Topic: Coterminal angles

Question: Which angle is coterminal with -150°?

Answer choices:

A 120°

B -420°

C 570°

D 230°

Solution: C

Let $\theta = -150^\circ$, and recall that if two angles are coterminal, then their measures (in degrees) differ by an integer multiple of 360°. Let α denote an angle which is coterminal with θ. Then there is some integer n such that

$$\alpha = \theta + n(360^\circ)$$

Putting this another way, there is some integer n such that

$$\frac{\alpha - \theta}{360^\circ} = n$$

Let's check each of the answer choices in turn, by substituting each of them as a value of α and determining whether there is some integer n that satisfies the equation $(\alpha - \theta)/(360^\circ) = n$.

$$\alpha = 120^\circ: \quad \alpha - \theta = 120^\circ - (-150^\circ) = 120^\circ + 150^\circ = 270^\circ \qquad \frac{270^\circ}{360^\circ} = \frac{3}{4}$$

$$\alpha = -420^\circ: \ \alpha - \theta = -420^\circ - (-150^\circ) = -420^\circ + 150^\circ = -270^\circ$$
$$\frac{-270^\circ}{360^\circ} = -\frac{3}{4}$$

$$\alpha = 570^\circ: \quad \alpha - \theta = 570^\circ - (-150^\circ) = 570^\circ + 150^\circ = 720^\circ \qquad \frac{720^\circ}{360^\circ} = 2$$

$$\alpha = 230^\circ: \quad \alpha - \theta = 230^\circ - (-150^\circ) = 230^\circ + 150^\circ = 380^\circ \qquad \frac{380^\circ}{360^\circ} = \frac{19}{18}$$

The only value of α for which $(\alpha - \theta)/(360^\circ)$ is equal to an integer is 570°.

Topic: Coterminal angles

Question: Which angle α is coterminal with $(71/16)\pi$ if the terminal side of α is reached from the terminal side of $(71/16)\pi$ by making two full rotations about the origin in the negative direction?

Answer choices:

A $\alpha = \frac{7}{16}\pi$

B $\alpha = -\frac{7}{16}\pi$

C $\alpha = \frac{39}{16}\pi$

D $\alpha = -\frac{25}{16}\pi$

Solution: A

Let $\theta = (71/16)\pi$, and recall that if two angles are coterminal, then their measures (in radians) differ by an integer multiple of 2π. Let α denote an angle which is coterminal with θ. Then there is some integer m such that

$$\alpha = \theta + m(2\pi)$$

Here, we want to determine the angle α which is coterminal with θ and whose terminal side is reached from the terminal side of θ by making a rotation of two full turns about the origin in the negative direction. Therefore, we must have $m = -2$, so we substitute $m = -2$ and solve for α:

$$\alpha = \theta + (-2)(2\pi)$$

$$\alpha = \frac{71}{16}\pi - 4\pi$$

$$\alpha = \left(\frac{71}{16} - 4\right)\pi$$

$$\alpha = \left(\frac{71 - 16(4)}{16}\right)\pi$$

$$\alpha = \left(\frac{71 - 64}{16}\right)\pi$$

$$\alpha = \frac{7}{16}\pi$$

Topic: Coterminal angles

Question: Which angle is not coterminal with $-(8/3)\pi$?

Answer choices:

A $\frac{4}{3}\pi$

B $-\frac{4}{3}\pi$

C $-\frac{26}{3}\pi$

D $\frac{22}{3}\pi$

Solution: B

Let $\theta = -(8/3)\pi$, and recall that if two angles are coterminal, then their measures (in radians) differ by an integer multiple of 2π. Let α denote an angle which is coterminal with θ. Then there is some integer m such that

$$\alpha = \theta + m(2\pi)$$

Putting this another way, there is some integer m such that

$$\frac{\alpha - \theta}{2\pi} = m$$

Let's check each of the answer choices in turn, by substituting each of them as a value of α and determining whether there is some integer m that satisfies the equation $(\alpha - \theta)/(2\pi) = m$.

$$\alpha = \frac{4}{3}\pi: \quad \alpha - \theta = \frac{4}{3}\pi - \left(-\frac{8}{3}\pi\right) = \left(\frac{4}{3} + \frac{8}{3}\right)\pi = \frac{12}{3}\pi = 4\pi \quad \frac{4\pi}{2\pi} = 2$$

$$\alpha = -\frac{4}{3}\pi: \quad \alpha - \theta = -\frac{4}{3}\pi - \left(-\frac{8}{3}\pi\right) = \left(-\frac{4}{3} + \frac{8}{3}\right)\pi = \frac{4}{3}\pi \quad \frac{\left(\frac{4}{3}\pi\right)}{2\pi} = \frac{2}{3}$$

$$\alpha = -\frac{26}{3}\pi: \quad \alpha - \theta = -\frac{26}{3}\pi - \left(-\frac{8}{3}\pi\right) = \left(-\frac{26}{3} + \frac{8}{3}\right)\pi = -\frac{18}{3}\pi = -6\pi \quad \frac{-6\pi}{2\pi} = -3$$

$$\alpha = \frac{22}{3}\pi: \quad \alpha - \theta = \frac{22}{3}\pi - \left(-\frac{8}{3}\pi\right) = \left(\frac{22}{3} + \frac{8}{3}\right)\pi = \frac{30}{3}\pi = 10\pi \quad \frac{10\pi}{2\pi} = 5$$

The only value of α for which $(\alpha - \theta)/(2\pi)$ is not equal to an integer is $-(4/3)\pi$.