# Theoretical framework

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## Abstract

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

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# Chapter 1

# Introduction

In macroeconomic theory, the investigation of business cycles and long-term growth trajectories traditionally unfolds in distinct academic silos, drawing a parallel to the distinct realms of quantum mechanics and Einstein's theory of relativity in physics. This academic segregation, however, obscures a fundamental and profound question: How do business cycles influence long-term economic growth? The exploration of this question is more than an academic exercise; it underpins the practical understanding of short-term economic policies, such as automatic stabilizers, and their profound long-term impacts on the economy.

Embarking on this exploration, my research primarily dwells in the realm of theory, supplemented by rigorous simulation and calibration exercises. The intricate complexity of business cycles, particularly evident during periods of economic downturn and recovery, challenges empirical approaches due to the plethora of confounding variables. Thus, a theoretical lens, rather than a purely empirical one, is employed to dissect and understand these phenomena.

Central to this theoretical framework is an examination of the role of financial market frictions during economic recessions. A key inquiry here is the investigation of policy interventions, such as demand stabilizers, and their potential effect in attenuating the 'cleansing effect' of recessions. This exploration is pivotal in understanding whether such policies might inadvertently lead to a reduced economic baseline or steady state in the long term.

The conceptual foundation of this investigation is inspired by an ecological analogy the cyclical dynamics observed between predator and prey populations in nature. This natural cycle, when observed over extended periods, reveals not just self-contained oscillations but also underlying trends of population growth for both predators and prey. This observation leads to a compelling analogy for economic cycles: while they appear as short-term fluctuations, they might be underpinned by long-term growth trajectories.

In natural ecosystems, interventions aimed at stabilizing these cycles such as protecting prey during times of increased predation might seem beneficial in the short term. However, such interventions often neglect the critical and natural process of selection. This interference disrupts evolutionary mechanisms, potentially leading to unforeseen consequences over time, such as the propagation of traits detrimental to the species' survival and adaptability in changing environments.

My thesis extends this analogy to the economic sphere, positing a similar selective mechanism at play in economic systems. The primary focus is on the recession's cleansing effect, which might be analogous to natural selection in ecology. This effect could potentially 'weed out' less productive firms, leaving a market landscape dominated by more efficient players. The exploration aims to decipher whether such an economic 'natural selection' mechanism exists and, if so, how it shapes the fabric of productivity, innovation, and growth in the long term. Through this lens, the research endeavors to contribute a nuanced understanding of the intricate interplay between short-term economic fluctuations and long-term economic evolution, offering insights into the design and implications of economic policies. In the following sections, we will delve deeper into specific theories that bridge the gap between business cycles and long-term economic growth. However, it is beneficial first to embark on a brief historical journey through the evolution of thought regarding business cycles, to understand the context and development of these interconnected economic theories. This exploration will provide a foundation for appreciating the diversity of perspectives and the progression of ideas that have shaped our understanding of the intricate relationship between short-term economic fluctuations and long-term growth trajectories.

# Chapter 2

# Literature review

## 2.1 Business cycle history

### 2.1.1 Theories Connecting Business Cycles to Long-Term Growth

In the domain of economic theory, the relationship between business cycles (BC) and long-term growth is dissected into two principal schools of thought. The conventional viewpoint suggests that long-term growth is chiefly propelled by technological progress. Within this framework, technological advancements are often viewed as exogenous—arising outside the economic model's explanatory scope, as highlighted in the seminal contributions of Solow [1956] and Swan [1956]. This perspective treats technological progress as an independent variable that exerts influence on economic growth without being influenced by the economy's internal dynamics.

Contrastingly, an alternative strand of theoretical work aims to endogenize technological progress, weaving it into the fabric of the economic process. These models embed factors such as incentives for innovation, the value of education, and the accumulation of knowledge through economic activities, epitomizing the 'learning by doing' paradigm. A prominent example of this approach is found in Stadler [1990], which posits technological progress as an emergent property of economic behavior and incentives, rather than a mysterious external force.

A critical aspect of the 'learning by doing' model is its premise that technological frontiers are contingent upon the existing knowledge base, which expands primarily through practical experience. Consequently, periods of economic expansion witness a sharp increase in the knowledge stock, driven by higher employment levels, whereas recessions tend to stabilize or even diminish this stock due to reduced employment rates. This dynamic suggests that economies devoid of cyclical fluctuations might attain superior steady-state growth, as employment remains consistently high, fostering continuous technological advancement. From this perspective, the concept of a 'cleansing effect'—whereby economic downturns eliminate low-productivity jobs and ostensibly strengthen the economy—is challenged. The elimination of even low-productivity roles can erode the overall knowledge base.

Such theories reframe the discourse on stabilization policies, particularly fiscal interventions, by highlighting their role in sustaining employment and, by extension, supporting the technological frontier even in downturns.

To illustrate this theory's implications more vividly, consider a nuanced example: an antiquated factory with limited land resources discovers an innovative method to utilize an old tractor more efficiently. Despite the ingenuity of this breakthrough, if the broader economy has moved beyond the technology that the tractor represents, the innovation might not significantly contribute to the overall knowledge stock or push the technological frontier forward. This scenario prompts a fundamental inquiry: does innovation at the lower end of the skill spectrum or within outdated technological contexts meaningfully advance the technological frontier? Or, would it be more beneficial for economic growth to transition such small-scale innovations into larger entities equipped with modern technologies?

One significant critique concerns the disparity in learning opportunities across different sectors and among individuals. The model's premise of uniform learning opportunities does not always align with the reality that some industries, such as the technology sector, offer rapid innovation and learning environments compared to more traditional manufacturing industries, where the pace of learning and innovation may be inherently slower due to the nature of the work processes involved.

Furthermore, the model may not adequately address the issue of structural unemployment that can arise from technological advancements. As certain workers benefit from "learning by doing," leading to increased productivity, others may find their skills becoming obsolete due to automation and other technological changes. This dynamic is evident in the automation of routine manufacturing jobs, which, while fostering "learning by doing" in fields like robotics and software engineering, simultaneously leads to structural unemployment for workers displaced by these technologies.

Another point of contention is the potential for diminishing returns to learning. The assumption that "learning by doing" continuously fuels growth may not hold up against the reality that initial rapid gains in productivity tend to taper off as workers gain proficiency, suggesting that the benefits of learning may diminish over time.

The model also potentially overlooks the externalities and spillover effects associated with "learning by doing." Technological advancements in one firm or sector do not automatically translate into broader economic growth if these advancements remain isolated and do not benefit other sectors or industries. This is illustrated by a software company that develops cutting-edge algorithms, enhancing its productivity but failing to contribute to the wider economy if the knowledge remains proprietary.

This nuanced exploration challenges the simplistic notion of 'learning by doing' by questioning the value and impact of incremental innovations within the broader economic and technological ecosystem. It underscores the complexity of technological progress and its interplay with economic dynamics, inviting a deeper investigation into the mechanisms that drive long-term growth and the role of policy in nurturing an environment conducive to continuous innovation and knowledge expansion.

The contemporary perspective on technological advancement, when viewed as a product of incremental contributions from every market participant, appears overly simplistic. A more accurate depiction of technological progress recognizes it as predominantly driven by those at the forefront of research. The expansion of the technology frontier is essentially shaped by the knowledge and breakthroughs of these leading-edge innovators. Other entities in the economy adopt these innovations at varying paces, influenced by the associated adoption costs. While firms that are not at the innovation frontier may achieve marginal efficiency gains through adoption, the impact of such improvements is often minimal. These marginal innovations are frequently a reflection of the adopting firms' constraints, particularly their inability to invest in more advanced and expensive technologies. Consequently, these incremental innovations have limited potential for widespread diffusion, as they stem from a position of necessity rather than pioneering advancement.

Another theory describe a recession as a period in which the opportunity cost of investing in a productivity enhancing projects is lower since the workforce is not fully in demand to produce goods. Doing this would lead in theory to higher productivity in the period of expansion. The key role here is that the productivity-enhancing activity is costly and thus divert capital and labor force from production as Hayek Hayek [1933] explained. For this view to be valid two key aspects should be true at the same time: in the first place the expectations about the length of the recession should reinter in the short-term otherwise it is cheaper to destroy some production units (labor and capital) to accommodate the slow in demand. The last condition is that internal resources must be less costly than external ones, however, it would be cheaper to higher more skilled workers and fire the low-skill one. An additional remark is the this theory describes all those initiatives like worker formation that affect only marginally the productivity of a firm. It misses the main mechanics in which a firm can increase its productivity sharply: thorough technical innovation, and to do so you need a research program where the workforce is fully dedicated to it and not diverted from production.

Another theory posits that recessions offer a period in which the opportunity cost of investing in productivity-enhancing projects are lower, primarily because the workforce is not fully engaged in producing goods. Theoretically, this would lead to higher productivity during subsequent periods of expansion. A crucial element in this perspective is the acknowledgment that productivity-enhancing activities are expensive, thereby diverting capital and labor away from immediate production, a concept Hayek Hayek [1933] elucidated.

For this viewpoint to hold, two critical conditions must be concurrently satisfied: firstly, expectations regarding the duration of the recession must be short-term. If the recession is anticipated to be prolonged, it becomes economically viable to dismantle some production units (both labor and capital) to adjust to reduced demand. Secondly, the cost of utilizing internal resources for such productivity-enhancing ventures must be lower than the cost of acquiring external resources. Otherwise, it might be more economical to hire more skilled workers and lay off less skilled ones.

An additional observation about this theory is that it accounts for initiatives like worker training, which only marginally affect a firm's productivity. This overlooks the primary mechanism through which a firm can significantly boost its productivity: through technical innovation. To achieve substantial innovation, a dedicated research program is essential, where the workforce is fully committed to innovation efforts rather than being diverted to current production tasks. This highlights a gap in the theory, suggesting that while reallocating resources during recessions may offer some productivity benefits, the most dramatic productivity improvements are likely achieved through focused innovation and research activities, not merely through the opportunistic reallocation of resources during economic downturns.

An intricate theory that elaborates on the dynamics of economic recessions and the associated lower opportunity costs is grounded in the concept of labor hoarding, as discussed in the seminal work by Clark Clark [1973]. This theory posits that firms maintain employment levels higher than what immediate efficiency metrics might dictate. The rationale behind such a strategy is to prepare for a potential surge in demand, ensuring that the firm can quickly ramp up production without the delays associated with recruiting and training new employees. However, this strategic maneuver towards the internal possibility frontier—where firms optimize their readiness for future demand—does not manifest as observable changes in employment rates. Consequently, this leads to discrepancies or residuals in the statistical series of employment, which do not align with what might be considered the level of optimal employment, a phenomenon further analyzed in the work of Burnside et al. [1993].

This labor hoarding theory offers a partial explanation for the strong pro-cyclically of measured productivity. During economic upturns, firms can immediately respond to increased demand using their hoarded labor, thereby appearing more productive. Conversely, during downturns, the reluctance to shed this excess labor, due to the anticipation of future demand recovery, results in lower observable productivity levels. This behavior underscores a strategic depth in firm management, navigating through the cyclical economic waves by balancing between immediate efficiency and long-term responsiveness.

Expanding on this foundation, it becomes evident that the decision to engage in productivity-enhancing activities during recessions is not merely a reaction to lower opportunity costs but also a strategic consideration influenced by expectations of the recession's duration and the comparative costs of internal versus external resources. If firms anticipate a short-lived recession, the logic of hoarding labor and investing in internal productivity initiatives becomes compelling. However, this strategy hinges on the assumption that improving the skill set of the existing workforce or reallocating resources towards innovation is less costly than the alternative—acquiring new, possibly more skilled labor post-recession.

The opportunity cost (OC) approach closely aligns with the theory of labor hoarding, which seeks to elucidate the pronounced procyclicality of measured productivity. This observation implies that during economic downturns, firms appear to retreat towards the interior of their production possibility frontier, opting for a strategic reduction in operational efficiency rather than workforce downsizing. This strategy is partly attributed to the invisible nature of one crucial input—effort—to statisticians, and the economic rationale that, given the costs associated with employee turnover, it proves more economically viable for firms to dial back effort during slumps instead of terminating employment.

An intriguing alternative to diminishing effort is the redirection of employee tasks from immediate production roles to undertakings that bolster the firm's long-term productivity. At first glance, this strategy bears a resemblance to labor hoarding but carries the added outcome that these so-called shadow activities, embraced during recessions, eventually manifest as enhancements in total factor productivity.

The concept of adjustment costs does not singularly confine firms from adapting their production factors according to operational necessities. This opportunity cost mechanism could theoretically extend to a macroeconomic scale, influencing individual entities via price adjustments. During periods marked by diminished production value, the immediate returns from production activities (e.g., wages for workers) decline in comparison to alternative endeavors, notably human capital accumulation, whose benefits are pegged to future earnings. This economic mechanism could precipitate a resource reallocation towards these alternative activities. The empirical observation that education durations tend to extend during economic recessions lends credence to this argument. Nevertheless, with the exception of leisure, most sectors shadow the movements of aggregate GDP. Thus, if productivity-enhancing activities (PEAs) are to occur during recessions, the resource reallocation process must predominantly unfold within the firms themselves.

One notable deviation might be labor reallocation. As demonstrated by Davis and Haltiwanger [1992], job destruction exhibits a more countercyclical pattern compared to job creation. Viewing job reallocation through the lens of both destruction and creation suggests a countercyclical trend, positing job reallocation as an investment in cultivating superior firm-worker matches, thereby sowing the seeds for heightened productivity in the future. Davis and Haltiwanger [1992]further postulate, within the framework of a model featuring a representative agent, that economic recessions present an optimal window for labor reallocation, highlighting a strategic dimension to workforce management during downturns that might ultimately contribute to long-term productivity gains.

The "lame ducks" theory, initially proposed by Caballero and Hammour [1994], offers an intriguing perspective on recessions as mechanisms that phase out less profitable, outdated capital. This theory delineates how the destruction of older units during downturns is more pronounced than the construction of new ones, characterizing recessions as periods marked by the systematic elimination of obsolete capital, hence the moniker "lame ducks" theory. Notably, this framework sheds light on observations documented by Davis and Haltiwanger [1992], positioning it as a prominent theoretical approach that will be delved into more thoroughly in subsequent discussions.

Despite its insights, this model lacks consideration of the financial dimensions of firms, an aspect addressed by the theoretical work of Osotimehin and Pappadà [2017]. Their research weaves the financial decision-making process into the broader context of intertemporal capital decisions, highlighting how financial frictions influence the lender's participation constraint. The study reveals that, despite financial frictions, the cleansing effect of recessions on productivity persists, potentially leading to a more pronounced productivity surge during expansion phases. This analysis serves as a pivotal foundation for the new theoretical framework introduced in this thesis, marking a significant departure from traditional views and emphasizing the multifaceted impact of recessions on firm productivity and economic recovery.

In the forthcoming sections, / we will explore the theoretical underpinnings that form the basis of the new, streamlined theoretical framework introduced in this thesis. Our examination begins with the insights of Caballero and Hammour [1994], focusing on the interplay between the destruction and creation margins in economic cycles. Subsequently, we will delve into the work of Osotimehin and Pappadà [2017] (2017), which sheds light on the financial dimensions, particularly how capital lending frictions can precipitate misallocations. These studies collectively inform the development of our theoretical framework, setting the stage for a comprehensive analysis of economic dynamics and firm behavior during recessions.

## 2.2 Literature review of theoretical models

### 2.2.1 Introduction

This section of the literature review examines two influential theoretical models that analyze the cleansing effect of recessions on the economy. The first model, introduced by Caballero and Hammour [1994], utilizes a vintage model of creative destruction to investigate how industries adapt to cyclical demand variations. This model is pioneering in its approach, highlighting how recessions can facilitate the removal of outdated and less efficient production units, potentially leading to an overall increase in industry productivity. The model's insights are derived from a framework where production units of varying vintages coexist, and the rate of technological adoption plays a crucial role in determining industry responses to economic fluctuations.

In contrast, the second model explored in this review, by Osotimehin and Pappadà [2017], introduces financial frictions into the analysis of the cleansing effect of recessions. This addition brings a new layer of complexity to the understanding of how economic downturns affect firm dynamics, particularly by influencing the selection process through which firms enter and exit the market. The model underscores the role of credit constraints in mediating the impact of recessions, suggesting that financial frictions can dampen the potential productivity gains that might otherwise arise from the purging effects of a downturn.

Both models rely on numerical methods to solve their respective frameworks, acknowledging the intricate dynamics and non-linearities inherent in their analyses. By comparing these models, this review seeks to illuminate the diverse mechanisms through which recessions can influence economic outcomes, as well as the varying implications of introducing different types of market frictions into theoretical frameworks.

### 2.2.2 The cleansing effect by Caballero

In the first paper that rationalizes the cleansing effect of recessions, Caballero and Hammour [1994] and published in the American Economic Review in 1994, the primary aim was to investigate how industries respond to cyclical variations in demand. They did this by employing a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations in two ways: by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model they used incorporated heterogeneous firms, where production units embodied the most advanced technology at the time of their creation. The costs associated with creating new units slowed down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a reduction in demand can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour's research Blanchard et al. [1990] offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard and Diamond Blanchard et al. [1990], as well as Steven Davis and John Haltiwanger Davis and Haltiwanger [1992], in their respective works from 1990. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study Davis and Haltiwanger [1992], where they assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986, it is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry. The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

#### Theoretical model

The model in question is a vintage model that simulates an industry experiencing exogenous technological progress. Within this model, production units are constructed using a fixed proportion of labor and capital, and they are continually being created and phased out. Notice that only the creation of new production units incurs a cost. This simplification is plausible, particularly in the context of the United States, where the expense associated with hiring is typically higher than the cost of termination, as demonstrated by Abdulkadiroğlu and Kranton (2003) Abowd and Kramarz [2003].

In this model, when a production unit is created at a specific time $t_0$, it embodies the most advanced technology available at that moment and consistently generates a uniform output represented by $A(t_0)$ throughout its operational lifetime. The productivity of this technology denoted as $A(t)$, experiences continuous growth at an exogenously determined constant rate $\delta \geq 0$. This growth in technology can be interpreted in two ways: either as the adoption of new technology or as a product innovation. In the latter scenario, a continuum of perfectly substitutable products can yield varying levels of output.

$$[f(a,t) \qquad 0 \leq a \leq \overline{a}(t)]$$

The above function represents the cross-sectional density of the production units aged $a$ at time $t$, where $\overline{a}(t)$ is the age of the oldest production unit at time $t$. The first assumption is that $f(a,t)$ and $\overline{a}(t)$ are continuous functions. The mass of production units at time $t$ is given by:

$$N(t) = \int_{\overline{a}(t)}^{0} f(a,t)da$$

$N(t)$ is a measure of either the industry's capital stock or its employment, due to a fixed share of capital and labor. Thus, the industry's output is given by:

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

The deterioration of production units involves both an exogenous depreciation rate $\delta$ and an endogenous destruction process, which impacts $f(a,t)$ at its limits. The count of production units surviving for $a$ years is expressed as:

$$f(a,t) = f(0,t-a)e^{-\delta a} \quad \text{where } 0 < a \leq \overline{a}(t)$$

The production flow is determined by:

$$\dot{N}(t) = f(0,t)[1 - \dot{\overline{a}}(t)] + \delta N(t)$$

Here, the first term represents the production rate, while the second term encapsulates the destruction rate, encompassing the obsolescence rate $f(\overline{a})(t)$, the technological obsolescence change over time $-f(\overline{a})(t)\dot{\overline{a}}(t)$, and the depreciation rate $\delta N(t)$.

The assumptions made by the authors are denoted as $\forall t \mid f(0,t) > 0 \cup \dot{\overline{a}}(t) <$.

The alteration in output concerning these flows is articulated as:

$$\dot{Q}(t) = A(t)f(0,t) - A(t-\overline{a}(t))f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)] + \delta Q(t)$$

The authors define a perfectly competitive industry in partial equilibrium, where supply is dictated by free entry and perfect equilibrium. Additionally, they introduce a cost function related to creating new production units:

$$c = c\left(f\left(f(0,t)\right)\right) \quad \text{where } c(\cdot) > 0,\, c'(\cdot) \leq 0$$

This cost function is contingent on the creation rate, implying that higher creation rates correspond to increased costs. The equilibrium condition is established by equating the cost of unit creation to the present discounted value of profits throughout its lifespan. The authors set the cost of a production unit to 1, and $P(t)$ is the price of a unit of output. Thus, the profits generated at time $t$ by a production unit aged $a$ are defined as:

$$\pi(a,t) = P(t)A(t-a) - 1$$

$$\overline{a}[t + T(t)] = T(t)$$

Here, $T(t)$ signifies the maximum lifetime of a unit created at $t$. At any given time $t$, the free entry condition is expressed as:

$$c(f(0,t)) = \int_{t+T(t)}^{t} \pi(s-t,t)e^{-(r+\delta)(s-t)\,ds}$$

In the above equation, where $r > 0$ represents the exogenously determined instantaneous interest rate, the determination of the exit of a production unit is contingent upon continuous $P(t)$ and the instance when the profits generated by a unit being destroyed reaches zero. This occurrence signifies the moment when the oldest unit operational at time $t$, denoted as $\overline{a(t)}$, must adhere to the equation:

$$P(t)A(t - \overline{a}(t)) = 1$$

The authors posit that $P(t)$ exhibits a decreasing trend due to the model's assumption regarding endogenous destruction, specifically $\overline{\dot{a}(t)} < 1$. To see, differentiate

$$\dot{P}(t) = -\gamma\left[1 - \dot{\overline{a}}P(t)\right]$$

Consequently, when the profits of a production unit diminish to zero for the first time, it will be subject to destruction.

On the demand side, the authors assume a unit-elastic demand function and consider the aggregate expenditure as exogenous $\overline{D}(t) = P(t)Q(t)$. The equilibrium is a path $\{f(0,t), \overline{a}(t), T(t), Q(t)\}_{t\geq 0}$ that satisfy the following conditions:

1.

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

2.

$$f(a,t) = f(0,t-a)e^{-\delta a}$$

3.

$$T(t) = \overline{a}\,(t + T(t))$$

4.

$$c(f(0,t)) = \int_t^{t+T(t)} \left[P(s)A(t) - 1\right] e^{-(r+\delta)(s-t)}\, ds$$

5.

$$P(t)A(t - \overline{a}(t)) = 1$$

6.

$$P(t)Q(t) = \overline{D}(t)$$

The first three equations (1, 2, 3) and the fifth one (5) suffice to delineate the trajectories of $T(t)$, $P(t)$, and $Q(t)$, which are determined by $\{f(0,t), \overline{a}(t)\}$. To affirm the robustness of the conditions expressed in equations 6 and 5, it is possible to derive these equations as first-order conditions for the maximization of a number of perfectly competitive firms holding production units.

To comprehend the functioning of endogenous destruction, let's consider a scenario with constant demand. In this case, both the destruction and creation rates change only due to supply factors. This steady state is characterized by a constant lifetime of production units $T(t) = \overline{a}(t) = \overline{a}^*$, resulting in a time-invariant age distribution $f(a,t) = f^*(a)$. Equation 5 implies that the price $P(t)$ must consistently decrease at a rate $\sigma$. Higher innovation rates lead to increased productivity, raising the supply and consequently lowering the price. Equation 2 reveals that the distribution of production units in the steady state follows a truncated exponential distribution:

$$f^*(a) = f^*(0)e^{-\delta a} \quad 0 \leq a \leq \overline{a}^*$$

Using free entry conditions (4) and the clearing condition (6), one can determine the creation and destruction ages $f^*(0)$ and $\overline{a}^*$. Equations 1 and 5 yield the cost function and productivity of a new production unit:

$$c(f^*(0)) = \frac{e^{\gamma\overline{a}^*} - e^{-(r+\delta)\overline{a}^*}}{\gamma + r + \sigma} - \frac{1 - e^{-(r+\delta)\overline{a}^*}}{r + \delta}$$

$$f(0) = \frac{(\sigma + \delta)\overline{D}^*}{e^{\sigma\overline{a}^* - e^{\delta\overline{a}^*}}}$$

The authors then normalize the creation rate:

$$N = f^*(0) \cdot (1 - e^{\delta\overline{a}^*})$$

In the steady state, this is given by:

$$(9)CC^* = \frac{\delta}{1 - e^{-\delta\overline{a}^*}}$$

Considering a special case where the creation cost is a constant $c$, i.e., $c(f^*(0)) = c$, substituting into equation 2.2.2 allows retrieval of $\overline{a}^*$. The effect of technological rate $\sigma$ on $\overline{a}^*$ is decreasing, as a higher innovation rate increases the opportunity cost of delayed renovation, while a higher cost of creating new units lowers the renovation rate. An optimal lifetime of production units increases with higher $r$ and $\delta$ as it becomes harder to recover creation costs.

Now, dropping the assumption of constant demand, we examine how the industry adjusts to demand fluctuations. Two ways are identified in which the industry adapts production to meet demand: by reducing the rate of creation $f(0,t)$ and by increasing the rate of endogenous destruction $f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)]$, thus reducing $\overline{a}$, the age at which units are demolished.

These two adjustments interact, leading to a reduction in demand causing the most outdated units to be scrapped, rendering them unprofitable. However, if the recession is partially accommodated by a reduction in the creation rate, the effect on the destruction margin is diminished. The authors argue that the extent to which creation will "insulate" existing units from variations in demand depends on the marginal cost of creating new units $c' f(0,t)$. When the marginal cost of creation is zero, demand fluctuations are entirely adjusted by the creation margin. This is exemplified in the case where $c(f(0,t)) = c$. In such instances, the insulation effect is complete, as there is no need to retire older units. Lowering $f(0,t)$ is sufficient, and it is cheaper than reducing the life of existing production units.

The insulation effect is not solely due to asymmetric adjustment costs on the creation and destruction margins. Complete insulation would occur even with linear adjusting costs. The creation rate in the case of constant creation cost is given by:

$$f(0,t) = \frac{\dot{\bar{D}}(t) + \delta \bar{D}(t) + P(t)A(t - \bar{a}(t)) f(\bar{a}(t),t)[1 - \dot{\bar{a}}(t)] - \dot{P}(t)Q(t)}{P(t)A(t)}$$

In the attained equilibrium, variations in demand are entirely offset by adjustments at the creation margin denoted as $f(0,t)$, with $\overline{a}(t)$ remaining steady at the destruction margin. The creation process effectively counteracts the impact of demand fluctuations on the price $P(t)$, effectively shielding existing units from demand changes. The price $P(t)$ experiences a constant decline at a rate represented by $\sigma$, reflecting the pace of technical progress. This consistent decline in $P(t)$ serves as a clear signal for production units to function optimally throughout their constant lifetime $\overline{a}(t)^*$.

In the aforementioned scenario, the destruction rate is not constant, but it does not respond to demand through variations in the age $\overline{a}(t)^*$ at which units are destroyed. Instead, variations in the creation rates have an impact on the number of units that reach obsolescence. If fewer units are created, fewer units become obsolete after $\overline{a}(t)^*$ periods. It is noteworthy that any modification leaving equations 3 to 5 independent of $\overline{D}(t)$ and $f(0,t)$ does not alter the full-insulation results.

Interestingly, assumptions such as perfect competition, industry-wide return to scale, and perfect foresight are not necessary for these conclusions. The latter is particularly noteworthy as it asserts that fully accommodating demand on the creation side only requires knowledge of current conditions. As long as the non-negativity constraint on $f(0,t)$ is never binding, implementing equilibrium behaviors does not necessitate expectations of future demand.

### Application of the model

The model undergoes calibration utilizing Job-flow data and Industry production data. The former facilitates the replication of job creation dynamics, while the latter is employed to mimic the behaviors of firm creation and destruction in the manufacturing industry. To capture these dynamics, the marginal cost of creating new production units is stipulated as positive $c'f(0,t)$. This allows for a partial insulation effect, and the destruction margin responds to demand fluctuations. However, introducing a dependency of $c$ on $f(0,t)$ compromises the analytical tractability of the system (Equations 1 - 6). Consequently, the authors resort to methods such as multiple shooting to ascertain the optimal equilibrium and subsequently employ an iterative procedure to converge to the correct expected creation rate.

For numerical solutions, the authors adopt a linear formulation:

$$c(f(0,t)) = c_0 + c_1 f(0,t)$$

To gain a deeper understanding of how creation and destruction respond to demand, the authors simulate sinusoidal demand using the equation:

$$\overline{D}(t) = 1 + 0.07\sin(t)$$

The results are visualized in the image below, depicting the feedback of normalized creation and destruction (CC and DD) to changes in demand.

The plot clearly illustrates that the insulation effect is only partial; otherwise, DD would have remained flat, as in the case with $c(f(0,t) = c)$. From a mathematical perspective, destruction responds to demand as equations 3-5 are no longer independent of the path $f(0,t)$ and demand. From an economic standpoint, increasing creation costs smoothen the creation process. In scenarios with a nearly flat innovation rate, firms during crises cannot fully accommodate lower demand, nullifying the adoption of new production units, as the marginal costs would exceed the reduction in existing production units.

In the considered model, production units integrate labor and capital in fixed proportions to generate output. Each unit can be conceptualized as contributing to job creation within the industry, and job-flow data serves as a metric for quantifying the flows of production units.

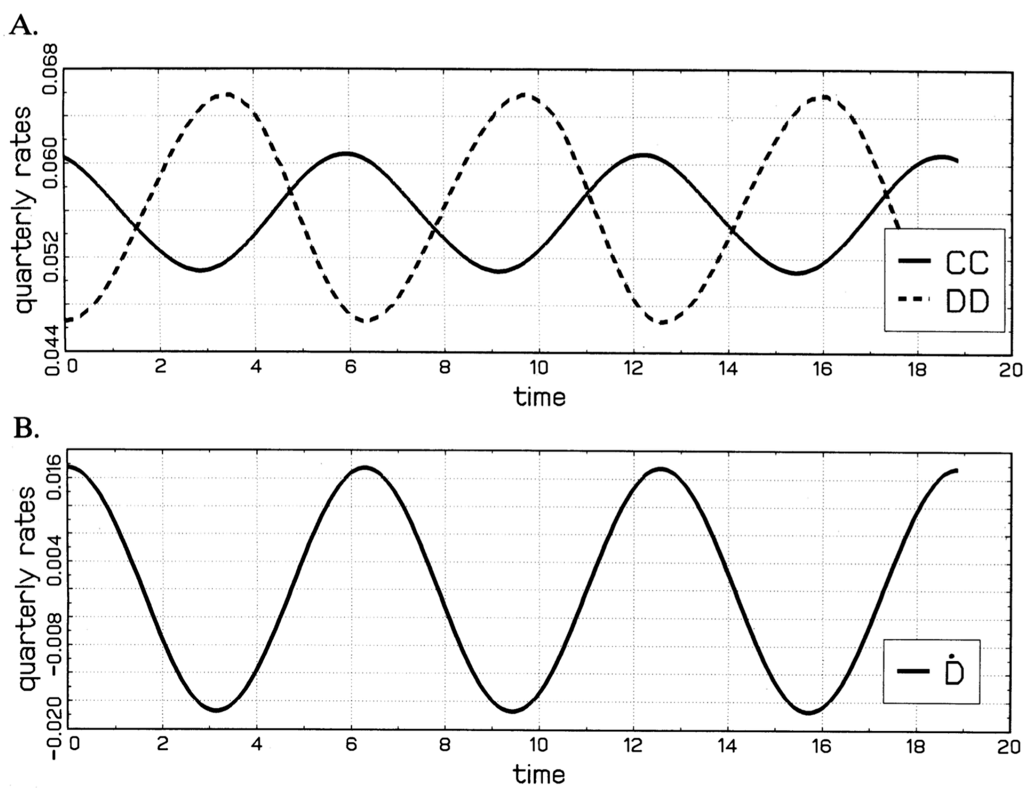


FIGURE 2. A) CREATION AND DESTRUCTION ($c_0 = 0.3$, $c_1 = 1.0$); B) CHANGE IN DEMAND (SYMMETRIC)

Figure 2.1: Figure 1. A Creation and destruction $c_0 = 0.3, c_1 = 1$ B Change in demand (Symmetric)

Datasets that closely align with the theoretical CC and DD series have been compiled by Davis and Haltiwanger [1990, 1992] and Blanchard et al. [1990], drawing from various sources. The primary focus lies on the dataset curated by Davis and Haltiwanger, who leverage the Longitudinal Research Database to construct quarterly series for U.S. manufacturing plants spanning the period 1972:2-1986:4.

In their empirical approach, **?**utilize output to empirically determine demand, employing the growth rate of the industrial production index as a proxy for output growth. Notably, in the foundational theoretical model, $Q(r)$ is smoothed by price movement, with the elasticity of demand determining the extent of smoothing, assumed to be equal to 1. While the theoretical model maintains a constant dividend wage, the authors acknowledge that considering a procyclical dividend wage, as in the case of general equilibrium with correlated industry shocks, may dampen the effect of demand shocks. However, they assert that this adjustment would alter only the magnitude, not the direction, of the analysis.

The figure below illustrates the data that the model seeks to replicate, showcasing job creation, job destruction, and growth.

To discern the characteristics of the series, the authors perform regression analysis on sectoral rates of job creation and job destruction against leads and lags of the corresponding rates of growth. They find that job creation is less responsive to demand fluctuations, while job destruction exhibits a more countercyclical behavior. The initial finding indicates that the rate of job destruction displays greater responsiveness to changes in sectoral activity compared to the rate of job creation. Specifically, the sums of coefficients are -0.384 for job destruction and 0.218 for job creation showed in the table 2.3, the same results as in Davis and Haltiwanger [1990, 1992] and in Blanchard et al. [1990]. The authors capitalize on a natural experiment rooted in the intrinsic asymmetric characteristics of business cycles. Recessions, marked by brevity but in-

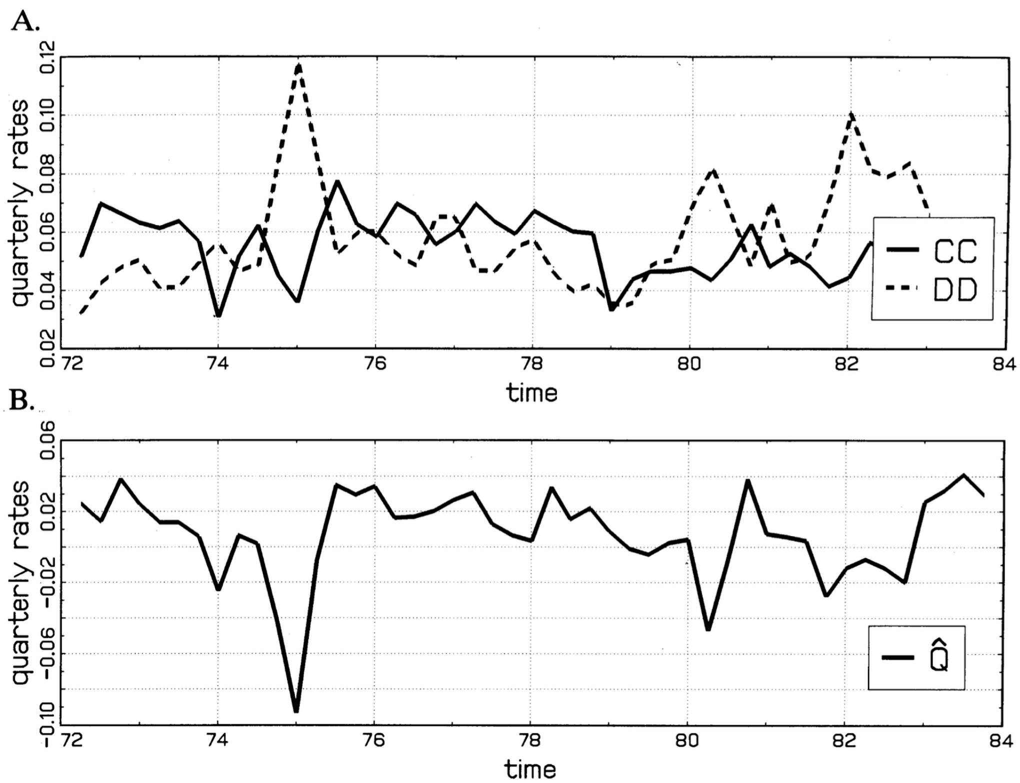


Figure 2.2: Figure 1. Job creation and job destruction in U.S. Manufacturing B Index of the industrial production

| Regressor | Timing | Creation: Coefficient | Creation: Standard deviation | Destruction: Coefficient | Destruction: Standard deviation |
|---|---|---|---|---|---|
| $\hat{Q}$ | 2 leads | 0.029 | 0.006 | 0.030 | 0.010 |
| | 1 lead | 0.065 | 0.007 | −0.068 | 0.010 |
| | contemporaneous | 0.108 | 0.007 | −0.185 | 0.010 |
| | 1 lag | 0.013 | 0.007 | −0.103 | 0.010 |
| | 2 lags | 0.003 | 0.006 | −0.058 | 0.010 |
| | Sum: | 0.218 | 0.013 | −0.384 | 0.017 |
| $\hat{Q}^{+}$ | 2 leads | 0.052 | 0.012 | 0.012 | 0.016 |
| | 1 lead | 0.102 | 0.012 | 0.002 | 0.016 |
| | contemporaneous | 0.131 | 0.012 | −0.065 | 0.016 |
| | 1 lag | 0.059 | 0.012 | −0.025 | 0.016 |
| | 2 lags | 0.055 | 0.012 | −0.008 | 0.016 |
| | Sum: | 0.399 | 0.026 | −0.066 | 0.023 |
| $\hat{Q}^{-}$ | 2 leads | 0.002 | 0.010 | 0.006 | 0.014 |
| | 1 lead | 0.022 | 0.011 | −0.149 | 0.014 |
| | contemporaneous | 0.093 | 0.012 | −0.293 | 0.015 |
| | 1 lag | −0.012 | 0.012 | −0.139 | 0.015 |
| | 2 lags | −0.021 | 0.012 | −0.059 | 0.015 |
| | Sum: | 0.084 | 0.020 | −0.634 | 0.024 |

Figure 2.3: Table 2.1. Job Creation and Job Destruction in U.S. Manufacturing Response to Output Growth

textitNotes: The table presents the reaction of job creation to the growth rate of the industrial production index. The latter is categorized into values above and below its mean (Q). The table encompasses quarterly observations for the two-digit SIC industries during the period 1972:2-1986:4. The coefficients are uniformly constrained to be equal across all sectors, with the exception of a constant (not shown).

tense contractions, provide the backdrop for the authors' model. This model endeavors to emulate the creation rate while concurrently mitigating the impact of asymmetric cyclicality inherent in business cycles. The empirical evidence supporting this model's behavior is encapsulated in Table 2.3, wherein two distinct scenarios are explored: output growth trajectories above $Q^+$ and below $Q^-$, relative to their respective means. The table meticulously delineates how creation and destruction rates respond to these deviations in output growth.

The salient observation emerges regarding creation rates, elucidating that they exhibit a more rapid and robust response in instances of vigorous output growth, as opposed to scenarios where the output growth rate experiences a reduction. On a contrasting note, the destruction margin, in line with the model's projections, manifests heightened sensitivity to a decline in output. This responsiveness is particularly pronounced from one quarter before the onset of the shock to one quarter after. Notably, during expansionary phases, the mean response of the destruction margin is -0.066, a notably milder reaction compared to the recessionary case where the mean response stands at -0.634.

These empirical outcomes seamlessly align with the predictions of the model. Specifically, the creation rate exhibits heightened responsiveness during expansionary phases, given their cyclical and symmetric nature. In contrast, the asymmetric and non-cyclical nature of recessions triggers a more substantial decline in the production unit rate, in line with the model's expectations.

In order to better understand the asymmetrical behavior the authors simulate an asymmetrical demand function:

$$\cdot\overline{D}(t) = 0.05[\cos(t) + \sin(t)] - 0.016\sin(2t) - 0.003\cos(3t)$$

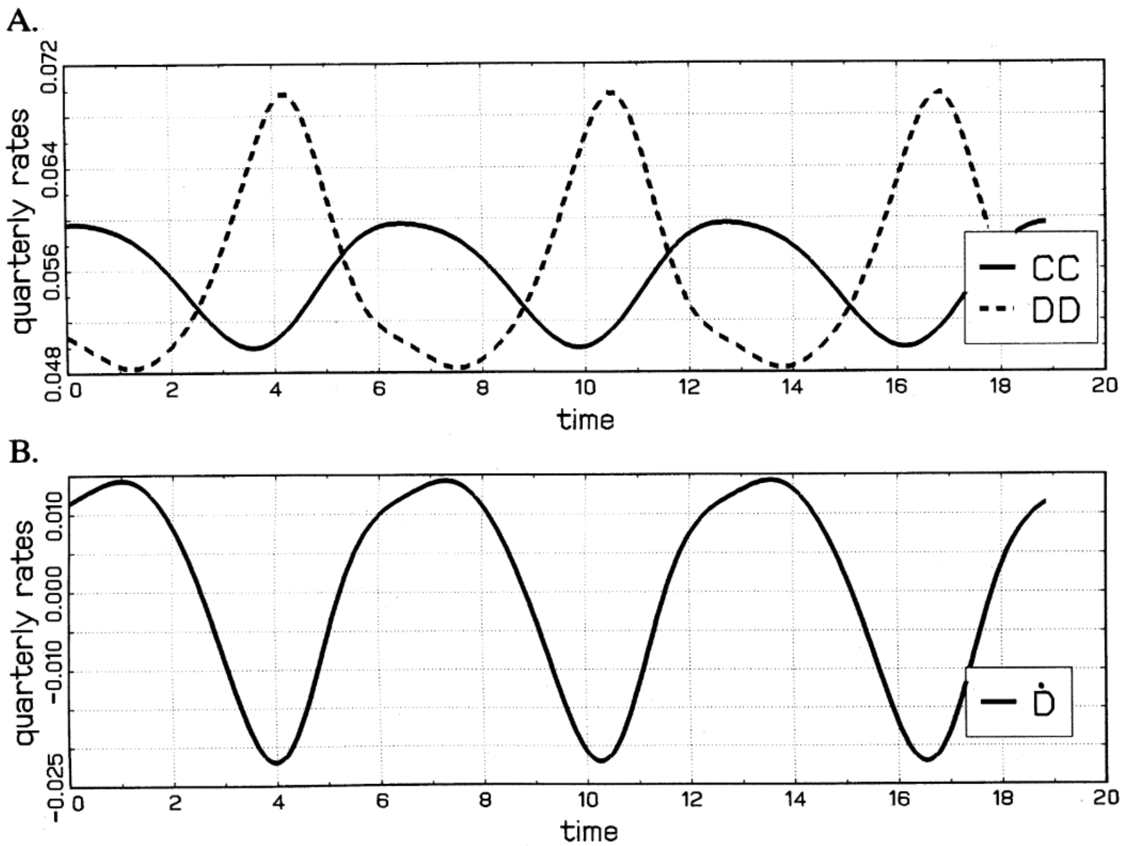


Figure 2.4: A. Creation and Destruction B. Output Growth
*Notes*: The figure depicts a simulation of asymmetrical supply growth.

$$\overline{D}(t) = 1 \quad r = 0.065, \delta = 0.15, \gamma = 0.028, c_0 = 0.3, c_1 = 1.0$$

The results are depicted in 2.4

From the plot 2.4, it is evident that firms use prediction in demand to smooth job creation to avoid big change, since they are too costly, by averaging the demand over the lifetime of a production unit. On the other hand, destruction depends only on current conditions, thus responding only to significant deviations from the demand prediction. It can be better understood thinking about a case in which creation rates respond only mildly to a sharp decrease in demand, and the equilibrium price falls leading to additional destruction since older units' profits go to 0. Indeed, destruction not only preserves but amplifies the asymmetry of demand.

**Frictionless economy** The authors culminate their study with a compelling calibration exercise using manufacturing series to exploit the model. This entails dissecting the observed net change in employment into destruction and creation rates, as well as applying the same approach to output production. The model is simulated for the duration of 1972:2-1983:4, with parameters as follows:

Table 2.1: Calibrated Parameters

| Variable | Symbol | Value |
|---|---|---|
| Interest rate | $r$ | 0.065 |
| Depreciation rate | $\delta$ | 0.150 |
| Rate of technical progress | $\gamma$ | 0.028 |
| Adjustment cost parameters | $c_0$ | 0.403 |
| | $c_1$ | 0.500 |

The technical progress is selected to attribute all the growth in employment and manufacturing to technological advancements, setting $\lambda$ as 2.8. The authors employ Equation 2.2.2, linking the steady state to the lifetime of jobs and job turnover ($CC^*$), determining $\overline{a}^* + 7.42$ years. Utilizing this information, they ascertain the steady state entry cost to be 0.525, equivalent to half a year's operating costs for production units. Subsequently, they employ ordinary least squares (OLS) to retrieve the value of $c_1$, the marginal cost of creating a new unit, which is found to be 0.5. This aligns with a small elasticity for the creation cost function, signifying the vulnerability of the insulation mechanism to breakdown. The model's simulations on employment and output, shown in Figure 2.5, reveal discrepancies with actual data, particularly in the smoother job creation trends, likely due to the model's exclusion of uncertainty. Nonetheless, it successfully captures the volatility in job creation and destruction patterns, along with job creation's greater symmetry, providing insights into employment and output fluctuations. The model's examination of the creation margin's response and its impact on the destruction margin offers a baseline for understanding the cleansing effect's role in production unit distribution.

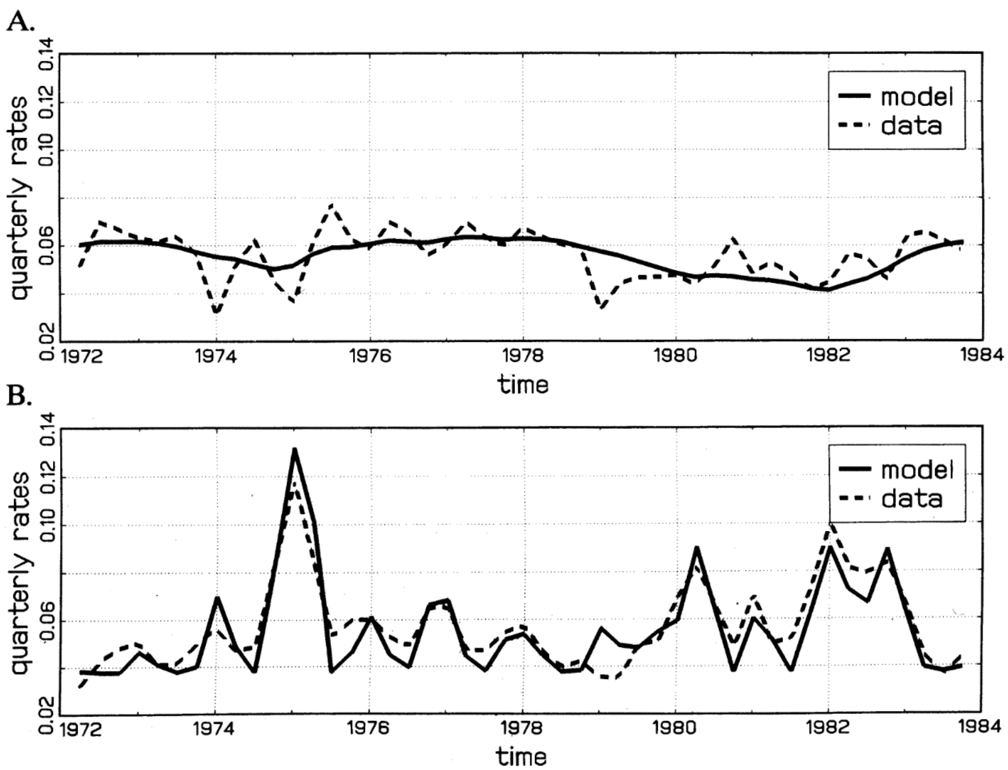


Figure 2.5: Figure 1. A employment driven job creation $c_0 = 0.403, c_1 = 0.5$ B Employment job destruction $c_0 = 0.403, c_1 = 0.5$

However, the model overlooks the impact of financial frictions, which could significantly affect both creation and destruction margins. It also entertains the notion of recessions as ”pit-stops” for strategic investment, adding depth to recession analysis. Despite the common view of procyclical labor productivity, the model, supported by Galí and Hammour [1992], suggests that recessions can indirectly boost long-term productivity through the cleansing effect.

A notable limitation is the assumption of constant marginal creation costs, which recent studies challenge, especially for larger firms known for substantial adjustments in response to demand drops. This observed behavior in larger firms, aligning with the model's predictions on downsizing, underscores its ability to reflect real-world dynamics despite its simplifications.

### 2.2.3 Cleansing effect in Osotimehin and Pappadà [2017]

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as $c$ to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as $e$, in the risk-free asset.

Firms opt to exit the market when expected profits no longer outweigh the fixed cost $c$, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r) - 1] e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to $q$. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors. In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as $k$. The production function, accounting for the fixed cost $c$, is expressed as follows: $Y = Z(\theta + \epsilon)k^\alpha$.
Key variables include:

- $Z$: Stochastic aggregate productivity common across firms.
- $\theta$: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- $\epsilon$: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \delta)$.
- $k^\alpha$: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

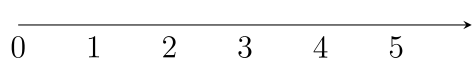


Figure 2.6: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of $Z, \theta, k^\alpha, e$ (where $e$ represents its endowment, different from $k$ since the firm can borrow money with $d = c + k - e$).
2. The firm computes the optimal $k$ to maximize the expected value of the firm, with $k$ ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.

3. At the end of the period, the firm observes $\epsilon$ and the aggregate shock.

4. The firm repays its debt and the fixed operating cost $(c+k-e)$, resulting in an end-of-period net worth $q$.

5. The firm decides on the amount of dividends to distribute $(q-e')$, observes the productivity shock $\theta', Z'$, and the process restarts from step 1.

**Frictionless economy** In a frictionless economy, firms have the option to borrow an amount denoted as $c+k-e$ at the risk-free interest rate $r=\frac{1}{\beta}-1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_k E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta+\epsilon)k^\alpha + (1-\delta)k - (1+r)(c+k-e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_k E \int [Z(\theta+\epsilon)k^\alpha - (1+r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market friction, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

**Economy with Credit Market Frictions** After production, the firm privately observes the temporary shock $\epsilon$, while financial intermediaries can only observe it at a cost of $\mu k^\alpha$. For one-period debt contracts, financial intermediaries observe $\epsilon$ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth $e$, current productivity $\theta$, and aggregate productivity value $Z$, all observable by both the financial intermediary and the firm at no additional cost.

**HP1 (Hypothesis 1):** The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\bar{\epsilon}$ is determined by the equation:

$$Z(\theta + \bar{\epsilon})k^\alpha + (1-\delta)k = (1+\widetilde{r})(c+k+e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component $\theta$.

The financial intermediary lends $(c + k - e)$ to the firm only if the expected income from the loan equals the the opportunity cost of the funds, as expressed by the inequality:

$$(1+\widetilde{r})(k+c+e)(1-\Phi(\bar{\epsilon})) + \int_{-\infty}^{\bar{\epsilon}} [Z(\theta+\bar{\epsilon})k^\alpha + (1-\delta)k - \mu k^\alpha]\, d\Phi(\epsilon) \geq (1+r)(c+k+e)$$

The financial contract is characterized by $(k, \bar{\epsilon})$. Given $Z, \theta, e$, the participation constraint indicates the default threshold $\bar{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given $\theta, Z$, there is a unique threshold $e_b(\theta, Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta + G(\bar{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\bar{\epsilon}_b) = (1+r)(k_b + c - \underline{e_b})$$

where $\bar{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta + \bar{\epsilon})k^\alpha + (1-\delta)k - (1+\widetilde{r})(k + c - e) & \text{if } \epsilon \geq \bar{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}); 0]$$

**The firm's problem** Define $V$ as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e, \theta, Z) = \max_{(k,\bar{\epsilon})} E\left\{\int I(q)q + (1 - I(q)) \max[q, \max_{e'} q - e' + \beta V(e', \theta', \zeta')]\, d\Phi(\epsilon)\right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1.

$$Z[\theta + G(\bar{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\bar{\epsilon}_b) \geq (1+r)(k_b + c - \underline{e_b})$$

2.

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}); 0]$$

3.

$$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.

**Findings**

This study investigates the complex interplay between credit frictions and the cleansing effect of recessions on firm dynamics. By integrating models of firm dynamics with credit frictions, the authors examine how these frictions influence the selection process for entering and exiting firms, potentially leading to the premature exit of some high-productivity entities. Despite the impact of credit frictions on firm selection, an increase in average productivity is observed following an aggregate productivity shock. Mirroring the dynamics in a frictionless economy, a negative aggregate productivity shock enhances average productivity by predominantly increasing the net exit rate of low-productivity firms.

The analysis underscores that the extent of the recession's cleansing effect significantly depends on the steady-state distributions of productivity and net worth among firms. The amplification in average productivity can be more substantial than in a frictionless context, depending on the severity of credit frictions and the productivity distribution of firms. Through calibration, the authors demonstrate that credit frictions generally mitigate the rise in average productivity, indicating a systematic diminution in the intensity of productivity enhancement for each percentage point increase in productivity, irrespective of the level of credit frictions and the productivity distribution.

Further exploration into the nature of economic shocks reveals that the type of shock critically influences the cleansing process. Specifically, while recessions can exert a cleansing effect, the intensity of this effect is attenuated when the downturn is triggered by a financial shock. This reduction is due to the financial shock affecting high-productivity firms as well, suggesting a nuanced relationship between the type of economic shock and the intensity of the cleansing effect: non-financial shocks tend to have a more significant cleansing impact compared to financial shocks.

Contributing to the literature on the cleansing effect of recessions, this study elucidates the role of credit frictions in affecting the average productivity of firms by facilitating the selective exit of low-productivity firms, even under credit constraints. It is imperative to note that the findings do not posit that recessions inherently enhance resource allocation efficiency. On the contrary, in the presence of credit frictions, many of the exiting firms could have remained operational in the absence of these constraints. This inefficiency is particularly evident during financial shocks, where the exit of potentially viable firms highlights the intricate dynamics governing the cleansing effect of recessions, emphasizing the importance of a nuanced understanding of economic downturns and their impact on firm dynamics.

# Chapter 3

# Theoretical model

## 3.1 Introduction

This thesis presents a partial equilibrium model in which firms maximize utility of dividends over an infinite period, under financial frictions, investigating how those frictions can affect the saddle paths of capital and dividends. Compare to Osotimehin and Pappadà [2017] and Caballero and Hammour [1994], this model allows to find a well defined optimal path for dividends and capital without using numerical approach. The subsequent sections delve into the formulation of the flow of funds and its dynamics. Following this, the focus shifts to scenarios where financial frictions are present, examining their implications on firm behavior and market outcomes.

## 3.2 Law of motion of capital and debt

This model is set within a partial equilibrium framework where firms are differentiated by their productivity levels. They have the option to fund their operations by obtaining loans from financial intermediaries, as outlined by Bernanke and Gertler [1995], or by retaining dividends. The capital at any time $t$ is calculated by adjusting the capital from the previous period for depreciation ($\delta$), then adding the net investment ($I$), thus the low of motion of the capital stock is:

$$k_t = k_{t-1}(1-\delta) + I_t$$

We can rearrange the above equation and get the investment function

$$I_t = k_t - k_{t-1}\,(1-\delta) \tag{3.1}$$

The 3.1 equation states the investment level at time t is equal to the increase in capital less the depreciated ones. The flow of funds constraint is:

$$I_t + Rb_{t-1} + d_t = f(k_{t-1}) + b_t \tag{3.2}$$

where $R$ denotes the gross interest rate and $b_{t-1}$ represents the debt from the preceding period. The components of the flow of funds (f-of-f) at time $t$ include:

1. $I_t$ the net investment at time t
2. $Rb_{t-1}$ repayment of debts (capital and interest) of the previous period
3. $d_t$ dividends distributed at time t

Conversely, the right-hand side details the sources of fund inflows:

1. $f(k_{t-1})$ output of production at time t
2. $b_t$ debt contracted at time t

The f-of-f constraints are further elaborated as the equation governing debt's movement, where $S_t = f(k_{t-1}) - d_t$ represents the earnings retained, with the application of equations:

$$b_t = Rb_{t-1} + I_t - S_t$$

This formulation clarifies that the debt level at time $t$ is the sum of the repayment for the previous period's debt (both capital and interest) and the net investment, adjusted for internal financing.

Utilizing these equations, we derive the law of motion for net worth as follows:

$$\begin{aligned} n_t = k_t - b_t &= k_{t-1}(1-\delta) + I_t - Rb_{t-1} - I_t + S_t \\ &= k_{t-1} - \delta k_{t-1} - b_{t-1} - rb_{t-1} + S_t \\ &= n_{t-1} - \delta k_{t-1} - rb_{t-1} + [f\,(k_t - 1) - d_t] \end{aligned}$$

Thus, the firm's net worth or equity at any given time is the previous period's net worth, adjusted for depreciation of capital and interest on the previous debt, increased by retained earnings. Therefore, a firm can enhance its net worth by either boosting retained earnings, which can be achieved by increasing output or reducing dividends.

### 3.2.1 Steady State

From 3.1 we can retrieve the locus in which capital ($k_{t-1} = k_t = \widehat{k}$) and debt ($b_{t-1} = b_t = \widehat{b}$) and for definition even dividends ($d_t = d_{t-1}$):

$$\widehat{k} = \widehat{k}\,(1-\delta) + \widehat{I} \tag{3.3}$$

$$\widehat{I} = \delta\widehat{k} \tag{3.4}$$

The 3.4 stated that in the steady state, the firm will invest only to substitute depreciated capital($\delta\widehat{k}$). From 3.3 substituting the stationary conditions, we get:

$$\widehat{b} = R\widehat{b} + \widehat{I} - \widehat{S} \tag{3.5}$$

$$\widehat{S} - \widehat{I} = r\widehat{b} \tag{3.6}$$

$$f\left(\widehat{k}\right) - \widehat{d} - \delta\widehat{k} = r\widehat{b} \tag{3.7}$$

The above equation 3.7 states that in the steady state, the retained earnings should be used only to repay matured interest over debt. Equation 3.7 can be rewritten as:

$$f(\widehat{k}) = \delta \cdot \widehat{k} + r \cdot \widehat{b} + \widehat{d} \tag{3.8}$$

The equation referenced as 3.8 indicates that, at equilibrium, the output must cover the costs of interest, dividends, and depreciation. To illustrate the equilibrium locus, one can refer to the graph in 3.1, which depicts the locus as defined by equation 3.8, employing the specified production function.

$$f(k_{t-1}) = Z \cdot k_{t-1}^{\alpha}, \tag{3.9}$$

with $Z$ indicating the firm's productivity level, and $k_t$ symbolizing capital as in the model by Caballero and Hammour [1994].

The figure illustrates the steady state relationships among debt $(\widehat{b})$, capital $(\widehat{k})$, and dividends $(\widehat{d})$ in a three-dimensional plot. The graph demonstrates how various combinations of debt and capital influence the distribution of dividends. It is evident that increasing the level of debt results in lower dividends, as a larger portion of resources is allocated towards servicing interest payments. Conversely, the relationship between capital and dividends is depicted as convex, highlighting an increase in dividends with

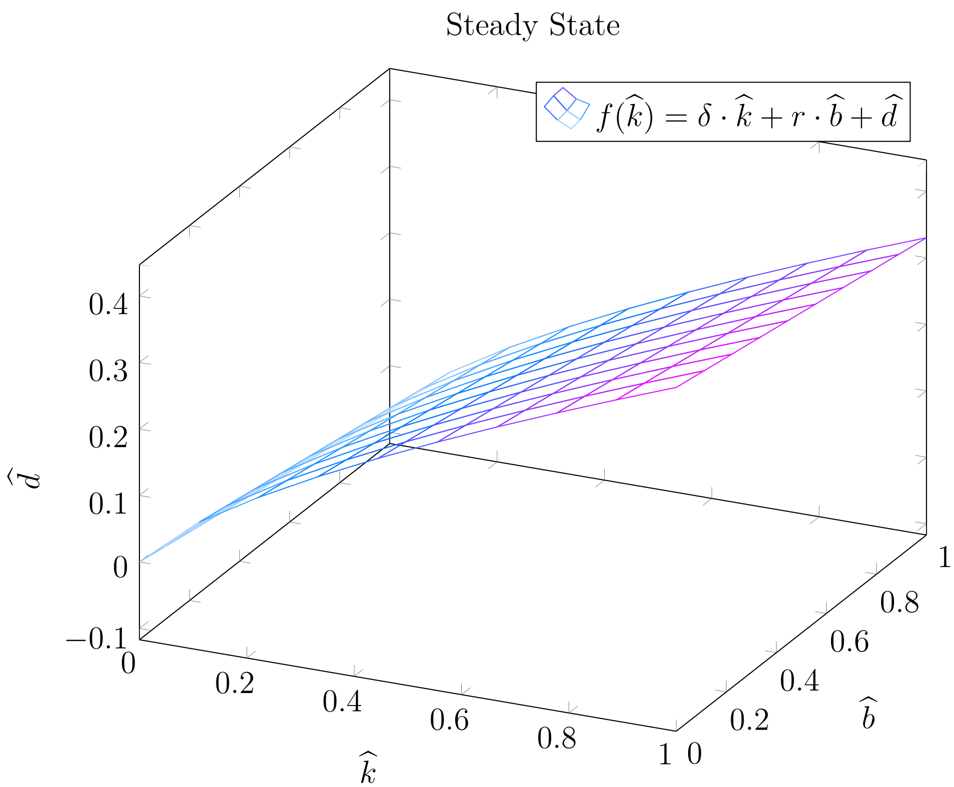


Figure 3.1: For this plot the following value has been used: $\delta = 0.1, r = 0.1, \alpha = 0.8, Z = 0.5$

higher capital levels, under the specified model parameters: $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, and $Z = 0.5$. For example, the firm starting with initial capital $k_0 = 0.2$ and debt $b_0 = 0.1$, to maintain a steady state for both capital and debt, the dividends should be equal to $\widehat{d} = 0.5 \times 0.2^{0.8} - 0.1 \times 0.2 - 0.1 \times 0.1$. This specific combination of $k = 0.2$, $b = 0.1$, $d \approx 0.11$represents a stationary point in the model.

### 3.2.2 Dynamics of capital

This section aims to elucidate the dynamics of capital in the face of deviations from steady-state conditions, with a particular focus on how alterations in dividend policies influence capital accumulation and distribution. In the simplified scenario where a firm operates without incurring debt ($b_t = 0$ for all $t$), the fundamental law of capital motion is modified as indicated by equation 3.2:

$$I_t + d_t = f(k_{t-1}) \tag{3.10}$$

$$I_t = S_t \tag{3.11}$$

This setup posits that in a debt-free environment, a firm's investment is solely financed through retained earnings, as specified in equation 3.11. From this premise, we derive a finite difference equation describing how capital evolves over time:

$$k_t = k_{t-1}(1 - \delta) + f(k_{t-1}) - d_{t-1} \tag{3.12}$$

To delve into the operational mechanics of this equation, we apply the production function introduced in 3.9 and differentiate with respect to $k_{t-1}$:

$$\frac{\partial k_t}{\partial k_{t-1}} = (1-\delta) + f'(k_{t-1}) \tag{3.13}$$

$$\frac{\partial k_t}{\partial k_{t-1}} = (1-\delta) + \alpha Z k_{t-1}^{\alpha-1} \tag{3.14}$$

Analysis reveals two distinct scenarios based on the derivative of capital with respect to $k_{t-1}$: "'latex

1. **Exploding Path When Derivative Is Greater Than 1**: If the derivative with respect to $k_{t-1}$ is greater than 1 and dividends are positive, the capital's trajectory becomes unsustainable. Specifically, if $k_{t-1}$ is below the steady-state level ($\widehat{k}$), capital dwindles to zero. Conversely, if $k_{t-1}$ exceeds $\widehat{k}$, capital increases without bound. The mathematical condition for this scenario is:

$$(1-\delta) + \alpha Z k_{t-1}^{\alpha-1} > 1, \tag{3.15}$$

$$\alpha Z k_{t-1}^{\alpha-1} > \delta, \tag{3.16}$$

$$k_{t-1} > \left(\frac{\delta}{\alpha Z}\right)^{\frac{1}{\alpha-1}}. \tag{3.17}$$

2. **Unbounded Path Without Steady State**: In cases where the partial derivative with respect to $k_{t-1}$ is less than 1, despite positive dividends, capital at time $t$ will be lower than at $t-1$, indicating a consistent decrease without achieving a

steady state. The condition for this behavior is:

$$(1-\delta) + \alpha Z k_{t-1}^{\alpha-1} < 1, \tag{3.18}$$

$$\alpha Z k_{t-1}^{\alpha-1} < \delta, \tag{3.19}$$

$$k_{t-1} < \left(\frac{\delta}{\alpha Z}\right)^{\frac{1}{\alpha-1}}. \tag{3.20}$$

"' These conditions illustrate the intricate relationship between capital's evolution and dividend policies under the absence of debt, clarifying the dynamics that lead to either unsustainable growth or contraction of capital based on the initial levels relative to a critical threshold. The following phase diagram represent the former case $\frac{\partial k_t}{\partial k_{t-1}} \leq 1$, using the following parameters: $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, $Z = 0.5$, and $d = 0.8$ The graph vividly displays the behavior of capital relative to a critical point, represented by a red dot, denoting the steady-state capital level $(\widehat{k})$. When a firm's capital at time $t$ is below this point, it signifies that the firm's capital is below the steady-state. This scenario leads to a diminishing capital trajectory, where the firm's capital gradually decreases. This reduction in capital is driven by operational outflows, such as depreciation and dividends, surpassing the firm's production output. In this context, maintaining constant and high dividend payouts necessitates a larger capital base to balance against production to prevent the depletion of capital towards zero.

Conversely, if the firm's capital surpasses the steady-state level, it finds itself in a state of overcapitalization, setting off an explosive growth trajectory where capital amplifies without limit. This pattern indicates that the firm's capital is in excess of the steady-state requirement, resulting in production significantly exceeding the operational outflows.

Additionally, the slope of the blue line in the graph indicates the productivity factor $Z$. A sharper slope signifies higher productivity, suggesting that firms with superior

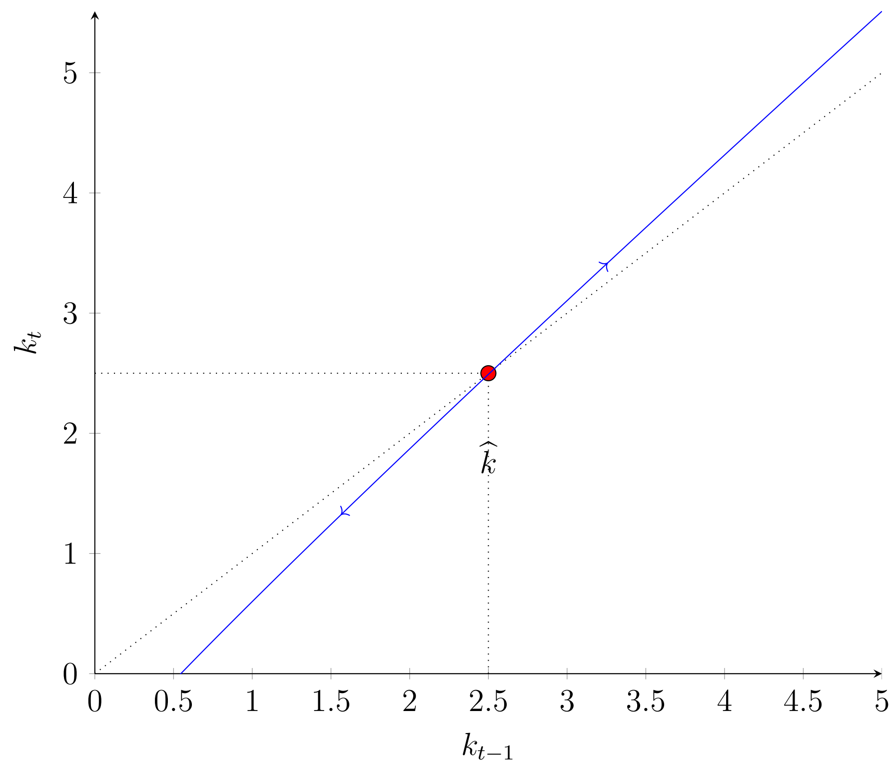


Figure 3.2: The graph illustrates the evolution of capital in a scenario devoid of debt, under the specific parameters $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, $Z = 0.5$, and $d = 0.8$. It features a blue line that represents the capital's trajectory, computed based on the formula $k_t = 0.5 \cdot k_{t-1}^{0.8} - 0.1 \cdot k_{t-1} + k_{t-1} - 0.8$, which encapsulates the dynamics of capital accumulation and its interaction with depreciation, return rate, and dividends within the system.

productivity can cover their operational expenses with less capital. This feature is particularly important for understanding a firm's ability to navigate through economic downturns, as it implies that more productive firms can sustain operations and remain viable with lower levels of capital, compared to less productive firms.

### 3.2.3 Dynamics of debts

To examine the dynamics of debt, consider a scenario where capital remains constant $k_t = k_{t-1} = \widehat{k}$, thus it is at the steady-state level. From equation 3.8 we get the finite difference equation for debt:

$$b_t = -f(\widehat{k}) + \delta\widehat{k} + Rb_{t-1} + d \tag{3.21}$$

Let's determine the condition for a stable path taking the partial derivatives with respect to $b_{t-1}$:

$$\frac{\partial b_t}{\partial b_{t-1}} = R \tag{3.22}$$

$$\tag{3.23}$$

Since $R > 1$, the partial derivative 3.21 will always be greater than one, thus the slope of the finite difference equation for debt will always be steeper than one. Adding a negative intercept due to positive dividends we get that under those conditions there exists a steady state of debt. Moreover, if the debt is below the steady state, the debt will shrink toward 0, while if the debt is over the steady state the dynamics of debt will explode toward $+\infty$. This is represented in the following phase diagram: The phase diagram illustrates the relationship between a firm's current debt ($b_t$) and its capacity

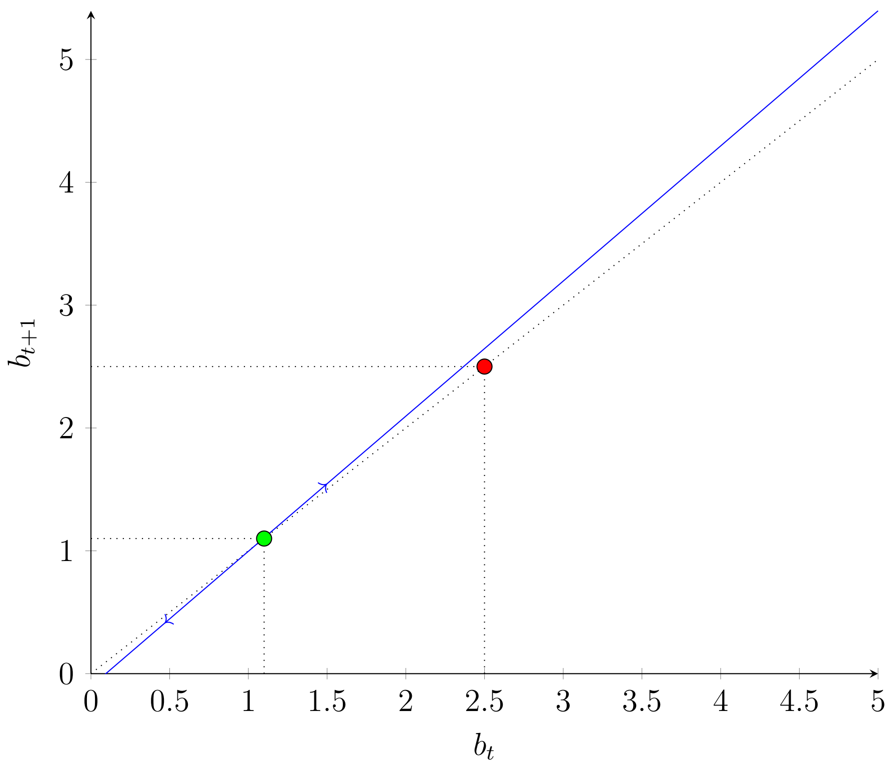


Figure 3.3: The phase diagram illustrates the progression of debt under the condition that the change in capital $(\Delta k)$ is zero, with parameters set at $\delta = 0.1$, $r = 0.1$, $\alpha = 0.8$, $Z = 0.5$, $d = 0.8$,$\widehat{k} = 3$ . The blue line represents the finite difference equation for debt as modeled by the equation $b_t = -f(\widehat{k}) + \delta \cdot \widehat{k} + R \cdot b_{t-1} + d_{t-1}$. The red dot marks the threshold beyond which debt cannot exceed capital, effectively serving as a limit on debt. The green dot signifies the steady state of the debt. The vertical or horizontal gap between the red and green dots quantifies the firm's equity.

for future operations ($k_{t+1}$), within the context of constant dividends. The steady state is indicated by the red dot, signifying the juncture at which the firm's output is precisely adequate to cover dividends, depreciation, and interest on its steady-state debt. In essence, the graph conveys how steady-state conditions are shaped by dividend policy and productivity, with the former influencing the firm's financial leverage and the latter determining its capital efficiency.

## 3.3 Free debt case: Ramsey-Cass-Kooopmans reinterpreted

This section outlines the intertemporal maximization problem faced by the firm in the free debt case, which is a Ramsey-Cass-Kooopmans model where there is a firm that seeks to maximize the utility of dividends instead of consumptions levels. The goal consists of maximizing the present value of future dividends, formulated as:

$$V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t),$$

where $U' > 0, U'' < 0$.

### 3.3.1 Steady State derivation

Consider a firm entirely financed by equity ($b_t = 0$ for all $t$), leading to a simplified flow-of-funds constrant equation:

$$k_{t+1} = k_t(1-\delta) + f(k_t) - d_t. \tag{3.24}$$

The maximization problem is tackled using a Lagrangian method, where the Lagrangian is defined as:

$$L_0 = \sum_{t=0}^{+\infty} \left[\beta^t U(d_t) - \beta^t \lambda_t \left[k_{t+1} - k_t(1-\delta) - f(k_t) + d_t\right]\right].$$

The first-order conditions for $d_t$, $k_{t+1}$, and $\lambda_t$ for all periods $t = 0, 1, \ldots$ yield:

$$U'(d_t) = \lambda_t, \quad \forall t,$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1}[f'(k_{t+1}) + (1-\delta)], \quad \forall t,$$

This approach delineates the optimal strategy for dividend distribution and capital allocation in a debt-free case. In the infinite horizon model, the transversality condition reads: $\lim_{T\to\infty} \beta^T U'(d_t)\, k_{T+1} = 0$. Thus, policies promoting accelerated capital accumulation are ruled out.

Each Lagrange multiplier $\lambda_t$ represents the marginal utility of dividends in period $t$.

From these first-order conditions (FOCs), we derive the Euler equation for dividends:

$$U'(d_t) = \beta U'(d_{t+1})[f'(k_{t+1}) + (1-\delta)] \tag{3.25}$$

indicating that the marginal utility of distributing dividends at time $t$ should match the discounted marginal utility of distributing dividends in the next period, adjusted for the net marginal product of capital after accounting for depreciation.

**Steady state condition for dividens** Imposing the steady state condition for dividends $d_t = d_{t+1} = \widehat{d}$ in 3.25 , we equate the marginal utilities across two consecutive periods:

$$U'(d_t) = U'(d_{t+1}) :$$

$$\frac{1}{\beta} = [f'(k_{t+1}) + (1 - \delta)],$$

This condition is satisfied if:

$$f'(k_{t+1}) = \frac{1}{\beta} - (1 - \delta),$$

Let's assume that:

$$y_{t+2} = Zk_{t+1}^{\alpha} = f(k_{t+1}) \tag{3.26}$$

then:

$$f'(k_{t+1}) = Z\alpha k_{t+1}^{\alpha-1}, \tag{3.27}$$

From 3.25 and 3.27, we get the steady state level of capital:

$$\widehat{k} = \left[\frac{\alpha\beta Z}{1 - \beta(1 - \delta)}\right]^{\frac{1}{1-\alpha}}. \tag{3.28}$$

**steady state condition for capital** Imposing steady state condition for capital $(k_t = k_{t-1} = \widehat{k})$ in the law of motion of capital 3.24 we get:

$$\widehat{d} = f(\widehat{k}) - \delta\widehat{k} \tag{3.29}$$

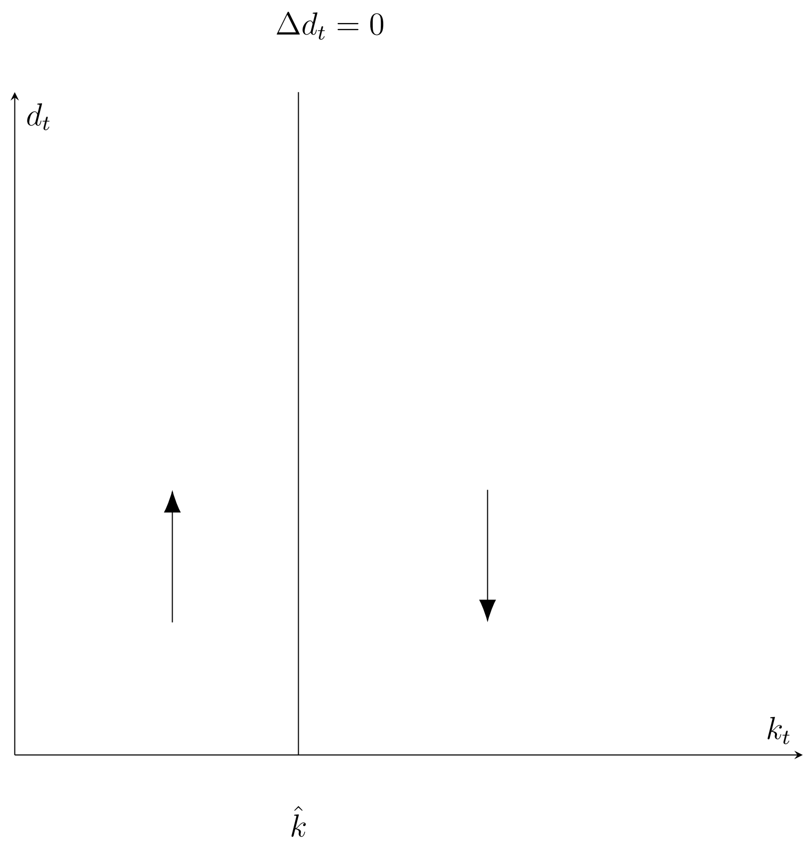


Figure 3.4: Phase diagram of dividends concerning capital, depicting the steady state level for dividends.

Using the s. s. level of capital 3.28 and 3.26 into 3.29, we get the s.s. dividend level $\widehat{d}$:

$$\widehat{d} = Z\left[\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}} \tag{3.30}$$

Thus we found the steady state level for capital and dividends.

### 3.3.2 Phase diagram

**Steady state for dividends** In this section, we will plot the phase diagram for capital and dividends exploiting steady-state conditions for capital and dividends.

The graph portrays the dynamics of dividends ($d_t$) in relation to the capital ($k_t$)

of a firm, with a particular focus on the behavior when capital is below or above the steady-state level, denoted by $\hat{k}$.

When the capital is below the steady-state level ($k_t < \hat{k}$), thus on the left of the vertical line, the firm is optimal to increase dividends over time ($d_t < d_{t+1}$) as represented by the arrow point upward. When instead ($k_t > \hat{k}$), dividends must shrink over time ($d_t > d_{t+1}$).

**Steady state for capital** Lets look at the locus in which capital is stationary $\Delta k = 0$ is given by the f-of-f constraint 3.29:

$$\widehat{d} = f(\widehat{k}) - \delta\widehat{k} \tag{3.31}$$

In our case, as obtained in the above section, the locus in which capital is stationary becomes 3.30:

$$\widehat{d} = Z\left[\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}} \tag{3.32}$$

This function starts at the origin since ($f(0) = 0$), with a maximum in $\underline{k}$ (defined as capital level such that $f'(\underline{k}) = \delta$). From equation 3.27 we can find the level of capital that maximizes dividends at the steady state of capital:

$$\underline{k} = \left[\frac{\alpha Z}{\delta}\right]^{\frac{1}{1-\alpha}} \tag{3.33}$$

While $\bar{k}$ is the capital level such that ($d_t = 0$), thus its obtained by solving ($f_(\bar{k}) - \delta\bar{k} = 0$) solving for the Cobb-Douglas production function 3.26, we get:

$$Z\bar{k}^{\alpha} = \delta\bar{k} \tag{3.34}$$

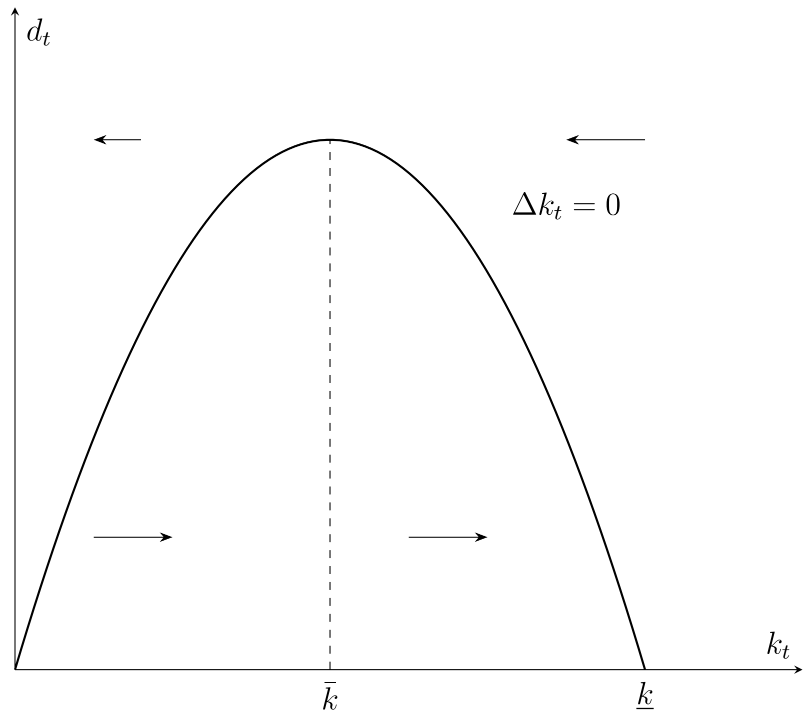


Figure 3.5: Phase diagram of dividends concerning capital, depicting the steady state level for capital.

In the case in which a capital level $\check{k} \in \left[0, \bar{k}\right]$, the corresponding dividends level that guarantee the stationarity of capital is:

$$\check{d} = f(\check{k}) - \delta\check{k} \tag{3.35}$$

If the firm distributes more dividends than $\check{d}$ the capital stock must decrease over time: since the dividends are too high the firm is consuming part of her capital. More precisely the firm is distributing more dividends than $\check{d}$, which guarantees that the difference between gross production, and dividends is exactly equal to capital depreciation. This behavior is represented by the arrows above the curve pointing to the left. If the firm consumes fewer dividends than $\check{d}$, the opposite happens: the firm increases its capital since there is a positive net investment. This behavior is represented by the arrows

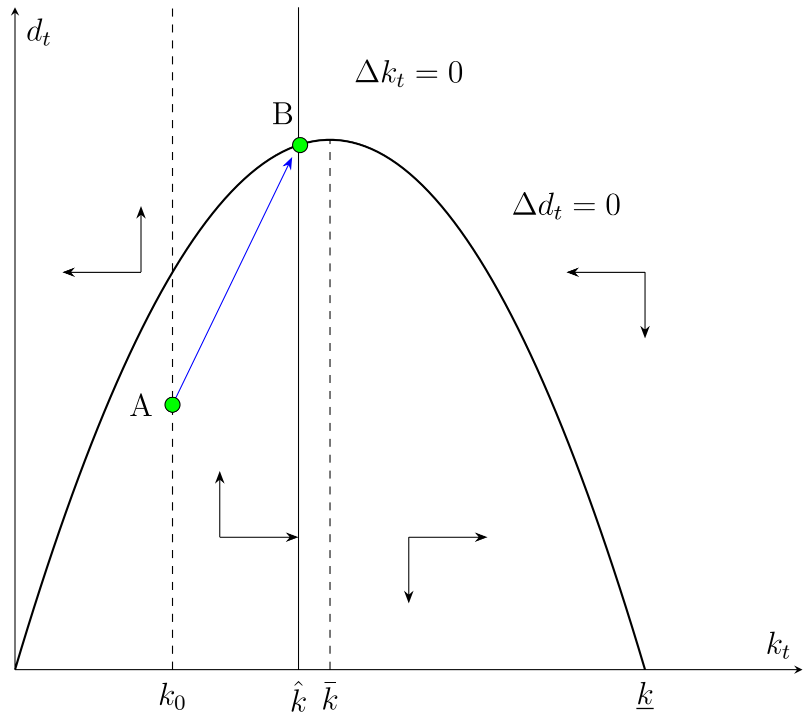


Figure 3.6: Dynamics of consumption concerning capital accumulation, showing the points of stability and instability along the curve.

below the curve pointing toward the right.

**Steady state for capital and dividend** Merging the plot above we get a phase diagram that represents the conditions for stationarity. Notice that there exits 3 steady states: one at the origins due to the assumption $f(0) = 0$, the point $(\bar{k}; 0)$, and finally point B. Point B was obtained in the previous paragraph 3.28 and 3.30 and represented the point in which dividends and capital are at a steady state, and both are strictly positive (this point is also referred to as saddlepoint). The blue line depicts a possible saddlepath towards the B. Starting at A, the firm chooses exactly the dividend level that leads to the stationary point B. This path not only fulfills the difference equations

3.25 and 3.29, but also, the transversality condition which states:

$$\lim_{T \to \infty} \beta^T U' (d_t) k_{T+1} = 0 \tag{3.36}$$

Indeed as $t \to \infty$, capital and dividends approach their steady-state level which are both positive and finite, thus the marginal utility of dividends at $\hat{d}$ is also finite, hence 3.36 is valid.

In conclusion, in this paragraph, we have derived the steady-state levels for both capital and dividends and plotted the steady-state conditions in a phase diagram. In the next section, we will repeat the same exercise introducing debt and financial friction depicting

## 3.4 Introducing financial frictions

In this section we tackle the infinite maximization problem of the firm, introducing the possibility of financing through debt and two types of financial frictions. The first financial friction is a financing constraint ($\forall t, b_t = lk_t$), which implies fixed leverage for the firm. The second financial friction is introducing a participation constraint with monitoring cost $1 - \mu$ for the financial intermediaries. The goal is to understand how those frictions affect the steady state of capital and dividends.

### 3.4.1 Participation constraint of the financial intermediaries

The subsection delves into the constraints facing financial intermediaries within the model, highlighting how firms can finance themselves either through retaining dividends or accruing debt. Initially, the model assumed an exogenous interest rate, unaffected by the volume of debt, leading to an unrealistic scenario where interest rates remain constant regardless of debt levels relative to equity. To address this, the model introduces a financial market where the interest rate is determined by market-clearing conditions, and financial intermediaries operate under perfect competition to maximize profits.

According to Bernanke and Gertler [1986], lending should yield a profit equivalent to the opportunity cost of capital. Lenders earn interest plus the principal if borrowers repay successfully (with probability $p$) or acquire the firm's production assets (net of monitoring cost: $1 - \mu$) in case of bankruptcy. The lender's participation constraint is formulated as:

$$R_t \cdot b_t p + (1 - p)\mu f(k_t) = R_f b_t,$$

where $r_f$ represents the risk-free rate, aligning the opportunity cost of capital with risk-free returns. This framework allows for the derivation of the interest rate as a function of $p$ and $f(k_t)$, assuming no financial frictions and perfect information for lenders to accurately estimate recoverable amounts in all firm states.

The revised participation constraint is expressed as:

$$R_t = \frac{R_f}{p} - \frac{1 - p}{p} \frac{\mu f(k_t)}{b_t}. \tag{3.37}$$

For illustration, consider parameters $mu = 1$, $p = \{0.95, 0.9\}$, $\delta = 0.1$, $\alpha = 0.8$, $Z = 0.5$, $d = 0.8$, $\widehat{k} = 4$, and $R_f = \{0.05, 0.1\}$. The graphical representation suggests that as debt levels increase, so do interest rates, reflecting the risk-return dilemma for lenders. A higher risk profile, denoted by a more elevated red line necessitates greater returns to compensate for default risks. It's important to note that while the the graph assumes constant capital, real-world scenarios often see debt increases leading to higher capital and, consequently, greater production capacities. This reasoning clarifies why

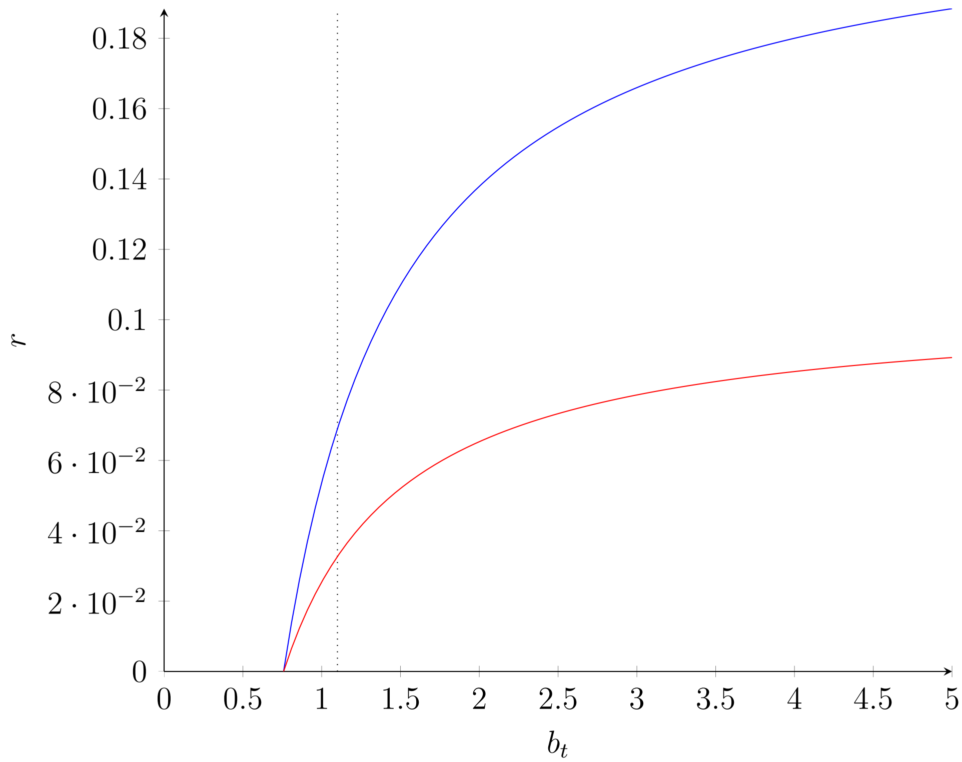


Figure 3.7: The figure presents a graphical analysis of the returns on loans as a function of the loan amount under a fixed capital level of $k = 4$. The red curve models the scenario where the default risk probability is $1 - p = 0.05$ , implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1 - p = 0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the debt stock grows. Notably, the opportunity cost of capital is maintained at 0.05 for the red one, while at 0.1 for the higher risk curve.

the curves do not start from the origin, as initial borrowing incorporates capital costs.

The graph 3.7 captures the dynamics between the debt stock $b_t$ and the return on capital $r$. An increase in the debt stock leads to a rise in the interest rate, reflecting the augmented risk perceived by lenders. Displayed are two distinct lines: one representing a riskier loan with a higher probability of default and the other indicating a safer loan with a lower default probability. As anticipated, the riskier loan scenario is characterized by a curve that lies above, dictating higher interest rates at each level of debt. The constant capital assumption underpins this model; however, in reality,

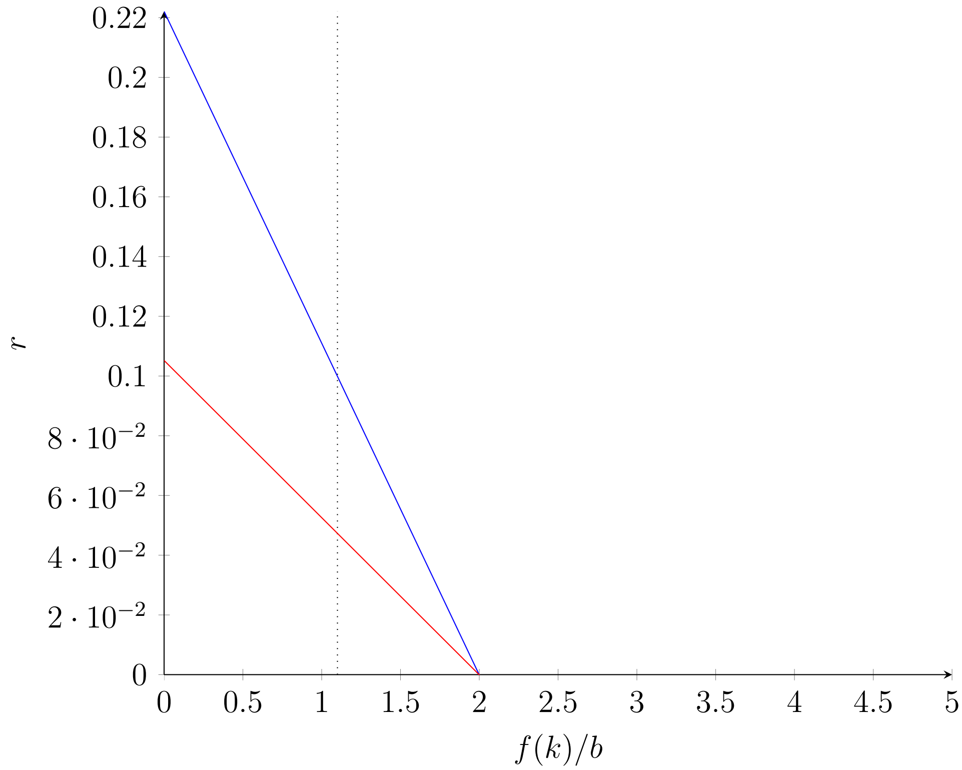


Figure 3.8: The figure presents a graphical analysis of the returns on loans as a function of the production over the debt level while keeping the level of capital at $k = 3$. The red curve models the scenario where the default risk probability is $1 - p = 0.05$ , implying a 5% chance of default, while the blue curve corresponds to a higher default risk at $1 - p = 0.1$, a 10% chance of default. Both curves reflect the increased interest rates required to compensate for the heightened risk as the production-debt ratio grows. Notably, the opportunity cost of capital is maintained at 0.05 for the red one, while at 0.1 for the higher risk curve.

an increase in debt usually translates into an increase in capital, thereby enhancing production potential. This factor accounts for the curves not starting at the origin.

Another way to visualize the participation constraint of the financial intermediaries is by defining $x = f(k)/b$. The graph 3.8 delineates a critical boundary within the participation constraint framework: as leverage approaches unsustainable levels, the interest rate escalates to a certain peak, signifying a cap on the maximum interest rate that deviates from the theoretical possibility of infinity. This ceiling on the rate is attributed to the fact that the probability of default, denoted by $p$, remains fixed and does not escalate alongside increasing leverage.

Ultimately, the participation constraint internalizes the interest rate of a loan as a function of the leverage, the opportunity cost of capital, and the default risk probability. By integrating this mechanism into the flow of funds model, the impact of debt on capital is mediated through the variable $r$, establishing a feedback loop where financial leverage influences and is influenced by the cost of borrowing.

### 3.4.2 Steady state and phase diagram

The firm's objective is to maximize its value through the optimal selection of dividends over time:

$$\max_{\{d_t\}_{t=0}^{+\infty}} V_0 = \sum_{t=0}^{+\infty} \beta^t U(d_t)$$

subject to:

1. the flow of funds constraint 3.2,
2. the investment function 3.1,
3. the financing constraint $b_t = lk_t \quad \forall t$
4. the participation constraint of borrower 3.37

Consolidating the constraints we get the flow of funds constraints:

$$\begin{aligned} k_t &= \left\{ k_{t-1}(1-\delta) - \left[ \frac{R_f}{p} - \frac{1-p}{p} \frac{\mu f(k_{t-1})}{lk_{t-1}} \right] \cdot lk_{t-1} + f(k_{t-1}) - d_t \right\} (1-l)^{-1} \\ k_t &= \left[ \frac{1+\mu-\mu p}{p} f(k_{t-1}) + \frac{p-\delta p - R_f l}{p} k_{t-1} - d_t \right] (1-l)^{-1} \end{aligned} \tag{3.38}$$

The Lagrangian for this optimization problem is formulated as:

$$L = \sum_{t=0}^{+\infty} \beta^t U(d_t) - \beta^t \lambda_t \left[ \frac{1+\mu-\mu p}{p} f(k_{t-1}) + \frac{p-\delta p - R_f l}{p} k_{t-1} - d_t \right] (1-l)^{-1}, \tag{3.39}$$

leading to the first-order conditions for optimizing dividends and capital over time:

$$U'(d_t) = \frac{\lambda_t}{(1-l)}, \quad \forall t, \tag{3.40}$$

and the dynamic optimality conditions for capital allocation:

$$\lambda_t = \beta \frac{\lambda_{t+1}}{(1-l)} \left[ f'(k_{t-1}) \frac{1+\mu-\mu p}{p} + \frac{p-\delta p - R_f l}{p} \right], \quad \forall t. \tag{3.41}$$

This formulation yields the Euler equation for dividends:

$$U'(d_t) = \frac{\beta}{(1-l)} U'(d_{t+1}) \left[ f'(k_{t-1}) \frac{1+\mu-\mu p}{p} + \frac{p-\delta p - R_f l}{p} \right], \tag{3.42}$$

imposing ($d_t = d_{t+1} = \hat{d}$), we get:

$$\begin{aligned} \frac{(1-l)\,p}{\beta} &= f'(\hat{k})\,(1+\mu-\mu p) + (p - \delta p - R_f l) \\ f'(\hat{k}) &= \frac{p - pl - \beta p + \beta \delta p + \beta R_f l}{\beta\,(1+\mu-\mu p)} \end{aligned} \tag{3.43}$$

using the Cobb Douglas production function 3.27 into 3.43 we get:

$$
\begin{aligned}
Z\alpha\hat{k}^{\alpha-1} &= \frac{p - pl - \beta p + \beta\delta p + \beta R_f l}{\beta\left(1+\mu-\mu p\right)} \\
\hat{k} &= \left[\frac{Z\alpha\beta\left(1+\mu-\mu p\right)}{p - pl - \beta p + \beta\delta p + \beta R_f l}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
\tag{3.44}
$$

As in the free debt case, if the firm has less capital than the steady-state level $\hat{k}$, the firm is optimal to increase her dividends over time. When instead $\hat{k} < k$,the firm should be better of shrinking the dividends over time. Indeed it's easy to prove that imposing monitoring cost $1-\mu = 1$, no debt $l = 0$, and no probability of default $1-p = 0$ we get the same capital level as in the debt-free case 3.28. Moreover its easy to see that the steady-state capital is higher compared to the debt-free:

$$
\begin{aligned}
\left[\frac{Z\alpha\beta(1+\mu-\mu p)}{p - pl - \beta p + \beta\delta p + \beta R_f l}\right]^{\frac{1}{1-\alpha}} &\geq \left[\frac{\alpha\beta Z}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}} \\
\frac{(1+\mu(1-p))(1-\beta(1-\delta))}{p(1-l)+\beta(R_f l - p(1-\delta))} &\geq 0 \\
(1+\mu(1-p))(1-\beta(1-\delta)) &\geq 0 \\
p(1-l)+\beta(R_f l - p(1-\delta)) &\geq 0
\end{aligned}
$$

However, if we consider the case on which has no cost of monitoring $\mu = 1$ and on which has $\mu = -.75$ ceteris paribus, $\hat{k}$ will be higher for the frictionless case.

**Steady state for capital** Imposing s.s. condition for capital ($k_t = k_{t+1} = \hat{k}$) into the flow of funds constraint 3.38:

$$
\widehat{d} = \frac{1+\mu-\mu p}{p} f(\hat{k}) - \left(\frac{lR_f + \delta p - lp}{p}\right)\hat{k}
\tag{3.45}
$$

It can be straightforwardly demonstrated that by setting the monitoring cost to $1-\mu = 1$, eliminating debt with $l = 0$, and removing the risk of default by setting $1-p = 0$, we arrive at an identical level of dividends as observed in the scenario without debt 3.29.

The dynamics are equal to the free debt case except for the fact that the coefficient that multiplied $f(\hat{k})$ is higher since:

$$\frac{1+\mu-\mu p}{p} > 1$$
$$1+\mu(1-p) > p$$

This is valid also for the coefficient of $\hat{k}$ in equation 3.45, which is higher than the free debt case:

$$\frac{lR_f+\delta p-lp}{p} > (1-\delta)$$
$$\frac{lR_f+\delta p-lp}{p(1-\delta)} > 0$$
$$R_f l > p(l-\delta)$$
$$p(1-\delta) > 0$$

### 3.4.3 Phase diagram

The goal of this section is to portray the phase diagram in two cases: one with monitoring costs and one without. However, we will use a less heuristic approach compared to the phase diagram of the free debt case, using the same value for parameters similar to Osotimehin and Pappadà [2017]:

| Parameter | Symbol | Value |
|---|---|---|
| Discount factor | $\beta$ | 0.956 |
| Risk-free rate | $R_f$ | 1.04 |
| Depreciation rate | $\delta$ | 0.07 |
| Returns to scale | $\alpha$ | 0.80 |
| Aggregate productivity | $\bar{Z}$ | 0.5 |
| Monitoring cost | $1-\mu$ | $0, 0.75$ |
| Productivity | $Z$ | 0.2 |
| Probability of default | $1-p$ | 0.6 |

Table 3.1: Parameters used in Osotimehin and Pappadà [2017]

Moreover, we assume a fixed leverage of $l = 0.8$, since for the moment we want to understand the effect of monitoring cost leaving all the other parameters equal.

The phase diagram illustrated in 3.9 depicts the capital accumulation dynamics under scenarios of fixed leverage and varying monitoring costs. While the overall dynamics remain consistent across both scenarios, the equilibrium capital level is notably reduced in firms that incur monitoring costs, in contrast to those without such costs. As a result, firms with monitoring costs settle into a steady state equilibrium for dividends, which leads to diminished dividend distributions compared to firms that do not bear these costs.

## 3.5 Finding optimal path

Addressing the dynamic optimization problem with an initial condition $k_0$, we employ a logarithmic utility function and frame the issue through a Bellman equation:

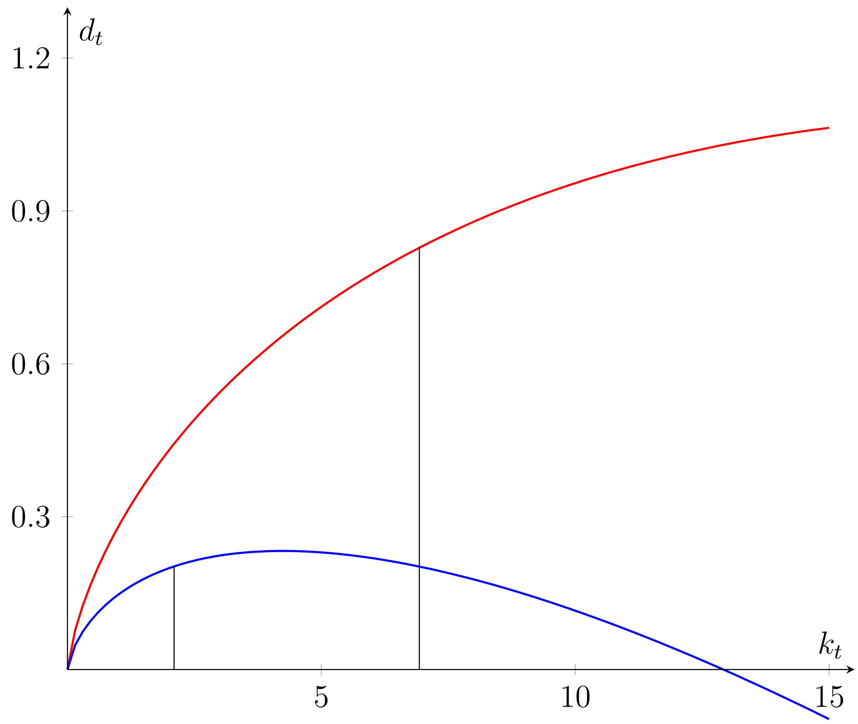


Figure 3.9: This phase diagram depicts the dividends dynamics as they relate to capital. The red line corresponds to a firm that is carrying debt without monitoring costs, whereas the blue-line represents the same firm but with monitoring costs

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U(d_0) + \beta \left[ \sum_{t=1}^{\infty} \beta^{t-1} U(c_t) \right] \right\}$$

subject to a dynamic capital accumulation constraint:

$$k_t = \left\{ k_{t-1}(1-\delta) - \left[ \frac{R_f}{p} - \frac{1-p}{p} \frac{f(k_{t-1})}{l \cdot k_t} \right] l k_{t-1} + f(k_{t-1}) - d_{t-1} \right\} (1-l)^{-1} \, \forall t.$$

The aim is to determine the optimal dividend strategy $d_t^*$ and the consequent capital levels $k_{t+1}^*$ across all periods. The optimal policy $\varphi(.)$ links dividends and capital in a time-invariant manner, deduced from the constraint:

$$k_t = \left\{ k_{t-1}(1-\delta) - \frac{R_f}{p} l k_{t-1} + \frac{f(k_{t-1})}{p} - \varphi(k_{t-1}) \right\} (1-l)^{-1} = \zeta(k_1).$$

Given the continuous and differentiable nature of capital and dividends, the optimal dividend path can be represented as a function of initial capital, thereby defining the maximum value function $V(k_1)$ in terms of overall utility maximization. The revised problem formulation becomes:

$$V(k_0) = \max_{c_0} \{U(d_0) + \beta V(k_1)\} \tag{3.46}$$

$$\text{s.t. } k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - d_1 \right\} (1-l)^{-1} \tag{3.47}$$

$$k_0 \text{ given.} \tag{3.48}$$

Before proceeding, it's critical to verify the solvability of the problem, adhering to the criteria for the existence and uniqueness of the solution:

1. $0 < \beta < 1$,
2. The utility function is continuous, bounded, and strictly concave,
3. The capital transition function is concave.

These conditions ensure the solution's uniqueness and strict concavity, although the logarithmic utility function $U(d_t) = \ln d_t$ might not strictly meet these criteria. Utilizing alternative theorems allows for the relaxation of the strict concavity requirement.

The optimal strategy is derived from the first order condition $U'(d_0^*) + \beta V'(k_1)\frac{\partial k_1}{\partial d_0} = 0$, with the capital transition function implying $\frac{\partial k_1}{\partial d_0} = -1$. The solution encompasses:

$$\begin{cases} V(k_0) = U(d_0^*) + \beta V(k_1), \\ k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - d_0^* \right\} (1-l)^{-1}, \\ U'(d_0^*) = \beta V'(k_1), \\ k_0 \text{ given.} \end{cases}$$

This system guides us towards the optimal path of dividends and capital levels, underpinning the dynamic economic analysis.

**Guess and verify** The method of "guess and verify" involves proposing a return function $U(d_t) = \ln d_t$ and working through a transition equation defined as $k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - d_0^* \right\} (1-l)^{-1}$. The first order condition (FOC) is specified as $d_0 = [\beta V'(k_1)]^{-1}$. When this FOC is incorporated into the transition equation, the formulation of the problem becomes a system of equations outlined as follows:

$$
\begin{cases}
V(k_0) = \ln(d_0^*) + \beta V(k_0), \\
k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - [\beta V'(k_{t+1})]^{-1} \right\} (1-l)^{-1}, \\
U'(d_0^*) = \beta V'(k_1), \\
k_0 \text{ given.}
\end{cases}
$$

Our initial guess for the solution is:

$$
V(k_t) = e + f \ln k_t,
$$

which leads to a refined system:

$$
\begin{cases}
e + f \ln k_0 = \ln\left(\frac{k_1}{\beta f}\right) + \beta[e + f \ln k_1], \\
k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - \left[\frac{k_1}{\beta f}\right] \right\} (1-l)^{-1}.
\end{cases}
$$

Assuming a condition to simplify the analysis, $p - \delta p - R_f l = 0$, we solve for $k_1$ and $d_1$, leading to expressions that relate capital and dividends directly to the parameters of the problem. These solutions indicate that dividends are a proportion of the output, dependent on the firm's productivity, leverage, and risk-free rate. The formulation highlights how dividends and capital evolve over time, with dividends being a constant share of the period's production.

Under conditions of no debt ($l = 0$ and thus $p = 1$) and ignoring depreciation, we obtain simplified expressions for capital and dividends in the steady state. This scenario suggests higher capital accumulation for a debt-free firm, as it does not bear interest expenses. The policy function derived reflects the relationship between dividends, firm productivity, and leverage, offering insights into the management of capital and dividends in different financial states of a firm.

### 3.5.1 Optimization Problem with Financial Frictions

Following the derivation of a closed-form solution for our policy function, we now integrate financial frictions stemming from information asymmetry between lenders and firms. We model this by introducing a discount factor $\mu$ on the perceived value of production, where $0 \leq \mu \leq 1$; a value closer to 0 indicates higher friction levels. Consequently, the lending activity modifies the optimization problem as follows:

$$\max_{\{d_t\}_{t=0}^{\infty}} V_0 = \max_{\{d_t\}_{t=0}^{\infty}} \left\{ U(d_0) + \beta \left[ \sum_{t=1}^{\infty} \beta^{t-1} U(c_t) \right] \right\}$$

subject to

$$k_t = \left\{ k_{t-1}(1-\delta) - \left[ \frac{R_f}{p} - \frac{1-p}{p} \frac{\mu f(k_{t-1})}{l \cdot k_t} \right] l k_{t-1} + f(k_{t-1}) - d_{t-1} \right\} \cdot (1-l)^{-1} \; \forall t.$$

Adapting our approach from the frictionless scenario, we propose:

$$V(k_0) = \ln(d_0^*) + \beta V(k_1),$$

where

$$k_1 = \left\{ k_0(1-\delta) - \frac{R_f}{p} l k_0 + \frac{f(k_0)}{p} - \left[ \beta V'(k_{t+1}) \right]^{-1} \right\} \cdot (1-l)^{-1},$$

and

$$U'(d_0^*) = \beta V'(k_1).$$

Our hypothetical solution takes the form:

$$V(k_t) = e + f\ln(k_t),$$

leading to:

$$e + f\ln(k_0) = \ln\left(\frac{k_1}{\beta f}\right) + \beta\left[e + f\ln(k_1)\right], \tag{3.49}$$

$$k_1 = \left\{k_0(1-\delta) - \frac{R_f}{p}lk_0 + \frac{f(k_0)}{p} - \left[\frac{k_1}{\beta f}\right]\right\} \cdot (1-l)^{-1}. \tag{3.50}$$

Assuming $p - \delta p - R_f l = 0$ simplifies to:

$$k_1 = \left[k_0(p - p\delta - R_f l) + \mu Z k_0^\alpha\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1}, \tag{3.51}$$

$$d_1 = \left[k_0(p - p\delta - R_f l) + \mu Z k_0^\alpha\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right), \tag{3.52}$$

$$\begin{aligned} e + f\ln(k_0) = \ln &\left\{\left[k_0(p - p\delta - R_f l) + \mu Z k_0^\alpha\right] \cdot \left(\frac{p}{\beta f - l\beta f + 1}\right)\right\} \\ &+ \beta\left[e + f\ln\left\{\left[k_0(p - p\delta - R_f l) + \mu Z k_0^\alpha\right] \cdot \left(\frac{\beta f}{\beta f - l\beta f + 1}\right) p^{-1}\right\}\right]. \end{aligned} \tag{3.53}$$

Hence, the transition and policy functions under financial frictions are formalized as:

$$k_1 = \frac{Z\mu k_0^\alpha(1-\delta)}{lR_f}\frac{\alpha\beta}{1-l\alpha\beta}, \tag{3.54}$$

$$d_0 = \frac{Z\mu k_0^\alpha(1-\delta)}{lR_f}\frac{1-\alpha\beta}{1-l\alpha\beta}\beta, \tag{3.55}$$

$$\widehat{k} = \left[\frac{Z\mu(1-\delta)}{lR_f}\frac{\alpha\beta}{1-l\alpha\beta}\right]^{\frac{1}{1-\alpha}}. \tag{3.56}$$

This analysis elucidates that financial frictions equate to a de facto reduction in productivity, which in turn diminishes dividends, capital levels, and the rate at which firms accumulate capital.

## 3.6 Simulation Study

To explore the distinctions between scenarios with and without financial frictions, we conduct a simulation exercise employing parameters consistent with those used in the Osotimehin and Pappadà [2017] study:

| Parameter | Symbol | Value |
|---|---|---|
| Discount factor | $\beta$ | 0.956 |
| Risk-free rate | $R_f$ | 1.04 |
| Depreciation rate | $\delta$ | 0.07 |
| Returns to scale | $\alpha$ | 0.70 |
| Aggregate productivity | $\bar{Z}$ | 1 |
| Monitoring cost | $1-\mu$ | 0.25 |

Table 3.2: Benchmark calibration

Our aim is to ascertain the impact of incorporating financial frictions into the model. Therefore, we set the leverage ratio ($l = 0.5$) and the initial capital ($k_0 = 1$). This allows us to examine capital evolution along the optimal path. For instance, we calculate $p = \frac{0.5 \cdot 1.04}{1-0.07} \approx 0.559$, with the outcomes depicted in the subsequent plots:

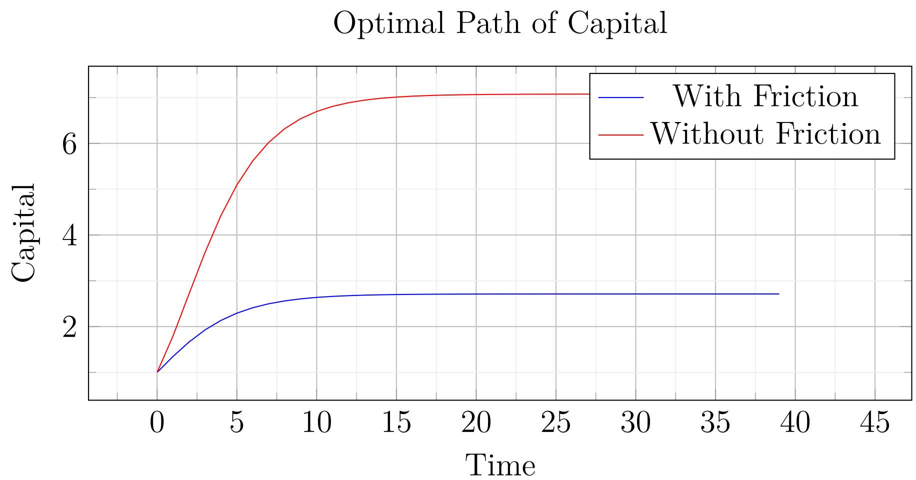


Figure 3.10: Evolution of capital over time.

The initial plot delineates the capital's transition function:

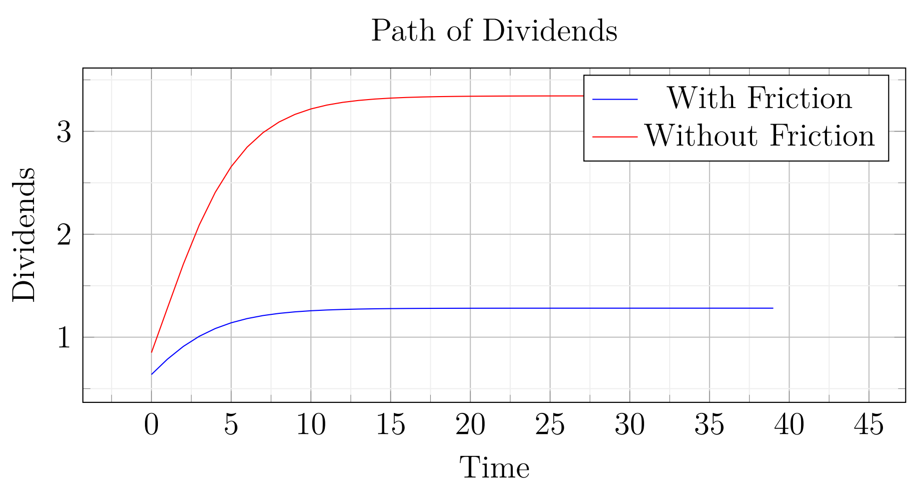


Figure 3.11: Evolution of dividends over time.

It is apparent that the trajectory of dividends is consistently higher in scenarios devoid of financial frictions, underscoring the impact of such frictions on diminishing returns. This comparison vividly demonstrates the differential outcomes in capital and dividend paths under varying financial conditions, highlighting the broader implications of financial frictions on economic performance and firm-level profitability.

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