Unit 6 Topic 4 H.W '24-25 (1)

page 287-289 #14, 16, 26, 27, 32, 42, 66, 69, 70

14] $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right) dx$

$= \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}\cdot 2x^{\frac{1}{2}} + C = \frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C = \frac{2}{3}x^{\frac{3}{2}} + \sqrt{x} + C$

$\frac{d}{dx}\left(\frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C\right) = \frac{2}{3}\cdot\frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} = \sqrt{x} + \frac{1}{2\sqrt{x}}$

16] $\int \left(\sqrt[4]{x^3} + 1\right) dx = \int \left(x^{\frac{3}{4}} + 1\right) dx = \frac{4}{7}x^{\frac{7}{4}} + x + C$

$\frac{d}{dx}\left(\frac{4}{7}x^{\frac{7}{4}} + x + C\right) = \frac{4}{7}\cdot\frac{7}{4}x^{\frac{3}{4}} + 1 = \sqrt[4]{x^3} + 1$

26] $\int (\sec y)(\tan y - \sec y)\, dy$

$= \int (\sec y \cdot \tan y)\, dy - \int \sec^2 y\, dy = \sec y - \tan y + C$

$\frac{d}{dy}\left(\sec y - \tan y + C\right) = \sec y \cdot \tan y - \sec^2 y$

$= \sec y(\tan y - \sec y)$

27] $\int (\tan^2 y + 1)\, dy = \int \sec^2 y\, dy = \tan y + C$

Pythagorean Identity

$\frac{d}{dy}\left(\tan y + C\right) = \sec^2 y = 1 + \tan^2 y$

32] $\int \left(\frac{4}{x} + \sec^2 x\right) dx = 4\int \frac{1}{x}\, dx + \int \sec^2 x\, dx$

$= 4\ln|x| + \tan x + C$

$\frac{d}{dx}\left(4\ln|x| + \tan x + C\right) = 4\cdot\frac{1}{x} + \sec^2 x$

$= \frac{4}{x} + \sec^2 x$

42] $f''(x) = \sin x, \quad f'(0) = 1, \quad f(0) = 6$

$f'(x) = \int \sin x \, dx = -\cos x + C$

$f'(0) = -\cos(0) + C = 1 \;;\; -1 + C = 1 \;;\; C = 2$

$f'(x) = -\cos x + 2$

$f(x) = \int -\cos x + 2 \, dx = -\sin x + 2x + D$

$f(0) = -\sin(0) + 2\cdot 0 + D = 6 \;;\; D = 6$

$\therefore f(x) = -\sin x + 2x + 6$

66] $f'(x) = \begin{cases} -1, & 0 \le x < 2 \\ 2, & 2 < x < 3 \\ 0, & 3 < x \le 4 \end{cases}$

$f(x) = \begin{cases} -x + C_1, & 0 \le x < 2 & \cdots (1) \\ 2x + C_2, & 2 < x < 3 & \cdots (2) \\ C_3, & 3 < x \le 4 & \cdots (3) \end{cases}$

f is continuous and $f(0) = 1$;

From (1), $0 + C_1 = 1 \quad \therefore f(x) = -x + 1, \; 0 \le x < 2$

f is continuous at $x = 2$,

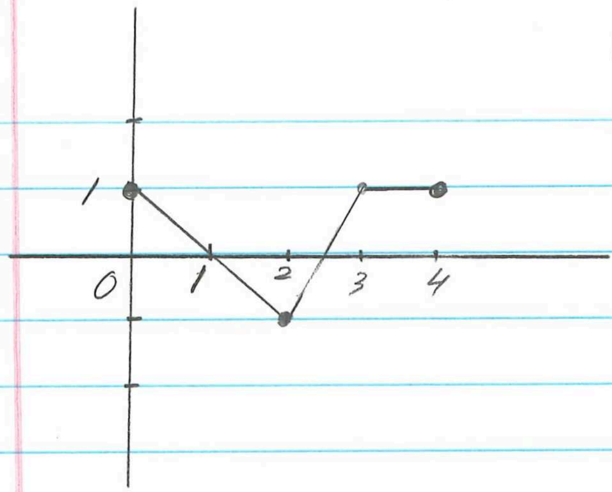
From (2), $2\cdot 2 + C_2 = 4 + C_2 = -2 + 1 = -1$

$\therefore C_2 = -5 \quad \therefore f(x) = 2x - 5, \; 2 < x < 3$

f is continuous at $x = 3$,

From (3), $C_3 = 2\cdot 3 - 5 = 1 \quad \therefore f(x) = 1, \; 3 < x \le 4$

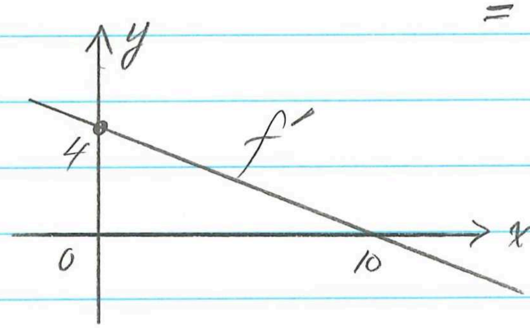
$$\therefore \quad f(x) = \begin{cases} -x+1, & 0 \le x < 2 \\ 2x-5, & 2 \le x < 3 \\ 1, & 3 \le x \le 4 \end{cases} \qquad ③$$



69] $$\int \sqrt{x}\,(10x-3)\,dx = \int x^{\frac{1}{2}}(10x-3)\,dx$$

$$= \int 10x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx = 10\cdot\frac{2}{5}x^{\frac{5}{2}} - 3\cdot\frac{2}{3}x^{\frac{3}{2}} + C$$

$$= 4x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + C \qquad Ⓓ$$

70] 

$f(0) = 3, \quad f(10) = ?$

$$f'(x) = -\frac{2}{5}x + 4$$

$$f(x) = \int\left(-\frac{2}{5}x + 4\right)dx = -\frac{2}{5}\cdot\frac{1}{2}\cdot x^2 + 4x + C$$

$$= -\frac{1}{5}x^2 + 4x + C$$

$$f(0) = C = 3$$

$$\therefore f(x) = -\frac{1}{5}x^2 + 4x + 3$$

$$f(10) = -\frac{1}{5}\cdot 10^2 + 4\cdot(10) + 3 = -20 + 40 + 3$$

$$= 23 \qquad Ⓓ$$