Price a European Up-and-out Call Option

Ng, Joe Hoong
ng_joehoong@hotmail.com

Nguyen, Dang Duy Nghia
nghia002@e.ntu.edu.sg

Ansari, Zain Us Sami Ahmed
zainussami@gmail.com

Thorne, Dylan
dylan.thorne@gmail.com

April. 8, 2020

Keywords: Exotic Options, European up-and-out call option, Black-Scholes-Merton model and Geometric Brownian Motion.

Abstract

In this report we discuss European up-and-out call option. In the first section we introduce a European up-and-out call option, in the second section we simulate paths for the underlying share and for the counterparty's firm value using varying sample sizes , in the third section we Determine Monte Carlo estimates of both the default-free value of the option and the Credit Valuation Adjustment (CVA), in the fourth section we calculate the Monte Carlo estimates for the price of the option incorporating counterparty risk, given by the default-free price less the CVA. We conclude the report by discussing ....

1 Introduction

A European up-and-out call option is type of a Barrier Option, these options are consider path-dependant because barrier option's payoff is based on the underlying stock's price path. The payoff of an up-and-out option at maturity T is given by,

$$Payoff_T = (S_T - K)^+ given, max_{t\in[0,T]} S_t < L$$

Where, where K is the strike of the option, L is the barrier level, and S_t is the share price at time t. This is a type of call option whose payoff is reduced to 0 if the share price crosses the barrier level.

The seminal work of Merton [1] pioneered the formula for pricing barrier options. This led pricing formulas under the geometric Brownian motion (GBM) framework for one-asset barrier options by Rich [2] and multi-asset barrier options by Wong and Kwok [3]. Although the return dynamics of underlying shares are not sufficiently well described by the GBM process proposed by Black and Scholes [4], we are going to assume both the stock and counterparty firm values follow GBM with constant drift and volatilities and default only occurs at maturity.

2 Lifetime simulations of the option.

3 Monte Carlo estimates of both the default-free value of the option and the Credit Valuation Adjustment (CVA).

4 Monte Carlo estimates for the price of the option incorporating counterparty risk, given by the default-free price less the CVA.

[1]:
```
import QuantLib as ql
from collections import namedtuple
import math
```

[2]:
```
today = ql.Date(15, ql.February, 2020);
settlement= ql.Date(19, ql.February, 2020);
ql.Settings.instance().evaluationDate = today;
term_structure = ql.YieldTermStructureHandle(
    ql.FlatForward(settlement,0.04875825,ql.Actual365Fixed())
    )
index = ql.Euribor1Y(term_structure)
```

```
--------------------------------------------------------------------------------
--
    Model Price     Market Price      Implied Vol       Market Vol        Rel Error
--------------------------------------------------------------------------------
--
        0.00871          0.00949          0.10531          0.11480         -0.08263
        0.00968          0.01008          0.10634          0.11080         -0.04018
        0.00867          0.00871          0.10652          0.10700         -0.00448
        0.00653          0.00625          0.10665          0.10210          0.04442
        0.00357          0.00334          0.10680          0.10000          0.06773
--------------------------------------------------------------------------------
--
Cumulative Error :         0.12288
```

5 Conclusion

References

[1] Merton, Robert C. "Theory of rational option pricing." The Bell Journal of economics and management science (1973): 141-183.

[2] Rich, Don R. "The mathematical foundations of barrier option-pricing theory." Advances in futures and options research 7 (1994).

[3] Wong, Hoi Ying, and YueâĂŘKuen Kwok. "Multi-asset barrier options and occupation time derivatives." Applied Mathematical Finance 10.3 (2003): 245-266.

[4] Black, Fisher, and Myron Scholes. "The pricing and Corportate Liabilities." Journal of Political Economy 81 (1973).

[5] Yousuf, M. "A fourth-order smoothing scheme for pricing barrier options under stochastic volatility." International Journal of Computer Mathematics 86.6 (2009): 1054-1067.