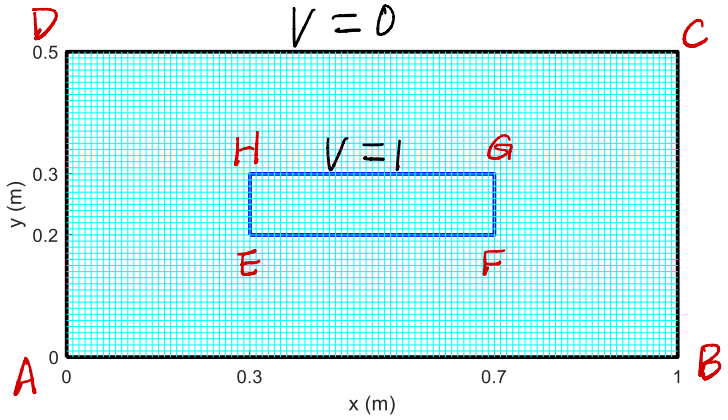
# Project 1 – Solve the Poisson Equation with The Finite Difference Method (release: Oct. 27, 2020; Due: Nov. 10, 2020)

*This is an extra homework. If you work on this, it will help you obtain as much as 2.5 points extra credits in your final grade. That is more than the average of one regular homework.*
*For this project, please submit both your solution and your code. Please submit your code in a zip file named with your name.*

Problem 1: (80 pts) Consider the 2D geometry as illustrated in the figure. The outer conductor is a rectangle with vertices $A(0,0)$, $B(1,0)$, $C(1,0.5)$ and $D(0,0.5)$. The inner conductor is a rectangle with vertices $E(0.3,0.2)$, $F(0.7,0.2)$, $G(0.7,0.3)$ and $H(0.3,0.3)$.




The outer conductor is grounded with potential $V = 0$, while the inner conductor has potential $V = 1\text{V}$.

Let us discretize the domain with $\Delta x = \Delta y = 0.01m$ as illustrated in the plot.

In between the two conductors, we have

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

And we want to solve $V(x,y)$ and $\bar{E}(x,y) = -\nabla V(x,y)$ using the finite difference method.

1) (30 pts) Following the discussion in the lecture notes, we use the iterative method to solve $V(x,y)$. Set the number of iteration $N_{\text{iter}} = 10000$. Plot out $V(x,y)$. You can use the function *imagesc*() for the plot, and please show *colorbar*.

2) (10 pts) Following step 1), compute $\bar{E}(x,y)$, and plot out $|\bar{E}(x,y)|$. Please comment on where $E$ is at its maxima.

3) (10 pts) Plot out the contour of $V(x,y)$ and the vector field $\bar{E}(x,y)$ in the same plot. You can use the function *contour*() for the contour plot and the function *quiver*() for the vector field plot, respectively. Please comment on the relation between the contour lines and the vector field.

4) (30 pts) We discussed in the lecture that we can either carry out the iteration synchronously or non-synchronously. Let us use the result in step 1) as the benchmark solution, and compare the convergence rate of the two methods of iteration. Please plot out the relative error of $V(x,y)$ as a function of iteration. You can set the y-axis (the relative error) in log-scale. Please comment on the results.

Let us define the relative error as

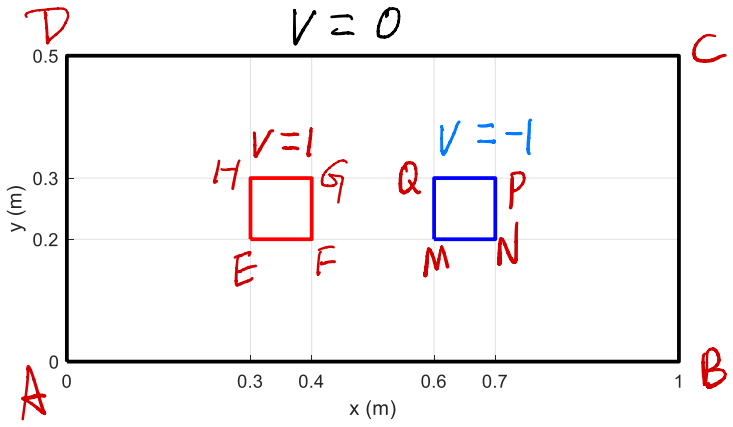
$$\text{err} = \frac{\|V - V_{\text{benchmark}}\|}{\|V_{\text{benchmark}}\|}$$

where the norm $\|V\|$ is defined as

$$\|V\| = \sqrt{\frac{1}{N}\sum_{n=1}^{N} V_n^2}$$

where $N$ is the total number of sampling points of $V(x,y)$ after discretization.

Problem 2: (20 pts) We split the inner conductor of Problem 1 into two square inner conductors with voltage $V = 1\text{V}$ and $-1\text{V}$ on the left and right, respectively. The outer conductor is still grounded with $V = 0$.




For this new configuration, please plot out the contour of $V(x,y)$ and the vector field $\bar{E}(x,y)$ in the same plot. You can use the function *contour*() for the contour plot and the function *quiver*() for the vector field plot, respectively.