**ECE 120 Second Midterm Exam**
**Fall 2016**

Tuesday, October 18, 2016

Name: ______________________________ NetID: ________________

Discussion Section:

| | | |
|---|---|---|
| 9:00 AM | | |
| 10:00 AM | | |
| 11:00 AM | [ ] AB1 | [ ] AB8 |
| 12:00 PM | [ ] AB2 | [ ] AB9 |
| 1:00 PM | [ ] AB3 | [ ] ABA |
| 2:00 PM | [ ] AB4 | [ ] ABB |
| 3:00 PM | [ ] AB5 | |
| 4:00 PM | [ ] AB6 | [ ] ABC |
| 5:00 PM | [ ] AB7 | [ ] ABD |

- **Be sure that your exam booklet has 11 pages.**
- **Write your name, netid and check discussion section on the title page.**
- **Do not tear the exam booklet apart, except for the last two pages.**
- **Use backs of pages for scratch work if needed.**
- **This is a closed book exam. You may not use a calculator.**
- **You are allowed one handwritten 8.5 x 11" sheet of notes (both sides).**
- **Absolutely no interaction between students is allowed.**
- **Clearly indicate any assumptions that you make.**
- **The questions are not weighted equally. Budget your time accordingly.**
- **Show your work.**

| | | |
|---|---|---|
| Problem 1 | 14 points | ________ |
| Problem 2 | 14 points | ________ |
| Problem 3 | 14 points | ________ |
| Problem 4 | 14 points | ________ |
| Problem 5 | 14 points | ________ |
| Problem 6 | 30 points | ________ |
| Total | 100 points | ________ |

**Problem 1 (14 points): K-maps and Don't Cares**

Consider the 4-variable function *f*(*a,b,c,d*), with the following K-map (drawn twice). **Note:** there are extra copies of this K-map on the last page of the exam. Use them for scratch work, but they will NOT be graded. Make sure you mark the two K-maps below correctly.

Minimal SOP

**f(a,b,c,d)** — cd

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal POS

**f(a,b,c,d)** — cd

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

1. **(4 points)** Give a **minimal SOP** expression for *f*(*a,b,c,d*) and show the corresponding loops on the **left map**.

   Minimal SOP: **f(a,b,c,d)=**________________________________________

2. **(3 points)** Is the solution to part 1 unique? If it is not, write a different minimal SOP solution.

   Circle one: **UNIQUE** **NOT UNIQUE**

   If not unique, write a different minimal SOP solution (but do not mark the loops):

   Minimal SOP: **f(a,b,c,d)=**________________________________________

3. **(4 points)** Give a **minimal POS** expression for *f*(*a,b,c,d*) and show the corresponding loops on the **right map**.

   Minimal POS: **f(a,b,c,d)=**________________________________________

4. **(3 points)** Is the solution to part 3 unique? If it is not, write a different minimal POS solution.

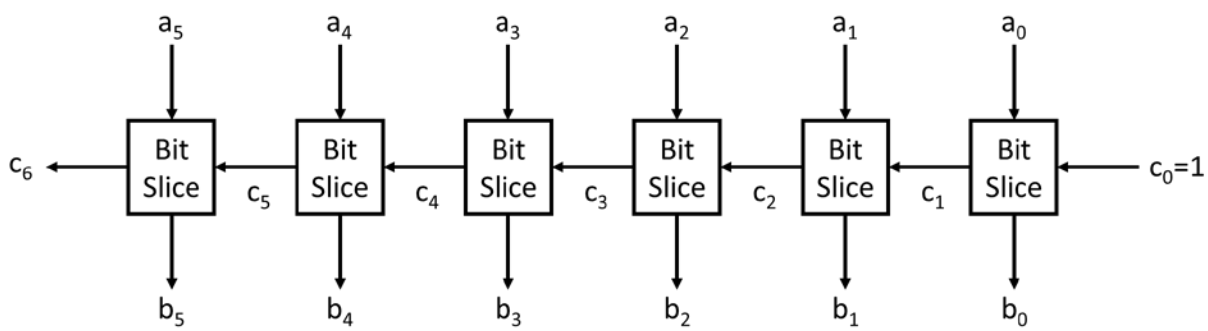
   Circle one: **UNIQUE** **NOT UNIQUE**

   If not unique, write a different minimal POS solution (but do not mark the loops):

   Minimal SOP: **f(a,b,c,d)=**________________________________________

**Problem 2 (14 points): Bit-sliced Design**

The circuit shown below is intended to negate a 6-bit 2's complement number. For example, if the input $a_5a_4a_3a_2a_1a_0$ = 110100, then the output $b_5b_4b_3b_2b_1b_0$ = 001100. Note that the initial carry-in bit $c_0$ is set to 1 in this design.




1. **(8 points)** Each bit slice is identical. Complete the truth table for a single bit slice and write the minimal SOP expressions.

| $c_i$ | $a_i$ | $c_{i+1}$ | $b_i$ |
|---|---|---|---|
| 0 | 0 | | |
| 0 | 1 | | |
| 1 | 0 | | |
| 1 | 1 | | |

Minimal SOP: **$c_{i+1}$=**____________________________________________

Minimal SOP: **$b_i$=**______________________________________________

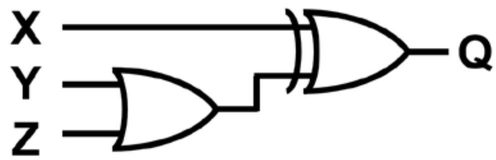
2. **(6 points)** Assume that the $b_i$ and $c_{i+1}$ outputs of each bit slice are implemented using 2-level AND-OR networks based on their minimal SOP expressions. **In this question, count AND and OR gates, but do not count NOT gates.**

How many gates are required in the entire circuit (all bit slices)? ________________________

How many gates are there along one of the longest paths from $a_0$ to $b_5$? __________________

**Problem 3 (14 points): Sequential Feedback Analysis**

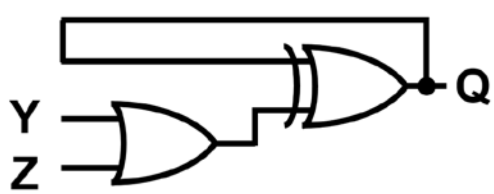
1. **(8 points)** Complete the truth table for the circuit shown below made out of one OR gate and one XOR gate.




| X | Y | Z | Q |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |

| X | Y | Z | Q |
|---|---|---|---|
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

2. **(6 points)** For the circuit shown below, which combinations of inputs YZ cause the output Q to oscillate between 0 and 1? In other words, which values of YZ prevent the output Q from holding a stable value?



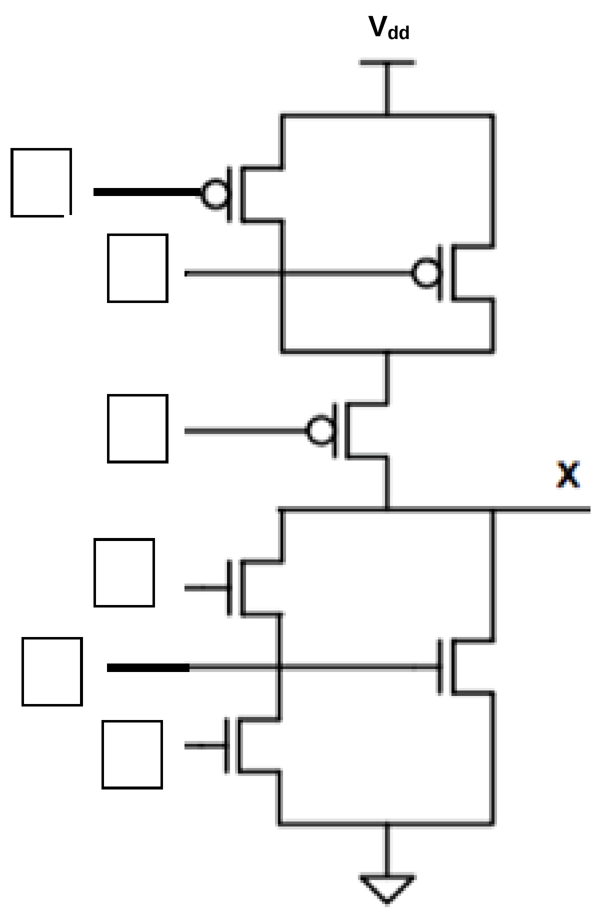

Circle all oscillating input combinations YZ: 00 01 10 11

**Problem 4 (14 points): CMOS Gates, Boolean Expressions and 2-Level Design**

1. **(6 points)** The transistor-level circuit below implements the logic equation:

$$X = \overline{AB + C}$$

Label the inputs to all transistors.



**Problem 4 (14 points), continued:**

2. **(4 points)** As a consultant to the company Dontel Corp., your task is to design a system that counts the number of 1s present in a 3-bit pattern ABC to output a two-bit number $X_1 X_0$ (representing that count in binary). Describe the behavior of your system as a truth table.

| A | B | C | $X_1$ | $X_0$ |
|---|---|---|---|---|
| 0 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 0 | 1 | 0 | | |
| 0 | 1 | 1 | | |
| 1 | 0 | 0 | | |
| 1 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

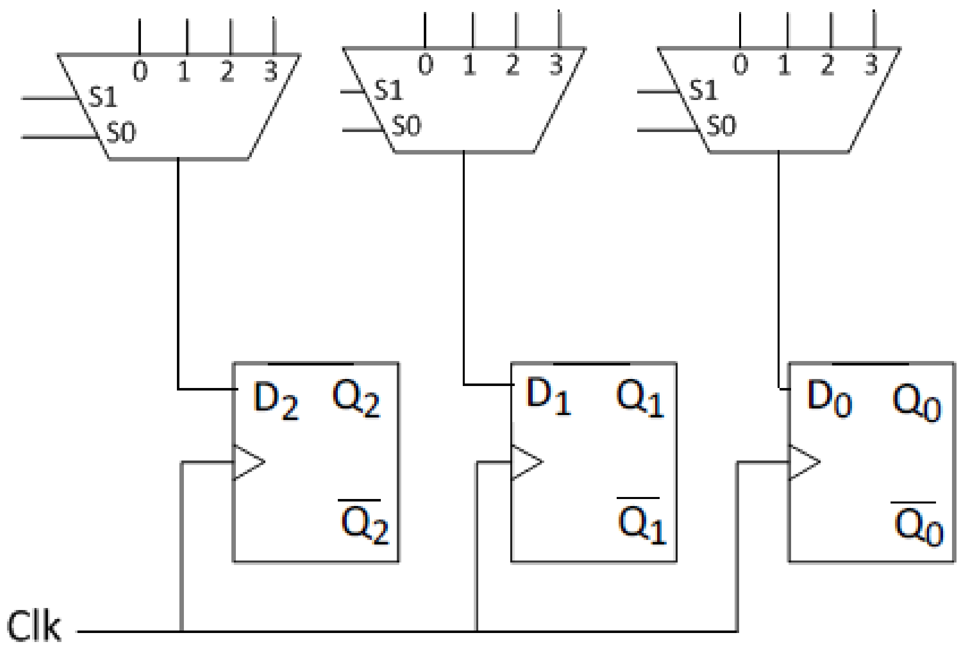
3. **(4 points)** Implement **f=wx+y'+xz** as a 2-level NAND-NAND circuit using the minimum number of gates.

**Problem 5 (14 points): Basic Registers**

Consider a 3-bit shift register that has the functionality specified in the table to the right.

1. **(10 points)** Label the inputs to the muxes to complete the design of this 3-bit register that performs the operations listed in the table. Parallel load input *i* is labeled as $P_i$. **Use labels and do not draw any additional gates or wires**.

| $F_1$ | $F_0$ | Operation |
|---|---|---|
| 0 | 0 | Arithmetic right shift |
| 0 | 1 | Circular shift left |
| 1 | 0 | Arithmetic shift left |
| 1 | 1 | Parallel load |




2. **(2 points)** If the shift register initially stores 110, what is stored in the register after one clock cycle when $F_1F_0 = 00$?

   Answer: ______________________________

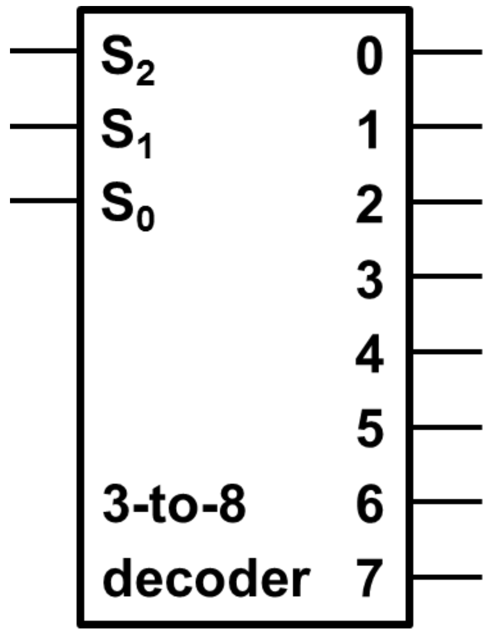
3. **(2 points)** Assume again that 110 is stored in the register. What is stored in the register after one clock cycle when $F_1F_0 = 10$?

   Answer: ______________________________

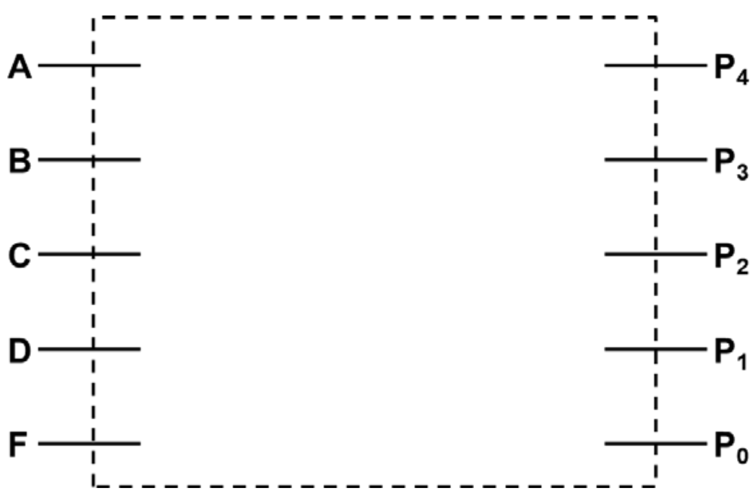
**Problem 6 (30 points): Designing Logic with Components**

Prof. Lumetta needs your help again. He is still fixated on calculating grades.
**Parts 1 through 4 do not depend on one another.**

1. **(8 points)** Help Prof. Lumetta use a decoder to translate the letter grade (A, B, C, D, or F) into a baseline GPA value. The typed ASCII letter is given as 7-bit input $G=G_6G_5G_4G_3G_2G_1G_0$. The letter ′A′ has ASCII code 0x41, ′ B′ is 0x42, and so forth. **Label decoder inputs and outputs as necessary on the 3-to-8 decoder to the right** to produce signals for the grades A, B, C, D, and F based on bits of input G. Leave any unused decoder outputs unlabeled.

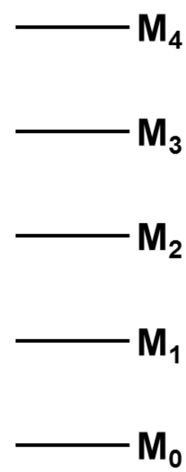



2. **(8 points)** Now use the grade signals A, B, C, D, and F to compute a 5-bit 2's complement GPA value P. The grade A has decimal value 12, B has value 9, C has value 6, D has value 3, and F has value 0. You may use **up to five gates** of any type (AND, OR, NOT, XOR, NAND, NOR, or XNOR) as well as **logic values 0 and 1**.



**Problem 6 (30 points), continued:**

3. **(6 points)** Grades can be modified by a + or a -. For example, B+ or A-. Process a second 7-bit ASCII character $Q=Q_6Q_5Q_4Q_3Q_2Q_1Q_0$, which is always one of the following: ′+′ (ASCII code 0x2B), ′ -′ (0x2D), or SPACE (0x20). Design a circuit to produce a 5-bit 2's complement GPA modifier M. M should have a decimal value of 1 when Q is ′ +′, a value of -1 when Q is ′ -′, and a value of 0 when Q is SPACE. Design your circuit to the right using only labels (no components, and no gates). In other words, **label each output bit with an appropriate input variable**.




4. **(8 points)** Prof. Lumetta stores a sum of GPA values in a 5-bit register, R. Using the values that you have computed, P and M, and the current sum R, you must now design a circuit to produce a new sum, T. **Use only combinational logic components** that you have seen in class, such as adders, comparators, muxes, and decoders. Draw your circuit in the box below, **labeling all inputs, outputs, and components clearly, including the number of bits in each signal**. **Do not worry about overflow for this problem.**

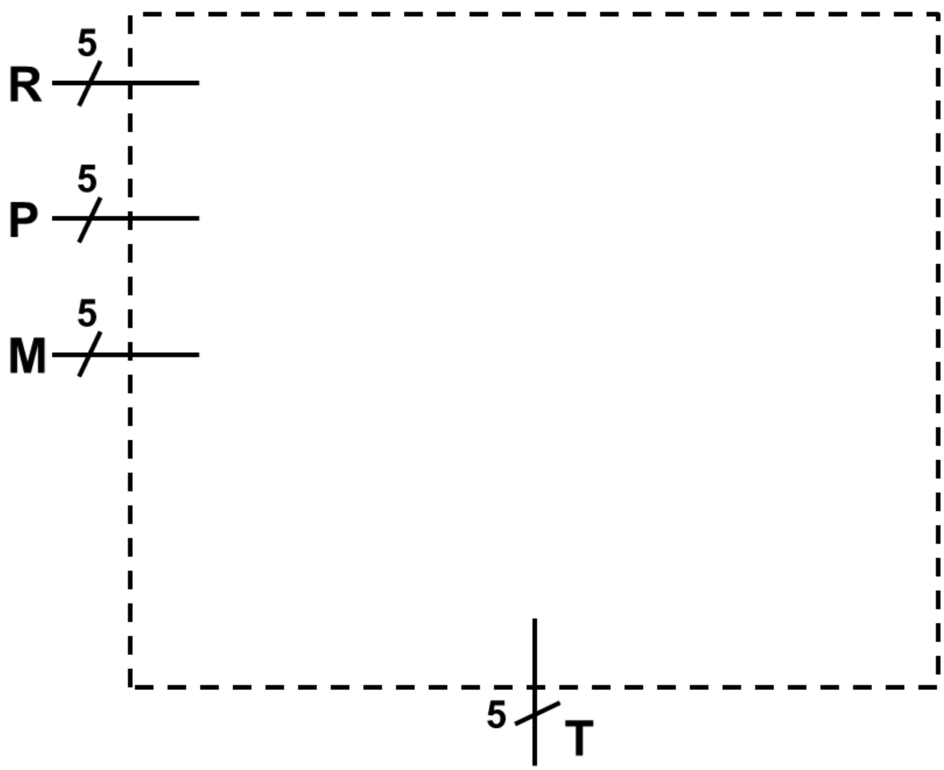


**Table of ASCII Characters**

| Char | Dec | Hex | Char | Dec | Hex | Char | Dec | Hex | Char | Dec | Hex |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (nul) | 0 | 00 | (sp) | 32 | 20 | @ | 64 | 40 | ` | 96 | 60 |
| (soh) | 1 | 01 | ! | 33 | 21 | A | 65 | 41 | a | 97 | 61 |
| (stx) | 2 | 02 | " | 34 | 22 | B | 66 | 42 | b | 98 | 62 |
| (etx) | 3 | 03 | # | 35 | 23 | C | 67 | 43 | c | 99 | 63 |
| (eot) | 4 | 04 | $ | 36 | 24 | D | 68 | 44 | d | 100 | 64 |
| (enq) | 5 | 05 | % | 37 | 25 | E | 69 | 45 | e | 101 | 65 |
| (ack) | 6 | 06 | & | 38 | 26 | F | 70 | 46 | f | 102 | 66 |
| (bel) | 7 | 07 | ' | 39 | 27 | G | 71 | 47 | g | 103 | 67 |
| (bs) | 8 | 08 | ( | 40 | 28 | H | 72 | 48 | h | 104 | 68 |
| (ht) | 9 | 09 | ) | 41 | 29 | I | 73 | 49 | i | 105 | 69 |
| (lf) | 10 | 0a | * | 42 | 2a | J | 74 | 4a | j | 106 | 6a |
| (vt) | 11 | 0b | + | 43 | 2b | K | 75 | 4b | k | 107 | 6b |
| (ff) | 12 | 0c | , | 44 | 2c | L | 76 | 4c | l | 108 | 6c |
| (cr) | 13 | 0d | - | 45 | 2d | M | 77 | 4d | m | 109 | 6d |
| (so) | 14 | 0e | . | 46 | 2e | N | 78 | 4e | n | 110 | 6e |
| (si) | 15 | 0f | / | 47 | 2f | O | 79 | 4f | o | 111 | 6f |
| (dle) | 16 | 10 | 0 | 48 | 30 | P | 80 | 50 | p | 112 | 70 |
| (dc1) | 17 | 11 | 1 | 49 | 31 | Q | 81 | 51 | q | 113 | 71 |
| (dc2) | 18 | 12 | 2 | 50 | 32 | R | 82 | 52 | r | 114 | 72 |
| (dc3) | 19 | 13 | 3 | 51 | 33 | S | 83 | 53 | s | 115 | 73 |
| (dc4) | 20 | 14 | 4 | 52 | 34 | T | 84 | 54 | t | 116 | 74 |
| (nak) | 21 | 15 | 5 | 53 | 35 | U | 85 | 55 | u | 117 | 75 |
| (syn) | 22 | 16 | 6 | 54 | 36 | V | 86 | 56 | v | 118 | 76 |
| (etb) | 23 | 17 | 7 | 55 | 37 | W | 87 | 57 | w | 119 | 77 |
| (can) | 24 | 18 | 8 | 56 | 38 | X | 88 | 58 | x | 120 | 78 |
| (em) | 25 | 19 | 9 | 57 | 39 | Y | 89 | 59 | y | 121 | 79 |
| (sub) | 26 | 1a | : | 58 | 3a | Z | 90 | 5a | z | 122 | 7a |
| (esc) | 27 | 1b | ; | 59 | 3b | [ | 91 | 5b | { | 123 | 7b |
| (fs) | 28 | 1c | < | 60 | 3c | \ | 92 | 5c | \| | 124 | 7c |
| (gs) | 29 | 1d | = | 61 | 3d | ] | 93 | 5d | } | 125 | 7d |
| (rs) | 30 | 1e | > | 62 | 3e | ^ | 94 | 5e | ~ | 126 | 7e |
| (us) | 31 | 1f | ? | 63 | 3f | _ | 95 | 5f | (del) | 127 | 7f |

## Boolean algebra properties

| | | |
|---|---|---|
| Commutativity | $x \cdot y = y \cdot x$ | $x + y = y + x$ |
| Associativity | $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ | $(x + y) + z = x + (y + z)$ |
| Distributivity | $x \cdot (y + z) = x \cdot y + x \cdot z$ | $x + y \cdot z = (x + y) \cdot (x + z)$ |
| Idempotence | $x \cdot x = x$ | $x + x = x$ |
| Identity | $x \cdot 1 = x$ | $x + 0 = x$ |
| Null | $x \cdot 0 = 0$ | $x + 1 = 1$ |
| Complementarity | $x \cdot x' = 0$ | $x + x' = 1$ |
| Involution | $(x')' = x$ | |
| DeMorgan's | $(x \cdot y)' = x' + y'$ | $(x + y)' = x' \cdot y'$ |
| Absorption | $x \cdot (x + y) = x$ | $x + x \cdot y = x$ |
| No-Name | $x \cdot (x' + y) = x \cdot y$ | $x + x' \cdot y = x + y$ |
| Consensus | $(x+y) \cdot (y+z) \cdot (x'+z) = (x+y) \cdot (x'+z)$ | $x \cdot y + y \cdot z + x' \cdot z = x \cdot y + x' \cdot z$ |

Feel free to tear this page off and use it as scratch paper.

**Extra copies of K-map for problem 1 (use as scratch copies, we will NOT grade them)**

Minimal SOP

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal POS

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal SOP

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal POS

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal SOP

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |

Minimal POS

**f(a,b,c,d)** cd

| ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | X | 0 | 0 | 0 |
| 11 | 0 | X | X | 0 |
| 10 | X | 1 | 1 | X |