Topic: Functions of negative angles

Question: Which of the following angles is coterminal with -116° and lies in the interval $[0^\circ, 360^\circ)$?

Answer choices:

A 116°

B 326°

C 360°

D 244°

Solution: D

Let $\theta = -116^\circ$, and let α be the angle that is coterminal with θ and lies in the interval $[0^\circ, 360^\circ)$. Then there is a unique integer n such that

$$\alpha = \theta + n(360^\circ)$$

Therefore,

$$0 \le \alpha < 360^\circ$$

$$0 \le \theta + n(360^\circ) < 360^\circ$$

Substituting $\theta = -116^\circ$:

$$0 \le -116^\circ + n(360^\circ) < 360^\circ$$

Adding 116° to all three quantities in these two inequalities, we obtain

$$116^\circ \le n(360^\circ) < 360^\circ + 116^\circ$$

$$116^\circ \le n(360^\circ) < 476^\circ$$

Dividing through by 360°,

$$\frac{116^\circ}{360^\circ} \le n < \frac{476^\circ}{360^\circ}$$

Note that

$$0 = \frac{0}{360} < \frac{116}{360} \le n < \frac{476}{360} < \frac{720}{360} = 2$$

From these inequalities, we find that n is an integer that satisfies $0 < n < 2$, so n must be equal to 1.

To obtain the measure of α, we'll substitute $n = 1$ in the expression $\theta + n(360^\circ)$:

$$\alpha = \theta + 1(360^\circ)$$

$$\alpha = -116^\circ + 360^\circ$$

$$\alpha = 244^\circ$$

Topic: Functions of negative angles

Question: Which of the following angles lies in the interval $[0,2\pi)$ and is coterminal with $-(9/4)\pi$?

Answer choices:

A $\frac{9}{4}\pi$

B $-\frac{7}{2}\pi$

C $\frac{7}{4}\pi$

D $\frac{5}{4}\pi$

Solution: C

Let $\theta = -(9/4)\pi$, and let α be the angle that is coterminal with θ and lies in the interval $[0,2\pi)$. Then there is a unique integer n such that

$$\alpha = \theta + n(2\pi)$$

Therefore,

$$0 \le \alpha < 2\pi$$

$$0 \le \theta + n(2\pi) < 2\pi$$

Substituting $\theta = -(9/4)\pi$:

$$0 \le -\frac{9}{4}\pi + n(2\pi) < 2\pi$$

Adding $(9/4)\pi$ to all three quantities in these two inequalities, we obtain

$$\frac{9}{4}\pi \le n(2\pi) < 2\pi + \frac{9}{4}\pi$$

$$\frac{9}{4}\pi \le n(2\pi) < \left(2 + \frac{9}{4}\right)\pi$$

$$\frac{9}{4}\pi \le n(2\pi) < \left[\frac{4(2)+9}{4}\right]\pi$$

$$\frac{9}{4}\pi \le n(2\pi) < \left(\frac{8+9}{4}\right)\pi$$

$$\frac{9}{4}\pi \le n(2\pi) < \frac{17}{4}\pi$$

Dividing through by 2π,

$$\frac{\left(\frac{9}{4}\pi\right)}{2\pi} \leq n < \frac{\left(\frac{17}{4}\pi\right)}{2\pi}$$

$$\frac{9}{4(2)} \leq n < \frac{17}{4(2)}$$

$$\frac{9}{8} \leq n < \frac{17}{8}$$

Now

$$1 = \frac{8}{8} < \frac{9}{8} \leq n < \frac{17}{8} < \frac{24}{8} = 3$$

Thus n is an integer that satisfies $1 < n < 3$, so n must be equal to 2.

To obtain the measure of α, we'll substitute $n = 2$ in the expression $\theta + n(2\pi)$:

$$\alpha = \theta + 2(2\pi)$$

$$\alpha = -\frac{9}{4}\pi + 2(2\pi)$$

$$\alpha = \left(-\frac{9}{4} + 4\right)\pi$$

$$\alpha = \left[\frac{-9 + 4(4)}{4}\right]\pi$$

$$\alpha = \left(\frac{-9 + 16}{4}\right)\pi$$

$$\alpha = \frac{7}{4}\pi$$

Topic: Functions of negative angles

Question: Which of the following angles lies in the interval $(-(3/5)\pi, (7/5)\pi]$ and is coterminal with $(67/5)\pi$?

Answer choices:

A $-\frac{2}{5}\pi$

B $-\frac{3}{5}\pi$

C $\frac{7}{5}\pi$

D $-\frac{1}{5}\pi$

Solution: C

Let $\theta = (67/5)\pi$, and let α be the angle that is coterminal with θ and lies in the interval $(-(3/5)\pi, (7/5)\pi]$. Note that the length of the interval $(-(3/5)\pi, (7/5)\pi]$ is

$$\frac{7}{5}\pi - \left(-\frac{3}{5}\pi\right) = \left(\frac{7}{5} + \frac{3}{5}\right)\pi = \left(\frac{7+3}{5}\right)\pi = \frac{10}{5}\pi = 2\pi$$

Thus there is a unique integer n such that

$$\alpha = \theta + n(2\pi)$$

Therefore,

$$-\frac{3}{5}\pi < \alpha \le \frac{7}{5}\pi$$

$$-\frac{3}{5}\pi < \theta + n(2\pi) \le \frac{7}{5}\pi$$

Substituting $\theta = (67/5)\pi$:

$$-\frac{3}{5}\pi < \frac{67}{5}\pi + n(2\pi) \le \frac{7}{5}\pi$$

Subtracting $(67/5)\pi$ from all three quantities in these two inequalities, we obtain

$$-\frac{3}{5}\pi - \frac{67}{5}\pi < n(2\pi) \le \frac{7}{5}\pi - \frac{67}{5}\pi$$

$$-\left(\frac{3}{5} + \frac{67}{5}\right)\pi < n(2\pi) \le \left(\frac{7}{5} - \frac{67}{5}\right)\pi$$

$$-\left(\frac{3+67}{5}\right)\pi < n(2\pi) \le \left(\frac{7-67}{5}\right)\pi$$

$$-\frac{70}{5}\pi < n(2\pi) \le -\frac{60}{5}\pi$$

$$-14\pi < n(2\pi) \le -12\pi$$

Dividing through by 2π,

$$\frac{-14\pi}{2\pi} < n \le \frac{-12\pi}{2\pi}$$

$$-7 < n \le -6$$

Thus n is an integer that satisfies $-7 < n \le -6$, so n must be equal to -6.

To obtain the measure of α, we'll substitute $n = -6$ in the expression $\theta + n(2\pi)$:

$$\alpha = \theta + (-6)(2\pi)$$

$$\alpha = \frac{67}{5}\pi - 12\pi$$

$$\alpha = \left(\frac{67}{5} - 12\right)\pi$$

$$\alpha = \left[\frac{67 + 5(-12)}{5}\right]\pi$$

$$\alpha = \left(\frac{67-60}{5}\right)\pi$$

$$\alpha = \frac{7}{5}\pi$$