Topic: Converting between degrees and DMS

Question: What is the measure, in DMS (degrees, minutes, and seconds), of 38.27°?

Answer choices:

A $38^\circ 0' 0.27''$

B $38^\circ 16' 12''$

C $38^\circ 0.27' 0''$

D $38^\circ 12' 16''$

Solution: B

First, note that

$$38.27^\circ = 38^\circ + 0.27^\circ = 38^\circ + \left(\frac{27}{100}\right)^\circ$$

Since 38° is the integer part of 38.27°, in DMS the degrees part of the measure of this angle is 38°. What remains is to convert the non-integer part of 38.27°, which is $(27/100)^\circ$, to minutes and seconds.

We'll first convert $(27/100)^\circ$ from degrees to minutes, so we'll multiply the angle measure by 1, written as the conversion factor $(60')/(1^\circ)$:

$$\left(\frac{27}{100}\right)^\circ = \left(\frac{27}{100}\right)^\circ (1)$$

$$\left(\frac{27}{100}\right)^\circ = \left(\frac{27}{100}\right)^\circ \left(\frac{60'}{1^\circ}\right)$$

On the right-hand side of this equation, the $^\circ$ in the numerator cancels against the $^\circ$ in the denominator, so we get

$$\left(\frac{27}{100}\right)^\circ = \left[\frac{27(60)}{100}\right]'$$

$$\left(\frac{27}{100}\right)^\circ = \left(\frac{1{,}620}{100}\right)'$$

$$\left(\frac{27}{100}\right)^\circ = \left(\frac{81}{5}\right)'$$

Note that

$$\frac{81}{5}=\frac{80+1}{5}=\frac{80}{5}+\frac{1}{5}=16+\frac{1}{5}=16\frac{1}{5}$$

Substituting this result, we obtain

$$\left(\frac{27}{100}\right)^{\circ}=\left(16\frac{1}{5}\right)'$$

Thus the minutes part of an angle of measure $(27/100)^{\circ}$ is equal to the integer part of

$$16\frac{1}{5}'$$

which is $16'$. To get the seconds part of that angle, we need to convert the non-integer part, namely $(1/5)'$, to seconds. To do that, we'll multiply by 1, written as the conversion factor $(60'')/(1')$:

$$\left(\frac{1}{5}\right)'=\left(\frac{1}{5}\right)'(1)$$

$$\left(\frac{1}{5}\right)'=\left(\frac{1}{5}\right)'\left(\frac{60''}{1'}\right)$$

On the right-hand side of this equation, the $'$ in the numerator cancels against the $'$ in the denominator, so we get

$$\left(\frac{1}{5}\right)'=\left(\frac{60}{5}\right)''$$

$$\left(\frac{1}{5}\right)' = 12''$$

Substituting this result, we find that

$$\left(\frac{27}{100}\right)^{\circ} = \left(16\frac{1}{5}\right)' = 16' + \left(\frac{1}{5}\right)' = 16' + 12''$$

Thus

$$38.27^{\circ} = 38^{\circ}16'12''$$

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Question: If the measure of an angle in DMS is $55^\circ 36' 18''$, what is its measure in decimal degrees?

Answer choices:

A 55.9°

B 55.54°

C 55.5°

D 55.605°

Solution: D

The degrees part of $55^\circ 36' 18''$ is 55°, so (in decimal degrees) the integer part of its measure is 55°. What we still need to do is convert the measures of the minutes and seconds parts of the measure of the given angle to decimal degrees. We'll do that by converting those two parts separately, and then adding the results to get the non-integer part of the measure of the given angle.

Since there are 60 minutes in one degree, we can convert the minutes part of the given angle (i.e., $36'$) to decimal degrees by multiplying it by 1, written as the conversion factor $(1^\circ)/(60')$:

$$36' = 36'(1)$$

$$36' = 36'\left(\frac{1^\circ}{60'}\right)$$

$$36' = \left(\frac{36}{60}\right)^\circ$$

$$36' = \left(\frac{3}{5}\right)^\circ$$

Note that $(3/5) = 0.6$, so $36' = 0.6^\circ$.

Since there are 60 minutes in one degree, and 60 seconds in one minute, we will use a product of two conversion factors, $(1^\circ)/(60')$ and $(1')/(60'')$, to convert the seconds part of the given angle (i.e., $18''$) to decimal degrees:

$$18'' = 18''(1)(1)$$

$$18'' = 18''\left(\frac{1^\circ}{60'}\right)\left(\frac{1'}{60''}\right)$$

On the right-hand side of this equation, the $''$ in the numerator cancels against the $''$ in the denominator, and the $'$ in the numerator cancels against the $'$ in the denominator, so we are left with the following:

$$18'' = \left[\frac{18}{60(60)}\right]^\circ$$

$$18'' = \left(\frac{18}{3{,}600}\right)^\circ$$

$$18'' = \left(\frac{1}{200}\right)^\circ$$

Note that $(1/200) = 0.005$, so $18'' = 0.005^\circ$.

Adding the measures of the minutes and seconds parts of the given angle, each of which is now expressed in units of decimal degrees, we have

$$36'18'' = 36' + 18'' = 0.6^\circ + 0.005^\circ = 0.605^\circ$$

When we include the integer part of the given angle, we obtain

$$55^\circ 36'18'' = 55^\circ + 36'18'' = 55^\circ + 0.605^\circ = 55.605^\circ$$

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Question: If a disc rotates $190^\circ 41' 58''$ about its center, followed immediately by a rotation of $135^\circ 56' 37''$ about its center, what is the total angle that the disc rotates?

Answer choices:

A $326^\circ 38' 35''$

B 327.483°

C 325.95°

D $325^\circ 79' 35''$

Solution: A

First, let's compute the sum of the degrees parts of the two rotations, and then do the same for the minutes parts and the seconds parts:

$$190^\circ + 135^\circ = 325$$

$$41' + 56' = 97'$$

$$58'' + 37'' = 95''$$

We have to adjust these in such a way that the minutes part of the total angle of rotation ends up being in the interval $[0', 59']$ and the seconds part ends up being in the interval $[0'', 60'')$.

Starting with the sum of the seconds parts of the two rotations, $95''$, we have to successively subtract $60''$ (i.e., $1'$) from $95''$ until we reach an angle that's in the interval $[0'', 60'')$. This will take just one subtraction:

$$95'' - 60'' = 35''$$

Therefore, the seconds part of the total angle of rotation is $35''$.

Next, we deal with the minutes. Note, however, that to make up for the $60''\,(= 1')$ that we subtracted from the sum of the seconds parts of the two rotations, we have to add $1'$ to the sum of the minutes parts of the two angles of rotation, which gives

$$97' + 1' = 98'$$

Now we have to successively subtract $60'$ (i.e., 1°) from $98'$ until we reach an angle that's in the interval $[0',59']$. A single subtraction of $60'$ yields an angle in that interval:

$$98' - 60' = 38'$$

Thus the minutes part of the total angle of rotation is $38'$.

Finally, to make up for the $60'\ (=1^\circ)$ that we subtracted from the sum of the minutes parts, we have to add 1° to the sum of the degrees parts. Therefore, the degrees part of the total angle of rotation is

$$325^\circ + 1^\circ = 326^\circ$$

What we have found is that the total angle through which the disc is rotated is $326^\circ 38' 35''$.