BC2

a) $\frac{g(-4)+8}{e^0-2+1}$ $\lim_{x\to -2}(g(2x)-4x)=0$

$\lim_{x\to -2}(e^{x^2-4}+x+1)=0$

$\lim_{x\to -2}\left(\frac{g(2x)-4x}{e^{x^2-4}+x+1}\right)=\lim_{x\to -2}\left(\frac{2g'(2x)-4}{(2x)(e^{x^2-4})+1}\right)=\frac{2g'(-4)-4}{(-4)(e^0)+1}=\frac{0-4}{-4+1}=$

$\boxed{\frac{4}{3}}$

b) $g'(x)=f(x)$

$-6, 0, 2$

$g(-6)=-\int_{-6}^{2} f(t)\,dt=3$

$g(0)=-\int_{0}^{2} f(t)\,dt=\boxed{2}$

$g(2)=\int_{2}^{2} f(t)\,dt=0$

Because $g'(x)>0$ from $[-4,0]$ and $g'(x)<0$ from $[0,2]$, and $g'(0)=0$, and $g(0)>g(-6)$ and $g(2)$, the endpoints, $2=g(0)$ is the absolute max of $g(x)$ on $[-6,2]$

c) $\int_{-3}^{1} x f'(2x)\,dx$

$u = x \quad dv = f'(2x)\,dx$

$du = dx \quad v = \frac{f(2x)}{2}$

$\frac{x f(2x)}{2} - \int \frac{f(2x)}{2}\,dx$

$\frac{x f(2x)}{2} - F(x) + C$

$\frac{1 f(2)}{2} - F(1) + F(-3) - \left(\frac{-3 f(-6)}{2}\right)$

$\frac{f(2)}{2} + F(-3) - F(1) + \frac{3}{2} f(-6)$

$u = 2x \quad dx = \frac{du}{2}$

$du = 2dx$

$F(-3) - F(1) = -\int_{-3}^{1} \frac{f(2x)}{2}\,dx = (-1)\int_{-6}^{2} \frac{f(u)}{4}\,du = -\frac{3}{4}$

$\frac{f(2)}{2} - \frac{3}{4} + \frac{3}{2} f(-6)$

$0 - \frac{3}{4} + \left(\frac{3}{2}\right)(-5) = \frac{-15}{2} - \frac{3}{4} = \frac{-30}{4} - \frac{3}{4} = \boxed{\frac{-33}{4}}$

BC2 continued

d) $\boxed{\int_2^5 (f(x))^2\,dx}$

e) $\sum_{n=5}^{\infty} (-4)\left(\frac{1}{2}\right)^{n-5}$

$$-4 \sum_{n=5}^{\infty} \frac{1}{2^{n-5}} = -4 \sum_{n=0}^{\infty} \frac{1}{2^n}$$

$$-4\left(\frac{1}{1-\frac{1}{2}}\right) = -4\left(\frac{1}{\frac{1}{2}}\right) = (-4)(2) = \boxed{-8}$$