

Homework 1: Representations and Bits

1. Number Conversion

- What value does the bit pattern 11011010110 represent in the unsigned representation? Write your answer in decimal.
- What value does the bit pattern 10101010010 represent in the 2's complement representation? Write your answer in decimal.
- Write the bit pattern for the decimal number -129 using a 10-bit 2's complement representation.
- Write the bit pattern for the decimal number 444 using a 10-bit unsigned representation.

2. Electronic Key Abstraction

Humans use abstraction layers in everyday life. In this problem, we ask you to think about the meaning of abstraction layers in a familiar context. The electronic key system used on our campus (based on our campus ID cards) provides an abstraction that simplifies our ability to enter buildings in a safe and controlled manner. Even people who do not know how such a device is implemented can make use of one.

- What function is provided to a human user by the human's ID card (when used as a key)?
- Briefly explain at least two ways in which this function can be implemented. (You may want to look up real implementations, or just be imaginative.) Include a simple diagram for each implementation.

3. Required Number of Bits

For each of the following, calculate the number of bits needed to uniquely represent the specified item. If you need to make any assumptions, write them down.

- A child in China.
- One of the 25 students in Prof. Lumetta's last ECE120 class.
- A building on the International Campus of ZJU in Haining.
- One PowerPoint slide among all of those available on our class website (those for the class only).

4. Ambiguity in Representations

When citing research papers, short labels are often created by merging the first letters of the authors' names with the last two digits of the year of publication. For example, a recent paper that includes Prof. Lumetta as an author might be cited as [RLKLC19], whereas one of the papers from his graduate student days might appear as [LKC95].

Give an example to explain why this mapping does not make a good representation for use by computers.

5. Ordering Numbers

- a. Order the following patterns in the 6-bit unsigned representation from smallest to largest represented value.
 - i. 101011_2 , 25_{10} , 25_{16}
 - ii. 001011_2 , 10_{10} , $0E_{16}$
- b. Order the following patterns from smallest to largest represented value, first assuming an 8-bit unsigned representation, then an 8-bit 2's complement representation.
 - i. 10010111_2 , 01101010_2 , $B5_{16}$, 14_{16}
 - ii. 00100000_2 , 10010101_2 , $C3_{16}$, 37_{16}

6. Binary Addition

Perform the following 8-bit additions. Perform the operation twice, first assuming that the patterns are in the unsigned representation, then again assuming a 2's complement representation. Indicate when overflow occurs. Show your work.

- a. $00100110 + 01000110$
- b. $10110111 + 01011011$
- c. $10010011 + 11011011$
- d. $00111011 + 10100001$

7. Binary Subtraction

Perform the following subtractions using the 8-bit 2's complement representation. *Hint: you may find it easier to negate the second number, then add.*

- a. $01001000 - 00110111$
- b. $10100101 - 10101100$
- c. $01011110 - 11110011$

8. Overflow on Other Operations

In class, we examined the overflow condition for unsigned addition, and later for 2's complement addition. Remember that overflow occurs when the resulting bit pattern does not represent the desired result for an operation.

- a. Consider a bit pattern $A_{N-1}A_{N-2}...A_1A_0$ representing an unsigned number A . Let's say that we shift the bits to the "left," dropping the leftmost and adding a 0 on the right (to obtain the bit pattern $A_{N-2}A_{N-3}...A_1A_00$). Assuming no overflow, what is the value of the resulting bit pattern in terms of the original value A ? What is the overflow condition in terms of the bits in the original bit pattern?
- b. Now consider shifting a bit pattern $A_{N-1}A_{N-2}...A_1A_0$, again representing an unsigned number A , one bit to the right and inserting a 0 on the left to produce the bit pattern $0A_{N-1}A_{N-2}...A_1$. Assuming no overflow, what is the value of the resulting bit pattern in terms of the original value A ? What is the overflow condition in terms of the bits in the original bit pattern?