry's capital stock or its employment, due to a fixed share of capital and labor. Thus, the industry's output is given by:

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

The deterioration of production units involves both an exogenous depreciation rate δ and an endogenous destruction process, which impacts $f(a,t)$ at its limits. The count of production units surviving for a years is expressed as:

$$f(a,t) = f(0,t-a)e^{-\delta a} \quad \text{where } 0 < a \leq \overline{a}(t)$$

The production flow is determined by:

$$\dot{N}(t) = f(0,t)[1 - \dot{\overline{a}}(t)] + \delta N(t)$$

Here, the first term represents the production rate, while the second term encapsulates the destruction rate, encompassing the obsolescence rate $f(\overline{a})(t)$, the technological obsolescence change over time $-f(\overline{a})(t)\dot{\overline{a}}(t)$, and the depreciation rate $\delta N(t)$.

The assumptions made by the authors are denoted as $\forall t \mid f(0,t) > 0 \cup \dot{\overline{a}}(t) <$.

The alteration in output concerning these flows is articulated as:

$$\dot{Q}(t) = A(t)f(0,t) - A(t - \overline{a}(t))f(\overline{a}(t), t) \cdot [1 - \dot{\overline{a}}(t)] + \delta Q(t)$$

The authors define a perfectly competitive industry in partial equilibrium, where supply is dictated by free entry and perfect equilibrium. Additionally, they introduce a cost function related to creating new production units:

$$c = c\left(f\left(f(0,t)\right)\right) \quad \text{where } c(\cdot) > 0,\ c'(\cdot) \leq 0$$

This cost function is contingent on the creation rate, implying that higher creation rates correspond to increased costs. The equilibrium condition is established by equating the cost of unit creation to the present discounted value of profits throughout its lifespan. The authors set the cost of a production unit to 1, and $P(t)$ is the price of a unit of output. Thus, the profits generated at time t by a production unit aged a are defined as:

$$\pi(a,t) = P(t)A(t-a) - 1$$

$$\overline{a}[t + T(t)] = T(t)$$

Here, $T(t)$ signifies the maximum lifetime of a unit created at t. At any given time t, the free entry condition is expressed as:

$$c(f(0,t)) = \int_{t+T(t)}^{t} \pi(s-t,t)e^{-(r+\delta)(s-t)\,ds}$$

In the above equation, where $r > 0$ represents the exogenously determined instantaneous interest rate, the determination of the exit of a production unit is contingent upon continuous $P(t)$ and the instance when the profits generated by a unit being destroyed reaches zero. This occurrence signifies the moment when the oldest unit operational at time t, denoted as $\overline{a(t)}$, must adhere to the equation:

$$P(t)A(t - \overline{a}(t)) = 1$$

The authors posit that $P(t)$ exhibits a decreasing trend due to the model's assumption regarding endogenous destruction, specifically $\overline{\dot{a}(t)} < 1$. To see, differentiate

$$\dot{P}(t) = -\gamma \left[1 - \bar{\dot{a}}P(t)\right]$$

Consequently, when the profits of a production unit diminish to zero for the first time, it will be subject to destruction.

On the demand side, the authors assume a unit-elastic demand function and consider the aggregate expenditure as exogenous $\overline{D}(t) = P(t)Q(t)$. The equilibrium is a path $\{f(0,t), \overline{a}(t), T(t), Q(t)\}_{t\geq 0}$ that satisfy the following conditions:

1. $$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

2.

$$f(a,t) = f(0,t-a)e^{-\delta a}$$

3.

$$T(t) = \overline{a}\,(t + T(t))$$

4.

$$c(f(0,t)) = \int_t^{t+T(t)} [P(s)A(t) - 1]\, e^{-(r+\delta)(s-t)}\, ds$$

5.

$$P(t)A(t - \overline{a}(t)) = 1$$

6.

$$P(t)Q(t) = \overline{D}(t)$$

The first three equations (1, 2, 3) and the fifth one (5) suffice to delineate the trajectories of $T(t)$, $P(t)$, and $Q(t)$, which are determined by $\{f(0,t), \overline{a}(t)\}$. To affirm the robustness of the conditions expressed in equations 6 and 5, it is possible to derive these equations as first-order conditions for the maximization of a number of perfectly competitive firms holding production units.

To comprehend the functioning of endogenous destruction, let's consider a scenario with constant demand. In this case, both the destruction and creation rates change only due to supply factors. This steady state is characterized by a constant lifetime of production units $T(t) = \overline{a}(t) = \overline{a}^*$, resulting in a time-invariant age distribution $f(a,t) = f^*(a)$. Equation 5 implies that the price $P(t)$ must consistently decrease at a rate σ. Higher innovation rates lead to increased productivity, raising the supply and consequently lowering the price. Equation 2 reveals that the distribution of production units in the steady state follows a truncated exponential distribution:

$$f^*(a) = f^*(0)e^{-\delta a} \quad 0 \leq a \leq \overline{a}^*$$

Using free entry conditions (4) and the clearing condition (6), one can determine the creation and destruction ages $f^*(0)$ and $\overline{a}^*$. Equations 1 and 5 yield the cost function and productivity of a new production unit:

$$c(f^*(0)) = \frac{e^{\gamma\overline{a}^*} - e^{-(r+\delta)\overline{a}^*}}{\gamma + r + \sigma} - \frac{1 - e^{-(r+\delta)\overline{a}^*}}{r + \delta}$$

$$f(0) = \frac{(\sigma + \delta)\overline{D}^*}{e^{\sigma\overline{a}^* - e^{\delta\overline{a}^*}}}$$

The authors then normalize the creation rate:

$$N = f^*(0) \cdot (1 - e^{\delta\overline{a}^*})$$

In the steady state, this is given by:

$$(9) CC^* = \frac{\delta}{1 - e^{-\delta\overline{a}^*}}$$

Considering a special case where the creation cost is a constant c, i.e., $c(f^*(0)) = c$, substituting into equation 2.2.2 allows retrieval of $\overline{a}^*$. The effect of technological rate σ on $\overline{a}^*$ is decreasing, as a higher innovation rate increases the opportunity cost of delayed renovation, while a higher cost of creating new units lowers the renovation rate. An optimal lifetime of production units increases with higher r and δ as it becomes harder to recover creation costs.

Now, dropping the assumption of constant demand, we examine how the industry adjusts to demand fluctuations. Two ways are identified in which the industry adapts production to meet demand: by reducing the rate of creation $f(0,t)$ and by increasing the rate of endogenous destruction $f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)]$, thus reducing $\overline{a}$, the age at which units are demolished.

These two adjustments interact, leading to a reduction in demand causing the most outdated units to be scrapped, rendering them unprofitable. However, if the recession is partially accommodated by a reduction in the creation rate, the effect on the destruction margin is diminished. The authors argue that the extent to which creation will "insulate" existing units from variations in demand depends on the marginal cost of creating new units $c'f(0,t)$. When the marginal cost of creation is zero, demand fluctuations are entirely adjusted by the creation margin. This is exemplified in the case where $c(f(0,t)) = c$. In such instances, the insulation effect is complete, as there is no need to retire older units. Lowering $f(0,t)$ is sufficient, and it is cheaper than reducing the life of existing production units.

The insulation effect is not solely due to asymmetric adjustment costs on the creation and destruction margins. Complete insulation would occur even with linear adjusting costs. The creation rate in the case of constant creation cost is given by:

$$f(0,t) = \frac{\dot{\bar{D}}(t) + \delta\bar{D}(t) + P(t)A(t-\bar{a}(t))f(\bar{a}(t),t)[1-\dot{\bar{a}}(t)] - \dot{P}(t)Q(t)}{P(t)A(t)}$$

In the attained equilibrium, variations in demand are entirely offset by adjustments at the creation margin denoted as $f(0,t)$, with $\overline{a}(t)$ remaining steady at the destruction margin. The creation process effectively counteracts the impact of demand fluctuations on the price $P(t)$, effectively shielding existing units from demand changes. The price $P(t)$ experiences a constant decline at a rate represented by σ, reflecting the pace of technical progress. This consistent decline in $P(t)$ serves as a clear signal for production units to function optimally throughout their constant lifetime $\overline{a}(t)^*$.

In the aforementioned scenario, the destruction rate is not constant, but it does not respond to demand through variations in the age $\overline{a}(t)^*$ at which units are destroyed. Instead, variations in the creation rates have an impact on the number of units that reach obsolescence. If fewer units are created, fewer units become obsolete after $\overline{a}(t)^*$ periods. It is noteworthy that any modification leaving equations 3 to 5 independent of $\overline{D}(t)$ and $f(0,t)$ does not alter the full-insulation results.

Interestingly, assumptions such as perfect competition, industry-wide return to scale, and perfect foresight are not necessary for these conclusions. The latter is particularly noteworthy as it asserts that fully accommodating demand on the creation side only requires knowledge of current conditions. As long as the non-negativity constraint on $f(0,t)$ is never binding, implementing equilibrium behaviors does not necessitate expectations of future demand.

Application of the model

The model undergoes calibration utilizing Job-flow data and Industry production data. The former facilitates the replication of job creation dynamics, while the latter is employed to mimic the behaviors of firm creation and destruction in the manufacturing industry. To capture these dynamics, the marginal cost of creating new production units is stipulated as positive $c'f(0,t)$. This allows for a partial insulation effect, and the destruction margin responds to demand fluctuations. However, introducing a dependency of c on $f(0,t)$ compromises the analytical tractability of the system (Equations 1 - 6). Consequently, the authors resort to methods such as multiple shooting to ascertain the optimal equilibrium and subsequently employ an iterative procedure to converge to the correct expected creation rate.

For numerical solutions, the authors adopt a linear formulation:

$$c(f(0,t)) = c_0 + c_1 f(0,t)$$

To gain a deeper understanding of how creation and destruction respond to demand, the authors simulate sinusoidal demand using the equation:

$$\overline{D}(t) = 1 + 0.07 \sin(t)$$

The results are visualized in the image below, depicting the feedback of normalized creation and destruction (CC and DD) to changes in demand.

The plot clearly illustrates that the insulation effect is only partial; otherwise, DD would have remained flat, as in the case with $c(f(0,t) = c)$. From a mathematical perspective, destruction responds to demand as equations 3-5 are no longer independent of the path $f(0,t)$ and demand. From an economic standpoint, increasing creation costs smoothen the creation process. In scenarios with a nearly flat innovation rate, firms during crises cannot fully accommodate lower demand, nullifying the adoption of new production units, as the marginal costs would exceed the reduction in existing production units.

In the considered model, production units integrate labor and capital in fixed proportions to generate output. Each unit can be conceptualized as contributing to job creation within the industry, and job-flow data serves as a metric for quantifying the flows of production units.

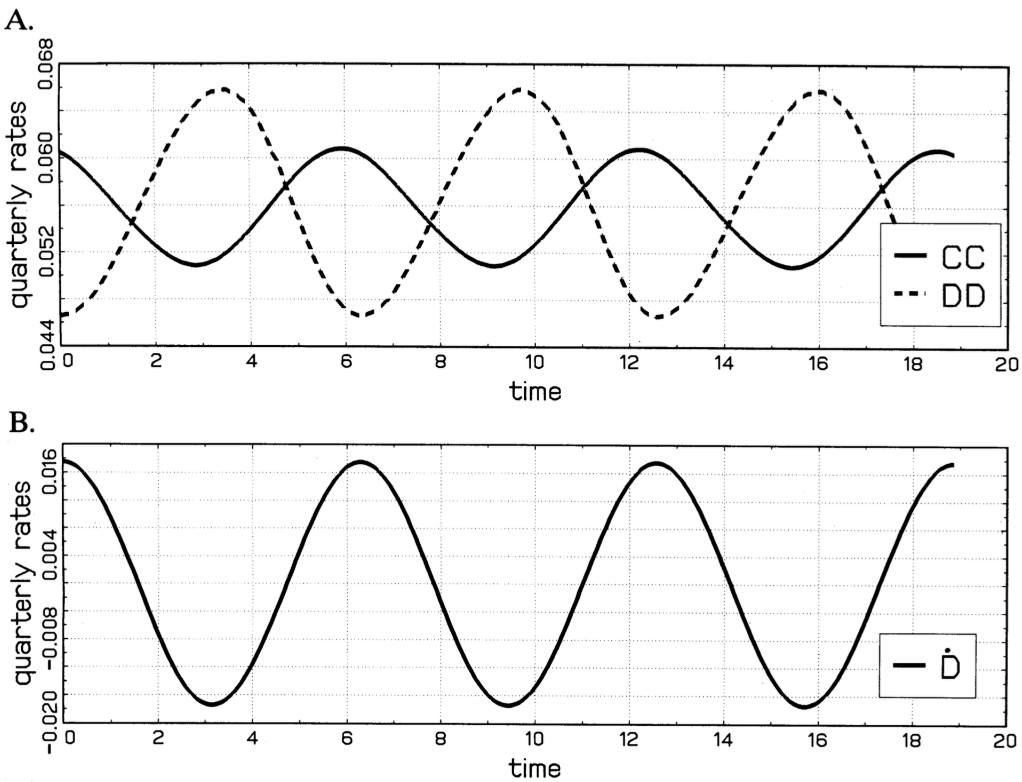


FIGURE 2. A) CREATION AND DESTRUCTION ($c_0 = 0.3$, $c_1 = 1.0$); B) CHANGE IN DEMAND (SYMMETRIC)

Figure 2.1: Figure 1. A Creation and destruction $c_0 = 0.3, c_1 = 1$ B Change in demand (Symmetric)

Datasets that closely align with the theoretical CC and DD series have been compiled by Davis and Haltiwanger [1990, 1992] and Blanchard et al. [1990], drawing from various sources. The primary focus lies on the dataset curated by Davis and Haltiwanger, who leverage the Longitudinal Research Database to construct quarterly series for U.S. manufacturing plants spanning the period 1972:2-1986:4.

In their empirical approach, **?**utilize output to empirically determine demand, employing the growth rate of the industrial production index as a proxy for output growth. Notably, in the foundational theoretical model, $Q(r)$ is smoothed by price movement, with the elasticity of demand determining the extent of smoothing, assumed to be equal to 1. While the theoretical model maintains a constant dividend wage, the authors acknowledge that considering a procyclical dividend wage, as in the case of general equilibrium with correlated industry shocks, may dampen the effect of demand shocks. However, they assert that this adjustment would alter only the magnitude, not the direction, of the analysis.

The figure below illustrates the data that the model seeks to replicate, showcasing job creation, job destruction, and growth.

To discern the characteristics of the series, the authors perform regression analysis on sectoral rates of job creation and job destruction against leads and lags of the corresponding rates of growth. They find that job creation is less responsive to demand fluctuations, while job destruction exhibits a more countercyclical behavior. The initial finding indicates that the rate of job destruction displays greater responsiveness to changes in sectoral activity compared to the rate of job creation. Specifically, the sums of coefficients are -0.384 for job destruction and 0.218 for job creation showed in the table 2.3, the same results as in Davis and Haltiwanger [1990, 1992] and in Blanchard et al. [1990]. The authors capitalize on a natural experiment rooted in the intrinsic asymmetric characteristics of business cycles. Recessions, marked by brevity but in-

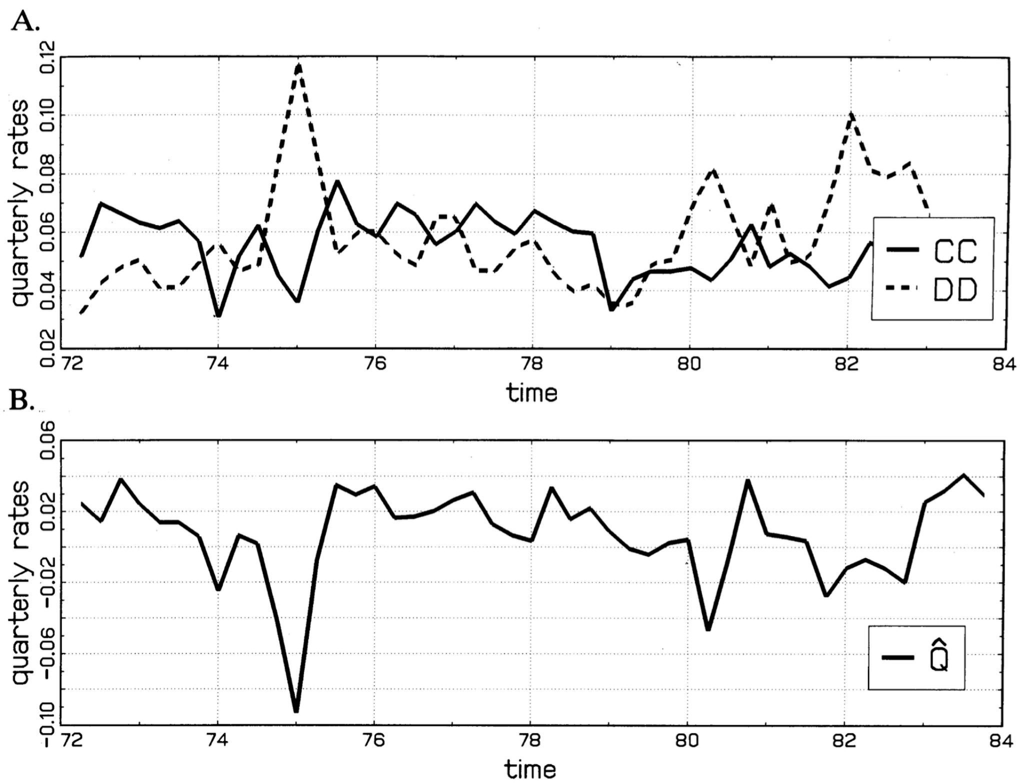


Figure 2.2: Figure 1. Job creation and job destruction in U.S. Manufacturing B Index of the industrial production

		Creation		Destruction	
Regressor	Timing	Coefficient	Standard deviation	Coefficient	Standard deviation
$\hat{Q}$	2 leads	0.029	0.006	0.030	0.010
	1 lead	0.065	0.007	−0.068	0.010
	contemporaneous	0.108	0.007	−0.185	0.010
	1 lag	0.013	0.007	−0.103	0.010
	2 lags	0.003	0.006	−0.058	0.010
	Sum:	0.218	0.013	−0.384	0.017
$\hat{Q}^+$	2 leads	0.052	0.012	0.012	0.016
	1 lead	0.102	0.012	0.002	0.016
	contemporaneous	0.131	0.012	−0.065	0.016
	1 lag	0.059	0.012	−0.025	0.016
	2 lags	0.055	0.012	−0.008	0.016
	Sum:	0.399	0.026	−0.066	0.023
$\hat{Q}^-$	2 leads	0.002	0.010	0.006	0.014
	1 lead	0.022	0.011	−0.149	0.014
	contemporaneous	0.093	0.012	−0.293	0.015
	1 lag	−0.012	0.012	−0.139	0.015
	2 lags	−0.021	0.012	−0.059	0.015
	Sum:	0.084	0.020	−0.634	0.024

Figure 2.3: Table 2.1. Job Creation and Job Destruction in U.S. Manufacturing Response to Output Growth

textitNotes: The table presents the reaction of job creation to the growth rate of the industrial production index. The latter is categorized into values above and below its mean (Q). The table encompasses quarterly observations for the two-digit SIC industries during the period 1972:2-1986:4. The coefficients are uniformly constrained to be equal across all sectors, with the exception of a constant (not shown).

tense contractions, provide the backdrop for the authors' model. This model endeavors to emulate the creation rate while concurrently mitigating the impact of asymmetric cyclicality inherent in business cycles. The empirical evidence supporting this model's behavior is encapsulated in Table 2.3, wherein two distinct scenarios are explored: output growth trajectories above Q^+ and below Q^-, relative to their respective means. The table meticulously delineates how creation and destruction rates respond to these deviations in output growth.

The salient observation emerges regarding creation rates, elucidating that they exhibit a more rapid and robust response in instances of vigorous output growth, as opposed to scenarios where the output growth rate experiences a reduction. On a contrasting note, the destruction margin, in line with the model's projections, manifests heightened sensitivity to a decline in output. This responsiveness is particularly pronounced from one quarter before the onset of the shock to one quarter after. Notably, during expansionary phases, the mean response of the destruction margin is -0.066, a notably milder reaction compared to the recessionary case where the mean response stands at -0.634.

These empirical outcomes seamlessly align with the predictions of the model. Specifically, the creation rate exhibits heightened responsiveness during expansionary phases, given their cyclical and symmetric nature. In contrast, the asymmetric and non-cyclical nature of recessions triggers a more substantial decline in the production unit rate, in line with the model's expectations.

In order to better understand the asymmetrical behavior the authors simulate an asymmetrical demand function:

$$\cdot\overline{D}(t) = 0.05[\cos(t) + \sin(t)] - 0.016\sin(2t) - 0.003\cos(3t)$$

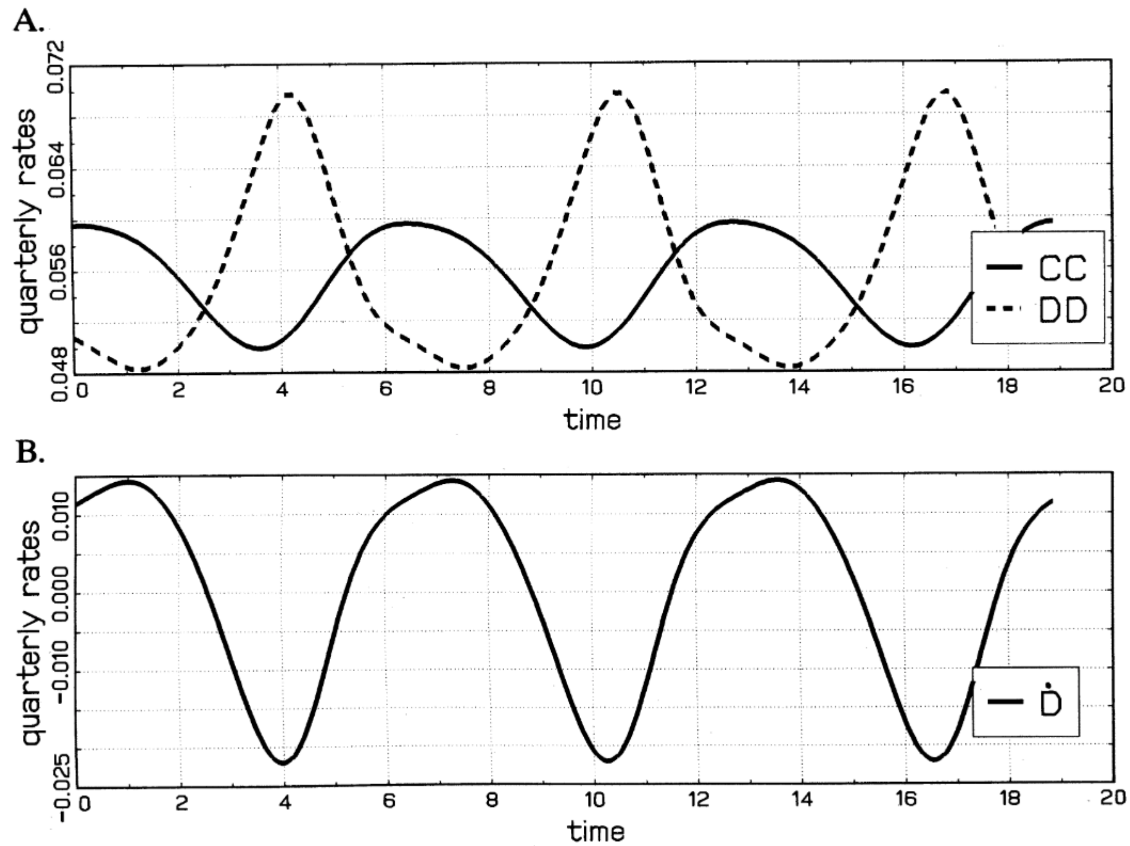


Figure 2.4: A. Creation and Destruction B. Output Growth
Notes: The figure depicts a simulation of asymmetrical supply growth.

$$\overline{D}(t) = 1 \quad r = 0.065, \delta = 0.15, \gamma = 0.028, c_0 = 0.3, c_1 = 1.0$$

The results are depicted in 2.4

From the plot 2.4, it is evident that firms use prediction in demand to smooth job creation to avoid big change, since they are too costly, by averaging the demand over the lifetime of a production unit. On the other hand, destruction depends only on current conditions, thus responding only to significant deviations from the demand prediction. It can be better understood thinking about a case in which creation rates respond only mildly to a sharp decrease in demand, and the equilibrium price falls leading to additional destruction since older units' profits go to 0. Indeed, destruction not only preserves but amplifies the asymmetry of demand.

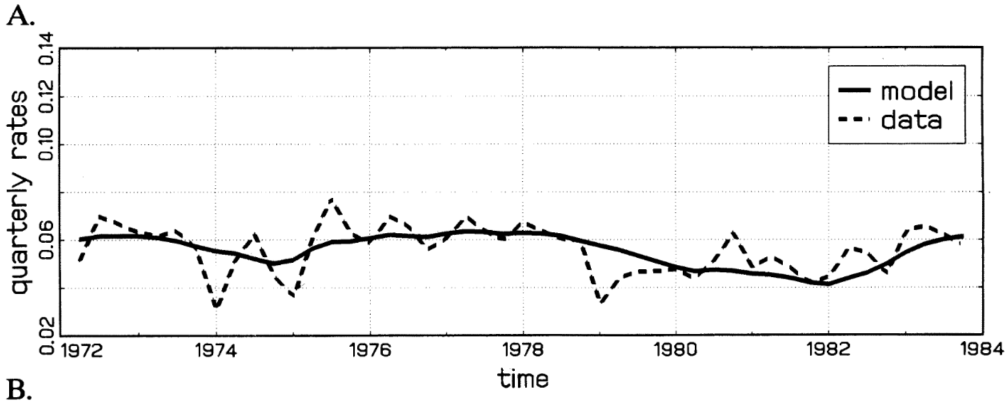
Frictionless economy The authors culminate their study with a compelling calibration exercise using manufacturing series to exploit the model. This entails dissecting the observed net change in employment into destruction and creation rates, as well as applying the same approach to output production. The model is simulated for the duration of 1972:2-1983:4, with parameters as follows:

Table 2.1: Calibrated Parameters

Variable	Symbol	Value
Interest rate	r	0.065
Depreciation rate	δ	0.150
Rate of technical progress	γ	0.028
Adjustment cost parameters	c_0	0.403
	c_1	0.500

The technical progress is selected to attribute all the growth in employment and manufacturing to technological advancements, setting λ as 2.8. The authors employ Equation 2.2.2, linking the steady state to the lifetime of jobs and job turnover (CC^*), determining $\overline{a}^* + 7.42$ years. Utilizing this information, they ascertain the steady state entry cost to be 0.525, equivalent to half a year's operating costs for production units. Subsequently, they employ ordinary least squares (OLS) to retrieve the value of c_1, the marginal cost of creating a new unit, which is found to be 0.5. This aligns with a small elasticity for the creation cost function, signifying the vulnerability of the insulation mechanism to breakdown. The model's simulations on employment and output, shown in Figure 2.5, reveal discrepancies with actual data, particularly in the smoother job creation trends, likely due to the model's exclusion of uncertainty. Nonetheless, it successfully captures the volatility in job creation and destruction patterns, along with job creation's greater symmetry, providing insights into employment and output fluctuations. The model's examination of the creation margin's response and its impact on the destruction margin offers a baseline for understanding the cleansing effect's role in production unit distribution.

A.



B.

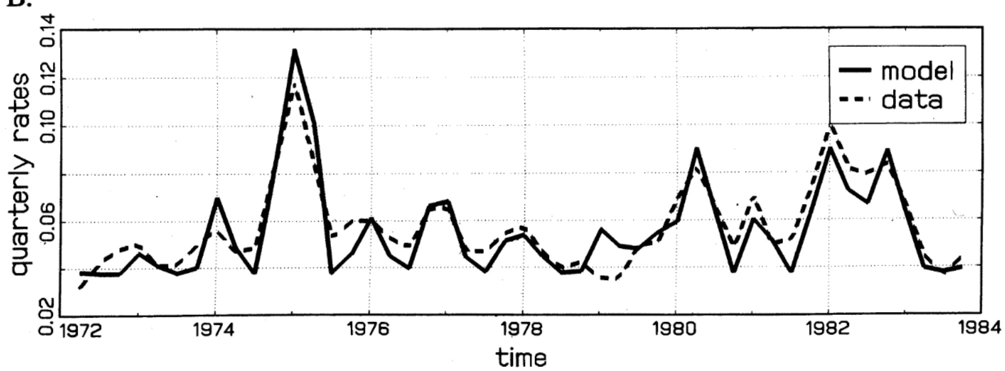


Figure 2.5: Figure 1. A employment driven job creation $c_0 = 0.403, c_1 = 0.5$ B Employment job destruction $c_0 = 0.403, c_1 = 0.5$

However, the model overlooks the impact of financial frictions, which could significantly affect both creation and destruction margins. It also entertains the notion of recessions as ”pit-stops” for strategic investment, adding depth to recession analysis. Despite the common view of procyclical labor productivity, the model, supported by Galí and Hammour [1992], suggests that recessions can indirectly boost long-term productivity through the cleansing effect.

A notable limitation is the assumption of constant marginal creation costs, which recent studies challenge, especially for larger firms known for substantial adjustments in response to demand drops. This observed behavior in larger firms, aligning with the model's predictions on downsizing, underscores its ability to reflect real-world dynamics despite its simplifications.

2.2.3 Cleansing effect in Osotimehin and Pappadà [2017]

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as c to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as e, in the risk-free asset.

Firms opt to exit the market when expected profits no longer outweigh the fixed cost c, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r) - 1] e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to q. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors. In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as k. The production function, accounting for the fixed cost c, is expressed as follows: $Y = Z(\theta + \epsilon)k^{\alpha}$. Key variables include:

- Z: Stochastic aggregate productivity common across firms.
- θ: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- ϵ: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \delta)$.
- k^{α}: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

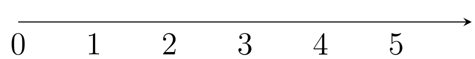


Figure 2.6: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of Z, θ, k^{α}, e (where e represents its endowment, different from k since the firm can borrow money with $d = c + k - e$).
2. The firm computes the optimal k to maximize the expected value of the firm, with k ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.

3. At the end of the period, the firm observes ϵ and the aggregate shock.

4. The firm repays its debt and the fixed operating cost $(c + k - e)$, resulting in an end-of-period net worth q.

5. The firm decides on the amount of dividends to distribute $(q - e')$, observes the productivity shock θ', Z', and the process restarts from step 1.

Frictionless economy In a frictionless economy, firms have the option to borrow an amount denoted as $c + k - e$ at the risk-free interest rate $r = \frac{1}{\beta} - 1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_{k} E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - (1 + r)(c + k - e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_{k} E \int [Z(\theta + \epsilon)k^{\alpha} - (1 + r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market friction, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

Economy with Credit Market Frictions After production, the firm privately observes the temporary shock ϵ, while financial intermediaries can only observe it at a cost of μk^α. For one-period debt contracts, financial intermediaries observe ϵ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth e, current productivity θ, and aggregate productivity value Z, all observable by both the financial intermediary and the firm at no additional cost.

HP1 (Hypothesis 1): The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\overline{\epsilon}$ is determined by the equation:

$$Z(\theta + \overline{\epsilon})k^\alpha + (1 - \delta)k = (1 + \widetilde{r})(c + k + e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component θ.

The financial intermediary lends $(c + k - e)$ to the firm only if the expected income from the loan equals the the opportunity cost of the funds, as expressed by the inequality:

$$(1+\widetilde{r})(k+c+e)(1-\Phi(\overline{\epsilon})) + \int_{-\infty}^{\overline{\epsilon}} [Z(\theta+\overline{\epsilon})k^\alpha + (1-\delta)k - \mu k^\alpha] \, d\Phi(\epsilon) \geq (1+r)(c+k+e)$$

The financial contract is characterized by $(k, \overline{\epsilon})$. Given Z, θ, e, the participation constraint indicates the default threshold $\overline{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given θ, Z, there is a unique threshold $e_b(\theta, Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta + G(\bar{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\bar{\epsilon}_b) = (1+r)(k_b + c - \underline{e_b})$$

where $\bar{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta+\bar{\epsilon})k^\alpha + (1-\delta)k - (1+\widetilde{r})(k+c-e) & \text{if } \epsilon \geq \bar{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}); 0]$$

The firm's problem Define V as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e,\theta,Z) = \max_{(k,\bar{\epsilon})} E\left\{\int I(q)q + (1-I(q))\max[q, \max_{e'} q - e' + \beta V(e',\theta',\zeta')]\, d\Phi(\epsilon)\right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1.

$$Z[\theta + G(\bar{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha\Phi(\bar{\epsilon}_b) \geq (1+r)(k_b + c - \underline{e_b})$$

2.

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}); 0]$$

3.

$$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.

Findings

This study investigates the complex interplay between credit frictions and the cleansing effect of recessions on firm dynamics. By integrating models of firm dynamics with credit frictions, the authors examine how these frictions influence the selection process for entering and exiting firms, potentially leading to the premature exit of some high-productivity entities. Despite the impact of credit frictions on firm selection, an increase in average productivity is observed following an aggregate productivity shock. Mirroring the dynamics in a frictionless economy, a negative aggregate productivity shock enhances average productivity by predominantly increasing the net exit rate of low-productivity firms.

The analysis underscores that the extent of the recession's cleansing effect significantly depends on the steady-state distributions of productivity and net worth among firms. The amplification in average productivity can be more substantial than in a frictionless context, depending on the severity of credit frictions and the productivity distribution of firms. Through calibration, the authors demonstrate that credit frictions generally mitigate the rise in average productivity, indicating a systematic diminution in the intensity of productivity enhancement for each percentage point increase in productivity, irrespective of the level of credit frictions and the productivity distribution.

Further exploration into the nature of economic shocks reveals that the type of shock critically influences the cleansing process. Specifically, while recessions can exert a cleansing effect, the intensity of this effect is attenuated when the downturn is triggered by a financial shock. This reduction is due to the financial shock affecting high-productivity firms as well, suggesting a nuanced relationship between the type of economic shock and the intensity of the cleansing effect: non-financial shocks tend to have a more significant cleansing impact compared to financial shocks.

Contributing to the literature on the cleansing effect of recessions, this study elucidates the role of credit frictions in affecting the average productivity of firms by facilitating the selective exit of low-productivity firms, even under credit constraints. It is imperative to note that the findings do not posit that recessions inherently enhance resource allocation efficiency. On the contrary, in the presence of credit frictions, many of the exiting firms could have remained operational in the absence of these constraints. This inefficiency is particularly evident during financial shocks, where the exit of potentially viable firms highlights the intricate dynamics governing the cleansing effect of recessions, emphasizing the importance of a nuanced understanding of economic downturns and their impact on firm dynamics.

Chapter 3

Theoretical model

3.1 Introduction

This thesis presents a partial equilibrium model in which firms maximize dividends over an infinite period, under financial frictions, investigating how those frictions can affect the saddle path of capital and dividends. Compared to Osotimehin and Pappadà [2017] and Caballero and Hammour [1994], this model allows us to find a closed-form solution optimal path for dividends and capital. The subsequent sections delve into the formulation of the flow of funds and its dynamics. Following this, the focus shifts to scenarios where financial frictions are present, examining their implications on firm behavior and market outcomes.

3.2 Law of motion of capital and debt

This model is set within a partial equilibrium framework where firms are differentiated by their productivity levels. They have the option to fund their operations by obtaining loans from financial intermediaries, as outlined by Bernanke and Gertler [1995], or by retaining dividends. The capital at any time t is calculated by adjusting the capital from the previous period for depreciation (δ), then adding gross investment (I), thus the low of motion of capital stock is:

$$k_{t+1} = k_t(1 - \delta) + I_t$$

We can rearrange the above equation and get gross investment at time t

$$I_t = k_{t+1} - k_t\,(1 - \delta) \tag{3.1}$$

The 3.1 equation states gross investment at time t is equal to the net capital formation plus replacement of depreciated capital. The flow of funds constraint is:

$$I_t + Rb_t + d_t = f\,(k_t) + b_{t+1} \tag{3.2}$$

where R denotes the gross interest rate and b_t represents the debt from period t. The components of the flow of funds (f-of-f) at time t include:

1. I_t gross investment at time t
2. Rb_t repayment of debts (principal and interest)
3. d_t dividends distributed at time t

Conversely, the right-hand side details the sources of fund inflows:

1. $f(k_t)$ output at time t+1
2. b_t debt at time t+1

The f-of-f constraint can be rewritten as the law of motion of debt:

$$b_{t+1} = Rb_t + I_t - S_t \tag{3.3}$$

where $S_t = f(k_t) - d_t$ represents earnings retained. This formulation clarifies that the debt level at time $t+1$ is the sum of the repayment for the previous period's debt (both principal and interest) and the net investment, adjusted for internal financing.

From equations 3.1, 3.3 and the definition of net worth $n_{t+1} = k_{t+1} - b_{t+1}$, we derive the law of motion for net worth as follows:

$$\begin{aligned} n_{t+1} = k_{t+1} - b_{t+1} &= k_t(1-\delta) + I_t - Rb_t - I_t + S_t \\ &= k_t - \delta k_t - b_t - rb_t + S_t \\ &= n_t - \delta k_t - rb_t + [f\,(k_t) - d_t] \end{aligned}$$

The net worth or equity of the firm is given by the net worth of the previous period less the depreciated capital, less the interest matured from the previous period augmented by the retained earnings. Therefore a firm can increase its net worth only through increasing the retained earnings levels, thus increasing output or decreasing dividends. Using equations 3.1 and 3.2, we get the flow of funds constraint for capital:

$$k_{t+1} = k_t(1-\delta) - Rb_t - d_t + f(k_t) + b_{t+1} \tag{3.4}$$

The above equation describes how capital evolves over time: capital at time $t+1$ is equal to capital at time t net of depreciation, less the repayment of debt (principal + interest), augmented by retained earnings.

3.2.1 Steady State

From 3.1, assuming $(k_{t+1} = k_t = \widehat{k})$, we get the steady state investment:

$$\begin{aligned} \widehat{k} &= \widehat{k}\,(1-\delta) + \widehat{I} \\ \widehat{I} &= \delta\widehat{k} \end{aligned} \tag{3.5}$$

3.5 states that in the steady state, the firm will invest only to substitute depreciated capital$(\delta\widehat{k})$. From 3.3, assuming $(b_{t+1} = b_t = \widehat{b})$ and $d_t = d_{t+1} = \hat{d}$, we get:

$$\widehat{b} = R\widehat{b} + \widehat{I} - \widehat{S} \tag{3.6}$$

where $\widehat{S} = r\widehat{b} - \widehat{I}$, therefore:

$$f\left(\widehat{k}\right) - \widehat{d} - \delta\widehat{k} = r\widehat{b} \tag{3.7}$$

Equation 3.7 states that in the steady state, retained earnings should be used to repay interest over debt. Equation 3.7 can be rewritten as:

$$f(\widehat{k}) = \delta \cdot \widehat{k} + r \cdot \widehat{b} + \widehat{d} \tag{3.8}$$

To illustrate the equilibrium locus, one can refer to the graph in 3.1, which depicts the locus as defined by equation 3.8, employing the following production function:

$$f(k_t) = Zk_t^{\alpha}, \tag{3.9}$$

with Z indicating the firm's productivity level, k_t indicating capital, $0 < \alpha < 1$.

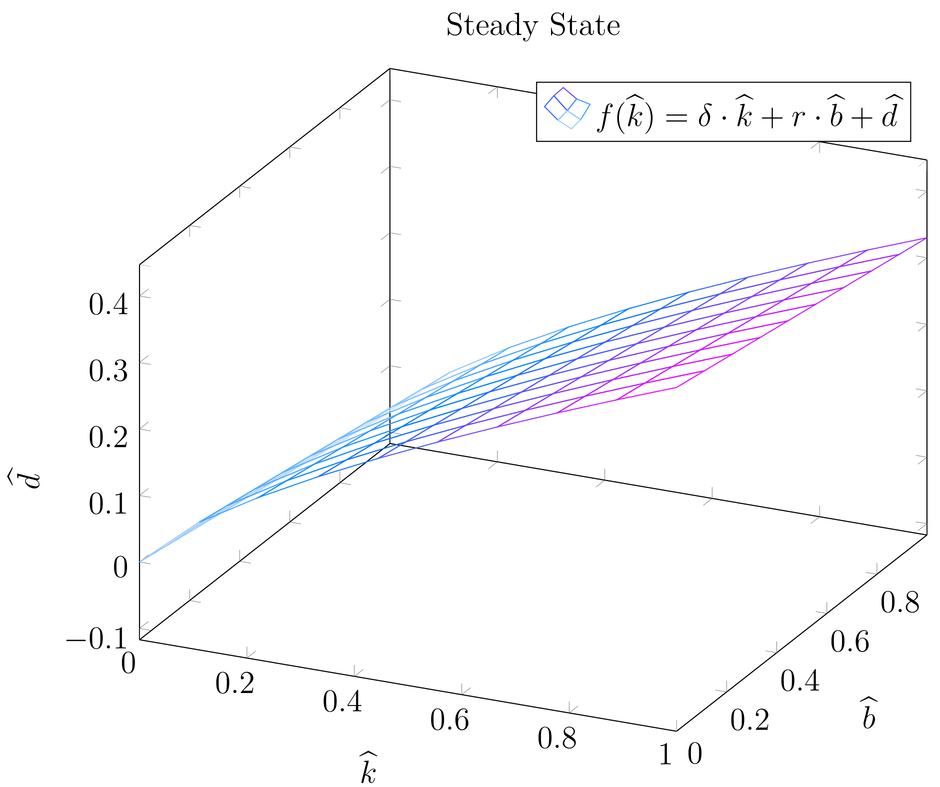


Figure 3.1: For this plot the following value has been used: $\delta = 0.1, r = 0.1, \alpha = 0.8, Z = 0.5$

The figure illustrates the steady state relationships among debt ($\widehat{b}$), capital ($\widehat{k}$), and dividends ($\widehat{d}$) in a three-dimensional plot. The graph demonstrates how various combinations of debt and capital influence the distribution of dividends. It is evident that, for any given level of $\hat{k}$, a higher level of debt results in lower dividends, as a larger portion of resources is allocated towards servicing interest payments. Conversely, the relationship between capital and dividends, given $\hat{b}$ is depicted as concave, highlighting an increase in dividends with higher capital levels, under the specified model parameters: $\delta = 0.1$, $\alpha = 0.8$, and $Z = 0.5$. For example, the firm starting with initial capital $k_0 = 0.2$ and debt $b_0 = 0.1$, to maintain a steady state for both capital and debt, the dividends should be equal to $\widehat{d} = 0.5 \times 0.2^{0.8} - 0.1 \times 0.2 - 0.1 \times 0.1$. This specific combination of $k = 0.2$, $b = 0.1$, $d \approx 0.11$ represents a stationary point in the model.

3.2.2 Dynamics of capital

In the simplified scenario where a firm operates without incurring debt ($b_t = 0$ for all t) and imposing constant and positive dividends ($d_t = d > 0$), the flow of funds constraint changes as follows:

$$I_t + d = f(k_t) \tag{3.10}$$

$$I_t = S_t \tag{3.11}$$

In the absence of debt, a firm's investment is solely financed through retained earnings, as specified in equation 3.11. From 3.11, we derive the difference equation describing the evolution of capital:

$$\Delta k_{t+1} = f(k_t) - d - \delta k_t + \tag{3.12}$$

Equation Equation 3.12 presented implies that variation in capital stock depends on balance between retained earnings and depreciated capital. Thus, for a positive increase in capital $\Delta k_{t+1} > 0$, it is necessary for a company to hold back earnings in excess of what is required to offset depreciated capital:

$$f(k_t) - d > \delta k_t$$

Similarly, the reverse scenario holds as well. To determine the steady-state level of capital, we apply the previously used production function (see equation 3.9), setting $k_{t+1} = k_t = \hat{k}$ within the difference equation (refer to equation 3.12):

$$Z\hat{k}^{\alpha} - d = \delta \hat{k} \tag{3.13}$$

The above equation can be solved as follows:

$$\hat{k} = \left[\frac{d}{Z(1-\delta)}\right]^{\frac{1}{\alpha}} \tag{3.14}$$

Recalling that the production function is monotonic and concave ($0 < \alpha < 1$), thus for a given set of parameters (Z, α, δ, d), there is a unique steady state for capital $\hat{k}$, defined by equation 3.14. The slope of the phase diagram is:

$$\frac{\partial k_{t+1}}{\partial k_t} = (1-\delta) + f'(k_t) = \tag{3.15}$$

$$= (1-\delta) + \alpha Z k_t^{\alpha-1} \tag{3.16}$$

Analysis reveals two distinct scenarios based on the derivative of capital with respect to k_t:

1. **Exploding Path When Derivative Is Greater Than 1**: If the partial deriva-

tive with respect to k_t is greater than 1, it suggests that small deviations from the steady-state level $(\widehat{k})$ will lead to increasing divergence rather than convergence back to $\widehat{k}$. This scenario can be characterized by potentially unstable dynamics. The condition is:

$$(1-\delta) + \alpha Z k_t^{\alpha-1} > 1, \quad \alpha Z k_t^{\alpha-1} > \delta, \quad k_t > \left(\frac{\alpha Z}{\delta}\right)^{\frac{1}{1-\alpha}}$$

2. **Convergence to Steady State**: Conversely, if the derivative is less than 1, this indicates that deviations from the steady-state level will diminish over time, leading to a stable equilibrium. This scenario reflects a stable path where the system tends to return to its steady state following a perturbation. The condition is:

$$(1-\delta) + \alpha Z k_t^{\alpha-1} < 1, \quad \alpha Z k_t^{\alpha-1} < \delta, \quad k_t < \left(\frac{\alpha Z}{\delta}\right)^{\frac{1}{1-\alpha}}$$

The following phase diagram represent the case $\frac{\partial k_t}{\partial k_{t-1}} > 1$, using the following parameters: $\delta = 0.1$, $\alpha = 0.8$, $Z = 0.5$, and $d = 0.8$

The graph distinctly demonstrates that when the capital at time t is below the red dot, it means that capital is less than the steady-state capital, leading to a diminishing trajectory in the firm's capital. Conversely, if capital is above the steady-state level, indicated by $\widehat{k}$, the firm is overcapitalized, and the trajectory becomes explosive, with capital increasing without bound.

If a firm's capital is less than the steady state, the outflows—such as the replacement of depreciated capital and dividends—are disproportionately high compared to its production. This dynamic will inevitably cause the firm's capital to go down towards zero. It's crucial to recognize that this path is predicated on the assumption of constant dividends.

Furthermore, the steeper the slope of the blue line, the higher the productivity factor

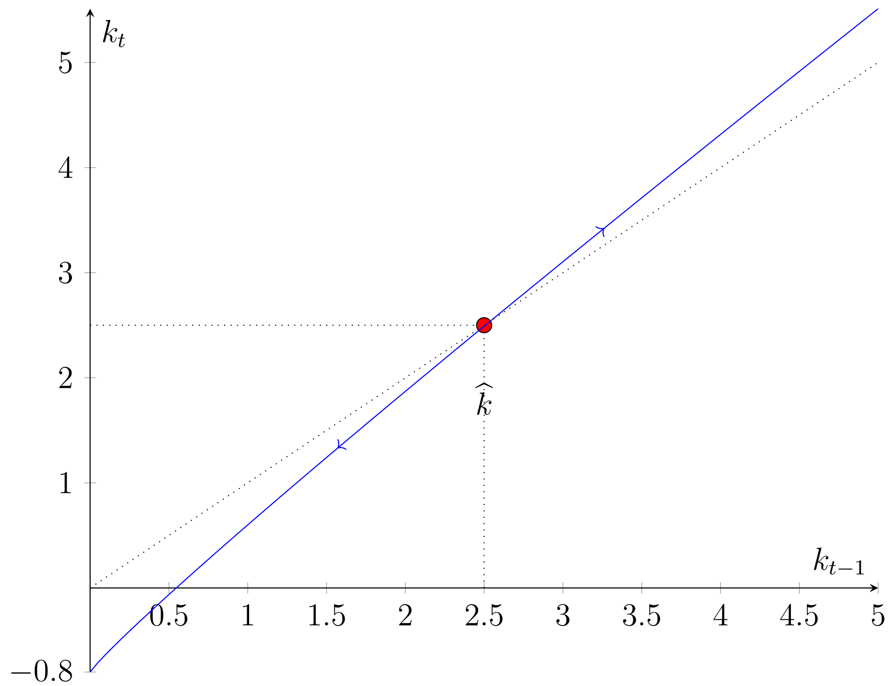


Figure 3.2: The phase diagram demonstrates the evolution of capital in a debt-free scenario, with the dynamics dictated by the specific parameters: depreciation rate ($\delta = 0.1$), capital's output elasticity ($\alpha = 0.8$), total factor productivity ($Z = 0.5$), and fixed dividends ($d = 0.8$). The blue trajectory line depicts the capital accumulation process, calculated with the formula $k_{t+1} = 0.5 \cdot k_t^{0.8} - 0.1 \cdot k_t + k_t - 0.8$, which captures the interplay between capital growth through production, the diminishing returns as capital increases (reflected by the concavity of the curve), and the outflow due to depreciation and dividends. The steady-state capital ($\widehat{k}$), marked by a red dot, indicates the level where the economy naturally gravitates over time. At this point, the firm's investment is precisely calibrated to replace depreciated capital and issue dividends, with no additional net investment. Notably, this equilibrium is a focal point of the system; below this level, capital accumulates, and above it, capital stock adjusts downward, converging back to this stable point. The diagram serves as a visual aid to comprehend the capital dynamics within this economic framework and underscores the balancing act between production, depreciation, and dividend distribution in long-term capital management.

Z, signifying a reduced need for capital.

3.2.3 Dynamics of debts

To examine the dynamics of debt, consider a scenario where capital remains constant $k_t = k_{t+1} = \widehat{k}$, thus it is at the steady-state level. From equation 3.8 we get the difference equation for debt:

$$\Delta b_{t+1} = d - f(\widehat{k}) + \delta\widehat{k} + rb_t \tag{3.17}$$

To derive the steady-state debt level, we'll look for the point where debt doesn't change from one period to the next, which means $\Delta b_{t+1} = 0$. This occurs when:

$$d - f(\widehat{k}) + \delta\widehat{k} + rb_t = 0 \tag{3.18}$$

Solving for the steady-state debt level $\widehat{b}$, we set $b_t = \widehat{b}$ and get:

$$d - f(\widehat{k}) + \delta\widehat{k} + r\widehat{b} = 0 \tag{3.19}$$

where $f(\widehat{k})$ is the output of the firm given the steady-state capital $\widehat{k}$. Since $f(k)$ follows a Cobb-Douglas production function 3.9, then:

$$d - Z\widehat{k}^{\alpha} + \delta\widehat{k} + r\widehat{b} = 0 \tag{3.20}$$

Isolating $\widehat{b}$ to find the steady-state level of debt, we get:

$$r\widehat{b} = Z\widehat{k}^{\alpha} - \delta\widehat{k} - d,$$
$$\widehat{b} = \frac{Z\widehat{k}^{\alpha} - \delta\widehat{k} - d}{r} \tag{3.21}$$

This equation gives us the steady-state level of debt $\widehat{b}$, assuming that the output of the firm is enough to cover depreciation and dividends, with the remaining used to service debt. If output is insufficient, the firm would need to borrow more, and the steady-state debt would be higher. If the output exceeds the depreciation and dividends, the firm can pay down the debt, and the steady-state debt would be lower. Let's determine the condition for a stable path taking the partial derivatives with respect to b_{t-1}:

$$\frac{\partial b_t}{\partial b_{t-1}} = 1 + r \tag{3.22}$$
$$\tag{3.23}$$

Since $r > 0$, the partial derivative will always be greater than one, thus the slope of the difference equation for debt will always be steeper than one. Adding a negative intercept due to positive dividends we get that under those conditions there exists a steady state of debt. Moreover, if the debt is below the steady state, the debt will shrink toward 0, while if the debt is over the steady state the dynamics of debt will explode toward $+\infty$. The phase diagram Figure 3.3 illustrates the relationship between a firm's current debt (b_t) and its capacity for future operations (k_{t+1}), within the context of constant dividends. The steady state is indicated by the red dot, signifying the juncture at which the firm's output is precisely adequate to cover dividends, depreciation, and interest on its steady-state debt. In essence, the graph conveys how steady-state conditions are shaped by dividend policy and productivity, with the former influencing the firm's

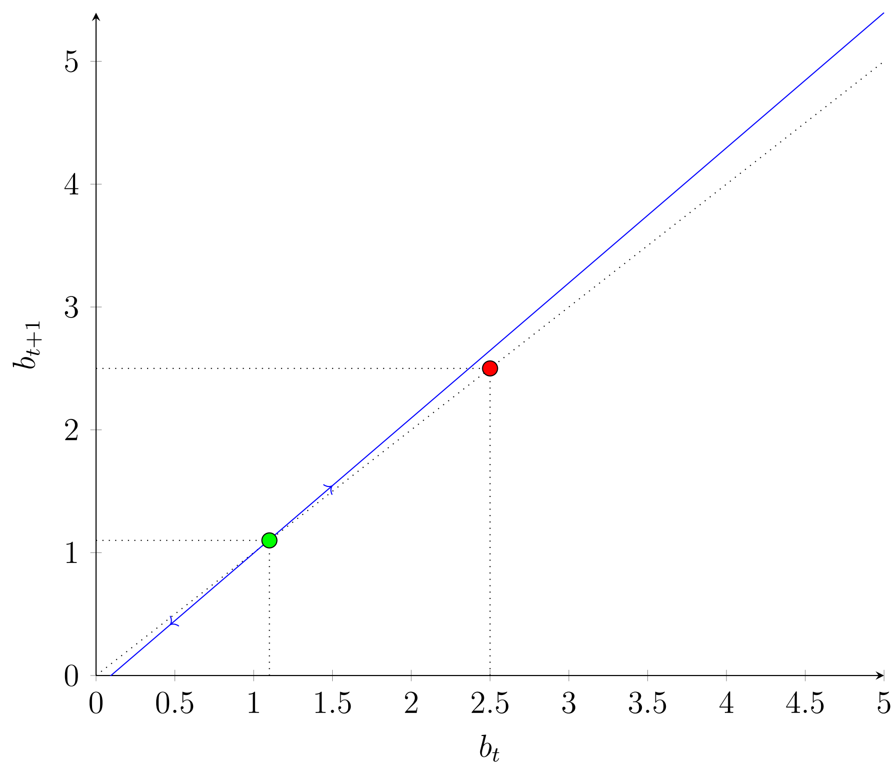


Figure 3.3: The phase diagram dynamically visualizes the firm's debt trajectory given a static capital stock ($\Delta k = 0$), operationalized within a model characterized by the parameters $\delta = 0.1$ (depreciation rate), $r = 0.1$ (interest rate), $\alpha = 0.8$ (capital output elasticity), $Z = 0.5$ (total factor productivity), $d = 0.8$ (constant dividend payout), and a steady-state capital ($\widehat{k} = 3$). The blue line embodies the trajectory of debt, guided by the finite difference equation $b_{t+1} = (1 + r) \cdot b_t - (f(\widehat{k}) - \delta\widehat{k} - d)$, where $f(\widehat{k})$ denotes the firm's output at the steady state of capital. This equation encapsulates the interplay between the interest on existing debt and the firm's obligations due to dividends and depreciation. The red dot marks the threshold beyond which debt cannot exceed capital, effectively serving as a limit on debt. The green dot represents the equilibrium or steady-state debt level, where the firm's financial obligations are perfectly balanced with its repayment capacity. The distance between the red and green dots illustrates the magnitude of the firm's equity buffer, serving as a measure of financial health and resilience against market fluctuations. This diagram is pivotal in understanding how dividend policies and productivity rates interlink to shape the firm's leverage strategy and long-term financial stability.

financial leverage and the latter determining its capital efficiency.

3.3 Ramsey-Cass-Kooopmans reinterpreted

This section outlines the intertemporal maximization problem faced by the firm in the absence of debt, which is a Ramsey-Cass-Kooopmans revisited model, where there is a firm that seeks to maximize the utility of dividends consumption of the shareholders. The objective function is:

$$V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t),$$

where $U' > 0, U'' < 0$.

3.3.1 Steady State derivation

Let's assume that the firm's investment is entirely financed by equity ($b_t = 0$ for all t), this leads to a simplified flow-of-funds constraint equation:

$$k_{t+1} = k_t(1-\delta) + f(k_t) - d_t. \tag{3.24}$$

The maximization problem is tackled using a Lagrangian method, where the Lagrangian is defined as:

$$L_0 = \sum_{t=0}^{+\infty} \left[\beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - k_t(1-\delta) - f(k_t) + d_t\right]\right].$$

The first-order conditions for d_t, k_t, for all periods $t = 0, 1, \ldots$ yield:

$$U'(d_t) = \lambda_t, \quad \forall t,$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1}[f'(k_{t+1}) + (1-\delta)], \quad \forall t,$$

This approach delineates the optimal strategy for dividend distribution and capital allocation in a debt-free case. In the infinite horizon model, the transversality condition reads:

$$\lim_{t\to\infty} \beta^T U'\left(d_t\right) k_{T+1} = 0 \tag{3.25}$$

From these first-order conditions (FOCs), we derive the Euler equation for dividends:

$$U'(d_t) = \beta U'(d_{t+1})[f'(k_{t+1}) + (1-\delta)] \tag{3.26}$$

indicating that the marginal utility of distributing 1 unit of output as dividends at time t should match the discounted marginal utility of not distributing dividends in t,saving, investing in t, using the additional capital to produce and distribute the corresponding dividends at time t+1.

Steady state condition for dividends Imposing the steady state condition for dividends $d_t = d_{t+1} = \widehat{d}$ in 3.26 , we equate the marginal utilities across two consecutive periods:

$$\begin{aligned} U'(d_t) &= U'(d_{t+1}), \\ \frac{1}{\beta} &= f'(k_{t+1}) + (1-\delta), \end{aligned} \tag{3.27}$$

This condition is satisfied if:

$$f'(k_{t+1}) = \frac{1}{\beta} - (1-\delta), \tag{3.28}$$

Using the Cobb Douglas production function and take the derivative respect to capital, we get:

$$f'(k_{t+1}) = Z\alpha k_{t+1}^{\alpha-1}, \tag{3.29}$$

From 3.27, we get:

$$\begin{aligned} \frac{1}{\beta} &= 1 - \delta + Z\alpha k^{\alpha-1}, \\ Z\alpha k^{\alpha-1} + (1-\delta)\beta &= 1, \\ \widehat{k} &= \left[\frac{\alpha\beta Z}{1-\beta\,(1-\delta)}\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{3.30}$$

The locus of points on the (k_t, d_t) plane such that dividends are constant is therefore $k_t = \hat{k}$, whose representation in the vertical line in figure 3.4.

Steady state condition for capital Imposing steady state condition for capital $(k_t = k_{t-1} = \widehat{k})$ in the law of motion of capital 3.24 we get:

$$\widehat{d} = f(k_t) - \delta k_t \tag{3.31}$$

This locus represents the set of points where the capital stock remains constant over time. Hence, we can determine the level of dividends that ensures both capital and dividends are maintained at steady-state levels. By incorporating the steady-state level of capital from Equation 3.30 and the production function from Equation 3.9 into the equilibrium condition for capital from Equation 3.31, the steady-state level of dividends, denoted as $\widehat{d}$, can be deduced:

$$\widehat{d} = Z\left(\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \tag{3.32}$$

In this manner, we ascertain the steady-state levels for both capital and dividends.

3.3.2 Phase diagram

In this section, we will plot the phase diagram for capital and dividends exploiting steady-state conditions for capital and dividends.

The graph Figure 3.4 portrays the dynamics of dividends (d_t) in relation to the capital (k_t) of a firm, with a particular focus on the behavior when capital is below or above the steady-state level, denoted by $\hat{k}$.

When the capital is below the steady-state level ($k_t < \hat{k}$), thus on the left of the vertical line, for the firm is optimal to increase dividends over time ($d_t < d_{t+1}$) as represented by the arrow pointing upward. When instead ($k_t > \hat{k}$), dividends must shrink over time ($d_t > d_{t+1}$). Lets look at the locus in which capital is stationary $\Delta k = 0$ is given by the f-of-f constraint 3.31:

$$d_t = f(k_t) - \delta k_t \tag{3.33}$$

In our case, as obtained in the above section, the locus in which capital is stationary becomes 3.31:

$$d_t = Zk_t^{\alpha} - \delta k_t \tag{3.34}$$

This function starts at the origin since ($f(0) = 0$), with a maximum in $\underline{k}$ (defined

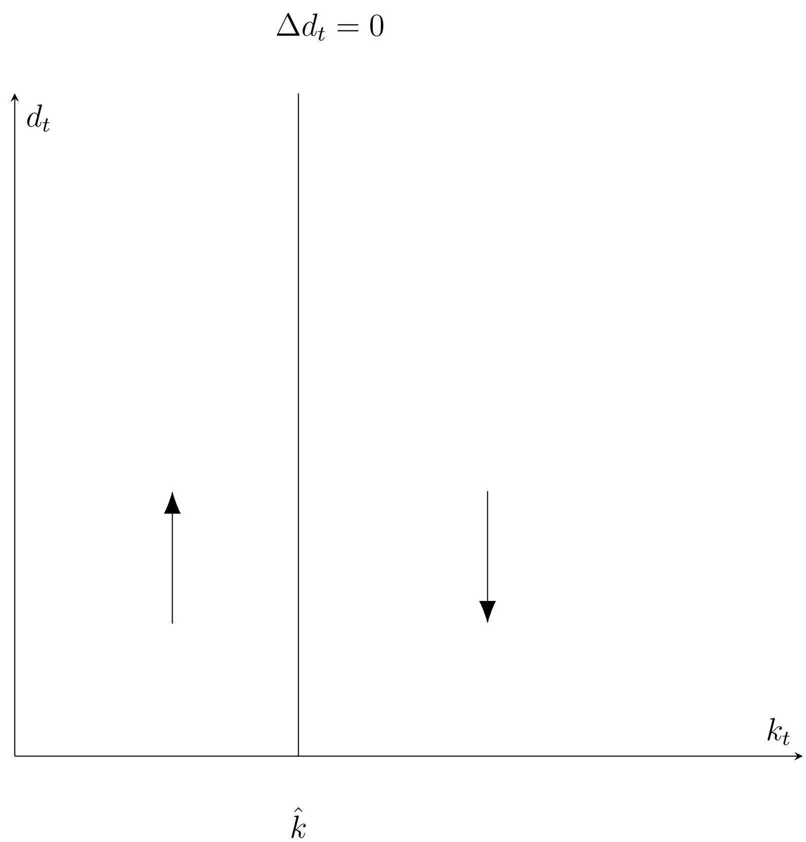


Figure 3.4: The phase diagram illustrates locus of points such that $d_t = d_{t-1}$. The vertical line at $\hat{k}$ represents the steady-state level of capital, where the rate of change in dividends Δd_t is zero. To the left of $\hat{k}$, where capital is below its steady-state level, the firm increases dividend payments. Conversely, to the right of $\hat{k}$, where capital exceeds the steady state, dividend payments decrease.

as capital level such that $f'(\underline{k}) = \delta$). The level of capital $\underline{k}$ is:

$$\underline{k} = \left[\frac{\alpha Z}{\delta}\right]^{\frac{1}{1-\alpha}} \tag{3.35}$$

It is easy to see that $\hat{k} < \underline{k}$:

$$\begin{aligned}
\frac{\alpha\beta Z}{1-\beta+\beta\delta} &< \frac{\alpha Z}{\delta}, \\
\frac{\beta}{1-\beta+\beta\delta} &< \frac{1}{\delta}, \\
\beta\delta &< 1-\beta+\beta\delta, \\
1-\beta &> 0 \\
\text{for definition} \quad &\blacksquare
\end{aligned}$$

We denote $\bar{k}$ the capital level such that $d_t = 0$, thus it's obtained by solving $f(\bar{k}) - \delta\bar{k} = 0$, using the Cobb-Douglas production function 3.9, we get:

$$\begin{aligned}
Z\bar{k}^{\alpha} &= \delta\bar{k}, \\
\bar{k} &= \left[\frac{Z}{\delta}\right]^{\frac{1}{1-\alpha}}
\end{aligned} \tag{3.36}$$

It is easy to see that $\underline{k} < \bar{k}$, since:

$$\begin{aligned}
\left[\frac{Z}{\delta}\right]^{\frac{1}{1-\alpha}} &> \left[\frac{\alpha Z}{\delta}\right]^{\frac{1}{1-\alpha}}, \\
\frac{Z}{\delta} &> \frac{\alpha Z}{\delta}, \\
1 &> \alpha \\
\text{for definition} \quad &\blacksquare
\end{aligned} \tag{3.37}$$

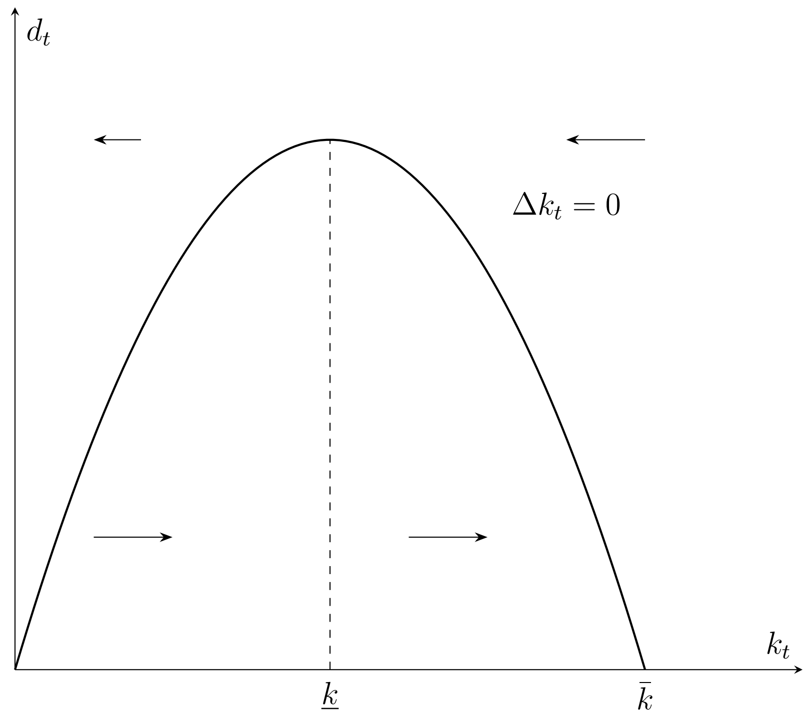


Figure 3.5: This phase diagram displays the set of points at which the capital stock k_t is unchanging from one period to the next ($k_t = k_{t-1}$). The capital level at $\underline{k}$ represents the maximum sustainable dividend payout without affecting the capital stock. Arrows above the curve pointing leftward indicate a reduction in capital resulting from dividend levels that exceed the sustainable steady state, while arrows pointing to the right below the curve signify the accumulation of capital due to dividend levels that are below the steady state, leading to an increase in capital stock over time.

Considering transitivity, the sequence for $\underline{k}, \hat{k}, \bar{k}$ is:

$$\hat{k} < \underline{k} < \bar{k}$$

For a given level of capital $k_0 \in [0, \bar{k}]$, the corresponding dividends level that guarantee the stationarity of capital is:

$$d_0 = f(k_0) - \delta k_0$$

If the firm distributes more dividends than d_0 the capital stock must decrease over time: since dividends are too high the firm is consuming part of her capital. More precisely the firm is distributing more dividends than d_0, which guarantees that the difference between production, and dividends is exactly equal to the replacement of depreciated capital. This behavior is represented by the arrows above the curve pointing to the left. If the firm distributes less dividends than d_0, the opposite happens: the firm increases its capital since there is a positive net investment. This behavior is represented by the arrows below the curve pointing toward the right.

Steady state for capital and dividends Plotting both loci we get a phase diagram that represents the condition for stationarity. Notice that there exits 3 steady states: one at the origin due to the assumption $f(0) = 0$, the point $(\bar{k}; 0)$, and finally point B $= (\hat{k}, \hat{d})$. Point B was obtained by equations 3.30 and 3.31 and represented the point in which dividends and capital are at a steady state, and both are strictly positive. This point is a saddlepoint. The blue line depicts a possible saddlepath towards B. Starting at A, the firm chooses exactly the dividend level that leads to the stationary point B. This path not only fulfills the difference equations 3.26 and 3.31, but also, the transversality condition 3.25 Indeed as $t \to \infty$, capital and dividends approach their steady-state level which are both positive and finite, thus the marginal utility of dividends at $\hat{d}$ is also finite, hence 3.25 is valid.

To conclude, this section has outlined the derivation of steady-state levels for capital and dividends, and these conditions have been visually represented in a phase diagram. The upcoming section will undertake a similar analysis but will incorporate debt and financial friction into the model. Additionally, it is important to note that in the steady state, where output $\hat{y}$ is given by $f(\hat{k})$, the production level is exclusively influenced by the productivity parameter Z, signifying that financial elements do not alter the

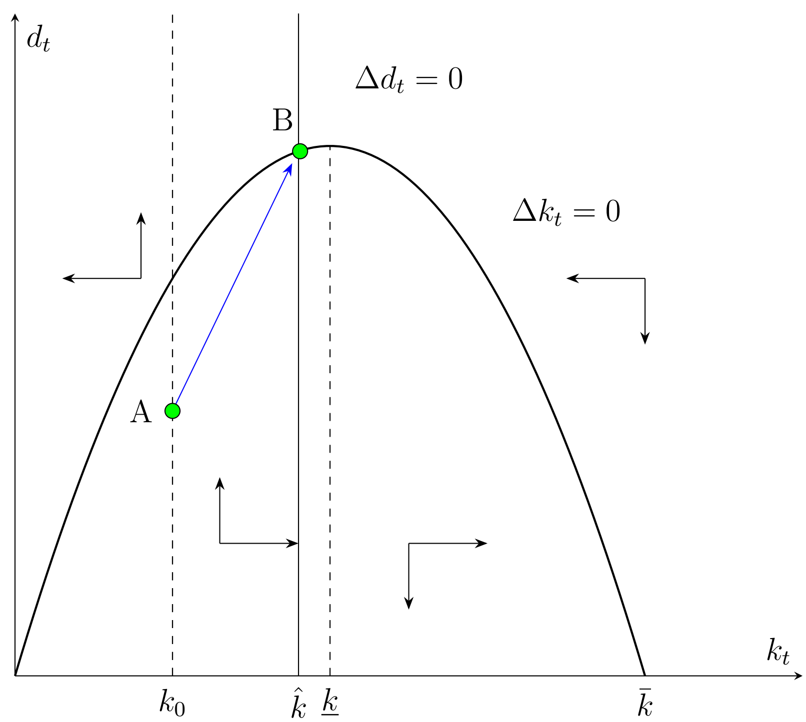


Figure 3.6: The phase diagram visualizes the relationship between dividends and capital. The vertical line marks where the dividend level remains constant, and the solid concave curve traces where the capital remains constant. Point B indicates the equilibrium where both dividends and capital are stationary, with the steady-state capital at $\hat{k}$. This value of $\hat{k}$ is notably lower than $\underline{k}$, which is the capital level that would maximize dividends while maintaining a steady capital stock. $\bar{k}$ represents the capital quantity at which the system reaches a stationary state for k with zero dividends. The arrows illustrates the dynamics of dividends and capital in case of the firm finding itself outside the stable loci. Point A is a possible starting point (k_0, d_0). The blue arrows indicate a potential trajectory, known as a saddlepath, leading towards the saddlepoint B.

fundamental connection between the production function and output.

3.4 Introducing financial frictions

In this section we tackle the maximization problem of the firm, introducing the possibility of financing through debt and two types of financial frictions. The first financial friction is a financing constraint, which implies fixed leverage for the firm. The second financial friction is introducing a participation constraint with monitoring cost for the financial intermediaries. The goal is to understand how those frictions affect the steady state of capital and dividends.

3.4.1 Participation constraint of the financial intermediaries

The subsection delves into the constraints facing financial intermediaries within the model. Initially, the model assumed an exogenous interest rate, unaffected by the volume of debt, leading to an unrealistic scenario where interest rates remain constant. To address this, the model incorporates a financial market in which the interest rate is set based on market-clearing conditions, with financial intermediaries functioning in a perfectly competitive environment aimed at profit maximization.

According to Bernanke and Gertler [1986], lending should yield a profit equivalent to the opportunity cost. Lenders earn interest plus the principal if borrowers repay successfully (with probability p) or acquire the firm's production assets (net of monitoring cost: $1-\mu$) in case of bankruptcy. Moreover, $(1-\mu)f(k_t)$represents monitoring cost. The lender's participation constraint is:

$$R_t \cdot b_t p + (1-p)\mu f(k_t) = R_f b_t,$$

where R_f represents the risk-free rate, the opportunity cost of lending. This framework allows for the derivation of the interest rate as a function of p,$f(k_t)$,μ,t and R_f.

The participation constraint can be rewritten as:

$$R_t = \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t}. \tag{3.38}$$

Investigating the asymptotic behavior of R_t with respect to p provides critical economic insights. Specifically, as $p \to 0$, indicating an increasingly likely default, R_t escalates without bound, reflecting the infinitely rising premium a lender would require to counterbalance the heightened risk. Conversely, as $p \to 1$, the default risk vanishes, and R_t converges to R_f, the risk-free rate, consonant with the absence of default risk necessitating no premium above the risk-free return. When examining the behavior of the interest rate formula as the debt amount b_t trends towards infinity, the resulting limit is given by:

$$\lim_{b_t \to +\infty} \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t} = \frac{R_f}{p}$$

As the denominator b_t, representing the total debt, increases without bound, the term involving the firm's productive output adjusted for recovery rate ($\mu f(k_t)/b_t$) diminishes to zero. This indicates that, in scenarios of very large debt volumes, the fraction of recoverable assets ($\mu f(k_t)$) compared to the outstanding debt becomes negligible in determining the interest rate R_t.

Therefore, under such conditions, the interest rate formula simplifies to $\frac{R_f}{p}$, underscoring that the loan's interest rate is fundamentally determined by the risk-free rate adjusted for the probability of successful repayment p, independent of the firm's asset recoverability or productivity levels. This elucidates an important economic principle: for vast sums of debt, the critical factors shaping lending rates pivot away from the specifics of asset recovery towards broader financial metrics—namely, the prevailing risk-free rate and the inherent risk of default.

The breakeven threshold for R_t, where the return on the loan is nonnegative, is formalized by the condition:

$$R_f b_t \geq b_t p + (1-p)\mu f(k_t). \tag{3.39}$$

This inequality dictates that to achieve a return rate on the loan that is greater than or equal to one, the opportunity cost of capital must be greater than expected profit from the firm's production, adjusted for default probability and asset recoverability post-default. This ensures the lender receives a return that compensates for the risk undertaken, juxtaposed with the risk-free alternative. Let's present a specific scenario for clarity: Imagine we have a constant capital level k, and the recovery rate in case of default μ is complete, meaning $\mu = 1$. In this instance, the depreciation rate δ is 0.1, the output elasticity of capital α is set at 0.8, while the technology factor Z is 0.5. Additionally, the firm's dividends level d is 0.8, and the capital stock at time t, k_t, equals 4. The risk-free rate R_f is given as 1.05.

Within this framework, consider two firms that are identical in every aspect except for their probability of returning profit, denoted by p. For the first firm, p is 0.95, suggesting a 5% default risk. For the second firm, p drops to 0.9, indicating a higher default risk of 10%. The graphical representation suggests that as debt levels increase, so do interest rates, reflecting the risk-return dilemma for lenders. A higher risk profile, denoted by a more elevated red line necessitates greater returns to compensate for default risks. It's important to note that while the the graph assumes constant capital, real-world scenarios often see debt increases leading to higher capital and, consequently, greater production capacities.

The analysis indicates that the blue curve, representing the rate of return for a riskier firm, is positioned above the red curve under a specific condition:

$$\mu f(k_t) < R_f b_t \tag{3.40}$$

This condition implies that the net output from employing capital k_t, after accounting for monitoring costs, must be less than the opportunity cost associated with lending the sum b_t. Should this not hold true, firms with a higher risk profile would end up incurring lower interest charges on their loans. To elucidate, consider the following scenario where p_h, denoting the probability of avoiding default for the less risky firm, is higher than p_l, the corresponding probability for the riskier firm. All other factors are assumed to remain constant across both firms:

$$\begin{aligned}
\frac{R_f}{p_h} - \frac{1-p_h}{p_h}\frac{\mu f(k_t)}{b_t} &< \frac{R_f}{p_l} - \frac{1-p_l}{p_l}\frac{\mu f(k_t)}{b_t}, \\
\left[R_f - (1-p_h)\mu\frac{f(k_t)}{b_t}\right] p_l &< \left[R_f - (1-p_l)\mu\frac{f(k_t)}{b_t}\right] p_h, \\
\left[-(1-p_h)p_l + (1-p_l)p_h\right]\mu\frac{f(k_t)}{b_t} &< R_f(p_h - p_l), \\
\mu\frac{f(k_t)}{b_t} &< R_f, \\
\mu f(k_t) &< R_f b_t.
\end{aligned}$$

This sequence of steps demonstrates that for the gross rate of return on loans to be positioned favorably for riskier firms, the expected output from the investment (after subtracting monitoring costs) must not exceed the lending opportunity cost.

The graph 3.7 captures the dynamics between the debt stock b_t and the interest rate applied to the loan r_t. An increase in the debt stock leads to a rise in the interest

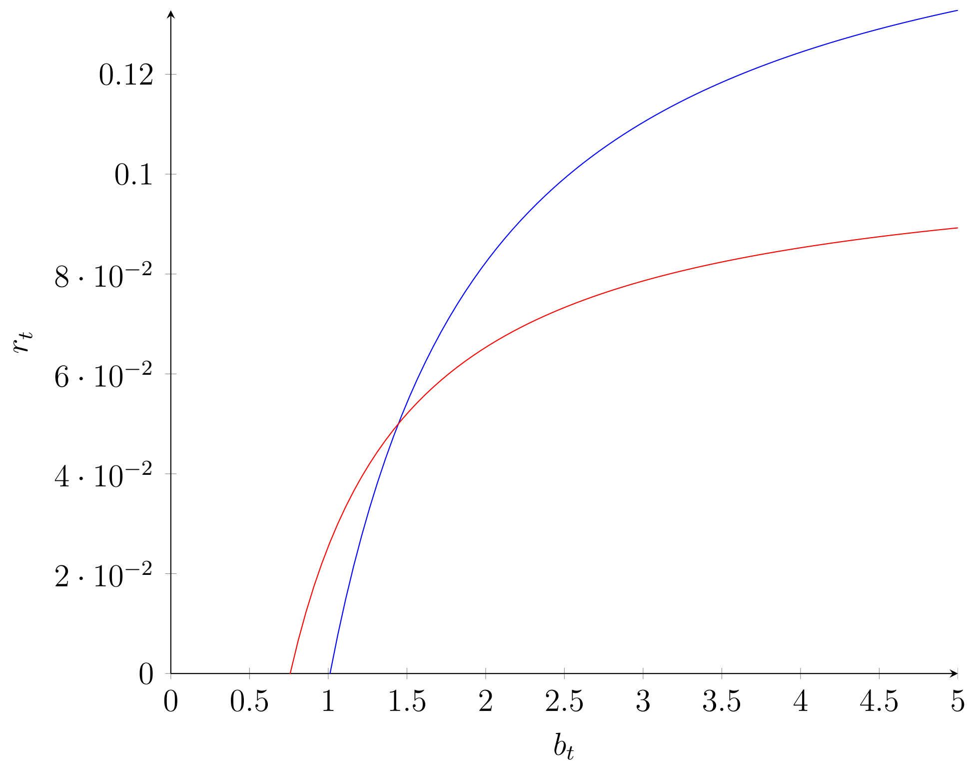


Figure 3.7: The figure presents a graphical analysis of the returns on loans as a function of the loan amount under a fixed capital level of $k = 4$. The red curve models the scenario where the default risk probability is $1 - p = 0.05$, implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1 - p = 0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the debt stock grows. Notably, the opportunity cost of capital is maintained at 0.05 for the red one, while at 0.1 for the higher risk curve.

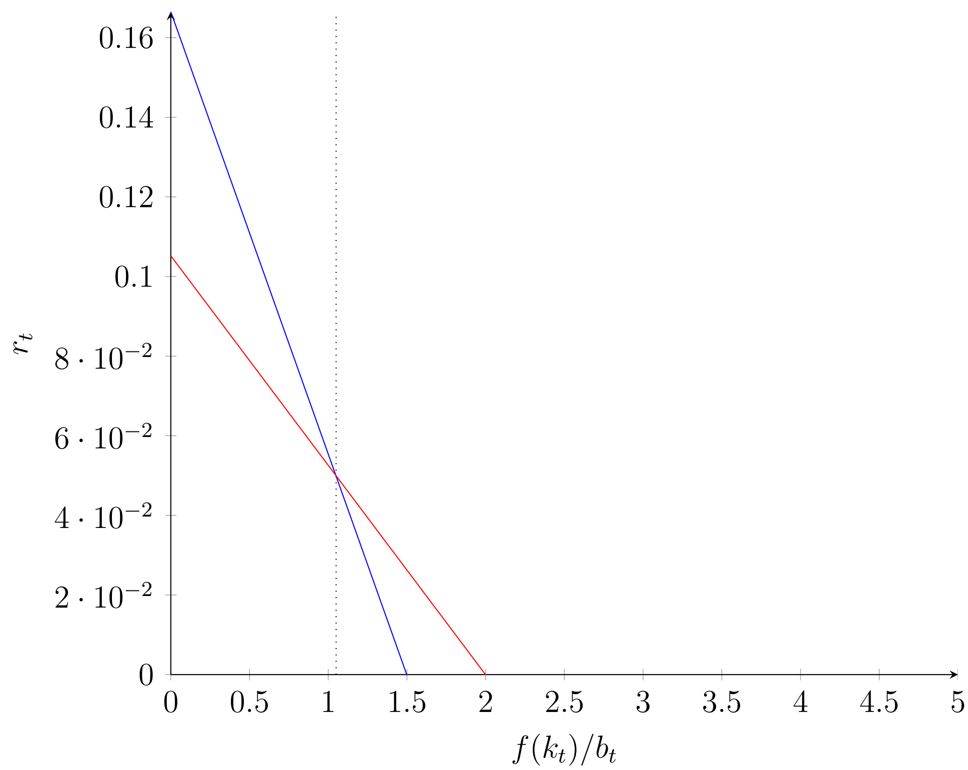


Figure 3.8: The figure presents a graphical analysis of the returns on loans as a function of the production over the debt level. The red curve models the scenario where the default risk probability is $1-p=0.05$, implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1-p=0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the production-debt ratio grows. Notably, the opportunity cost of capital is maintained at 0.05.

rate, reflecting the augmented risk perceived by lenders. Displayed are two distinct lines: one representing a riskier loan with a higher probability of default and the other indicating a safer loan with a lower default probability. As anticipated, the riskier loan scenario is characterized by a curve that lies above, dictating higher interest rates, if $\mu f(k_t) < R_f b_t$ holds.

An alternative approach to illustrating the participation constraint of financial intermediaries involves graphing the gross interest rate, R, against the ratio of output to debt, $\frac{f(k)}{b}$, directly. The graph 3.8 delineates a critical boundary within the participation constraint framework: as leverage approaches unsustainable levels, the interest rate escalates to a certain peak, signifying a cap on the maximum interest rate that deviates from the theoretical possibility of infinity. This ceiling on the rate is attributed to the fact that the probability of default, denoted by p, remains fixed and does not escalate alongside increasing leverage.

Ultimately, the participation constraint internalizes the interest rate of a loan as a function of the leverage, the opportunity cost of capital, and the default risk probability. By integrating this mechanism into the flow of funds model, the impact of debt on capital is mediated through the variable R, establishing a feedback loop where financial leverage influences and is influenced by the cost of borrowing.

3.4.2 The problem of the firm in presence of financial frictions

The firm's objective is to maximize:

$$\max_{\{d_t\}_{t=0}^{+\infty}} V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t)$$

subject to:

1. the flow of funds constraint: $I_t + Rb_t + d_t = f(k_t) + b_{t+1}$
2. the investment function $I_t = k_{t+1} - k_t\,(1-\delta)$
3. the financing constraint $b_t = lk_t$
4. the participation constraint of borrower $R_t = \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t}$

Consolidating the constraints we get the flow of funds constraints:

$$\begin{aligned} k_{t+1} &= \left\{ k_t(1-\delta) - \left[\frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{lk_t}\right] \cdot lk_t + f(k_t) - d_t \right\} (1-l)^{-1} \\ k_{t+1} &= \left[\frac{p+\mu-\mu p}{p} f(k_t) + \frac{p-\delta p - R_f l}{p} k_t - d_t\right](1-l)^{-1} \end{aligned} \tag{3.41}$$

The Lagrangian for this optimization problem is formulated as:

$$L = \sum_{t=0}^{+\infty} \beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - \frac{p+\mu-\mu p}{p} f(k_t) - \frac{p-\delta p - R_f l}{p} k_t + d_t \right] (1-l)^{-1}, \tag{3.42}$$

leading to the first-order conditions for optimizing dividends and capital over time:

$$U'(d_t) = \frac{\lambda_t}{(1-l)}, \quad \forall t, \tag{3.43}$$

and the dynamic optimality conditions for capital allocation:

$$\lambda_t = \beta \frac{\lambda_{t+1}}{(1-l)} \left[f'(k_t) \frac{p+\mu-\mu p}{p} + \frac{p-\delta p - R_f l}{p} \right], \quad \forall t. \tag{3.44}$$

This formulation yields the Euler equation for dividends:

$$U'(d_t) = \frac{\beta}{(1-l)} U'(d_{t+1}) \left[f'(k_t) \frac{p+\mu-\mu p}{p} + \frac{p-\delta p - R_f l}{p} \right], \tag{3.45}$$

imposing ($d_t = d_{t+1} = \hat{d}$), we get:

$$\begin{aligned} \frac{(1-l)\,p}{\beta} &= f'(\hat{k})\,(p+\mu-\mu p) + (p - \delta p - R_f l) \\ f'(\hat{k}) &= \frac{p - pl - \beta p + \beta \delta p + \beta R_f l}{\beta\,(p+\mu-\mu p)} \end{aligned} \tag{3.46}$$

using the Cobb Douglas production function 3.29 into 3.46 we get:

$$Z\alpha\hat{k}^{\alpha-1} = \frac{p - pl - \beta p + \beta\delta p + \beta R_f l}{\beta\left(p + \mu - \mu p\right)}$$
$$\hat{k} = \left[\frac{Z\alpha\beta\left(p + \mu - \mu p\right)}{p - pl - \beta p + \beta\delta p + \beta R_f l}\right]^{\frac{1}{1-\alpha}} \tag{3.47}$$

Similar to the scenario without debt, when the firm's capital falls below the steady-state threshold $\hat{k}$, it is advantageous for the firm to incrementally raise its dividend payouts over time. Conversely, when the firm's capital surpasses $\hat{k}$, it would be more beneficial for the firm to gradually reduce dividends. It can be readily demonstrated that when monitoring costs become entirely ineffective $(1-\mu = 1)$, the firm operates without debt $(l = 0)$, and the probability of default is eliminated $(1 - p = 0)$, the resulting capital level aligns with that of the debt-free situation as specified in equation 3.30:

$$\hat{k} = \left[\frac{Z\alpha\beta\left(1 + 1 - 1\right)}{1 - 0 - \beta + \beta\delta + 0}\right]^{\frac{1}{1-\alpha}},$$
$$\hat{k} = \left[\frac{Z\alpha\beta}{1 - \beta(1 - \delta)}\right]^{\frac{1}{1-\alpha}}. \quad \blacksquare$$

Imposing s.s. condition for capital $(k_t = k_{t+1} = \hat{k})$ into the flow of funds constraint 3.41:

$$\widehat{d} = \frac{p + \mu - \mu p}{p} f(\hat{k}) - \left(\frac{lR_f + \delta p - lp}{p}\right)\hat{k} \tag{3.48}$$

It can be straightforwardly demonstrated that by setting the monitoring cost to $1-\mu = 1$, eliminating debt with $l = 0$, and removing the risk of default by setting $1-p = 0$, we arrive at an identical level of dividends as observed in the scenario without debt 3.31.

3.4.3 Phase diagram

The goal of this section is to portray the phase diagram in two cases: one with monitoring costs and one without. However, we will use a less heuristic approach compared to the phase diagram of the free debt case, using parameters similar to Osotimehin and Pappadà [2017]:

Parameter	Symbol	Value
Discount factor	β	0.956
Risk-free rate	R_f	1.04
Depreciation rate	δ	0.07
Returns to scale	α	0.80
Aggregate productivity	$\bar{Z}$	0.5
Monitoring cost	$1-\mu$	$0, 0.75$
Productivity	Z	0.2
Probability of default	$1-p$	0.6

Table 3.1: Parameters

Moreover, we assume a fixed leverage of $l = 0.8$, since for the moment we want to understand the effect of monitoring cost leaving all the other parameters equal.

The phase diagram illustrated in 3.9 depicts the capital accumulation dynamics under scenarios of fixed leverage and varying monitoring costs. While the overall dynamics remain consistent across both scenarios, the equilibrium capital level is notably reduced in firms that incur monitoring costs, in contrast to those without such costs. As a result, firms with monitoring costs settle into a steady state equilibrium for dividends, which leads to diminished dividend distributions compared to firms that do not bear these costs.

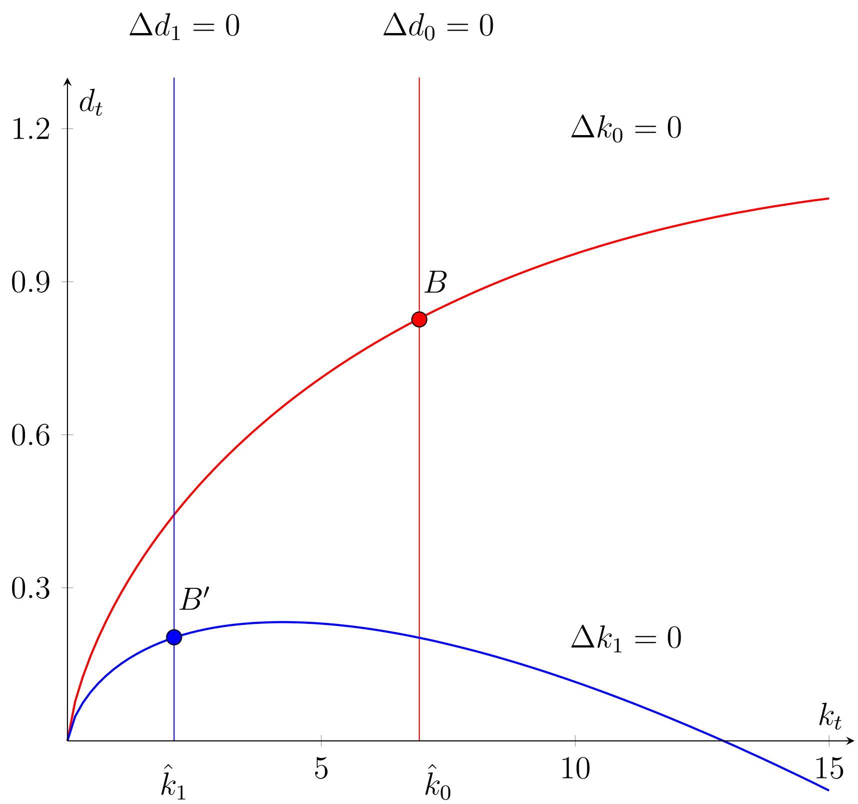


Figure 3.9: This phase diagram illustrates the relationship between dividends (d_t) and capital (k_t) for two different scenarios of a firm. The red curve represents a firm (Firm 0) that has debt but incurs no monitoring costs, while the blue curve represents another firm (Firm 1) that does have monitoring costs. The curves, labeled $\Delta d_1 = 0$ and $\Delta d_0 = 0$, represent the loci where dividends remain constant. The vertical lines, labeled $\Delta k_1 = 0$ and $\Delta k_0 = 0$, denote the loci where capital is stationary. The intersection point B on the red curve indicates the equilibrium for Firm 0 where both capital and dividends are stationary, located at $(7.25, 0.9)$. In contrast, the intersection point B' on the blue curve represents the corresponding equilibrium for Firm 1 with monitoring costs, positioned at $(2.5, 0.28)$. It is evident that Firm 1, which bears monitoring costs, sustains lower dividends while keeping capital constant, and also requires a lower level of capital to maintain constant dividends when compared to Firm 0.

3.5 Finding optimal path

Addressing the dynamic optimization problem with an initial condition k_0, we employ a logarithmic utility function and frame the issue through a Bellman equation:

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U(d_0) + \beta \left[\sum_{t=1}^{\infty} \beta^{t-1} U(d_t) \right] \right\}$$

subject to a dynamic capital accumulation constraint:

$$k_{t+1} = \left[\frac{p + \mu - \mu p}{p} f(k_t) + \frac{p - \delta p - R_f l}{p} k_t - d_t \right] \cdot (1 - l)^{-1} \ \forall t,$$

The aim is to determine the optimal dividend strategy d_t^* and the consequent capital levels k_{t+1}^* across all periods. The optimal policy $d_t^\star = \varphi(k_t)$ links dividends and capital in a time-invariant manner, deduced from the constraint:

$$k_{t+1} = \left[\frac{p + \mu - \mu p}{p} f(k_t) + \frac{p - \delta p - R_f l}{p} k_t - \varphi(k_t) \right] \cdot (1 - l)^{-1}$$

Given the continuous and differentiable nature of capital and dividends, the optimal dividend path can be represented as a function of initial capital, thereby defining the maximum value function $V(k_1)$ in terms of overall utility maximization. The revised problem formulation becomes:

$$
\begin{aligned}
&V(k_0) = \max_{d_0} \{U(d_0) + \beta V(k_1)\} \\
&\text{s.t. } k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - \varphi(k_0)\right] \cdot (1-l)^{-1} \\
&k_0 \text{ given.}
\end{aligned}
\tag{3.49}
$$

The method of "guess and verify" involves working through the transition equation defined as:

$$
k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - d_0^*\right] \cdot (1-l)^{-1}
$$

The first order condition (FOC) is specified as $d_0^* = [\beta V'(k_1)]^{-1}$. When this FOC is incorporated into the transition equation, the formulation of the problem becomes a system of equations outlined as follows:

$$
\begin{cases}
V(k_0) = U(d_0^*) + \beta V(k_1), \\
k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - d_0^*\right] \cdot (1-l)^{-1}, \\
d_0^* = [\beta V'(k_1)]^{-1}, \\
k_0 \text{ given.}
\end{cases}
$$

Our initial guess for the solution is:

$$
V(k_t) = e + f \ln k_t,
$$

leading to a refined system:

$$e + f\ln(k_0) = \ln\left(\frac{k_1}{\beta f}\right) + \beta\left[e + f\ln(k_1)\right] \tag{3.50}$$

$$k_1 = \left\{\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - \left[\frac{k_1}{\beta f}\right]\right\} \cdot (1-l)^{-1}.$$

Solving the above system we find:

$$k_1 = \frac{\beta f}{\beta f(1-l)+1}\left(\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0\right) \tag{3.51}$$

Using $d_0^* = k_1(\beta f)^{-1}$ in the above equation we get:

$$d_1 = \frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0\right\} \tag{3.52}$$

Assuming $p - \delta p - R_f l = 0$, and using the Cobb-Douglas production function 3.11sim-plifies to:

$$\begin{aligned}
k_1 &= \frac{\beta f}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\}\\
d_1 &= \frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\}\\
e + f\ln(k_0) &= \ln\left\{\frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\}\right\}\\
&\quad + \beta\left\{e + f\ln\left[\frac{\beta f}{\beta f(1-l)+1}\left(\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right)\right]\right\},\\
e + f\ln(k_0) &= -\ln\left(1+(1-l)\beta f\right) + \ln\left(p+\mu-\mu p\right) - \ln p +\\
&+ (\alpha\ln Z + \alpha\ln k_0) + \beta e - \beta f\ln\left(1+(1-l)\beta f\right) +\\
&+ \beta f\ln\left(p+\mu-\mu p\right) - \beta f\ln p + \beta f\ln(\beta f) + \beta f(\alpha\ln Z + \alpha\ln k_0)
\end{aligned}$$

The above equation must be satisfied for any k_0 and for any admissible value of the parameters $Z, \beta, \alpha, \mu, p, R_f, l$. Therefore, must be true that:

$$f = \frac{\alpha}{1-\alpha\beta} \tag{3.53}$$

$$e = [-\ln(1+(1-l)\beta f) + \ln(p+\mu-\mu p) - \ln(p)$$
$$+ \alpha\ln(Z) - \beta f \ln(1+(1-l)\beta f)$$
$$+\beta f \ln(p+\mu-\mu p) - \beta f \ln(p) + \beta f(\alpha\ln(Z)) + \beta f \ln(\beta f)]\,(1-\beta)^{-1}$$

Therefore, the transition and policy functions under financial frictions can be articulated as follows:

$$k_1 = \left[\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right] \frac{\alpha\beta}{1-l\alpha\beta}, \tag{3.54}$$

Utilizing $d_0^* = k_1(\beta f)^{-1}$ and in conjunction with Equation 3.53, we can deduce:

$$d_0^* = \left[\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right] \frac{1-\alpha\beta}{1-l\alpha\beta}, \tag{3.55}$$

By setting $k_1 = k_0 = \hat{k}$ in Equation 3.54, the equation simplifies to:

$$\hat{k} = \left(\frac{Z\alpha\beta(p+\mu-\mu p)}{p(1-\alpha\beta l)}\right)^{\frac{1}{1-\alpha}} \tag{3.56}$$

This expression elucidates the steady-state capital level, $\hat{k}$, in relation to the parameters α, β, l, p, μ, and Z.

To derive the optimal trajectories for capital and dividends, as well as the steady-state locus for capital in the context of no debt and financial friction (assuming $l = 0$,

$\mu = 1$, and $p = 1$), the following expressions are obtained:

$$k_1 = Zk_0^{\alpha}\alpha\beta, \tag{3.57}$$

$$d_0^* = Zk_0^{\alpha}(1 - \alpha\beta), \tag{3.58}$$

$$\hat{k} = [Z\alpha\beta]^{\frac{1}{1-\alpha}}. \tag{3.59}$$

In conclusion, the optimal capital and dividend paths have been derived under specific financial conditions. Key assumptions, particularly $p - \delta p - R_f l = 0$, critically simplify the dynamics, implying that the net effect of the discount rate, the probability of repayment, and the cost of debt on capital growth is neutral. This assumption allows for a direct relationship between the capital level and the dividend policy, as depicted by the transition and policy functions. The closed-form expressions for k_1, d_1, and $\hat{k}$ emphasize the steady-state capital level's sensitivity to these parameters, underscoring the pivotal role of p in shaping the firm's financial trajectory. Under the no-debt and no-financial-friction scenario ($l = 0$, $\mu = 1$, $p = 1$), the model further reflects how the absence of these frictions leads to straightforward growth dynamics, reinforcing the importance of these financial parameters in determining the firm's optimal strategies.

3.6 Simulation Study

To explore the distinctions between scenarios with and without financial frictions, we conduct a simulation exercise employing parameters similar with those used in the Osotimehin and Pappadà [2017] study:

Parameter	Symbol	Value
Discount factor	β	0.956
Risk-free rate	R_f	1.04
Depreciation rate	δ	0.07
Returns to scale	α	0.70
Aggregate productivity	$\bar{Z}$	1

Table 3.2: Benchmark calibration

This section presents a simulation study to compare scenarios with and without financial frictions, employing parameters from the Osotimehin and Pappadà [2017] study. The simulation explores the effects of financial frictions by setting different monitoring costs for two firms, with firm 0 facing no monitoring cost and firm 1 experiencing a monitoring cost of $25\%\mu = 0.75$. The key assumption for the simulation is $p - \delta p - R_f l = 0$, leading to a calculated probability of repayment (p) of approximately 0.559 for both firms.

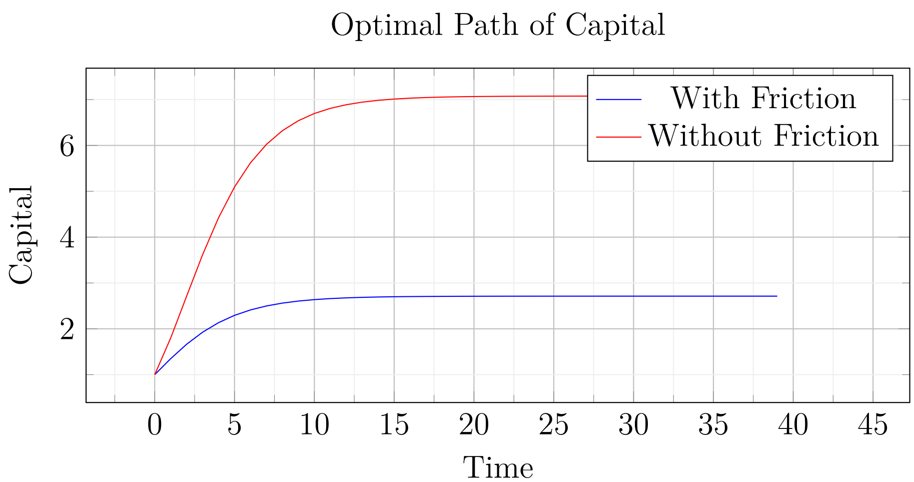


Figure 3.10: The plot depicts the optimal path for capital following equation 3.54. The red line, which is above the blue line for all periods, represents the optimal path for captial toward the steady state level, in absence of monitoring costs. While the optimal trajectory for the firm facing 25% of monitoring cost is depicted by the blue line. All the other parameters, exception for monitoring cost, are left equal. Clearly monitoring costs affects affects the steady state by lowering it.

Figure 3.10 explicitly demonstrates the impact of monitoring costs on reducing the steady-state capital level. The observed discrepancy in steady-state capital levels between the two firms is quantified as follows:

$$\hat{k_0} = 7.077,$$

$$\hat{k_1} = 2.713,$$

$$\Delta\hat{k} = \hat{k_0} - \hat{k_1} = 4.364.$$

Hence, imposing a 25% monitoring cost on the output, under a fixed leverage ratio of 0.5, results in a 69% reduction in the steady-state level of capital.

Additionally, Figure 3.11 illustrates the optimal dividend paths for the same two firms, one burdened by monitoring costs and the other not. Following the policy function detailed in Equation 3.55, derived from the solution to the Bellman equation.

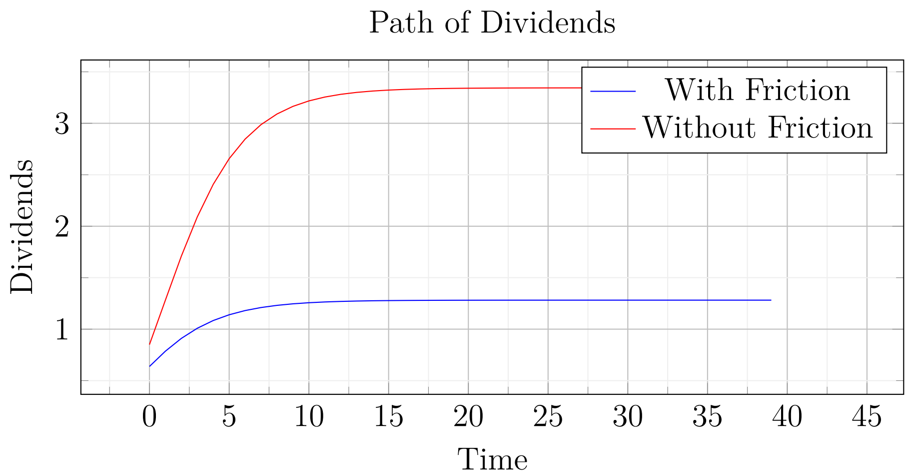


Figure 3.11: The plot depicts the optimal path for dividends following equation 3.55. The red line, which is above the blue line for all periods, represents the optimal path toward the steady state level, in absence of monitoring costs. While the optimal trajectory for the firm facing 25% of monitoring cost is depicted by the blue line. All the other parameters, exception for monitoring cost, are left equal. Clearly monitoring costs affects affects the steady state by lowering it.

The dividend trajectory is notably higher in scenarios without financial frictions, emphasizing the significant role these frictions play in reducing returns. In a steady-state analysis, monitoring costs lead to a 78% decrease in dividend levels, given a constant leverage ratio of 0.5. In the forthcoming sections, the analysis will expand to simulate an economy with heterogeneous agents, aiming to dissect the broader implications of financial frictions on overall output. This simulation will contrast environments with and without financial frictions, with a particular focus on the cleansing effect that such frictions may exert in economic systems. Through this comparative approach, we aim to deepen our understanding of how financial frictions influence economic resilience, performance, and the potential for innovation and growth amidst challenges. Next, we will simulate an economy with heterogeneous agents to assess the overall output in environments with and without financial frictions. This analysis aims to explore the cleansing effect of financial frictions, providing insights into their influence on economic performance and growth potential.

3.6.1 Heterogeneity and Aggregation Mechanism

To refine our model, we introduce heterogeneity among firms, marking a departure from uniform productivity. Specifically, productivity levels (Z_i) now vary across firms, introducing a spectrum of efficiency within the model. Additionally, we diversify leverage ratios, ensuring no direct correlation between a firm's productivity and its leverage. This heterogeneity is captured from the outset by simulating the initial distribution of capital, leverage, and productivity, setting the stage for a dynamic interplay of firm characteristics. The aggregate output at time t for a population of N firms is :

$$K_t = \sum_{i=0}^{N} Z_{i,t} k_{i,t^{\alpha}} \tag{3.60}$$

The business cycle is modeled through a sine function, impacting the productivity factor Z uniformly across all firms in the economy. Furthermore, the maximum influence the business cycles can exert on productivity is limited to 5% of Z. Consequently, the cycle is represented as follows:

$$\Delta Z_t = 1 + 0.05 \sin(t) \tag{3.61}$$

The firm determines an optimal level of output at time t_0 for the subsequent time t_1, based on equation 3.54. However, the actual output may deviate from this optimal level due to fluctuations in the productivity factor Z, which is influenced by the business cycle, as described in Equation 3.61. To clearly differentiate between these two scenarios, the term ”optimal” will refer to the firm’s chosen level of output, while ”actual” will describe the output level that the firm ultimately achieves.

Productivity (Z) and leverage (l) follow truncated normal distributions:

$$l \sim \mathcal{N}(0.05, 0.1), \qquad 0.01 \leq l \leq 1, \tag{3.62}$$

$$Z - 1 \sim \mathcal{N}(0.5, 0.1), \qquad 1.01 \leq Z \leq 1.1. \tag{3.63}$$

The following analysis includes 100 Monte Carlo simulations of the model, during which we calculate the sample mean and variance for each variable. The subsequent plot Figure 3.12 illustrate both the overall optimal production, as defined by equation 3.60, and the overall actual production, presented in terms of levels.

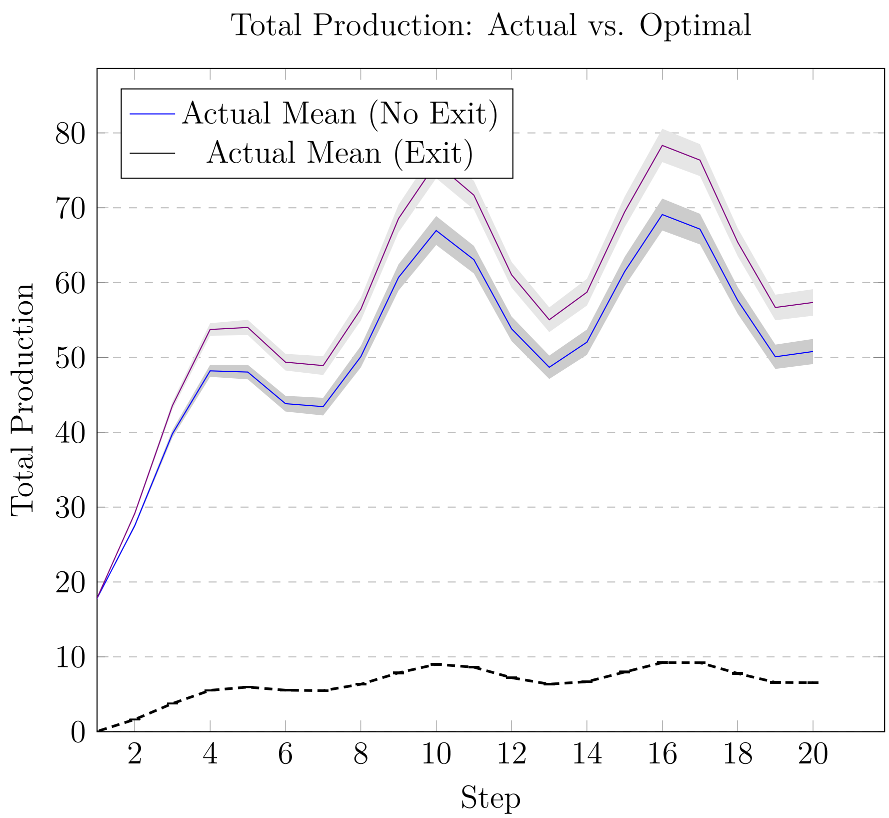


Figure 3.12: Illustrating a 20-step simulation of 10 firms, this plot compares the chosen overall optimal output (K) (red line) against the overall actual capital (k) post-business cycle adjustment (blue line). The confidence intervals are set at 95%.

Following this, the distribution of dividends, based on actual capital, is visualized:

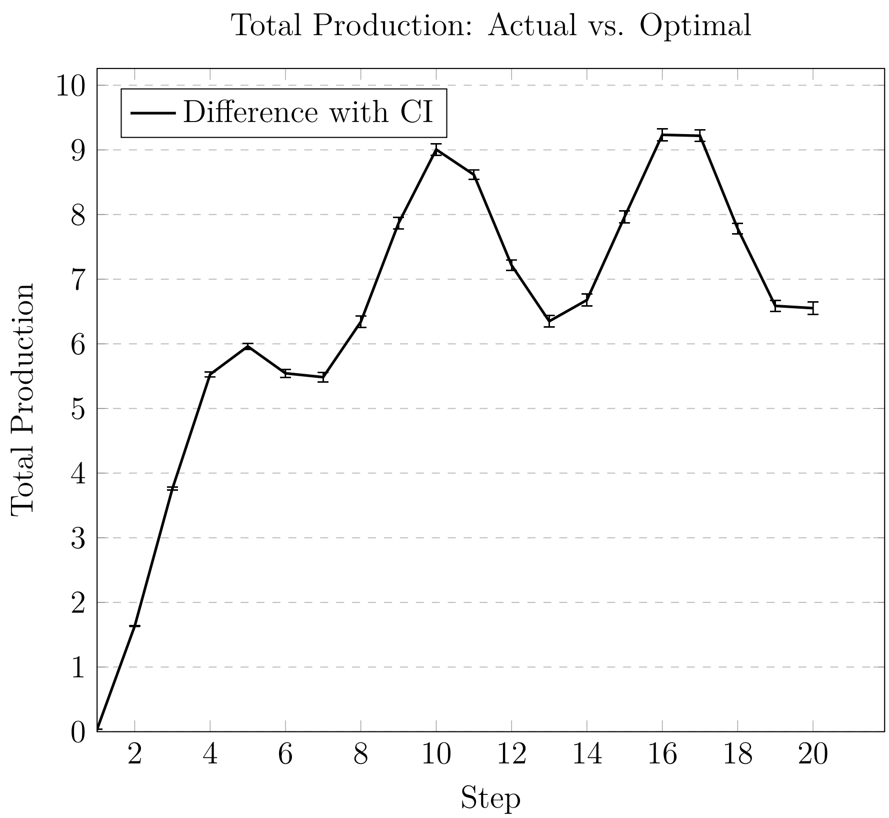


Figure 3.13: Displaying a 20-step simulation for 10 firms, this plot highlights dividends (d) and the actual capital (k) after business cycle adjustments.

Figure 3.13 distinctly shows that firms with superior productivity yield higher dividends. Moreover, upon achieving a steady state, both capital and dividends exhibit oscillations around a rising mean.

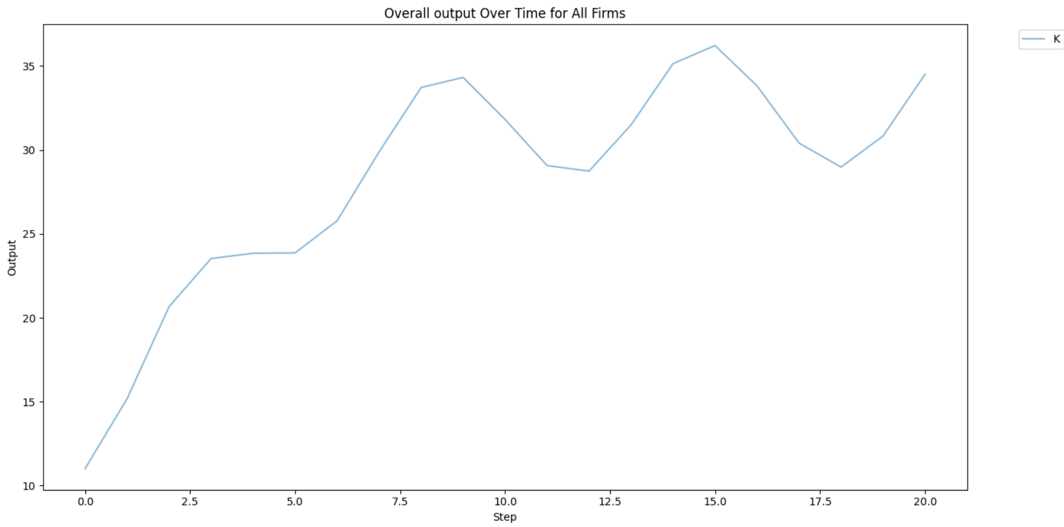


Figure 3.14: Showcasing a 20-step simulation of 10 firms, this plot reveals the adjusted output (K) following the business cycle impact.

3.6.2 The Exit Mechanism

A firm is prompted to exit the market if its return on capital ranks as the lowest within the firm distribution, making way for a newcomer who inherits the minimum capital from the existing pool. Subsequently, the exiting firm's capital is reallocated proportionally among the remaining firms, based on their respective returns on capital. The return on capital for firm i at time t is defined as:

$$R_{i,t} = \frac{d_{i,t-1}}{k_{i,t}} + \frac{k_{i,t} - k_{i,t-1}}{k_{i,t}} \tag{3.64}$$

Should a firm exhibit the min R, it is compelled to exit the market due to possessing the lowest return on capital among all firms.[1]

In the ensuing simulation, while all parameters remain consistent with prior ex-

[1]This criterion is enforced at each step, effectively merging the concepts of reallocation and exit mechanisms henceforth.

amples, the distinctive feature now is the market exit of firms. The subsequent figure illustrates both the optimal and actual capital paths for each firm:

Contrasting with previous outcomes, the introduction of an exit mechanism and the redistribution of residual capital—proportionate to returns, yet ensuring the new entrant retains the minimum capital from the firm distribution—markedly influences capital trajectory. These mechanisms facilitate incumbent firms in accruing additional capital, thus hastening their approach to a stationary state, particularly benefiting the most productive entities. This dynamic underscores the cleansing effect of recessions, as capital reallocation during economic downturns favors business continuity. The impact of reallocation manifests in the subsequent graph, which delineates a higher dividends trajectory compared to scenarios devoid of reallocation mechanisms:

Ultimately, examining overall production reveals an uptick attributable to capital reallocation when juxtaposed with prior simulations, underscoring the efficiency of the exit and reallocation strategies in fostering economic resilience and growth.

Integrating the exit and reallocation mechanisms notably enhances both the dividends' trajectory and the aggregate output, distinctly highlighting how the cleansing effect bolsters productivity.

3.6.3 Incorporating Financial Frictions into the Model

In keeping with the methodology set forth in Osotimehin and Pappadà [2017], financial frictions are incorporated into the model, parameterized as $1 - \mu = 0.25$. To discern the cleansing effect amidst financial frictions, an initial simulation is run where capital remains static due to the absence of a reallocation or exit mechanism. Subsequently, a contrasting simulation is performed where capital reallocation is possible, with both iterations subjected to financial frictions.

The following plot illustrates the progression of total output over time, displaying two distinct scenarios. The red line delineates the case without financial frictions, and the blue line portrays the scenario with frictions in place.

Observations indicate that financial frictions attenuate overall production, akin to an effective reduction in productivity when firms operate with leverage. Although the cleansing effect is evident without financial frictions, it is imperative to examine whether this effect is sustained when frictions are introduced. To this end, the subsequent plot juxtaposes the overall output for two comparable financial friction scenarios: one with capital reallocation and one without.

Figure **??** demonstrates that, even under financial frictions where $1 - \mu = 0.25$, there is a productivity-enhancing mechanism facilitated by capital reallocation due to firm exits. This occurs despite the presence of asymmetric information between financial intermediaries and firms, as discussed in Osotimehin and Pappadà [2017]. Furthermore, the cleansing effect on overall production appears to be cumulative, with its impact amplifying over time, as evidenced by the trend in the graph. Thus, it can be concluded that two economies, identical in their distribution of productivity and capital among firms and initialized with the same seed[2], will diverge in terms of output and productivity if one allows for capital reallocation through firm exits. This divergence is not only distinct but also grows as time progresses.

[2] All simulations were conducted with the same seed for consistency.

Bibliography

Sophie Osotimehin and Francesco Pappadà. Credit frictions and the cleansing effect of recessions. *The Economic Journal*, 127(602):1153–1187, 2017. doi: https://doi.org/10.1111/ecoj.12319. URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/ecoj.12319`.

Robert M. Solow. A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956. ISSN 00335533, 15314650. URL `http://www.jstor.org/stable/1884513`.

T. W. Swan. Economic growth and capital accumulation. *Economic Record*, 32(2):334–361, 1956. doi: https://doi.org/10.1111/j.1475-4932.1956.tb00434.x. URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1475-4932.1956.tb00434.x`.

George W Stadler. Business cycle models with endogenous technology. *American Economic Review*, 80(4):763–78, 1990. URL `https://EconPapers.repec.org/RePEc:aea:aecrev:v:80:y:1990:i:4:p:763-78`.

Friedrich A. von (Friedrich August) Hayek. *Monetary theory and the trade cycle / by Friedrich A. Hayek; translated from the German by N. Kaldor and H.M. Croome.* The Bedford series of economic handbooks. Economic theory section. J. Cape, London, 1933.

C Scott Clark. Labor hoarding in durable goods industries. *The American Economic Review*, pages 811–824, 1973.

Craig Burnside, Martin Eichenbaum, and Sergio Rebelo. Labor hoarding and the business cycle. *Journal of Political Economy*, 101(2):245–273, 1993. doi: 10.1086/261875. URL `https://doi.org/10.1086/261875`.

Steven J. Davis and John Haltiwanger. Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics*, 107(3):819–863, 1992. ISSN 00335533, 15314650. URL `http://www.jstor.org/stable/2118365`.

Ricardo J. Caballero and Mohamad L. Hammour. The cleansing effect of recessions. *The American Economic Review*, 84(5):1350–1368, 1994. ISSN 00028282. URL `http://www.jstor.org/stable/2117776`.

Olivier Jean Blanchard, Peter Diamond, Robert E. Hall, and Kevin Murphy. The cyclical behavior of the gross flows of u.s. workers. *Brookings Papers on Economic Activity*, 1990(2):85–155, 1990. ISSN 00072303, 15334465. URL `http://www.jstor.org/stable/2534505`.

F. A. Hayek and Bruce Caldwell. *The Pure Theory of Capital*. Number 9780226320991 in University of Chicago Press Economics Books. University of Chicago Press, January 2007. ISBN ARRAY(0x3cfcacd8). URL `https://ideas.repec.org/b/ucp/bkecon/9780226320991.html`.

John M Abowd and Francis Kramarz. The costs of hiring and separations. *Labour Economics*, 10(5):499–530, 2003. ISSN 0927-5371. doi: https://doi.org/10.1016/S0927-5371(03)00017-4. URL `https://www.sciencedirect.com/science/article/pii/S0927537103000174`.

Steven J. Davis and John Haltiwanger. Gross job creation and destruction: Microeconomic evidence and macroeconomic implications. *NBER Macroeconomics Annual*, 5:123–168, 1990. ISSN 08893365, 15372642. URL http://www.jstor.org/stable/3585137.

Jordi Galí and J.L. Hammour. Long run effects of business cycles. Working papers, Columbia - Graduate School of Business, 1992. URL https://EconPapers.repec.org/RePEc:fth:colubu:92-26.

Ben S Bernanke and Mark Gertler. Inside the black box: the credit channel of monetary policy transmission. *Journal of Economic perspectives*, 9(4):27–48, 1995.

Ben S Bernanke and Mark Gertler. Agency costs, collateral, and business fluctuations. Working Paper 2015, National Bureau of Economic Research, September 1986. URL http://www.nber.org/papers/w2015.