Homework 14: LC-3 Control Signals and Error Control Coding

In this homework, you must implement control signals for an LC-3 processor. For problems 1-4, you should consider only the abbreviated list given in Section 4.1.1 (p. 133) of the notes. The full set in Appendix C of Patt and Patel includes several aspects of LC-3 functionality that we do not discuss in our class. For examples of mapping RTL to these control signals, see Sections 4.1.2 and 4.1.3.

1. Control Signals for ST

Write the full set of control signals necessary to implement an ST instruction from fetch through execution. Use the state numbers in Patt and Patel's FSM diagram to identify each state (for example, the first fetch state is #18, and the decode state is #32).

2. Control Signals for BR

Write the full set of control signals necessary to implement the execution (not fetch nor decode) of a BR instruction. Again, use the state numbers to identify the states.

3. Adding a New Instruction

Prof. Lumetta is annoyed with having to load values into registers whenever he needs to use an immediate operand of more than 5 bits. Help Prof. Lumetta by adding a new instruction that allows a 16-bit immediate field to the LC-3. In particular, your new instruction must require two memory locations, and the second memory location will contain the 16-bit immediate value. Your implementation should load the immediate value into R7, advance the PC again, and use the existing ADD and AND execution states to perform the operation as though the instruction is using a second register operand. Do not worry about overwriting R7.

- a. Draw your instruction format in the style used in the class handouts (and the textbook). The new instruction should use opcode 1101 to reach your sequence of states. The DR and SR1 fields from AND and ADD are, of course, still needed. You also need one bit—IR[3]—to select between ADD (0) and AND (1). You may assume that the LC-3 ignores IR[3] (as well as IR[15:12]!) when executing ADD and AND. However, you must configure the remaining 5 bits of your new instruction format to make these states operate correctly with the 16-bit immediate value.
- b. Draw the sequence of FSM states needed to implement your new instruction. The first state should be state #13 (the DECODE state sends your any instruction with opcode 1101 to that state). Number the other states in your diagram starting at 90 (these are not valid state numbers, but are needed for part (c) below), but reuse the ADD and AND FSM execution states—numbers 1 and 5, respectively—to make use of the ALU with your new instruction (do not reimplement these two states). Be sure to label which bits are used to make transition decisions if your states have more than one next state.
- c. For each state in your new sequence, write the 25 control signals necessary to implement that state. Use the state numbers to identify the states.

4. Microsequencer Bits

For each of the FSM states in problems 1 and 2 (complete processing of an ST instruction, and execution of a BR instruction), write the sequencing control bits J, COND, and IRD for the Patt and Patel microsequencer, as described in Appendix C.4.

5. Pair Codes?

A friend of yours is quite excited about having developed a new class of codes: pair codes. Pair codes are a generalization of the 2-out-of-5 representation discussed in Section 4.2.1 of the notes. In a pair code, each code word has exactly two 1 bits. However, one can define a pair code on any number of bits N. For example, if N=100, one has 100 bits in the code words, and exactly two 1 bits.

Your friend points out that as N grows, the fraction of valid code words drops dramatically. For N=100, for example, there are only 4950 valid code words (100 times 99 divided by 2), but there are 2¹⁰⁰ bit patterns. Your friend argues that error correction capabilities for pair codes must be quite powerful, since the codes are so sparse.

- a. What is the Hamming distance of the pair code with 6-bit code words? Use an example to prove that your answer is correct.
- b. What is the Hamming distance of the pair code on 100-bit code words? Explain how you can again prove that your answer is correct (please avoid writing 100-bit numbers).
- c. How many bits can be corrected using a pair code with N-bit code words?

6. Hamming Codes

The notes provided details on 7-bit Hamming codes. Consider the use of Hamming codes on more bits.

- a. If a Hamming code is used with 10 bits, how many of the bits are parity check bits?
- b. If a Hamming code is used with 100 bits, how many of the bits are parity check bits?
- c. Why does it make little sense to use a Hamming code with 128 bits? Explain your answer.

7. SEC-DED Codes

A communication system uses a SEC-DED code to protect transmissions. The code is based on a 7-bit Hamming code extended with an odd parity bit (the first bit). Using the notation from the notes, each received word consists of the parity bit followed by the 7-bit Hamming code word $x_7x_6x_5x_4x_3x_2x_1$. For each of the following received words, extract the 4-bit data word when possible, or mention that the received word had uncorrectable errors.

- a. 00101101
- b. 10011110
- c. 10101001
- d. 10010001
- e. 00111001