Theoretical framework

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Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

1 Introduction

In macroeconomic theory, the investigation of business cycles and long-term growth trajectories traditionally unfolds in distinct academic silos, drawing a parallel to the distinct realms of quantum mechanics and Einstein's theory of relativity in physics. This academic segregation, however, obscures a fundamental and profound question: How do business cycles influence long-term economic growth? The exploration of this question is more than an academic exercise; it underpins the practical understanding of short-term economic policies, such as automatic stabilizers, and their profound long-term impacts on the economy.

Embarking on this exploration, my research primarily dwells in the realm of theory, supplemented by rigorous simulation and calibration exercises. The intricate complexity of business cycles, particularly evident during periods of economic downturn and recovery, challenges empirical approaches due to the plethora of confounding variables. Thus, a theoretical lens, rather than a purely empirical one, is employed to dissect and understand these phenomena.

Central to this theoretical framework is an examination of the role of financial market frictions during economic recessions. A key inquiry here is the investigation of policy interventions, such as demand stabilizers, and their potential effect in attenuating the 'cleansing effect' of recessions. This exploration is pivotal in understanding whether such policies might inadvertently lead to a reduced economic baseline or steady state in the long term.

The conceptual foundation of this investigation is inspired by an ecological analogy the cyclical dynamics observed between predator and prey populations in nature. This natural cycle, when observed over extended periods, reveals not just self-contained oscillations but also underlying trends of population growth for both predators and prey. This observation leads to a compelling analogy for economic cycles: while they appear as short-term fluctuations, they might be underpinned by long-term growth trajectories.

In natural ecosystems, interventions aimed at stabilizing these cycles such as protecting prey during times of increased predation might seem beneficial in the short term. However, such interventions often neglect the critical and natural process of selection. This interference disrupts evolutionary mechanisms, potentially leading to unforeseen consequences over time, such as the propagation of traits detrimental to the species' survival and adaptability in changing environments.

My thesis extends this analogy to the economic sphere, positing a similar selective mechanism at play in economic systems. The primary focus is on the recession's cleansing effect, which might be analogous to natural selection in ecology. This effect could potentially 'weed out' less productive firms, leaving a market landscape dominated by more efficient players. The exploration aims to decipher whether such an economic 'natural selection' mechanism exists and, if so, how it shapes the fabric of productivity, innovation, and growth in the long term. Through this lens, the research endeavors to contribute a nuanced understanding of the intricate interplay between short-term economic fluctuations and long-term economic evolution, offering insights into the design and implications of economic policies. In the following sections, we will delve deeper into specific theories that bridge the gap between business cycles and long-term economic growth. However, it is beneficial first to embark on a brief historical journey through the evolution of thought regarding business cycles, to understand the context and development of these interconnected economic theories. This exploration will provide a foundation for appreciating the diversity of perspectives and the progression of ideas that have shaped our understanding of the intricate relationship between short-term economic fluctuations and long-term growth trajectories.

2 Business cycle history

The exploration of business cycle theories represents a cornerstone in the history of economic thought. A prominent exponent in this realm was Friedrich Hayek, who articulated the complexities of business cycles in relation to economic equilibrium theory. Hayek's perspective is encapsulated in his own words:

> "The incorporation of cyclical phenomena into the system of economic equilibrium theory, with which they are in apparent contradiction, remains the crucial problem of Trade Cycle theory; By 'equilibrium theory' we primarily understand the modern theory of the general interdependence of all economic quantities, which has been perfectly expressed by the Lausanne School of theoretical economics." Hayek [1933]

In the turbulent era of the early 1930s, the exploration of business cycles was prominently shaped by the contrasting viewpoints of Friedrich Hayek and John Maynard Keynes, two of the twentieth century's most influential economists. Hayek's examination of business cycles, as delineated in his seminal work "Monetary Theory and the Trade Cycle," Hayek [1933], and further elaborated in "The Pure Theory of Capital" Hayek and Caldwell [1941], provides a profound analysis through the lens of monetary theory and its ramifications on capital structure. Hayek posited that economic distortions, notably those stemming from the artificial depression of interest rates by central banks, precipitate malinvestments during periods of economic expansion. These malinvestments, especially prevalent in capital-intensive sectors, were deemed unsustainable, leading inevitably to economic downturns characterized by the correction of these misallocations.

Conversely, Keynes, in his groundbreaking "The General Theory of Employment, Interest, and Money" **?**, approached the business cycle issue from an equilibrium perspective, focusing on the role of aggregate demand in determining overall economic activity levels. Keynes argued that a shortfall in aggregate demand could lead to protracted periods of high unemployment, advocating for active government intervention to stimulate demand and re-establish economic equilibrium.

Hayek's theoretical framework underscores the intricate relationship between capital investment and monetary disequilibrium within the business cycle. He elucidates this connection through the concept of inter-temporal preferences, which dictate the pace of capital investment and the extent of capital accumulation. The decision-making process regarding the allocation of resources between present dividend and future investment is critical to understanding the cyclical nature of the economy. Hayek asserts:

$$I_t = f(r_t, r_n)$$

$$r_t < r_n \rightarrow \text{Malinvestment}$$

where I_t signifies investment at time t, r_t represents the market interest rate, and r_n denotes the natural rate of interest. A divergence between r_t and r_n, particularly when r_t is artificially maintained below r_n, leads to malinvestments. This discrepancy between the market and natural rates of interest, exacerbated by the expansion of bank credit, serves as the cornerstone of Hayek's business cycle theory. Hayek further elaborates on the dynamic and temporal aspects of production and investment in "The Pure Theory of Capital" Hayek and Caldwell [1941], where he critically assesses the equilibrium-based economic theories and emphasizes the importance of understanding the temporal structure of capital.

Hayek's analysis of monetary disequilibrium and its impact on the business cycle is complemented by his insights into the mechanisms of bank credit creation and its influence on the natural state of equilibrium in the market for loanable funds. The expansion of bank credit, which decouples the market rate of interest from the natural rate, instigates cycles of over-investment and mal-investment, ultimately culminating in inter-temporal economic instability.

These theoretical perspectives offered by Hayek provide a nuanced understanding of the complexities inherent in the business cycle, challenging the Keynesian emphasis on aggregate demand and fiscal policy interventions. Hayek's contributions, particularly in "The Pure Theory of Capital," highlight the significance of capital theory in analyzing monetary disequilibria and underscore the dynamic and inter-temporal nature of economic activities.

Thus, while Keynes emphasized stabilizing aggregate demand to achieve equilibrium and mitigate business cycles, Hayek focused on the inherent dynamism and complexity of economic systems, criticizing equilibrium models for their oversimplification of the intricate processes that drive economic activities. This divergence in views marked a fundamental debate in economic theory, shaping the discourse on how economies respond to and recover from periods of economic downturns.

Joseph Schumpeter, another influential economist of the early 20th century, brought a unique perspective to the discussion of business cycles, one that diverged significantly from both Hayek and Keynes. Schumpeter's approach, primarily outlined in his theory of "creative destruction," emphasized the role of innovation and entrepreneurial spirit in economic development and business cycles.

Schumpeter viewed business cycles as inherent and vital to capitalist economies, driven by the process of innovation. According to Schumpeter, the entrepreneur is the agent of change, introducing new technologies, products, and methods, which disrupt existing market equilibria. This process of innovation leads to the destruction of outdated industries and economic structures, paving the way for new ones. In Schumpeter's framework, the cyclical nature of the economy was a reflection of this ongoing process of creative destruction, where periods of economic downturns were seen not just as phases of correction, as Hayek might argue, or as failures of demand, as per Keynes, but as essential for clearing away the old to make way for the new.

While Hayek focused on the distortions in capital structure caused by monetary interventions and Keynes emphasized the role of aggregate demand and government intervention in stabilizing the economy, Schumpeter's perspective highlighted the evolutionary nature of capitalist economies. He argued that economic fluctuations were natural and necessary, a process through which economies evolve and adapt over time. Schumpeter's theory thus provided a more dynamic view of capitalism, recognizing the disruptive yet progressive role of innovation and entrepreneurship in shaping economic landscapes.

Schumpeter's contribution added a dimension of evolutionary change to the understanding of business cycles, contrasting with Hayek's emphasis on monetary theory and capital structure, and Keynes's focus on equilibrium and aggregate demand. Schumpeter's insights into the transformative power of innovation offered a more optimistic view of economic downturns, seeing them as opportunities for new growth and advancements.

In the aftermath of World War II, the landscape of macroeconomic theory experienced a paradigmatic shift, significantly influenced by the ascendancy of Keynesian economics. The widespread devastation of the war necessitated a thorough reevaluation of economic policies and theories, setting the stage for Keynesian principles to take a dominant position in shaping government approaches to economic policy, especially concerning business cycles.

Central to the Keynesian doctrine is the advocacy for proactive government intervention to stabilize economic cycles. This perspective gained substantial traction in the post-war era, a period marked by the reconstruction efforts of numerous nations. The foundational principle of Keynesianism, as articulated in Keynes's seminal work "The General Theory of Employment, Interest, and Money," Keynes [1960]is that government spending can act as a catalyst to stimulate demand during economic downturns. This tenet represented a stark divergence from the pre-war classical economic thought, which predominantly favored limited government intervention in the economy.

A key element in Keynesian policy is the concept of fiscal multipliers, which posits that government spending has a multiplied effect on overall economic output. The multiplier effect can be conceptualized as follows:

$$\Delta Y = \text{Multiplier} \times \Delta G$$

where ΔY represents the change in total output, and ΔG is the change in government spending. The 'Multiplier' is defined as:

$$\text{Multiplier} = \frac{1}{1 - c(1 - t) + m}$$

Here, c is the marginal propensity to consume, t is the tax rate, and m is the marginal propensity to import. This formula illustrates how an increase in government spending (ΔG) can lead to a larger increase in total output (ΔY), thereby stimulating economic activity during recessions.

The immediate post-war period witnessed the successful implementation of Keynesian policies, evidenced by stable economic growth and reduced unemployment rates. This success solidified the influence of Keynesian economics, particularly in the United States and Western Europe, where governments widely adopted strategies such as fiscal stimulus, interest rate manipulation, and welfare state expansion to regulate economic cycles and mitigate recessionary impacts.

The era also saw the rise of the concept of "fine-tuning" the economy. Policymakers and economists believed that through judicious management of fiscal and monetary policies, it was possible to circumvent severe economic downturns and maintain consistent growth. The underlying idea was that by modulating government spending, tax policies, and interest rates, governments could bolster and sustain demand, thereby smoothing out the fluctuations inherent in business cycles.

The post-war dominance of Keynesian economics encountered a significant challenge in the 1970s with the onset of stagflation, a term coined to describe the unprecedented combination of high inflation and high unemployment, accompanied by stagnant demand. This phenomenon posed a critical challenge to the Keynesian framework, which had not anticipated such a simultaneous occurrence of inflation and stagnation, fundamentally questioning its policy prescriptions.

The Keynesian model, as traditionally understood, posited an inverse relationship between inflation and unemployment, often illustrated by the Phillips Curve Phillips [1958]. This relationship suggested that higher inflation could help reduce unemployment by stimulating demand, and vice versa. However, the stagflation of the 1970s, where high inflation coexisted with high unemployment, contradicted this established Keynesian principle. The phenomenon was first brought to widespread attention by economists such as Milton Friedman and Edmund Phelps, who argued that the Phillips Curve could only function in the short term, and that long-term attempts to exploit this trade-off would lead to ever-increasing rates of inflation without reducing unemployment Friedman [1972].

The stagflation era was marked by several critical global events that influenced economic conditions. The 1973 oil crisis, triggered by an oil embargo by OPEC nations, led to a dramatic increase in oil prices, contributing significantly to inflationary pressures worldwide. This external shock, coupled with the already expansionary fiscal and monetary policies in many Western economies, exacerbated inflation without spurring economic growth or reducing unemployment.

The Keynesian approach, which advocated for increased government spending and lower interest rates to combat recessions, appeared ineffective in addressing the simultaneous challenges of stagnation and inflation. This inadequacy led to the rise of alternative economic theories, such as monetarism and supply-side economics, which focused more on controlling inflation and stimulating supply rather than managing demand. Monetarism, championed by Milton Friedman, emphasized the importance of controlling the money supply to manage inflation, arguing that inflation was primarily a monetary phenomenonFriedman [1972]. Meanwhile, supply-side economics advocated for reducing tax rates and decreasing regulation to encourage production and investment.

The emergence of stagflation and the subsequent paradigm shift in economic theory underscored the complexity of economic systems and the limitations of existing models. It marked a transition in macroeconomic thought and policy, from a predominantly Keynesian consensus to a more diverse array of approaches, including monetarism and supply-side economics, each offering different perspectives on how to achieve economic stability and growth. This period of economic rethinking paved the way for significant policy shifts in the 1980s, particularly in the United States and the United Kingdom, where there was a move towards deregulation, reduction of government intervention, and a greater emphasis on monetary policy to control inflation.

The narrative of economic thought in the latter half of the 20th century took a pivotal turn with the introduction of the Lucas Critique, formulated by economist Robert Lucas in 1976 Lucas [1976]. This critique fundamentally challenged the prevailing Keynesian orthodoxy and ushered in a new era in the understanding of economic policy and business cycles.

The Lucas Critique posited that traditional Keynesian macroeconomic models, which were used to design economic policies, were fundamentally flawed because they did not take into account the way people's expectations and decisions would change in response to policy shifts. Lucas argued that individuals and firms adjust their behavior based on their expectations of future policy, which in turn alters the effectiveness of that policy. This meant that historical data, which Keynesian models heavily relied upon, could not reliably predict the outcomes of economic policies, as the very implementation of these policies would change the economic environment and behavior of agents within it.

Consider a simple Keynesian macroeconomic model where output is determined by:

$$Y_t = \alpha + \beta G_t + \epsilon_t \tag{1}$$

where Y_t is the output, G_t is the government expenditure, α and β are parameters, and ϵ_t is a random error term.

Lucas argued that the parameters α and β in such models are not constant but change with policy. Thus, a policy change that alters G_t will also change α and β, rendering predictions based on historical data unreliable. This is because the parameters are not deep structural parameters but are themselves functions of agents' expectations and behaviors, which are influenced by policy.

The critique can be formalized as follows:

$$Y_t = f(G_t, \theta_t) + \epsilon_t \tag{2}$$

where θ_t represents the agents' expectations and behaviors, which are influenced by policies. Therefore, any policy change that alters G_t also changes θ_t, and thus the relationship between Y_t and G_t.

The Lucas Critique led to the development of models with microeconomic foundations and rational expectations, where policy effectiveness is assessed by considering how policies alter agents' expectations and decision-making processes. This shift was instrumental in the evolution of New Keynesian economics, which integrates rational expectations with menu costs, nominal rigidities, and other market imperfections.

The implications of the Lucas Critique were profound. It suggested that policies based on historical models might be ineffective or even counterproductive. This critique was a significant factor in shifting the focus of macroeconomic research from Keynesian models, which emphasized aggregate demand and fiscal policy, to new classical models, which focused more on microeconomic foundations and the role of rational expectations.

Lucas's ideas were met with resistance and debate. Opponents, particularly those in the Keynesian camp, argued that while the critique had merit, it did not invalidate the use of all macroeconomic modeling. They contended that while expectations are important, other factors like market imperfections and rigidities also play a critical role in the functioning of the economy. These Keynesian economists argued for the continued relevance of fiscal policy and government intervention, especially in situations like recessions, where private demand is insufficient.

The debate sparked by the Lucas Critique led to significant advancements in economic theory. It pushed economists to develop new models that incorporated rational expectations and more robust microeconomic foundations. This period saw the rise of New Keynesian economics, which attempted to merge Keynesian concepts with microeconomic foundations, including rational expectations and market imperfections.

In the years following the Lucas Critique, economic thought around business cycle theories underwent significant evolution, particularly with the integration of various types of frictions into macroeconomic models. This period saw a shift from idealized notions of perfect competition and complete information to models that better reflected the complexities of real-world economies.

The late 1980s and early 1990s marked the rise of New Keynesian economics, which emphasized the role of nominal rigidities, such as sticky prices and wages, in the economy. Pioneering works in this field, like those of Gregory Mankiw and David Romer Mankiw and Romer [1991], established theoretical foundations where these market imperfections were central to explaining why economies do not self-correct efficiently after disturbances.

Simultaneously, Real Business Cycle (RBC) theory, emerging from the seminal work of Finn E. Kydland and Edward C. Prescott, particularly their influential paper ”Time to Build and Aggregate Fluctuations” Kydland and Prescott [1982], focused on real (non-monetary) shocks and frictions. RBC theory emphasized aspects such as technology shocks and constraints in capital accumulation, offering insights grounded in neoclassical microfoundations.

Another significant development was the exploration of credit market frictions, especially after the financial crisis of 2007-2008. The work of Bernanke and Gertler, particularly "Agency Costs, Net Worth, and Business Fluctuations" Bernanke and Gertler [1986], brought to light how asymmetric information in credit markets could amplify business cycles, linking the financial health of firms to their investment and production decisions.

Further, the introduction of search and matching frictions in labor markets, attributed to the work of economists like Diamond, Mortensen, and Pissarides, highlighted how inefficiencies in matching workers with jobs could lead to persistent unemployment, thus influencing the business cycle. Their research, including papers like "Job Creation and Job Destruction in the Theory of Unemployment" (1994)Mortensen and Pissarides [1994], was pivotal in integrating labor market dynamics into business cycle theories.

The culmination of these developments is seen in the formulation of Dynamic Stochastic General Equilibrium (DSGE) models that incorporate various frictions. These models, blending insights from New Keynesian and RBC theories, integrate sticky prices, financial frictions, and labor market imperfections. They have become a standard tool for economic policy analysis, particularly among central banks, representing a significant advance in the ability to simulate and understand economic fluctuations and policy impacts.

3 Theories Connecting Business Cycles to Long-Term Growth

In the domain of economic theory, the relationship between business cycles (BC) and long-term growth is dissected into two principal schools of thought. The conventional viewpoint suggests that long-term growth is chiefly propelled by technological progress. Within this framework, technological advancements are often viewed as exogenous—arising outside the economic model's explanatory scope, as highlighted in the seminal contributions of Solow [1956] and Swan [1956]. This perspective treats technological progress as an independent variable that exerts influence on economic growth without being influenced by the economy's internal dynamics.

Contrastingly, an alternative strand of theoretical work aims to endogenize technological progress, weaving it into the fabric of the economic process. These models embed factors such as incentives for innovation, the value of education, and the accumulation of knowledge through economic activities, epitomizing the 'learning by doing' paradigm. A prominent example of this approach is found in Stadler [1990], which posits technological progress as an emergent property of economic behavior and incentives, rather than a mysterious external force.

A critical aspect of the 'learning by doing' model is its premise that technological frontiers are contingent upon the existing knowledge base, which expands primarily through practical experience. Consequently, periods of economic expansion witness a sharp increase in the knowledge stock, driven by higher employment levels, whereas recessions tend to stabilize or even diminish this stock due to reduced employment rates. This dynamic suggests that economies devoid of cyclical fluctuations might attain a superior steady-state growth, as employment remains consistently high, fostering continuous technological advancement. From this perspective, the concept of a 'cleansing effect'—whereby economic downturns eliminate low-productivity jobs and ostensibly strengthen the economy—is challenged. The elimination of even low-productivity roles can erode the overall knowledge base.

Such theories reframe the discourse on stabilization policies, particularly fiscal interventions, by highlighting their role in sustaining employment and, by extension, supporting the technological frontier even in downturns.

To illustrate this theory's implications more vividly, consider a nuanced example: an antiquated factory with limited land resources discovers an innovative method to utilize an old tractor more efficiently. Despite the ingenuity of this breakthrough, if the broader economy has moved beyond the technology that the tractor represents, the innovation might not significantly contribute to the overall knowledge stock or push the technological frontier forward. This scenario prompts a fundamental inquiry: does innovation at the lower end of the skill spectrum or within outdated technological contexts meaningfully advance the technological frontier? Or, would it be more beneficial for economic growth to transition such small-scale innovations into larger entities equipped with modern technologies?

One significant critique concerns the disparity in learning opportunities across different sectors and among individuals. The model's premise of uniform learning opportunities does not always align with the reality that some industries, such as the technology sector, offer rapid innovation and learning environments compared to more traditional manufacturing industries, where the pace of learning and innovation may be inherently slower due to the nature of the work processes involved.

Furthermore, the model may not adequately address the issue of structural unemployment that can arise from technological advancements. As certain workers benefit from "learning by doing," leading to increased productivity, others may find their skills becoming obsolete due to automation and other technological changes. This dynamic is evident in the automation of routine manufacturing jobs, which, while fostering "learning by doing" in fields like robotics and software engineering, simultaneously leads to structural unemployment for workers displaced by these technologies.

Another point of contention is the potential for diminishing returns to learning. The assumption that "learning by doing" continuously fuels growth may not hold up against the reality that initial rapid gains in productivity tend to taper off as workers gain proficiency, suggesting that the benefits of learning may diminish over time.

The model also potentially overlooks the externalities and spillover effects associated with "learning by doing." Technological advancements in one firm or sector do not automatically translate into broader economic growth if these advancements remain isolated and do not benefit other sectors or industries. This is illustrated by a software company that develops cutting-edge algorithms, enhancing its productivity but failing to contribute to the wider economy if the knowledge remains proprietary.

This nuanced exploration challenges the simplistic notion of 'learning by doing' by questioning the value and impact of incremental innovations within the broader economic and technological ecosystem. It underscores the complexity of technological progress and its interplay with economic dynamics, inviting a deeper investigation into the mechanisms that drive long-term growth and the role of policy in nurturing an environment conducive to continuous innovation and knowledge expansion.

The contemporary perspective on technological advancement, when viewed as a product of incremental contributions from every market participant, appears overly simplistic. A more accurate depiction of technological progress recognizes it as predominantly driven by those at the forefront of research. The expansion of the technology frontier is essentially shaped by the knowledge and breakthroughs of these leading-edge innovators. Other entities in the economy adopt these innovations at varying paces, influenced by the associated adoption costs. While firms that are not at the innovation frontier may achieve marginal efficiency gains through adoption, the impact of such improvements is often minimal. These marginal innovations are frequently a reflection of the adopting firms' constraints, particularly their inability to invest in more advanced and expensive technologies. Consequently, these incremental innovations have limited potential for widespread diffusion, as they stem from a position of necessity rather than pioneering advancement.

Another theory describe recession as a period in which the opportunity cost of investing in a producitvity enhancing projects is lower, since the work force is not fully demdand to produce goods. Doing this would lead in theory to higher productivity in the period of expansion. The key role here is that the productivity enhancing activity are costly thus divert capital and labour force from production as Hayek Hayek [1933] explained. For this view to be valid there are two key aspects that should be true at the same time: in first place the expectations about the length of the recession should rienter in the short-term otherwise it is cheaper to destruct some production units (labor and capital) to accommodate the slow in demand. The last condition is that internal resources must be less costly than external one, however it would be cheaper to higher more skilled workers and fire the low skill one. An additional remark is the this theory describe the all those inniziatives like worker formation that affects only marginally the productivity of a firm. It miss the main mechanics in which a firm can increase his productivity sharply: thorough technical innovation, and in order to do so you need a research program where the workforce is fully dedicated into it and not diverted from production.

Another theory posits that recessions offer a period in which the opportunity cost of investing in productivity-enhancing projects is lower, primarily because the workforce is not fully engaged in producing goods. Theoretically, this would lead to higher productivity during subsequent periods of expansion. A crucial element in this perspective is the acknowledgment that productivity-enhancing activities are expensive, thereby diverting capital and labor away from immediate production, a concept Hayek Hayek [1933] elucidated.

For this viewpoint to hold, two critical conditions must be concurrently satisfied: firstly, expectations regarding the duration of the recession must be short-term. If the recession is anticipated to be prolonged, it becomes economically viable to dismantle some production units (both labor and capital) to adjust to reduced demand. Secondly, the cost of utilizing internal resources for such productivity-enhancing ventures must be lower than the cost of acquiring external resources. Otherwise, it might be more economical to hire more skilled workers and lay off less skilled ones.

An additional observation about this theory is that it accounts for initiatives like worker training, which only marginally affects a firm's productivity. This overlooks the primary mechanism through which a firm can significantly boost its productivity: through technical innovation. To achieve substantial innovation, a dedicated research program is essential, where the workforce is fully committed to innovation efforts rather than being diverted to current production tasks. This highlights a gap in the theory, suggesting that while reallocating resources during recessions may offer some productivity benefits, the most dramatic improvements in productivity are likely achieved through focused innovation and research activities, not merely through the opportunistic reallocation of resources during economic downturns.

An intricate theory that elaborates on the dynamics of economic recessions and the associated lower opportunity costs is grounded in the concept of labor hoarding, as discussed in the seminal work by Clark Clark [1973]. This theory posits that firms maintain employment levels higher than what immediate efficiency metrics might dictate. The rationale behind such a strategy is to prepare for a potential surge in demand, ensuring that the firm can quickly ramp up production without the delays associated with recruiting and training new employees. However, this strategic maneuver towards the internal possibility frontier—where firms optimize their readiness for future demand—does not manifest as observable changes in employment rates. Consequently, this leads to discrepancies or residuals in the statistical series of employment, which do not align with what might be considered the level of optimal employment, a phenomenon further analyzed in the work of Burnside and Eichenbaum Burnside et al. [1993].

This labor hoarding theory offers a partial explanation for the strong pro-cyclicality of measured productivity. During economic upturns, firms can immediately respond to increased demand using their hoarded labor, thereby appearing more productive. Conversely, during downturns, the reluctance to shed this excess labor, due to the anticipation of future demand recovery, results in lower observable productivity levels. This behavior underscores a strategic depth in firm management, navigating through the cyclical economic waves by balancing between immediate efficiency and long-term responsiveness.

Expanding on this foundation, it becomes evident that the decision to engage in productivity-enhancing activities during recessions is not merely a reaction to lower opportunity costs but also a strategic consideration influenced by expectations of the recession's duration and the comparative costs of internal versus external resources. If firms anticipate a short-lived recession, the logic of hoarding labor and investing in internal productivity initiatives becomes compelling. However, this strategy hinges on the assumption that improving the skill set of the existing workforce or reallocating resources towards innovation is less costly than the alternative—acquiring new, possibly more skilled labor post-recession.

The opportunity cost (OC) approach closely aligns with the theory of labor hoarding, which seeks to elucidate the pronounced procyclicality of measured productivity. This observation implies that during economic downturns, firms appear to retreat towards the interior of their production possibility frontier, opting for a strategic reduction in operational efficiency rather than workforce downsizing. This strategy is partly attributed to the invisible nature of one crucial input—effort—to statisticians, and the economic rationale that, given the costs associated with employee turnover, it proves more economically viable for firms to dial back effort during slumps instead of terminating employment.

An intriguing alternative to diminishing effort is the redirection of employee tasks from immediate production roles to undertakings that bolster the firm's long-term productivity. At first glance, this strategy bears resemblance to labor hoarding but carries the added outcome that these so-called shadow activities, embraced during recessions, eventually manifest as enhancements in total factor productivity.

The concept of adjustment costs does not singularly confine firms from adapting their production factors according to operational necessities. This opportunity cost mechanism could theoretically extend to a macroeconomic scale, influencing individual entities via price adjustments. During periods marked by diminished production value, the immediate returns from production activities (e.g., wages for workers) decline in comparison to alternative endeavors, notably human capital accumulation, whose benefits are pegged to future earnings. This economic mechanism could precipitate a resource reallocation towards these alternative activities. The empirical observation that education durations tend to extend during economic recessions lends credence to this argument. Nevertheless, with the exception of leisure, most sectors shadow the movements of aggregate GDP. Thus, if productivity-enhancing activities (PEAs) are to occur during recessions, the resource reallocation process must predominantly unfold within firms themselves.

One notable deviation might be labor reallocation. As demonstrated by Davis and Haltiwanger [1992], job destruction exhibits a more countercyclical pattern compared to job creation. Viewing job reallocation through the lens of both destruction and creation suggests a countercyclical trend, positing job reallocation as an investment in cultivating superior firm-worker matches, thereby sowing the seeds for heightened productivity in the future. Davis and Haltiwanger [1992]further postulate, within the framework of a model featuring a representative agent, that economic recessions present an optimal window for labor reallocation, highlighting a strategic dimension to workforce management during downturns that might ultimately contribute to long-term productivity gains.

The ”lame ducks” theory, initially proposed by Caballero and Hammour [1994], offers an intriguing perspective on recessions as mechanisms that phase out less profitable, outdated capital. This theory delineates how the destruction of older units during downturns is more pronounced than the construction of new ones, characterizing recessions as periods marked by the systematic elimination of obsolete capital, hence the moniker ”lame ducks” theory. Notably, this framework sheds light on observations documented by Davis and Haltiwanger [1992], positioning it as a prominent theoretical approach that will be delved into more thoroughly in subsequent discussions.

Despite its insights, this model lacks consideration of the financial dimensions of firms, an aspect addressed by the theoretical work of Osotimehin and Pappadà [2017]. Their research weaves the financial decision-making process into the broader context of intertemporal capital decisions, highlighting how financial frictions influence the lender's participation constraint. The study reveals that, despite financial frictions, the cleansing effect of recessions on productivity persists, potentially leading to a more pronounced productivity surge during expansion phases. This analysis serves as a pivotal foundation for the new theoretical framework introduced in this thesis, marking a significant departure from traditional views and emphasizing the multifaceted impact of recessions on firm productivity and economic recovery.

In the forthcoming sections, we will delve into the empirical evidence presented by Davis and Haltiwanger [1992] in their seminal works from 1990 and 1992, which lay the groundwork for our discussion. Following this, we will explore the theoretical underpinnings that form the basis of the new, streamlined theoretical framework introduced in this thesis. Our examination begins with the insights of Caballero and Hammour

[1994], focusing on the interplay between the destruction and creation margins in economic cycles. Subsequently, we will delve into the work of Osotimehin and Pappadà [2017] (2017), which sheds light on the financial dimensions, particularly how capital lending frictions can precipitate misallocations. These studies collectively inform the development of our theoretical framework, setting the stage for a comprehensive analysis of economic dynamics and firm behavior during recessions.

4 Emprical findings

To rephrase the introduction while adhering to your guidelines:

This analysis juxtaposes two pivotal studies that dissect business cycle dynamics through the lens of labor market fluctuations and job reallocation. The first, by Davis and Haltiwanger [1992], scrutinizes job reallocation during recessions, employing data from the U.S. manufacturing sector. The second study, by Blanchard et al. [1990], concentrates on labor flows throughout business cycles. Both studies utilize labor data from the Current Population Survey (1968-1986) and manufacturing data from the Federal Bureau of Labor Statistics up to 1981, later complemented by the Longitudinal Research Database. These works collectively illuminate the dynamic interplay of job creation and destruction, offering a nuanced understanding of labor market volatility and its implications for economic cycles. Further exploration will include an analysis of Haltiwanger's findings and their application to understanding the Great Recession's impact on the labor market in his paper Foster et al. [2016]

4.1 Reallocation cleansing or not?

This study meticulously investigates the variance in employment changes at the establishment level within the U.S. manufacturing sector from 1972 to 1986, focusing on the gross creation and destruction of jobs and the rate of job reallocation across plants. Leveraging an extensive dataset from the Annual Survey of Manufactures, it provides a detailed examination of how job and worker reallocation contribute to understanding employment dynamics and the cyclical nature of the labor market. The findings reveal significant rates of job turnover within specific industry sectors and elucidate the relationship between job reallocation and various plant characteristics such as age, size, and ownership type. Moreover, the research discusses the persistence and concentration of job creation and destruction, highlighting their implications for long-term employment trends and worker mobility. Through analyzing the interplay between establishment-level job flows and broader labor market states, this paper contributes valuable insights into the mechanisms of labor reallocation and the structural factors driving employment heterogeneity in the manufacturing sector.

The key concept behind the Davis and Haltiwanger [1990, 1992] studies is that measure of reallocation is given by the sum of job or capital creation and distruction. In first place lets get some definitons that are used in the paper: The analysis in this paper establishes a straightforward link between the gross job flow metrics and the size-weighted distribution of establishment growth rates. Gross job creation is determined by aggregating employment increases in both growing and newly established firms within a sector. In a similar vein, gross job destruction is assessed by totaling employment declines in contracting and ceasing operations. This methodological approach allows for a comprehensive evaluation of employment dynamics, highlighting the pivotal role of establishment size and growth in understanding sectoral labor market fluctuations. To measure Total Factor Productivity (TFP) at the establishment level, they construct an index following the methodology similar to Baily et al. [1992] and subsequent studies. The formula for the index is:

$$\ln(\text{TFP}_{et}) = \ln(Q_{et}) - a_K \ln(K_{et}) - a_L \ln(L_{et}) - a_M \ln(M_{et})$$

In this equation, Q represents real output, K stands for real capital, L denotes labor input, and M signifies real materials used, with a representing the factor elasticities. The subscript e refers to individual establishments, while t denotes time, allowing for a dynamic analysis of productivity changes across establishments. The methodology for measuring Total Factor Productivity (TFP) at the establishment level involves detailed component measurements. Nominal output is calculated by summing total shipments with inventory changes, adjusted by industry-specific deflators from the NBER-CES Manufacturing Industry Database. Capital is quantified using the perpetual inventory method for structures and equipment, while labor input comprises total hours worked by both production and nonproduction workers. Materials, including physical materials and energy, are measured and deflated by industry-specific indices, with all values expressed in 1997 constant dollars. Factor elasticities are determined using industry-level cost shares, with adjustments made to account for industry differences in TFP calculations, ensuring comparisons control for sectoral disparities. The first question they try to address is Did Reallocation Dynamics Change in the great recession? The graph also depicts the fluctuations in the unemployment rate, clearly demonstrating that periods of rising unemployment are typically marked by increased job losses and decreased job creation. However, the trend took a notable turn during the Great Recession. Specifically, while job losses did surge in the 2008-2009 period, the most remarkable change was the significant drop in job creation beginning in 2007 and extending through 2010. Additionally, there is an observable long-term decline in job flow dynamics, a topic for further discussion.

To corroborate these findings and delve into more granular data, they utilized the Business Employment Dynamics (BED) statistics for the U.S. private sector. The

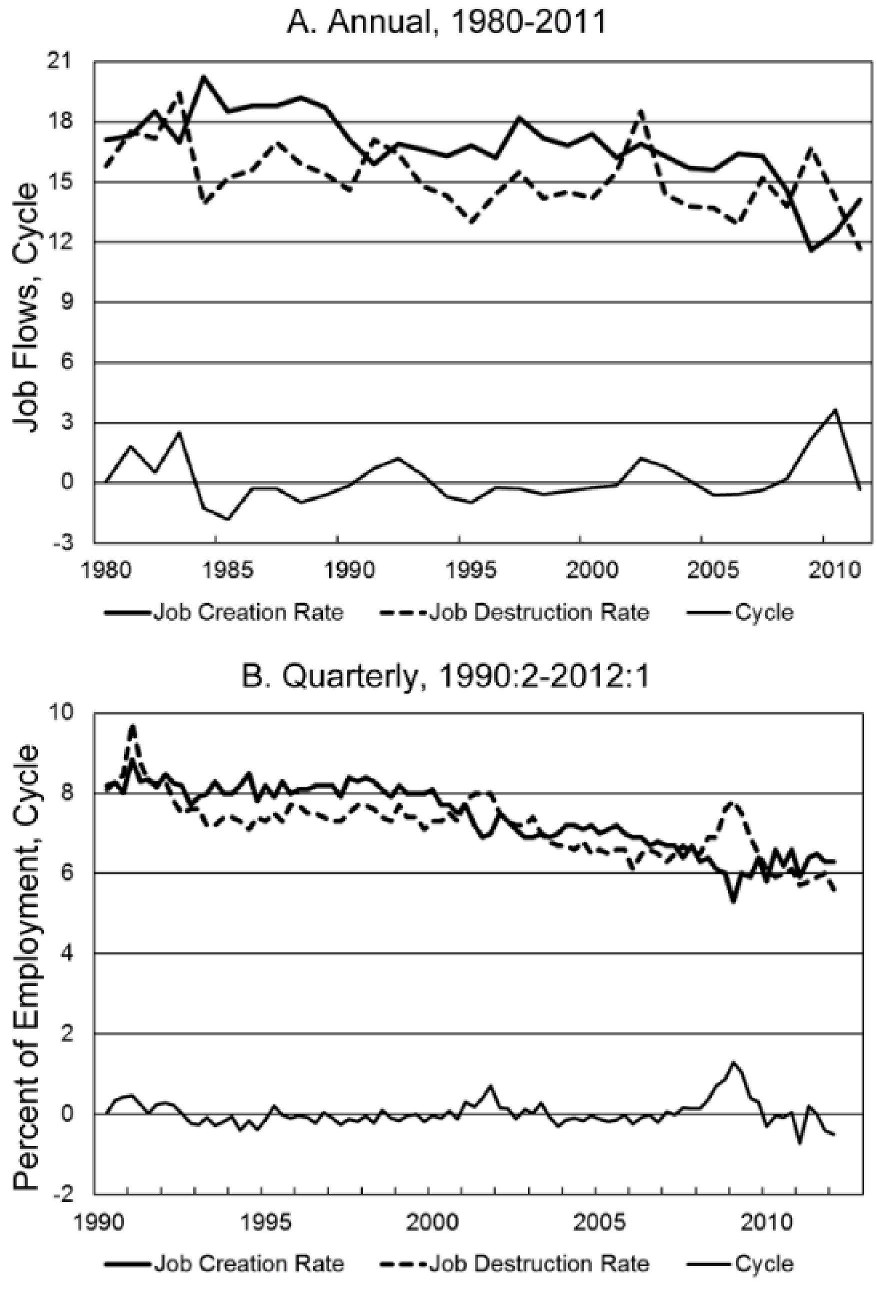


Figure 1: Figure Job flows and the business cycle. Authors' calculations using Business Dynamics Statistics (annual), Business Employment Dynamics (quarterly), and the Current Population Survey. Cycle is the change in the unemployment rate.

BED's quarterly data from the second quarter of 1990 to the first quarter of 2012 (Panel B of the figure) underscores the annual trends, showing that recessions typically involve higher job destruction and lower job creation rates. The period starting in 2007 stands out for a particularly steep decline in job creation, a trend that is more pronounced in the BED data. The BED series also highlights that the sluggish recovery from the Great Recession up to early 2012 is primarily attributable to weak job creation rather than enduringly high job destruction rates. This pattern, confirmed by other datasets such as the Job Openings and Labor Turnover Survey (JOLTS), persisted beyond the first quarter of 2012.

Adding to this, job creation during the Great Recession was as low as it has been at any time in the past 30 years, as illustrated in the figure. Moreover, job reallocation (the sum of job creation and destruction) reached its lowest point in 30 years during the Great Recession and its immediate aftermath. When comparing the Great Recession to the early 1980s recession, the rate of job reallocation was 28* in 2009, in stark contrast to 35* in 1983 (both periods represent peaks in job destruction and are measured using March-to-March Business Dynamics Statistics data). These patterns are partly driven by significant downward trends in job flows, as evidenced in both the Business Dynamics Statistics (BDS) and the BED data. Although exploring the causes of these declining trends in job flows is beyond the scope of this analysis, it is clear that these trends are significant and have been taken into account in the analysis.

Moreover the authors show how the patterns change considerbaly in the great recession, in particularly regarding how the creation and distruction rate og jobs respond to the sharp fall in demand. They found as expected from Caballero and Hammour [1994] that the creation rate of new jobs slower before the crisis spread out and than the distruction rate increses when it is not possible to fully accommodate the decrease in demand only stopping new labour hiring. They asses that during the Great Recession this patterns changes in particularly the fraction of net employment contraction accounted by a job reduction is higeger than 0.5, while for all the previous recession it was way below 0.4 as shown in the table.

Table 1: Share of Change in Net Employment Growth Due to Change in Job Creation in Periods of Net Contraction

	National		State
Period	BDS (Annual)	BED (Quarterly)	BDS (Annual)
Pre-Great Recession	.21	.28	.39
Post-2007	.61	.59	.65

This analysis is based on the authors' computations using data from the Business Dynamics Statistics (BDS) and Business Employment Dynamics (BED). The methodology hinges on the principle that net employment change equals job creation minus job destruction. For any period(s) of net employment decrease lasting at least one period, both the total change in net employment growth and the total change in job creation are aggregated over the full duration of consecutive net contraction periods. Furthermore, these aggregated changes are then further combined within the periods outlined in the analysis. The proportion mentioned refers to the fraction of the total aggregated change in net employment growth during the specified timeframe that is attributed to the total change in job creation within the same timeframe. Specifically, the BDS data spans from 1981 to 2007 for the pre-Great Recession era and from 2008 to 2011 for the post-2007 era. For the BED, the pre-Great Recession period covers from the second quarter of 1990 to the third quarter of 2007, and the post-2007 era from the fourth quarter of 2007 to the first quarter of 2012. It's important to note that these calculations are applied solely to periods experiencing a net decrease in employment growth. For instance, this applies to the period from the fourth quarter of 2007 to the first quarter of 2010 for the BED data. Regarding the BDS National Annual data, there are six years of net employment contraction, with two of those years occurring

Table 2: Job Flows and Change in the Unemployment Rate at the State-Level (Annual), 1981-2011

	Job Creation Rate	Job Destruction Rate	Reallocation Rate
Cycle	$-.631***$ (.046)	$1.194***$ (.053)	$.563***$ (.068)
GR × cycle	$-.371***$ (.079)	$-.421^{*}**$ (.079)	$-.793***$ (.128)
Trend	$-.168***$ (.010)	$-.136***$ (.011)	$-.304***$ (.020)

[1]

after 2007. In the BED Quarterly data, there are twenty-two quarters of net contraction, with nine quarters following 2008. For the BDS State Annual data, there are 393 state-year instances of net contraction, with 112 of those instances occurring after 2007.

Another way to see this change in patterns is regressing job creation rate, job destruction rate and Reallocation rate to Cyle an iteration between a Great Recession dummy and the cycle and a trend. The results are shown in the table 3. The authors delve into the nuances of state-level job flow dynamics by leveraging the variation in the relationship between cyclical economic indicators and job flows across different states. Their methodological approach is rooted in conducting descriptive regressions that not only link job flows to a selected cyclical indicator but also incorporate an interaction term that captures the distinctive economic conditions of the Great Recession period alongside the cyclical variable. To achieve this, they meticulously utilize changes in the unemployment rate at the state level as a proxy for economic cycles. Recognizing the persistent negative trend in job flows across the board, the authors judiciously incorporate a linear trend into their regression models to adequately account for this overarching pattern.

The findings from these regressions are illuminating, revealing that cyclical fluctuations predominantly explain a substantial portion of the variance observed in job flows, particularly with respect to the job destruction rate. This observation is in line with the theoretical framework posited by Caballero and Hammour [1994], which suggests that the margin of job destruction exhibits a higher sensitivity to cyclical downturns compared to the job creation margin. A striking aspect of their analysis is the significant and negative interaction observed between the cyclical variable and the Great Recession dummy variable, indicating that this period was characterized by a pronounced decrease in the rate of new job openings, whereas the rate of job layoffs was not as severely affected as it had been in other recessions. This pattern underscores the profound and distinctive impact of the Great Recession, which led to a dramatic downturn in job creation, surpassing the declines observed in job destruction rates and deviating markedly from the trends seen in prior economic downturns.

Furthermore, the analysis sheds light on the behavior of the job reallocation rate during the Great Recession, which, in a departure from its traditionally pro-cyclical nature observed in earlier recessions, switched signs, indicating a contraction in job reallocation. This reversal is particularly noteworthy as it signifies a shift towards less job reallocation, thereby challenging the conventional understanding of job flow dynamics during economic downturns.

An important contribution to this discourse comes from studies that emphasize the significant role of young businesses in the observed decline in job creation rates, particularly highlighted in the research by Fort et al. [2013]. These young businesses, defined as entities with less than five years of operation, exhibit a markedly higher responsiveness in their job creation margin, especially in the aftermath of the 2007 financial crisis. This responsiveness is contrasted with the behavior of older firms, which, although also experiencing shifts in their job creation and destruction margins, do not demonstrate the same level of sensitivity as their younger counterparts. The distinction between young and old firms becomes even more pronounced when examining the destruction margin, where young firms again show a greater responsiveness, underscoring the dif-

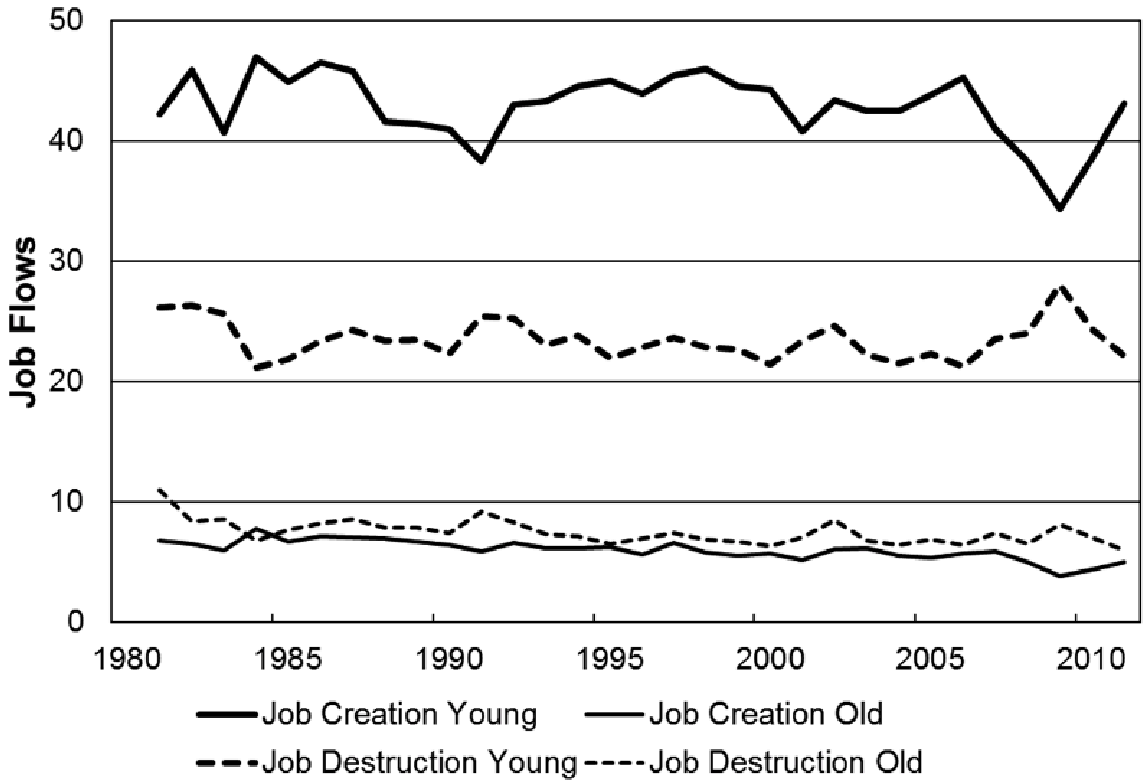


Figure 2: Job flows by age, 1981-2011. Authors' calculations on Business Dy- namics Statistics. Young is for establishments owned by firms less than 5 years old. Mature is for establishments owned by firms 5 or more years old. Job flows are establishment based and are classified by firm age characteristics.

ferential impact of economic cycles based on firm age. The second question that the authors try to adress was: Did Cleansing Effect change in the great recession? In order to address this question the authors use a regression which examine the relation between growth and survival dynamics of the incumbent esthablisment to productivity. The specification that they imply is the following The specification is defined as follows:

$$Y_{es,t+1} = \lambda_s + \lambda_{t+1} + \beta(\mathrm{TFP}_{es,t}) + \gamma(\mathrm{Cycle}_{s,t+1}) + \delta(\mathrm{TFP}_{es,t} \times \mathrm{Cycle}_{s,t+1}) + X'_{es,t}\mathbf{V} + \epsilon_{es,t+1};$$

where e represents the establishment, s denotes the state, and Y encompasses a series of outcomes. TFP refers to the deviations in total factor productivity from the average within each industry by year, and Cycle represents the variation in the state-specific unemployment rate from time t to $t+1$. The model assesses three distinct outcomes (all calculated from t to $t+1$): "Overall Growth" (combining continuing operations and exits), "Exit," and "Conditional Growth" (considering only those that continue, i.e., continuers). They used data from 1981-2010 controlling per year and state effects. In order to asses if the Great recession was different respect to previous recessions the add iteration terms with a dummy variable representing the Great Recession. The specification became the following: The refined model is expressed as:

$$Y_{es,t+1} = \lambda_s + \lambda_{t+1} + \beta(\mathrm{TFP}_{es,t}) + \gamma(\mathrm{Cycle}_{s,t+1}) + \delta(\mathrm{TFP}_{es,t} \times \mathrm{Cycle}_{s,t+1}) + \xi(\mathrm{GR}_{t+1} \times \mathrm{TFP}_{es,t}) + \mu(\mathrm{GR}_{t+1} \times \mathrm{Cy}$$

where GR signifies a dummy variable for the Great Recession, assigned a value of 1 during the years 2007 to 2009. This model takes into account the impact of the Great Recession by including interaction terms between the Great Recession dummy (GR), changes in the state-specific unemployment rate (Cycle), and deviations in total factor productivity (TFP) from the industry-year averages. The results are shown in the following table The regression analysis presented in the table offers insightful observations on how the Great Recession exacerbated the effects on overall growth rates compared to other periods of economic downturns. A critical aspect to highlight is the interaction between the Great Recession dummy variable (GR) and the cycle, which denotes changes in the state-year unemployment rate. This interaction is particularly important and statistically significant, with one coefficient significant at the 5% level and the other at the 1% level. This suggests that the economic dynamics during the Great Recession had a distinct and profound impact on reallocation and productivity across businesses.

The coefficient for the GR $\times$ cycle interaction being significant implies that the

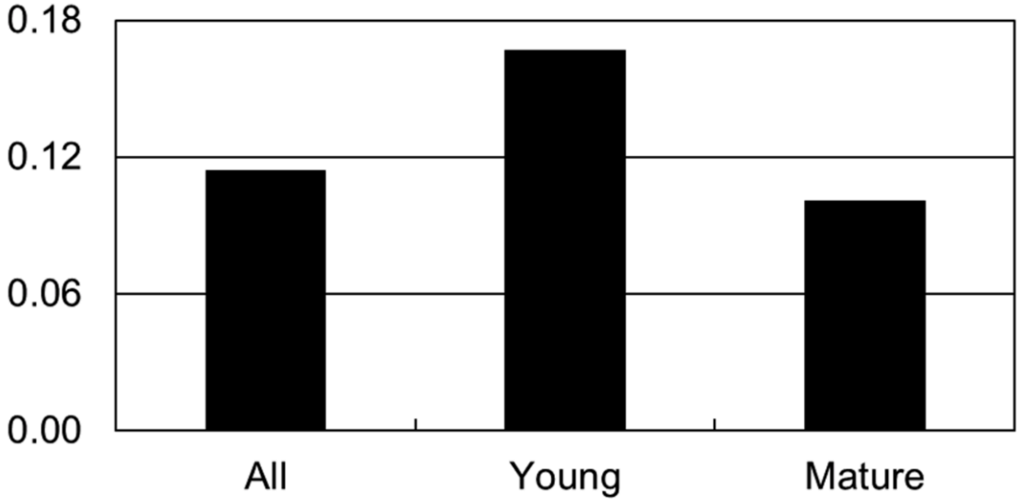
Table 3: Reallocation and Productivity over the Business Cycle

	Overall Gr.Rat. (Cont + Exit)		Exit		Cond. Gr.Rat.(Cont.)	
	prova (1)	(2)	(3)	(4)	(5)	(6)
TFP	.157***	159***	−.060***	−.060**	.041**	.042*
	(.006)	(.006)	(.003)	(.003)	(.003)	(.003)
Cycle	−3.307**	−2.961*	.671*	.497*	−2.143**	−2.128
	(.459)	(.483)	(.176)	(.179)	(.247)	(.286)
TFP × cycle	1.542***	2.182***	−.655***	−.927**	.494	.534
	(.643)	(.862)	(.226)	(.265)	(.412)	(.567)
GR × TFP		.030		−.018*		−.005
		(.023)		(.011)		(.011)
GR × cycle		−3.116*		1.581		−.126
		(1.349)		(.523)		(.770)
GR × TFP × cycle		−2.961*		1.466***		.066
		(1.619)		(.684)		(.764)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm size class FE	Yes	Yes	Yes	Yes	Yes	Yes
N (millions)	2.2	2.2	2.2	2.2	2.1	2.1

2 3

Great Recession intensified the negative effects of the business cycle on overall growth rates. This finding is notable because it underscores the unique severity of the Great Recession, distinguishing it from previous economic downturns. Specifically, the negative coefficient indicates that during the Great Recession, the adverse effects of an economic downturn on job creation and overall business growth were more pronounced. This could be attributed to a range of factors, including tighter credit conditions, greater uncertainty, and possibly more significant structural shifts in the economy that affected businesses more severely during this period.

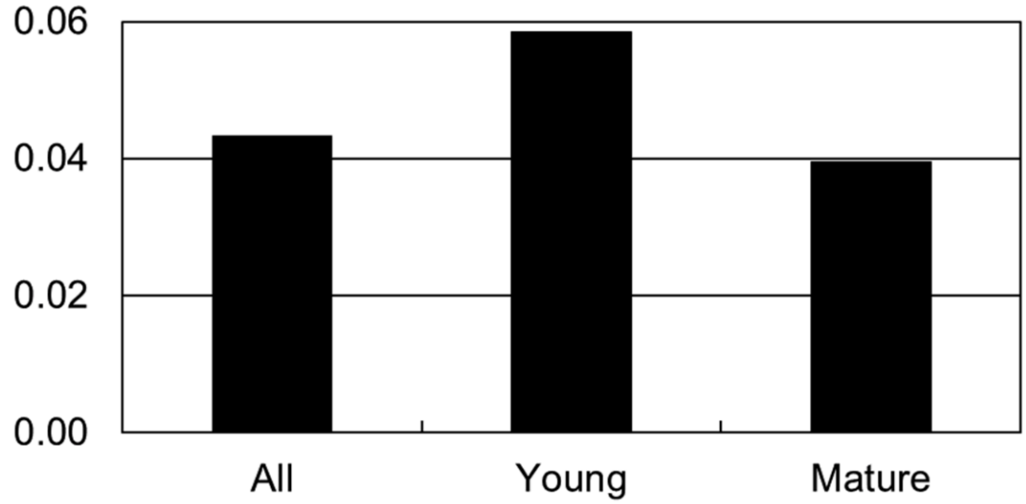
Moreover, the presence of significant interaction terms involving TFP (total factor productivity), the cycle, and the Great Recession dummy highlights the complex relationship between productivity, economic cycles, and major economic crises. The analysis suggests that the Great Recession not only impacted the rate of job creation ht A. Overall Growth(Cont. + Exiters)



and destruction but also influenced how productivity interacts with the business cycle to affect economic outcomes.

To have a better comprhension the authors use the regression coefficents to estimate the difference in the outcomes between an establishment firm with one standard deviation above and below the industry year mean, differing between older and younger firms the results are depicted in the following plot. From the graph, it is evident that there is an 11 percentage point difference in overall growth between firms that are one standard deviation more productive than the average and those one standard deviation below the mean. When distinguishing between mature and younger firms, it becomes apparent that younger firms are more responsive to productivity differences. Specifically, younger firms that are more productive experience an overall growth that is 18 percentage points higher compared to less productive counterparts. In contrast, this relationship between productivity and growth is weaker for mature firms, where those with higher productivity only see an overall growth of 0.11 percentage points more than their less productive peers. The initial graph encompasses all firms, including those that survived the Great Recession and those that exited.

ht B.Exit Rates



ht C.Conditional Growth (Continuers only)

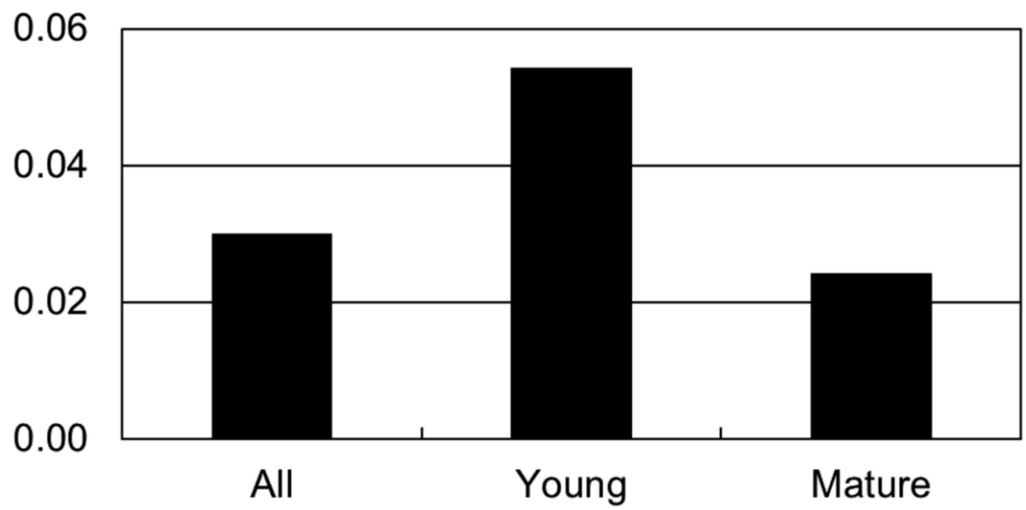


Figure 3: Differences in growth rates between high-productivity and low- productivity establishments, normal times. Authors' calculations on Annual Survey of Manufactures, Census of Manufactures, and Longitudinal Business Database. Depicted is the predicted difference in growth rates (panels A and C, high minus low) and the predicted difference in probability of exit (panel B, low minus high) between an establishment one standard deviation above industry-by-year mean productivity and an establishment one standard deviation below industry-by-year mean productivity. Normal is zero change in state-level unemployment.

Focusing on exit rates, being more productive offers a smaller advantage in survival rate probabilities. While the patterns are consistent with the previous graph, the benefit of higher productivity is reduced across all categories. Notably, younger firms with higher productivity have a 6 percentage point greater chance of survival compared to their less productive counterparts, whereas for mature firms, this productivity premium is reduced by 2 percentage points.

Looking exclusively at firms that did not exit, the growth rate is significantly less influenced by productivity, showing nearly half the sensitivity compared to the initial analysis, which includes exiters. This indicates that less productive firms are more likely to exit the market, especially if they are younger. This is largely consistent with Osotimehin and Pappadà [2017] since younger firms are less capitalaside compare to older companies and this meachanism that is not included in the Caballero and Hammour [1994] can modify the strict relations between producity and survivial rate. A final question remains from the empircal point of view; Did these patterns change in the great Recession? That is an important question since as exposed in the model in this thesis financial frictions can reduce the cleansing effect of the economic downturn. An answer to this question can be found in the regressions results exposed in the table 3, in particualar one can see that the interaction effects between cycle and TFP is larger for period previous to the Great Recession. Indeed the three-way interactions terms between TFP, cycle and Great Recession dummy is negative and statistically significant for exit and overall growth. Thus, instead of the cycle enhancing the impact of TFP on overall growth, it tends to diminish it on the margin in the Great Recession. A similar pattern is observed for exit. The estimated three-way interaction effect is positive and larger in magnitude than the two-way interaction effect of TFP and the cycle. Instead of the cycle enhancing the impact of TFP on exit, it tends to diminish it on the margin in the Great Recession.

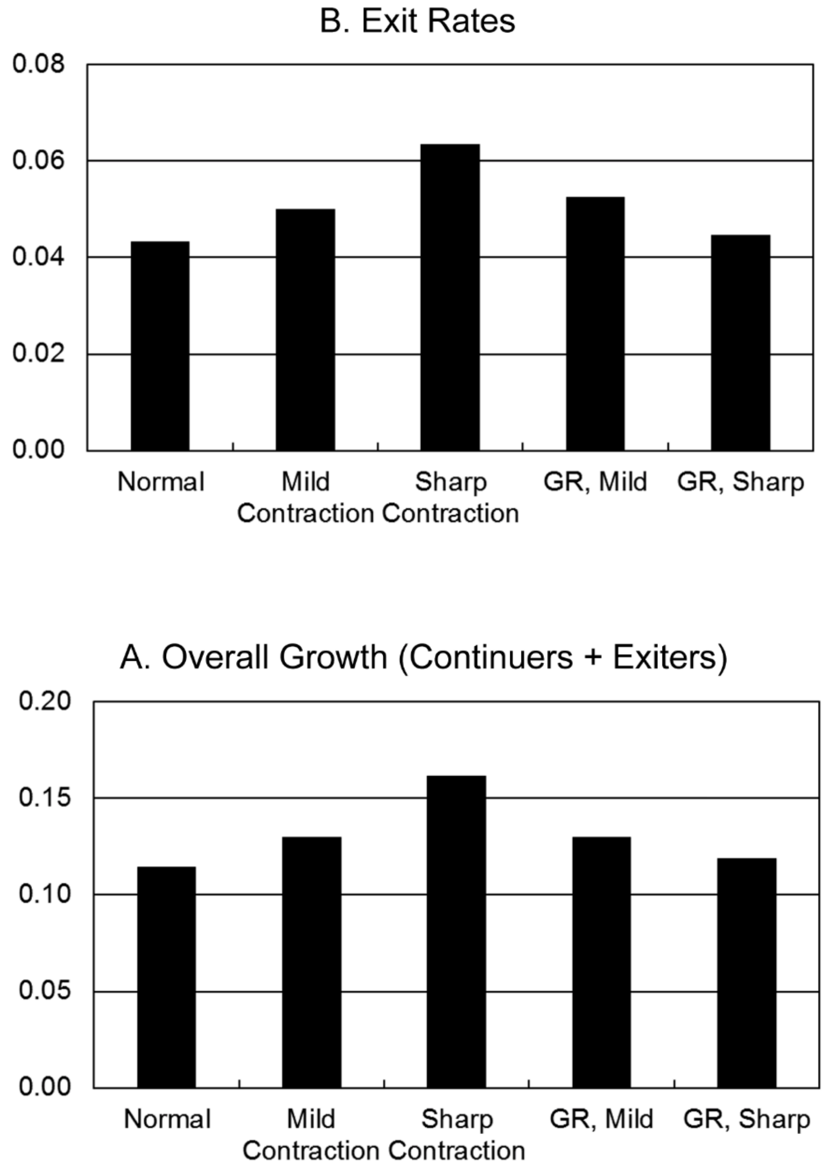


Figure 4: Differences in growth and exit rates between high-productivity and low-productivity establishments over the business cycle. Authors' calculations on Annual Survey of Manufactures, Census of Manufactures, and Longitudinal Busi- ness Database. Depicted is the predicted difference in growth rates (panel A, high minus low) and the predicted difference in probability of exit (panel B, low minus high) between an establishment one standard deviation above industry-by-year mean productivity and an establishment one standard deviation below industry-by- year mean productivity. Normal is zero change in state-level unemployment, mild contraction is 1 percentage point increase in state level unemployment, sharp contraction is 3 percentage point increase in state-level unemployment, and GR is for the period 2007-9.

5 Osotimehin and Pappadà [2017]

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as c to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as e, in the risk-free asset.

Firms opt to exit the market when expected profits no longer outweigh the fixed cost c, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r) - 1] e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to q. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors.

In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as k. The production function, accounting for the fixed cost c, is expressed as follows: $Y = Z(\theta + \epsilon)k^{\alpha}$.

Key variables include:

- Z: Stochastic aggregate productivity common across firms.

- θ: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- ϵ: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \delta)$.
- k^{α}: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

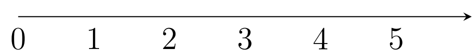


Figure 5: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of Z, θ, k^{α}, e (where e represents its endowment, different from k since the firm can borrow money with $d = c + k - e$).
2. The firm computes the optimal k to maximize the expected value of the firm, with k ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.
3. At the end of the period, the firm observes ϵ and the aggregate shock.
4. The firm repays its debt and the fixed operating cost $(c + k - e)$, resulting in an end-of-period net worth q.
5. The firm decides on the amount of dividends to distribute $(q - e')$, observes the productivity shock θ', Z', and the process restarts from step 1.

5.1 Frictionless economy

In a frictionless economy, firms have the option to borrow an amount denoted as $c+k-e$ at the risk-free interest rate $r = \frac{1}{\beta} - 1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_{k} E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - (1 + r)(c + k - e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_{k} E \int [Z(\theta + \epsilon)k^{\alpha} - (1 + r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market frictions, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

5.2 Economy with Credit Market Frictions

After production, the firm privately observes the temporary shock ϵ, while financial intermediaries can only observe it at a cost of μk^{α}. For one-period debt contracts, financial intermediaries observe ϵ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth e, current productivity θ, and aggregate productivity value Z, all observable by both the financial intermediary and the firm at no additional cost.

HP1 (Hypothesis 1): The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\overline{\epsilon}$ is determined by the equation:

$$Z(\theta+\overline{\epsilon})k^{\alpha}+(1-\delta)k=(1+\widetilde{r})(c+k+e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component θ.

The financial intermediary lends $(c+k-e)$ to the firm only if the expected income from the loan equals the opportunity cost of the funds, as expressed by the inequality:

$$(1+\widetilde{r})(k+c+e)(1-\Phi(\overline{\epsilon}))+\int_{-\infty}^{\overline{\epsilon}}[Z(\theta+\overline{\epsilon})k^{\alpha}+(1-\delta)k-\mu k^{\alpha}]\,d\Phi(\epsilon)\geq(1+r)(c+k+e)$$

The financial contract is characterized by $(k,\overline{\epsilon})$. Given Z,θ,e, the participation constraint indicates the default threshold $\overline{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given θ,Z, there is a unique threshold $e_b(\theta,Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta+G(\overline{\epsilon}_b)]k^{\alpha}+(1-\delta)k-uk_b^{\alpha}\Phi(\overline{\epsilon}_b)=(1+r)(k_b+c-\underline{e_b})$$

where $\overline{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q=\begin{cases}Z(\theta+\overline{\epsilon})k^{\alpha}+(1-\delta)k-(1+\widetilde{r})(k+c-e) & \text{if } \epsilon\geq\overline{\epsilon}\\ 0 & \text{otherwise}\end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^{\alpha}(\epsilon - \overline{\epsilon}); 0]$$

5.3 The firm's problem

Define V as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e, \theta, Z) = \max_{(k,\overline{\epsilon})} E\left\{ \int I(q)q + (1 - I(q)) \max[q, \max_{e'} q - e' + \beta V(e', \theta', \zeta')] \, d\Phi(\epsilon) \right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1.

$$Z[\theta + G(\overline{\epsilon}_b)]k^{\alpha} + (1 - \delta)k - uk_b^{\alpha}\Phi(\overline{\epsilon}_b) \geq (1 + r)(k_b + c - \underline{e_b})$$

2.

$$q = \max[Zk^{\alpha}(\epsilon - \overline{\epsilon}); 0]$$

3.

$$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.

6 The cleansing effect by Caballero

6.1 Introduction

In the first paper that rationalize the cleansing effect of recessions, authored by Ricardo J. Caballero and Mohamad L. Hammour Caballero and Hammour [1994] and published in the American Economic Review in 1998, the primary aim was to investigate how industries respond to cyclical variations in demand. They did this by employing a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations in two ways: by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model they used incorporated heterogeneous firms, where production units embodied the most advanced technology at the time of their creation. The costs associated with creating new units slowed down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a reduction in demand can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour's research Blanchard et al. [1990] offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard and Diamond Blanchard et al. [1990], as well as Steven Davis and John Haltiwanger Davis and Haltiwanger [1992], in their respective works from 1990. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study Davis and Haltiwanger [1992], where they assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986, it is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry. The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

6.2 Theoretical model

The model in question is a vintage model that simulates an industry experiencing exogenous technological progress. Within this model, production units are constructed using a fixed proportion of labor and capital, and they are continually being created and phased out. Notice that only the creation of new production units incurs a cost. This simplification is plausible, particularly in the context of the United States, where the expense associated with hiring is typically higher than the cost of termination, as demonstrated by Abdulkadiroğlu and Kranton (2003) Abowd and Kramarz [2003].

In this model, when a production unit is created at a specific time t_0, it embodies the most advanced technology available at that moment and consistently generates a uniform output represented by $A(t_0)$ throughout its operational lifetime. The productivity of this technology, denoted as $A(t)$, experiences continuous growth at an exogenously determined constant rate $\delta \geq 0$. This growth in technology can be interpreted in two ways: either as the adoption of new technology or as a product innovation. In the latter scenario, a continuum of perfectly substitutable products can yield varying levels of output.

$$[f(a,t) \qquad 0 \leq a \leq \overline{a}(t)]$$

The above function represents the cross-sectional density of the production units aged a at time t, where $\overline{a}(t)$ is the age of the oldest production unit at time t. The first assumption is that $f(a,t)$ and $\overline{a}(t)$ are continuous functions. The mass of production units at time t is given by:

$$N(t) = \int_{\overline{a}(t)}^{0} f(a,t)da$$

$N(t)$ is a measure of either the industry's capital stock and its employment, due to a fixed share of capital and labor. Thus, the industry's output is given by:

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

The deterioration of production units involves both an exogenous depreciation rate δ and an endogenous destruction process, which impacts $f(a,t)$ at its limits. The count of production units surviving for a years is expressed as:

$$f(a,t) = f(0,t-a)e^{-\delta a} \quad \text{where } 0 < a \leq \overline{a}(t)$$

The production flow is determined by:

$$\dot{N}(t) = f(0,t)[1 - \dot{\overline{a}}(t)] + \delta N(t)$$

Here, the first term represents the production rate, while the second term encapsulates the destruction rate, encompassing the obsolescence rate $f(\overline{a})(t)$, the technological obsolescence change over time $-f(\overline{a})(t)\dot{\overline{a}}(t)$, and the depreciation rate $\delta N(t)$.

The assumptions made by the authors are denoted as $\forall t \mid f(0,t) > 0 \cup \dot{\overline{a}}(t) <$.

The alteration in output concerning these flows is articulated as:

$$\dot{Q}(t) = A(t)f(0,t) - A(t-\overline{a}(t))f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)] + \delta Q(t)$$

The authors define a perfectly competitive industry in partial equilibrium, where supply is dictated by free entry and perfect equilibrium. Additionally, they introduce a cost function related to creating new production units:

$$c = c\left(f\left(f(0,t)\right)\right) \quad \text{where } c(\cdot) > 0,\ c'(\cdot) \leq 0$$

This cost function is contingent on the creation rate, implying that higher creation rates correspond to increased costs. The equilibrium condition is established by equating the cost of unit creation to the present discounted value of profits throughout its lifespan. The authors set the cost of a production unit to 1, and $P(t)$ is the price of a unit of output. Thus, the profits generated at time t by a production unit aged a are defined as:

$$\pi(a,t) = P(t)A(t-a) - 1$$

$$\overline{a}[t + T(t)] = T(t)$$

Here, $T(t)$ signifies the maximum lfetime of a unit created at t. At any given time t, the free entry condition is expressed as:

$$c(f(0,t)) = \int_{t+T(t)}^{t} \pi(s-t,t)e^{-(r+\delta)(s-t)\,ds}$$

In the above equation, where $r > 0$ represents the exogenously determined instantaneous interest rate, the determination of the exit of a production unit is contingent upon continuous $P(t)$ and the instance when the profits generated by a unit being destroyed reach zero. This occurrence signifies the moment when the oldest unit operational at time t, denoted as $\overline{a(t)}$, must adhere to the equation:

$$P(t)A(t - \overline{a}(t)) = 1$$

The authors posit that $P(t)$ exhibits a decreasing trend due to the model's assumption regarding endogenous destruction, specifically $\overline{\dot{a}(t)} < 1$. To see, differentiate

$$\dot{P}(t) = -\gamma \left[1 - \bar{\dot{a}}P(t)\right]$$

Consequently, when the profits of a production unit diminish to zero for the first time, it will be subject to destruction.

On the demand side, the authors assume a unit-elastic demand function and consider the aggregate expenditure as exogenous $\overline{D}(t) = P(t)Q(t)$. The equilibrium is a path $\{f(0,t), \overline{a}(t), T(t), Q(t)\}_{t\geq 0}$ that satisfy the following conditions:

1. $Q(t) = \displaystyle\int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$

2. $f(a,t) = f(0, t-a)e^{-\delta a}$

3. $T(t) = \overline{a}\,(t + T(t))$

4. $c(f(0,t)) = \displaystyle\int_{t}^{t+T(t)} \left[P(s)A(t) - 1\right] e^{-(r+\delta)(s-t)}\, ds$

5. $P(t)A(t - \overline{a}(t) = 1)$

6. $P(t)Q(t) = \overline{D}(t)$

The first three equations (1, 2, 3) and the fifth one (5) suffice to delineate the trajectories of $T(t)$, $P(t)$, and $Q(t)$, which are determined by $\{f(0,t), \overline{a}(t)\}$. To affirm the robustness of the conditions expressed in equations 6 and 5, it is possible to derive these equations as first-order conditions for the maximization of a number of perfectly competitive firms holding production units.

To comprehend the functioning of endogenous destruction, let's consider a scenario with constant demand. In this case, both the destruction and creation rates change only due to supply factors. This steady state is characterized by a constant lifetime of production units $T(t) = \overline{a}(t) = \overline{a}^*$, resulting in a time-invariant age distribution $f(a,t) = f^*(a)$. Equation 5 implies that the price $P(t)$ must consistently decrease at a rate σ. Higher innovation rates lead to increased productivity, raising the supply and consequently lowering the price. Equation 2 reveals that the distribution of production units in the steady state follows a truncated exponential distribution:

$$f^*(a) = f^*(0)e^{-\delta a} \quad 0 \leq a \leq \overline{a}^*$$

Using free entry conditions (4) and the clearing condition (6), one can determine the creation and destruction ages $f^*(0)$ and $\overline{a}^*$. Equations 1 and 5 yield the cost function and productivity of a new production unit:

$$c(f^*(0)) = \frac{e^{\gamma\overline{a}^*} - e^{-(r+\delta)\overline{a}^*}}{\gamma + r + \sigma} - \frac{1 - e^{-(r+\delta)\overline{a}^*}}{r + \delta}$$

$$f(0) = \frac{(\sigma + \delta)\overline{D}^*}{e^{\sigma\overline{a}^* - e^{\delta\overline{a}^*}}}$$

The authors then normalize the creation rate:

$$N = f^*(0) \cdot (1 - e^{\delta\overline{a}^*})$$

In the steady state, this is given by:

$$(9) CC^* = \frac{\delta}{1 - e^{-\delta\overline{a}^*}}$$

Considering a special case where the creation cost is a constant c, i.e., $c(f^*(0)) = c$, substituting into equation 6.2 allows retrieval of $\overline{a}^*$. The effect of technological rate σ on $\overline{a}^*$ is decreasing, as a higher innovation rate increases the opportunity cost of delayed renovation, while a higher cost of creating new units lowers the renovation rate. Optimal lifetime of production units increases with higher r and δ as it becomes harder to recover creation costs.

Now, dropping the assumption of constant demand, we examine how the industry adjusts to demand fluctuations. Two ways are identified in which the industry adapts production to meet demand: by reducing the rate of creation $f(0,t)$ and by increasing the rate of endogenous destruction $f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)]$, thus reducing $\overline{a}$, the age at which units are demolished.

These two adjustments interact, leading to a reduction in demand causing the most outdated units to be scrapped, rendering them unprofitable. However, if the recession is partially accommodated by a reduction in the creation rate, the effect on the destruction margin is diminished. The authors argue that the extent to which creation will ”insulate” existing units from variations in demand depends on the marginal cost of creating new units $c'f(0,t)$. When the marginal cost of creation is zero, demand fluctuations are entirely adjusted by the creation margin. This is exemplified in the case where $c(f(0,t)) = c$. In such instances, the insulation effect is complete, as there is no need to retire older units. Lowering $f(0,t)$ is sufficient, and it is cheaper than reducing the life of existing production units.

The insulation effect is not solely due to asymmetric adjustment costs on the creation and destruction margins. Complete insulation would occur even with linear adjusting costs. The creation rate in the case of constant creation cost is given by:

$$f(0,t) = \frac{\dot{\bar{D}}(t) + \delta\bar{D}(t) + P(t)A(t - \bar{a}(t))f(\bar{a}(t),t)[1 - \dot{\bar{a}}(t)] - \dot{P}(t)Q(t)}{P(t)A(t)}$$

In the attained equilibrium, variations in demand are entirely offset by adjustments at the creation margin denoted as $f(0,t)$, with $\overline{a}(t)$ remaining steady at the destruction margin. The creation process effectively counteracts the impact of demand fluctuations on the price $P(t)$, effectively shielding existing units from demand changes. The price $P(t)$ experiences a constant decline at a rate represented by σ, reflecting the pace of technical progress. This consistent decline in $P(t)$ serves as a clear signal for production units to function optimally throughout their constant lifetime $\overline{a}(t)^*$.

In the aforementioned scenario, the destruction rate is not constant, but it does not respond to demand through variations in the age $\overline{a}(t)^*$ at which units are destroyed. Instead, variations in the creation rates have an impact on the number of units that reach obsolescence. If fewer units are created, fewer units become obsolete after $\overline{a}(t)^*$ periods. It is noteworthy that any modification leaving equations 3 to 5 independent of $\overline{D}(t)$ and $f(0,t)$ does not alter the full-insulation results.

Interestingly, assumptions such as perfect competition, industry-wide return to scale, and perfect foresight are not necessary for these conclusions. The latter is particularly noteworthy as it asserts that fully accommodating demand on the creation side only requires knowledge of current conditions. As long as the non-negativity constraint on $f(0,t)$ is never binding, implementing equilibrium behaviors does not necessitate expectations of future demand.

6.3 Application of the model

The model undergoes calibration utilizing Job-flow data and Industry production data. The former facilitates the replication of job creation dynamics, while the latter is employed to mimic the behaviors of firm creation and destruction in the manufacturing industry. To capture these dynamics, the marginal cost of creating new production units is stipulated as positive $c'f(0,t)$. This allows for a partial insulation effect, and the destruction margin responds to demand fluctuations. However, introducing a dependency of c on $f(0,t)$ compromises the analytical tractability of the system (Equations 1 - 6). Consequently, the authors resort to methods such as multiple shooting to ascertain the optimal equilibrium and subsequently employ an iterative procedure to converge to the correct expected creation rate.

For numerical solutions, the authors adopt a linear formulation:

$$c(f(0,t)) = c_0 + c_1 f(0,t)$$

To gain a deeper understanding of how creation and destruction respond to demand, the authors simulate sinusoidal demand using the equation:

$$\overline{D}(t) = 1 + 0.07\sin(t)$$

The results are visualized in the image below, depicting the feedback of normalized creation and destruction (CC and DD) to changes in demand.

The plot clearly illustrates that the insulation effect is only partial; otherwise, DD would have remained flat, as in the case with $c(f(0,t) = c)$. From a mathematical perspective, destruction responds to demand as equations 3-5 are no longer independent of the path $f(0,t)$ and demand. From an economic standpoint, increasing creation costs smoothen the creation process. In scenarios with a nearly flat innovation rate, firms during crises cannot fully accommodate lower demand, nullifying the adoption of new production units, as the marginal costs would exceed the reduction in existing production units.

In the considered model, production units integrate labor and capital in fixed proportions to generate output. Each unit can be conceptualized as contributing to job creation within the industry, and job-flow data serves as a metric for quantifying the

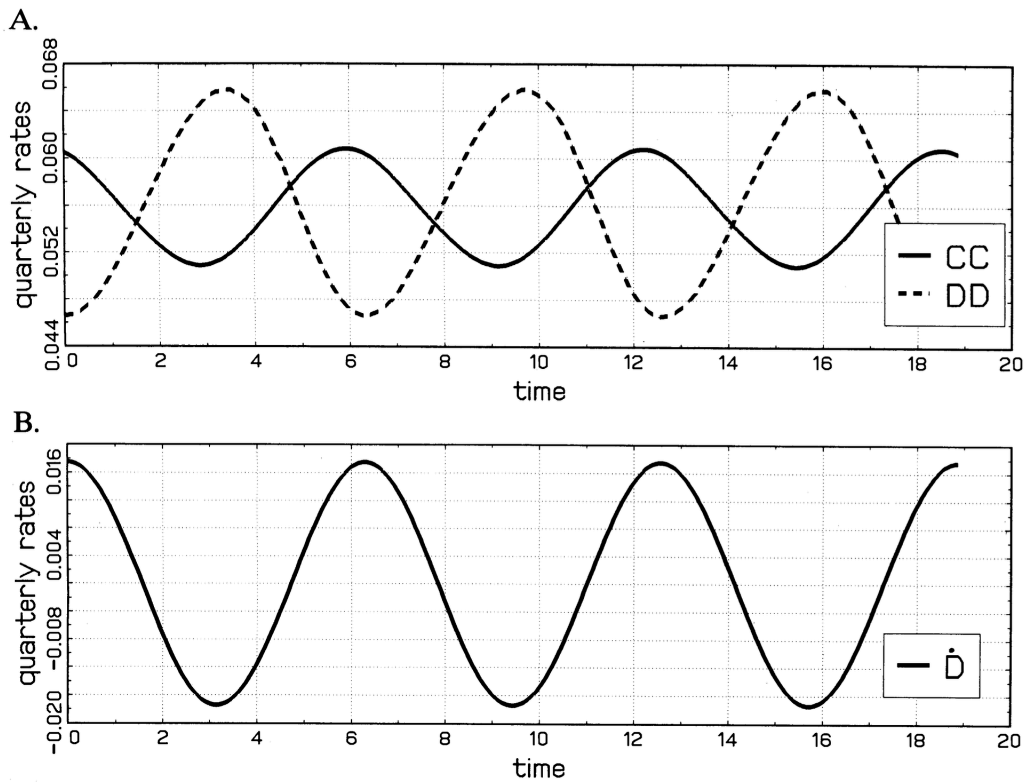


FIGURE 2. A) CREATION AND DESTRUCTION ($c_0 = 0.3$, $c_1 = 1.0$); B) CHANGE IN DEMAND (SYMMETRIC)

Figure 6: Figure 1. A Creation and destruction $c_0 = 0.3, c_1 = 1$ B Change in demand (Symmetric)

flows of production units.

Datasets that closely align with the theoretical CC and DD series have been compiled by Davis and Haltiwanger Davis and Haltiwanger [1990, 1992] and Blanchard and Diamond Blanchard et al. [1990], drawing from various sources. The primary focus lies on the dataset curated by Davis and Haltiwanger, who leverage the Longitudinal Research Database to construct quarterly series for U.S. manufacturing plants spanning the period 1972:2-1986:4.

In their empirical approach, **?**utilize output to empirically determine demand, employing the growth rate of the industrial production index as a proxy for output growth. Notably, in the foundational theoretical model, $Q(r)$ is smoothed by price movement, with the elasticity of demand determining the extent of smoothing, assumed to be equal to 1. While the theoretical model maintains a constant dividend-wage, the authors acknowledge that considering a procyclical dividend-wage, as in the case of general equilibrium with correlated industry shocks, may dampen the effect of demand shocks. However, they assert that this adjustment would alter only the magnitude, not the direction, of the analysis.

The figure below illustrates the data that the model seeks to replicate, showcasing job creation, job destruction, and growth.

To discern the characteristics of the series, the authors perform regression analysis on sectoral rates of job creation and job destruction against leads and lags of the corresponding rates of growth. They find that job creation is less responsive to demand fluctuations, while job destruction exhibits a more countercyclical behavior. The initial finding indicates that the rate of job destruction displays greater responsiveness to changes in sectoral activity compared to the rate of job creation. Specifically, the sums of coefficients are -0.384 for job destruction and 0.218 for job creation showed in the table 8, the same results as in Davis and Haltiwanger [1990, 1992] and in Blanchard

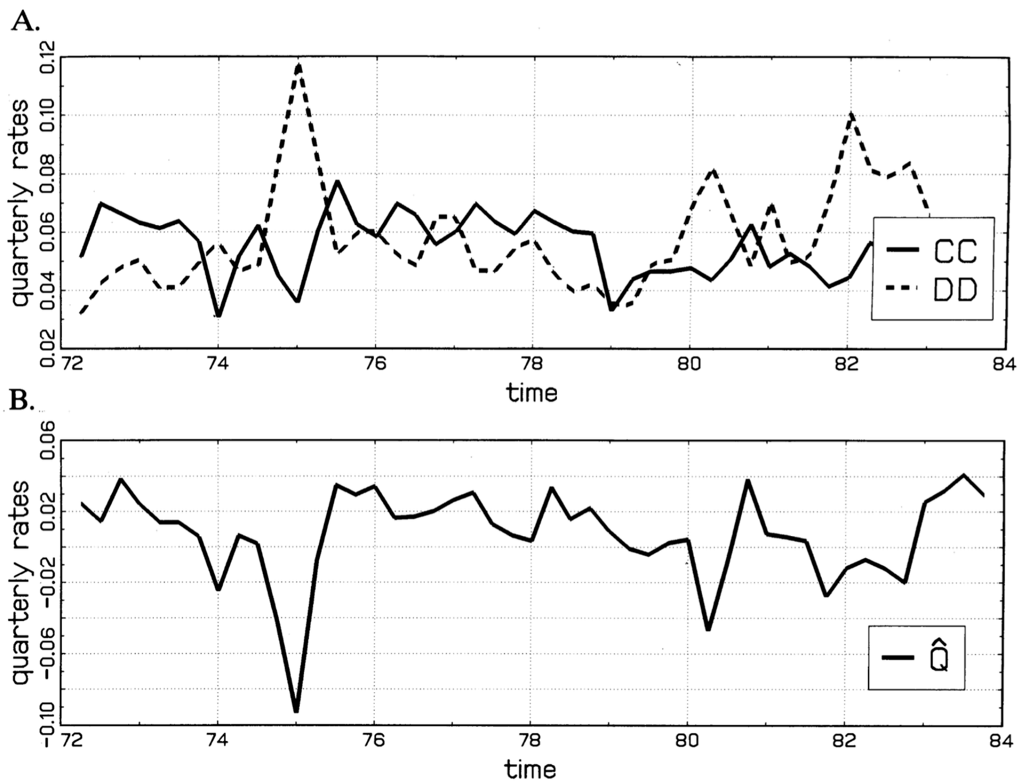


Figure 7: Figure 1. Job creation and job destruction in U.S. Manufacturing B Index of the industrial production

Regressor	Timing	Creation: Coefficient	Creation: Standard deviation	Destruction: Coefficient	Destruction: Standard deviation
$\hat{Q}$	2 leads	0.029	0.006	0.030	0.010
	1 lead	0.065	0.007	−0.068	0.010
	contemporaneous	0.108	0.007	−0.185	0.010
	1 lag	0.013	0.007	−0.103	0.010
	2 lags	0.003	0.006	−0.058	0.010
	Sum:	0.218	0.013	−0.384	0.017
$\hat{Q}^+$	2 leads	0.052	0.012	0.012	0.016
	1 lead	0.102	0.012	0.002	0.016
	contemporaneous	0.131	0.012	−0.065	0.016
	1 lag	0.059	0.012	−0.025	0.016
	2 lags	0.055	0.012	−0.008	0.016
	Sum:	0.399	0.026	−0.066	0.023
$\hat{Q}^-$	2 leads	0.002	0.010	0.006	0.014
	1 lead	0.022	0.011	−0.149	0.014
	contemporaneous	0.093	0.012	−0.293	0.015
	1 lag	−0.012	0.012	−0.139	0.015
	2 lags	−0.021	0.012	−0.059	0.015
	Sum:	0.084	0.020	−0.634	0.024

Figure 8: Table 2.1. Job Creation and Job Destruction in U.S. Manufacturing Response to Output Growth

Notes: The table presents the reaction of job creation to the growth rate of the industrial production index. The latter is categorized into values above and below its mean (Q). The table encompasses quarterly observations for the two-digit SIC industries during the period 1972:2-1986:4. The coefficients are uniformly constrained to be equal across all sectors, with the exception of a constant (not shown).

et al. [1990]. The authors capitalize on a natural experiment rooted in the intrinsic asymmetric characteristics of business cycles. Recessions, marked by brevity but intense contractions, provide the backdrop for the authors' model. This model endeavors to emulate the creation rate while concurrently mitigating the impact of asymmetric cyclicality inherent in business cycles. The empirical evidence supporting this model's behavior is encapsulated in Table 8, wherein two distinct scenarios are explored: output growth trajectories above Q^+ and below Q^-, relative to their respective means. The table meticulously delineates how creation and destruction rates respond to these deviations in output growth.

The salient observation emerges regarding creation rates, elucidating that they exhibit a more rapid and robust response in instances of vigorous output growth, as opposed to scenarios where the output growth rate experiences a reduction. On a contrasting note, the destruction margin, in line with the model's projections, manifests heightened sensitivity to a decline in output. This responsiveness is particularly pronounced from one quarter before the onset of the shock to one quarter after. Notably, during expansionary phases, the mean response of the destruction margin is -0.066, a notably milder reaction compared to the recessionary case where the mean response stands at -0.634.

These empirical outcomes seamlessly align with the predictions of the model. Specifically, the creation rate exhibits heightened responsiveness during expansionary phases, given their cyclical and symmetric nature. In contrast, the asymmetric and non-cyclical nature of recessions triggers a more substantial decline in the production unit rate, in line with the model's expectations.

In oder to better understand the asymmetrical behavior the authors simulate an

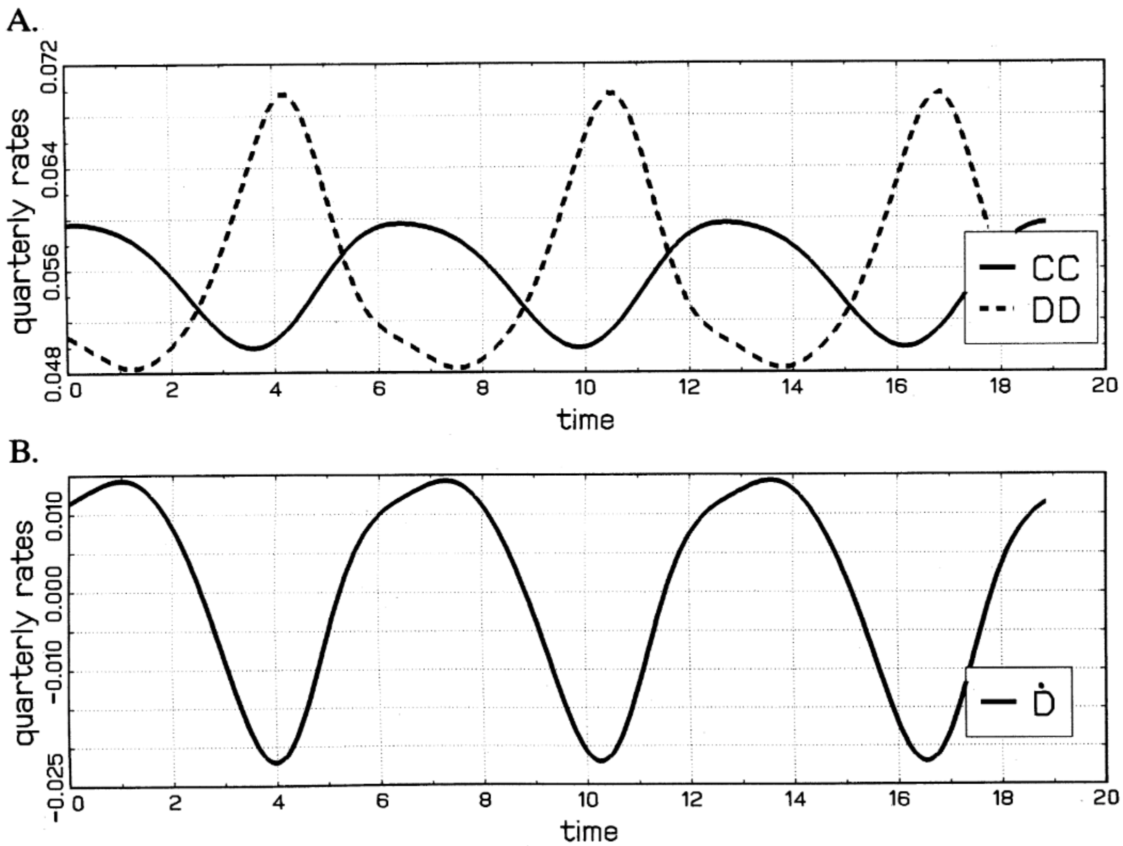


Figure 9: A. Creation and Destruction B. Output Growth
Notes: The figure depicts a simulation of asymmetrical supply growth.

asymmetrical demand function:

$$\cdot\overline{D}(t) = 0.05[\cos(t) + \sin(t)] - 0.016\sin(2t) - 0.003\cos(3t)$$

$$\overline{D}(t) = 1 \quad r = 0.065, \delta = 0.15, \gamma = 0.028, c_0 = 0.3, c_1 = 1.0$$

The results are depicted in 9

From the plot 9, its evident that firms use prediction in demand to smooth job creation in order to avoid big change, since they are too costly,by avaraging the demand over the lifetime of a production unit ove. On the other hand, destruction depends only on current conditions, thus responding only to significant deviations from the demand prediction . It can be better undestand thinking about a case in which creation rates respond only mildy to a sharp deacrease in demand, the equilibrium price falls leading to additional distruction, since older units' profits go to 0. Indeed, destruction not only preserves , but amplifies the assymetry of demand.

7 Frictionless economy

The authors culminate their study with a compelling calibration exercise using manufacturing series to exploit the model. This entails dissecting the observed net change in employment into destruction and creation rates, as well as applying the same approach to output production. The model is simulated for the duration of 1972:2-1983:4, with parameters as follows:

Table 2.1 - Calibrated Parameters

Variable	Symbol	Value
Interest rate	r	0.065
Depreciation rate	δ	0.150
Rate of technical progress	γ	0.028
Adjustment cost parameters	c_0	0.403
	c_1	0.500

(3)

The technical progress is selected to attribute all the growth in employment and manufacturing to technological advancements, setting λ as 2.8. The authors employ Equation 6.2, linking the steady state to the lifetime of jobs and job turnover (CC^*), determining $\overline{a}^* + 7.42$ years. Utilizing this information, they ascertain the steady state entry cost to be 0.525, equivalent to half a year's operating costs for production units. Subsequently, they employ ordinary least squares (OLS) to retrieve the value of c_1,

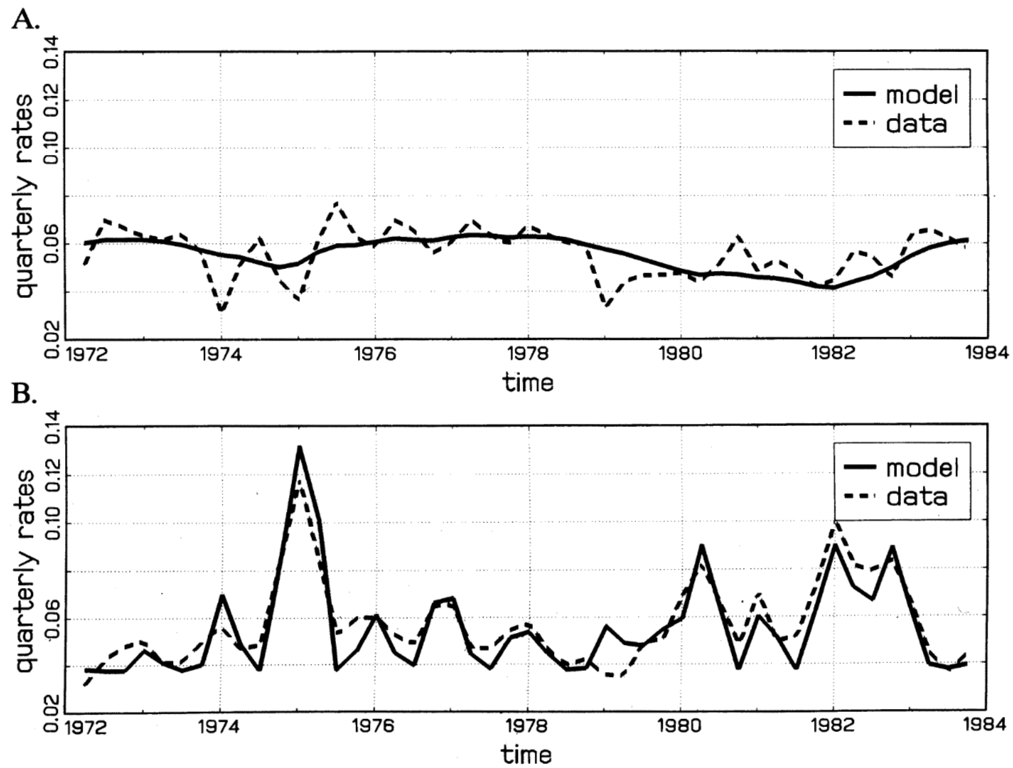


Figure 10: Figure 1. A employment driven job creation $c_0 = 0.403, c_1 = 0.5$ B Employment job destruction $c_0 = 0.403, c_1 = 0.5$

the marginal cost of creating a new unit, which is found to be 0.5. This aligns with a small elasticity for the creation cost function, signifying the vulnerability of the insulation mechanism to breakdown. The outcomes stemming from the simulations driven by employment and output are disclosed and contrasted with the data in Figure 10. Notably, the simulation of job creation displays a level of smoothness that diverges from the observed data, with this discrepancy being attributed, in part, to the inherent absence of uncertainty in our model. Despite this, the model effectively elucidates the relative volatility discernible in the patterns of job creation and destruction. Moreover, it successfully captures the greater symmetry observed in the former, offering insights into the nuanced dynamics at play in employment and output fluctuations.

The model provides intriguing insights as it elucidates certain empirical findings found in Davis and Haltiwanger [1990, 1992]. Specifically, it delves into the dynamics of how the response of the creation margin contributes to an insulating effect on the destruction margin. The model's salient features lie in its incorporation of heterogeneity across production units and their turnover, rendering it a meaningful baseline for comprehending how the cleansing effect influences the distribution of production units.

However, it's essential to note that the model, in its current formulation, does not account for the potential impact of financial frictions arising from asymmetric information between borrowers and lenders. Such frictions could conceivably influence both the destruction and creation margins, introducing a layer of complexity not considered in the current framework.

An alternative perspective on recessions is captured by the concept of a "pit-stop," where a recession is characterized as a period during which improvement investments in production are undertaken due to temporarily low opportunity costs, as posited by Davis and Haltiwanger [1990]. This viewpoint adds nuance to the understanding of recessions, emphasizing them as periods conducive to strategic investments.

One potential objection to the notion that recessions are times of cleansing is rooted in the implication of countercyclical productivity. Notably, labor productivity is often observed to be procyclical. However, this apparent inconsistency can be attributed to frictions, as suggested by Galí and Hammour [1992]. Their findings provide evidence supporting the notion that the cleansing effect enhances productivity in the long term, offering a nuanced perspective on the relationship between economic downturns and productivity dynamics.

A crucial observation in the aforementioned model is the authors' reliance on a constant marginal cost of creation. Yet, recent literature has raised concerns about the reliability of this assumption, especially for larger firms. The dynamics of the business environment in recent years suggest that significant firms tend to favor substantial adjustments, particularly in terms of downsizing.

Interestingly, this deviation from the constant marginal adjustment cost for bigger firms can be interpreted as a validation of the model's predictions. When firms opt not to fully insulate themselves from a decline in demand using the creation margin, they tend to respond with intense layoffs. This alignment between the model's predictions and the observed behavior of larger firms underlines the model's relevance and its capacity to capture real-world dynamics.

8 Theoretical model

8.1 Introduction

The model exposed in this thesis is similar to the theoretical framework of Osotimehin and Pappadà [2017], since the main scope of this model is understand how the financial frictions affects the distribution of exiters with respect to productivity in particular if this mechanism is responsible to the difference between previous crisis and the Great recession. The innovation to this model due to differnce in the flow of funds and in the partecipartion constrant of the financial intermediaries, moreover the intertemporal maximization problem of the firms is more tractable from the mathematical point of view. In the following paragraphs I dwelve first in the new formulation of flow of funds and its dynamics than I proced first solving the maximization problem in case of a frictionless economy, then i procedd with the case in which frictions are present. Finally I will simulate with a sinusoidal demand the distribution of exiters in relation with capital and productivity.

8.2 Dynamics of Financial Flows

This model is set within a partial equilibrium framework where firms are differentiated by their productivity levels. They have the option to fund their operations by either obtaining loans from financial intermediaries, as outlined by Bernanke and Gertler [1995], or by channeling dividends into capital growth. The capital at any time t is calculated by adjusting the capital from the previous period for depreciation (δ), then adding the investment and including the interest on previous debt:

$$k_t = k_{t-1}(1-\delta) - r \cdot b_{t-1} + I,$$

where r denotes the interest rate and b_{t-1} represents the debt from the preceding period.

Investment is articulated as the aggregate of the preceding period's debt principal, the newly incurred debt, and the net proceeds from production after the distribution of dividends:

$$I = b_t + f(k_{t-1}) - d_{t-1},$$

with d_{t-1} indicating the dividends disbursed at time t.

The production function is:

$$f(k_{t-1}) = Z \cdot k_t^{\alpha},$$

with Z indicating the firm's productivity level, and k_t symbolizing capital, which may also reflect units of production as in the model by Caballero and Hammour (1994). This setup captures the intricate relationship among capital growth, investment choices, and productivity-driven firm output.

Explicating the investment function within the financial flow framework, we have:

$$k_t = k_{t-1}(1-\delta) - R \cdot b_{t-1} + b_t + f(k_{t-1}) - d_{t-1},$$

where $R = 1 + r$. This expression can be simplified to:

$$k_t - k_{t-1}(1-\delta) = f(k_{t-1}) - d_{t-1} + b_t - R \cdot b_{t-1},$$

leading to:

$$\Delta k_t = f(k_{t-1}) - \delta \cdot k_{t-1} - R \cdot b_{t-1} + b_t - d_{t-1}$$

Imposing a stationary condition for capital ($\Delta k = 0$):

$$0 = f(k_{t-1}) - \delta \cdot k_{t-1} - R \cdot b_{t-1} + b_t - d_{t-1},$$

$$f(\widehat{k_t}) = \delta \cdot k + r \cdot b_{t-1} - \Delta b_t + d_{t-1}.$$

To satisfy the stationary capital condition, the firm's output must equal the sum of capital depreciation, debt interest, and dividend payouts, minus any debt increment. Analyzing the scenario where both debt and capital remain unchanged ($b_t = b_{t+1}$):

$$\Delta b_t = \Delta k_t - f(k_{t-1}) + \delta \cdot k_{t-1} + r \cdot b_{t-1} + d_{t-1},$$

$$0 = \Delta k_t - f(k_{t-1}) + \delta \cdot k_{t-1} + r \cdot b_{t-1} + d_{t-1},$$

$$\Delta k_t = f(k_{t-1}) - \delta \cdot k_{t-1} - r \cdot b_{t-1} - d_{t-1}.$$

Hence, for capital to remain constant, production must offset the sum of capital depreciation, the interest payable on debt, and dividends, less any increase in debt.

Considering a state where both debt and capital are steady, we define:

$$\begin{cases} 0 = \Delta k_t - f(k_{t-1}) + \delta \cdot k_{t-1} + r \cdot b_{t-1} + d_{t-1}, \\ 0 = f(k_{t-1}) - \delta \cdot k_{t-1} - R \cdot b_{t-1} + b_t - d_{t-1}. \end{cases} \tag{4}$$

Consequently, the equilibrium condition is:

$$f(\widehat{k}) = \delta \cdot \widehat{k} + r \cdot \widehat{b} + d_{t-1}.$$

It is demonstrable that when capital and debt stock are in equilibrium, dividends too reach a steady state:

$$d_{t-1} = f(\widehat{k}) - \delta \cdot \widehat{k} - r \cdot \widehat{b}.$$

Projecting one period forward:

$$d_{t+1} = f(\widehat{k}) - \delta \cdot \widehat{k} - r \cdot \widehat{b},$$

therefore:

$$\Delta d = f(\widehat{k}) - \delta \cdot \widehat{k} - r \cdot \widehat{b} - \left[f(\widehat{k}) - \delta \cdot \widehat{k} - r \cdot \widehat{b} \right],$$

$$\Delta d = 0 \quad \blacksquare$$

Thus, the condition simplifies to:

$$f(\widehat{k}) = \delta \cdot \widehat{k} + r \cdot \widehat{b} + \widehat{d}.$$

This refined analysis delineates the equilibrium conditions under which firms manage their capital and debt to navigate financial flows and productivity-driven growth, illustrated by a steady state where both debt and capital, alongside dividends, are balanced.

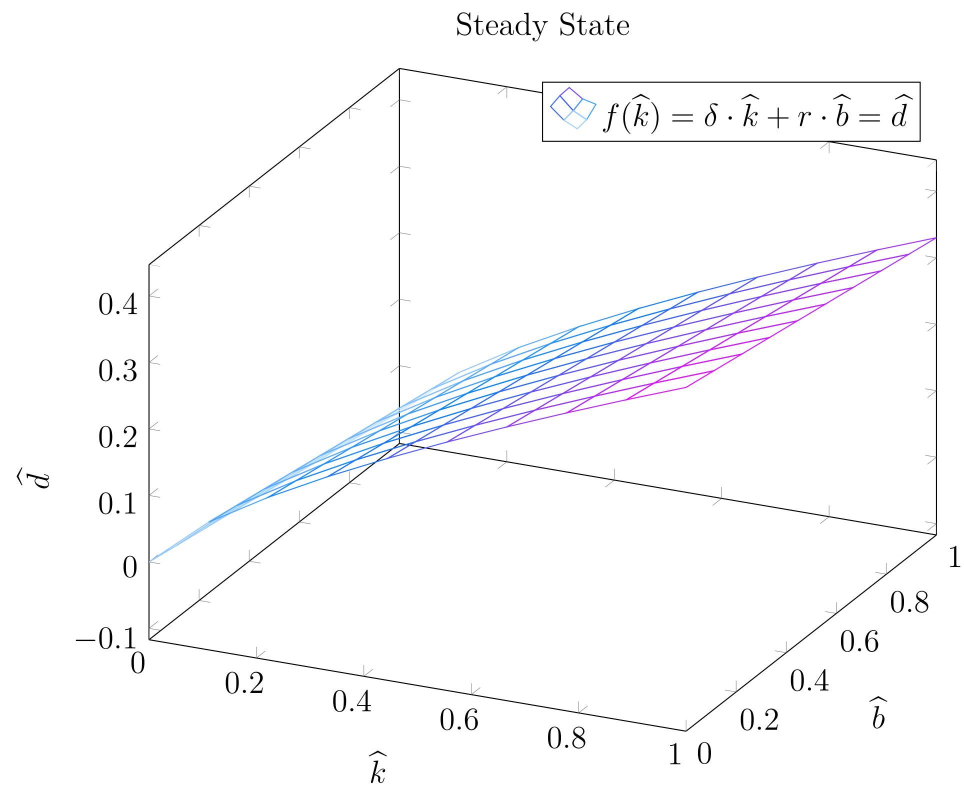


Figure 11: For this plot the following value has been used: $\delta = 0.1, r = 0.1, \alpha = 0.8, Z = 0.5$

The graph plots different steady states with different combination of debt b, capital k and dividends d, its clear that if we increse the stock of debt the dividends became less, since major resources are put into paying interest. While clearly the relation between capital and dividens is a convex one as illutrared in the graph. Lets consider a firm wich has $k_0 = 0.2, b_0 = 0.1$ in order to met this condition of stationary for both capital and debt, the dividens whould be equal to $\widehat{d} = 0.5 * 0.02^{0.8} - 0.1 * 0.2 - 0.1 * 0.1$. This point $k_0 = 0.2, b_0 = 0.1$ is a stationary ponints. Up to now we are focus only in stationary condition, but what happend if we perturbate the sytem, starting from the relation between capital and dividens. What happends if the dividens are higher compare the stationary path in which capital remains constan over time. For the moment lets assume

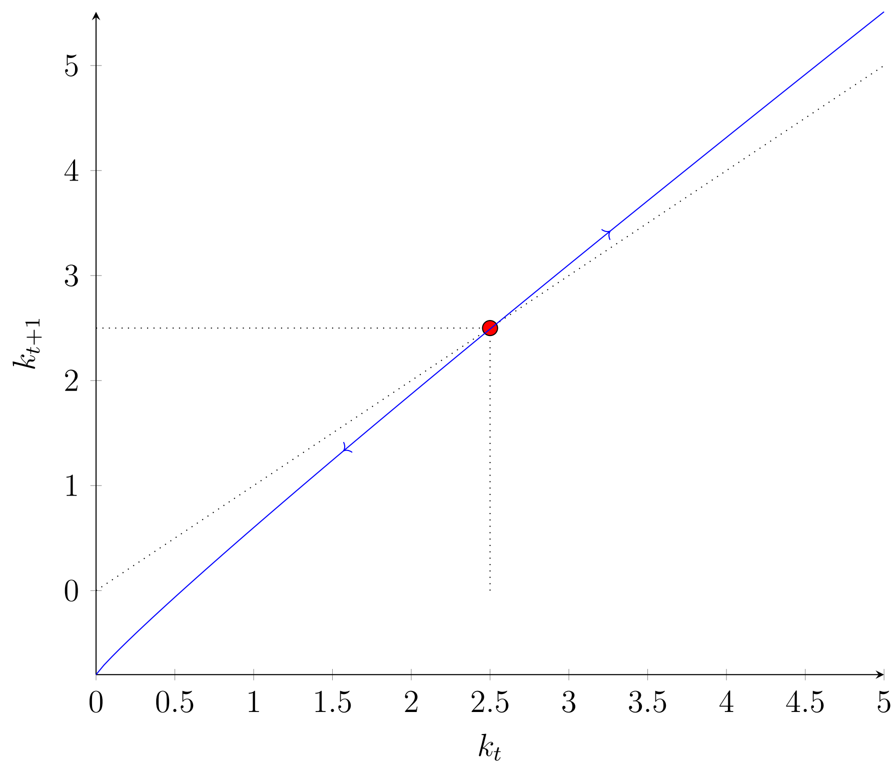


Figure 12: The graph depicts the trajectory of capital when there is no debt involved, given the parameters $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, $Z = 0.5$, and $d = 0.8$. The blue line traces the flow of funds according to the equation $k_{t+1} = 0.5 \cdot k_t^{0.8} - 0.1 \cdot k_t + k_t - 0.8$.

the firm is free of debt. Thus the flow of funds became ,

$$\Delta k_t = f(k_{t-1}) - \delta \cdot k_{t-1} - d_{t-1}$$

$$k_{t+1} = f(k_{t-1}) + (1 - \delta)k_t - d_{t-1}$$

The graph distinctly demonstrates that when the capital at time t is below the red dot, it signifies that the capital is less than the steady-state capital, leading to a diminishing trajectory in the firm's capital. Conversely, if the capital is above the steady-state level, indicated by $\widehat{k}$, the firm is overcapitalized, and the trajectory becomes explosive, with capital increasing without bound.

If a firm's capital is less than the steady-state, meaning it has less than the optimal amount, the outflows—such as depreciation and constant dividends—are disproportionately high compared to its production. This dynamic will inevitably cause the firm's capital to deplete towards zero. It's crucial to recognize that this path is predicated on the assumption of constant dividends; the higher the dividend payout, the greater the capital necessary to ensure that production can meet the outflows.

Furthermore, the steeper the slope of the blue line, the higher the productivity factor Z, signifying a reduced need for capital. This plays a significant role since firms with greater productivity can sustain their expenses with less capital, which correlates with a higher likelihood of enduring economic downturns.

To examine the dynamics of debt, consider a scenario where the capital remains constant, signifying that it is at the steady-state level.

$$b_t = -f(k_{t-1}) + \delta \cdot \widehat{k} + R \cdot b_{t-1} + d_{t-1}$$

The graph illustrates the relationship between a firm's current debt (b_t) and its capacity for future operations (k_{t+1}), within the context of constant dividends. The steady state is indicated by the red dot, signifying the juncture at which the firm's output is precisely adequate to cover dividends, depreciation, and interest on its steady-state debt.

If dividends were to increase, this would necessitate a higher debt level to maintain the steady state, as the firm would have less equity. This change would be represented graphically by an elevated intercept on the curve, resulting in an increased debt burden.

As for productivity, firms with superior productivity require less debt to produce the same amount of dividends, as they operate more efficiently. This is depicted by their position on the b_t axis for a given k_{t+1}. However, when the goal is to maximize dividends, highly productive firms will need more capital to reach the optimal dividend payout, a point that will be elaborated upon while discussing the maximization problem

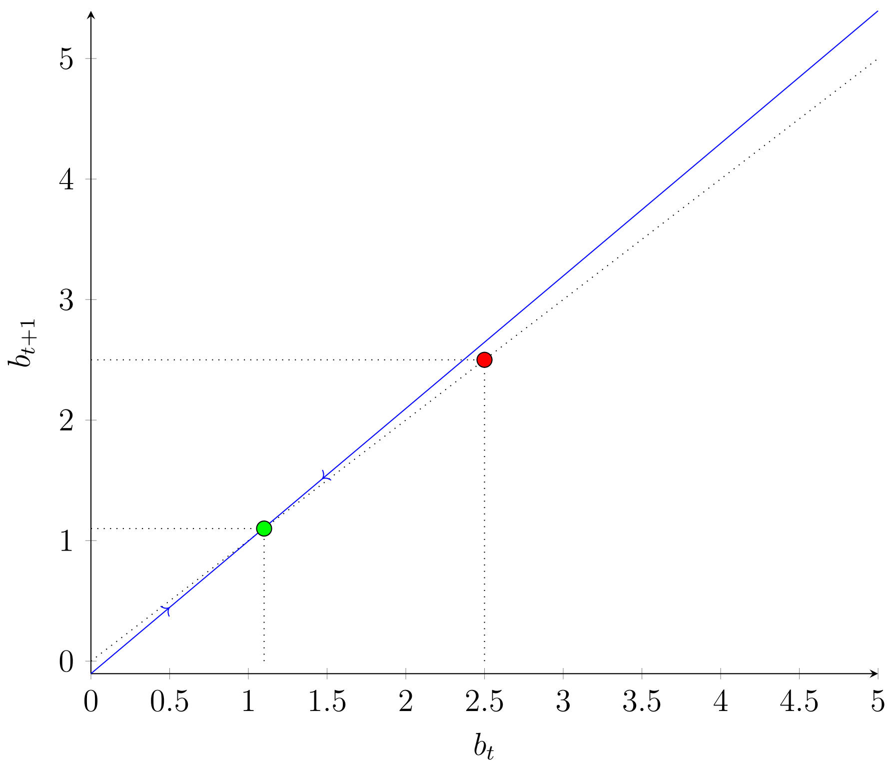


Figure 13: The graph illustrates the progression of debt under the condition that the change in capital (Δk) is zero, with parameters set at $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, $Z = 0.5$, $d = 0.8$,$\widehat{k} = 3$. The blue line represents the flow of funds as modeled by the equation $b_t = -f(k_{t-1}) + \delta \cdot \widehat{k} + R \cdot b_{t-1} + d_{t-1}$. The red dot marks the threshold beyond which debt cannot exceed capital, effectively serving as a limit on debt. The green dot signifies the steady state of the debt. The vertical or horizontal gap between the red and green dots quantifies the firm's equity.

later in the analysis.

In essence, the graph conveys how steady-state conditions are shaped by dividend policy and productivity, with the former influencing the firm's financial leverage and the latter determining its capital efficiency.

8.3 Partecipation constraint of the financial intermidiaries

The firms in this model coul finace himself in two way retaing dividends or increasing debts, up to now the interest was exogenous and indipendent from the quantity of the debt. This rise a problem since with the exogenous rate formulation if the debt goes to infinity compare to the equity the interest remain the same. In ordet to make the model more realistic, there shold be a fincial market in which interest rate is determined by the clearing condition. Moreover the financial intermidiaries maxise its proifts in a perfect competition enviroment. The results as in the Bernanke and Gertler [1986] is that the lennding activity should be proffitable as the opportunity cost of money: the lender with a probabilities p that is the successful repayment of the debt she will get the interest mature plus the capital, while in the case in which firms declare bankrupvy she will get the capital of the production of the firms and the capital net of depriciation. Thus the partecipation constriant of the lender is the following:

$$(1 + r)(c + k_t - e_t)p + (1 - p)f(k_{t-1}) = (1 + r_f)(c + k_t - e_t)$$

Here the opportunity cost of capital is set as risk free. Using this partecipartion constraint its possible to express the interest rate as a function of $p, f(k)$. Here there are no financial frictions since the lender can sicure the entire production, and there are no assymetric information thus the lender can exactly estimates the amouts that can retrive in both state of the firm. Rewriting the constraint, we get:

$$R = \frac{R_f}{p} - \frac{1-p}{p}\frac{f(k_{t-1})}{b_t}$$

Lets draw a graph with the following paramenters $p = \{0.95, 0.9\}, \delta = 0.1, \alpha = 0.8, Z = 0.5, d = 0.8, \widehat{k} = 4, R_f = 0.05, 0.1$. The visualization conveys that higher levels of debt are associated with increased interest rates, underscoring the risk-return tradeoff faced by lenders. A riskier financial profile, as indicated by the elevated position of the red line, demands higher returns to offset the potential for default. It is critical to recognize that the capital remains constant in this depiction; yet in practical scenarios, an escalation in debt typically results in augmented capital, thereby boosting production capabilities. This dynamic explains why the curves do not originate from the origin, as the initial loan amounts already incorporate the cost of capital.

The graph captures the dynamics between the debt stock b_t and the return on capital r. It is clear that an increase in the debt stock leads to a rise in the interest rate, reflecting the augmented risk perceived by lenders. Displayed are two distinct lines: one representing a riskier loan with a higher probability of default and the other indicating a safer loan with a lower default probability. As anticipated, the riskier loan scenario is characterized by a curve that lies above, dictating higher interest rates at each level of debt. The constant capital assumption underpins this model; however, in reality, an increase in debt usually translates into an increase in capital, thereby enhancing production potential. This factor accounts for the curves not starting at the origin.

Another way to visualize the partecipartion constraint of the financial intermediaries is by the definig $x = f(k)/b$. The graph delineates a critical boundary within the participation constraint framework: as leverage approaches unsustainable levels, the

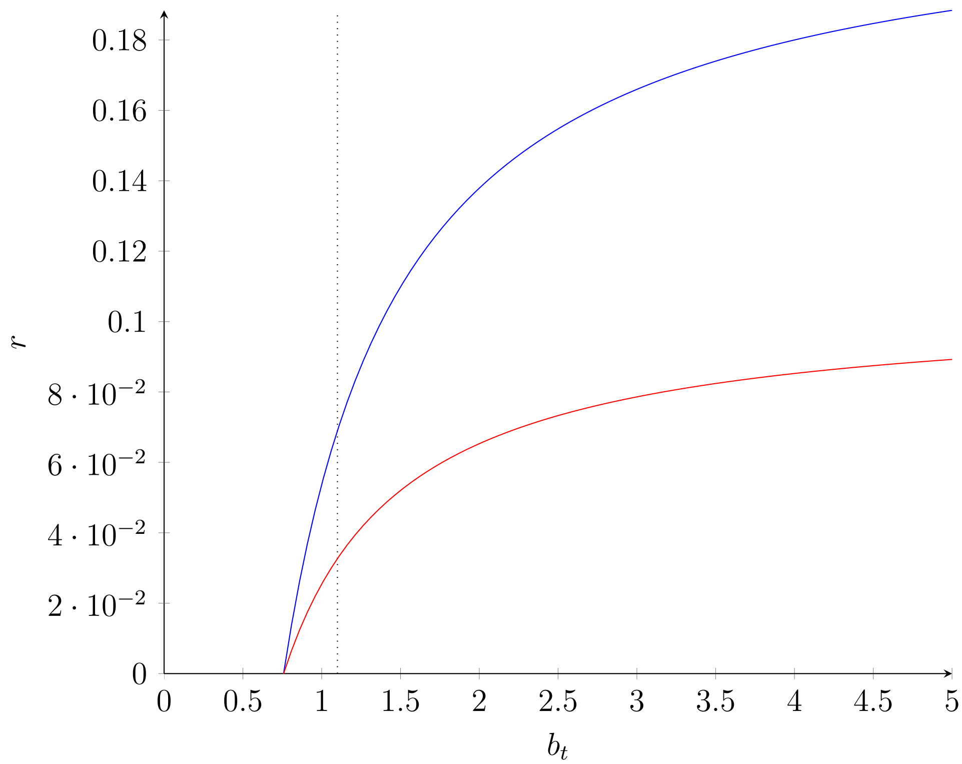


Figure 14: The figure presents a graphical analysis of the returns on loans as a function of the loan amount under a fixed capital level of $k = 3$. The red curve models the scenario where the default risk probability is $1 - p = 0.05$, implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1 - p = 0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the debt stock grows. Notably, the opportunity cost of capital is maintained at 0.05 for the red one, while at 0.1 for the higher risk curve.

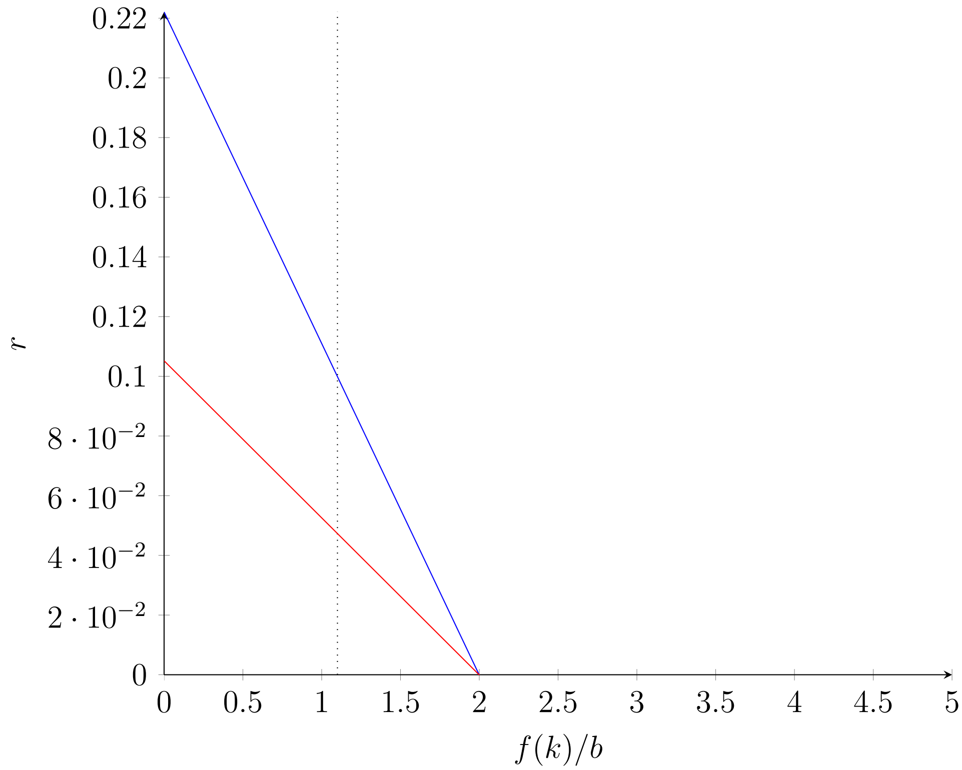


Figure 15: The figure presents a graphical analysis of the returns on loans as a function of the production over the debt level, while kepping level of capital at $k = 3$. The red curve models the scenario where the default risk probability is $1 - p = 0.05$, implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1 - p = 0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the production-debt ratio grows. Notably, the opportunity cost of capital is maintained at 0.05 for the red one, while at 0.1 for the higher risk curve.

interest rate escalates to a certain peak, signifying a cap on the maximum interest rate that deviates from the theoretical possibility of infinity. This ceiling on the rate is attributed to the fact that the probability of default, denoted by p, remains fixed and does not escalate alongside increasing leverage.

Ultimately, the participation constraint internalizes the interest rate of a loan as a function of the leverage, the opportunity cost of capital, and the default risk probability. By integrating this mechanism into the flow of funds model, the impact of debt on capital is mediated through the variable r, establishing a feedback loop where financial leverage influences and is influenced by the cost of borrowing.

8.4 The intertemporal maximization problem of the firm

In this paragraph will be exposed the intertemporal maximization problem of the firm using a lagrangian approach. The firm maximizes dividends over time, thus the intertemporal problem is defined as follow:

$$V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_{t-1})$$

Our problem becames

$$\max_{\{d_t\}_{t_0}^{+\infty}} V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t)$$

the constrants are the following:

$$R = \frac{R_f}{p} - \frac{1-p}{p}\frac{f(k_{t-1})}{b_t}$$

$$f(k_{t-1}) = Z \cdot k_t^{\alpha}$$

$$k_t = k_{t-1}(1-\delta) - R \cdot b_{t-1} + b_t + f(k_{t-1}) - d_{t-1}$$

The debt free case Assuming Modgliani Miller theorem holds since we are in the case of no assymetric information between firms and lender, we can assume that the firms is complitely financed with equity $b_t = 0 \quad \forall t$, thus the flow-of-funds becames:

$$k_t = k_{t-1}(1-\delta) + f(k_{t-1}) - d_{t-1}$$

Lets solve the maximization problem with the lagrangia approach, the lagrangian is defined as follows:

$$L_0 = \sum_{t=0}^{+\infty} \left[\beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - k_t(1-\delta) + f(k_t) - d_t\right]\right]$$

We take the first order conditions with respect to d_t, k_{t+1}, λ_t for $t = 0, 1, ...$ obtaning:

$$U'(d_t) = \lambda_t, \forall t$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1} \left[f'(k_{t+1}) + (1-\delta)\right], \forall t,$$

and, of course:

$$k_{t+1} = f(k_t) + (1-\delta)k_t - d_t, \forall t$$

In the infinite horizon model, the role of this final condition is played by the so-called transversality condition: $\lim_{T\to\infty} \beta^T U'(d_t) k_{T+1} = 0$. Policies promoting accelerated capital accumulation are ruled out, as suggested in the literature by Femm et al. Differentiating equation (1.5) concerning dividend levels at time $T+1$ yields the following set of first-order conditions:

$$
\begin{aligned}
U'(c_0) &= \lambda_0, \\
U'(c_1) &= \lambda_1, \\
&\vdots \\
U'(d_t) &= \lambda_t, \\
&\vdots \\
U'(d_t) &= \lambda_T.
\end{aligned} \tag{5}
$$

Each Lagrange multiplier λ_t represents the marginal utility of dividends in period t. If the multipliers were not adjusted by β^t, they would represent the marginal utilities from the viewpoint of period 0.

Optimization with respect to the capital levels at $T+1$ (ranging from k_1 to k_{T+1}) results in the following conditions:

$$
\begin{aligned}
\lambda_0 &= \beta\lambda_1 \left[f'(k_1) + (1-\delta)\right], \\
\beta\lambda_1 &= \beta^2\lambda_2 \left[f'(k_2) + (1-\delta)\right], \\
&\vdots \\
\beta^t\lambda_t &= \beta^{t+1}\lambda_{t+1} \left[f'(k_{t+1}) + (1-\delta)\right], \\
&\vdots \\
\beta^{T-1}\lambda_{T-1} &= \beta^T\lambda_T \left[f'(k_T) + (1-\delta)\right], \\
\beta^T\mu &= \beta^T\lambda_T.
\end{aligned} \tag{6}
$$

Deriving equation (1.5) with respect to the Lagrange multipliers λ_t for $t = 0, 1, 2, \ldots, T$ results in the system of constraints (1.3). Additionally, derivation concerning μ yields the constraint (1.4); given that (1.4) is an inequality constraint, the complementary slackness condition must be considered:

$$\beta^T \mu k_{T+1} = 0 \quad \text{and} \quad \mu \geq 0.$$

From the FOCs above we can retrive the dividends' euler equation:

$$U'\left(d_t\right) = \beta U'\left(d_{t+1}\right)\left[f'\left(k_{t+1}\right) + (1-\delta)\right]$$

The euler divideds equation suggests the marginal utility of dividend distribuoited at time should be equalt to the discounted marginal utility of dividends a period the next period multiplied by the resudual marginal value of the production less the depriciation. Now in order to drow the phase diagram lets study the locus in which dividends are stationary $\Delta d_t = 0$, thus $U'(d_t) = U'(d_{t+1})$:

$$\frac{1}{\beta} = \left[f'\left(k_{t+1}\right) + (1-\delta)\right]$$

$$f'\left(k_{t+1}\right) = \frac{1}{\beta} - (1-\delta)$$

$$Z\widehat{k}^{\alpha} = \frac{1-\beta+\delta}{\beta}$$

$$\widehat{k} = \left[\frac{1-\beta+\delta}{\beta Z}\right]^{\frac{1}{\alpha}}$$

The graph portrays the dynamics of dividends (c_t) in relation to the capital (k_t) of a firm, with a particular focus on the behavior when capital is below or above the steady-state level, denoted by $\hat{k}$.

When the capital is below the steady-state level $(k_t < \hat{k})$, as indicated by the left side of the vertical line, the arrows point to the right, towards the steady-state. This suggests that if a firm's capital is less than the optimal steady-state level, the firm's

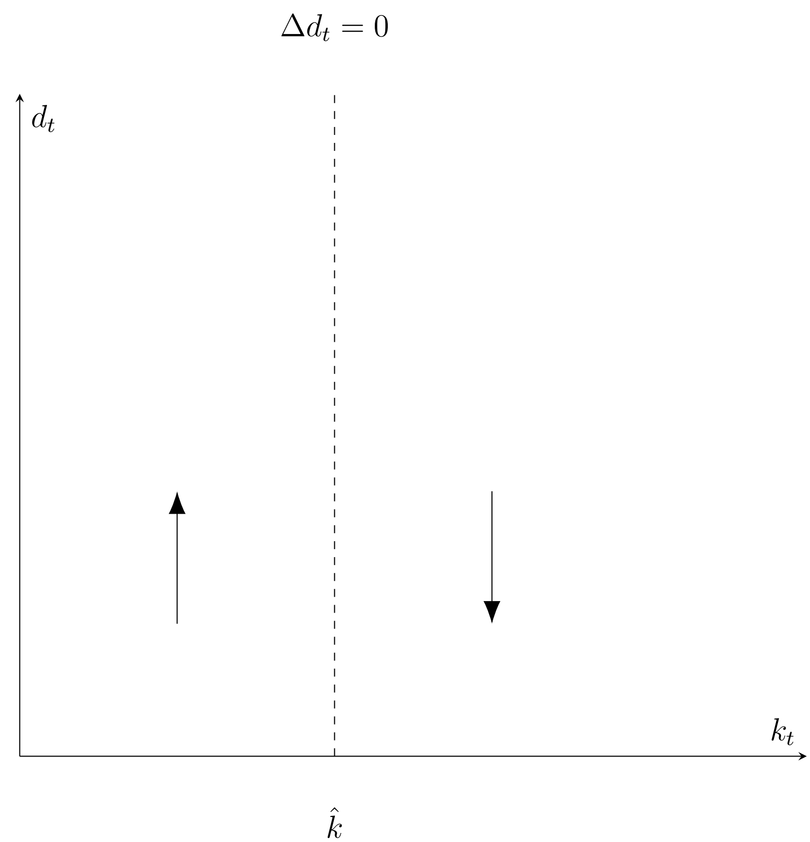


Figure 16: Dynamics of dividend with respect to capital accumulation.

dividend and investment decisions will likely be oriented towards increasing capital. This could be due to reinvestment of profits or acquisition of additional capital to reach the more efficient steady-state, where no further changes in dividend are needed ($\Delta d_t = 0$).

Conversely, when capital is above the steady-state level ($k_t > \hat{k}$), as shown on the right side of the vertical line, the arrows point left, back towards the steady-state. This implies that if a firm has more capital than what is optimal at the steady-state, it might be incurring unnecessary costs or facing diminishing returns on the excess capital. In such a scenario, the firm would adjust by reducing capital, either through disinvestment or by channeling funds into dividends or other expenditures, until it returns to the steady-state level.

The steady-state itself is characterized by a vertical line where $\Delta d_t = 0$, indicating that at this level of capital, the firm has no incentive to change its dividend—any movement away from this point is self-correcting, driving the firm back to steady-state. This equilibrium is crucial for the firm's long-term planning and operational efficiency, providing a target for managing its capital structure and dividends patterns.

Lets look at the locus in which capital is stationary $\Delta k = 0$ is given by the partecipetion constraint:

$$d_t = f\left(k_t\right) - \delta k_t$$

The graph represents the dynamics of dividends (d_t) with respect to capital (k_t). The depicted parabolic shape indicates that as capital initially increases from zero, dividends also increase, up to a certain point where the change in capital (Δk_t) is zero. This point is likely to represent the optimal level of capital that maximizes dividend payout, beyond which any additional capital does not translate into higher dividends.

Arrows on either side of the apex of the parabola suggest the movement of capital

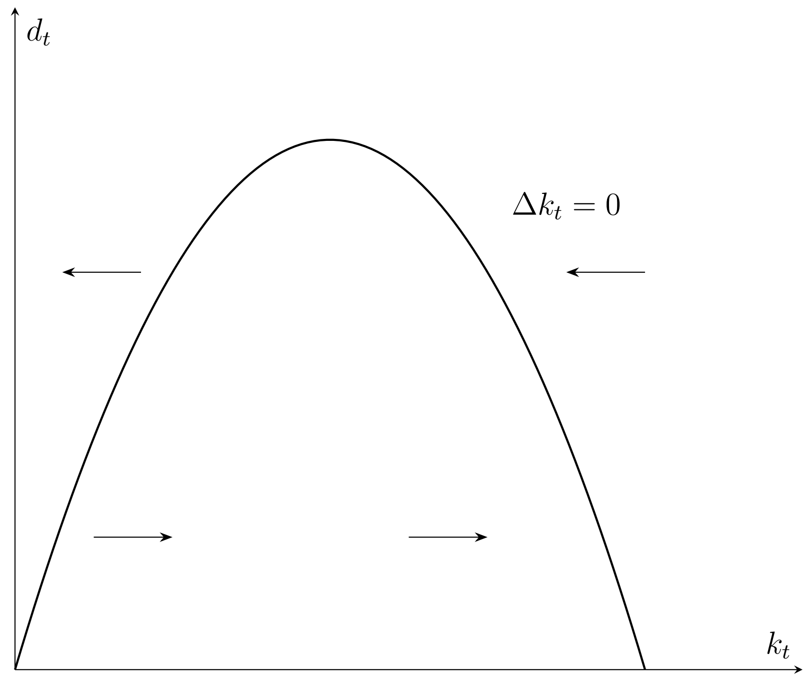


Figure 17: Dynamics of dividends with respect to capital .

towards this optimal point. To the left of the apex, where capital is below the optimal level, the arrows point to the right, indicating that it is beneficial to increase capital to reach the peak. On the right side of the apex, where capital exceeds the optimal level, the arrows point to the left, suggesting that there is an incentive to reduce capital as it would be in excess of what is needed to maximize dividends.

This dynamic equilibrium at the apex where $\Delta k_t = 0$ shows that there is no net investment in capital; firms are neither looking to acquire more capital nor to divest from their current levels. The model assumes that firms are profit-maximizing and will adjust their capital to reach the level that maximizes their dividend payout to shareholders.

The graph illustrates the trajectory of debt d_t in relation to capital k_t with the blue line representing an optimal path of capital accumulation. Starting from point A, located at initial capital k_0, the blue line ascends along the curve towards point B, which lies at the peak of the parabola where $\Delta k_t = 0$. This peak signifies the steady-

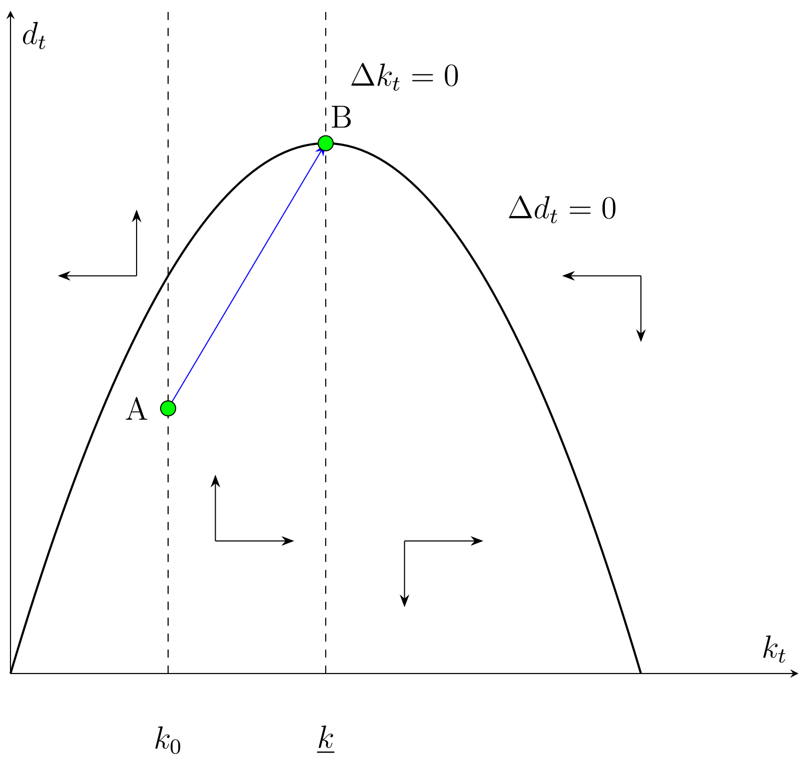


Figure 18: Dynamics of consumption with respect to capital accumulation, showing the points of stability and instability along the curve.

state level of capital, $\hat{k}$, where no further net investments are made as the capital stock does not change.

The arrows indicate the dynamic behavior of the system; to the left of the steady-state, capital is accumulating as firms move towards a higher level of capital, guided by the optimal path. To the right of the steady-state, the arrows show that if capital exceeds the steady-state level, it tends to decrease, moving back towards the steady-state. This adjustment occurs because capital levels above the steady-state are not sustainable due to decreasing marginal returns or other economic pressures that incentivize the firm to reduce capital back to $\hat{k}$.

The optimal path, marked in blue, is the desirable route for firms to follow as it leads to the steady-state level of capital where the economy can sustain a constant level of consumption or investment without necessitating further changes in capital.

In conclusion there is an optimal path of dividends and capital, however in order to find the close solution is necessary to use a belman equation. However before finding the exact solution, we will examine what happends if we include debt. The same procedure will be followed in the next paragraph.

The case with debt Now lets consider the possibility to use debt to increase capital, thus the intertemporal maximization problem of the firm is exactly as defined at the beging of this chapter, and lets assume a fixed leverage $b_t = l \cdot k_t \quad 0 \leq l \leq 1$. Our problem becames

$$\max_{\{d_t\}_{t_0}^{+\infty}} V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t)$$

the constrants are the following:

$$R = \frac{R_f}{p} - \frac{1-p}{p} \frac{f(k_{t-1})}{b_t}$$

$$f(k_{t-1}) = Z \cdot k_t^{\alpha}$$

$$k_t = k_{t-1}(1-\delta) - R \cdot b_{t-1} + b_t + f(k_{t-1}) - d_{t-1}$$

$b_t = l \cdot k_t \quad 0 \leq l \leq 1$ Consolidating the constraints:

$$k_t = \frac{k_{t-1}(1-\delta) - \left[\frac{R_f}{p} - \frac{1-p}{p}\frac{f(k_{t-1})}{l \cdot k_t}\right] \cdot lk_{t-1} + f(k_{t-1}) - d_{t-1}}{(1-l)}$$

The lagrangian fanction is the following:

$$L = \sum_{t=0}^{+\infty} \beta^t U(d_t) - \beta^t \lambda_t \left[k_{t-1}(1-\delta) - \left(\frac{R_f}{p} - \frac{1-p}{p}\frac{f(k_{t-1})}{lk_{t-1}}\right) \cdot lk_{t-1} + f(k_{t-1}) - d_{t-1}\right](1-l)^{-1}$$

We optimize for $d_t, k_t \quad \forall t$, we get the following focs:

$$\begin{aligned} U'(d_0) &= \frac{\lambda_0}{(1-l)}, \\ U'(d_1) &= \frac{\lambda_1}{(1-l)}, \\ &\vdots \\ U'(d_t) &= \frac{\lambda_t}{(1-l)}, \\ &\vdots \\ U'(d_T) &= \frac{\lambda_T}{(1-l)}. \end{aligned} \tag{7}$$

$$
\begin{aligned}
\lambda_0 &= \beta\frac{\lambda_1}{(1-l)}\left[f'(k_1)+(1-\delta)+\frac{1-p}{p}f'(k_1)-\frac{R_f}{p}l\right],\\
\beta\lambda_1 &= \beta^2\frac{\lambda_2}{(1-l)}\left[f'(k_2)+(1-\delta)+\frac{1-p}{p}f'(k_2)-\frac{R_f}{p}l\right],\\
&\vdots\\
\beta^t\lambda_t &= \beta^{t+1}\frac{\lambda_{t+1}}{(1-l)}\left[f'(k_{t+1})+(1-\delta)+\frac{1-p}{p}f'(k_{t+1})-\frac{R_f}{p}l\right],\\
&\vdots\\
\beta^{T-1}\lambda_{T-1} &= \beta^T\frac{\lambda_T}{(1-l)}\left[f'(k_T)+(1-\delta)+\frac{1-p}{p}f'(k_T)-\frac{R_f}{p}l\right],\\
\beta^T\mu &= \beta^{T+1}\frac{\lambda_T}{(1-l)}.
\end{aligned}
\tag{8}
$$

thus the euler dividens equation becames:

$$
U'(d_t)=\frac{\beta}{(1-l)}U'(d_{t+1})\left[\frac{f'(k_{t+1})}{p}+(1-\delta)-\frac{R_f}{p}l\right]
$$

consindering the locus of costant dividens, the capital should be:

$$
f'(k_{t+1})=(l-\beta+\beta\delta)\frac{p}{\beta}+R_f
$$

$$
\widehat{k}=\left\{\left[(l-\beta+\beta\delta)\frac{p}{\beta}+R_f\right]Z^{-1}\right\}^{\frac{1}{\alpha}}
$$

Compare to the previus steady state the capital is lower the case free of debt, since there is interest to pay. Moreover Considering the case in which capital is less than $\widehat{k}$, thus the marginal producitvity will be higher than the marginal cost of capital. While the locus in which capital is at the steady state:

$$
k_t=\left[k_{t-1}(1-\delta)-\left[\frac{R_f}{p}-\frac{1-p}{p}\frac{f(k_{t-1})}{l\cdot k_t}\right]\cdot lk_{t-1}+f(k_{t-1})-d_{t-1}\right](1-l)^{-1}
$$

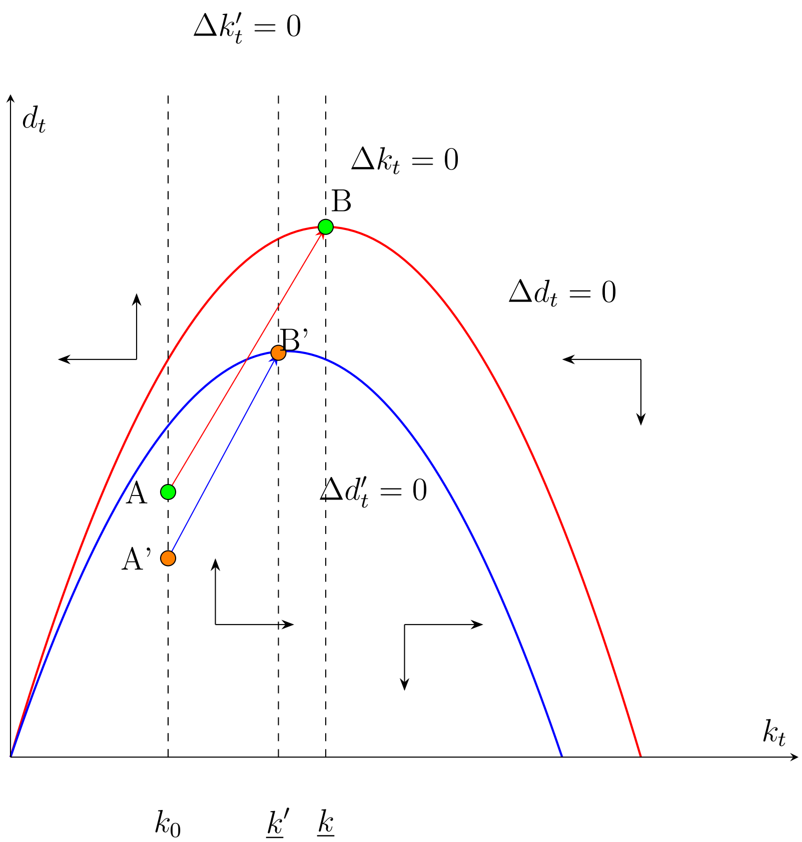


Figure 19: This diagram depicts the consumption dynamics as they relate to capital accumulation, highlighting areas of stability and instability. The red line corresponds to a firm that is carrying debt, whereas the blue line represents a firm with fixed leverage. The red arrow traces a potential saddle path for the indebted firm, beginning from k_0, while the blue arrow illustrates a conceivable saddle path for a firm with fixed leverage.

$$\widehat{d} = \frac{f(k)}{p} - k\left(\delta + \frac{R_f}{p}l - l\right)$$

In order to represent the phase diagram, lets consider the case in which dividends are higher then steady state, in that case $k_t < k_{t-1}$, thus the capital will decrease over time, while in the opposite case the capital will increase. The phase diagram 19 illustrates the dynamics of capital accumulation in scenarios with fixed leverage and without debt. Although the dynamics are similar in both cases, the steady-state level of capital is lower for firms with debt due to their obligation to service the debt. Consequently, the trajectory that firms with debt follow, known as the saddle path, results in lower dividend payouts compared to debt-free firms.

This distinction is particularly significant in economic downturns. Assuming two firms have identical production functions but different levels of leverage, the firm with higher debt may face a greater risk of exiting the market during a recession. This is because its lower dividend levels may not be sufficient to compensate shareholders for the increased risk associated with its leverage. Therefore, in times of economic stress, firms with higher leverage are at a disadvantage due to their reduced financial flexibility and the burden of debt repayments.

8.5 Solving with the Belman

Now thats the dynamics are clear we can solve dynamic problem finding the optimal path given an initial condition k_0. In order to do so we assume that the utility function is a logaritmic function, and we rewrite the problem using a bellman equation:

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U\left(d_0\right) + \beta \left[\sum_{t=1}^{\infty} \beta^{d-1} U\left(c_t\right) \right] \right\}$$
$$s.t. \tag{9}$$
$$k_t = \left\{ k_{t-1}(1-\delta) - \left[\frac{R_f}{p} - \frac{1-p}{p} \frac{f(k_{t-1})}{l \cdot k_t} \right] \cdot lk_{t-1} + f(k_{t-1}) - d_{t-1} \right\} (1-l)^{-1} \forall t$$

In other words, we wish to determine the optimal dividend d_t^*, and the implied capital level k_{t+1}^*, for $t = 0, 1, \ldots$. The optimal trajectory is a function $\varphi(.)$ relating the dividends with the capital levels, such relation should be stationary, thus do not depend over time, using constraint is possible to define such a function:

$$k_t = \left\{ k_{t-1}(1-\delta) - \frac{R_f}{p} lk_{t-1} + \frac{f(k_{t-1})}{p} - \varphi(k_{t-1}) \right\} (1-l)^{-1} = \zeta(k_1) \tag{10}$$

Since the capital and dividends are constinous and differentiable, thusunction of capital at time t-1:

$$k_1 : d_2^* = \varphi\left(k_2^*\right) = \varphi\left(\zeta\left(k_1\right)\right)$$

riterating the process we can express a the optimal path of dividens as a function of capital at time 1, thus we can define $V(k_1)$ as the maximum value function:maximum overall utility $\varphi(.)$ should be continous and differentiable. In this way er can express consumption at time t as a fat time 1 given the optimal path of dividends:

$$\sum_{t=1}^{\infty} \beta^{t-1} U\left(d_t^*\right) = V\left(k_1\right)$$

. We can rewrite the problem as follows:

$$\begin{aligned} & V\left(k_0\right) = \max_{c_0} \left\{U\left(c_0\right) + \beta V\left(k_1\right)\right\} \\ & \text{ s.t. } k_1 = f\left(k_0\right) + (1-\delta)k_0 - c_0 \\ & k_0 given. \end{aligned} \tag{11}$$

Before solving the belman is necessary to determine under which condition the problem is solvable, in this sense the theorem exposed in Stokey et al. [1989] which states: the solution exists and it is unique if the following conditons are met:

1. $\beta \in (0, 1)$
2. the return function is continous, boundend and strictly concave
3. the transition function is concave

Then the maximum value $V(k_t)$ exits, its unique and strictly concave. Howeve this condition rules out the logaritmic return function $U(d_t) = \ln d_t$. However if we are willing to leave the strictly concave results we can use the Thompson [2004] theorem.

Now that our problem have a unique solution, lets find the optimal path. The first order condition is the following:

$$U'\left(d_0^*\right) + \beta V'\left(k_1\right) \frac{\partial k_1}{\partial d_0} = 0$$

from the transition function $\frac{\partial k_1}{\partial d_0} = -1$. The optimal path is thus given by the followinf system:

$$\begin{cases} V\left(k_0\right) = U\left(d_0^*\right) + \beta V\left(k_1\right), \\ k_1 = \left\{k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - d_0^*\right\}(1-l)^{-1}, \\ U'\left(d_0^*\right) = \beta V'\left(k_1\right), \\ k_0 \text{ given.} \end{cases} \tag{12}$$

Guess and verify method The return function is $U(d_t) = \ln d_t$, the transition function is $k_1 = \left\{k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - d_0^*\right\}(1-l)^{-1}$. The Foc becames $d_0 = \left[\beta V'(k_1)\right]^{-1}$, Substituting into the transition function, the problem can be written as

$$\begin{cases} V\left(k_0\right) = \ln\left(d_0^*\right) + \beta V\left(k_0\right), \\ k_1 = \left\{k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - \left[\beta V'(k_{t+1})\right]^{-1}\right\}(1-l)^{-1}, \\ U'\left(d_0^*\right) = \beta V'\left(k_1\right), \\ k_0 \text{ given.} \end{cases} \tag{13}$$

Our guess solution is:

$$V(k_t) = e + f \ln k_t \tag{14}$$

thus:

$$\begin{cases} e + f \ln k_0 = \ln\left(\frac{k_1}{\beta f}\right) + \beta\left[e + f \ln k_1\right], \\ k_1 = \left\{k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - \left[\frac{k_1}{\beta f}\right]\right\}(1-l)^{-1}. \end{cases} \tag{15}$$

In order to be tractable lets assume $p - \delta p - R_f l = 0$. Solving for k_1:

$$k_1 = \left[k_0\left(p - p\delta - R_f l\right) + Zk_0^\alpha\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1}, \tag{16}$$

$$d_1 = \left[k_0\left(p - p\delta - R_f l\right) + Zk_0^\alpha\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right), \tag{17}$$

$$\begin{aligned} e + f \ln k_0 = {} & \ln\left\{\left[k_0\left(p - p\delta - R_f l\right) + Zk_0^\alpha\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right)\right\} \\ & + \beta\left[e + f\ln\left\{\left[k_0\left(p - p\delta - R_f l\right) + Zk_0^\alpha\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1}\right\}\right]. \end{aligned} \tag{18}$$

Now is possible to retrieve parameters e and f:

$$f = \frac{\alpha}{1 - \alpha\beta}, \tag{19}$$

$$\begin{aligned} e = {} & \left\{\frac{1 - \alpha\beta - \alpha}{1 - \alpha\beta}\left[\ln Z - \ln\left(1 - l\alpha\beta\right)\right]\right. \\ & + \frac{1 - \alpha\beta}{1 - \alpha\beta}\ln p + \ln\left(1 - \alpha\beta\right) \\ & \left. + \frac{\alpha}{1 - \alpha\beta}\left(\ln\alpha + \ln\beta\right)\right\}\left(1 - \beta\right)^{-1}. \end{aligned} \tag{20}$$

f is positive so $V'(k) > 0$, thus higher dividens path leads to higher value of the return function. Moreover is important to notice that since $\beta \in (0, 1)$ the maximum value function do not explode to infinity. The guess is verified since e, f are indipendent from k. Substituting e, f into (16), (17):

$$k_1 = \frac{Zk_0^\alpha}{p}\frac{\alpha\beta}{1 - l\alpha\beta}$$

$$d_0 = \frac{Zk_0^\alpha}{p}\frac{1 - \alpha\beta}{1 - l\alpha\beta}\beta$$

the steady state for capital is $\widehat{k} = \left[\frac{Z}{p}\frac{\alpha\beta}{1-l\alpha\beta}\right]^{\frac{1}{1-\alpha}}$. Using $p = \frac{R_f l}{1-\delta}$:

$$k_1 = \frac{Zk_0^\alpha(1-\delta)}{lR_f}\frac{\alpha\beta}{1-l\alpha\beta} \tag{21}$$

$$d_0 = \frac{Zk_0^\alpha(1-\delta)}{lR_f}\frac{1-\alpha\beta}{1-l\alpha\beta}\beta \tag{22}$$

$$\widehat{k} = \left[\frac{Z(1-\delta)}{lR_f}\frac{\alpha\beta}{1-l\alpha\beta}\right]^{\frac{1}{1-\alpha}} \tag{23}$$

Locunik at the policy function (22) its interesting to notice that dividends are a constant share of production of the same period. Moreover dividends depend negatively on firm'slavarage and on the risk free rate if the firms is not debt free. While more productive is the firm higher its level of capital and the dividends level. Notice that in the case of firm free of debt the solution does not hold since l=0 and thus p=1. Hower is easy to find the solution assuming no depreciation:

$$k_1 = Zk_0^\alpha(1-\delta)\alpha\beta \tag{24}$$

$$d_0 = Zk_0^\alpha(1-\delta)1-\alpha\beta \tag{25}$$

$$\widehat{k} = [Z\alpha\beta]^{\frac{1}{1-\alpha}} \tag{26}$$

It easy to show that under no debts case the firm has higher capital at the steady state since it does not need to pay interest as it is shown in 19.

8.6 Optimization problem with frictions

Now that we have found a closed form solution to our policy function, lets introduce financial friction due to assymetry of information between the lender and the firms. Lets assume that the lender discount an uncertainty value about the value of production $\mu \quad 0 \leq \mu \leq 1$: close is to 0 higher is the level of frictions. the landing activity thus the problem will became

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U(d_0) + \beta \left[\sum_{t=1}^{\infty} \beta^{d-1} U(c_t) \right] \right\}$$

$$s.t. \tag{27}$$

$$k_t = \left\{ k_{t-1}(1-\delta) - \left[\frac{R_f}{p} - \frac{1-p}{p} \frac{\mu f(k_{t-1})}{l \cdot k_t} \right] \cdot l k_{t-1} + f(k_{t-1}) - d_{t-1} \right\} (1-l)^{-1} \forall t$$

Using the same method in the case without frictions:

$$\begin{cases} V(k_0) = \ln(d_0^*) + \beta V(k_0), \\ k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - [\beta V'(k_{t+1})]^{-1} \right\} (1-l)^{-1}, \\ U'(d_0^*) = \beta V'(k_1), \\ k_0 \text{ given.} \end{cases} \tag{28}$$

Our guess solution is:

$$V(k_t) = e + f \ln k_t \tag{29}$$

$$\begin{cases} e + f \ln k_0 = \ln\left(\frac{k_1}{\beta f}\right) + \beta \left[e + f \ln k_1 \right], \\ k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - \left[\frac{k_1}{\beta f} \right] \right\} (1-l)^{-1}. \end{cases} \tag{30}$$

In order to be tractable lets assume $p - \delta p - R_f l = 0$. Solving for k_1:

$$k_1 = \left[k_0 \left(p - p\delta - R_f l\right) + \mu Z k_0^{\alpha}\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1}, \tag{31}$$

$$d_1 = \left[k_0 \left(p - p\delta - R_f l\right) + \mu Z k_0^{\alpha}\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right), \tag{32}$$

$$\begin{aligned} e + f \ln k_0 = \ln & \left\{ \left[k_0 \left(p - p\delta - R_f l\right) + \mu Z k_0^{\alpha}\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right) \right\} \\ & + \beta \left[e + f \ln \left\{ \left[k_0 \left(p - p\delta - R_f l\right) + \mu Z k_0^{\alpha}\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1} \right\} \right]. \end{aligned} \tag{33}$$

$$f = \frac{\alpha}{1 - \alpha\beta}, \tag{34}$$

$$\begin{aligned} e = & \left\{ \frac{1 - \alpha\beta - \alpha}{1 - \alpha\beta} \left[\ln Z + ln\mu - \ln\left(1 - l\alpha\beta\right)\right] \right. \\ & + \frac{1 - \alpha\beta}{1 - \alpha\beta} \ln p + \ln\left(1 - \alpha\beta\right) \\ & \left. + \frac{\alpha}{1 - \alpha\beta} (\ln\alpha + \ln\beta) \right\} (1 - \beta)^{-1}. \end{aligned} \tag{35}$$

$$\tag{36}$$

Thus the transition function and policy function in the case of financial friction are:

$$k_1 = \frac{Z\mu k_0^{\alpha}(1 - \delta)}{lR_f} \frac{\alpha\beta}{1 - l\alpha\beta} \tag{37}$$

$$d_0 = \frac{Z\mu k_0^{\alpha}(1 - \delta)}{lR_f} \frac{1 - \alpha\beta}{1 - l\alpha\beta} \beta \tag{38}$$

$$\widehat{k} = \left[\frac{Z\mu(1 - \delta)}{lR_f} \frac{\alpha\beta}{1 - l\alpha\beta}\right]^{\frac{1}{1-\alpha}} \tag{39}$$

Thus frictions as the same effect as a reduction in productivity which lowers the dividens, the capital level and velocity to which firms accumulate capital.

Parameter	Symbol	Value
Discount factor	β	0.956
Risk-free rate	R_f	1.04
Depreciation rate	δ	0.07
Returns to scale	α	0.70
Aggregate productivity	$\bar{Z}$	1
Monitoring cost	$1-\mu$	0.25

Table 4: Benchmark calibration

8.7 Simulation example

In order to grasp the differnces between the friction case and the frictionless, lets implement a simulation exercise in which I'll use the same parameters as in in the Osotimehin and Pappadà [2017] paper: Since we want to understand which are the effect of including friction we fix the leverage $l = 0.5$ and $k_0 = 1$. Let's take a look to the transtion function, thus how the capital evolves following the optimal path. We can compute $p = \frac{0.5 \cdot 1.04}{1-0.07} \approx 0.559$. The results are shown in the plots below: The first plot shows the

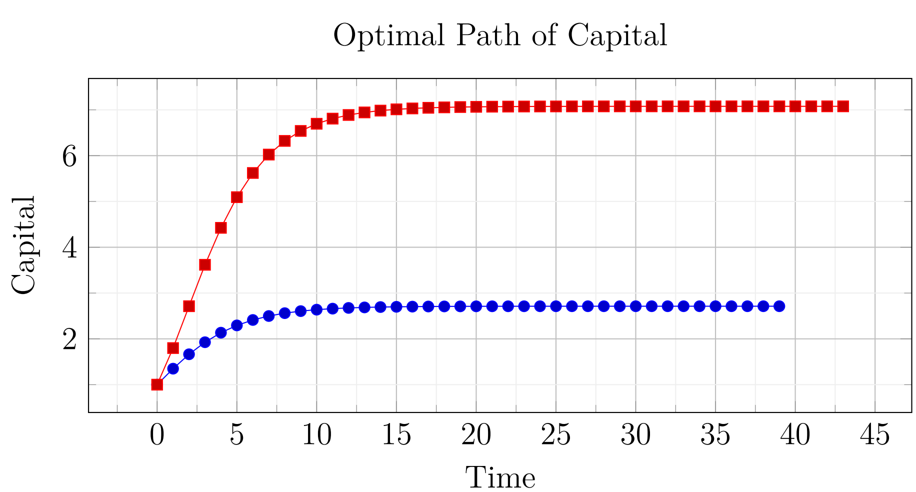


Figure 20: Evolution of capital over time

transition function: Clearly, the path of dividends is higher in the case of the absence of friction

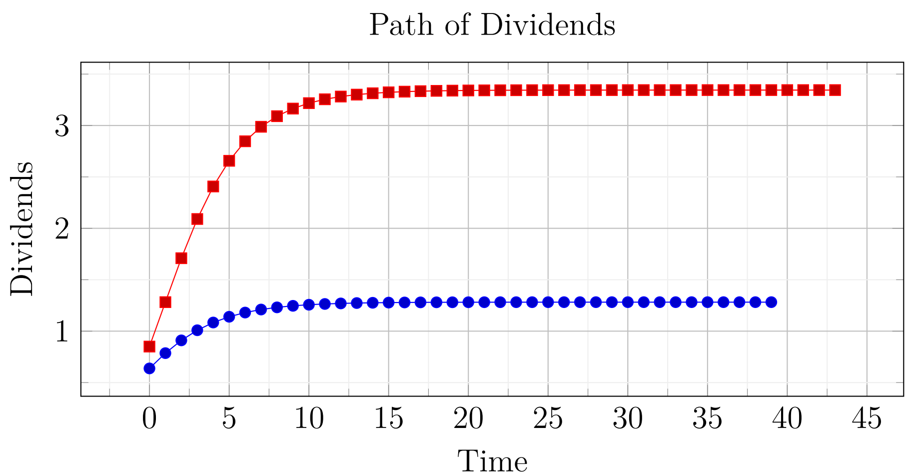


Figure 21: Evolution of dividends over time

8.8 Simulation exercises

To enhance our model, we now introduce heterogeneity across firms. Specifically, the productivity level, denoted as Z_i, varies from one firm to another. Additionally, leverage ratios are no longer uniform across firms. We simulate the initial distribution of capital, leverage, and productivity in a manner that ensures the correlation between productivity and leverage is zero. The aggregate production is equal to:

$$\overline{K} = \int_{\underline{K}}^{\overline{k}} Z_i k_{i,t}^{\alpha} \, di$$

Business cycles are introduced through variations in capital, where each firm experiences changes proportionate to their existing capital levels. Less efficient firms face larger decreases during downturns, whereas in periods of economic upswing, all firms benefit from increased capital availability. Additionally, firms may exit the market under two conditions: voluntary exit when returns on equity fall below the risk-free rate, and bankruptcy, characterized by an inability to cover debts and depreciation. Initially, the simulation incorporates a sinusoidal business cycle to modulate output:

$$\Delta \overline{K}_t = 1 + 0.05 \sin(t)$$

Meanwhile, the distributions for productivity (Z) and leverage (l) are defined as truncated normal:

$$l \sim \mathcal{N}(0.5, 0.02), \quad 0.01 \leq l \leq 2, \tag{40}$$

$$Z - 1 \sim \mathcal{N}(0.05, 0.02), \quad 1.01 \leq Z \leq 1.1. \tag{41}$$

A firm exits the market if its return on capital is less than the risk-free rate, being replaced by a new firm with capital equivalent to the lowest in the existing pool. The remaining capital is then proportionally reinvested among the other firms, based on their returns on capital. The return on capital for firm i at time t is calculated as follows:

$$R_{i,t} = \frac{d_{i,t}}{k_{i,t}}$$

If $R_f \geq R$, the firm exits the market due to its return on capital being lower than the risk-free rate.[4]

The figure below illustrates the outcome of a 20-step simulation involving 10 firms, depicting the optimal and actual capital levels. The actual capital is adjusted according to the defined business cycle:

$$\Delta \overline{K}_t = 1 + 0.05 \sin(t)$$

The subsequent graph displays the distribution of dividends calculated based on the

[4]This criterion is applied only after 5 periods to highlight the differences between scenarios with and without the cleansing effect in action.

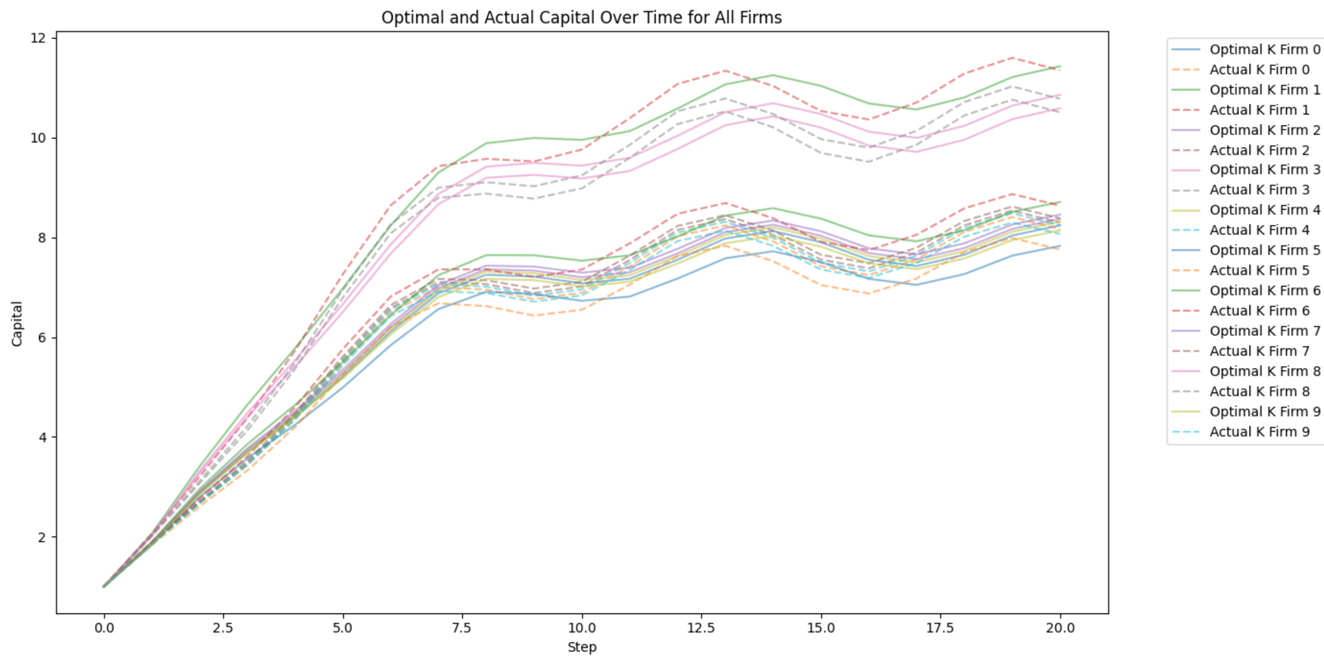


Figure 22: The plot shows the result of a 20-step simulation with 10 firms: the optimal k is the capital choice of the firm, while the actual k is the capital after applying the business cycle effect.

actual capital of each firm.

This rewritten section maintains the integrity of your original description while enhancing readability and precision, particularly in the mathematical modeling and the explanation of the simulation's framework and outcomes.

8.9 Solving with the Belman for the frictions

$$V(k_t) = \max_{k_{t+1}, e_{t+1}} d_t + \beta V(k_{t+1})$$

$$s.t.$$

$$f(k_t) = Z k_t^\alpha$$

$$f(k_t) = d_t + (c + k_{t-1} - e_{t-1})(1 + r) + k_t - (c + k_t - e_t) - k_{-1}(1 - \delta)$$

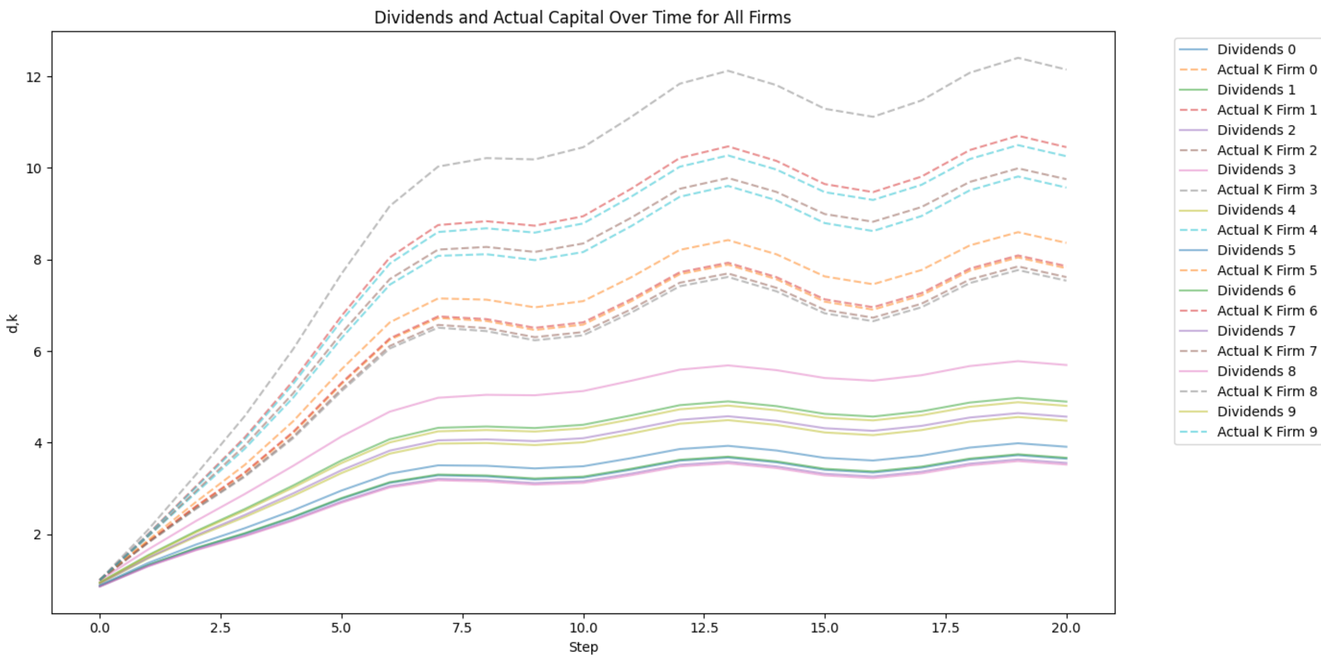


Figure 23: This plot presents the results of a 20-step simulation with 10 firms, showcasing dividends (d) and the actual capital (k) post-adjustment for the business cycle effect.

$$(1+r)(c+k_t-e_t)p+(1-p)f(k_t)=(1+r_f)(c+k_t-e_t)$$

$$B_t=c+k_t-e_t; R=1+r; R_f=1+r_f;$$

$$R=\frac{R_f}{p}-\frac{1-p}{p}\frac{f(k_t)}{D_t}$$

In order to understand the mechanism behind this optimization problem, I firstly solve the three times problem working backward. The value function in $t=2$ is

$$V_{t+2}=\max d_{t+2}$$

Since there firm will not exists in t+2, there are no investiment $B_{t+2}=0$, thus $0=k_{t+2}+c-e_{t+2}$ as consequence $k_{t+2}=e_{t+2}-c$. Then we can rewrite the value function:

$$V_{t+2}=\max Z(e_{t+2}-c)^{\alpha}-(c+k_{t+1}-e_{t+1})(1+r_{t+1})-e_{t+2}+c+(c+e_{t+2}-c-e_{t+2})+k_{t+1}(1-\delta)$$

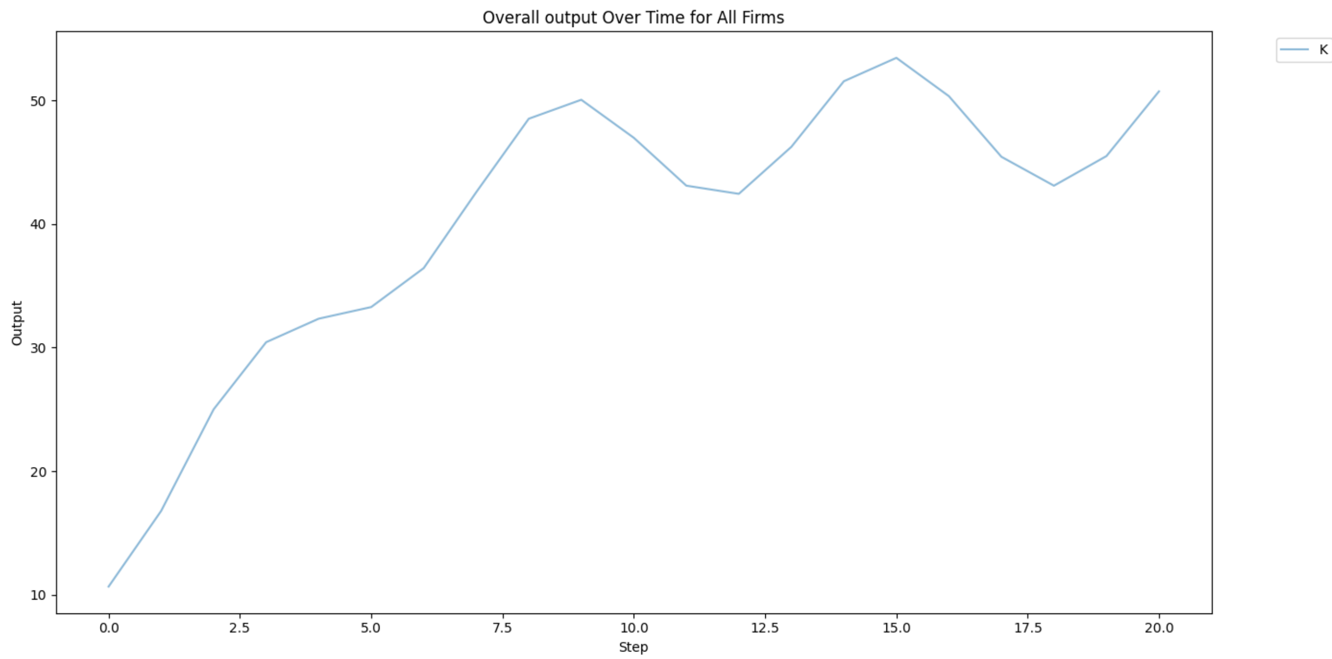


Figure 24: This plot presents the results of a 20-step simulation with 10 firms, showcasing output (K) post-adjustment for the business cycle effect.

$$V_{t+2} = \max_{e_{t+2}} Z(e_{t+2} - c)^{\alpha} - B_{t+1}R_{l,t+1} - e_{t+2} + c + k_{t+1}(1-\delta)$$

FOC:

$$\frac{\partial V_{t+2}}{\partial e_{t+2}} = Z\alpha(e_{t+2} - c)^{\alpha-1} - 1 = 0$$

$$(e_{t+2} - c)^{\alpha-1} = (Z\alpha)^{-1}$$

$$e_{t+2} = (Z\alpha)^{\frac{1}{1-\alpha}} + c$$

Thus:

$$d_{t+2} = Z\left[(Z\alpha)^{\frac{\alpha}{1-\alpha}}\right] - B_{t+1}R_{t+1} - \left[(Z\alpha)^{\frac{1}{1-\alpha}}\right] + k_{t+1}(1-\delta)$$

$$V_{t+2} = Z\left[(Z\alpha)^{\frac{\alpha}{1-\alpha}}\right] - B_{t+1}R_{t+1} - \left[(Z\alpha)^{\frac{1}{1-\alpha}}\right] + k_{t+1}(1-\delta)$$

Writing the problem in t+1:

$$V_{t+1} = \max_{e_{t+1},k_{t+1}} d_{t+1} + \beta V_{t+2}$$

$$d_{t+1} = Zk_{t+1}^{\alpha} - B_t R_L - k_{t+1} + B_{t+1} + k_t(1-\delta)$$

FOCs:

$$\begin{cases} \frac{\partial V_{t+1}}{\partial e_{t+1}} = \frac{\partial d_{t+1}}{\partial e_{t+1}} + \beta \frac{\partial V_{t+2}}{\partial e_{t+1}} = 0 \\ \frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial d_{t+1}}{\partial k_{t+1}} + \beta \frac{\partial V_{t+2}}{\partial K_{t+1}} = 0 \end{cases} \tag{42}$$

solving $\frac{\partial d_{t+1}}{\partial e_{t+1}}$:

$$\frac{\partial d_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial B_{t+1}}{\partial e_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial e_{t+1}} = \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

$$\frac{\partial d_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

solving $\frac{\partial V_{t+2}}{\partial e_{t+1}}$:

$$\frac{\partial V_{t+2}}{\partial e_{t+1}} = -\left[\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right]$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \frac{\partial B_{t+1}}{\partial e_{t+1}}R_{t+1} + B_{t+1}\frac{\partial R_{t+1}}{\partial e_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial e_{t+1}} = \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

$$\frac{\partial R_{t+1}}{\partial e_{t+1}} = -\frac{1-p}{p}\left\{\left[Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \delta\frac{\partial k_{t+1}}{\partial e_{t+1}}\right]B_{t+1} - \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[Zk_{t+1}^{\alpha} - \delta k_{t+1}\right]\right\}B_{t+1}^{-2}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right]R_{t+1} - \frac{1-p}{p}\left\{\left[Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \delta\frac{\partial k_{t+1}}{\partial e_{t+1}}\right]B_{t+1} - \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[Zk_{t+1}^{\alpha} - \delta k_{t+1}\right]\right.$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[\left(zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\frac{1-p}{p}B_{t+1}^{-1} + R_{t+1}\right] - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial V_{t+2}}{\partial e_{t+1}} = -\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right) R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right]$$

Substituting into the first FOC, we get:

$$\frac{\partial V_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1 - \beta\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right) R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1 -$$

second FOC:

solving $\frac{\partial d_{t+1}}{\partial k_{t+1}}$:

$$\frac{\partial d_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - 1 + \frac{\partial B_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial k_{t+1}} = 1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial d_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

solving $\frac{\partial V_{t+2}}{\partial k_{t+1}}$:

$$\frac{\partial V_{t+2}}{\partial k_{t+1}} = -\left[\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} - (1-\delta)\right]$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \frac{\partial B_{t+1}}{\partial k_{t+1}}R_{t+1} + B_{t+1}\frac{\partial R_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial k_{t+1}} = 1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial R_{t+1}}{\partial k_{t+1}} = -\frac{1-p}{p}\left[\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) B_{t+1} - \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\right] B_{t+1}^{-2}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right) R_{t+1} + \left\{\frac{1-p}{p}\left[(Z\alpha k_{t+1}^{\alpha-1} - \delta)B_{t+1} - \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\right] B_{t+1}^{-1}\right\}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+}}{\partial k_{t+1}}\right)\left[R_{t+1} + \frac{1-p}{p}\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right) B_{t+1}^{-1}\right] - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right) R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial V_{t+2}}{\partial k_{t+1}} = -\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right) R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right]$$

Substituting into the FOC:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}} - \beta\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right) R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right] = 0$$

thus the FOCs are:

$$\frac{\partial V_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1 - \beta\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right) R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right] = 0$$

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}} - \beta\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right) R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right] = 0$$

rearranging $\frac{\partial V_{t+1}}{\partial k_{t+1}}$ to isolate k_{t+1}:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial e_{t+1}}{\partial k_{t+1}}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1}$$

rearranging $\frac{\partial V_{t+1}}{\partial e_{t+1}}$ to isolate k_{t+1}:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z\alpha}$$

Equating the two equations:

$$\left[\frac{\partial e_{t+1}}{\partial k_{t+1}}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z\alpha}$$

From this equation, you can isolate $\frac{\partial e_{t+1}}{\partial k_{t+1}}$ to solve for it explicitly.

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = -\left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z\alpha}\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}(1-\beta R_f)^{-1} - \beta\left(r_f + \frac{\delta}{p}\right)$$

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = -\left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \delta\frac{1-p}{p}\right]\frac{\beta p + \beta - p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)(1-\beta R_f)^{-1}$$

Since $\frac{\partial e_{t+1}}{\partial k_{t+1}}$ is the reciprocal of $\frac{\partial k_{t+1}}{\partial e_{t+1}}$, we can compute the optimal path of the networth as a function of the capital. Defining $y = \frac{\partial e_{t+1}}{\partial k_{t+1}}$ and ths $\frac{y^{-1}=\partial e_{t+1}}{\partial k_{t+1}}$:

$$y = -\left[\frac{1}{y}\left(1-\beta R_f\right) + \delta\frac{1-p}{p}\right]\frac{\beta p+\beta-p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)\left(1-\beta R_f\right)^{-1}$$

To solve the given equation

$$y = -\left[\frac{1}{y}\left(1-\beta R_f\right) + \delta\frac{1-p}{p}\right]\frac{\beta p+\beta-p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)\left(1-\beta R_f\right)^{-1}$$

Use Python and the sympy library to solve the equation:

```python
from sympy import symbols, Eq, solve

# Define the symbols
y, beta, R_f, delta, p, r_f = symbols('y beta R_f delta p r_f')

# Define the equation
equation = Eq(y, - ((1/y) * (1 - beta * R_f) + delta * (1 - p) / p) * (beta * p + beta - p) / (1 - beta * R_f) - beta * (r_f + delta / p) / (1 - beta * R_f))

# Solve for y
solution = solve(equation, y)
```

The equation for y is given by:

$$y = \frac{\beta\delta \pm \sqrt{\Delta}}{2p(\beta R_f - 1)}$$

In the solutions provided, Δ represents the discriminant of the quadratic equation that was formed during the solution process. It is the expression under the square root in the solutions. The discriminant Δ in this case is a complex expression involving the variables R_f, β, p, δ, and r_f. Specifically, Δ is given by:

$$\Delta = (-\beta\delta - \beta p^2\delta\left(\frac{1}{p} - 1\right) + p^2\delta\left(\frac{1}{p} - 1\right) \tag{43}$$

$$- \beta p\delta\left(\frac{1}{p} - 1\right) - \beta p r_f)^2 \tag{44}$$

$$- 4(\beta p R_f - p)(\beta^2 p^2 R_f - \beta p^2 R_f + \beta^2 p R_f \tag{45}$$

$$- \beta p^2 + p^2 - \beta p) \tag{46}$$

The discriminant Δ is given by:

$$\Delta = [\text{complex expression involving } R_f,\ \beta,\ p,\ \delta, \text{ and } r_f] \tag{47}$$

The sign of the solutions depends on:

- The values of β, δ, p, R_f, and r_f.
- The value and sign of Δ.

Since Δ involves these parameters in a complex manner, the sign of the solutions can be:

- Real and positive, real and negative, or complex (depending on the sign and magnitude of Δ and other parameters).
- Determined specifically only when actual values for the parameters are provided.

Since now we have a partial derivative as a function of parametrs we can retrive the relation between equity and capital. Rewriting the solutions as:

$$y = \frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D}$$

Given the partial derivative:

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D} \tag{48}$$

where N and D are constants with respect to k, we want to integrate this with respect to k.

Since N and D do not depend on k, the integral is straightforward:

$$\int \frac{N}{D} \, dk \tag{49}$$

Integrating a constant with respect to k yields:

$$\int \frac{N}{D} \, dk = \frac{N}{D} \cdot k + C \tag{50}$$

where C is the constant of integration.

Thus, the set of possible solutions is:

$$e_{t+1} = \frac{N}{D} \cdot k_{t+1} + C \tag{51}$$

where C is determined based on boundary conditions or initial values. Given the relationship:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial e_{t+1}}{\partial k_{t+1}} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p}\right)\right] \left\{Z\alpha \left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} \tag{52}$$

we can retrieve k_{t+1}.

Substituting $\frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D}$ into the equation, we get:

$$k_{t+1}^{\alpha-1} = \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p}\right)\right] \left\{Z\alpha \left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} \tag{53}$$

Sine now we have the optimal k we can retrieve the optimal path of networth:

$$e_{t+1} = \frac{N}{D} \cdot \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + C$$

Thus the debt at time t+1 is:

$$\begin{aligned} B_{t+1} = & \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + c \\ & - \left\{ \frac{N}{D} \cdot \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + C \right\} \end{aligned} \tag{54}$$

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