ECE329 Project #2 Report

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1 Problem 1

(20 pts) Please derive the analytical solution of $v(z,t)$ and $i(v,t)$ assuming $v_s(t) = V_0 sin(\omega t)$ so that you do not need to worry about the transient effects. Please show your derivation.

Since $\beta l = \beta d = \beta \frac{\lambda}{4} = \beta \frac{2\pi}{4\beta} = \frac{\pi}{2}$, we have.

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{25-50}{25+50} = -\frac{1}{3} \tag{1}$$

Also, for Z_{in} we compute it from the formula:

$$Z_{\text{in}} = Z(-d) = Z_C \frac{Z_L + jZ_C \tan\beta d}{Z_C + jZ_L \tan\beta d} = \frac{Z_C^2}{Z_L} = 100\Omega \tag{2}$$

Then, using KVL we have:

$$V(-d) = V_s \frac{Z_{\text{in}}}{Z_{\text{in}} + R_s} = \frac{1}{2} V_s = -j\frac{1}{2} V_0 \tag{3}$$

Substitute this value into $\tilde{V}(z) = \tilde{V}^+(e^{-j\beta z} + \Gamma_0 e^{j\beta z})$, we have that $\tilde{V}^+ = -\frac{3}{8}$. Similarly we can get the expression for current. Finally:

$$\tilde{V}(z) = -\frac{3}{8} e^{-j\beta z} + \frac{1}{8} e^{j\beta z}$$
$$\tilde{I}(z) = -\frac{3}{400} e^{-j\beta z} - \frac{1}{400} e^{j\beta z}$$

or,

$$v(z,t) = -\frac{3}{8}\cos(\omega t - \beta z) + \frac{1}{8}\cos(\omega t + \beta z)$$
$$i(z,t) = -\frac{3}{400}\cos(\omega t - \beta z) - \frac{1}{400}\cos(\omega t + \beta z)$$

2 Problem 2

Please find the load voltage $v_L(t) = v(z=0,t)$, for $0 \le t \le 10T$ using the FDTD method given $v_S(t) = V_0 sin(\omega t)u(t)$.

2.1 (i)

To normalsize the results, assume that $T = 10$s and $\lambda = 4$m. Also, to help simplify the parameters, let $\Delta z_0 = \frac{1}{20}\lambda$ and $\Delta t_0 = \frac{\Delta z}{v_p}$. Here are the results of the simulations:

For $\Delta z = 2 * \Delta z_0$ and $\Delta t = 2 * \Delta t_0$:

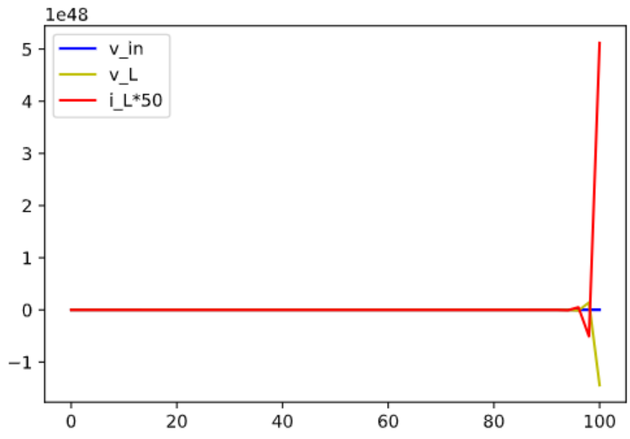

Figure 1: Plot for $\Delta z = 2 * \Delta z_0$ and $\Delta t = 2 * \Delta t_0$

For $\Delta z = \Delta z_0$ and $\Delta t = \Delta t_0$:

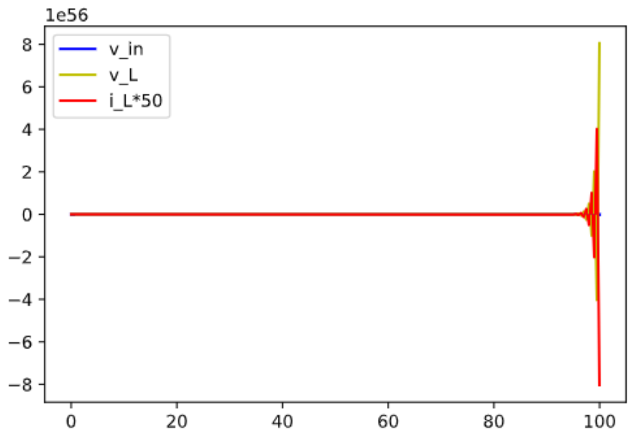

Figure 2: Plot for $\Delta z = \Delta z_0$ and $\Delta t = \Delta t_0$

For $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \Delta t_0$:

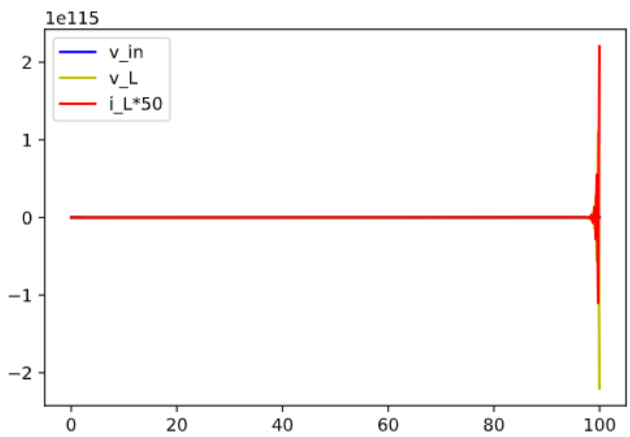

Figure 3: Plot for $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \Delta t_0$

For $\Delta z = \Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$:

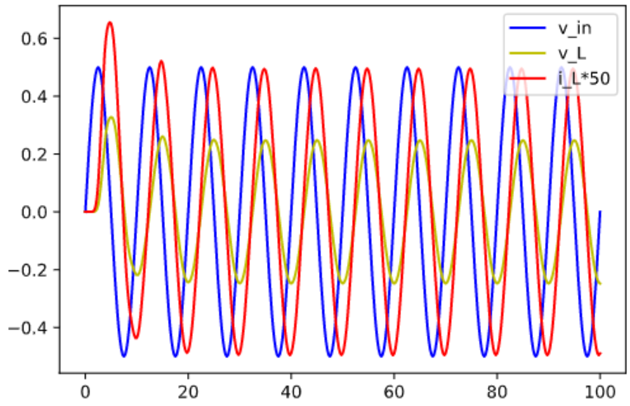

Figure 4: Plot for $\Delta z = \Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$

For $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$:

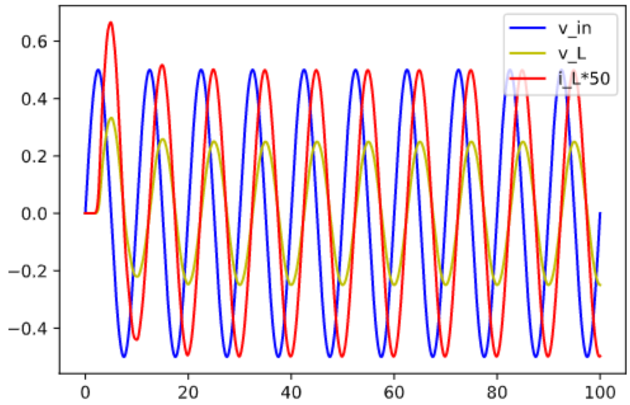

Figure 5: Plot for $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$

For $\Delta z = \frac{1}{5}\Delta z_0$ and $\Delta t = \frac{1}{5}\Delta t_0$:

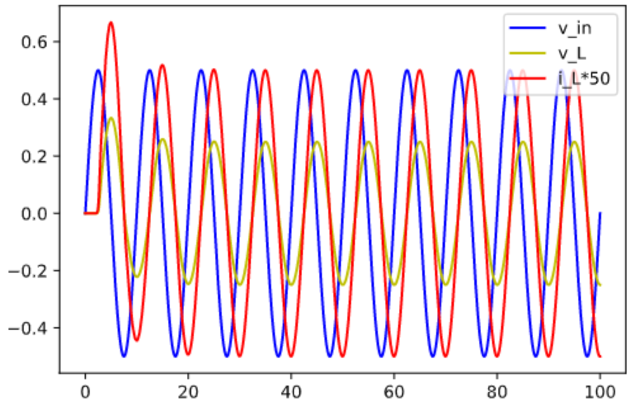

Figure 6: Plot for $\Delta z = \frac{1}{5}\Delta z_0$ and $\Delta t = \frac{1}{5}\Delta t_0$

From above we can see that as Δz and Δt decrease, the accuracy and stability both increase. Also, it seems that Δt is more significant then Δz when concerning stability. Considering accuracy, stability and efficiency, we choose $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$. The comparison between the numerical results and the analytical results are shown below:

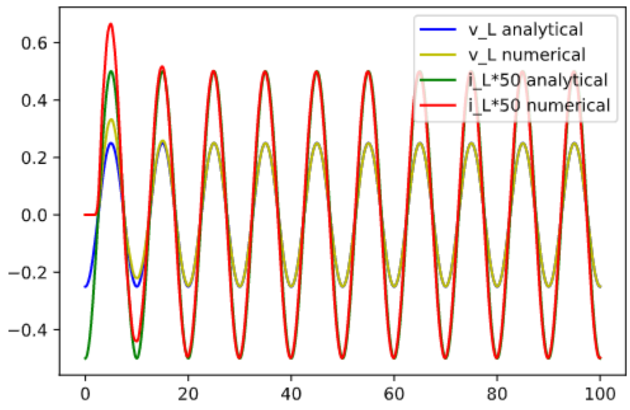

Figure 7: Comparison between Numerical and Analytical Results using $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$

It's obvious that they converge well after only about 1.5 cycles.

2.2 (ii)

The comparison result is shown as Figure 7. The numerical solution converges well.

2.3 (iii)

If $d = 0$, then $V_L(t) = V_s \frac{R_L}{R_L + R_s} = \frac{1}{5}\sin(\omega t)u(t)$, and $I_L(t) = \frac{V_L}{R_L} = \frac{1}{125}\sin(\omega t)u(t)$.

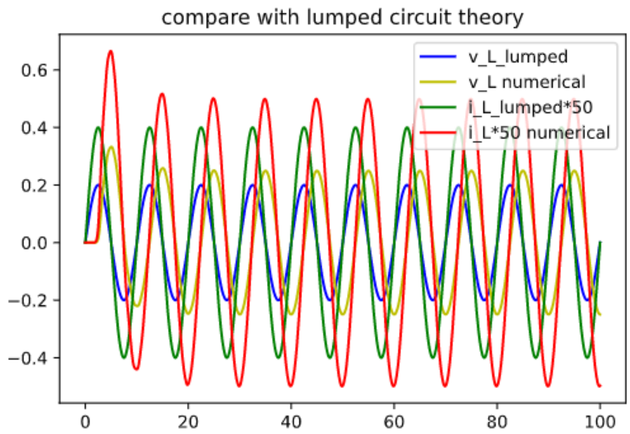

Figure 8: Comparison between the Results with the Lumped Circuit Theory Results using $\Delta z = \frac{1}{2}\Delta z_0$ and $\Delta t = \frac{1}{2}\Delta t_0$

It shows that not only the amplitude is decreases from the lumped circuit theory results to the numerical results, the phase is also shifted.

3 Problem 3

All the results are shown below:

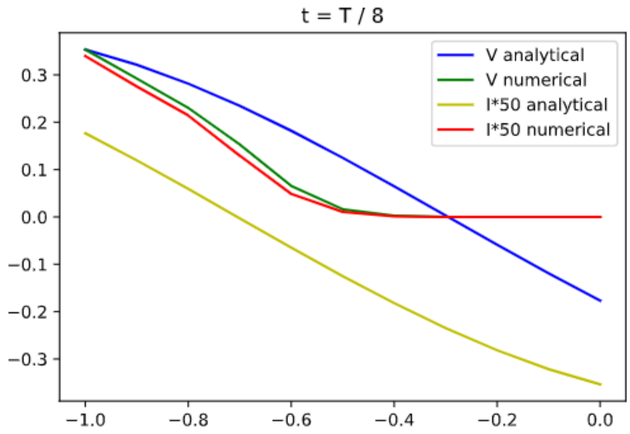

Figure 9: Comparison between Numerical and Analytical with $t = \frac{T}{8}$

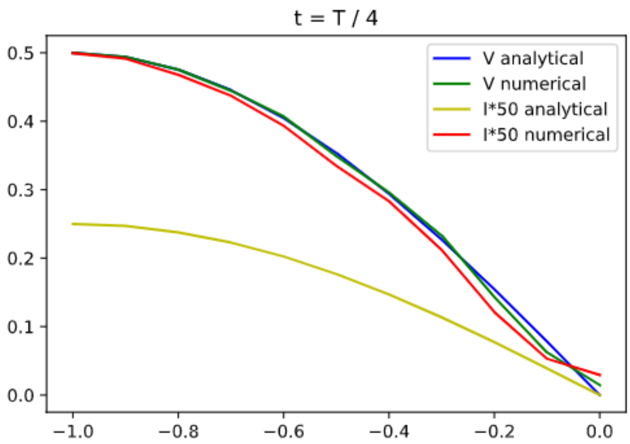

Figure 10: Comparison between Numerical and Analytical with $t = \frac{T}{4}$

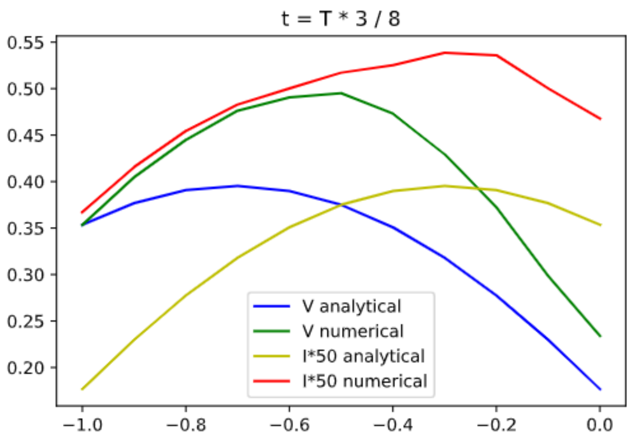

Figure 11: Comparison between Numerical and Analytical with $t = \frac{3T}{8}$

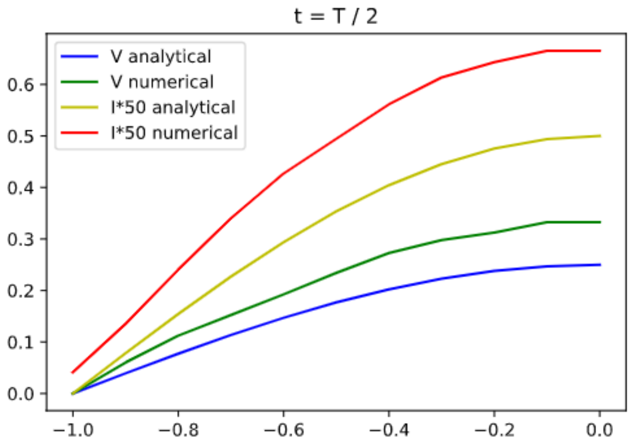

Figure 12: Comparison between Numerical and Analytical with $t = \frac{T}{2}$

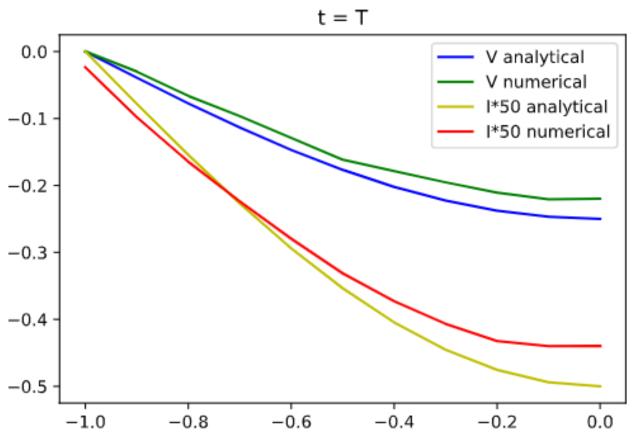

Figure 13: Comparison between Numerical and Analytical with $t = T$

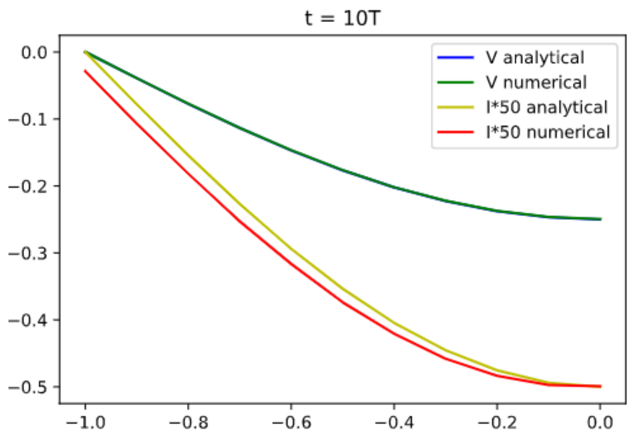

Figure 14: Comparison between Numerical and Analytical with $t = 10T$

As shown above, it takes some time for the simulation to be stable. About 1 period is enough.