Price a Vanilla European Call Option

Ng, Joe Hoong
ng_joehoong@hotmail.com

Nguyen, Dang Duy Nghia
nghia002@e.ntu.edu.sg

Ansari, Zain Us Sami Ahmed
zainussami@gmail.com

Thorne, Dylan
dylan.thorne@gmail.com

April. 26, 2020

Keywords: European Options, Heston model, Constant Elasticity of Variance (CEV) model and Stochastic Volatility.

Abstract

In this paper, we price a vanilla European call option under the Heston model and then simulate the monthly share price over a year using the Constant Elasticity of Variance (CEV) model, with the assumption of constant volatility each month. Monte Carlo simulations with varying sample sizes are run and the results are plotted against the closed form value for comparison.

1 Introduction

We initialize most variables as given by the question.

- Option maturity is one year
- The option is struck at-the-money
- The current share price is $100
- The risk-free continuously compounded interest rate is 8
- The volatility for the underlying share is 30

2 Fourier pricing technique under Heston model dynamics

Black-Scholes option pricing models assumed volatility of the underlying assets to be constant or a predetermined function of time, we will now implement a model which features instantaneous variance of asset price using volatility that evolves stochastically in time. Although there are several models incorporating stochastic volatility including introduced by Hull and White [?], Stein and Stein [?] and Heston [?] [?], we will implement Heston's constant interest rate model.

With the assumption that the underlying share follows the Heston model dynamics, the additional parameters required are specified as follows:

- $\nu_0 = 0.06$
- $\kappa = 9$
- $\theta = 0.06$
- $\rho = -0.4$

The Characteristic function is implemented using a function presented by Albrecher et al [**?**]. The function is written as:

$$\phi_{S_T} = exp(C(\tau; u) + D(\tau; u)v_t + iu\log(S_t))$$

Where,

$$C(\tau; u) = ri\tau u + \theta\kappa[\tau x_{\_} - \frac{1}{a}\log(\frac{1 - ge^{d\tau}}{1 - g})],$$

$$D(\tau; u) = (\frac{1 - e^{d\tau}}{1 - ge^{d\tau}})x_{\_},$$

$$\tau = T - t,$$

$$g = \frac{x_{\_}}{x_{+}},$$

$$x_{\pm} = \frac{b \pm d}{2a},$$

$$d = \sqrt{b^2 - 4ac},$$

$$c = -\frac{u^2 + ui}{2},$$

$$b = \kappa - \rho\sigma iu,$$

$$a = \frac{\sigma^2}{2}$$

[ ]:
```
#Characteristic function code

a = sigma**2/2

def b(u):
    return kappa - rho*sigma*1j*u

def c(u):
```

```
    return -(u**2+1j*u)/2

def d(u):
    return np.sqrt(b(u)**2-4*a*c(u))

def xminus(u):
    return (b(u)-d(u))/(2*a)

def xplus(u):
    return (b(u)+d(u))/(2*a)

def g(u):
    return xminus(u)/xplus(u)

def C(u):
    val1 = T*xminus(u)-np.log((1-g(u)*np.exp(-T*d(u)))/(1-g(u)))/a
    return r*T*1j*u + theta*kappa*val1

def D(u):
    val1 = 1-np.exp(-T*d(u))
    val2 = 1-g(u)*np.exp(-T*d(u))
    return (val1/val2)*xminus(u)

def log_char(u):
    return np.exp(C(u) + D(u)*v0 + 1j*u*np.log(S0))

def adj_char(u):
    return log_char(u-1j)/log_char(-1j)
```

Now we vectorize the code, calculate an estimate for integrals and calculate the Fourier estimate of our call price.

[]:
```
delta_t = t_max/N
from_1_to_N = np.linspace(1,N,N)
t_n = (from_1_to_N-1/2)*delta_t

#Integral calculations
first_integral = sum((((np.exp(-1j*t_n*k_log)*adj_char(t_n)).imag)/t_n)*delta_t)
second_integral = sum((((np.exp(-1j*t_n*k_log)*log_char(t_n)).imag)/t_n)*delta_t)

#Call value
fourier_call_val = S0*(1/2 + first_integral/np.pi)-np.exp(-r*T)*K*(1/2 +
 ↪second_integral/np.pi)
fourier_call_val
```

[]: 13.734895692109077

To see the effectiveness of the pricing option under Heston dynamics we will also price the call option under Black-Scholes assumption.

[]:
```
# Code for analytical solution for vanilla European Call option
d_1_stock = (np.log(S0/K)+(r + sigma**2/2)*(T))/(sigma*np.sqrt(T))
d_2_stock = d_1_stock - sigma*np.sqrt(T)

analytic_callprice = S0*norm.cdf(d_1_stock)-K*np.exp(-r*(T))*norm.cdf(d_2_stock)
analytic_callprice
```

[]:
```
15.711312547892973
```

3 Simulate a share price path using CEV Model

4 Monte Carlo estimates

5 Conclusion

References

[1] Hull, J. and White, A. (1987). The pricing of options on assets with stochastic volatilities. The journal of finance, 42(2):281–300.

[2] Stein, E. M. and Stein, J. C. (1991). Stock price distributions with stochastic volatility: an analytic approach. Review of financial Studies, 4(4):727–752.

[3] Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of financial studies, 6(2):327–343.

[4] Heston, S. L. (1997). A simple new formula for options with stochastic volatility.

[5] Albrecher, H., Mayer, P., Schoutens, W. and Tistaert, J. (2007). "The Little Heston Trap", Wilmott (1): 83–92.