# Theoretical framework

Riccardo Dal Cero

January 11, 2024

**Abstract**

The main idea is to study how and whether the asymmetry of information have an impact on the cleansing effect of recession, replicating the model in computer simulation.

## 1 Theoretical framework

The economy comprises risk-neutral firms with a constant discount rate represented by $0 < \beta < 1$. These firms exhibit heterogeneity in productivity and net worth. They employ a production technology that relies solely on capital (or production units) as input, featuring diminishing returns to scale.

In each period, firms incur a fixed production cost denoted as $c$ to initiate production. After production, they decide how to allocate profits for the next period. The remaining profits are invested in a risk-free asset. Firms face a choice: they can either continue operating and reinvest their profits or exit the market, investing their entire net worth, denoted as $e$, in the risk-free asset.

Firms opt to exit the market when expected profits no longer outweigh the fixed cost $c$, or when the value of production becomes inferior to the value they could gain by investing in the risk-free asset.

The value obtained from investing in the risk-free asset is given by:

$$q_t + \sum_{s=0}^{+\infty} \beta^s [\beta(1+r) - 1] e_{t+s+1}.$$

Notably, when the condition $\beta(1+r) \leq 1$ holds, this value simplifies to $q$. In such cases, firms are either indifferent regarding the timing of dividend distributions or have a preference for distributing their end-of-period net worth to shareholders or investors. In this economic model, the agents are the firms themselves, aiming to maximize their value over time by selecting an optimal level of capital denoted as $k$. The production function, accounting for the fixed cost $c$, is expressed as follows: $Y = Z(\theta + \epsilon)k^\alpha$. Key variables include:

- $Z$: Stochastic aggregate productivity common across firms.
- $\theta$: Persistent firm-specific productivity shock (modeled as a Markov Chain).
- $\epsilon$: Firm-specific productivity shock with $\epsilon \sim \mathcal{N}(0, \delta)$.
- $k^\alpha$: Capital or production units, as in Caballero and Hammour (AER).

The timeline of events is as follows:

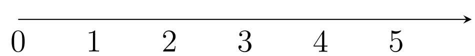


Figure 1: Timeline of Events

The sequence of events includes:

1. The firm possesses knowledge of $Z, \theta, k^\alpha, e$ (where $e$ represents its endowment, different from $k$ since the firm can borrow money with $d = c + k - e$).
2. The firm computes the optimal $k$ to maximize the expected value of the firm, with $k$ ranging from $[0, +\infty]$. If $k = 0$, it indicates the firm's decision to exit.

3. At the end of the period, the firm observes $\epsilon$ and the aggregate shock.

4. The firm repays its debt and the fixed operating cost $(c + k - e)$, resulting in an end-of-period net worth $q$.

5. The firm decides on the amount of dividends to distribute $(q - e')$, observes the productivity shock $\theta', Z'$, and the process restarts from step 1.

## 1.1 Frictionless economy

In a frictionless economy, firms have the option to borrow an amount denoted as $c+k-e$ at the risk-free interest rate $r = \frac{1}{\beta} - 1$. Therefore, at the start of the period, the firm's value is determined by the following expression:

$$V_{FL} = \max_k E \int \max[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z'))] \, d\Phi(\epsilon)$$

where the end of period net worth is equal to:

$$q = Z(\theta + \epsilon)k^\alpha + (1 - \delta)k - (1 + r)(c + k - e)$$

Under the condition of survival, it can be demonstrated that:

$$\widehat{V}_{FL}(\theta, Z) = \max_k E \int [Z(\theta + \epsilon)k^\alpha - (1 + r)c \, d\Phi(\epsilon)] + \beta \max[0, \widehat{V}_{FL}(\theta', Z')]$$

In the absence of market frictions, firms choose to exit when their productivity reaches a certain threshold. Specifically, they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined as the value for which $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

## 1.2 Economy with Credit Market Frictions

After production, the firm privately observes the temporary shock $\epsilon$, while financial intermediaries can only observe it at a cost of $\mu k^{\alpha}$. For one-period debt contracts, financial intermediaries observe $\epsilon$ only if the firm faces financial distress, which occurs when the private shock is insufficient to repay its debt. The terms of the financial contract depend on the firm's net worth $e$, current productivity $\theta$, and aggregate productivity value $Z$, all observable by both the financial intermediary and the firm at no additional cost.

**HP1 (Hypothesis 1):** The risk-free interest rate is $\beta < \frac{1}{1+r}$, which implies a lower risk-free rate in an economy with credit frictions compared to a frictionless one. It also ensures that firms do not always reinvest their profits.

When a firm defaults, the financial intermediary incurs verification costs and seizes all of the firm's income. The default threshold $\bar{\epsilon}$ is determined by the equation:

$$Z(\theta + \bar{\epsilon})k^{\alpha} + (1 - \delta)k = (1 + \widetilde{r})(c + k + e)$$

Default results in a zero net worth but does not necessarily force the firm to exit the market, depending on its persistent productivity component $\theta$.

The financial intermediary lends $(c + k - e)$ to the firm only if the expected income from the loan equals the opportunity cost of the funds, as expressed by the inequality:

$$(1+\widetilde{r})(k+c+e)(1-\Phi(\bar{\epsilon})) + \int_{-\infty}^{\bar{\epsilon}} [Z(\theta+\bar{\epsilon})k^{\alpha} + (1-\delta)k - \mu k^{\alpha}]\, d\Phi(\epsilon) \geq (1+r)(c+k+e)$$

The financial contract is characterized by $(k, \bar{\epsilon})$. Given $Z, \theta, e$, the participation constraint indicates the default threshold $\bar{\epsilon}$ required by the financial intermediary to lend a given amount. For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given $\theta, Z$, there is a unique threshold $e_b(\theta, Z)$ below which the financial intermediary refuses to lend any amount:

$$Z[\theta + G(\overline{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\overline{\epsilon}_b) = (1+r)(k_b + c - \underline{e_b})$$

where $\overline{\epsilon}_b$ maximizes the expected income of the financial intermediary. When the firm has a net worth below $\underline{e_b}$, the firm defaults.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta + \overline{\epsilon})k^\alpha + (1-\delta)k - (1+\widetilde{r})(k + c - e) & \text{if } \epsilon \geq \overline{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

Using the default condition we can rewrite as

$$q = \max[Zk^\alpha(\epsilon - \overline{\epsilon}); 0]$$

## 1.3 The firm's problem

Define $V$ as the firm's value at the start of the period, which hinges on investment outcomes and exit decisions. If the end-of-period net worth falls below a threshold ($q < e_b(\theta', Z')$), the firm exits. Otherwise, it compares its continuing value to the end-of-period net worth ($q \geq e_b(\theta', Z')$) and exits if the continuing value is lower.

The firm's value function is given by:

$$V(e, \theta, Z) = \max_{(k, \overline{\epsilon})} E\left\{ \int I(q)q + (1 - I(q)) \max[q, \max_{e'} q - e' + \beta V(e', \theta', \zeta')] \, d\Phi(\epsilon) \right\}$$

Where:

$$I(q) = \begin{cases} 0 & \text{if } q \geq e_b(\theta', Z') \\ 1 & \text{if } q < e_b(\theta', Z') \end{cases}$$

Subject to the following constraints:

1.

$$Z[\theta + G(\bar{\epsilon}_b)]k^\alpha + (1-\delta)k - uk_b^\alpha \Phi(\bar{\epsilon}_b) \geq (1+r)(k_b + c - \underline{e_b})$$

2.

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}); 0]$$

3.

$$\overline{e_b}(\theta', Z) \leq e' \leq q$$

The firm aims to maximize expected dividends while complying with the financial intermediary's participation constraint (constraint 1). Equation (constraint 2) characterizes the firm's end-of-period net worth, and Equation (constraint 3) ensures that the net worth is sufficiently high to satisfy the participation constraint.

Furthermore, the firm is prohibited from issuing new shares and can only augment its net worth by reinvesting profits. This limitation presents a trade-off: increasing capital boosts production capacity but also raises the risk of default, as the default threshold set by the financial intermediary increases with borrowed amounts.

# 2 The cleansing effect by Caballero

## 2.1 Introduction

In the first paper that rationalize the cleansing effect of recessions, authored by Ricardo J. Caballero and Mohamad L. Hammour Caballero and Hammour [1994] and published in the American Economic Review in 1998, the primary aim was to investigate how industries respond to cyclical variations in demand. They did this by employing a vintage model of creative destruction. The underlying concept postulates that the processes of creation and destruction within an industry partially explain business cycles. Industries continuously experiencing creative destruction can adapt to demand fluctuations in two ways: by adjusting the rate at which they produce new units embodying advanced techniques or by altering the rate at which outdated units are retired. The model they used incorporated heterogeneous firms, where production units embodied the most advanced technology at the time of their creation. The costs associated with creating new units slowed down technology adoption, resulting in the coexistence of production units with varying vintages.

Key to understanding how firms adapt to business cycles are the concepts of the creative margin and the destruction margin. For example, a reduction in demand can be accommodated either by reducing the rate of technology adoption or by retiring older production units. One of the primary factors determining which margin is more responsive to business cycles is the adjustment cost. When this cost follows a linear pattern, the study shows that insulation is complete, and the industry's response relies exclusively on its creation margin. Consequently, the creation margin becomes smoother over time in comparison to the destruction margin, which exhibits greater responsiveness to the business cycle.

Crucially, Caballero and Hammour's research Blanchard et al. [1990] offers theoretical insights supported by empirical evidence. Their findings on the cyclical nature of the destruction margin align with the studies conducted by Blanchard and Diamond Blanchard et al. [1990], as well as Steven Davis and John Haltiwanger Davis and Haltiwanger [1992], in their respective works from 1990. This convergence between theoretical framework and empirical substantiation underscores the importance of comprehending the dynamic interplay between creative destruction and business cycles, which significantly influences how industries respond to economic fluctuations.

In their study Davis and Haltiwanger [1992], where they assess the heterogeneity of employment changes at the establishment level in the U.S. manufacturing sector from 1972 to 1986, it is revealed that job destruction exhibits procyclical tendencies, responding more robustly to downturns in the economic cycle compared to the creation rate, in line with the theoretical model proposed by Caballero and Hammour Caballero and Hammour [1994]. The authors leverage a natural experiment inherent in the data to examine whether the structure of adjustment costs can account for the behavior of these two margins. This natural experiment arises from the asymmetric nature of business cycles, with recessions being shorter but more severe than expansions. The theoretical model predicts that these differences should be attenuated in the creation process, a prediction that is substantiated by the data since creation exhibits relative symmetry around its mean, while destruction displays a high degree of asymmetry. The underlying concept driving the behavior of the destruction margin can be traced back to the theories of Schumpeter and Hayek. They suggest that recessions represent periods during which unprofitable or outdated techniques are pruned from the economy, leaving behind the most efficient firms Hayek and Caldwell [2007].

## 2.2 Theoretical model

The model in question is a vintage model that simulates an industry experiencing exogenous technological progress. Within this model, production units are constructed using a fixed proportion of labor and capital, and they are continually being created and phased out. Notice that only the creation of new production units incurs a cost. This simplification is plausible, particularly in the context of the United States, where the expense associated with hiring is typically higher than the cost of termination, as demonstrated by Abdulkadiroğlu and Kranton (2003) Abowd and Kramarz [2003].

In this model, when a production unit is created at a specific time $t_0$, it embodies the most advanced technology available at that moment and consistently generates a uniform output represented by $A(t_0)$ throughout its operational lifetime. The productivity of this technology, denoted as $A(t)$, experiences continuous growth at an exogenously determined constant rate $\delta \geq 0$. This growth in technology can be interpreted in two ways: either as the adoption of new technology or as a product innovation. In the latter scenario, a continuum of perfectly substitutable products can yield varying levels of output.

$$[f(a,t) \qquad 0 \leq a \leq \overline{a}(t)]$$

The above function represents the cross-sectional density of the production units aged $a$ at time $t$, where $\overline{a}(t)$ is the age of the oldest production unit at time $t$. The first assumption is that $f(a,t)$ and $\overline{a}(t)$ are continuous functions. The mass of production units at time $t$ is given by:

$$N(t) = \int_{\overline{a}(t)}^{0} f(a,t)da$$

$N(t)$ is a measure of either the industry's capital stock and its employment, due to a fixed share of capital and labor. Thus, the industry's output is given by:

$$Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$$

The deterioration of production units involves both an exogenous depreciation rate $\delta$ and an endogenous destruction process, which impacts $f(a,t)$ at its limits. The count of production units surviving for $a$ years is expressed as:

$$f(a,t) = f(0,t-a)e^{-\delta a} \quad \text{where } 0 < a \leq \overline{a}(t)$$

The production flow is determined by:

$$\dot{N}(t) = f(0,t)[1 - \dot{\overline{a}}(t)] + \delta N(t)$$

Here, the first term represents the production rate, while the second term encapsulates the destruction rate, encompassing the obsolescence rate $f(\overline{a})(t)$, the technological obsolescence change over time $-f(\overline{a})(t)\dot{\overline{a}}(t)$, and the depreciation rate $\delta N(t)$.

The assumptions made by the authors are denoted as $\forall t \mid f(0,t) > 0 \cup \dot{\overline{a}}(t) <$.

The alteration in output concerning these flows is articulated as:

$$\dot{Q}(t) = A(t)f(0,t) - A(t-\overline{a}(t))f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)] + \delta Q(t)$$

The authors define a perfectly competitive industry in partial equilibrium, where supply is dictated by free entry and perfect equilibrium. Additionally, they introduce a cost function related to creating new production units:

$$c = c\left(f\left(f(0,t)\right)\right) \quad \text{where } c(\cdot) > 0,\ c'(\cdot) \leq 0$$

This cost function is contingent on the creation rate, implying that higher creation rates correspond to increased costs. The equilibrium condition is established by equating the cost of unit creation to the present discounted value of profits throughout its lifespan. The authors set the cost of a production unit to 1, and $P(t)$ is the price of a unit of output. Thus, the profits generated at time $t$ by a production unit aged $a$ are defined as:

$$\pi(a,t) = P(t)A(t-a) - 1$$

$$\overline{a}[t + T(t)] = T(t)$$

Here, $T(t)$ signifies the maximum lfetime of a unit created at $t$. At any given time $t$, the free entry condition is expressed as:

$$c(f(0,t)) = \int_{t+T(t)}^{t} \pi(s-t,t)e^{-(r+\delta)(s-t)\,ds}$$

In the above equation, where $r > 0$ represents the exogenously determined instantaneous interest rate, the determination of the exit of a production unit is contingent upon continuous $P(t)$ and the instance when the profits generated by a unit being destroyed reach zero. This occurrence signifies the moment when the oldest unit operational at time $t$, denoted as $\overline{a(t)}$, must adhere to the equation:

$$P(t)A(t - \overline{a}(t)) = 1$$

The authors posit that $P(t)$ exhibits a decreasing trend due to the model's assumption regarding endogenous destruction, specifically $\overline{\dot{a}(t)} < 1$. To see, differentiate

$$\dot{P}(t) = -\gamma \left[1 - \bar{\dot{a}}P(t)\right]$$

Consequently, when the profits of a production unit diminish to zero for the first time, it will be subject to destruction.

On the demand side, the authors assume a unit-elastic demand function and consider the aggregate expenditure as exogenous $\overline{D}(t) = P(t)Q(t)$. The equilibrium is a path $\{f(0,t), \overline{a}(t), T(t), Q(t)\}_{t\geq 0}$ that satisfy the following conditions:

1. $Q(t) = \int_{\overline{a}(t)}^{0} A(t-a)f(a,t)da$

2. $f(a,t) = f(0, t-a)e^{-\delta a}$

3. $T(t) = \overline{a}\left(t + T(t)\right)$

4. $c(f(0,t)) = \int_{t}^{t+T(t)} \left[P(s)A(t) - 1\right] e^{-(r+\delta)(s-t)}\, ds$

5. $P(t)A(t - \overline{a}(t) = 1)$

6. $P(t)Q(t) = \overline{D}(t)$

The first three equation 123 and the fifth one 5 are sufficient to describe the paths of $T(t), P(t)$ and $Q(t)$ that is determined by $\{f(0,t), \overline{a}(t)\}$. To confirm the strenght of the formulation of the condition express in the equations 6 and **??**, its possible to derive those equations as first order condition for maximization of a number of pefectly competitive firms that hold production units.

In order to understand how endogenous distruction works, lets start considering a constant demand, thus the distruction and the creation rate change only due to a supply factors. This steady state is characterized by constant lifetime of production units $T(t) = \overline{a}(t) = \overline{a}\star$, so even the age distribution is time-invariant $f(a,t) = f\star(a)$, by 5 the price $P(t)$ must be decreasing a constant rate $\sigma$. Indeed higher innovation rates leads higher productivity that increase supply, finally leading to a lower price if the demand is constant or increaseing a lower rate. From equation 2 follows that the distribution of production units in the steady state is a truncated exponatial distributions:

$$f^*(a) = f^*(0)e^{-\delta a} \quad 0 <\leq \overline{a}^*$$

Using free entry conditions 4 and the clearing condition 6 it is possible to determine the creation and destruction age $f^*(0), \overline{a}^*$. Using equation 1 and 5, we get the cost function and the productivity of a new production unit:

$$c(f^*(0)) = \frac{e^{\gamma \overline{a}^*} - e^{-(r+\delta)\overline{a}^*}}{\gamma + r + \sigma} - \frac{1 - e^{-(r+\delta)\overline{a}^*}}{r + \delta}$$

$$f(0) = \frac{(\sigma + \delta)\overline{D}^*}{e^{\sigma \overline{a}^* - e^{\delta \overline{a}*}}}$$

The authors then normalized the creation rate:

$$N = f^*(0) * (1 - e^{\delta \overline{a}^*})$$

in the steady state is given by

$$CC^* = \frac{\delta}{1 - e^{-\delta \overline{a}^*}}$$

Considering a special case in when the creation cost is a constant c, thus $c(f^*(0)) = c$, substituting into 2.2, one can retrive $\overline{a}^*$ substituing into 2.2. Regarding the effect of technological rate $\sigma$ on $\overline{a}^*$, it is decreasing since higher the innovation rate, higher will be the oppurtinity cost of delay renovation, while higher is the cost of creating new units lower will be the renovation rate. finally, optimal lifetime of production units increases with higher $rand\delta$ since it would make harder to recover creation costs.

Now we can drop the constant demand, in order to understand how the industry adjust to the fluctuations of demand. There are two ways in which industry can adjust production to meet demand:

- reducing the rate of creation $f(0,t)$
- increasing the rate of endogenous destruction $f(\overline{a}(t),t) \cdot [1 - \dot{\overline{a}}(t)]$, thus reducing $\overline{a}$ the age at which units are demolited

These two margins interact with each other leading: a reduction in demand the most outdated units are scrapped since are made unproffitable, but if the recession is partially accommodated by a reduction in the creation rate the effect on the distruction margin is reduced. The authors argue that the extent to which creation will "insulate" existing units from a variations in demand depends on the marginal cost of creating a new units $c'f(0,t)$. In the case in which the marginal cost of creation is 0 then the demand fluctuations are completly adjusted by the creation margin. This is the case above $c(f(0,t)) = c$. In this case the insulation effects is complete since there is non need to drop older units, its enough to lower $f(0,t)$, since is cheaper to reduce creation rate instead of reducing the life of already existing production units. The insulation effect is not due to assymetric adjustemnt cost on the creation and destruction margins, indeed the insulation would be complete even with linear adjusting costs. The creation rate in the case of costant creation cost is given by:

- solve for $\overline{a}$, using the free-entry condition 6 together with 3 and **??**, the solution is the same as in constant demand case
- Solve for the creation rate $f(0,t)$ to satisfy the market equilibrium condition 6, using 1 and 6.

Finally you get that the creation rate:

# References

Ricardo J. Caballero and Mohamad L. Hammour. The cleansing effect of recessions. *The American Economic Review*, 84(5):1350–1368, 1994. ISSN 00028282. URL http://www.jstor.org/stable/2117776.

Olivier Jean Blanchard, Peter Diamond, Robert E. Hall, and Kevin Murphy. The cyclical behavior of the gross flows of u.s. workers. *Brookings Papers on Economic Activity*, 1990(2):85–155, 1990. ISSN 00072303, 15334465. URL http://www.jstor.org/stable/2534505.

Steven J. Davis and John Haltiwanger. Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics*, 107(3):819–863, 1992. ISSN 00335533, 15314650. URL http://www.jstor.org/stable/2118365.

F. A. Hayek and Bruce Caldwell. *The Pure Theory of Capital*. Number 9780226320991 in University of Chicago Press Economics Books. University of Chicago Press, January 2007. ISBN ARRAY(0x3cfcacd8). URL https://ideas.repec.org/b/ucp/bkecon/9780226320991.html.

John M Abowd and Francis Kramarz. The costs of hiring and separations. *Labour Economics*, 10(5):499–530, 2003. ISSN 0927-5371. doi: https://doi.org/10.1016/S0927-5371(03)00017-4. URL https://www.sciencedirect.com/science/article/pii/S0927537103000174.