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Theoretical framework

Riccardo Dal Cero

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Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

1 Introduction

In macroeconomic theory, the investigation of business cycles and long-term growth trajectories traditionally unfolds in distinct academic silos, drawing a parallel to the distinct realms of quantum mechanics and Einstein's theory of relativity in physics. This academic segregation, however, obscures a fundamental and profound question: How do business cycles influence long-term economic growth? The exploration of this question is more than an academic exercise; it underpins the practical understanding of short-term economic policies, such as automatic stabilizers, and their profound long-term impacts on the economy.

Embarking on this exploration, my research primarily dwells in the realm of theory, supplemented by rigorous simulation and calibration exercises. The intricate complexity of business cycles, particularly evident during periods of economic downturn and recovery, challenges empirical approaches due to the plethora of confounding variables. Thus, a theoretical lens, rather than a purely empirical one, is employed to dissect and understand these phenomena.

Central to this theoretical framework is an examination of the role of financial market frictions during economic recessions. A key inquiry here is the investigation of policy interventions, such as demand stabilizers, and their potential effect in attenuating the 'cleansing effect' of recessions. This exploration is pivotal in understanding whether such policies might inadvertently lead to a reduced economic baseline or steady state in the long term.

The conceptual foundation of this investigation is inspired by an ecological analogy the cyclical dynamics observed between predator and prey populations in nature. This natural cycle, when observed over extended periods, reveals not just self-contained oscillations but also underlying trends of population growth for both predators and prey. This observation leads to a compelling analogy for economic cycles: while they appear as short-term fluctuations, they might be underpinned by long-term growth trajectories.

In natural ecosystems, interventions aimed at stabilizing these cycles such as protecting prey during times of increased predation might seem beneficial in the short term. However, such interventions often neglect the critical and natural process of selection. This interference disrupts evolutionary mechanisms, potentially leading to unforeseen consequences over time, such as the propagation of traits detrimental to the species' survival and adaptability in changing environments.

My thesis extends this analogy to the economic sphere, positing a similar selective mechanism at play in economic systems. The primary focus is on the recession's cleansing effect, which might be analogous to natural selection in ecology. This effect could potentially 'weed out' less productive firms, leaving a market landscape dominated by more efficient players. The exploration aims to decipher whether such an economic 'natural selection' mechanism exists and, if so, how it shapes the fabric of productivity, innovation, and growth in the long term. Through this lens, the research endeavors to contribute a nuanced understanding of the intricate interplay between short-term economic fluctuations and long-term economic evolution, offering insights into the design and implications of economic policies. In the following sections, we will delve deeper into specific theories that bridge the gap between business cycles and long-term economic growth. However, it is beneficial first to embark on a brief historical journey through the evolution of thought regarding business cycles, to understand the context and development of these interconnected economic theories. This exploration will provide a foundation for appreciating the diversity of perspectives and the progression of ideas that have shaped our understanding of the intricate relationship between short-term economic fluctuations and long-term growth trajectories.

2 Business cycle history

The exploration of business cycle theories represents a cornerstone in the history of economic thought. A prominent exponent in this realm was Friedrich Hayek, who articulated the complexities of business cycles in relation to economic equilibrium theory. Hayek's perspective is encapsulated in his own words:

> "The incorporation of cyclical phenomena into the system of economic equilibrium theory, with which they are in apparent contradiction, remains the crucial problem of Trade Cycle theory; By 'equilibrium theory' we primarily understand the modern theory of the general interdependence of all economic quantities, which has been perfectly expressed by the Lausanne School of theoretical economics." [Hayek, 1933]

In the turbulent era of the early 1930s, the investigation of business cycles was particularly driven by the contrasting views of Friedrich Hayek and John Maynard Keynes, two of the most influential economists of the 20th century. Hayek, in his seminal work "Monetary Theory and the Trade Cycle," Hayek [1933] examined business cycles through the prism of monetary theory and its influence on the structure of capital. He posited that distortions in the economy, especially those resulting from the artificial lowering of interest rates by central banks, lead to malinvestments during economic booms. These malinvestments, particularly prevalent in capital-intensive sectors, were deemed unsustainable, culminating inevitably in economic downturns characterized by corrections of these misallocations.

In stark contrast stood Keynes, whose groundbreaking work "The General Theory of Employment, Interest, and Money"Keynes [1960] offered a different vantage point. Keynes approached the issue of business cycles from the standpoint of equilibrium. He concentrated on the role of aggregate demand in dictating overall economic activity levels. According to Keynes, a deficiency in aggregate demand could result in extended periods of high unemployment. He advocated for proactive government intervention to invigorate demand and re-establish economic equilibrium, thus shifting the analytical focus from isolated markets to the economy's aggregate behavior.

To illustrate the technical differences between Keynes and Hayek's approaches to business cycle theory, it's helpful to represent their ideas through simplified equations. These equations, while not exhaustive, capture the essence of their theoretical perspectives.

1. Hayek's Business Cycle Theory: Hayek's theory, centered on the Austrian School's ideas, emphasizes the role of capital structure and monetary policy. A key aspect is how artificial lowering of interest rates leads to a misallocation of capital, or malinvestments. A simplified representation could be:

$$I_t = f(r_t, r_n)$$

$$r_t < r_n \rightarrow \text{Malinvestment}$$

Here, I_t represents investment at time t, r_t is the artificially lowered interest rate, and r_n is the natural rate of interest. Hayek argued that when r_t is set below r_n by central banks, it leads to malinvestments, particularly in capital-intensive sectors, ultimately causing economic distortions and a business cycle.

2. Keynes's Business Cycle Theory: In contrast, Keynes's theory in "The General Theory of Employment, Interest, and Money" focuses on aggregate demand and its role in economic cycles. The basic Keynesian model can be represented as:

$$Y = C(Y - T) + I(r) + G + NX$$

$$Y < Y_{full} \rightarrow \text{Increase } G$$

Here, Y is the national income, C is consumption (a function of disposable income $Y - T$, where T is taxes), I is investment (a function of the interest rate r), G is government spending, and NX is net exports. Keynes argued that when Y is less than the full employment output (Y_{full}), it indicates insufficient aggregate demand, warranting an increase in government spending G to stimulate the economy.

These equations summarize the core differences in how Hayek and Keynes viewed the drivers of business cycles. Hayek focused on the distortions created by monetary policy in capital investment, leading to cycles of booms and busts. In contrast, Keynes emphasized the role of aggregate demand in driving economic activity, advocating for fiscal policy interventions to manage these cycles. Their divergent views laid the foundation for much of the economic debate and policy formulation in the 20th century, with each theory offering distinct insights into the mechanisms of economic fluctuations. These divergent views laid the foundation for subsequent economic theories and policy formulations, deeply influencing how successive generations of economists and policymakers would grapple with the complexities of business cycles and their implications for long-term economic growth and stability.

Responding to Keynes, Hayek further developed his ideas in "The Pure Theory of Capital," Hayek and Caldwell [1941] where he provided a detailed critique of equilibrium-based economic theories. Hayek's argument centered on the complexity of economic systems, highlighting how the notion of equilibrium oversimplified the dynamic interplay of different markets, particularly the capital goods market. He stressed that economic equilibrium failed to account for the temporal structure of capital and the dynamic processes that govern investment and production over time. Hayek posited that economic activities are inherently dynamic and subject to constant change, influenced by numerous factors including individual preferences, technological changes, and policy decisions. This view was a stark departure from Keynesian equilibrium, suggesting that attempts to artificially maintain economic stability could lead to unintended consequences, exacerbating economic fluctuations rather than smoothing them.

Thus, while Keynes emphasized stabilizing aggregate demand to achieve equilibrium and mitigate business cycles, Hayek focused on the inherent dynamism and complexity of economic systems, criticizing equilibrium models for their oversimplification of the intricate processes that drive economic activities. This divergence in views marked a fundamental debate in economic theory, shaping the discourse on how economies respond to and recover from periods of economic downturns.

Joseph Schumpeter, another influential economist of the early 20th century, brought a unique perspective to the discussion of business cycles, one that diverged significantly from both Hayek and Keynes. Schumpeter's approach, primarily outlined in his theory of "creative destruction," emphasized the role of innovation and entrepreneurial spirit in economic development and business cycles.

Schumpeter viewed business cycles as inherent and vital to capitalist economies, driven by the process of innovation. According to Schumpeter, the entrepreneur is the agent of change, introducing new technologies, products, and methods, which disrupt existing market equilibria. This process of innovation leads to the destruction of outdated industries and economic structures, paving the way for new ones. In Schumpeter's framework, the cyclical nature of the economy was a reflection of this ongoing process of creative destruction, where periods of economic downturns were seen not just as phases of correction, as Hayek might argue, or as failures of demand, as per Keynes, but as essential for clearing away the old to make way for the new.

While Hayek focused on the distortions in capital structure caused by monetary interventions and Keynes emphasized the role of aggregate demand and government intervention in stabilizing the economy, Schumpeter's perspective highlighted the evolutionary nature of capitalist economies. He argued that economic fluctuations were natural and necessary, a process through which economies evolve and adapt over time. Schumpeter's theory thus provided a more dynamic view of capitalism, recognizing the disruptive yet progressive role of innovation and entrepreneurship in shaping economic landscapes.

Schumpeter's contribution added a dimension of evolutionary change to the understanding of business cycles, contrasting with Hayek's emphasis on monetary theory and capital structure, and Keynes's focus on equilibrium and aggregate demand. Schumpeter's insights into the transformative power of innovation offered a more optimistic view of economic downturns, seeing them as opportunities for new growth and advancements.

In the aftermath of World War II, the landscape of macroeconomic theory experienced a paradigmatic shift, significantly influenced by the ascendancy of Keynesian economics. The widespread devastation of the war necessitated a thorough reevaluation of economic policies and theories, setting the stage for Keynesian principles to take a dominant position in shaping government approaches to economic policy, especially concerning business cycles.

Central to the Keynesian doctrine is the advocacy for proactive government intervention to stabilize economic cycles. This perspective gained substantial traction in the post-war era, a period marked by the reconstruction efforts of numerous nations. The foundational principle of Keynesianism, as articulated in Keynes's seminal work "The General Theory of Employment, Interest, and Money," is that government spending can act as a catalyst to stimulate demand during economic downturns. This tenet represented a stark divergence from the pre-war classical economic thought, which predominantly favored limited government intervention in the economy.

A key element in Keynesian policy is the concept of fiscal multipliers, which posits that government spending has a multiplied effect on overall economic output. The multiplier effect can be conceptualized as follows:

$$\Delta Y = \text{Multiplier} \times \Delta G$$

where ΔY represents the change in total output, and ΔG is the change in government spending. The 'Multiplier' is defined as:

$$\text{Multiplier} = \frac{1}{1 - c(1 - t) + m}$$

Here, c is the marginal propensity to consume, t is the tax rate, and m is the marginal propensity to import. This formula illustrates how an increase in government spending (ΔG) can lead to a larger increase in total output (ΔY), thereby stimulating economic activity during recessions.

The immediate post-war period witnessed the successful implementation of Keynesian policies, evidenced by stable economic growth and reduced unemployment rates. This success solidified the influence of Keynesian economics, particularly in the United States and Western Europe, where governments widely adopted strategies such as fiscal stimulus, interest rate manipulation, and welfare state expansion to regulate economic cycles and mitigate recessionary impacts.

The era also saw the rise of the concept of ”fine-tuning” the economy. Policymakers and economists believed that through judicious management of fiscal and monetary policies, it was possible to circumvent severe economic downturns and maintain consistent growth. The underlying idea was that by modulating government spending, tax policies, and interest rates, governments could bolster and sustain demand, thereby smoothing out the fluctuations inherent in business cycles.

The post-war dominance of Keynesian economics encountered a significant challenge in the 1970s with the onset of stagflation, a term coined to describe the unprecedented combination of high inflation and high unemployment, accompanied by stagnant demand. This phenomenon posed a critical challenge to the Keynesian framework, which had not anticipated such a simultaneous occurrence of inflation and stagnation, fundamentally questioning its policy prescriptions.

The Keynesian model, as traditionally understood, posited an inverse relationship between inflation and unemployment, often illustrated by the Phillips Curve. This relationship suggested that higher inflation could help reduce unemployment by stimulating demand, and vice versa. However, the stagflation of the 1970s, where high inflation coexisted with high unemployment, contradicted this established Keynesian principle. The phenomenon was first brought to widespread attention by economists such as Milton Friedman and Edmund Phelps, who argued that the Phillips Curve could only function in the short term, and that long-term attempts to exploit this trade-off would lead to ever-increasing rates of inflation without reducing unemployment.

The stagflation era was marked by several critical global events that influenced economic conditions. The 1973 oil crisis, triggered by an oil embargo by OPEC nations, led to a dramatic increase in oil prices, contributing significantly to inflationary pressures worldwide. This external shock, coupled with the already expansionary fiscal and monetary policies in many Western economies, exacerbated inflation without spurring economic growth or reducing unemployment.

The Keynesian approach, which advocated for increased government spending and lower interest rates to combat recessions, appeared ineffective in addressing the simultaneous challenges of stagnation and inflation. This inadequacy led to the rise of alternative economic theories, such as monetarism and supply-side economics, which focused more on controlling inflation and stimulating supply rather than managing demand. Monetarism, championed by Milton Friedman, emphasized the importance of controlling the money supply to manage inflation, arguing that inflation was primarily a monetary phenomenon. Meanwhile, supply-side economics advocated for reducing tax rates and decreasing regulation to encourage production and investment.

The emergence of stagflation and the subsequent paradigm shift in economic theory underscored the complexity of economic systems and the limitations of existing models. It marked a transition in macroeconomic thought and policy, from a predominantly Keynesian consensus to a more diverse array of approaches, including monetarism and supply-side economics, each offering different perspectives on how to achieve economic stability and growth. This period of economic rethinking paved the way for significant policy shifts in the 1980s, particularly in the United States and the United Kingdom, where there was a move towards deregulation, reduction of government intervention, and a greater emphasis on monetary policy to control inflation.

The narrative of economic thought in the latter half of the 20th century took a pivotal turn with the introduction of the Lucas Critique, formulated by economist Robert Lucas in 1976. This critique fundamentally challenged the prevailing Keynesian orthodoxy and ushered in a new era in the understanding of economic policy and business cycles.

The Lucas Critique posited that traditional Keynesian macroeconomic models, which were used to design economic policies, were fundamentally flawed because they did not take into account the way people's expectations and decisions would change in response to policy shifts. Lucas argued that individuals and firms adjust their behavior based on their expectations of future policy, which in turn alters the effectiveness of that policy. This meant that historical data, which Keynesian models heavily relied upon, could not reliably predict the outcomes of economic policies, as the very implementation of these policies would change the economic environment and behavior of agents within it.

Consider a simple Keynesian macroeconomic model where output is determined by:

$$Y_t = \alpha + \beta G_t + \epsilon_t \tag{1}$$

where Y_t is the output, G_t is the government expenditure, α and β are parameters, and ϵ_t is a random error term.

Lucas argued that the parameters α and β in such models are not constant but change with policy. Thus, a policy change that alters G_t will also change α and β, rendering predictions based on historical data unreliable. This is because the parameters are not deep structural parameters but are themselves functions of agents' expectations and behaviors, which are influenced by policy.

The critique can be formalized as follows:

$$Y_t = f(G_t, \theta_t) + \epsilon_t \tag{2}$$

where θ_t represents the agents' expectations and behaviors, which are influenced by policies. Therefore, any policy change that alters G_t also changes θ_t, and thus the relationship between Y_t and G_t.

The Lucas Critique led to the development of models with microeconomic foundations and rational expectations, where policy effectiveness is assessed by considering how policies alter agents' expectations and decision-making processes. This shift was instrumental in the evolution of New Keynesian economics, which integrates rational expectations with menu costs, nominal rigidities, and other market imperfections.

The implications of the Lucas Critique were profound. It suggested that policies based on historical models might be ineffective or even counterproductive. This critique was a significant factor in shifting the focus of macroeconomic research from Keynesian models, which emphasized aggregate demand and fiscal policy, to new classical models, which focused more on microeconomic foundations and the role of rational expectations.

Lucas's ideas were met with resistance and debate. Opponents, particularly those in the Keynesian camp, argued that while the critique had merit, it did not invalidate the use of all macroeconomic modeling. They contended that while expectations are important, other factors like market imperfections and rigidities also play a critical role in the functioning of the economy. These Keynesian economists argued for the continued relevance of fiscal policy and government intervention, especially in situations like recessions, where private demand is insufficient.

The debate sparked by the Lucas Critique led to significant advancements in economic theory. It pushed economists to develop new models that incorporated rational expectations and more robust microeconomic foundations. This period saw the rise of New Keynesian economics, which attempted to merge Keynesian concepts with microeconomic foundations, including rational expectations and market imperfections.

In the years following the Lucas Critique, economic thought around business cycle theories underwent significant evolution, particularly with the integration of various types of frictions into macroeconomic models. This period saw a shift from idealized notions of perfect competition and complete information to models that better reflected the complexities of real-world economies.

The late 1980s and early 1990s marked the rise of New Keynesian economics, which emphasized the role of nominal rigidities, such as sticky prices and wages, in the economy. Pioneering works in this field, like those of Gregory Mankiw and David Romer Mankiw and Romer [1991], established theoretical foundations where these market imperfections were central to explaining why economies do not self-correct efficiently after disturbances.

Simultaneously, Real Business Cycle (RBC) theory, emerging from the seminal work of Finn E. Kydland and Edward C. Prescott, particularly their influential paper "Time to Build and Aggregate Fluctuations" Kydland and Prescott [1982], focused on real (non-monetary) shocks and frictions. RBC theory emphasized aspects such as technology shocks and constraints in capital accumulation, offering insights grounded in neoclassical microfoundations.

Another significant development was the exploration of credit market frictions, especially after the financial crisis of 2007-2008. The work of Bernanke and Gertler, particularly "Agency Costs, Net Worth, and Business Fluctuations" Bernanke and Gertler [1986], brought to light how asymmetric information in credit markets could amplify business cycles, linking the financial health of firms to their investment and production decisions.

Further, the introduction of search and matching frictions in labor markets, attributed to the work of economists like Diamond, Mortensen, and Pissarides, highlighted how inefficiencies in matching workers with jobs could lead to persistent unemployment, thus influencing the business cycle. Their research, including papers like ”Job Creation and Job Destruction in the Theory of Unemployment” (1994), was pivotal in integrating labor market dynamics into business cycle theories.

The culmination of these developments is seen in the formulation of Dynamic Stochastic General Equilibrium (DSGE) models that incorporate various frictions. These models, blending insights from New Keynesian and RBC theories, integrate sticky prices, financial frictions, and labor market imperfections. They have become a standard tool for economic policy analysis, particularly among central banks, representing a significant advance in the ability to simulate and understand economic fluctuations and policy impacts.

3 Theories Connecting Business Cycles to Long-Term Growth

In the realm of economic theory, approaches that link business cycles (BC) to long-term growth can broadly be categorized into two distinct groups. The traditional perspective posits that long-term growth is predominantly driven by technological progress. In many of these theories, technological advancement is treated as an exogenous factor, a trend in progress not explained within the model itself, as exemplified in the works of Solow [1956] and Swan [1956]. These models imply that technological progress is an external, independent force influencing economic growth.

On the other hand, a contrasting body of theories seeks to endogenize technological progress. These theories integrate mechanisms that explain technological advancement as part of the economic process itself. They incorporate factors such as incentives for innovation, the pursuit of education, and the acquisition of knowledge through economic activities, including the concept of 'learning by doing.' This approach is well-represented in studies like Stadler [1990], where technological progress is seen as a result of economic actions and incentives, rather than an external, unexplained variable.

4 Theoretical framework

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as c to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as e, in the risk-free asset.

Firms opt to exit the market when expected profits no longer outweigh the fixed cost c, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r) - 1] e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to q. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors. In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as k. The production function, accounting for the fixed cost c, is expressed as follows: $Y = Z(\theta + \epsilon)k^{\alpha}$. Key variables include:

- Z: Stochastic aggregate productivity common across firms.
- θ: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- ϵ: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \delta)$.
- k^{α}: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

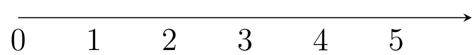

Figure 1: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of Z, θ, k^{α}, e (where e represents its endowment, different from k since the firm can borrow money with $d = c + k - e$).

2. The firm computes the optimal k to maximize the expected value of the firm, with k ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.

3. At the end of the period, the firm observes ϵ and the aggregate shock.

4. The firm repays its debt and the fixed operating cost $(c + k - e)$, resulting in an end-of-period net worth q.

5. The firm decides on the amount of dividends to distribute $(q - e')$, observes the productivity shock θ', Z', and the process restarts from step 1.

4.1 Frictionless economy

In a frictionless economy, firms have the option to borrow an amount denoted as $c+k-e$ at the risk-free interest rate $r = \frac{1}{\beta} - 1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_{k} E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - (1 + r)(c + k - e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_{k} E \int [Z(\theta + \epsilon)k^{\alpha} - (1 + r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market frictions, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

4.2 Economy with Credit Market Frictions

After production, the firm privately observes the temporary shock ϵ, while financial intermediaries can only observe it at a cost of μk^{α}. For one-period debt contracts, financial intermediaries observe ϵ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth e, current productivity θ, and aggregate productivity value Z, all observable by both the financial intermediary and the firm at no additional cost.

HP1 (Hypothesis 1): The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\bar{\epsilon}$ is determined by the equation:

$$Z(\theta + \bar{\epsilon})k^{\alpha} + (1 - \delta)k = (1 + \widetilde{r})(c + k + e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component θ.

The financial intermediary lends $(c + k - e)$ to the firm only if the expected income from the loan equals the opportunity cost of the funds, as expressed by the inequality:

$$(1+\widetilde{r})(k+c+e)(1-\Phi(\bar{\epsilon}))+\int_{-\infty}^{\bar{\epsilon}}[Z(\theta+\bar{\epsilon})k^{\alpha}+(1-\delta)k-\mu k^{\alpha}]\,d\Phi(\epsilon) \geq (1+r)(c+k+e)$$

The financial contract is characterized by $(k, \bar{\epsilon})$. Given Z, θ, e, the participation constraint indicates the default threshold $\bar{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given θ, Z, there is a unique threshold $e_b(\theta, Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta + G(\bar{\epsilon}_b)]k^{\alpha} + (1 - \delta)k - uk_b^{\alpha}\Phi(\bar{\epsilon}_b) = (1 + r)(k_b + c - \underline{e_b})$$

where $\bar{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta + \overline{\epsilon})k^{\alpha} + (1-\delta)k - (1+\widetilde{r})(k + c - e) & \text{if } \epsilon \geq \overline{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^{\alpha}(\epsilon - \overline{\epsilon}); 0]$$

4.3 The firm's problem

Define V as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e, \theta, Z) = \max_{(k, \overline{\epsilon})} E \left\{ \int I(q)q + (1 - I(q)) \max[q, \max_{e'} q - e' + \beta V(e', \theta', \zeta')] \, d\Phi(\epsilon) \right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1.

$$Z[\theta + G(\overline{\epsilon}_b)]k^{\alpha} + (1-\delta)k - uk_b^{\alpha}\Phi(\overline{\epsilon}_b) \geq (1+r)(k_b + c - \underline{e_b})$$

2.

$$q = \max[Zk^{\alpha}(\epsilon - \bar{\epsilon}); 0]$$

3.

$$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.

5 The cleansing effect by Caballero

5.1 Introduction

In the first paper that rationalize the cleansing effect of recessions, authored by Ricardo J. Caballero and Mohamad L. Hammour Caballero and Hammour [1994] and published in the American Economic Review in 1998, the primary aim was to investigate how industries respond to cyclical variations in demand. They did this by employing a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations in two ways: by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model they used incorporated heterogeneous firms, where production units embodied the most advanced technology at the time of their creation. The costs associated with creating new units slowed down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a reduction in demand can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour's research Blanchard et al. [1990] offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard and Diamond Blanchard et al. [1990], as well as Steven Davis and John Haltiwanger Davis and Haltiwanger [1992], in their respective works from 1990. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study Davis and Haltiwanger [1992], where they assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986, it is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry. The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

5.2 Theoretical model

The model in question is a vintage model that simulates an industry experiencing exogenous technological progress. Within this model, production units are constructed using a fixed proportion of labor and capital, and they are continually being created and phased out. Notice that only the creation of new production units incurs a cost. This simplification is plausible, particularly in the context of the United States, where the expense associated with hiring is typically higher than the cost of termination, as demonstrated by Abdulkadiroğlu and Kranton (2003) Abowd and Kramarz [2003].

In this model, when a production unit is created at a specific time t_0, it embodies the most advanced technology available at that moment and consistently generates a uniform output represented by $A(t_0)$ throughout its operational lifetime. The productivity of this technology, denoted as $A(t)$, experiences continuous growth at an exogenously determined constant rate $\delta \geq 0$. This growth in technology can be interpreted in two ways: either as the adoption of new technology or as a product innovation. In the latter scenario, a continuum of perfectly substitutable products can yield varying levels of output.

$$[f(a,t) \qquad 0 \leq a \leq \overline{a}(t)]$$

The above function represents the cross-sectional density of the production units aged a at time t, where $\overline{a}(t)$ is the age of the oldest production unit at time t. The first assumption is that $f(a,t)$ and $\overline{a}(t)$ are continuous functions. The mass of production units at time t is given by:

$$N(t) = \int_{\overline{a}(t)}^{0} f(a,t)da$$

$N(t)$ is a measure of either the industry's capital stock and its employment, due to a fixed share of capital and labor. Thus, the industry's output is given by:

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

The deterioration of production units involves both an exogenous depreciation rate δ and an endogenous destruction process, which impacts $f(a,t)$ at its limits. The count of production units surviving for a years is expressed as:

$$f(a,t) = f(0,t-a)e^{-\delta a} \quad \text{where } 0 < a \leq \overline{a}(t)$$

The production flow is determined by:

$$\dot{N}(t) = f(0,t)[1 - \dot{\overline{a}}(t)] + \delta N(t)$$

Here, the first term represents the production rate, while the second term encapsulates the destruction rate, encompassing the obsolescence rate $f(\overline{a})(t)$, the technological obsolescence change over time $-f(\overline{a})(t)\dot{\overline{a}}(t)$, and the depreciation rate $\delta N(t)$.

The assumptions made by the authors are denoted as $\forall t \mid f(0,t) > 0 \cup \dot{\overline{a}}(t) <$.

The alteration in output concerning these flows is articulated as:

$$\dot{Q}(t) = A(t)f(0,t) - A(t-\overline{a}(t))f(\overline{a}(t),t) \cdot [1-\dot{\overline{a}}(t)] + \delta Q(t)$$

The authors define a perfectly competitive industry in partial equilibrium, where supply is dictated by free entry and perfect equilibrium. Additionally, they introduce a cost function related to creating new production units:

$$c = c\left(f\left(f(0,t)\right)\right) \quad \text{where } c(\cdot) > 0,\, c'(\cdot) \leq 0$$

This cost function is contingent on the creation rate, implying that higher creation rates correspond to increased costs. The equilibrium condition is established by equating the cost of unit creation to the present discounted value of profits throughout its lifespan. The authors set the cost of a production unit to 1, and $P(t)$ is the price of a unit of output. Thus, the profits generated at time t by a production unit aged a are defined as:

$$\pi(a,t) = P(t)A(t-a) - 1$$

$$\overline{a}[t+T(t)] = T(t)$$

Here, $T(t)$ signifies the maximum lfetime of a unit created at t. At any given time t, the free entry condition is expressed as:

$$c(f(0,t)) = \int_{t+T(t)}^{t} \pi(s-t,t)e^{-(r+\delta)(s-t)\,ds}$$

In the above equation, where $r > 0$ represents the exogenously determined instantaneous interest rate, the determination of the exit of a production unit is contingent upon continuous $P(t)$ and the instance when the profits generated by a unit being destroyed reach zero. This occurrence signifies the moment when the oldest unit operational at time t, denoted as $\overline{a(t)}$, must adhere to the equation:

$$P(t)A(t - \overline{a}(t)) = 1$$

The authors posit that $P(t)$ exhibits a decreasing trend due to the model's assumption regarding endogenous destruction, specifically $\overline{\dot{a}(t)} < 1$. To see, differentiate

$$\dot{P}(t) = -\gamma \left[1 - \dot{\overline{a}} P(t)\right]$$

Consequently, when the profits of a production unit diminish to zero for the first time, it will be subject to destruction.

On the demand side, the authors assume a unit-elastic demand function and consider the aggregate expenditure as exogenous $\overline{D}(t) = P(t)Q(t)$. The equilibrium is a path $\{f(0,t), \overline{a}(t), T(t), Q(t)\}_{t \geq 0}$ that satisfy the following conditions:

1. $Q(t) = \displaystyle\int_{\overline{a}(t)}^{0} A(t-a) f(a,t) da$
2. $f(a,t) = f(0, t-a)e^{-\delta a}$
3. $T(t) = \overline{a}\,(t + T(t))$
4. $c(f(0,t)) = \displaystyle\int_{t}^{t+T(t)} \left[P(s)A(t) - 1\right] e^{-(r+\delta)(s-t)}\, ds$
5. $P(t)A(t - \overline{a}(t) = 1)$
6. $P(t)Q(t) = \overline{D}(t)$

The first three equations (1, 2, 3) and the fifth one (5) suffice to delineate the trajectories of $T(t)$, $P(t)$, and $Q(t)$, which are determined by $\{f(0,t), \overline{a}(t)\}$. To affirm the robustness of the conditions expressed in equations 6 and 5, it is possible to derive these equations as first-order conditions for the maximization of a number of perfectly competitive firms holding production units.

To comprehend the functioning of endogenous destruction, let's consider a scenario with constant demand. In this case, both the destruction and creation rates change only due to supply factors. This steady state is characterized by a constant lifetime of production units $T(t) = \overline{a}(t) = \overline{a}^*$, resulting in a time-invariant age distribution $f(a,t) = f^*(a)$. Equation 5 implies that the price $P(t)$ must consistently decrease at a rate σ. Higher innovation rates lead to increased productivity, raising the supply and consequently lowering the price. Equation 2 reveals that the distribution of production units in the steady state follows a truncated exponential distribution:

$$f^*(a) = f^*(0)e^{-\delta a} \quad 0 \le a \le \overline{a}^*$$

Using free entry conditions (4) and the clearing condition (6), one can determine the creation and destruction ages $f^*(0)$ and $\overline{a}^*$. Equations 1 and 5 yield the cost function and productivity of a new production unit:

$$c(f^*(0)) = \frac{e^{\gamma \overline{a}^*} - e^{-(r+\delta)\overline{a}^*}}{\gamma + r + \sigma} - \frac{1 - e^{-(r+\delta)\overline{a}^*}}{r + \delta}$$

$$f(0) = \frac{(\sigma + \delta)\overline{D}^*}{e^{\sigma \overline{a}^* - e^{\delta \overline{a}^*}}}$$

The authors then normalize the creation rate:

$$N = f^*(0) \cdot (1 - e^{\delta \overline{a}^*})$$

In the steady state, this is given by:

$$(9) CC^* = \frac{\delta}{1 - e^{-\delta \overline{a}^*}}$$

Considering a special case where the creation cost is a constant c, i.e., $c(f^*(0)) = c$, substituting into equation 5.2 allows retrieval of $\overline{a}^*$. The effect of technological rate σ on $\overline{a}^*$ is decreasing, as a higher innovation rate increases the opportunity cost of delayed renovation, while a higher cost of creating new units lowers the renovation rate. Optimal lifetime of production units increases with higher r and δ as it becomes harder to recover creation costs.

Now, dropping the assumption of constant demand, we examine how the industry adjusts to demand fluctuations. Two ways are identified in which the industry adapts production to meet demand: by reducing the rate of creation $f(0, t)$ and by increasing the rate of endogenous destruction $f(\overline{a}(t), t) \cdot [1 - \dot{\overline{a}}(t)]$, thus reducing $\overline{a}$, the age at which units are demolished.

These two adjustments interact, leading to a reduction in demand causing the most outdated units to be scrapped, rendering them unprofitable. However, if the recession is partially accommodated by a reduction in the creation rate, the effect on the destruction margin is diminished. The authors argue that the extent to which creation will "insulate" existing units from variations in demand depends on the marginal cost of creating new units $c' f(0, t)$. When the marginal cost of creation is zero, demand fluctuations are entirely adjusted by the creation margin. This is exemplified in the case where $c(f(0, t)) = c$. In such instances, the insulation effect is complete, as there is no need to retire older units. Lowering $f(0, t)$ is sufficient, and it is cheaper than reducing the life of existing production units.

The insulation effect is not solely due to asymmetric adjustment costs on the creation and destruction margins. Complete insulation would occur even with linear adjusting costs. The creation rate in the case of constant creation cost is given by:

$$f(0,t) = \frac{\dot{\bar{D}}(t) + \delta \bar{D}(t) + P(t)A(t-\bar{a}(t))f(\bar{a}(t),t)[1-\dot{\bar{a}}(t)] - \dot{P}(t)Q(t)}{P(t)A(t)}$$

In the attained equilibrium, variations in demand are entirely offset by adjustments at the creation margin denoted as $f(0,t)$, with $\overline{a}(t)$ remaining steady at the destruction margin. The creation process effectively counteracts the impact of demand fluctuations on the price $P(t)$, effectively shielding existing units from demand changes. The price $P(t)$ experiences a constant decline at a rate represented by σ, reflecting the pace of technical progress. This consistent decline in $P(t)$ serves as a clear signal for production units to function optimally throughout their constant lifetime $\overline{a}(t)^*$.

In the aforementioned scenario, the destruction rate is not constant, but it does not respond to demand through variations in the age $\overline{a}(t)^*$ at which units are destroyed. Instead, variations in the creation rates have an impact on the number of units that reach obsolescence. If fewer units are created, fewer units become obsolete after $\overline{a}(t)^*$ periods. It is noteworthy that any modification leaving equations 3 to 5 independent of $\overline{D}(t)$ and $f(0,t)$ does not alter the full-insulation results.

Interestingly, assumptions such as perfect competition, industry-wide return to scale, and perfect foresight are not necessary for these conclusions. The latter is particularly noteworthy as it asserts that fully accommodating demand on the creation side only requires knowledge of current conditions. As long as the non-negativity constraint on $f(0,t)$ is never binding, implementing equilibrium behaviors does not necessitate expectations of future demand.

5.3 Application of the model

The model undergoes calibration utilizing Job-flow data and Industry production data. The former facilitates the replication of job creation dynamics, while the latter is employed to mimic the behaviors of firm creation and destruction in the manufacturing industry. To capture these dynamics, the marginal cost of creating new production units is stipulated as positive $c'f(0,t)$. This allows for a partial insulation effect, and the destruction margin responds to demand fluctuations. However, introducing a dependency of c on $f(0,t)$ compromises the analytical tractability of the system (Equations 1 - 6). Consequently, the authors resort to methods such as multiple shooting to ascertain the optimal equilibrium and subsequently employ an iterative procedure to converge to the correct expected creation rate.

For numerical solutions, the authors adopt a linear formulation:

$$c(f(0,t)) = c_0 + c_1 f(0,t)$$

To gain a deeper understanding of how creation and destruction respond to demand, the authors simulate sinusoidal demand using the equation:

$$\overline{D}(t) = 1 + 0.07\sin(t)$$

The results are visualized in the image below, depicting the feedback of normalized creation and destruction (CC and DD) to changes in demand.

The plot clearly illustrates that the insulation effect is only partial; otherwise, DD would have remained flat, as in the case with $c(f(0,t) = c)$. From a mathematical perspective, destruction responds to demand as equations 3-5 are no longer independent of the path $f(0,t)$ and demand. From an economic standpoint, increasing creation costs smoothen the creation process. In scenarios with a nearly flat innovation rate,

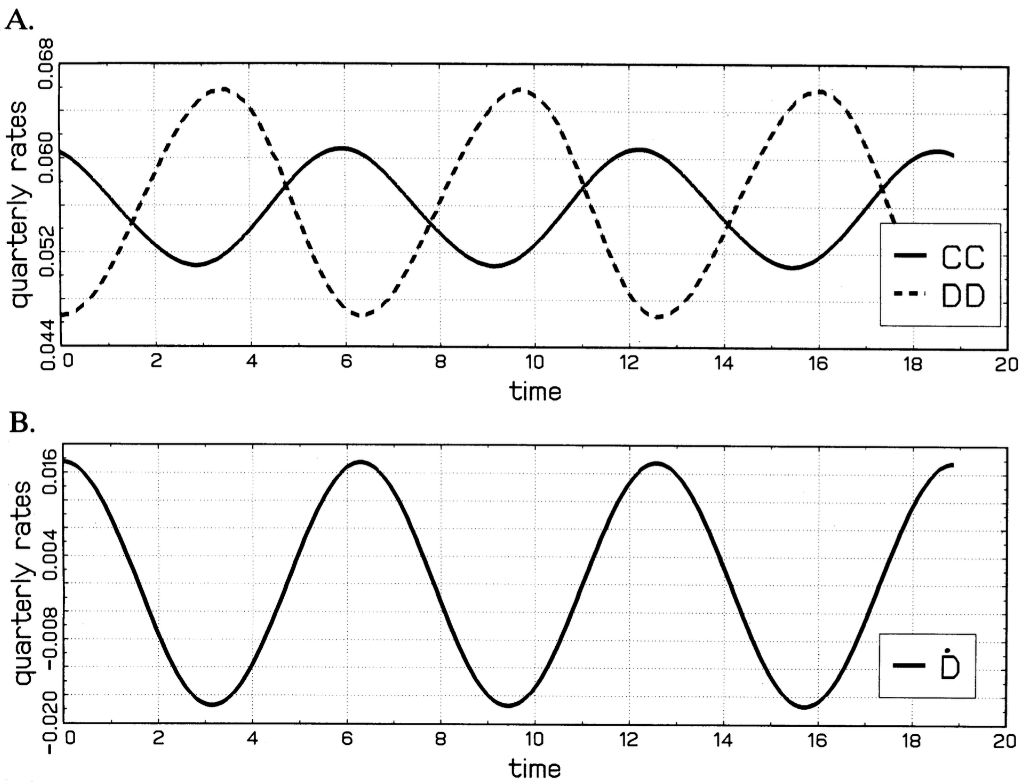

FIGURE 2. A) CREATION AND DESTRUCTION ($c_0 = 0.3$, $c_1 = 1.0$); B) CHANGE IN DEMAND (SYMMETRIC)

Figure 2: Figure 1. A Creation and destruction $c_0 = 0.3, c_1 = 1$ B Change in demand (Symmetric)

firms during crises cannot fully accommodate lower demand, nullifying the adoption of new production units, as the marginal costs would exceed the reduction in existing production units.

In the considered model, production units integrate labor and capital in fixed proportions to generate output. Each unit can be conceptualized as contributing to job creation within the industry, and job-flow data serves as a metric for quantifying the flows of production units.

Datasets that closely align with the theoretical CC and DD series have been compiled by Davis and Haltiwanger Davis and Haltiwanger [1990, 1992] and Blanchard and Diamond Blanchard et al. [1990], drawing from various sources. The primary focus lies on the dataset curated by Davis and Haltiwanger, who leverage the Longitudinal Research Database to construct quarterly series for U.S. manufacturing plants spanning the period 1972:2-1986:4.

In their empirical approach, Davis and Haltiwanger utilize output to empirically determine demand, employing the growth rate of the industrial production index as a proxy for output growth. Notably, in the foundational theoretical model, $Q(r)$ is smoothed by price movement, with the elasticity of demand determining the extent of smoothing, assumed to be equal to 1. While the theoretical model maintains a constant consumption-wage, the authors acknowledge that considering a procyclical consumption-wage, as in the case of general equilibrium with correlated industry shocks, may dampen the effect of demand shocks. However, they assert that this adjustment would alter only the magnitude, not the direction, of the analysis.

The figure below illustrates the data that the model seeks to replicate, showcasing job creation, job destruction, and growth.

To discern the characteristics of the series, the authors perform regression analysis on sectoral rates of job creation and job destruction against leads and lags of the cor-

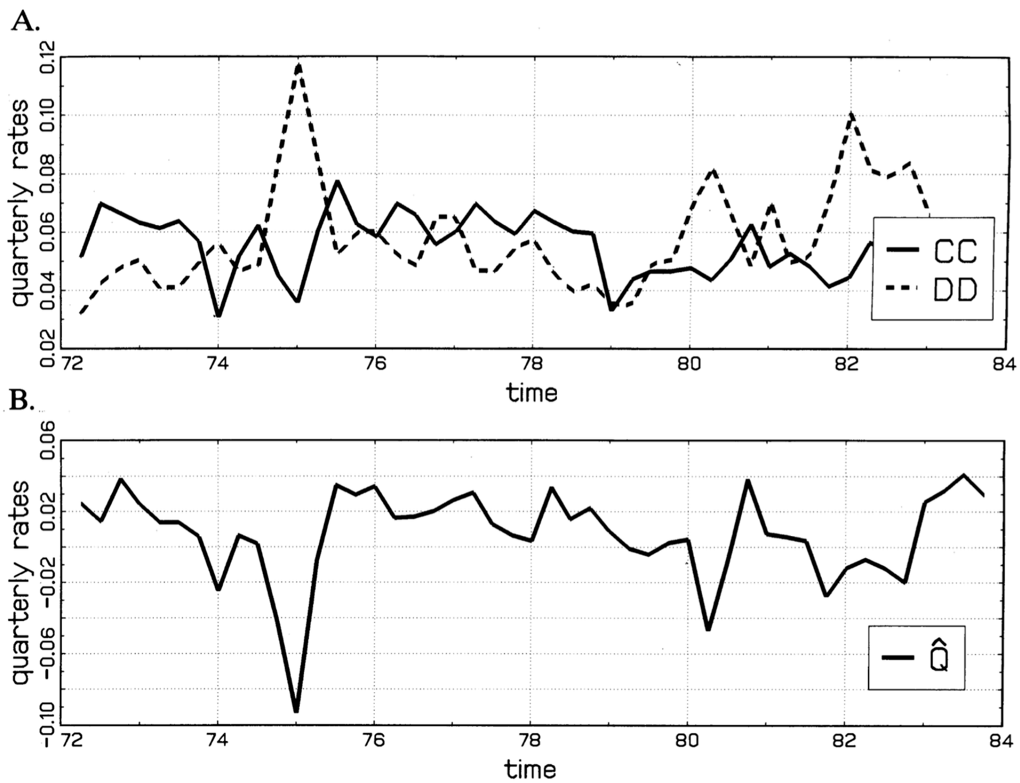

Figure 3: Figure 1. Job creation and job destruction in U.S. Manufacturing B Index of the industrial production

responding rates of growth. They find that job creation is less responsive to demand fluctuations, while job destruction exhibits a more countercyclical behavior. The initial finding indicates that the rate of job destruction displays greater responsiveness to changes in sectoral activity compared to the rate of job creation. Specifically, the sums of coefficients are -0.384 for job destruction and 0.218 for job creation showed in the table 4, the same results as in Davis and Haltiwanger [1990, 1992] and in Blanchard et al. [1990]. The authors capitalize on a natural experiment rooted in the intrinsic asymmetric characteristics of business cycles. Recessions, marked by brevity but intense contractions, provide the backdrop for the authors' model. This model endeavors to emulate the creation rate while concurrently mitigating the impact of asymmetric cyclicality inherent in business cycles. The empirical evidence supporting this model's behavior is encapsulated in Table 4, wherein two distinct scenarios are explored: output growth trajectories above Q^+ and below Q^-, relative to their respective means. The table meticulously delineates how creation and destruction rates respond to these deviations in output growth.

The salient observation emerges regarding creation rates, elucidating that they exhibit a more rapid and robust response in instances of vigorous output growth, as opposed to scenarios where the output growth rate experiences a reduction. On a contrasting note, the destruction margin, in line with the model's projections, manifests heightened sensitivity to a decline in output. This responsiveness is particularly pronounced from one quarter before the onset of the shock to one quarter after. Notably, during expansionary phases, the mean response of the destruction margin is -0.066, a notably milder reaction compared to the recessionary case where the mean response stands at -0.634.

These empirical outcomes seamlessly align with the predictions of the model. Specifically, the creation rate exhibits heightened responsiveness during expansionary phases,

Regressor	Timing	Creation: Coefficient	Creation: Standard deviation	Destruction: Coefficient	Destruction: Standard deviation
$\hat{Q}$	2 leads	0.029	0.006	0.030	0.010
	1 lead	0.065	0.007	−0.068	0.010
	contemporaneous	0.108	0.007	−0.185	0.010
	1 lag	0.013	0.007	−0.103	0.010
	2 lags	0.003	0.006	−0.058	0.010
	Sum:	0.218	0.013	−0.384	0.017
$\hat{Q}^+$	2 leads	0.052	0.012	0.012	0.016
	1 lead	0.102	0.012	0.002	0.016
	contemporaneous	0.131	0.012	−0.065	0.016
	1 lag	0.059	0.012	−0.025	0.016
	2 lags	0.055	0.012	−0.008	0.016
	Sum:	0.399	0.026	−0.066	0.023
$\hat{Q}^-$	2 leads	0.002	0.010	0.006	0.014
	1 lead	0.022	0.011	−0.149	0.014
	contemporaneous	0.093	0.012	−0.293	0.015
	1 lag	−0.012	0.012	−0.139	0.015
	2 lags	−0.021	0.012	−0.059	0.015
	Sum:	0.084	0.020	−0.634	0.024

Figure 4: Table 2.1. Job Creation and Job Destruction in U.S. Manufacturing Response to Output Growth

Notes: The table presents the reaction of job creation to the growth rate of the industrial production index. The latter is categorized into values above and below its mean (Q). The table encompasses quarterly observations for the two-digit SIC industries during the period 1972:2-1986:4. The coefficients are uniformly constrained to be equal across all sectors, with the exception of a constant (not shown).

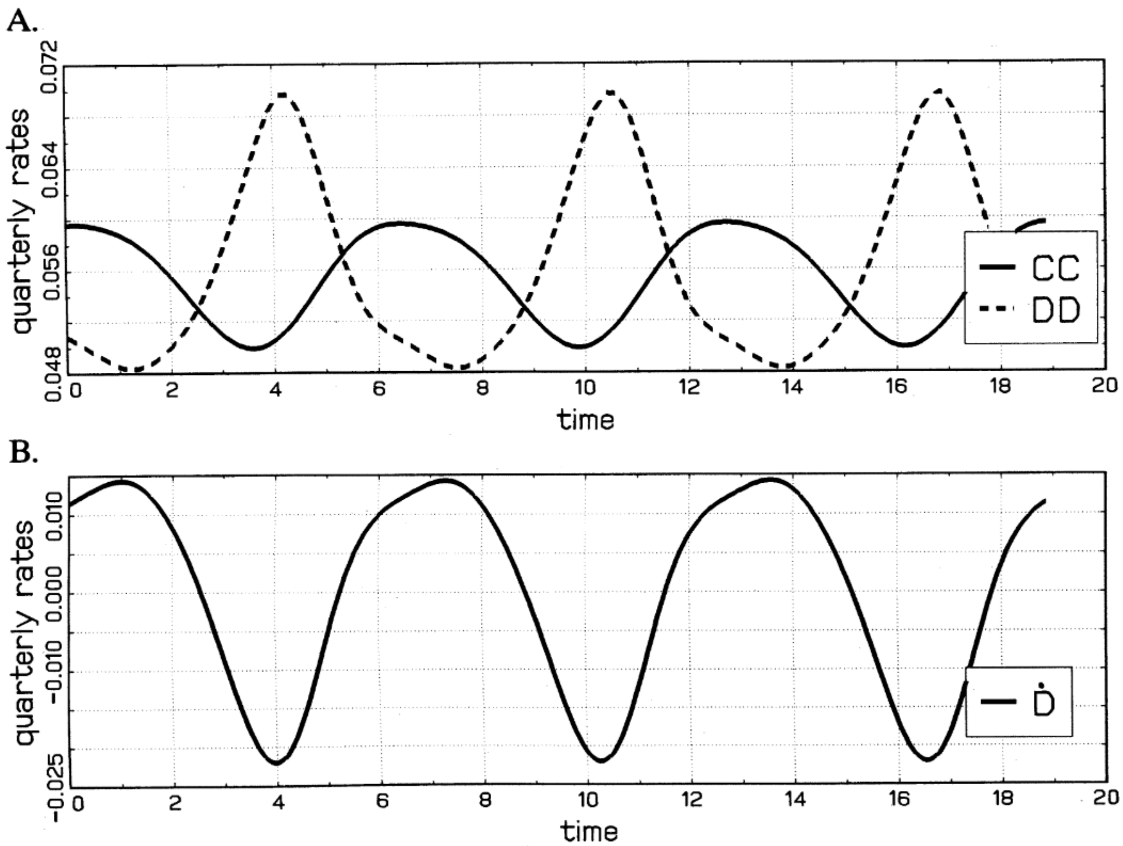

Figure 5: A. Creation and Destruction B. Output Growth
Notes: The figure depicts a simulation of asymmetrical supply growth.

given their cyclical and symmetric nature. In contrast, the asymmetric and non-cyclical nature of recessions triggers a more substantial decline in the production unit rate, in line with the model's expectations.

In oder to better understand the asymmetrical behavior the authors simulate an asymmetrical demand function:

$$\cdot\overline{D}(t) = 0.05[\cos(t) + \sin(t)] - 0.016\sin(2t) - 0.003\cos(3t)$$

$$\overline{D}(t) = 1 \quad r = 0.065, \delta = 0.15, \gamma = 0.028, c_0 = 0.3, c_1 = 1.0$$

The results are depicted in 5

From the plot 5, its evident that firms use prediction in demand to smooth job creation in order to avoid big change, since they are too costly,by avaraging the demand over the lifetime of a production unit ove. On the other hand, destruction depends only on current conditions, thus responding only to significant deviations from the demand prediction . It can be better undestand thinking about a case in which creation rates respond only mildy to a sharp deacrease in demand, the equilibrium price falls leading to additional distruction, since older units' profits go to 0. Indeed, destruction not only preserves , but amplifies the assymetry of demand.

6 Frictionless economy

The authors culminate their study with a compelling calibration exercise using manufacturing series to exploit the model. This entails dissecting the observed net change in employment into destruction and creation rates, as well as applying the same approach to output production. The model is simulated for the duration of 1972:2-1983:4, with parameters as follows:

Table 2.1 - Calibrated Parameters

Variable	Symbol	Value
Interest rate	r	0.065
Depreciation rate	δ	0.150
Rate of technical progress	γ	0.028
Adjustment cost parameters	c_0	0.403
	c_1	0.500

(3)

The technical progress is selected to attribute all the growth in employment and manufacturing to technological advancements, setting λ as 2.8. The authors employ

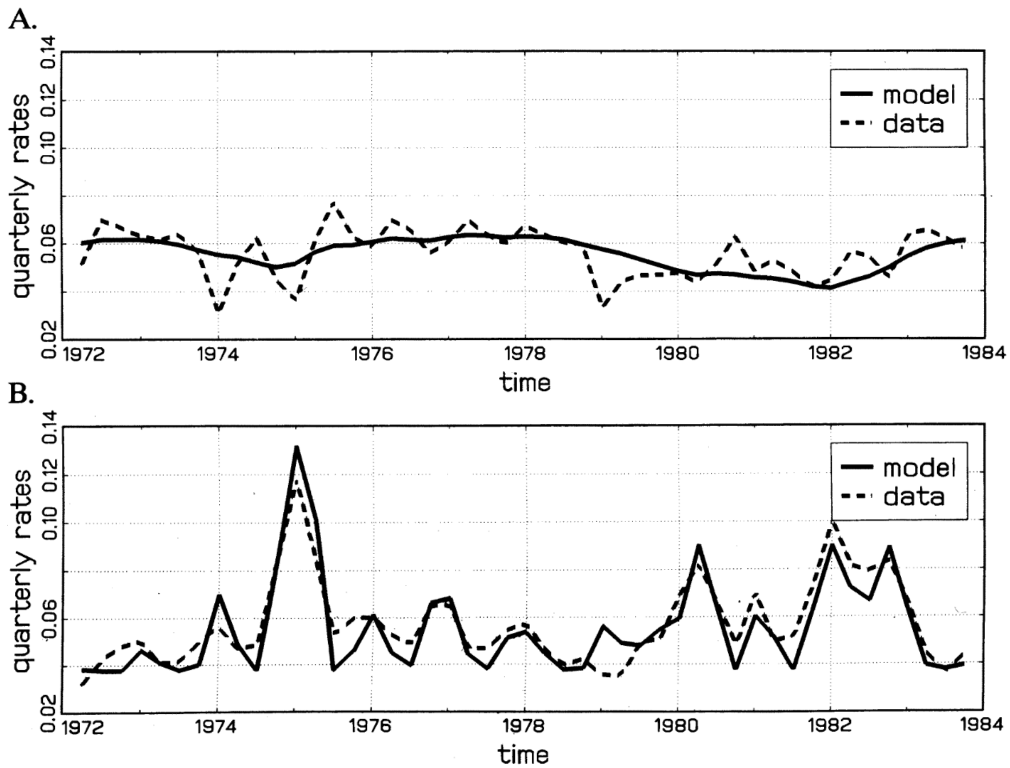

Figure 6: Figure 1. A employment driven job creation $c_0 = 0.403, c_1 = 0.5$ B Employment job destruction $c_0 = 0.403, c_1 = 0.5$

Equation 5.2, linking the steady state to the lifetime of jobs and job turnover (CC^*), determining $\overline{a}^* + 7.42$ years. Utilizing this information, they ascertain the steady state entry cost to be 0.525, equivalent to half a year's operating costs for production units. Subsequently, they employ ordinary least squares (OLS) to retrieve the value of c_1, the marginal cost of creating a new unit, which is found to be 0.5. This aligns with a small elasticity for the creation cost function, signifying the vulnerability of the insulation mechanism to breakdown. The outcomes stemming from the simulations driven by employment and output are disclosed and contrasted with the data in Figure 6. Notably, the simulation of job creation displays a level of smoothness that diverges from the observed data, with this discrepancy being attributed, in part, to the inherent absence of uncertainty in our model. Despite this, the model effectively elucidates the relative volatility discernible in the patterns of job creation and destruction. Moreover, it successfully captures the greater symmetry observed in the former, offering insights into the nuanced dynamics at play in employment and output fluctuations.

The model provides intriguing insights as it elucidates certain empirical findings found in Davis and Haltiwanger [1990, 1992]. Specifically, it delves into the dynamics of how the response of the creation margin contributes to an insulating effect on the destruction margin. The model's salient features lie in its incorporation of heterogeneity across production units and their turnover, rendering it a meaningful baseline for comprehending how the cleansing effect influences the distribution of production units.

However, it's essential to note that the model, in its current formulation, does not account for the potential impact of financial frictions arising from asymmetric information between borrowers and lenders. Such frictions could conceivably influence both the destruction and creation margins, introducing a layer of complexity not considered in the current framework.

An alternative perspective on recessions is captured by the concept of a "pit-stop," where a recession is characterized as a period during which improvement investments in production are undertaken due to temporarily low opportunity costs, as posited by Davis and Haltiwanger [1990]. This viewpoint adds nuance to the understanding of recessions, emphasizing them as periods conducive to strategic investments.

One potential objection to the notion that recessions are times of cleansing is rooted in the implication of countercyclical productivity. Notably, labor productivity is often observed to be procyclical. However, this apparent inconsistency can be attributed to frictions, as suggested by Galí and Hammour [1992]. Their findings provide evidence supporting the notion that the cleansing effect enhances productivity in the long term, offering a nuanced perspective on the relationship between economic downturns and productivity dynamics.

A crucial observation in the aforementioned model is the authors' reliance on a constant marginal cost of creation. Yet, recent literature has raised concerns about the reliability of this assumption, especially for larger firms. The dynamics of the business environment in recent years suggest that significant firms tend to favor substantial adjustments, particularly in terms of downsizing.

Interestingly, this deviation from the constant marginal adjustment cost for bigger firms can be interpreted as a validation of the model's predictions. When firms opt not to fully insulate themselves from a decline in demand using the creation margin, they tend to respond with intense layoffs. This alignment between the model's predictions and the observed behavior of larger firms underlines the model's relevance and its capacity to capture real-world dynamics.

7 Solving

Given that the evolution of the net worth is the following:

$$e_{t+1} = Z(\theta + \epsilon)k_t^\alpha + (1-\delta)k_t - (1+r)(c + k_t - e_t)$$

The value of the firm at time t is given by:

$$V_t(e_t) = \max_{k_t} e_t + \beta V_{t+1}(e_{t+1})$$

Solving the Belman equation, the FOC are given by:

$$\frac{\vartheta V_t(e_t)}{\vartheta k_t} = \frac{\vartheta e_t}{\vartheta k_t} + \beta \frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta k_t}$$

Strong assumption a change in k_t does not have an impact on e_t only on e_{t+1}:

$$\frac{\vartheta e_t}{\vartheta k_t} = 0$$

Using the envelope theorem:

$$\frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta k_t} = \frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta e_{t+1}} \frac{\vartheta e_{t+1}}{\vartheta k_t}$$

$$\frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta e_{t+1}} = 1 + \beta(1+r)$$

$$\frac{\vartheta e_{t+1}}{\vartheta k_t} = Z(\theta+\epsilon)\alpha k_t^{\alpha-1} - (\delta + r)$$

Thus:

$$\frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta k_t} = [1+\beta(1+r)][Z(\theta+\epsilon)\alpha k_t^{\alpha-1} - (\delta+r)]$$

So the FOC became:

$$\frac{\vartheta V_t(e_t)}{\vartheta k_t} = 0 + \beta[1+\beta(1+r)][Z(\theta+\epsilon)\alpha k_t^{\alpha-1} - (\delta+r)] = 0$$

The optimal level of capital at time t is:

$$\widehat{k}_t = \frac{\beta(1+r)Z(\theta+\epsilon)\alpha}{\delta+r}^{\frac{1}{1-\alpha}}$$

Plotting the graph in (assuming $X = Z(\theta+\epsilon)$) and $y = \widehat{k}_t$

$$y = \frac{0.8x}{0.03+0.1}^{\frac{1}{1-0.8}}$$

Its interesting to see that if there is an increase in productivity the firm need more optimal capital K, while the most low productivity firms need less capital to operate

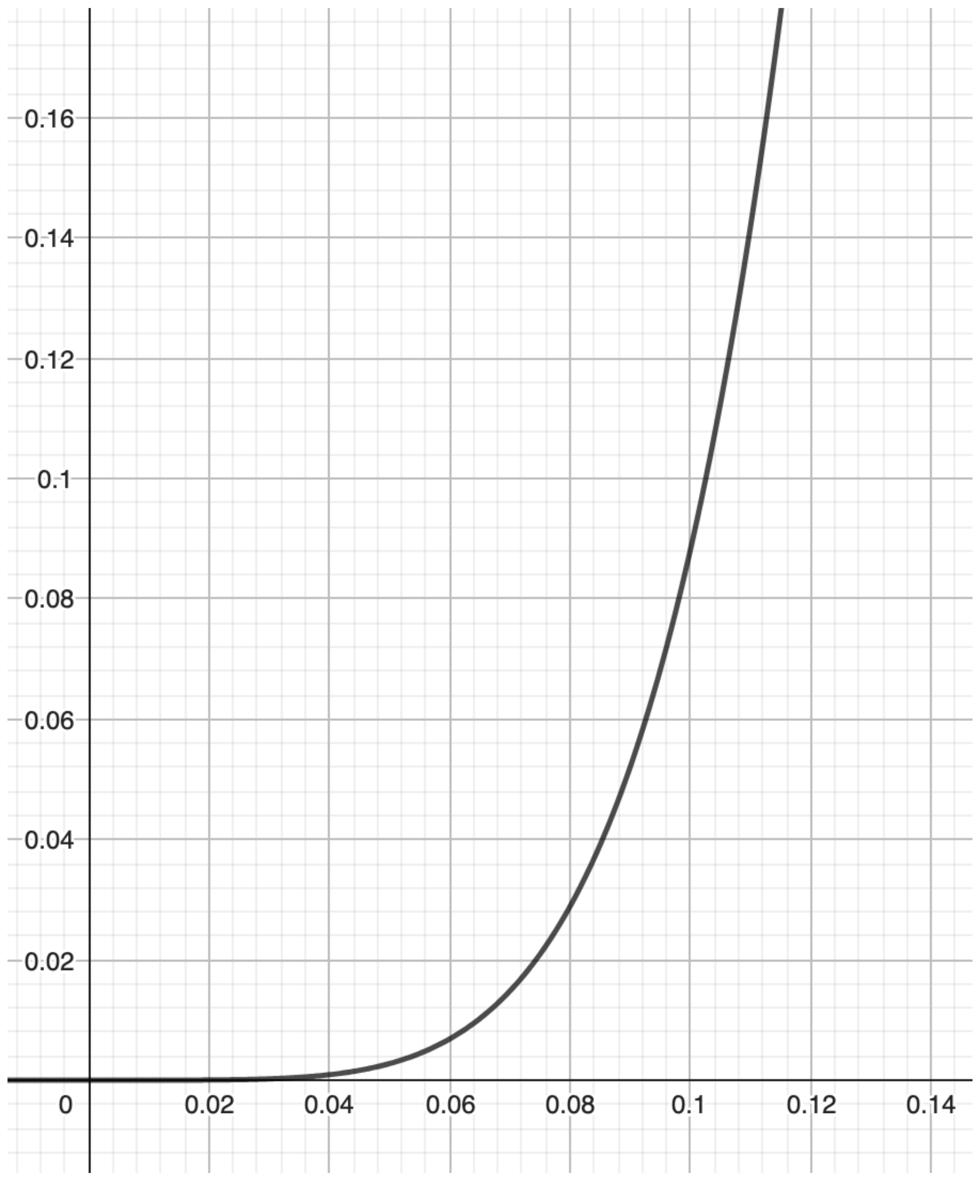

Figure 7: $X = Z(\theta + \epsilon)$ and $y = \widehat{k}_t$

7.1 Frictions

Now I consider the participation constraint of the borrower given that she could observe ϵ only paying μk^{α} so:

$$(1+r)(k+c+e)(1-\Phi(\bar{\epsilon}))+\int_{-\infty}^{\bar{\epsilon}}[Z(\theta+\bar{\epsilon})k^{\alpha}+(1-\delta)k-\mu k^{\alpha}]\,d\Phi(\epsilon)\geq(1+r)(c+k+e)$$

r is the rate of interest that makes equal the expected value of borrowing to the opportunity cost of capital. rewriting became

$$Z[\theta+G(\bar{\epsilon})]k_t^{\alpha}+(1-\delta)k_t-uk_t^{\alpha}\Phi(\bar{\epsilon})=(1+r)(k_t+c-e_t)$$

where

$$G(\bar{\epsilon})=(1-\Phi(\varepsilon)\bar{\varepsilon}+\int_{-\infty}^{\bar{\varepsilon}}\epsilon\,d\Phi(\epsilon))$$

While the firm participation constraint is $q_t \geq 0$ so the end-of-period net worth must be greater than 0, thus the problem of the firm becomes: The value of the firm at time t is given by:

$$V_t(e_t)=\max_{k_t} q_t-e_{t+1}+\beta V_{t+1}(e_{t+1})$$

$$s.t.$$

$$q_t=Z(\theta+\epsilon)k_t^{\alpha}+(1-\delta)k_t-(1+r)(c+k_t-e_t)$$

$$Z[\theta+G(\bar{\epsilon})]k_t^{\alpha}+(1-\delta)k_t-uk_t^{\alpha}\Phi(\bar{\epsilon})=(1+r)(k_t+c-e_t)$$

so we can rewrite the second constraint in order to get r:

$$r=\frac{Z[\theta+G(\bar{\epsilon})]k_t^{\alpha}+(1-\delta)k_t-uk_t^{\alpha}\Phi(\bar{\epsilon})}{k_t+c-e_t}-1$$

FOC:

$$\frac{\vartheta V_t(e_t)}{\vartheta k_t} = \frac{\vartheta q_t}{\vartheta k_t} - \frac{\vartheta e_t}{\vartheta k_t} + \beta \frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta k_t} = 0$$

by envelope theorem:

$$\frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta k_t} = \frac{\vartheta V_{t+1}(e_{t+1})}{\vartheta e_{t+1}} \frac{\vartheta e_{t+1}}{\vartheta k_t}$$

Strong HP:

$$\frac{\vartheta e_{t+1}}{\vartheta k_t} = 0$$

Qui non so se ha senso continuare perchè non ho nessun meccanismo che mi trasformi il net worth al tempo t in net worth al periodo t+1, almeno che non includa la defınzione di dividendo come $d_t = q_t - e_{t+1}$, allora in questo caso dovrei risolvere il problema con un langrangiana dato che la firm dovrebbe scegliere sia k che e net worth.

8 Redefining the problem

$$V(k_t) = \max_{k_{t+1}, e_{t+1}} d_t + \beta V(k_{t+1})$$

$$s.t.$$

$$f(k_t) = Z k_t^{\alpha}$$

$$f(k_t) = d_t + (c + k_{t-1} - e_{t-1})(1 + r) + k_t - (c + k_t - e_t) - k_{-1}(1 - \delta)$$

$$(1 + r)(c + k_t - e_t)p + (1 - p)f(k_t) = (1 + r_f)(c + k_t - e_t)$$

$$B_t = c + k_t - e_t; R = 1 + r; R_f = 1 + r_f;$$

$$R = \frac{R_f}{p} - \frac{1 - p}{p} \frac{f(k_t)}{D_t}$$

In order to understand the mechanism behind this optimization problem, I firstly solve the three times problem working backward. The value function in $t = 2$ is

$$V_{t+2} = \max d_{t+2}$$

Since there firm will not exists in t+2, there are no investiment $B_{t+2} = 0$, thus $0 = k_{t+2} + c - e_{t+2}$ as consequence $k_{t+2} = e_{t+2} - c$. Then we can rewrite the value function:

$$V_{t+2} = \max Z(e_{t+2}-c)^{\alpha}-(c+k_{t+1}-e_{t+1})(1+r_{t+1})-e_{t+2}+c+(c+e_{t+2}-c-e_{t+2})+k_{t+1}(1-\delta)$$

$$V_{t+2} = \max_{e_{t+2}} Z(e_{t+2} - c)^{\alpha} - B_{t+1}R_{l,t+1} - e_{t+2} + c + k_{t+1}(1 - \delta)$$

FOC:

$$\frac{\partial V_{t+2}}{\partial e_{t+2}} = Z\alpha(e_{t+2} - c)^{\alpha-1} - 1 = 0$$

$$(e_{t+2} - c)^{\alpha-1} = (Z\alpha)^{-1}$$

$$e_{t+2} = (Z\alpha)^{\frac{1}{1-\alpha}} + c$$

Thus:

$$d_{t+2} = Z\left[(Z\alpha)^{\frac{\alpha}{1-\alpha}}\right] - B_{t+1}R_{t+1} - \left[(Z\alpha)^{\frac{1}{1-\alpha}}\right] + k_{t+1}(1 - \delta)$$

$$V_{t+2} = Z\left[(Z\alpha)^{\frac{\alpha}{1-\alpha}}\right] - B_{t+1}R_{t+1} - \left[(Z\alpha)^{\frac{1}{1-\alpha}}\right] + k_{t+1}(1 - \delta)$$

Writing the problem in t+1:

$$V_{t+1} = \max_{e_{t+1},k_{t+1}} d_{t+1} + \beta V_{t+2}$$

$$d_{t+1} = Zk_{t+1}^{\alpha} - B_tR_L - k_{t+1} + B_{t+1} + k_t(1 - \delta)$$

FOCs:

$$\begin{cases} \frac{\partial V_{t+1}}{\partial e_{t+1}} = \frac{\partial d_{t+1}}{\partial e_{t+1}} + \beta \frac{\partial V_{t+2}}{\partial e_{t+1}} = 0 \\ \frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial d_{t+1}}{\partial k_{t+1}} + \beta \frac{\partial V_{t+2}}{\partial K_{t+1}} = 0 \end{cases} \tag{4}$$

solving $\frac{\partial d_{t+1}}{\partial e_{t+1}}$:

$$\frac{\partial d_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial B_{t+1}}{\partial e_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial e_{t+1}} = \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

$$\frac{\partial d_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

solving $\frac{\partial V_{t+2}}{\partial e_{t+1}}$:

$$\frac{\partial V_{t+2}}{\partial e_{t+1}} = -\left[\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right]$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \frac{\partial B_{t+1}}{\partial e_{t+1}}R_{t+1} + B_{t+1}\frac{\partial R_{t+1}}{\partial e_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial e_{t+1}} = \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1$$

$$\frac{\partial R_{t+1}}{\partial e_{t+1}} = -\frac{1-p}{p}\left\{\left[Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \delta\frac{\partial k_{t+1}}{\partial e_{t+1}}\right]B_{t+1} - \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[Zk_{t+1}^{\alpha} - \delta k_{t+1}\right]\right\}B_{t+1}^{-2}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right]R_{t+1} - \frac{1-p}{p}\left\{\left[Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \delta\frac{\partial k_{t+1}}{\partial e_{t+1}}\right]B_{t+1} - \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[Zk_{t+1}^{\alpha} - \delta k_{t+1}\right.$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)\left[\left(zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\frac{1-p}{p}B_{t+1}^{-1} + R_{t+1}\right] - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial e_{t+1}} = \left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial V_{t+2}}{\partial e_{t+1}} = -\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right]$$

Substituting into the first FOC, we get:

$$\frac{\partial V_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - \frac{\partial k_{t+1}}{\partial e_{t+1}} + \frac{\partial k_{t+1}}{\partial e_{t+1}} - 1 - \beta\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1 -\right.$$

second FOC:

solving $\frac{\partial d_{t+1}}{\partial k_{t+1}}$:

$$\frac{\partial d_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - 1 + \frac{\partial B_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial k_{t+1}} = 1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial d_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

solving $\frac{\partial V_{t+2}}{\partial k_{t+1}}$:

$$\frac{\partial V_{t+2}}{\partial k_{t+1}} = -\left[\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} - (1-\delta)\right]$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \frac{\partial B_{t+1}}{\partial k_{t+1}}R_{t+1} + B_{t+1}\frac{\partial R_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial B_{t+1}}{\partial k_{t+1}} = 1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}$$

$$\frac{\partial R_{t+1}}{\partial k_{t+1}} = -\frac{1-p}{p}\left[\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)B_{t+1} - \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\right]B_{t+1}^{-2}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)R_{t+1} + \left\{\frac{1-p}{p}\left[(Z\alpha k_{t+1}^{\alpha-1} - \delta)B_{t+1} - \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right)\right]B_{t+1}^{-1}\right\}$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+}}{\partial k_{t+1}}\right)\left[R_{t+1} + \frac{1-p}{p}\left(Zk_{t+1}^{\alpha} - \delta k_{t+1}\right)B_{t+1}^{-1}\right] - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial B_{t+1}R_{t+1}}{\partial k_{t+1}} = \left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right)$$

$$\frac{\partial V_{t+2}}{\partial k_{t+1}} = -\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right]$$

Substituting into the FOC:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}} - \beta\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right] = 0$$

thus the FOCs are:

$$\frac{\partial V_{t+1}}{\partial e_{t+1}} = Z\alpha k_{t+1}^{\alpha-1}\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1 - \beta\left[\left(\frac{\partial k_{t+1}}{\partial e_{t+1}} - 1\right)R_f - \frac{1-p}{p}\frac{\partial k_{t+1}}{\partial e_{t+1}}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - \frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\delta)\right] = 0$$

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = Z\alpha k_{t+1}^{\alpha-1} - \frac{\partial e_{t+1}}{\partial k_{t+1}} - \beta\left[\left(1 - \frac{\partial e_{t+1}}{\partial k_{t+1}}\right)R_f - \frac{1-p}{p}\left(Z\alpha k_{t+1}^{\alpha-1} - \delta\right) - (1-\delta)\right] = 0$$

rearranging $\frac{\partial V_{t+1}}{\partial k_{t+1}}$ to isolate k_{t+1}:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial e_{t+1}}{\partial k_{t+1}}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1}$$

rearranging $\frac{\partial V_{t+1}}{\partial e_{t+1}}$ to isolate k_{t+1}:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z\alpha}$$

Equating the two equations:

$$\left[\frac{\partial e_{t+1}}{\partial k_{t+1}}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} = \left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z}$$

From this equation, you can isolate $\frac{\partial e_{t+1}}{\partial k_{t+1}}$ to solve for it explicitly.

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = -\left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \beta(r_f + \delta) + \delta\frac{1-p}{p}\right]\frac{p}{Z\alpha}\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}(1-\beta R_f)^{-1} - \beta\left(r_f + \frac{\delta}{p}\right)$$

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = -\left[\frac{\partial k_{t+1}}{\partial e_{t+1}}(1-\beta R_f) + \delta\frac{1-p}{p}\right]\frac{\beta p + \beta - p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)(1-\beta R_f)^{-1}$$

Since $\frac{\partial e_{t+1}}{\partial k_{t+1}}$ is the reciprocal of $\frac{\partial k_{t+1}}{\partial e_{t+1}}$, we can compute the optimal path of the networth as a function of the capital. Defining $y = \frac{\partial e_{t+1}}{\partial k_{t+1}}$ and ths $\frac{y^{-1}=\partial e_{t+1}}{\partial k_{t+1}}$:

$$y = -\left[\frac{1}{y}(1-\beta R_f) + \delta\frac{1-p}{p}\right]\frac{\beta p + \beta - p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)(1-\beta R_f)^{-1}$$

To solve the given equation

$$y = -\left[\frac{1}{y}(1-\beta R_f) + \delta\frac{1-p}{p}\right]\frac{\beta p + \beta - p}{1-\beta R_f} - \beta\left(r_f + \frac{\delta}{p}\right)(1-\beta R_f)^{-1}$$

Use Python and the sympy library to solve the equation:

```python
from sympy import symbols, Eq, solve

# Define the symbols
y, beta, R_f, delta, p, r_f = symbols('y beta R_f delta p r_f')

# Define the equation
equation = Eq(y, - ((1/y) * (1 - beta * R_f) + delta * (1 - p) / p) * (
    beta * p + beta - p) / (1 - beta * R_f) - beta * (r_f + delta / p) /
    (1 - beta * R_f))

# Solve for y
solution = solve(equation, y)
```

The equation for y is given by:

$$y = \frac{\beta\delta \pm \sqrt{\Delta}}{2p(\beta R_f - 1)}$$

In the solutions provided, Δ represents the discriminant of the quadratic equation that was formed during the solution process. It is the expression under the square root in the solutions. The discriminant Δ in this case is a complex expression involving the variables R_f, β, p, δ, and r_f. Specifically, Δ is given by:

$$\Delta = (-\beta\delta - \beta p^2\delta\left(\frac{1}{p} - 1\right) + p^2\delta\left(\frac{1}{p} - 1\right) \tag{5}$$

$$- \beta p\delta\left(\frac{1}{p} - 1\right) - \beta p r_f)^2 \tag{6}$$

$$- 4(\beta p R_f - p)(\beta^2 p^2 R_f - \beta p^2 R_f + \beta^2 p R_f \tag{7}$$

$$- \beta p^2 + p^2 - \beta p) \tag{8}$$

The discriminant Δ is given by:

$$\Delta = [\text{complex expression involving } R_f,\ \beta,\ p,\ \delta, \text{ and } r_f] \tag{9}$$

The sign of the solutions depends on:

- The values of β, δ, p, R_f, and r_f.
- The value and sign of Δ.

Since Δ involves these parameters in a complex manner, the sign of the solutions can be:

- Real and positive, real and negative, or complex (depending on the sign and magnitude of Δ and other parameters).
- Determined specifically only when actual values for the parameters are provided.

Since now we have a partial derivative as a function of parametrs we can retrive the relation between equity and capital. Rewriting the solutions as:

$$y = \frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D}$$

Given the partial derivative:

$$\frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D} \tag{10}$$

where N and D are constants with respect to k, we want to integrate this with respect to k.

Since N and D do not depend on k, the integral is straightforward:

$$\int \frac{N}{D}\, dk \tag{11}$$

Integrating a constant with respect to k yields:

$$\int \frac{N}{D}\, dk = \frac{N}{D} \cdot k + C \tag{12}$$

where C is the constant of integration.

Thus, the set of possible solutions is:

$$e_{t+1} = \frac{N}{D} \cdot k_{t+1} + C \tag{13}$$

where C is determined based on boundary conditions or initial values. Given the relationship:

$$k_{t+1}^{\alpha-1} = \left[\frac{\partial e_{t+1}}{\partial k_{t+1}}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} \tag{14}$$

we can retrieve k_{t+1}.

Substituting $\frac{\partial e_{t+1}}{\partial k_{t+1}} = \frac{N}{D}$ into the equation, we get:

$$k_{t+1}^{\alpha-1} = \left[\frac{N}{D}(1-\beta R_f) + \beta\left(r_f + \frac{\delta}{p}\right)\right]\left\{Z\alpha\left[(1-\beta) - \frac{\beta}{p}\right]\right\}^{-1} \tag{15}$$

Sine now we have the optimal k we can retrieve the optimal path of networth:

$$e_{t+1} = \frac{N}{D} \cdot \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + C$$

Thus the debt at time t+1 is:

$$\begin{aligned} B_{t+1} = & \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + c \\ & - \left\{ \frac{N}{D} \cdot \left[\frac{N}{D} (1 - \beta R_f) + \beta \left(r_f + \frac{\delta}{p} \right) \right] \left\{ Z\alpha \left[(1 - \beta) - \frac{\beta}{p} \right] \right\}^{-1} + C \right\} \end{aligned} \tag{16}$$