Concepts this Week

Relevant Lectures for Discussion Session:

- Lecture 10: Center of mass

- Lecture 12: Conservation of Momentum
  1. Definition of momentum
  2. Momentum conservation
  3. Center of mass frame sees a total momentum of zero

Current PreLectures: Prelecture 10 and PreLecture 12

Key concepts this week:

- Definition of center of mass $\vec{R}_{cm} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$
- Newton's law for CM $\vec{a}_{CM} = \dfrac{\vec{F}_{Net,External}}{M_{Total}}$
- Center of mass equation $\Delta\left(\frac{1}{2} M V_{CM}^2\right) = \int \vec{F}_{Net,External} \cdot d\vec{l}_{CM}$
- Definition of momentum ($\vec{p} = m\vec{v}$)
- Conservation of momentum
  - Total momentum does not change so long as the next external force is zero (i.e. $\vec{F}_{\text{net, external}} = 0 \;\Rightarrow\; \vec{P}_{\text{total}} = \sum_i \vec{p}_i = \text{constant}$)
  - Particularly easy to use in the center of mass frame where $\vec{P}_{\text{total}} = 0$
  - In general, kinetic energy is NOT conserved in a collision (i.e. most collisions are "inelastic")

Roller Coaster

A roller coaster car has a mass of 840 kg. It is launched horizontally from a giant spring, with spring constant 31,000 N/m into a frictionless vertical loop-the-loop track of radius 6.2m. What is the minimum amount that the spring must be compressed if the car is to stay on the track?

Romeo and Juliet

Romeo, who is sitting in the rear of their boat in still water, entertains Juliet by playing his guitar. After the serenade, Juliet, who was sitting in the front of the boat, carefully moves to the rear to plant a kiss on Romeo's cheek. The 80-kg boat is facing shore and the 55-kg Juliet moves 2.7 m towards the 77-kg Romeo. How far does the boat move? Does it move toward or away from the shore?

Along Came a Spider...

A man and a woman are sitting in a sleigh that is at rest on frictionless ice. The mass of the man is 80 kg, that of the woman is 60 kg, and that of the sleigh is 120 kg. The people suddenly see a poisonous spider on the floor of the sleigh and jump out. The man jumps to the left with a velocity of 5.0 m/s relative to the ground at 30^o above the horizontal. The woman jumps to the right at 9.0 m/s relative to the ground at 37^o above the horizontal. What is the velocity (magnitude and direction) of the sleigh after they have both jumped out?

Run the Plank

In frozen Minnesota the Winter Sports Carnival includes some unusual events. Since it is dangerous to run on ice, each runner runs on a heavy (240 kg) and long (40 m) wooden plank, which itself rests on the smooth and horizontal ice. One of the competitors is a 60 kg woman who runs the length of the plank in 4.4 seconds, quite an impressive time. Her performance is viewed by a crowd huddled on the ice. The performance that they see is less impressive. With what speed do they see her moving?

Space Shuttle Emergency

You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 m from the shuttle and moving with it (i.e., she is not moving with respect to the shuttle). She is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10-kg tool kit and throw it away with all her strength, so that it has a speed of 8 m/s *relative to her*. In the script, she survives, but is this correct? Her mass, including her space suit but not her tool kit, is 80 kg.

Kinematics

$g = 9.81\,\frac{\text{m}}{\text{s}^2} = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$\vec{v} = \vec{v}_0 + \vec{a}t$

$x = x_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$v^2 = v_0^2 + 2a\left(x - x_0\right)$

$\vec{v}_{A,B} = \vec{v}_{A,C} + \vec{v}_{C,B}$

Uniform Circular Motion

$a = \frac{v^2}{r} = \omega^2 r$

$v = \omega r$

Dynamics

$\vec{F}_{\text{net}} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_{A,B} = -\vec{F}_{B,A}$

$F = mg$ (near Earth's surface)

$F_{\text{gravity}} = G\dfrac{m_1 m_2}{r^2}$ (in general)

(where $G = 6.67 \times 10^{-11}$)

$F_{\text{spring}} = -kx$

Friction

$f = \mu_k N$ (kinetic)

$f \leq \mu_s N$ (static)

Work & Kinetic Energy

$W = \int \vec{F} \bullet d\vec{l}$

$W = \vec{F} \bullet \Delta\vec{r} = F\Delta r\cos\theta$

(constant force)

$W_{\text{grav}} = -mg\Delta y$

$W_{\text{spring}} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$

$K = \frac{1}{2}mv^2$

$W_{\text{NET}} = \Delta K$

Potential Energy

$U_{\text{grav}} = mgy$ (near Earth)

$U_{\text{grav}} = -G\dfrac{Mm}{r}$ (general)

$U_{\text{spring}} = \frac{1}{2}kx^2$

$\Delta E = \Delta K + \Delta U = W_{\text{NC}}$

System of Particles

$\vec{R}_{\text{CM}} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

$\sum \vec{F}_{\text{ext}} = M_{\text{total}}\vec{A}_{\text{CM}}$

$K_{system,lab} = K_{\text{relative to CM}} + K_{\text{CM}}$

Momentum

$\vec{P}_{\text{total}} = M_{\text{total}}\vec{V}_{\text{CM}}$

$\dfrac{d\vec{P}_{\text{total}}}{dt} = \vec{F}_{\text{net,external}}$

$\int \vec{F}_{\text{net}}\,dt = \Delta\vec{p}$

If $\vec{F}_{\text{net,external}} = 0$,

then $\vec{P}_{\text{total}}$ remains constant