UNIVERSITÀ CATTOLICA DEL SACRO CUORE

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# Financial Frictions and the Cleansing Effects of Recessions

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**Abstract**

This thesis develops a theoretical framework to explore how financial frictions affect firm's decision on the optimal trajectory for capital and dividends. Initially, the model excludes the possibility of debt financing for investment, akin to the Ramsey-Cass-Koopmans model, with dividends instead of consumption levels. By employing a Lagrangian method, the Euler equation for dividends is derived. In the next stage, firms are allowed to finance investments through debt and retained earnings. The model introduces asymmetric information via monitoring costs included in the participation constraint of financial intermediaries. Additionally, a borrowing constraint that sets a fixed leverage ratio for firms represents another financial friction. The solution process starts with a Lagrangian approach to formulate the Euler equations for dividends, followed by a Bellman equation with a guess-and-verify method to derive a closed-form solution for the optimal paths of dividends and capital. An important finding is that monitoring costs lead to lower steady-state levels of capital and dividends. The thesis concludes with Monte Carlo simulations that introduce heterogeneity across firms and continuous sinusoidal productivity shocks. These simulations suggest that financial frictions reduce the cleansing effect of recession, but the latter remains positively significant, aligning with the findings of Osotimehin and Pappadà [2017].

*This is a shortened version of the original thesis. The full thesis is available here:*

*Master Thesis on GitHub*

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# Chapter 1

# Introduction

Recessions are characterized by increased instances of bankruptcies and business shutdowns. Notably, during the Great Recession, the annual establishment exit rate in the US escalated from 11.8% to 13.5% from March 2008 to March 2009. This surge in firm exit rates during economic contractions has propelled the notion that recessions serve a 'cleansing' role in the economy: inefficient firms, becoming unprofitable, are phased out, thereby facilitating the redistribution of resources to more productive entities.

This traditional perspective, highlighted in the work of Caballero and Hammour [1994], presupposes that markets inherently favor the survival of the most productive firms. Nonetheless, this premise has been contested by numerous studies indicating that a firm's likelihood of exiting is influenced not just by its productivity but also by its ability to secure credit. These insights imply that, amidst credit frictions, even highly productive firms that are financially strained might be compelled to leave the market. Consequently, credit frictions could potentially undermine the productivity-boosting impact of recessions as found in Osotimehin and Pappadà [2017].

This thesis investigates the impact of credit frictions on the cleansing effect during recessions by examining firm dynamics and credit constraints. Initially, it introduces a theoretical model to analyze how financial frictions influence firm decisions on capital, debt, and dividends. Monte Carlo simulations are then applied to assess potential implications for the cleansing effect of recessions.

The model is divided into two scenarios: in the first, investment is funded solely through retained earnings; in the second, firms also have the option to finance through debt. Without the ability to borrow, the model aligns with Ramsey-Cass-Koopmans frameworks, substituting family savings with firm savings and consumption with dividends. Firms face capital depreciation and can only replace it by retaining dividends, depicted in the funds flow constraint. Using a Lagrangian function, the Euler equation for dividends is derived, leading to steady-state conditions for dividends and capital, visualized in a phase diagram.

The second part introduces financial frictions and debt financing for investments, aiming to find closed-form solutions for the optimal paths of dividends and capital. Credit restrictions are shaped by asymmetric information and monitoring costs. By incorporating a one-period financial contract, akin to the approach by Bernanke and Gertler [1986], into the firm dynamics model, it outlines borrowing constraints and interest rates set by financial intermediaries based on each firm's productivity and leverage. The model also applies a borrowing constraint, fixing leverage over time.

The novelty introduced in this thesis to the literature is the development of a closed-form solution for the optimal trajectories of dividends and capital under financial frictions. This is achieved using a Bellman equation combined with a guess-and-verify approach, employing a logarithmic utility function. The derived solution demonstrates the significant impact of monitoring costs, which reduce both capital and dividends, thereby increasing the vulnerability of firms irrespective of their productivity.

Following the theoretical developments, this thesis employs Monte Carlo simulations to analyze the behavior of firms differentiated by their productivity and leverage ratios in the face of business cycle fluctuations. These fluctuations are modeled as continuous sinusoidal shocks to the productivity component, simulating the economic cycles. A key feature of the model is the exit mechanism, which forces firms to exit the market if their return on capital falls below the risk-free rate, directly engaging with the discourse on the cleansing effect of recessions.

The heterogeneity in firm characteristics, in terms of productivity and leverage, allows for an examination of how heterogeneous firms respond to economic cycles and credit constraints. Firms with higher leverage may face greater financial stress during downturns, influencing their survival and contributing to the market's cleansing process. Similarly, variations in productivity affect firms' ability to withstand economic shocks, further delineating the selection process during recessions.

The simulations suggest that recessions enhance the economy's overall output by reallocating resources towards more efficient firms, particularly in scenarios without monitoring costs. However, the introduction of such costs lowers this efficiency gain, underscoring the role that financial frictions play in influencing the economy's resilience during downturns. This observed cleansing effect, albeit modulated by the presence of financial frictions, aligns with the findings in Osotimehin and Pappadà [2017]. Despite the dampening influence of credit constraints, I found that recessions retain an inherent capacity to purge the economy of less efficient firms.

## 1.1 Literature review

Two influential theoretical models analyze the cleansing effect of recessions on the economy. The first model, introduced by Caballero and Hammour [1994], utilizes a vintage model of creative destruction to investigate how industries adapt to cyclical demand variations. This model highlights how recessions can facilitate the removal of outdated and less efficient production units, potentially leading to an overall increase in industry productivity. The model's insights are derived from a framework where production units of varying vintages coexist, and the rate of technological adoption plays a crucial role in determining industry responses to economic fluctuations.

In contrast, the second model explored in this review, by Osotimehin and Pappadà [2017], introduces financial frictions into the analysis of the cleansing effect of recessions. This addition brings a new layer of complexity to the understanding of how economic downturns affect firm dynamics, particularly by influencing the selection process through which firms enter and exit the market. The model underscores the role of credit constraints in mediating the impact of recessions, suggesting that financial frictions can dampen the potential productivity gains that might otherwise arise from the purging effects of a downturn.

Both models rely on numerical methods to solve their respective frameworks, acknowledging the intricate dynamics and non-linearities inherent in their analyses. By comparing these models, this review seeks to illuminate the diverse mechanisms through which recessions can influence economic outcomes, as well as the varying implications of introducing different types of market frictions into theoretical frameworks.

In the first paper that rationalizes the cleansing effect of recessions, Caballero and Hammour [1994] investigate how industries respond to cyclical variations in demand using a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model incorporates heterogeneous firms, where production units embody the most advanced technology at the time of their creation. The costs associated with creating new units slow down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a demand reduction can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour [1994]'s research offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard et al. [1990] and Davis and Haltiwanger [1992]. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study, Davis and Haltiwanger [1992] assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986. It is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry.

The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

The second model, by Osotimehin and Pappadà [2017], introduces financial frictions into the analysis of the cleansing effect of recessions. This model highlights how credit constraints can influence firm dynamics during economic downturns. Financial frictions, such as borrowing constraints and monitoring costs, can dampen the potential productivity gains that might otherwise arise from the purging effects of a downturn. The model suggests that financial frictions can lead to the premature exit of some high-productivity firms, thereby reducing the overall productivity gains from recessions. The model incorporates a framework where firms face a choice between continuing operations and exiting the market based on their productivity and financial health. Firms with higher leverage may face greater financial stress during downturns, influencing their survival and contributing to the market's cleansing process. Similarly, variations in productivity affect firms' ability to withstand economic shocks, further delineating the selection process during recessions. The findings from this model suggest that while financial frictions reduce the cleansing effect of recessions, the latter remains positively significant. This aligns with the empirical evidence provided by Osotimehin and Pappadà [2017], which shows that despite the dampening influence of credit constraints, recessions retain an inherent capacity to purge the economy of less efficient firms.

This thesis contributes to the literature by developing a theoretical framework that explores how financial frictions affect firms' decisions on the optimal trajectory for capital and dividends. Initially, the model excludes the possibility of debt financing for investment, akin to the Ramsey-Cass-Koopmans model, with dividends instead of consumption levels. By employing a Lagrangian method, the Euler equation for dividends is derived. In the next stage, firms are allowed to finance investments through debt and retained earnings. The model introduces asymmetric information via monitoring costs included in the participation constraint of financial intermediaries. Additionally, a borrowing constraint that sets a fixed leverage ratio for firms represents another financial friction. The solution process starts with a Lagrangian approach to formulate the Euler equations for dividends, followed by a Bellman equation with a guess-and-verify method to derive a closed-form solution for the optimal paths of dividends and capital. An important finding is that monitoring costs lead to lower steady-state levels of capital and dividends. The thesis concludes with Monte Carlo simulations that introduce heterogeneity across firms and continuous sinusoidal productivity shocks. These simulations suggest that financial frictions reduce the cleansing effect, but the latter remains positively significant, aligning with the findings of Osotimehin and Pappadà [2017].

By comparing these two models and introducing a new theoretical framework, this literature review highlights the diverse mechanisms through which recessions can influence economic outcomes. The vintage model of creative destruction by Caballero and Hammour [1994] emphasizes the role of technological adoption and the coexistence of production units with varying vintages, while the model by Osotimehin and Pappadà [2017] underscores the impact of financial frictions on firm dynamics. Together, these models, along with the contributions of this thesis, provide a comprehensive understanding of the cleansing effect of recessions and the varying implications of introducing different types of market frictions into theoretical frameworks.

# Chapter 2

# The Firm's Problem

## 2.1 Introduction

This thesis presents a partial equilibrium model in which firms maximize dividends over an infinite period, under financial frictions, investigating how those frictions can affect the saddle path of capital and dividends. Compared to Osotimehin and Pappadà [2017] and Caballero and Hammour [1994], this model allows us to find a closed-form solution optimal path for dividends and capital. The subsequent sections delve into the formulation of the flow of funds and its dynamics. Following this, the focus shifts to scenarios where financial frictions are present, examining their implications on firm behavior and market outcomes.

## 2.2 Law of motion of capital and debt

This model is set within a partial equilibrium framework where firms are differentiated by their productivity levels. They have the option to fund their operations by obtaining loans from financial intermediaries, as outlined by Bernanke and Gertler [1995], or by retaining dividends. The capital at any time $t$ is calculated by adjusting capital from the previous period for depreciation ($\delta$), then adding gross investment ($I$), thus the law of motion of capital stock is:

$$k_{t+1} = k_t(1-\delta) + I_t$$

We can rearrange the above equation and get gross investment at time t

$$I_t = k_{t+1} - k_t\,(1-\delta) \tag{2.1}$$

The 2.1 equation states gross investment at time t is equal to the net capital formation plus replacement of depreciated capital. The flow of funds constraint is:

$$I_t + Rb_t + d_t = f(k_t) + b_{t+1} \tag{2.2}$$

where $R$ denotes the gross interest rate and $b_t$ represents the debt from period t. The components of the flow of funds (f-of-f) at time $t$ include:

1. $I_t$ gross investment at time t
2. $Rb_t$ repayment of debts (principal and interest)
3. $d_t$ dividends distributed at time t

Conversely, the right-hand side details the sources of fund inflows:

1. $f(k_t)$ output at time t+1
2. $b_t$ debt at time t+1

The f-of-f constraint can be rewritten as the law of motion of debt:

$$b_{t+1} = Rb_t + I_t - S_t \tag{2.3}$$

where $S_t = f(k_t) - d_t$ represents retained earnings. This formulation clarifies that the debt level at time $t+1$ is the sum of the repayment for the previous period's debt (both principal and interest) and the net investment, adjusted for internal financing.

From equations 2.1, 2.3 and the definition of net worth $n_{t+1} = k_{t+1} - b_{t+1}$, we derive the law of motion for net worth as follows:

$$
\begin{aligned}
n_{t+1} = k_{t+1} - b_{t+1} &= k_t(1-\delta) + I_t - Rb_t - I_t + S_t \\
&= k_t - \delta k_t - b_t - rb_t + S_t \\
&= n_t - \delta k_t - rb_t + [f\left(k_t\right) - d_t]
\end{aligned}
$$

The net worth or equity of the firm is given by the net worth of the previous period less the depreciated capital, less the interest matured from the previous period augmented by the retained earnings. Therefore a firm can increase its net worth only through increasing the retained earnings levels, thus increasing output or decreasing dividends. Using equations 2.1 and 2.2, we get the flow of funds constraint for capital:

$$k_{t+1} = k_t(1-\delta) - Rb_t - d_t + f(k_t) + b_{t+1} \tag{2.4}$$

The above equation describes how capital evolves: capital at time $t+1$ is equal to capital at time t net of depreciation, less the repayment of debt (principal + interest), augmented by retained earnings.

### 2.2.1 Steady State

From 2.1, assuming $(k_{t+1} = k_t = \widehat{k})$, we get the steady state investment:

$$
\begin{aligned}
\widehat{k} &= \widehat{k}\left(1-\delta\right) + \widehat{I} \\
\widehat{I} &= \delta\widehat{k}
\end{aligned}
\tag{2.5}
$$

2.5 states that in the steady state, the firm will invest only to substitute depreciated capital($\delta\widehat{k}$). From 2.3, assuming ($b_{t+1} = b_t = \widehat{b}$) and $d_t = d_{t+1} = \hat{d}$, we get:

$$\widehat{b} = R\widehat{b} + \widehat{I} - \widehat{S} \tag{2.6}$$

or $\widehat{S} = f(\hat{k}) - \hat{d}$, therefore:

$$f\left(\widehat{k}\right) - \widehat{d} - \delta\widehat{k} = r\widehat{b} \tag{2.7}$$

Equation 2.7 states that in the steady state, retained earnings should be used to repay interest over debt. Equation 2.7 can be rewritten as:

$$f(\widehat{k}) = \delta \cdot \widehat{k} + r \cdot \widehat{b} + \widehat{d} \tag{2.8}$$

To illustrate the equilibrium locus, one can refer to the graph in 2.1, which depicts the locus as defined by equation 2.8, employing the following production function:

$$f(k_t) = Zk_t^{\alpha}, \tag{2.9}$$

with $Z$ indicating the firm's productivity level, $0 < \alpha < 1$.

The figure illustrates the steady state relationships among debt ($\widehat{b}$), capital ($\widehat{k}$), and dividends ($\widehat{d}$) in a three-dimensional plot. The graph demonstrates how various combinations of debt and capital influence the distribution of dividends. It is evident that, for any given level of $\hat{k}$, a higher level of debt results in lower dividends, as a larger portion of resources is allocated towards servicing interest payments.

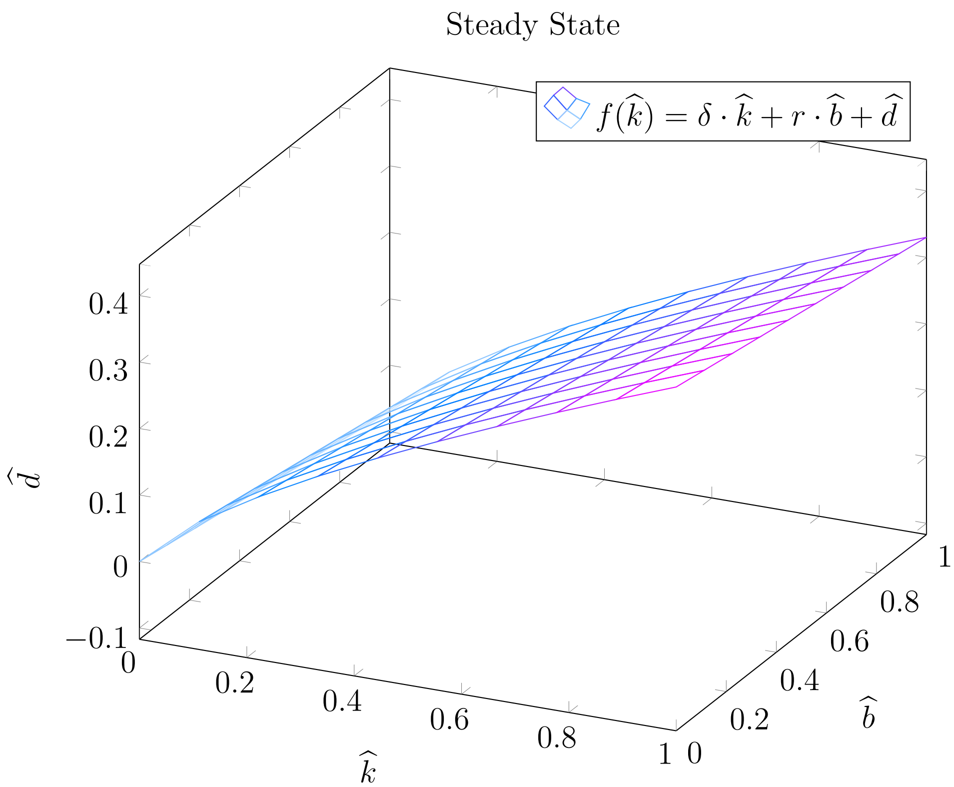


Figure 2.1: The three-dimensional plot delineates the steady state locus as defined by equation 2.8, where the production function parameter values are set as $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, and $Z = 0.5$. It highlights how at any steady-state level of capital $\widehat{k}$, an increment in debt $\widehat{b}$ necessitates a decline in dividends $\widehat{d}$, illustrating the trade-off between debt servicing and dividend distribution.

## 2.3 Ramsey-Cass-Koopmans reinterpreted

This section outlines the intertemporal maximization problem faced by the firm in the absence of debt, which is a Ramsey-Cass-Koopmans revisited model, where there is a firm that seeks to maximize the utility of consumption of the shareholders (= dividends). The objective function is:

$$V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t),$$

where $U' > 0, U'' < 0$. Let's assume that the firm's investment is entirely financed by equity ($b_t = 0$ for all $t$), this leads to a simplified flow-of-funds constraint equation:

$$k_{t+1} = k_t(1-\delta) + f(k_t) - d_t. \tag{2.10}$$

The maximization problem is tackled using a Lagrangian method, where the Lagrangian is defined as:

$$L_0 = \sum_{t=0}^{+\infty} \left[\beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - k_t(1-\delta) - f(k_t) + d_t\right]\right].$$

The first-order conditions for $d_t$, $k_t$, for all periods $t = 0, 1, \ldots$ yield:

$$U'(d_t) = \lambda_t, \quad \forall t,$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1} [f'(k_{t+1}) + (1-\delta)], \quad \forall t,$$

This approach delineates the optimal strategy for dividend distribution and capital allocation in a debt-free case. In the infinite horizon model, the transversality condition reads:

$$\lim_{t\to\infty} \beta^T U'(d_t)\, k_{T+1} = 0 \tag{2.11}$$

From these first-order conditions (FOCs), we derive the Euler equation for dividends:

$$U'(d_t) = \beta U'(d_{t+1})[f'(k_{t+1}) + (1-\delta)] \tag{2.12}$$

indicating that the marginal utility of distributing 1 unit of output as dividends at time $t$ should match the discounted marginal utility of not distributing dividends in t, saving, investing in t, using the additional capital to produce and distribute the corresponding dividends at time t+1.

**Steady state condition for dividends** Imposing the steady state condition for dividends $d_t = d_{t+1} = \widehat{d}$ in 2.12 , we equate the marginal utilities across two consecutive periods:

$$\begin{aligned} U'(d_t) &= U'(d_{t+1}), \\ \frac{1}{\beta} &= f'(k_{t+1}) + (1-\delta), \end{aligned} \tag{2.13}$$

This condition is satisfied if:

$$f'(k_{t+1}) = \frac{1}{\beta} - (1-\delta), \tag{2.14}$$

Using the Cobb-Douglas production function and taking the derivative with respect to capital, we get:

$$f'(k_{t+1}) = Z\alpha k_{t+1}^{\alpha-1}, \tag{2.15}$$

From 2.13, we get:

$$
\begin{aligned}
\frac{1}{\beta} &= 1 - \delta + Z\alpha k^{\alpha-1}, \\
Z\alpha k^{\alpha-1} + (1-\delta)\beta &= 1, \\
\widehat{k} &= \left[\frac{\alpha\beta Z}{1-\beta\,(1-\delta)}\right]^{\frac{1}{1-\alpha}}.
\end{aligned}
\tag{2.16}
$$

The locus of points on the $(k_t, d_t)$ plane such that dividends are constant is therefore $k_t = \hat{k}$, whose representation in the vertical line in figure 2.2.

**Steady state condition for capital** Imposing the steady state condition for capital $(k_t = k_{t+1} = \widehat{k})$ in the law of motion of capital 2.10 we get:

$$
d_t = f(k_t) - \delta k_t \tag{2.17}
$$

This locus represents the set of points where capital stock remains constant over time. Hence, we can determine the level of dividends that ensures both capital and dividends are maintained at steady-state levels. By incorporating the steady-state level of capital from Equation 2.16 and the production function from Equation 2.9 into the equilibrium condition for capital from Equation 2.17, the steady-state level of dividends, can be deduced:

$$
\widehat{d} = Z\left(\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \tag{2.18}
$$

In this manner, we ascertain the steady-state levels for both capital and dividends.

### 2.3.1 Phase diagram

In this section, we will plot the phase diagram for capital and dividends exploiting steady-state conditions for capital and dividends.

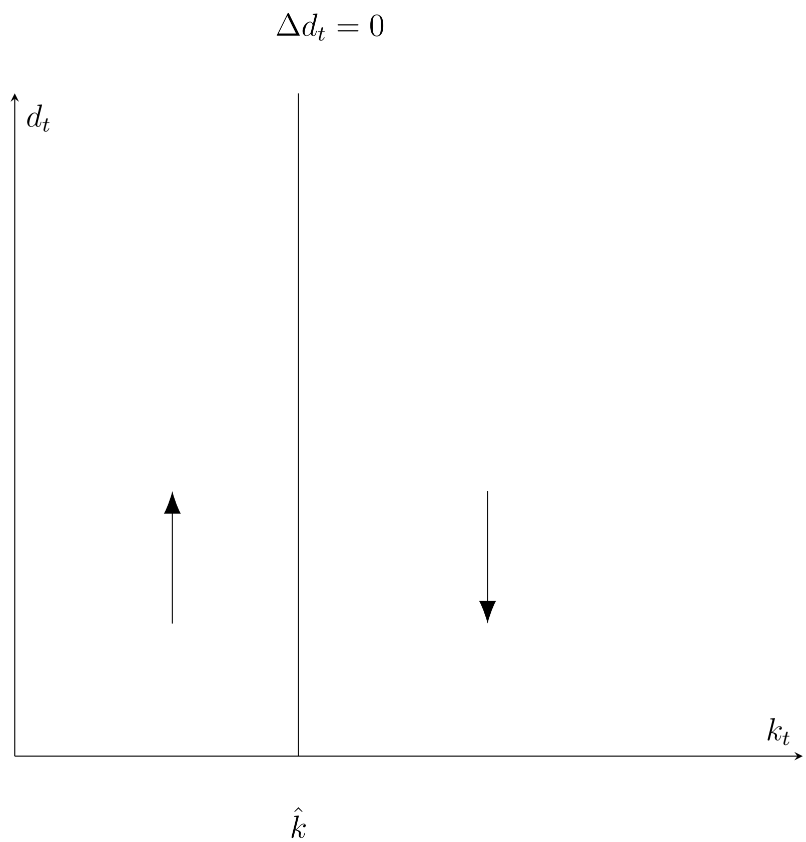


Figure 2.2: The phase diagram illustrates locus of points such that $d_t = d_{t-1}$. The vertical line at $\hat{k}$ represents the steady-state level of capital, where the rate of change in dividends $\Delta d_t$ is zero. To the left of $\hat{k}$, where capital is below its steady-state level, the firm increases dividend payments. Conversely, to the right of $\hat{k}$, where capital exceeds the steady state, dividend payments decrease.

The 2.2 portrays the dynamics of dividends ($d_t$) in relation to the capital ($k_t$) of a firm, with a particular focus on the behavior when capital is below or above the steady-state level, denoted by $\hat{k}$.

When capital is below the steady-state level ($k_t < \hat{k}$), thus on the left of the vertical line, for the firm it is optimal to increase dividends over time ($d_t < d_{t+1}$) as represented by the arrow pointing upward. When instead $k_t > \hat{k}$, dividends must shrink over time ($d_t > d_{t+1}$). Let's look at the locus in which capital is stationary $\Delta k = 0$ is given by the f-of-f constraint 2.17:

$$d_t = f(k_t) - \delta k_t \tag{2.19}$$

In this case, as obtained in the above section, the locus in which capital is stationary becomes 2.17:

$$d_t = Zk_t^{\alpha} - \delta k_t \tag{2.20}$$

This function starts at the origin since ($f(0) = 0$), with a maximum in $\underline{k}$ (defined as capital level such that $f'(\underline{k}) = \delta$). The level of capital $\underline{k}$ is:

$$\underline{k} = \left[\frac{\alpha Z}{\delta}\right]^{\frac{1}{1-\alpha}} \tag{2.21}$$

It is easy to see that $\hat{k} < \underline{k}$:

$$\begin{aligned}
\frac{\alpha\beta Z}{1-\beta+\beta\delta} &< \frac{\alpha Z}{\delta}, \\
\frac{\beta}{1-\beta+\beta\delta} &< \frac{1}{\delta}, \\
\beta\delta &< 1-\beta+\beta\delta, \\
1-\beta &> 0
\end{aligned}$$

We denote $\bar{k}$ the capital level such that $d_t = 0$, thus it's obtained by solving $f(\bar{k}) - \delta\bar{k} =$

0, using the Cobb-Douglas production function 2.9, we get:

$$\begin{aligned} Z\bar{k}^{\alpha} &= \delta\bar{k}, \\ \bar{k} &= \left[\frac{Z}{\delta}\right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{2.22}$$

It is easy to see that $\underline{k} < \bar{k}$, since:

$$\begin{aligned} \left[\frac{Z}{\delta}\right]^{\frac{1}{1-\alpha}} &> \left[\frac{\alpha Z}{\delta}\right]^{\frac{1}{1-\alpha}}, \\ \frac{Z}{\delta} &> \frac{\alpha Z}{\delta}, \\ 1 &> \alpha \end{aligned}$$

Considering transitivity, the sequence for $\underline{k}, \hat{k}, \bar{k}$ is:

$$\hat{k} < \underline{k} < \bar{k}$$

For a given level of capital $k_0 \in [0, \bar{k}]$, the corresponding dividends level that guarantee the stationarity of capital is:

$$d_0 = f(k_0) - \delta k_0$$

If the firm distributes more dividends than $d_0$ the capital stock must decrease over time: since dividends are too high the firm is consuming part of her capital. More precisely the firm is distributing more dividends than $d_0$, which guarantees that the difference between production, and dividends is exactly equal to the replacement of depreciated capital. This behavior is represented by the arrows above the curve pointing to the left. If the firm distributes fewer dividends than $d_0$, the opposite happens: the firm increases its capital since there is a positive net investment. This behavior is represented by the arrows below the curve pointing toward the right.

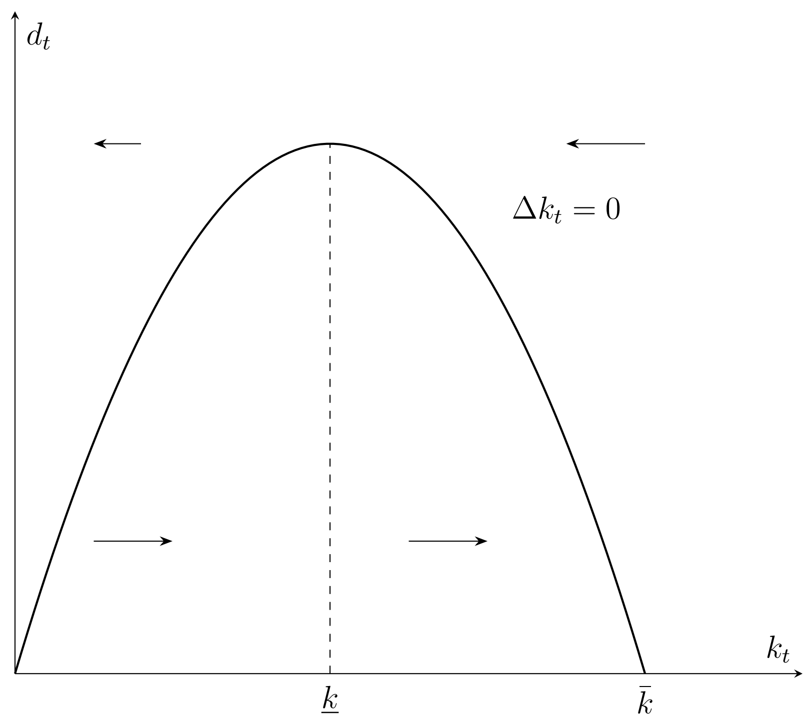


Figure 2.3: This phase diagram displays the set of points at which the capital stock $k_t$ is unchanging from one period to the next ($k_t = k_{t-1}$). The capital level at $\underline{k}$ represents the maximum sustainable dividend payout without affecting the capital stock. Arrows above the curve pointing leftward indicate a reduction in capital resulting from dividend levels that exceed the sustainable steady state, while arrows pointing to the right below the curve signify the accumulation of capital due to dividend levels that are below the steady state, leading to an increase in capital stock over time.

**Steady state for capital and dividends** Plotting both loci we get a phase diagram that represents the condition for stationarity. Notice that there exits 3 steady states: one at the origin due to the assumption $f(0) = 0$, the point $(\bar{k}; 0)$, and finally point B $= (\hat{k}, \hat{d})$. Point B was obtained by equations 2.16 and 2.17 and represented the point in which dividends and capital are at a steady state, and both are strictly positive. This point is a saddle point. The blue line depicts a possible saddle path towards B. Starting at A, the firm chooses exactly the dividend level that leads to the stationary point B. This path not only fulfills the difference equations 2.12 and 2.17, but also, the transversality condition 2.11 Indeed as $t \to \infty$, capital and dividends approach their steady-state level, which are both positive and finite, thus the marginal utility of dividends at $\hat{d}$ is also finite, hence 2.11 is valid.

To conclude, this section has outlined the derivation of steady-state levels for capital and dividends, and these conditions have been visually represented in a phase diagram. The upcoming section will undertake a similar analysis but will incorporate debt and financial friction into the model. Additionally, it is important to note that in the steady state, where output $\hat{y}$ is given by $f(\hat{k})$, the production level is exclusively influenced by the productivity parameter $Z$, signifying that financial elements do not alter the fundamental connection between the production function and output.

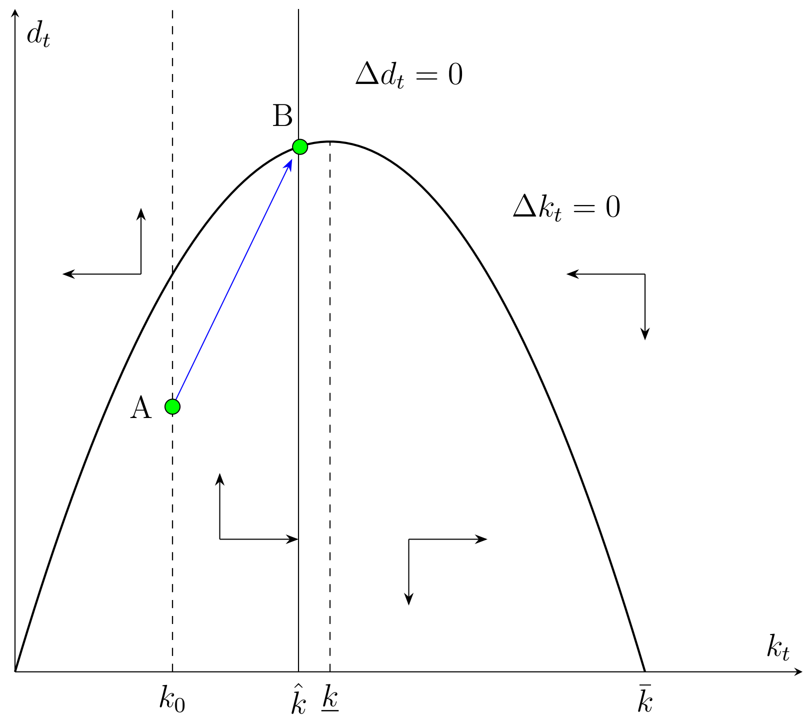


Figure 2.4: The phase diagram visualizes the relationship between dividends and capital. The vertical line marks where the dividend level remains constant, and the solid concave curve traces where the capital remains constant. Point B indicates the equilibrium where both dividends and capital are stationary, with the steady-state capital at $\hat{k}$. This value of $\hat{k}$ is notably lower than $\underline{k}$, which is the capital level that would maximize dividends while maintaining a steady capital stock. $\bar{k}$ represents the capital quantity at which the system reaches a stationary state for k with zero dividends. The arrows illustrate the dynamics of dividends and capital in case of the firm finding itself outside the stable loci. Point A is a possible starting point $(k_0, d_0)$. The blue arrows indicate a potential trajectory, known as a saddle path, leading towards saddle point B.

# Chapter 3

# Optimization under Financial Frictions

This chapter delves into a firm's optimization in a context where debt financing is entwined with financial frictions. We dissect two pivotal constraints:

The fixed leverage constraint restricts a firm's debt-to-equity ratio, thereby shaping its capital structure and investment choices. This fixed ratio ($0 < l < 1$), where $b_t = lk_t$, is enforced to mitigate default risk. It remains invariant over time, disallowing leverage adjustment in response to productivity shocks, thereby reducing the optimization problem to capital and dividend decisions.

We also explore the participation constraint with monitoring costs for financial intermediaries. These intermediaries incur costs to ensure borrowers' compliance and solvency, influencing the cost of borrowing and affecting loan terms and credit access.

Incorporating these frictions, the model aims to mirror the real-world financial conditions of firms. We investigate the impact of these frictions on firms' steady-state behavior, particularly on capital and dividend dynamics, vital for understanding the broader economic impact of financial frictions.

Following the analysis of participation constraints, we address the firm's maximization problem in the face of financial frictions. Using a Lagrangian formulation, we determine the stationary conditions for dividends and capital, visualized in a phase diagram. The firm's problem is then reformulated via a Bellman equation and solved using iterative techniques.

The chapter concludes with Monte Carlo simulations to model firm heterogeneity in leverage and productivity, incorporating continuous productivity shocks and an exit condition based on a return on capital threshold. This exit mechanism serves as a test of the 'cleansing effect' on the economy's output, both with and without monitoring costs, to determine if a cleansing effect persists in the presence of financial frictions.

## 3.1 Participation constraint of the financial intermediaries

The subsection delves into the constraints facing financial intermediaries within the model. Initially, the model assumed an exogenous interest rate, unaffected by the volume of debt, leading to an unrealistic scenario where interest rates remain constant. To address this, the model incorporates a financial market in which the interest rate is set based on market-clearing conditions, with financial intermediaries functioning in a perfectly competitive environment aimed at profit maximization.

According to Bernanke and Gertler [1986], lending should yield a profit equivalent to the opportunity cost. Lenders earn interest plus the principal if borrowers repay successfully (with probability $p$) or acquire the firm's production assets (net of monitoring cost: $1-\mu$) in case of bankruptcy. Moreover, $(1-\mu)f(k_t)$represents monitoring cost. The lender's participation constraint is:

$$R_t \cdot b_t p + (1-p)\mu f(k_t) = R_f b_t,$$

where $R_f$ represents the risk-free rate, the opportunity cost of lending. This framework allows for the derivation of the interest rate as a function of $p$,$f(k_t)$,$\mu$,$b_t$, and $R_f$.

The participation constraint can be rewritten as:

$$R_t = \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t}. \tag{3.1}$$

Investigating the asymptotic behavior of $R_t$ with respect to $p$ provides critical economic insights. Specifically, as $p \to 0$, indicating an increasingly likely default, $R_t$ escalates without bound, reflecting the infinitely rising premium a lender would require to counterbalance the heightened risk. Conversely, as $p \to 1$, the default risk vanishes, and $R_t$ converges to $R_f$, the risk-free rate, consonant with the absence of default risk necessitating no premium above the risk-free return. When examining the behavior of the interest rate formula as the debt amount $b_t$ trends towards infinity, the resulting limit is given by:

$$\lim_{b_t \to +\infty} \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t} = \frac{R_f}{p}$$

As the denominator $b_t$, representing the total debt, increases without bound, the term involving the firm's productive output adjusted for recovery rate ($\mu f(k_t)/b_t$) diminishes to zero. This indicates that, in scenarios of very large debt volumes, the fraction of recoverable assets ($\mu f(k_t)$) compared to the outstanding debt becomes negligible in determining the interest rate $R_t$.

Therefore, under such conditions, the interest rate formula simplifies to $\frac{R_f}{p}$, underscoring that the loan's interest rate is fundamentally determined by the risk-free rate adjusted for the probability of successful repayment $p$, independent of the firm's asset recoverability or productivity levels. This elucidates an important economic principle: for vast sums of debt, the critical factors shaping lending rates pivot away from the specifics of asset recovery towards broader financial metrics—namely, the prevailing risk-free rate and the inherent risk of default.

## 3.2 The problem of the firm in the presence of financial frictions

The firm's objective is to maximize:

$$\max_{\{d_t\}_{t=0}^{+\infty}} V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t)$$

subject to:

1. the flow of funds constraint: $I_t + Rb_t + d_t = f(k_t) + b_{t+1}$
2. the investment function $I_t = k_{t+1} - k_t\,(1-\delta)$
3. the financing constraint $b_t = lk_t$
4. the participation constraint of lenders $R_t = \frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{b_t}$

Consolidating the constraints we get the flow of funds constraints:

$$\begin{aligned}
k_{t+1} &= \left\{k_t(1-\delta) - \left[\frac{R_f}{p} - \frac{1-p}{p}\frac{\mu f(k_t)}{lk_t}\right]\cdot lk_t + f(k_t) - d_t\right\}(1-l)^{-1} \\
k_{t+1} &= \left[\frac{p+\mu-\mu p}{p} f(k_t) + \frac{p-\delta p - R_f l}{p} k_t - d_t\right](1-l)^{-1} \qquad (3.2)
\end{aligned}$$

The Lagrangian for this optimization problem is :

$$L = \sum_{t=0}^{+\infty} \beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - \frac{p+\mu-\mu p}{p} f(k_t) - \frac{p-\delta p - R_f l}{p} k_t + d_t\right](1-l)^{-1}, \qquad (3.3)$$

leading to the first-order conditions for optimizing dividends and capital over time:

$$U'(d_t) = \frac{\lambda_t}{(1-l)}, \quad \forall t, \qquad (3.4)$$

and the dynamic optimality conditions for capital allocation:

$$\lambda_t = \beta \frac{\lambda_{t+1}}{(1-l)} \left[ f'(k_t) \frac{p + \mu - \mu p}{p} + \frac{p - \delta p - R_f l}{p} \right], \quad \forall t. \tag{3.5}$$

This formulation yields the Euler equation for dividends:

$$U'(d_t) = \frac{\beta}{(1-l)} U'(d_{t+1}) \left[ f'(k_t) \frac{p + \mu - \mu p}{p} + \frac{p - \delta p - R_f l}{p} \right], \tag{3.6}$$

imposing ($d_t = d_{t+1} = \hat{d}$), we get:

$$\begin{aligned} \frac{(1-l)\,p}{\beta} &= f'(\hat{k})\,(p + \mu - \mu p) + (p - \delta p - R_f l) \\ f'(\hat{k}) &= \frac{p - pl - \beta p + \beta \delta p + \beta R_f l}{\beta\,(p + \mu - \mu p)} \end{aligned} \tag{3.7}$$

using the Cobb Douglas production function 2.15 into 3.7 we get:

$$\begin{aligned} Z\alpha \hat{k}^{\alpha-1} &= \frac{p - pl - \beta p + \beta \delta p + \beta R_f l}{\beta\,(p + \mu - \mu p)} \\ \hat{k} &= \left[ \frac{Z\alpha\beta\,(p + \mu - \mu p)}{p - pl - \beta p + \beta \delta p + \beta R_f l} \right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{3.8}$$

Similar to the scenario without debt, when the firm's capital falls below the steady-state threshold $\hat{k}$, it is advantageous for the firm to raise its dividend payouts over time. Conversely, when the firm's capital surpasses $\hat{k}$, it would be more beneficial for the firm to gradually reduce dividends. It can be readily demonstrated that when monitoring costs become entirely ineffective ($1 - \mu = 0$), the firm operates without debt ($l = 0$), and the probability of default is eliminated ($1 - p = 0$), the resulting capital level aligns with that of the debt-free situation as specified in equation 2.16:

$$\hat{k} = \left[\frac{Z\alpha\beta\,(1+1-1)}{1-0-\beta+\beta\delta+0}\right]^{\frac{1}{1-\alpha}},$$
$$\hat{k} = \left[\frac{Z\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}}.$$

Imposing s.s. condition for capital ($k_t = k_{t+1} = \hat{k}$) into the flow of funds constraint 3.2:

$$\widehat{d} = \frac{p+\mu-\mu p}{p} f(\hat{k}) - \left(\frac{lR_f + \delta p - lp}{p}\right)\hat{k} \tag{3.9}$$

It can be straightforwardly demonstrated that by setting the monitoring cost to $1-\mu = 0$, eliminating debt with $l = 0$, and removing the risk of default by setting $1-p = 0$, we arrive at an identical level of dividends as observed in the scenario without debt 2.17.

## 3.3 Phase diagram

The goal of this section is to portray the phase diagram in two cases: one with monitoring costs and one without. However, we will use a less heuristic approach compared to the phase diagram of the free debt case, using parameters similar to Osotimehin and Pappadà [2017]:

| Parameter | Symbol | Value |
|---|---|---|
| Discount factor | $\beta$ | 0.956 |
| Risk-free rate | $R_f$ | 1.04 |
| Depreciation rate | $\delta$ | 0.07 |
| Returns to scale | $\alpha$ | 0.80 |
| Aggregate productivity | $\bar{Z}$ | 0.5 |
| Monitoring cost | $1-\mu$ | $0, 0.75$ |
| Productivity | $Z$ | 0.2 |
| Probability of default | $1-p$ | 0.6 |

Table 3.1: Parameters

Moreover, we assume a fixed leverage of $l = 0.8$, since for the moment we want to understand the effect of monitoring cost leaving all the other parameters equal.

The phase diagram illustrated in 3.1 depicts the capital accumulation dynamics under scenarios of fixed leverage and varying monitoring costs. While the overall dynamics remain consistent across both scenarios, the equilibrium capital level is notably reduced in firms that incur monitoring costs, in contrast to those without such costs. As a result, firms with monitoring costs settle into a steady state equilibrium for dividends, which leads to diminished dividend distributions compared to firms that do not bear these costs.

## 3.4 Finding the optimal path

Addressing the dynamic optimization problem with an initial condition $k_0$, we employ a logarithmic utility function and frame the issue through a Bellman equation:

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U(d_0) + \beta \left[ \sum_{t=1}^{\infty} \beta^{t-1} U(d_t) \right] \right\}$$

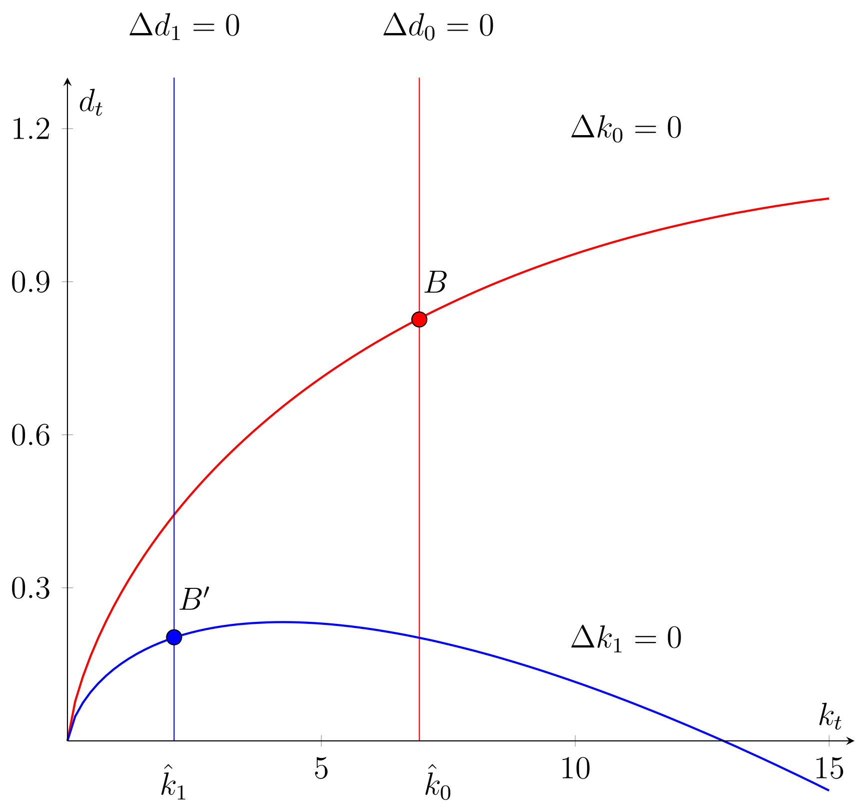


Figure 3.1: This phase diagram illustrates the relationship between dividends ($d_t$) and capital ($k_t$) for two different scenarios of a firm. The red curve represents a firm (Firm 0) that has debt but incurs no monitoring costs, while the blue curve represents another firm (Firm 1) that does have monitoring costs. The curves, labeled $\Delta d_1 = 0$ and $\Delta d_0 = 0$, represent the loci where dividends remain constant. The vertical lines, labeled $\Delta k_1 = 0$ and $\Delta k_0 = 0$, denote the loci where capital is stationary. The intersection point B on the red curve indicates the equilibrium for Firm 0 where both capital and dividends are stationary, located at $(7.25, 0.9)$. In contrast, the intersection point $B'$ on the blue curve represents the corresponding equilibrium for Firm 1 with monitoring costs, positioned at $(2.5, 0.28)$. It is evident that Firm 1, which bears monitoring costs, sustains lower dividends while keeping capital constant, and also requires a lower level of capital to maintain constant dividends when compared to Firm 0.

subject to a dynamic capital accumulation constraint:

$$k_{t+1} = \left[\frac{p+\mu-\mu p}{p} f(k_t) + \frac{p-\delta p - R_f l}{p} k_t - d_t\right] \cdot (1-l)^{-1} \ \forall t,$$

The aim is to determine the optimal dividend strategy $d_t^*$ and the consequent capital levels $k_{t+1}^*$ across all periods. The optimal policy $d_t^* = \varphi(k_t)$ links dividends and capital in a time-invariant manner, deduced from the constraint:

$$k_{t+1} = \left[\frac{p+\mu-\mu p}{p} f(k_t) + \frac{p-\delta p - R_f l}{p} k_t - \varphi(k_t)\right] \cdot (1-l)^{-1}$$

Given the continuous and differentiable nature of capital and dividends, the optimal dividend path can be represented as a function of initial capital, thereby defining the maximum value function $V(k_1)$ in terms of overall utility maximization. The revised problem formulation becomes:

$$\begin{aligned}
&V(k_0) = \max_{d_0} \{U(d_0) + \beta V(k_1)\} \\
&\text{s.t. } k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - \varphi(k_0)\right] \cdot (1-l)^{-1} \\
&k_0 \text{ given.}
\end{aligned} \tag{3.10}$$

The method of 'guess and verify' involves working through the transition equation defined as:

$$k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - d_0^*\right] \cdot (1-l)^{-1}$$

The first order condition (FOC) is specified as $d_0^* = [\beta V'(k_1)]^{-1}$. When this FOC is incorporated into the transition equation, the formulation of the problem becomes a system of equations outlined as follows:

$$\begin{cases} V(k_0) = U(d_0^*) + \beta V(k_1), \\ k_1 = \left[\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - d_0^*\right] \cdot (1-l)^{-1}, \\ d_0^* = [\beta V'(k_1)]^{-1}, \\ k_0 \text{ given.} \end{cases}$$

The initial guess for the solution is:

$$V(k_t) = e + f \ln k_t,$$

leading to a refined system:

$$e + f\ln(k_0) = \ln\left(\frac{k_1}{\beta f}\right) + \beta\left[e + f\ln(k_1)\right] \tag{3.11}$$
$$k_1 = \left\{\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0 - \left[\frac{k_1}{\beta f}\right]\right\} \cdot (1-l)^{-1}.$$

Solving the above system we find:

$$k_1 = \frac{\beta f}{\beta f(1-l)+1}\left(\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0\right) \tag{3.12}$$

Using $d_0^* = k_1(\beta f)^{-1}$ in the above equation we get:

$$d_1 = \frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} f(k_0) + \frac{p-\delta p - R_f l}{p} k_0\right\} \tag{3.13}$$

Assuming $p - \delta p - R_f l = 0$, and using the Cobb-Douglas production function **??**simplifies to:

$$
\begin{aligned}
k_1 &= \frac{\beta f}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\} \\
d_1 &= \frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\} \\
e + f\ln(k_0) &= \ln\left\{\frac{1}{\beta f(1-l)+1}\left\{\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right\}\right\} \\
&\quad + \beta\left\{e + f\ln\left[\frac{\beta f}{\beta f(1-l)+1}\left(\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right)\right]\right\}, \\
e + f\ln(k_0) &= -\ln\left(1+(1-l)\beta f\right) + \ln\left(p+\mu-\mu p\right) - \ln p + \\
&+ (\alpha\ln Z + \alpha\ln k_0) + \beta e - \beta f\ln\left(1+(1-l)\beta f\right) + \\
&+ \beta f\ln\left(p+\mu-\mu p\right) - \beta f\ln p + \beta f\ln\left(\beta f\right) + \beta f(\alpha\ln Z + \alpha\ln k_0)
\end{aligned}
$$

The above equation must be satisfied for any $k_0$ and for any admissible value of the parameters $Z, \beta, \alpha, \mu, p, R_f, l$. Therefore, must be true that:

$$
f = \frac{\alpha}{1-\alpha\beta} \tag{3.14}
$$

$$
\begin{aligned}
e = &\left[-\ln(1+(1-l)\beta f) + \ln(p+\mu-\mu p) - \ln(p)\right. \\
&+ \alpha\ln(Z) - \beta f\ln(1+(1-l)\beta f) \\
&\left.+\beta f\ln(p+\mu-\mu p) - \beta f\ln(p) + \beta f(\alpha\ln(Z)) + \beta f\ln\left(\beta f\right)\right](1-\beta)^{-1}
\end{aligned}
$$

Therefore, the transition and policy functions under financial frictions can be articulated as follows:

$$
k_1 = \left[\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right]\frac{\alpha\beta}{1-l\alpha\beta}, \tag{3.15}
$$

Utilizing $d_0^* = k_1(\beta f)^{-1}$ and in conjunction with 3.14, we can deduce:

$$
d_0^* = \left[\frac{p+\mu-\mu p}{p} Z k_0^{\alpha}\right]\frac{1-\alpha\beta}{1-l\alpha\beta}, \tag{3.16}
$$

By setting $k_1 = k_0 = \hat{k}$ in 3.15, the equation simplifies to:

$$\hat{k} = \left( \frac{Z\alpha\beta(p + \mu - \mu p)}{p(1 - \alpha\beta l)} \right)^{\frac{1}{1-\alpha}} \tag{3.17}$$

This expression elucidates the steady-state capital level, $\hat{k}$, in relation to the parameters $\alpha$, $\beta$, $l$, $p$, $\mu$, and $Z$.

Taking the derivation of $\hat{k}$ with respect to $\mu$ we get:

$$\frac{\partial \hat{k}}{\partial \mu} = \frac{1}{1-\alpha} \left( \frac{Z\alpha\beta(p(1-\mu) + u)}{p(1 - \alpha\beta l)} \right)^{\frac{\alpha}{1-\alpha}} \frac{Z\alpha\beta(1-p)}{[(1 - \alpha\beta l)]} \tag{3.18}$$

Given that the partial derivatives in equation 3.18 are positive, it's clear that monitoring costs $(1 - \mu)$ diminish the steady-state level of capital, and this effect intensifies with increased productivity. Hence, firms with higher productivity face greater adverse impacts from monitoring costs. A potential rationale for this phenomenon is that, typically, more productive firms could negotiate lower interest rates due to their ability to present higher outputs as collateral to financial intermediaries in case of default. However, monitoring costs erode this advantage. In situations where monitoring costs reach 100%, the differential impact on firms of varying productivity levels disappears, as monitoring costs uniformly affect all firms.

Another interesting aspect to explore is the effect on the steady-state level of capital when there is an increase in the leverage ratio. To investigate this, we consider the derivative of $\hat{k}$ with respect to leverage $(l)$, under the assumption that $p = R_f l/(1 - \delta)$.

$$\frac{\partial \hat{k}}{\partial l} = \frac{1}{1-\alpha}\left(\frac{Z\alpha\beta(p(1-\mu)+\mu)}{p(1-\alpha\beta l)}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\left(\frac{Z\alpha\beta(1-\mu)p'p(1-\alpha\beta l)-(p'(1-\alpha\beta l)+p\alpha\beta)Z\alpha\beta(p(1-\mu)+\mu)}{(p(1-\alpha\beta l))^2}\right)$$
$$\frac{\partial \hat{k}}{\partial l} = \frac{1}{1-\alpha}\left(\frac{Z\alpha\beta(R_f l(1-\delta)^{-1}(1-\mu)+\mu)}{R_f l(1-\delta)^{-1}(1-\alpha\beta l)}\right)^{\frac{\alpha}{1-\alpha}}.$$
$$\Big(-\frac{(1-\delta)\mu+\alpha\beta Z R_f(1-\mu)l}{R_f l^2(1-\alpha\beta l)}$$
$$+\frac{\alpha\beta(1-\delta)\mu+\alpha^2\beta^2 R_f(1-\mu)Zl}{R_f l(1-\alpha\beta l)^2}+\frac{\alpha\beta Z(1-\mu)}{l(1-\alpha\beta l)}\Big) \tag{3.19}$$

The determination of whether the partial derivative mentioned in equation 3.19 is positive primarily hinges on the second term of the derivative. To clarify, we can state the condition for the partial derivative to be positive as follows:

$$l(\alpha\beta(1-\delta)\mu+\alpha^2\beta^2 R_f(1-\mu)Zl)+\alpha\beta Z(1-\mu)\geq$$
$$((1-\delta)\mu+\alpha\beta Z R_f(1-\mu)l)(1-\alpha\beta l) \tag{3.20}$$

If the condition outlined in equation 3.20 is satisfied, then increasing leverage will actually increase the steady-state level of capital. This might seem unexpected because typically, higher leverage means a higher risk of default. However, if the assumption ($p-\delta p-R_f l=0$) and condition 3.20 hold, it suggests that the chance of default decreases enough with increased leverage to lower the interest rate, which compensates for the typical rise in interest rates seen with higher leverage. This balancing act continues only up to a certain point of leverage; beyond this, the detrimental effects of additional leverage surpasses the benefits, resulting in a decrease in steady-state capital.

The graph referenced as 3.2 displays this interesting dynamic. For lower levels of leverage, the impact on steady-state capital is positive, which aligns with the condition being met. As leverage increases beyond this favorable range, the graph shows that the partial derivative becomes negative, which means that more leverage reduces the

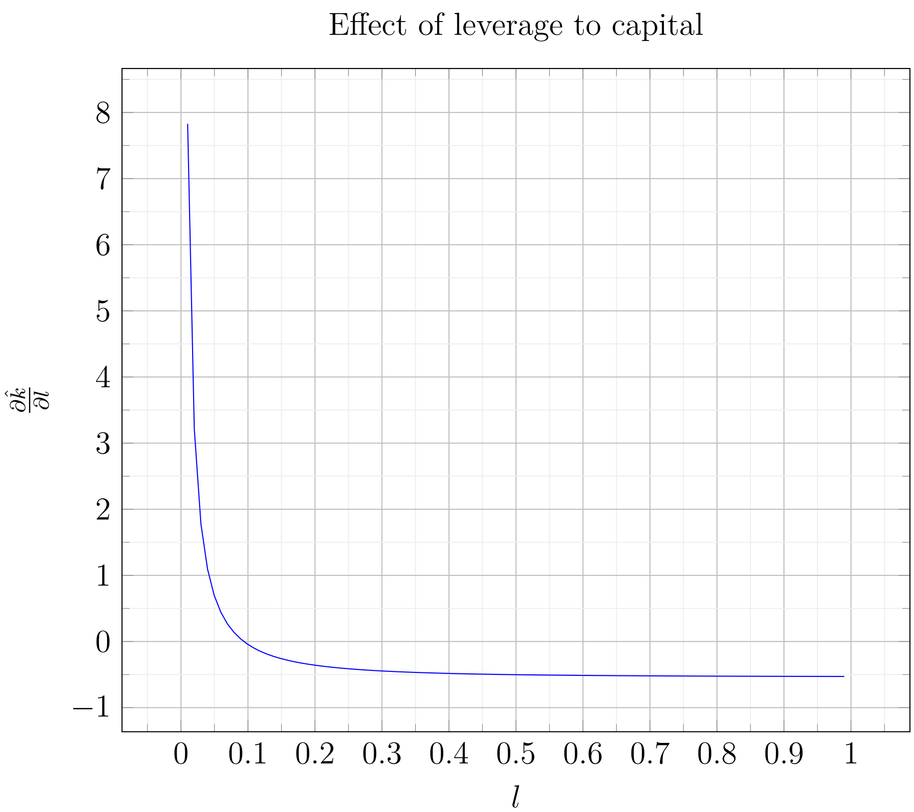


Figure 3.2: The graph depicts the relationship between the partial derivative of $\hat{k}$ with respect to leverage and the leverage itself, under the parameters $R_f = 1.4$, $\mu = 0.75$, $\alpha = 0.7$, $\beta = 0.956$, $\delta = 0.07$, and $Z = 1$. It can be observed that when the leverage is greater than 0.1, the partial derivative becomes negative, signifying that an increase in leverage results in a decrease in steady-state capital. Conversely, when the leverage is below this threshold, the partial derivative sharply rises towards $+\infty$.

capital level. This dynamic is captured in the graph where, below a leverage point around 0.1, leverage has a positive effect on capital, skyrocketing as it approaches zero.

Finally, we can derive the optimal trajectories for capital and dividends, as well as the steady-state locus for capital in the context of no debt and financial friction (assuming $l = 0$, $\mu = 1$, and $p = 1$), the following expressions are obtained:

$$k_1 = Zk_0^\alpha \alpha\beta, \tag{3.21}$$

$$d_0^* = Zk_0^\alpha(1 - \alpha\beta), \tag{3.22}$$

$$\hat{k} = [Z\alpha\beta]^{\frac{1}{1-\alpha}}. \tag{3.23}$$

In conclusion, the optimal capital and dividend paths have been derived under specific financial conditions. Key assumptions, particularly $p - \delta p - R_f l = 0$, critically simplify the dynamics, implying that the net effect of the discount rate, the probability of repayment, and the cost of debt on capital growth is neutral. This assumption allows for a direct relationship between the capital level and the dividend policy, as depicted by the transition and policy functions. The closed-form expressions for $k_1$, $d_1$, and $\hat{k}$ emphasize the steady-state capital level's sensitivity to these parameters, underscoring the pivotal role of $p$ in shaping the firm's financial trajectory. Under the no-debt and no-financial-friction scenario ($l = 0$, $\mu = 1$, $p = 1$), the model further reflects how the absence of these frictions leads to straightforward growth dynamics, reinforcing the importance of these financial parameters in determining the firm's optimal strategies.

## 3.5 Monte Carlo Simulations

To explore the distinctions between scenarios with and without financial frictions, we conduct a simulation exercise employing parameters similar to those used in the Osotimehin and Pappadà [2017] study:

| Parameter | Symbol | Value |
|---|---|---|
| Discount factor | $\beta$ | 0.956 |
| Risk-free rate | $R_f$ | 1.04 |
| Depreciation rate | $\delta$ | 0.07 |
| Returns to scale | $\alpha$ | 0.70 |
| Firm productivity | $Z$ | 1 |

Table 3.2: Benchmark calibration

This section presents a simulation study to compare scenarios with and without financial frictions, employing parameters from the Osotimehin and Pappadà [2017] study. The simulation explores the effects of financial frictions by setting different monitoring costs for two firms, with firm 0 facing no monitoring cost and firm 1 experiencing a monitoring cost of $25\%\mu = 0.75$. The key assumption for the simulation is $p - \delta p - R_f l = 0$, leading to a calculated probability of repayment ($p$) of approximately 0.559 for both firms.

Figure 3.3 explicitly demonstrates the impact of monitoring costs on reducing the steady-state capital level. The observed discrepancy in steady-state capital levels between the two firms is quantified as follows:

$$
\begin{aligned}
\hat{k_0} &= 7.077, \\
\hat{k_1} &= 2.713, \\
\Delta\hat{k} &= \hat{k_0} - \hat{k_1} = 4.364.
\end{aligned}
$$

Hence, imposing a 25% monitoring cost on the output, under a fixed leverage ratio of 0.5, results in a 69% reduction in the steady-state level of capital.

Additionally, Figure 3.4 illustrates the optimal dividend paths for the same two firms, one burdened by monitoring costs and the other not. Following the policy function detailed in 3.16, derived from the solution to the Bellman equation.

In the 3.4 The dividend trajectory is notably higher in scenarios without financial

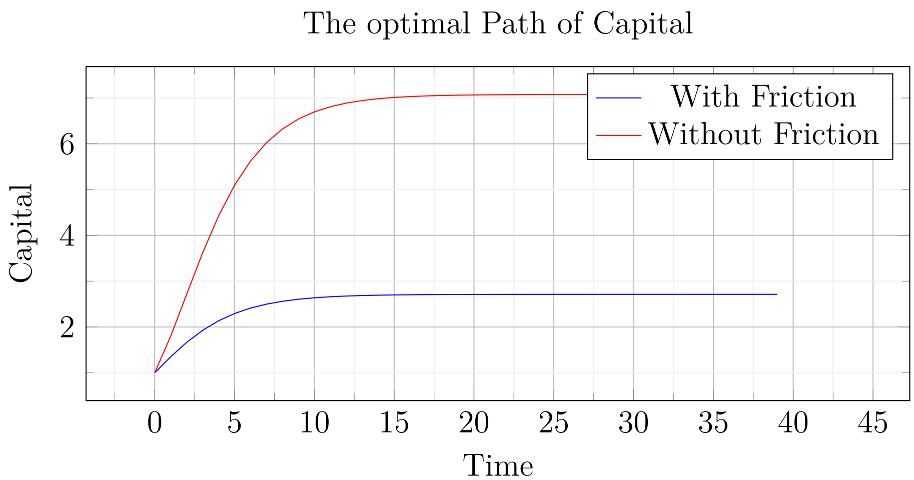


Figure 3.3: The graph illustrates the optimal trajectory for capital as defined by equation 3.15. It features a red line that remains above the blue line across all periods, symbolizing the optimal capital path towards the steady state in the absence of monitoring costs. Conversely, the blue line delineates the optimal path for firms subjected to a 25% monitoring cost. Apart from the monitoring cost, all other parameters are held constant. It is evident from the graph that monitoring costs have a significant impact on the steady state, notably reducing it.

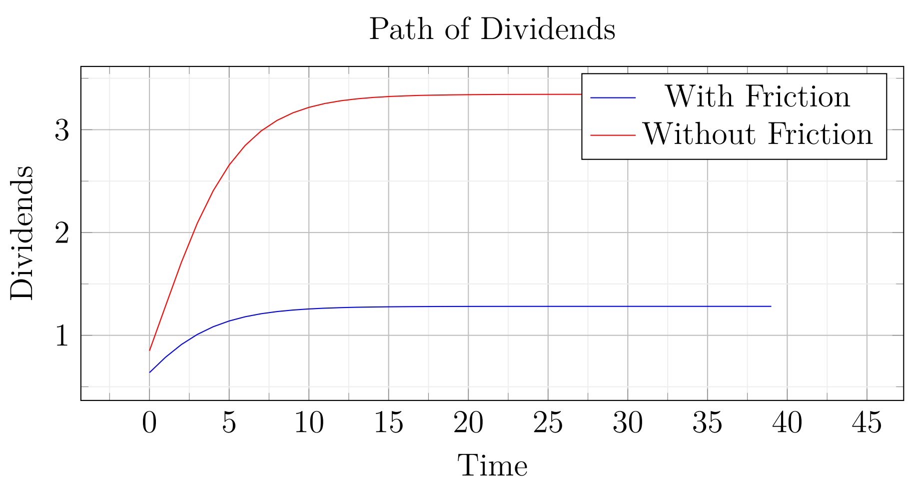


Figure 3.4: The figure illustrates the optimal trajectory of dividends as derived from equation 3.16. Here, the red line, consistently above the blue across all periods, indicates the optimal progression toward the steady-state level when monitoring costs are not factored in. In contrast, the blue line represents the trajectory under the influence of a 25% monitoring cost. With all other parameters held constant, it is evident that monitoring costs exert a downward pressure on the steady-state level of dividends.

frictions, emphasizing the significant role these frictions play in reducing returns. In a steady-state analysis, monitoring costs lead to a 78% decrease in dividend levels, given a constant leverage ratio of 0.5. In the forthcoming sections, the analysis will expand to simulate an economy with heterogeneous agents, aiming to dissect the broader implications of financial frictions on overall output. This simulation will contrast environments with and without financial frictions, with a particular focus on the cleansing effect that such frictions may exert in economic systems. Through this comparative approach, we aim to deepen the understanding of how financial frictions influence economic resilience, performance, and the potential for innovation and growth amidst challenges. Next, we will simulate an economy with heterogeneous agents to assess the overall output in environments with and without financial frictions. This analysis aims to explore the cleansing effect of financial frictions, providing insights into their influence on economic performance and growth potential.

### 3.5.1 Heterogeneity and Aggregation Mechanism

To refine the model, we introduce heterogeneity among firms, marking a departure from uniform productivity. Specifically, productivity levels $(Z_i)$ now vary across firms, introducing a spectrum of efficiency within the model. Additionally, we diversify leverage ratios, ensuring no direct correlation between a firm's productivity and its leverage. This heterogeneity is captured from the outset by simulating the initial distribution of capital, leverage, and productivity, setting the stage for a dynamic interplay of firm characteristics. The aggregate output at time t for a population of N firms is :

$$K_t = \sum_{i=0}^{N} Z_{i,t} k_{i,t^{\alpha}} \tag{3.24}$$

The business cycle is modeled through a sine function, impacting the productivity factor $Z$ uniformly across all firms in the economy. Furthermore, the maximum influence the business cycles can exert on productivity is limited to 10% of $Z$. Consequently, the cycle is represented as follows:

$$\Delta Z_t = 1 + 0.1 \sin(t) \tag{3.25}$$

The firm determines an optimal level of output at time $t_0$ for the subsequent time $t_1$, based on equation 3.15. However, the actual output may deviate from this optimal level due to fluctuations in the productivity factor $Z$, which is influenced by the business cycle, as described in Equation 3.25. To differentiate between these two scenarios, the term 'optimal' will refer to the firm's chosen level of output, while 'actual' will describe the output level that the firm ultimately achieves.

Productivity ($Z$) and leverage ($l$) follow truncated normal distributions:

$$l \sim \mathcal{N}(0.05, 0.1), \qquad 0.01 \leq l \leq 1, \tag{3.26}$$

$$Z - 1 \sim \mathcal{N}(0.5, 0.1), \qquad 1.01 \leq Z \leq 1.1. \tag{3.27}$$

The exit mechanism for firms in this model is based on criteria where a firm's exit is contingent on its return on capital being below the risk-free rate and also being among the lowest two within the entire firm population. This approach ensures that at each evaluation point, the potential for market exit is limited to the two firms with the least efficient capital utilization, provided their returns are less than the risk-free rate. Consequently, this mechanism inherently caps the maximum proportion of firms that can exit the market at any given time to prevent unrealistic, explosive market dynamics.

By introducing a natural cap that theoretically limits maximum exits to 20% of the firm population, the market maintains a dynamic equilibrium, facilitating the exit of the least productive firms and preventing scenarios where excessive market exits lead to implausible economic conditions.

For a firm $i$ at time $t$, the return on capital is calculated as follows:

$$R_{i,t} = \frac{d_{i,t-1}}{k_{i,t}} + \frac{k_{i,t} - k_{i,t-1}}{k_{i,t}} \tag{3.28}$$

A firm is mandated to exit the market if its return on capital ranks among the lowest two and falls below the risk-free rate ($R_f$). This criterion is applied systematically at each simulation step, aligning the market's evolutionary process with economic efficiency principles.[1] Through this mechanism, the model illustrates a market that rewards efficiency while ensuring a balanced impact of exits on the economic landscape.

### 3.5.2 Findings

In the presented investigation, financial frictions are integrated into the economic model, characterized by a monitoring cost parameter $1 - \mu = 0.15$. The aim is to discern the repercussions of such frictions on the economic dynamics. To this end, we conduct two distinct simulations within the environment of financial frictions: one where an economy is subject to an exit and capital reallocation mechanism, and another where this mechanism is absent, inhibiting the movement of capital.

The analysis is robust, comprising 100 Monte Carlo simulations to determine the sample mean and variance of each variable. Central to our inquiry is whether a recession induces a 'cleansing effect,' wherein less efficient firms exit the market, leading to a potential increase in aggregate productivity. Accordingly, we simulate two economic setups: one where firms operate without an enforced exit, and another where firms are mandated to exit under certain conditions. In both setups, firms face financial frictions manifested as monitoring costs, consistent with $1 - \mu = 0.15$. The ensuing figures present the sample mean and 95% confidence intervals for total output, comparing scenarios where exits are permitted to those where no reallocation occurs.

Visual representation (3.5) of total output over a 20-step horizon, including 100

[1] The exit criteria, focusing on the two lowest returns below the risk-free rate, are assessed at each step of the simulation to identify firms for market exit.

iterations among a group of 10 firms, illustrates the comparative trajectories. Under financial frictions, the economy with the exit mechanism in play consistently registers a superior aggregate output level. This finding corroborates the notion of a market-driven 'cleansing effect'—the process through which less productive firms are culled, freeing up resources for redistribution to more efficient entities. Notably, this effect is not merely sustained amidst financial frictions; it is measurably impactful, enhancing production output. This observed phenomenon underscores the potential for policy-driven capital reallocation mechanisms to bolster economic efficiency, even when financial barriers are present.

The presence of a forced reallocation mechanism contributes to a higher overall capital ($k$) at each step, substantiating the notion that a cleansing effect persists even amidst financial frictions, resulting in greater production compared to scenarios where firms are barred from exiting. Evidently, these variations in overall capital are markedly pro-cyclical, aligning with the findings in Davis and Haltiwanger [1992]. In the subsequent figure 3.6, we delve into the nuances of this impact by examining the mean log difference in total production between the two scenarios, accompanied by a 95% confidence interval. The data underscores a positive and a statistically significant difference in overall output, indicative of the beneficial role played by the exit mechanism in the context of financial frictions. This effect echoes the theoretical insights discussed earlier, where the exit mechanism serves as a pivotal driver of economic efficiency and productivity.

Graph 3.6 illustrates the effect of firm exits and capital reallocation on total output in two different economic scenarios, factoring in the impact of monitoring costs. The red line represents economies with monitoring costs equating to 15% of recoverable assets ($1 - \mu = 0.15$), while the blue line depicts economies without any monitoring costs ($1 - \mu = 0$). The contrast between the two lines showcases the 'cleansing effect' of firm exits: in the absence of monitoring costs, this effect prompts a 20% rise in output, whereas, with monitoring costs, the output boost is more modest, at about 7%. The error bars on the graph denote a 95% confidence interval, affirming the statistical

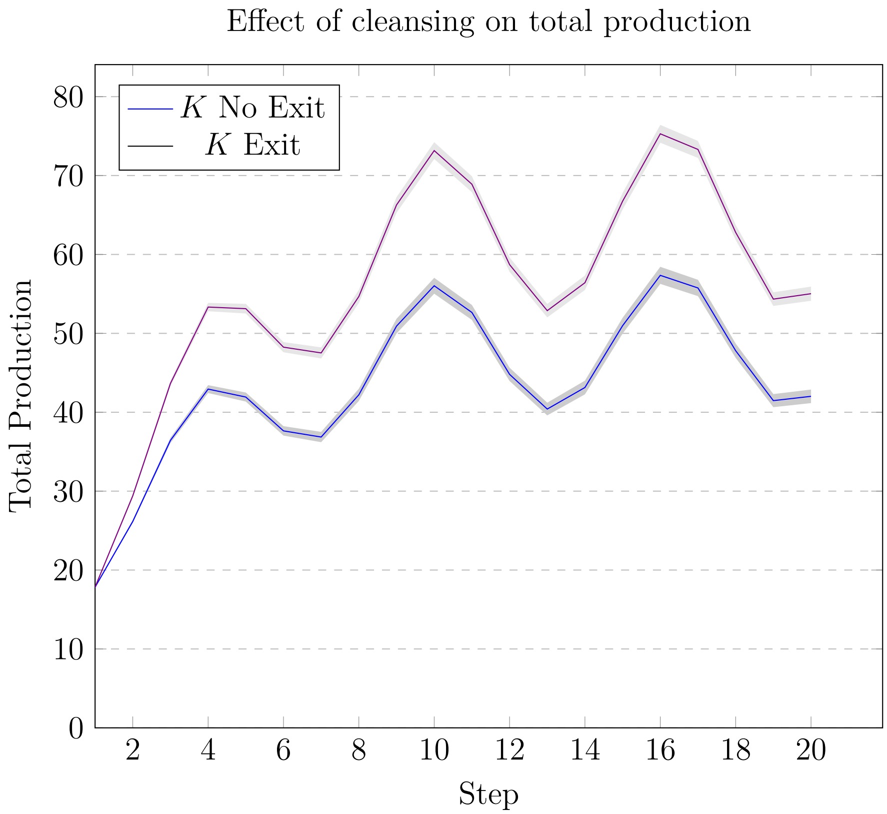


Figure 3.5: The plot presents the outcomes of a 20-step simulation involving 10 firms, contrasting the total output ($K$) in an economy with an exit and reallocation mechanism (red line) against an economy lacking such mechanisms (blue line). Both scenarios are subject to 15% monitoring costs and experience a sinusoidal continuous shock affecting productivity. The visualization illustrates that the incorporation of exit and reallocation strategies enhances the overall output of the economy despite the presence of financial frictions. Given that the total number of firms is held constant at ten, the aggregate production can be interpreted as average productivity by dividing by ten. The data suggest the presence of a cleansing effect within the economy, even when financial frictions are factored in. The confidence intervals are depicted at a 95% level, derived from 100 Monte Carlo simulation iterations.

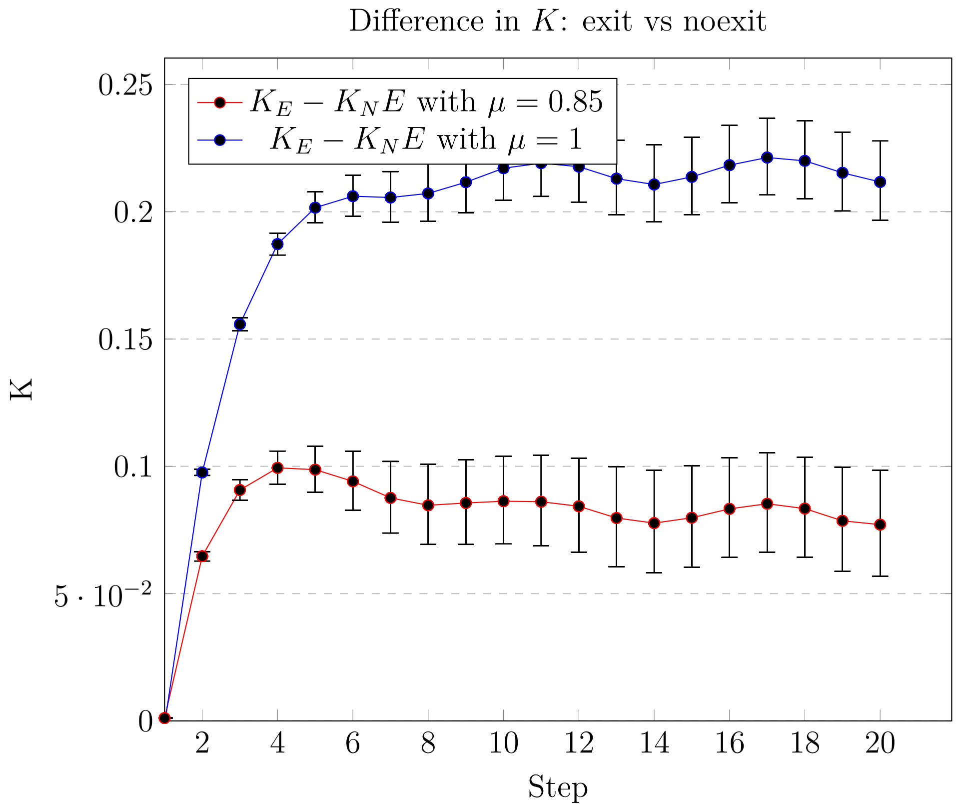


Figure 3.6: The graph presents the percentage difference in total output between two economic scenarios: one with firm exit and capital reallocation and another with no firm exit, with a consideration of monitoring costs. The red line indicates economies where monitoring costs amount to 15% of recoverable assets ($1 - \mu = 0.15$), and the blue line represents economies without monitoring costs ($1 - \mu = 0$). The 'cleansing effect' of market exit on overall output is depicted by the difference between these lines. In the absence of monitoring costs, the cleansing effect results in a 20% increase in output, while in the presence of monitoring costs, the increase in output is positive but smaller, at approximately 7%. Additionally, the error bars indicate a 95% confidence interval (CI), highlighting the statistical reliability of the observed differences.

significance of the results.

Figure 3.6 affirms that capital reallocation following firm exits can enhance productivity, even under financial constraints represented by a 15% monitoring cost. Despite the asymmetric information challenges between financial intermediaries and firms, the positive impact on production is not only present but also accumulates over time, suggesting a magnifying effect. Therefore, two otherwise identical economies in terms of productivity and capital distribution will ultimately diverge in output if one incorporates the mechanism of firm exits and capital reallocation, despite the existence of financial frictions.

# Chapter 4

# Conclusions

In the usual course of recessions, as Davis and Haltiwanger [1992] noted, there's typically an uptick in the number of firms shutting down and a boost in the movement of workers—factors that are often followed by an increase in average productivity. However, the Great Recession deviated from this pattern in the U.S., with a sluggish reallocation rate and a persistently low rate of new firm creation, unlike what was seen in prior downturns. The research from Foster et al. [2016] points out that the reallocation trends that were common in past recessions didn't hold up during the Great Recession. In this context, Osotimehin and Pappadà [2017] took a fresh look at the key theoretical model of the cleansing effect from the existing literature, specifically the model by Caballero and Hammour [1994], by incorporating the concept of financial frictions to probe whether these frictions were at play in altering the reallocation behavior of firms during such challenging economic times. This thesis revisits the established model by incorporating borrowing constraints that limit the amount of debt a firm can incur. Through this adjustment of the theoretical framework, it has been possible to derive a closed-form solution for the optimal trajectories of dividends and capital. This solution enables a direct examination of how monitoring costs and a predetermined leverage ratio influence steady-state capital.

The findings from the model reveal that monitoring costs, which financial intermediaries face when a firm defaults, reduce the optimal trajectory and the steady state of capital and dividends. This makes firms more susceptible to the adverse effects of economic downturns. Notably, this impact is more pronounced for highly productive firms, which feel a greater effect on their steady-state capital and dividends. The reason behind this is linked to the assets that can be collateralized in this model, represented by the ratio of output to debt. More productive firms, capable of generating more output with the same amount of capital, would typically benefit from lower interest rates. However, because monitoring costs scale with output, they diminish the positive impact that productivity has on interest rates. In scenarios where monitoring costs match the output-to-debt ratio, the advantage of higher productivity vanishes entirely. Another noteworthy result pertains to the influence of leverage on steady-state capital. This effect is ambiguous, as the partial derivatives are sensitive to the actual leverage level: the derivative becomes negative when leverage surpasses a specific threshold, indicating that excessive leverage reduces steady-state capital. Conversely, within the bounds of stringent borrowing constraints, thus below the leverage level that makes derivative positive, firms might find advantages in increasing their leverage. Consequently, in regulatory environments that cap leverage at a low level, there's a potential risk of firms operating with suboptimal capital. Relaxing these constraints to permit slightly higher leverage could, under certain conditions, enhance the firm's financial health and stability. It's crucial to understand that the financial frictions of borrowing constraints and monitoring costs are interconnected, and their impacts are cumulative, sometimes even counteracting each other. For instance, as demonstrated, when leverage is initially low, an increase in leverage can actually boost steady-state capital and production. Yet, it's more common and practical to consider situations where leverage exceeds this minimal threshold, reversing the positive effect on the partial derivative. Reflecting on the Great Recession, if we look at tighter leverage restrictions combined with heightened monitoring costs, these two factors together contribute to a significant decrease in steady-state capital, as well as in overall output and dividends. This observation may explain why the Great Recession differed from previous downturns in terms of firm reallocation patterns, as Foster et al. [2016] have highlighted. Through Monte Carlo simulations, it was possible to model an economy under continuous sinusoidal productivity shocks, featuring firms varied by leverage and productivity. An exit mechanism was introduced to simulate the cleansing effect, forcing firms to exit the market if their return on capital was among the lowest or fell below the risk-free rate. Notably, economies facing monitoring costs showed a diminished cleansing effect on average productivity, in contrast to those without such costs, which experienced an enhanced effect. These simulations underscore that during financial recessions with intensified financial frictions, the advantages of capital reallocation through firm exits are reduced, corroborating the findings of Osotimehin and Pappadà [2017]. In conclusion, this theoretical model has shed some light on how financial frictions, like borrowing constraints and monitoring costs, affect the paths and steady states for capital and dividends. The Monte Carlo simulations hint at why financial crises might show different reallocation patterns than other recessions, providing a starting point for understanding these complex dynamics.

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