Trigonometry Formulas

Right triangle trigonometry (SOH CAH TOA)

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

Trigonometric functions
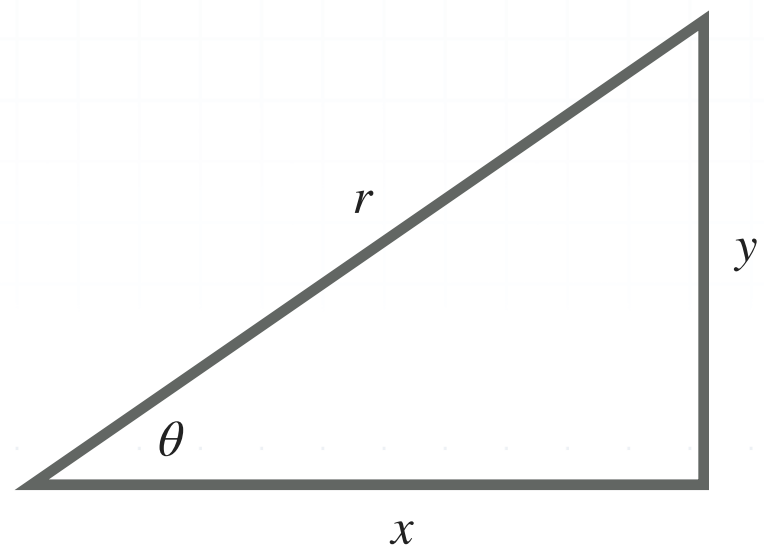
$$\sin \theta = \frac{y}{r} \qquad \csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r} \qquad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$$

Transformations

Vertical and horizontal shifting

Assume $c > 0$

to shift a function c units upward, find $y = f(x) + c$

to shift a function c units downward, find $y = f(x) - c$

to shift a function $\quad$ c units to the right, find $\quad$ $y = f(x - c)$

to shift a function $\quad$ c units to the left, find $\quad$ $y = f(x + c)$

Vertical and horizontal stretching and reflecting
Assume $c > 1$

to stretch $\quad$ vertically by a factor of c, find $\quad$ $y = cf(x)$

to shrink $\quad$ vertically by a factor of c, find $\quad$ $y = (1/c)f(x)$

to stretch $\quad$ horizontally by a factor of c, find $\quad$ $y = f(x/c)$

to shrink $\quad$ horizontally by a factor of c, find $\quad$ $y = f(cx)$

to reflect $\quad$ about the x-axis, find $\quad$ $y = -f(x)$

to reflect $\quad$ about the y-axis, find $\quad$ $y = f(-x)$

Reciprocal identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad\qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad\qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad\qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Even-odd identities

$$\sin(-\theta) = -\sin\theta \qquad\qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad\qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad\qquad \cot(-\theta) = -\cot\theta$$

Co-function identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad\qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad\qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad\qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Sum-difference identities

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Double-angle identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad \cos 2x = 2 \cos^2 x - 1 \qquad \cos 2x = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-angle (power reducing) identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} = \frac{1}{2}(1 + \cos 2x)$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product-to-sum identities

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\cos x \sin y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

Sum-to-product identities

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of sines for area of a triangle

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2}ac \sin B$$

$$\text{Area} = \frac{1}{2}bc \sin A$$

Heron's formula for area of a triangle

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c are the lengths of the sides of the triangle and

$$s = \frac{1}{2}(a + b + c)$$

The Unit Circle

$(0,1)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{2}$

$90°$

$\frac{2\pi}{3}$

$120°$

$\frac{3\pi}{4}$

$135°$

$\pi/3$

$60°$

$\pi/4$

$45°$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{5\pi}{6}$

$150°$

$\pi/6$

$30°$

$(-1,0)$ π $180°$ $0°$ $0, 2\pi$ $(1,0)$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{7\pi}{6}$ $210°$

$330°$ $\frac{11\pi}{6}$

$\frac{5\pi}{4}$ $225°$

$315°$ $\frac{7\pi}{4}$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\frac{4\pi}{3}$ $240°$

$300°$ $\frac{5\pi}{3}$

$270°$

$\frac{3\pi}{2}$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$(0, -1)$