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Switching Investments Can Be a Bad Idea When Parrondo's Paradox Applies

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Many studies have indicated that a buy-and-hold investment strategy is superior to a trading strategy. This is thought to be true because trading incurs transaction costs that lower net returns compared to a buy-and-hold strategy. We propose a behavioral finance argument to illustrate that merely switching between positive expected return assets can lead to a long-run negative expected return, even when transaction costs are ignored. This counterintuitive result may obtain because of Parrondo's Paradox. We provide a stylized theoretical example that demonstrates how a trader can lose money by trading between assets with positive long-run expected returns. We also present simulation results to support our example. Thus, long-run negative results from trading may not be due entirely to transaction costs. A trading strategy may prove inferior to buy-and-hold for agents simply because of their singular trading patterns, as we outline in the paper.

Many studies indicate that a buy-and-hold investing strategy is superior to a trading strategy. Brock, Lakonishok, and LeBaron [1992] and Malkiel [1995] show that institutional money managers who trade actively underperform relative to market indices. This is thought to be true because trading incurs transaction costs that lower net returns compared to a buy-and-hold strategy. Moreover, overconfidence in market timing or in data interpretation can lead to excessive trading and poor performance (e.g., see Odean [1999] and Gervais and Odean [2001]).

This article illustrates that merely switching between positive expected return assets can lead to a long-run negative return even when transaction costs are ignored and traders are not overconfident. Thus, we give you a Ms. Jane Doe who assumes she will make money because she is only buying assets with positive expected returns and trading between these assets. She pays no transactions costs and does not overestimate the positive expected return of any of her holdings. Yet she loses money over the long haul!

This counterintuitive result may obtain because of Parrondo's Paradox. Thus, long-run negative trading results may not be due entirely to transaction costs or to traders' inflated views of their own abilities. A trading strategy may prove inferior to a buy-and-hold strategy simply because of agents' singular trading patterns.

The rise in trading volume on the major exchanges implies that investors prefer to trade assets and switch positions quite often. Investors continually reallocate among investments for many reasons. One rationale drawn from behavioral finance is that traders are overconfident in their own abilities to find investments that will provide large returns. Their overconfidence leads them to trade often in search of superior returns. Odean [1999] demonstrates that investors with discount brokerage accounts trade excessively with poor results, and he argues that this activity may be due to overconfidence.

In this paper we do not attempt to assign a reason for active trading by market participants. Our focus is on the possible result of trading, regardless of the motivation.

The original Parrondo paradox was devised by Juan Parrondo in 1997 to help explain the phenomena known in physics as flashing ratchets. Harmer and Abbott [1999a, 1999b] and Harmer, Abbott, and Taylor [2000] introduced the paradox to the literature as an interesting example to help explain counterintuitive results. The paradox uses two losing gambling games that, when played in alternation, lead to a winning result. Thus, the paradox obtains.

Harmer et al. [2000] suggest that the paradox can be extended to other fields such as stochastic signaling processing, biology, and economics. This paper extends the paradox to investments. We adapt the variation of the game found in Parrondo, Harmer, and Abbott [2000] (PHA) in a stylized example to illustrate our result. Our presentation of the gambling game is the same as PHA's game, except we use financial assets with probabilistic outcomes in place of coin tosses. We keep similar notation.

In our example, we propose five assets, A, and B_i, i = 1…4. Consider asset A to be the S&P Index and asset B_i to be an individual stock. The investor trades between the S&P Index and an individual stock, the

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choice of which depends on the investor's recent gains or losses. Over the time period under consideration, asset A offers an investment return (win) of a unit with probability $p = ½$, and an investment return (loss) of a unit with probability $1 - p$. Gains and losses in any investment are one unit. Asset B_i has a slightly more complicated return: The probabilities for obtaining a positive investment return "win" or "loss" of one unit are shown in Table 1.

We choose the probabilities for asset B and the rationale for the choice of the switching rules strictly for mathematical reasons. The justification is not necessarily to present a glimpse into reality, but to provide a stylized example to demonstrate that the complexity of human behavior may lead to bizarre investment returns.

We assume completely random investing between assets A and B_i. The investor is investing either in the index, A, or a particular asset B_i at all times. At the end of each period, the investor sells the current investment, makes a gain or loss of one unit, and then chooses asset A or B_i strictly randomly. As for which asset B_i is chosen, we again assume the choice is motivated solely by recent investment experience.

The psychological reasons are not examined here, but we assume the investor's reasoning is affected by whether her recent investments were gains or losses. Thus, the investor either does not learn from, or is not influenced by, the underlying probabilities of a gain from the different investments. If, over the past two periods, the investor experienced two losses, asset B_1 is chosen. If the investor experienced a loss followed by a gain, B_2 is chosen. If a gain was followed by a loss, B_3 is chosen. B_4 is chosen if the investor has had two consecutive gains.

The investment rule is stylized, but is not far from real world behavior. This rule is somewhat akin to when a gambler has a couple of wins, believes he is on a lucky streak, and gambles accordingly. Or when a gambler changes the type of wager after several losses. Thus, recent changes in the investor's capital, defined as $X(t)$, impact the choice of asset B_i. Note that our model does not have to rely on recent investment results; it may be defined on long past returns. We use recent results because the latest gains or losses are assumed to have more impact on current investment decisions.

Although PHA [2000] provide a simulation result, we demonstrate our own simulation result here. The mathematics for deriving the parameters of the simulation are given in the Appendix and follow those of PHA. The mathematics provide the theoretical underpinning for the trading dynamics, but they are not necessary to follow the results of this paper. It is more important to note that investor trading behavior may lead to unusual and unexpected investment returns.

Table 1. *Selection of Asset Bi By Recent Trades and the Associated Win Probabilities*

		Probability of …	
Recent Trades	**Asset**	**Win**	**Loss**
Loss, Loss	B_1	p_1	$1 - p_1$
Loss, Gain	B_2	p_2	$1 - p_2$
Gain, Loss	B_3	p_3	$1 - p_3$
Gain, Gain	B_4	p_4	$1 - p_4$

Table 2. *Capital From Switching Investments or Individual Investments*

	Initial Conditions				
Investment	**1, –1**	**–1, 1**	**–1, –1**	**1, 1**	**Number of Switches**
A&B	0.18246	0.08316	–0.38478	–0.3817	20
	0.0508	–0.03409	0.50503	0.50082	40
	–0.0674	–0.14857	–0.62667	–0.62654	60
	–0.18998	–0.27729	–0.7491	–0.74626	80
	–0.31388	–0.3952	–0.86385	–.86452	100
B	0.71193	0.5365	–.78924	–.17996	20
	0.83898	0.65592	–0.66172	–.06299	40
	0.96456	0.78303	–0.52769	0.07362	60
	1.07807	0.92451	–.40782	0.20088	80
	1.21862	1.03505	–0.28061	0.32785	100
A	0.10544	0.12475	0.10614	0.11769	20
	0.24	0.23327	0.23655	0.22821	40
	0.35657	0.34689	0.35115	0.36076	60
	.47318	0.48267	0.46618	0.47532	80
	.59392	0.59306	0.59231	0.59321	100

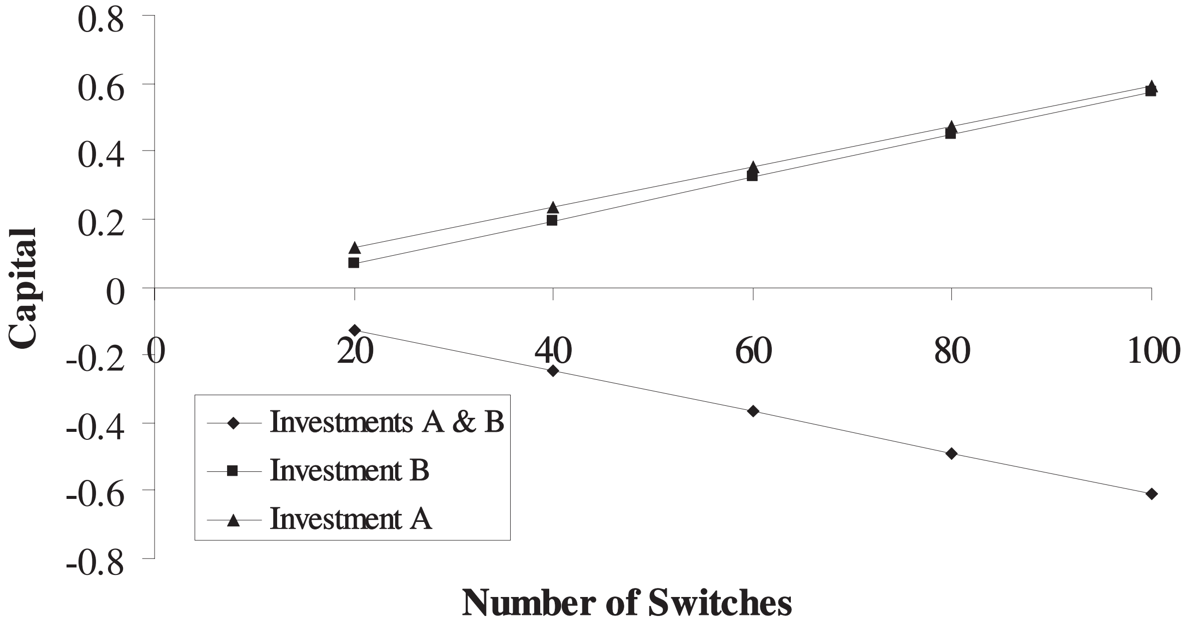
Note: Initial conditions are described as (-1, -1) for two losses, (-1, 1) for a loss followed by a gain, and so on. For each initial condition, gains or losses are calculated starting from zero capital for 500,000 repetitions of each number of switches. Values are shown for investments B and A without switches, and for switching between A and B. The parameter value is 0.003.

The simulation results in Figure 1 show clearly that investing in asset A or B_i without switching between them is a winning strategy. Increasing the number of investment opportunities increases the amount of capital gained, because these assets have a greater probability of gains than losses. When the investor switches between asset A and assets B_i, however, losses paradoxically mount. The chart shows that the more switches made, the more losses result.

There are four possible initial conditions: (loss, loss), (loss, win), (win, loss), and (win, win), which we designate as (–1, –1), (–1, 1), (1, –1), and (1, 1). The effect of the initial condition on gains and losses is worthy of examination even in our stylized example.

Table 2 presents the simulation results by initial condition. As Odean [1999] claims, investors seem not to learn from past mistakes because equity markets are too noisy. In our example also, we see that market participants have a difficult time learning from investment results. Note in Table 2 that under initial conditions of

FIGURE 1
Change in Capital with Switches



Note: The upper two lines show that individual investments A and B gain when there are no switches between them. Note that investment B consists of four different assets in which investment is made one at a time. The bottom line depicts the surprising decline in capital when random switching between A and B occurs. Simulations are done with a parameter value of 0.003. Results are from 500,000 ensembles at 20, 40, 60, 80, and 100 switch levels. There are four possible initial conditions that affect only the intercepts, but not the slopes. Each curve is the average of the four initial conditions.

(–1, –1), investment B_i alone is a loser after 100 investment opportunities. Of course, the slope moves in the predicted direction, but the (–1, –1) condition is not readily overcome even for this asset, which is likely to deliver positive returns in the long run. Switching between A and B is also sensitive to the initial conditions. An investor starting with a (win, loss) scenario is likely to have a positive outcome after twenty to forty switches and feel confident, not knowing the long-run negative effect of switching. In certain scenarios, the investor would not learn this for quite some time.

We have demonstrated the paradox that investors who trade from one asset to another may exhibit long-run losses even if the assets have expected long-run positive returns. We constrain the trader's investment policy here to be dependent on past successes and failures, but we could also make it dependent on capital amount (see Harmer and Abbott [1999a]).

Although it is true that inordinate trading may be due to overconfidence and may lead to poor net returns because of excessive transaction costs, our results do not come from this motivation. The paradox obtains regardless of transaction costs. We present recent trade results as another behavioral motivation that may lead to less than optimal investing. Our example is stylized, but what trader is immune from the effects of recent trades?

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Appendix

Although we allow random switching between assets A and B_i, the results follow when there are reasons for switching between these assets as long as a sufficient number of switches occurs in the time frame examined. See PHA [2000] for the effect of other than random switching behavior in this model.

Now define the vector

$$Y(t) = \begin{pmatrix} X(t) - X(t-1) \\ X(t-1) - X(t-2) \end{pmatrix} \tag{1}$$

This vector makes a Markov chain and can take four values (±1, ±1). The transition probabilities are obtained from the investor's behavior toward assets B_i. We let $\pi_1(t)$, $\pi_2(t)$, $\pi_3(t)$, and $\pi_4(t)$ be the probabilities that Y(t) is (–1, –1), (1, –1), (–1, 1), and (1, 1), respectively. We are interested in finding the stationary distribution π_{st}, a row vector, of these probabilities. The transition matrix A is given as

$$\mathbf{A} = \begin{pmatrix} 1-p_1 & 0 & 1-p_3 & 0 \\ p_1 & 0 & p_3 & 0 \\ 0 & 1-p_2 & 0 & 1-p_4 \\ 0 & p_2 & 0 & p_4 \end{pmatrix} \tag{2}$$

We know that $\pi_i(t+1) = \pi_i(t)A$, since A is the transition matrix. The stationary distribution π_{st} vector implies that $\pi_{st}A = \pi_{st}$. The stationary distribution can be determined as

$$\pi_{st} = \frac{1}{N} \begin{pmatrix} (1-p_3)(1-p_4) \\ (1-p_4)p_1 \\ (1-p_4)p_1 \\ p_1 p_2 \end{pmatrix} \tag{3}$$

where N is a normalization constant.

The probability for a gain in the long run is

$$p_{win} = \sum_{i=1}^{4} \pi_{st,i} p_i = \frac{p_1(p_2 + 1 - p_4)}{(1-p_4)(2p_1 + 1 - p_3) + p_1 p_2} \tag{4}$$

Or written as $p_{win} = 1/(2 + c/s)$, where $s = p_1(p_2 + 1 - p_4)$, which is positive for all p_i, and $c = (1 - p_4)(1 - p_3) - p_1p_2$. We define gaining (losing) as X(t), having an expectation that it is an increasing (decreasing) function of t. For investment in asset B_i to show, on average, a gain, break-even, or loss depends on the sign of c. This is true because for an investment to show, on average, a gain (loss), we must have $p_{win} >$ (<) ½. Thus, over the long run, if $c < 0$ the investment is likely to show a gain, if $c = 0$ it will show no gain or loss, and if $c > 0$ it is likely to show a loss.

The random switching between assets A and B_i does not alter the Markov chain property of vector Y(t). The switching between investments according to our rules changes only the probabilities of having a gain or loss through the blending of the probabilities of the investment in A and B_i. The probabilities of a gain are now $p_1' = (p_i + p)/2$. To demonstrate the paradox, we need to produce values of p and p_i (i = 1, 2, 3, 4) so that the following conditions are satisfied:

$$\begin{aligned} &1 - p < p, \\ &(1-p_4)(1-p_3) < p_1 p_2, \\ &(2 - p_4 - p)(2 - p_3 - p) > (p_1 + p)(p_2 + p), \end{aligned} \tag{5}$$

The first inequality indicates that investment solely in A will produce a gain on average. The second inequality indicates that investment solely in B_i will produce a gain on average. The third inequality is the result of substituting p_i' and reversing the inequality so that switching investments between assets A and B_i will result in a long-run loss.

The next step is to select probabilities so that the paradox is exhibited. Clearly, given $\varepsilon > 0$, asset A is gaining if $p = ½ + \varepsilon$, as the first inequality will be satisfied. In order for asset B_i to be gaining, we allow assets B_2 and B_3 to be "good" assets. That is, we set $p_1 = 3/10 + \varepsilon$, $p_2 = p_3 = 3/4 + \varepsilon$, and $p_4 = 1/10 + \varepsilon$. The second condition is also met for any $\varepsilon > 0$. The third condition is met if $\varepsilon < 1/168$ for the parameter values we have chosen. These values are derived from those used by PHA [2000] by simple subtraction so that our ε condition is the same as theirs.