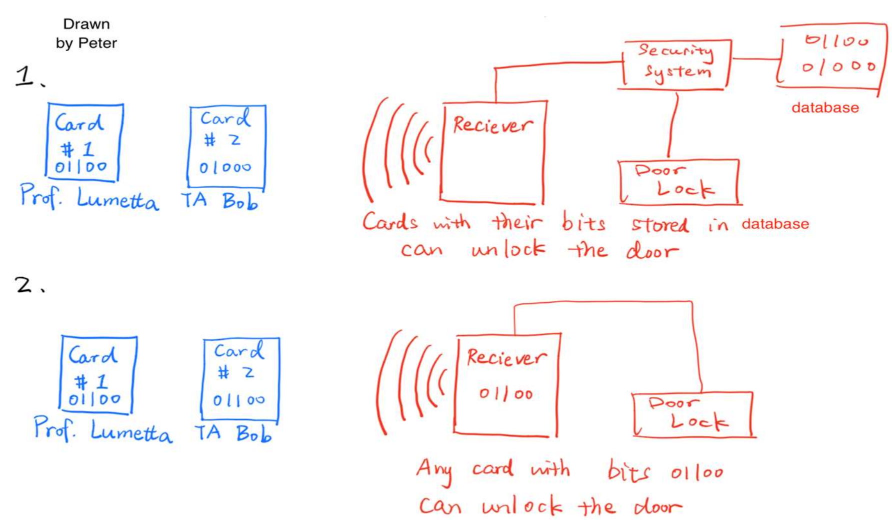
ECE 120: Introduction to Computing Spring 2019 ZJUI

**due at start of section on Friday 1 March 2019**

# Homework 1: Representations and Bits

1. **Number Conversion**
   a. What value does the bit pattern 11011010110 represent in the unsigned representation? Write your answer in decimal. 1750
   b. What value does the bit pattern 10101010010 represent in the 2's complement representation? Write your answer in decimal. -686
   c. Write the bit pattern for the decimal number -129 using a 10-bit 2's complement representation. 1101111111
   d. Write the bit pattern for the decimal number 444 using a 10-bit unsigned representation. 0110111100

2. **Electronic Key Abstraction**
   Humans use abstraction layers in everyday life. In this problem, we ask you to think about the meaning of abstraction layers in a familiar context. The electronic key system used on our campus (based on our campus ID cards) provides an abstraction that simplifies our ability to enter buildings in a safe and controlled manner. Even people who do not know how such a device is implemented can make use of one.
   a. What function is provided to a human user by the human's ID card (when used as a key)? Open the door.
   b. Briefly explain at least two ways in which this function can be implemented. (You may want to look up real implementations, or just be imaginative.) Include a simple diagram for each implementation. 1. Every card is assigned with a distinct bit pattern, which can be read through electromagnetic waves. If the bit pattern in the card matches a pattern that is allowed access, the door is opened. 2. All cards store the same bit pattern, and the receiver recognizes only that pattern. If matched, door will open.



**3. Required Number of Bits**

For each of the following, calculate the number of bits needed to uniquely represent the specified item. If you need to make any assumptions, write them down.

a. A child in China. There are around 0.3 billion children in China, and ceil ($\log_2$ (0.3 billion)) = 28 bits.

b. One of the 25 students in Prof. Lumetta's last ECE120 class. Ceil ($\log_2$ (25)) = 5 bits.

c. A building on the International Campus of ZJU in Haining. According to the campus map, there are 26 buildings, and ceil ($\log_2$ (26)) = 5 bits.

d. One PowerPoint slide among all of those available on our class website (those for the class only). For part 1 of the class, there are 189 slides, and the class has four parts, so around 800 slides, and ceil ($\log_2$ (800)) = 10 bits.

**4. Ambiguity in Representations**

When citing research papers, short labels are often created by merging the first letters of the authors' names with the last two digits of the year of publication. For example, a recent paper that includes Prof. Lumetta as an author might be cited as [RLKLC19], whereas one of the papers from his graduate student days might appear as [LKC95].

Give an example to explain why this mapping does not make a good representation for use by computers. For example, if the same group of people publishes two publications in a single year, the two labels are identical, leading to ambiguity.

**5. Ordering Numbers**

a. Order the following patterns in the 6-bit unsigned representation from smallest to largest represented value.

i. $101011_2$, $25_{10}$, $25_{16}$ $\quad 25_{10} < 25_{16} < 101011_2$

ii. $001011_2$, $10_{10}$, $0E_{16}$ $\quad 10_{10} < 001011_2 < 0E_{16}$

b. Order the following patterns from smallest to largest represented value, first assuming an 8-bit unsigned representation, then an 8-bit 2's complement representation.

i. $10010111_2$, $01101010_2$, $B5_{16}$, $14_{16}$ unsigned: $14_{16} < 01101010_2 < 10010111_2 < B5_{16}$ ;
2's complement: $10010111_2 < B5_{16} < 14_{16} < 01101010_2$

ii. $00100000_2$, $10010101_2$, $C3_{16}$, $37_{16}$ unsigned: $00100000_2 < 37_{16} < 10010101_2 < C3_{16}$;
2's complement: $10010101_2 < C3_{16} < 00100000_2 < 37_{16}$

**6. Binary Addition**

Perform the following 8-bit additions. Perform the operation twice, first assuming that the patterns are in the unsigned representation, then again assuming a 2's complement representation. Indicate when overflow occurs. Show your work. (Result is same for both representations.)

a. 00100110 + 01000110=01101100(unsigned); (2's complement)

b. 10110111 + 01011011=00010010(unsigned, overflow); (2's complement)

c. 10010011 + 11011011=01101110(unsigned, overflow); (2's complement, overflow)

d. 00111011 + 10100001=11011100(unsigned); (2's complement)

7. **Binary Subtraction**

Perform the following subtractions using the 8-bit 2's complement representation. *Hint: you may find it easier to negate the second number, then add.*

a. 01001000 – 00110111=01001000 + 11001001=00010001
b. 10100101 – 10101100=10100101 + 01010100=11111001
c. 01011110 – 11110011=01011110 + 00001101=01101011

8. **Overflow on Other Operations**

In class, we examined the overflow condition for unsigned addition, and later for 2's complement addition. Remember that overflow occurs when the resulting bit pattern does not represent the desired result for an operation.

a. Consider a bit pattern $A_{N-1}A_{N-2}\ldots A_1A_0$ representing an unsigned number A. Let's say that we shift the bits to the "left," dropping the leftmost and adding a 0 on the right (to obtain the bit pattern $A_{N-2}A_{N-3}\ldots A_1A_00$). Assuming no overflow, what is the value of the resulting bit pattern in terms of the original value A? What is the overflow condition in terms of the bits in the original bit pattern? Shift left produces 2A, and overflows iff $A_{N-1}$ is 1.
b. Now consider shifting a bit pattern $A_{N-1}A_{N-2}\ldots A_1A_0$, again representing an unsigned number A, one bit to the right and inserting a 0 on the left to produce the bit pattern $0A_{N-1}A_{N-2}\ldots A_1$. Assuming no overflow, what is the value of the resulting bit pattern in terms of the original value A? What is the overflow condition in terms of the bits in the original bit pattern? Shift right produces floor (A/2) (rounded down) and never overflows.