Dataset

I. Single Satellite with SDMA

A. Network Model

we consider a downlink satellite and terrestrial transmission model, where a N_{T}-antenna LEO satellite serves K single-antenna ground users. Let $\mathbb{E}\{|s_k|^2\} = 1$ where $k \in \mathcal{K} = \{1, ..., K\}$. Thus, the transmit signal is given by

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{w}_k s_k, \tag{1}$$

where $\mathbf{w}_k \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ denotes the beamforming vector associated with the stream s_k. Then, the total required transmit power at the LEO satellite is expressed as

$$P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right) = \mu \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 + P_{\mathrm{C}}, \tag{2}$$

where $\mu \in [1, \infty)$ and P_{C} denote the power amplifier efficiency factor and the constant power consumption by circuit modules, respectively.

The considered system operates in a time-descretized manner [**?**], where the time is divided into a set of slots with small intervals. In each time slot, the received signal at the k-th user is expressed as

$$y_k = \sum_{k=1}^{K} \mathbf{h}_k^H \mathbf{w}_k s_k + n_k, \tag{3}$$

where $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$ denotes the direct channel between the LEO satellite and the k-th ground user. n_k with power of σ_k^2 denotes the corresponding Gaussian noise.

The received information rate at the i-th user are respectively given by

$$R_k\left(\{\mathbf{w}_k\}\right) = \log_2\left(1 + \frac{\left|\mathbf{h}_k^H \mathbf{w}_k\right|^2}{\sum\limits_{k' \neq k}^{K} \left|\mathbf{h}_k^H \mathbf{w}_{k'}\right|^2 + \sigma_k^2}\right). \tag{4}$$

Then, the sum achievable rate of the considered system is expressed as

$$R\left(\{\mathbf{w}_k\}\right) = \sum_{k=1}^{K} R_k\left(\{\mathbf{w}_k\}\right). \tag{5}$$

As EE is defined as the ratio of the sum achievable rate to the total power consumption, the EE of the considered system is given by

$$\mathrm{EE}\left(\{\mathbf{w}_k\}\right) = \frac{R\left(\{\mathbf{w}_k\}\right)}{P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right)}. \tag{6}$$

B. Time-varying Channel Model

As the wireless environments changes fast in practice, the time-varying channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k (\frac{c}{4\pi f_c d_s})^2} \mathbf{g}(t), \tag{7}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the small-scale fading vector. For notation simplification, we omit the index of $\mathbf{h}$ and $\mathbf{h}$ here.

Following the Jakes fading model [**?**], the small-scale flat fading is modeled as a first-order complex Gauss Markov process, which is given by

$$\mathbf{g}(t) = \rho \mathbf{g}(t-1) + \sqrt{1 - \rho^2} \mathbf{e}, \tag{8}$$

where $\mathbf{g}(0) \sim \mathcal{CN}(0, \mathbf{I})$ and $\mathbf{e} \sim \mathcal{CN}(0, \mathbf{I})$ denote the Rician fading vector and the additive complex Gaussian noise, respectively, and the correlation coefficient ρ is determined by

$$\rho = J_0(2\pi f_d T_s), \tag{9}$$

where $J_0(\cdot)$, T_s and f_d denote the first kind zero-order Bessel function, the time interval between adjacent instants, and the maximum Doppler frequserncy[1], respectively.

C. Fixed Channel Model

The channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k (\frac{c}{4\pi f_c d_s})^2} \mathbf{g}(t), \tag{10}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the small-scale fading vector with Rician. For notation simplification, we omit the index of $\mathbf{h}$ and $\mathbf{h}$ here.

D. Sum-rate maximization problem formulation

We aim to maximize the system sum rate of the considered system under the constraints of the power budget at the LEO satellite and the QoS requirements of information communication at each ground user. Therefore, the corresponding optimization problem is mathematically formulated as

$$\begin{aligned} \mathbf{P}_0 : & \max_{\{\mathbf{w}_k\}} R\left(\{\mathbf{w}_k\}\right) \\ \text{s.t. } & R_k\left(\{\mathbf{w}_k\}\right) \geq \xi_k, \end{aligned} \tag{11a}$$

[1]In practice, the GGUs and LGUs may have some movements sometimes. For example, the users may inadvertently shake or vibrate the wearable wireless sensor devices or smart phones when they move their bodies, then the scenarios with some Doppler frequserncy shifts may appear.

$$
\begin{aligned}
& P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right) \leq P_{\max}, && \text{(11b)}\\
& \forall k \in \mathcal{K}.
\end{aligned}
$$

In constraints (39b), constants ξ_k denotes the achievable information rate requirement of the i-th user. In constraint (39c), $P_{\max}$ denotes the maximum power budget at the LEO satellite.

E. EE problem formulation

We aim to maximize the system EE of the considered system under the constraints of the power budget at the LEO satellite and the QoS requirements of both information communication at ground each user. Therefore, the corresponding optimization problem is mathematically formulated as

$$
\begin{aligned}
\mathbf{P}_0 : & \max_{\{\mathbf{w}_k\}} \mathrm{EE}\left(\{\mathbf{w}_k\}\right) \\
\text{s.t. } & R_k\left(\{\mathbf{w}_k\}\right) \geq \xi_k, && \text{(12a)}\\
& P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right) \leq P_{\max}, && \text{(12b)}\\
& \forall k \in \mathcal{K}.
\end{aligned}
$$

In constraints (39b), constants ξ_k denotes the achievable information rate requirement of the i-th user, respectively. In constraint (39c), $P_{\max}$ denotes the maximum power budget at the LEO satellite.

II. Multi-satellite design with SDMA

A. Network Model

For the heterogeneous scenarios, we consider a downlink heterogeneous satellite network composed of one N_{M}-antenna GEO satellite and one N_{T}-antenna LEO satellite, where these satellites use the same spectrum. The GEO satellite serves M single-antenna GEO ground users (GGUs) and each LEO satellite serves K LGUs. For clarity, Let $\mathbb{E}\{|s_k|^2\} = 1$ and $\mathbb{E}\{|s_m|^2\} = 1$. We use 0 to denote the index of the GEO satellite, $m \in \mathcal{M} = \{1, 2, ..., M\}$ to denote the index of the m-th GGU and $k \in \mathcal{K} = \{1, 2, ..., K\}$ to denote the index of the k-th LGU. Thus, the transmit signals of GEO and LEO are, respectively, given by

$$
\mathbf{x}_{\mathrm{G}} = \sum_{m=1}^{M} \mathbf{w}_m s_m, \tag{13}
$$

and

$$
\mathbf{x}_{\mathrm{L}} = \sum_{k=1}^{K} \mathbf{w}_k s_k. \tag{14}
$$

The network is running in the time-descretized manner, where the time is divided into a set of slots with small intervals. In each time slot, the channel state and network state are assumed to be unchanged and they may change from one slot to its next randomly. At the t-th time slot, the transmit signals generated by the GEO satellite and the n-th LEO satellite are respectively given by

$$
\mathbf{x}_0 = \sum_{m=1}^{M} \mathbf{w}_{0,m} s_{0,m},
$$

and

$$
\mathbf{x}_n = \sum_{k=1}^{K} \mathbf{w}_{n,k} s_{n,k},
$$

where $s_{0,m} \in \mathbb{C}$ and $s_{n,k} \in \mathbb{C}$ denote the desired steams for the m-th GGU in the GEO satellite area with $\mathbb{E}\{|s_{0,m}|^2\} = 1$ and the k-th LGU in the n-th LEO satellite area with $\mathbb{E}\{|s_{n,k}|^2\} = 1$, respectively. $\mathbf{w}_{0,m} \in \mathbb{C}^{N_{\mathrm{M}} \times 1}$ denotes the GEO satellite transmit beamforming vectors for the m-th GGU in the GEO satellite area. $\mathbf{w}_{n,k}^{\mathrm{p}} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ denotes the LEO satellite beamforming vectors for the k-th LGU in the n-th LEO satellite area.

Let $\mathbf{h}_{i,0,m} \in \mathbb{C}^{N_{\mathrm{M}} \times 1}$ and $\mathbf{h}_{i,n,k} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ denote the channel vector from i-th BS (with $i = 0$ denoting the GEO satellite and $i \in \mathcal{N}$ denoting the n-th LEO satellite) to the m-th GGU and k-th LGU in the n-th LEO satellite area, respectively. At the t-th time slot, the received signals at the m-th GGU in the GEO satellite area $y_{0,m}$ and the k-th LGU in the n-th LEO satellite area $y_{n,k}$ are respectively given by

$$
\begin{aligned}
y_{0,m} &= \mathbf{h}_{0,0,m}^{H} \mathbf{x}_0 + \sum_{n=1}^{N} \mathbf{h}_{n,0,m}^{H} \mathbf{x}_n + z_{0,m} \\
&= \mathbf{h}_{0,0,m}^{H} \mathbf{w}_{0,m} s_{0,m} + \sum_{m' \neq m}^{M} \mathbf{h}_{0,0,m}^{H} \mathbf{w}_{0,m'} s_{0,m'} \\
&+ \sum_{n=1}^{N} \sum_{k=1}^{K} \mathbf{h}_{n,0,m}^{H} \mathbf{w}_{n,k} s_{n,k} + z_{0,m},
\end{aligned}
$$

and

$$
\begin{aligned}
y_{n,k} &= \sum_{n=1}^{N} \mathbf{h}_{n,n,k}^{H} \mathbf{x}_n + \mathbf{h}_{0,n,k}^{H} \mathbf{x}_0 + z_{n,k} \\
&= \mathbf{h}_{n,n,k}^{H} \left(\sum_{k=1}^{K} \mathbf{w}_{n,k} s_{n,k} \right) + \sum_{n' \neq n}^{N} \mathbf{h}_{n',n,k}^{H} \left(\sum_{k=1}^{K} \mathbf{w}_{n',k} s_{n',k} \right) \\
&+ \sum_{m=1}^{M} \mathbf{h}_{0,n,k}^{H} \mathbf{w}_{0,m} s_{0,m} + z_{n,k},
\end{aligned}
$$

where $z_{0,m} \sim \mathcal{CN}(0, \sigma_a^2)$ and $z_{n,k} \sim \mathcal{CN}(0, \sigma_b^2)$ denote the additive white Gaussian noises (AWGN) with both σ_a^2 and σ_b^2 denoting the noise power at the GGU and LGU, respectively.

For the m-th GGU, the achievable information rate with normalized bandwidth is given by

$$
R_{0,m}\left(\{\mathbf{w}_{n,k}^{\mathrm{p}}\}\right) = \log_2(1 + \Gamma_{0,m}). \tag{15}
$$

where $\Gamma_{0,m}$ denotes the received signal to interference and noise ratio (SINR), which is expressed by (41).

The received information rate at the k-th LGU in the n-th LEO satellite area are respectively given by

$$
R_{n,k}\left(\{\mathbf{w}_{n,k}, \mathbf{w}_n^{\mathrm{c}}\}\right) = \log_2(1 + \Gamma_{n,k}), \tag{16}
$$

where $\Gamma_{n,k}$ denotes corresponding SINR, which are given by (45).

$$\Gamma_{0,m} = \frac{|\mathbf{h}_{0,0,m}^{H}\mathbf{w}_{0,m}|^2}{\sum\limits_{m' \neq m}^{M} |\mathbf{h}_{0,0,m}^{H}\mathbf{w}_{0,m'}|^2 + \sum\limits_{n=1}^{N}\sum\limits_{k=1}^{K} |\mathbf{h}_{n,0,m}^{H}\mathbf{w}_{n,k}^{\mathrm{p}}|^2 + \sigma_a^2}, \tag{17}$$

$$\Gamma_{n,k} = \frac{|\mathbf{h}_{n,n,k}^{H}\mathbf{w}_{n,k}|^2}{\sum\limits_{k' \neq k}^{K} |\mathbf{h}_{n,n,k}^{H}\mathbf{w}_{n,k'}|^2 + \sum\limits_{n' \neq n}^{N}\sum\limits_{k'=1}^{K} |\mathbf{h}_{n',n,k}^{H}\mathbf{w}_{n',k'}|^2 + \sum\limits_{m=1}^{M} |\mathbf{h}_{0,n,k}^{H}\mathbf{w}_{0,m}|^2 + \sigma_b^2}, \tag{18}$$

B. Time-varying Channel Model

As the wireless environments changes fast in practice, the time-varying channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k (\frac{c}{4\pi f_c d_s})^2}\mathbf{g}(t), \tag{19}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the small-scale fading vector. For notation simplification, we omit the index of $\mathbf{h}_{i,0,m}$ and $\mathbf{h}_{i,n,k}$ here.

Following the Jakes fading model [**?**], the small-scale flat fading is modeled as a first-order complex Gauss Markov process, which is given by

$$\mathbf{g}(t) = \rho\mathbf{g}(t-1) + \sqrt{1-\rho^2}\mathbf{e}, \tag{20}$$

where $\mathbf{g}(0) \sim \mathcal{CN}(0, \mathbf{I})$ and $\mathbf{e} \sim \mathcal{CN}(0, \mathbf{I})$ denote the Rician fading vector and the additive complex Gaussian noise, respectively, and the correlation coefficient ρ is determined by

$$\rho = J_0(2\pi f_d T_s), \tag{21}$$

where $J_0(\cdot)$, T_s and f_d denote the first kind zero-order Bessel function, the time interval between adjacent instants, and the maximum Doppler frequserncy[2], respectively.

C. Fixed Channel Model

The channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k (\frac{c}{4\pi f_c d_s})^2}\mathbf{g}(t), \tag{22}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the small-scale fading vector with Rician. For notation simplification, we omit the index of $\mathbf{h}$ and $\mathbf{h}$ here.

[2]In practice, the GGUs and LGUs may have some movements sometimes. For example, the users may inadvertently shake or vibrate the wearable wireless sensor devices or smart phones when they move their bodies, then the scenarios with some Doppler frequserncy shifts may appear.

D. Sum rate maximization Problem Formulation

To improve the spectrum resource utilization of the described heterogeneous mega-constellation network, we aim to maximize the achievable sum information rate of N LEO satellite areas by jointly optimizing the transmit beamforming vectors $\{\mathbf{w}_{n,k}\}$ for all LGUs, subject to the rate requirement constraints at GGUs and LGUs. Therefore, the optimization problem is mathematically expressed by

$$\max_{\{\mathbf{w}_{n,k}\}} \quad \sum_{n=1}^{N}\sum_{k=1}^{K} R_{n,k}\left(\{\mathbf{w}_{n,k}\}\right) \tag{23a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \left\|\mathbf{w}_{n,k}^{\mathrm{p}}\right\|^2 \le P_n, \tag{23b}$$

$$R_{0,m}\left(\{\mathbf{w}_{n,k}\}\right) \ge \xi_{\mathrm{GGU}}, \tag{23c}$$

$$R_{n,k}\left(\{\mathbf{w}_{n,k}\}\right) \ge \xi_{\mathrm{LGU}}, \tag{23d}$$

$$\forall m \in \mathcal{M}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}. \tag{23e}$$

In constraint (52b), P_n denotes the maximum power budget at the n-th LEO satellite. In constraints (52c) and (52d), ξ_{GGU} and ξ_{LGU} denote the minimum required achievable information rate threshold of each GGU and LGU, respectively.

E. EE maximization Problem Formulation

For the considered heterogeneous mega-constellation network, the system EE is denoted by

$$\mathrm{EE}\left(\{\mathbf{w}_{n,k}\}\right) = \sum_{n=1}^{N} \frac{\sum\limits_{k=1}^{K} R_{n,k}\left(\{\mathbf{w}_{n,k}\}\right)}{\mu\left(\sum\limits_{k=1}^{K} \left\|\mathbf{w}_{n,k}\right\|^2\right) + P_{\mathrm{C}}}, \tag{24}$$

where $\mu \in [1, \infty)$ and P_{C} denote the power amplifier efficiency factor and the constant power consumption by circuit modules, respectively.

We aim to maximize the achievable EE of N LEO satellite areas by jointly optimizing the transmit beamforming vectors $\{\mathbf{w}_{n,k}\}$ for all LGUs, subject to the rate requirement constraints at GGUs and LGUs. Therefore, the optimization problem is mathematically expressed by

$$\max_{\{\mathbf{w}_{n,k}\}} \quad \mathrm{EE}\left(\{\mathbf{w}_{n,k}\}\right) \tag{25a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \left\|\mathbf{w}_{n,k}^{\mathrm{p}}\right\|^2 \le P_n, \tag{25b}$$

$$R_{0,m}\left(\{\mathbf{w}_{n,k}\}\right) \ge \xi_{\mathrm{GGU}}, \tag{25c}$$

$$R_{n,k}\left(\{\mathbf{w}_{n,k}\}\right) \geq \xi_{\mathrm{LGU}}, \tag{25d}$$

$$\forall m \in \mathcal{M}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}. \tag{25e}$$

In constraint (52b), P_n denotes the maximum power budget at the n-th LEO satellite. In constraints (52c) and (52d), ξ_{GGU} and ξ_{LGU} denote the minimum required achievable information rate threshold of each GGU and LGU, respectively.

III. Single Satellite with RSMA

A. Network Model

we consider a downlink satellite and terrestrial transmission model, where a N_{T}-antenna LEO satellite serves K single-antenna ground users. To enhance the communication quality and mitigate the co-channel interference, the 1-layer RSMA scheme is adopted.

With the 1-layer RSMA scheme at the LEO satellite, the message associated with the k-th user W_k is split into two parts, i.e., the common part $W_k^{(\mathrm{C})}$ and the private part $W_k^{(\mathrm{P})}$. Then, the common parts of all the ground users are combined as $W^{(\mathrm{C})}$, which is encoded as a common stream s_0. The private part of the k-th ground user is encoded as the private stream s_k. Without loss of generality, let $\mathbb{E}\{|s_k|^2\} = 1$ where $k \in \mathcal{K} = \{0, 1, ..., K\}$. Thus, the transmit signal includes $(K+1)$ symbols is given by

$$\mathbf{x} = \sum_{k=0}^{K} \mathbf{w}_k s_k, \tag{26}$$

where $\mathbf{w}_k \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ denotes the beamforming vector associated with the stream s_k. Then, the total required transmit power at the LEO satellite is expressed as

$$P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right) = \mu \sum_{k=0}^{K} \|\mathbf{w}_k\|^2 + P_{\mathrm{C}}, \tag{27}$$

where $\mu \in [1, \infty)$ and P_{C} denote the the power amplifier efficiency factor and the constant power consumption by circuit modules, respectively.

The considered system operates in a time-descretized manner [**?**], where the time is divided into a set of slots with small intervals. In each time slot, the received signal at the i-th user ($i \in \mathcal{I} = \{1, 2, ..., K\}$) is expressed as

$$y_i = \sum_{k=0}^{K} \mathbf{h}_i^H \mathbf{w}_k s_k + n_i, \tag{28}$$

where $\mathbf{h}_i \in \mathbb{C}^{N \times 1}$ denotes the direct channel between the LEO satellite and the i-th ground user. n_i with power of σ_i^2 denotes the corresponding Gaussian noise.

With the RSMA scheme, each user first decodes the common stream by treating all the private streams as interference, and then decodes its private stream after removing the common stream by SIC. The received information rates of the common stream and the private stream at the i-th user are respectively given by

$$R_i^{(\mathrm{C})}\left(\{\mathbf{w}_k\}\right) = \log_2\left(1 + \frac{\left|\mathbf{h}_i^H \mathbf{w}_0\right|^2}{\sum\limits_{k=1}^{K}\left|\mathbf{h}_i^H \mathbf{w}_k\right|^2 + \sigma_i^2}\right), \tag{29}$$

and

$$R_i^{(\mathrm{P})}\left(\{\mathbf{w}_k\}\right) = \log_2\left(1 + \frac{\left|\mathbf{h}_i^H \mathbf{w}_i\right|^2}{\sum\limits_{k=1, k\neq i}^{K}\left|\mathbf{h}_i^H \mathbf{w}_k\right|^2 + \sigma_i^2}\right). \tag{30}$$

To guarantee that the common message can be successfully decoded at all the users, the rate of the common message should be chosen as $\min_i R_i^{(\mathrm{C})}(\{\mathbf{w}_k\})$. Denote c_i as the data rate for receiving the common message at the i-th user. It satisfies that

$$\sum_{i=1}^{K} c_i \leq \min_i R_i^{(\mathrm{C})}\left(\{\mathbf{w}_k\}\right). \tag{31}$$

Then, the sum achievable rate of the considered system is expressed as

$$R\left(\{\mathbf{w}_k, c_i\}\right) = \sum_{i=1}^{K}\left(c_i + R_i^{(\mathrm{P})}\left(\{\mathbf{w}_k\}\right)\right). \tag{32}$$

As EE is defined as the ratio of the sum achievable rate to the total power consumption, the EE of the considered system is given by

$$\mathrm{EE}\left(\{\mathbf{w}_k, c_i\}\right) = \frac{R\left(\{\mathbf{w}_k, c_i\}\right)}{P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right)}. \tag{33}$$

B. Time-varying Channel Model

As the wireless environments changes fast in practice, the time-varying channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k \left(\frac{c}{4\pi f_c d_s}\right)^2} \mathbf{g}(t), \tag{34}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the small-scale fading vector. For notation simplification, we omit the index of **h** and **h** here.

Following the Jakes fading model [**?**], the small-scale flat fading is modeled as a first-order complex Gauss Markov process, which is given by

$$\mathbf{g}(t) = \rho \mathbf{g}(t-1) + \sqrt{1-\rho^2}\mathbf{e}, \tag{35}$$

where $\mathbf{g}(0) \sim \mathcal{CN}(0, \mathbf{I})$ and $\mathbf{e} \sim \mathcal{CN}(0, \mathbf{I})$ denote the Rician fading vector and the additive complex Gaussian noise, respectively, and the correlation coefficient ρ is determined by

$$\rho = J_0(2\pi f_d T_s), \tag{36}$$

where $J_0(\cdot)$, T_s and f_d denote the first kind zero-order Bessel function, the time interval between adjacent instants, and the maximum Doppler frequserncy[3], respectively.

[3]In practice, the GGUs and LGUs may have some movements sometimes. For example, the users may inadvertently shake or vibrate the wearable wireless sensor devices or smart phones when they move their bodies, then the scenarios with some Doppler frequserncy shifts may appear.

C. Fixed Channel Model

The channel model is considered, which is depicted by

$$\mathbf{h}=\sqrt{G_sG_k(\frac{c}{4\pi f_c d_s})^2}\mathbf{g}(t), \tag{37}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g}\in\mathbb{C}^{N_{\mathrm{T}}\times 1}$ is the small-scale fading vector with Rician. For notation simplification, we omit the index of $\mathbf{h}$ and $\mathbf{h}$ here.

D. Sum-rate maximization problem formulation

We aim to maximize the system sum rate of the considered system under the constraints of the power budget at the LEO satellite and the QoS requirements of both information communication at each ground user. Therefore, the corresponding optimization problem is mathematically formulated as

$$\begin{aligned}\mathbf{P}_0: &\max_{\{\mathbf{w}_k,c_i\}} R\left(\{\mathbf{w}_k,c_i\}\right)\\ \text{s.t.}\ &\sum_{i=1}^{K}c_i\leq\min_i R_i^{(\mathrm{C})}\left(\{\mathbf{w}_k\}\right), &\text{(38a)}\\ &c_i+R_i^{(\mathrm{P})}\left(\{\mathbf{w}_k\}\right)\geq\xi_i, &\text{(38b)}\\ &P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right)\leq P_{\max}, &\text{(38c)}\\ &c_i\geq 0, &\text{(38d)}\\ &\forall i\in\mathcal{I},\forall k\in\mathcal{K}.\end{aligned}$$

Constraints (39a) and (39d) guarantee that the common message is able to be successfully decoded by each user. In constraints (39b), constants ξ_i denotes the achievable information rate requirement of the i-th user, respectively. In constraint (39c), $P_{\max}$ denotes the maximum power budget at the LEO satellite.

E. EE problem formulation

We aim to maximize the system EE of the considered system under the constraints of the power budget at the LEO satellite and the QoS requirements of both information communication at ground each user. Therefore, the corresponding optimization problem is mathematically formulated as

$$\begin{aligned}\mathbf{P}_0: &\max_{\{\mathbf{w}_k,c_i\}} \mathrm{EE}\left(\{\mathbf{w}_k,c_i\}\right)\\ \text{s.t.}\ &\sum_{i=1}^{K}c_i\leq\min_i R_i^{(\mathrm{C})}\left(\{\mathbf{w}_k\}\right), &\text{(39a)}\\ &c_i+R_i^{(\mathrm{P})}\left(\{\mathbf{w}_k\}\right)\geq\xi_i, &\text{(39b)}\\ &P_{\mathrm{T}}\left(\{\mathbf{w}_k\}\right)\leq P_{\max}, &\text{(39c)}\\ &c_i\geq 0, &\text{(39d)}\\ &\forall i\in\mathcal{I},\forall k\in\mathcal{K}.\end{aligned}$$

Constraints (39a) and (39d) guarantee that the common message is able to be successfully decoded by each user. In constraints (39b), constants ξ_i denotes the achievable information rate requirement of the i-th user, respectively. In constraint (39c), $P_{\max}$ denotes the maximum power budget at the LEO satellite.

IV. MULTI-SATELLITE DESIGN WITH RSMA

A. Network Model

Taking the optimization of scenarios and access methods as an example, when a human user selects a heterogeneous satellite scenario and decides to use RSMA as the access technology, the generative AI agent will provide corresponding network modeling accordingly, as shown follows.

Consider a heterogeneous satellite network composed of one N_{M}-antenna GEO satellite and one N_{T}-antenna LEO satellite. The GEO satellite serves M single-antenna GEO ground users (GGUs) and each LEO satellite serves K single-antenna LEO ground users (LGUs). By accommodating multiple LEO satellites under the coverage of the GEO satellite, the wireless coverage of the GEO satellite and the information rate of users, especially for the users at the edge areas and dead points, may be greatly enhanced. In order to fully utilize the limited frequency band resources, the LEO satellites are allowed to utilize to the same licensed spectrum of the GEO satellite without adverse impact on the GGUs. For clarity, we use 0 to denote the index of the GEO satellite, $m\in\mathcal{M}=\{1,2...,M\}$ to denote the index of the m-th GGU and $k\in\mathcal{K}=\{1,2...,K\}$ to denote the index of the k-th LGU, respectively.

Furthermore, to enhance the communication quality and mitigate the co-channel interference, the 1-layer RSMA scheme is adopted at the LEO satellite. With the 1-layer RSMA scheme, the message associated with the k-th LGU denoted W_k is split into two parts, i.e., the common part $W_k^{(\mathrm{C})}$ and the private part $W_k^{(\mathrm{P})}$. Then, the common parts of all the LGUs are combined as $W^{(\mathrm{C})}$, which is encoded as a common stream s^{c} with $\mathbb{E}\{|s^{\mathrm{c}}|^2\}=1$. The private part of the k-th LGU is encoded as the private stream s_k^{p} with $\mathbb{E}\{|s_k^{\mathrm{p}}|^2\}=1$.

The network is running in a time-discretized manner, where the time is divided into a set of slots with small intervals. In each time slot, the channel state and network state are assumed to be unchanged and they may change from one slot to the next randomly. At the t-th time slot, the transmit signals generated by the GEO satellite and the LEO satellite are respectively given by

$$\mathbf{x}_{\mathrm{G}}=\sum_{m=1}^{M}\mathbf{w}_m s_m,$$

and

$$\mathbf{x}_{\mathrm{L}}=\mathbf{w}^{\mathrm{c}}s^{\mathrm{c}}+\sum_{k=1}^{K}\mathbf{w}_k^{\mathrm{p}}s_k^{\mathrm{p}},$$

where $s_m\in\mathbb{C}$ denotes the desired steam for the m-th GGU in the GEO satellite area with $\mathbb{E}\{|s_m|^2\}=1$. $\mathbf{w}_m\in\mathbb{C}^{N_{\mathrm{M}}\times 1}$ denotes the GEO satellite transmit beamforming vectors for the m-th GGU in the GEO satellite area. $\mathbf{w}_n^{\mathrm{c}}\in\mathbb{C}^{N_{\mathrm{T}}\times 1}$ and $\mathbf{w}_k^{\mathrm{p}}\in\mathbb{C}^{N_{\mathrm{T}}\times 1}$ denote the LEO satellite beamforming vectors associated with the stream s^{c} and s_k^{p}, respectively.

Let $\mathbf{h}_{i,0,m}\in\mathbb{C}^{N_{\mathrm{M}}\times 1}$ and $\mathbf{h}_{i,n,k}\in\mathbb{C}^{N_{\mathrm{T}}\times 1}$ denote the channel vector from i-th BS (with $i=0$ denoting the GEO satellite and $i\in\mathcal{N}$ denoting the n-th LEO satellite) to the m-th GGU and k-th LGU , respectively. At the t-th time slot, the received signals at the m-th GGU in the GEO satellite area $y_{0,m}$ and the k-th LGU $y_{n,k}$ are respectively given by

$$\begin{aligned} y_m &= \mathbf{h}_{\mathrm{G},m}^H \mathbf{x}_\mathrm{G} + \mathbf{h}_{\mathrm{L},m}^H \mathbf{x}_\mathrm{L} + z_m \\ &= \mathbf{h}_{\mathrm{G},m}^H \mathbf{w}_m s_m + \sum_{m' \neq m}^{M} \mathbf{h}_{\mathrm{L},m}^H \mathbf{w}_{m'} s_{m'} \\ &+ \sum_{k=1}^{K} \mathbf{h}_{\mathrm{L},m}^H \mathbf{w}_k^\mathrm{p} s_k^\mathrm{p} + \mathbf{h}_{\mathrm{L},m}^H \mathbf{w}^\mathrm{c} s^\mathrm{c} + z_{0,m}, \end{aligned}$$

and

$$\begin{aligned} y_k &= \mathbf{h}_{\mathrm{L},k}^H \mathbf{x}_\mathrm{L} + \mathbf{h}_{\mathrm{G},k}^H \mathbf{x}_\mathrm{G} + z_k \\ &= \mathbf{h}_{\mathrm{L},k}^H \mathbf{w}^\mathrm{c} s^\mathrm{c} + \mathbf{h}_{\mathrm{L},k}^H \mathbf{w}_k^\mathrm{p} s_k^\mathrm{p} + z_k \\ &+ \sum_{k' \neq k}^{K} \mathbf{h}_{\mathrm{L},k}^H \mathbf{w}_{k'}^\mathrm{p} s_{k'}^\mathrm{p} + \sum_{m=1}^{M} \mathbf{h}_{\mathrm{G},k}^H \mathbf{w}_m s_m, \end{aligned}$$

where $z_m \sim \mathcal{CN}(0, \sigma_a^2)$ and $z_k \sim \mathcal{CN}(0, \sigma_b^2)$ denote the additive white Gaussian noises (AWGN) with both σ_a^2 and σ_b^2 denoting the noise power at the GGU and LGU, respectively.

For the m-th GGU, the achievable information rate with normalized bandwidth is given by

$$R_m\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right) = \log_2(1 + \Gamma_m). \tag{40}$$

where Γ_m denotes the received signal to interference and noise ratio (SINR), which is expressed by

$$\Gamma_m = \frac{|\mathbf{h}_{\mathrm{G},m}^H \mathbf{w}_m|^2}{\sum\limits_{m' \neq m}^{M} |\mathbf{h}_{\mathrm{G},m}^H \mathbf{w}_{m'}|^2 + \sum\limits_{k=1}^{K} |\mathbf{h}_{\mathrm{L},m}^H \mathbf{w}_k^\mathrm{p}|^2 + |\mathbf{h}_{\mathrm{L},m}^H \mathbf{w}^\mathrm{c}|^2 + \sigma_a^2}, \tag{41}$$

With the RSMA scheme, each LGU first decodes the common stream by treating all the private streams as interference and then decodes its private stream after removing the common stream by SIC. The received information rates of the common stream and the private stream at the k-th LGU are respectively given by

$$R_k^\mathrm{c}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right) = \log_2(1 + \Gamma_k^\mathrm{c}), \tag{42}$$

and

$$R_k^\mathrm{p}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right) = \log_2(1 + \Gamma_k^\mathrm{p}), \tag{43}$$

where $\Gamma_{n,k}^\mathrm{c}$ and $\Gamma_{n,k}^\mathrm{p}$ denote corresponding SINR, which are respectively given by

$$\Gamma_k^\mathrm{c} = \frac{|\mathbf{h}_{\mathrm{L},k}^H \mathbf{w}^\mathrm{c}|^2}{\sum\limits_{k'=1}^{K} |\mathbf{h}_{\mathrm{L},k}^H \mathbf{w}_{n,k'}^\mathrm{p}|^2 + \sum\limits_{m=1}^{M} |\mathbf{h}_{\mathrm{G},k}^H \mathbf{w}_m|^2 + \sigma_b^2}, \tag{44}$$

and

$$\Gamma_k^\mathrm{p} = \frac{|\mathbf{h}_{\mathrm{L},k}^H \mathbf{w}_k^\mathrm{p}|^2}{\sum\limits_{k' \neq k}^{K} |\mathbf{h}_{\mathrm{L},k}^H \mathbf{w}_{k'}^\mathrm{p}|^2 + \sum\limits_{m=1}^{M} |\mathbf{h}_{\mathrm{G},k}^H \mathbf{w}_m|^2 + \sigma_b^2}. \tag{45}$$

Furthermore, to guarantee that the common message can be successfully decoded at all LUGs in the same area, the rate of the common message should be chosen as $\min_k R_k^\mathrm{c}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right)$. Denote c_k as the data rate for receiving the common message at the k-th LGU. It satisfies that

$$\sum_{k=1}^{K} c_k \leq \min_k R_k^\mathrm{c}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right). \tag{46}$$

Then, the sum achievable rate at the k-th LGU is expressed as

$$R_k\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}, c_k\}\right) = \left(c_k + R_k^\mathrm{p}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}\}\right)\right). \tag{47}$$

B. Channel Model

Similarly, when a human user considers a time-varying channel, the generative AI agent will provide corresponding channel models accordingly, as shown below.

As the wireless environments change fast in practice, the time-varying channel model is considered, which is depicted by

$$\mathbf{h} = \sqrt{G_s G_k \left(\frac{c}{4\pi f_c d_s}\right)^2} \mathbf{g}(t), \tag{48}$$

with G_s and G_k the satellite antenna gain and the user antenna gain, respectively. c is the light speed, f_c is the carrier frequency, d_s is the distance between the corresponding satellite and the user. $\mathbf{g} \in \mathbb{C}^{N_\mathrm{T} \times 1}$ is the small-scale fading vector. For notation simplification, we omit the index of $\mathbf{h}_{i,0,m}$ and $\mathbf{h}_{i,n,k}$ here.

Following the Jakes fading model [**?**], the small-scale flat fading is modeled as a first-order complex Gauss Markov process, which is given by

$$\mathbf{g}(t) = \rho \mathbf{g}(t-1) + \sqrt{1-\rho^2}\mathbf{e}, \tag{49}$$

where $\mathbf{g}(0) \sim \mathcal{CN}(0, \mathbf{I})$ and $\mathbf{e} \sim \mathcal{CN}(0, \mathbf{I})$ denote the Rician fading vector and the additive complex Gaussian noise, respectively, and the correlation coefficient ρ is determined by

$$\rho = J_0(2\pi f_d T_s), \tag{50}$$

where $J_0(\cdot)$, T_s and f_d denote the first kind zero-order Bessel function, the time interval between adjacent instants, and the maximum Doppler frequency, respectively.

C. Problem Formulation

Similarly, when a human user considers the LEO satellite system EE as optimization goal, the generative AI agent will provide a corresponding formulated optimization problem accordingly, as shown below.

For the considered heterogeneous satellite network, the system EE is denoted by

$$\mathrm{EE}\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}, c_k\}\right) = \frac{\sum\limits_{k=1}^{K} R_k\left(\{\mathbf{w}_k^\mathrm{p}, \mathbf{w}^\mathrm{c}, c_k\}\right)}{\mu\left(\|\mathbf{w}^\mathrm{c}\|^2 + \sum\limits_{k=1}^{K} \|\mathbf{w}_k^\mathrm{p}\|^2\right) + P_\mathrm{C}}, \tag{51}$$

where $\mu \in [1, \infty)$ and P_C denote the power amplifier efficiency factor and the constant power consumption by circuit modules, respectively.

We aim to maximize the achievable EE of the LEO satellite areas by jointly optimizing the transmit private beamforming vectors $\{\mathbf{w}_k^\mathrm{p}\}$, common beamforming vectors $\{\mathbf{w}^\mathrm{c}\}$, and common rate $\{c_k\}$, subject to the rate requirement constraints at GGUs and LGUs. Therefore, the optimization problem is mathematically expressed by

$$\max_{\{\mathbf{w}_k^{\mathrm{p}}, \mathbf{w}^{\mathrm{c}}, c_k\}} \ \mathrm{EE}\left(\{\mathbf{w}_k^{\mathrm{p}}, \mathbf{w}^{\mathrm{c}}, c_k\}\right) \tag{52a}$$

$$\text{s.t.} \quad \|\mathbf{w}^{\mathrm{c}}\|^2 + \sum_{k=1}^{K} \|\mathbf{w}_n^{\mathrm{p}}\|^2 \leq P_{\max}, \tag{52b}$$

$$R_m\left(\{\mathbf{w}_k^{\mathrm{p}}, \mathbf{w}_n^{\mathrm{c}}\}\right) \geq \xi_{\mathrm{GGU}}, \tag{52c}$$

$$R_k\left(\{\mathbf{w}_k^{\mathrm{p}}, \mathbf{w}_n^{\mathrm{c}}, c_k\}\right) \geq \xi_{\mathrm{LGU}}, \tag{52d}$$

$$\sum_{k=1}^{K} c_k \leq \min_k R_k^{\mathrm{c}}\left(\{\mathbf{w}_k^{\mathrm{p}}, \mathbf{w}_n^{\mathrm{c}}\}\right) \tag{52e}$$

$$c_k \geq 0, \tag{52f}$$

$$\forall m \in \mathcal{M}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}. \tag{52g}$$

In constraint (52b), P_n denotes the maximum power budget at the n-th LEO satellite. In constraints (52c) and (52d), ξ_{GGU} and ξ_{LGU} denote the minimum required achievable information rate threshold of each GGU and LGU, respectively. constraints (52e) and (52f) guarantee that the common message is able to be successfully decoded by each LGU.

References