$$X^T X = \begin{pmatrix} \vec{x}_0^T \vec{x}_0 & \vec{x}_0^T \vec{x} \\ \vec{x}^T \vec{x}_0 & \vec{x}^T \vec{x} \end{pmatrix} = \begin{pmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix}$$

So $(X^T X)^{-1} = \dfrac{1}{\sum x_i^2 - (\sum x_i)^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & N \end{pmatrix}$, therefore

$$\vec{x}_i (X^T X)^{-1} \vec{x}_i^T = \frac{1}{\sum x_i^2 - (\sum x_i)^2} \left(\sum x_i^2 - 2N^2 \sum x_i^2 + N x_i^2 \right) = h_i$$

If we let $X^{(i)}$ be the data matrix where the ith row is deleted,

the $\hat{\beta}^{(i)} = (X^{(i)T} X^{(i)})^{-1} X^{(i)T} Y^{(i)}$ and $err^{(i)} = y_i - x_i \hat{\beta}^{(i)}$

Also $X^{(i)T} X^{(i)} = X^T X - x_i^2$

$$(X_i^T X_i)^{-1} = (X^T X)^{-1} + \frac{(X^T X)^{-1} x_i x_i^T (X^T X^{-1})}{1 - h_i}$$

(Sherman-Morrisun-Woodbury theorem)

Therefore, $\hat{\beta}^{(i)} = \hat{\beta} - \dfrac{(X^T X)^{-1} x_i e_i}{1 - h_i}$

$$err_i = y_i - x_i^T \hat{\beta}_i$$

$$= \frac{e_i}{1 - h_i}$$