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EE 379K Lab 2 Written Questions

1) $Z \sim N(\mu, \sigma^2)$ $\quad n$ samples $(z_1, \ldots, z_n)$

$$Z_{avg} = \sum_{i=1}^{n} \frac{z_i}{n}$$

a) $Z \sim N(0,1)$ $\quad n = 10000$

(i) $P(Z_{avg} > 0.1)$

$\Downarrow$

area under gaussian curve greater than 0.1

$\mu_{Z_{avg}} = 0$

$\sigma_{Z_{avg}} = \frac{1}{\sqrt{10000}} = \frac{1}{100} = 0.01$

$\downarrow$

$Z_{avg} \sim N(0, 0.01^2)$

Using the table, $\boxed{P(Z_{avg} > 0.1) = 0}$

(ii) $\boxed{P(Z_{avg} > 0.01) = 0.1587}$

(iii) $\boxed{P(Z_{avg} > 0.001) = 0.4602}$

b) $Z \sim N(\mu, \sigma^2)$

(i) $P(Z_{avg} - \mu > n^{-1/3})$

$\mu_{Z_{avg}} = \mu$

$\sigma_{Z_{avg}} = \frac{\sigma}{\sqrt{n}}$

Subtracting the mean standardizes the RV to be centered at 0.

$Z_{avg} \sim N(\mu, \frac{\sigma^2}{n})$

$Z = \frac{n^{-1/3}}{\sigma}$ ← Find area to the right of Z to get the probability that it is greater.

$$P(Z_{avg} - \mu > n^{-1/3}) = \boxed{\int_{\frac{n^{-1/3}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\sigma^2 \pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

(ii) $$P(Z_{avg} - \mu > n^{-1/2}) = \boxed{\int_{\frac{n^{-1/2}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\sigma^2 \pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

(iii) $$P(Z_{avg} - \mu > n^{-2/3}) = \boxed{\int_{\frac{n^{-2/3}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\sigma^2 \pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

2) $y_i = x_i\beta + e_i$

a) $\min_{\beta} : \frac{1}{n}\sum_{i=1}^{n}(x_i\beta - y_i)^2$

$\frac{1}{n}((x_1\beta - y_1)^2 + (x_2\beta - y_2)^2 + \dots + (x_n\beta - y_n)^2)$

$\frac{1}{n}((x_1^2\beta^2 - x_1y_1\beta - x_1y_1\beta + y_1^2) + \dots + (x_n^2\beta^2 - 2x_ny_n\beta + y_n^2))$

$\frac{1}{n}((x_1^2\beta^2 - 2x_1y_1\beta + y_1^2) + \dots + (x_n^2\beta^2 - 2x_ny_n\beta + y_n^2))$

$\frac{1}{n}((x_1^2\beta^2 + x_n^2\beta^2 - 2x_1y_1\beta - 2x_ny_n\beta + y_1^2 + y_n^2 + \dots))$

$\frac{1}{n}((x_1^2 + \dots + x_n^2)\beta^2 - 2(x_1y_1 + \dots + x_ny_n)\beta + (y_1^2 + \dots + y_n^2))$

$\min_{\beta} : \frac{(x_1^2 + \dots + x_n^2)}{n}\beta^2 - \frac{2}{n}(x_1y_1 + \dots + x_ny_n)\beta + \frac{1}{n}(y_1^2 + \dots + y_n^2)$

$\boxed{\min_{\beta} : \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\beta^2 - \left(\frac{2}{n}\sum_{i=1}^{n} x_iy_i\right)\beta + \left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right)}$ ← quadratic in $\beta$ clearly

b) $\boxed{A = \frac{1}{n}\sum_{i=1}^{n} x_i^2}$ $\boxed{B = -\frac{2}{n}\sum_{i=1}^{n} x_iy_i}$ $\boxed{C = \frac{1}{n}\sum_{i=1}^{n} y_i^2}$

$A \geq 0$ ?

This must be true because with any values for $x_i$, the value is squared, meaning it will be positive. Also, $n > 0$ because you cannot have fewer than 0 samples. So, all terms in A are positive which means $A \geq 0$.

c) $\frac{d}{d\beta}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\beta^2 - \left(\frac{2}{n}\sum_{i=1}^{n} x_iy_i\right)\beta + \left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) = 0$

$\hookrightarrow \left(\frac{2}{n}\sum_{i=1}^{n} x_i^2\right)\beta - \frac{2}{n}\sum_{i=1}^{n} x_iy_i = 0$

$\left(\frac{2}{n}\sum_{i=1}^{n} x_i^2\right)\beta = \frac{2}{n}\sum_{i=1}^{n} x_iy_i$

$$\hat{\beta} = \frac{\cancel{2/n}\sum_{i=1}^{n} x_iy_i}{\cancel{2/n}\sum_{i=1}^{n} x_i^2} = \boxed{\frac{\sum_{i=1}^{n} x_iy_i}{\sum_{i=1}^{n} x_i^2}}$$

d) $$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i(x_i\beta + e_i)}{\sum_{i=1}^{n} x_i^2} = \frac{\sum_{i=1}^{\hat{}} x_i^2\beta + x_ie_i}{\sum_{i=1}^{\hat{}} x_i^2} = \frac{\beta\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{\hat{}} x_ie_i}{\sum_{i=1}^{\hat{}} x_i^2}$$

$$\hat{\beta} = \beta + \frac{\sum_{i=1}^{\hat{}} x_ie_i}{\sum_{i=1}^{\hat{}} x_i^2} = \beta + \frac{\vec{x}\cdot\vec{e}}{\vec{x}\cdot\vec{x}} = \beta + \frac{\vec{x}\cdot\vec{e}}{\|\vec{x}\|^2}$$

$$\boxed{\hat{\beta} = \beta + \frac{\vec{x}}{\|\vec{x}\|^2}\cdot\vec{e}}$$

$$\boxed{z = \frac{\vec{x}}{\|\vec{x}\|^2}}$$