Name____________________ Student ID____________________ TA's name________________

Math 231 Homework 4

Due Apr 8th Submit at the beginning of the class (Put into the box with your TA's name)
Do the calculation and write the numbers during the process

1. (a) Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the n^{th} month? Show the Fibonacci Sequence is $f(n)$.

   (b) Let $a_n = f_{n+1}/f_n$ and show that $a_{n-1} = 1 + 1/a_{n-2}$. Assuming that $\{a_n\}$ is convergent, find its limit.

2. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

   (a) $\{ 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\}$

   (b) $\{5, 8, 11, 14, 17, \ldots\}$

   (c) $\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots\}$

3. Determine whether the following sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \frac{n^2}{\sqrt{n^3+4n}}$, (b) $\{\frac{(2n-1)!}{(2n+1)!}\}$, (c) $a_n = \frac{cos^2 n}{2^n}$ (d) $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\}$

4. Let a and b positive numbers with $a > b$. let a_1 be their arithmetic mean and b_1 their geometric mean:

   $$a_1 = \frac{a+b}{2} \qquad b_1 = \sqrt{ab}$$

   Repeat this process so that, in general,

   $$a_{n+1} = \frac{a_n+b_n}{2} \qquad b_{n+1} = \sqrt{a_n b_n}$$

   (a) Use mathematical induction to show that.

   $$a_n > a_{n+1} > b_{n+1} > b_n$$

   (b) Deduce that both $\{a_n\}$ and $\{b_n\}$ are convergent.

   (c) Show that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$. Gauss called the common value of these limits the **arithmetic-geometric mean** of numbers a and b.

5. When money is spent on goods and services, those who receive the money also spend some of it. The people receiving some of the twice-spent money will spend some of that, and so on. Economists call this chain reaction the *multiplier effect*. In a hypothetical isolated community, the local government begins the process by spending D dollars. Suppose that each recipient of spent money spends $100c\%$ and saves $100s\%$ of the money that he or she receives. The values c and s are called the *marginal propensity to consume* and the *marginal propensity to save* and, of course, $c+s=1$.

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(a) Let S_n be the total spending that has been generated after n transactions. Find an equation for S_n.

(b) Show that $\lim_{n\to} S_n = kD$, where $k=1/s$. The number k is called the *multiplier*.

What is the multiplier if the marginal propensity to consume is 80% ?

Note: The federal government uses this principle to justify deficit spending. Banks use this principle to justify lending a large percentage of the money that they receive in deposits.