FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE - CS161
Spring 2021

Assignment 5: due Wednesday, May 5th, 2021,11:55pm

Please submit a digital copy of your solution on CCLE. Submitted files can be in either PDF or plain text. You may also submit a scanned PDF of a handwritten solution, but please ensure that the scanned file is clearly legible.

1. (10 pts) Use truth tables (worlds) to show that the following pairs of sentences are equivalent:
   - $P \Rightarrow \neg Q, \; Q \Rightarrow \neg P$
   - $P \Leftrightarrow \neg Q, \; ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

a)

Sentence 1(s_1): $P \Rightarrow \neg Q \quad \rightarrow \quad \neg P \vee \neg Q$

Sentence 2(s_2): $Q \Rightarrow \neg P \quad \rightarrow \quad \neg Q \vee \neg P$

	P	Q	$\neg P$	¬Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
1	T	T	F	F	F	F
2	T	F	F	T	T	T
3	F	T	T	F	T	T
4	F	F	T	T	T	T

Obviously, $M(s_1)$ = $M(s_2)$ = {2, 3, 4}. i.e., $M(s_1) \subseteq M(s_2)$ and $M(s_1) \supseteq M(s_2)$.

Therefore, the sentences are equivalent.

b)

s_1: $P \Leftrightarrow \neg Q \quad \rightarrow \quad (P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P) \quad \rightarrow \quad (\neg P \vee \neg Q) \wedge (Q \vee P)$

s_2: $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

	P	Q	$\neg P \vee \neg Q$	$Q \vee P$	$(\neg P \vee \neg Q) \wedge (Q \vee P)$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
1	T	T	F	T	F	F	F	F
2	T	F	T	T	T	T	F	T
3	F	T	T	T	T	F	T	T
4	F	F	T	F	F	F	F	F

Obviously, $M(s_1)$ = $M(s_2)$ = {2, 3}. i.e., $M(s_1) \subseteq M(s_2)$ and $M(s_1) \supseteq M(s_2)$.

Therefore, the sentences are equivalent.

2. (20 pts) Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

- (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
- (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
- ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Justify your answer using truth tables (worlds).

Denote Smoke as event S, Fire as event F, Heat as event H.

S1:

$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$

$\equiv (\neg S \vee F) \Rightarrow (S \vee \neg F)$

$\equiv \neg(\neg S \vee F) \vee (S \vee \neg F)$

$\equiv (\neg(\neg S) \wedge \neg F) \vee (S \vee \neg F)$

$\equiv (S \wedge \neg F) \vee (S \vee \neg F)$

	S	F	$S \wedge \neg F$	$S \vee \neg F$	$(S \wedge \neg F) \vee (S \vee \neg F)$
w_1	T	T	F	T	T
w_2	T	F	T	T	T
w_3	F	T	F	F	F
w_4	F	F	F	T	T

$\therefore M(s_1) = \{w_1, w_2, w_4\}$

The sentence is not valid. Since it does not satisfy all worlds. (3 out of 4)

$M(\alpha) \neq \emptyset$ and $M(\alpha) \neq$all worlds.

And since it is satisfied by some truth assignemnt, it is neither unsatisfiable or valid.

s_2: ((Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

S_2:

$$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$$

$$\equiv (\neg S \vee F) \Rightarrow (\neg (S \vee H) \vee F)$$

$$\equiv \neg(\neg S \vee F) \vee (\neg(S \vee H) \vee F)$$

$$\equiv (S \wedge \neg F) \vee ((\neg S \wedge \neg H) \vee F)$$

$$\equiv (S \wedge \neg F) \vee F \vee (\neg S \wedge \neg H)$$

World	S	F	H	S ∧¬ F	¬S ∧¬H	(S ∧¬ F) ∨ F ∨ (¬S ∧¬ H)
1	T	T	T	F	F	T
2	T	T	F	F	F	T
3	T	F	T	T	F	T
4	T	F	F	T	F	T
5	F	T	T	F	F	T
6	F	T	F	F	T	T
7	F	F	T	F	F	F
8	F	F	F	F	T	T

From the table, we can know that s_2 is not valid. Since it does not satisfy by all worlds. (Does not satisfy by world 7) . $M(\alpha) \neq \emptyset$ and $M(\alpha) \neq$all worlds.

And since it satisfied by some truth assignemnt, it is neither unsatisfiable or valid.

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s_3:

$$((S \wedge H) \Rightarrow F) \Longleftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$$

$$\equiv (\neg(S \wedge H) \vee F) \Longleftrightarrow ((\neg S \vee F) \vee (\neg H \vee \cancel{F}))$$

$$\equiv (\neg S \vee F \vee \neg H) \Longleftrightarrow (\neg S \vee F \vee \neg H)$$

World	**S**	**F**	**H**	$(S \wedge H) \Rightarrow F$	**S** $\Rightarrow$ **F**	**H** $\Rightarrow$ **F**	**(S** $\Rightarrow$ **F)** $\vee$ **(H** $\Rightarrow$ **F)**	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
1	T	T	T	T	T	T	T	T
2	T	T	F	T	T	T	T	T
3	T	F	T	F	F	F	F	T
4	T	F	F	T	F	T	T	T
5	F	T	T	T	T	T	T	T
6	F	T	F	T	T	T	T	T
7	F	F	T	T	T	F	T	T
8	F	F	F	T	T	T	T	T

From the table, we can know s_3 is valid since it is satisfied by all worlds, it is not unsatisfible.

3. (30 pts) Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).
(b) Convert the knowledge base into CNF.
(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

a) Through the above statements, we can get propositional logic statement as below in KB Δ :

P1. Mythical $\Rightarrow \neg$ Mortal

P2. $\neg$ Mythical $\Rightarrow$ Mortal $\wedge$ Mammal

P3. ($\neg$ Mortal $\vee$ Mammal) $\Rightarrow$ Horned

P4. Horned $\Rightarrow$ Magical

b)Denote immortal as $\neg$ Mortal

Convert into CNF clauses:

P_1 : $\neg$ Mythical $\vee$ $\neg$ Mortal

P_2: Mythical $\vee$ (Mortal $\wedge$ Mammal) $\equiv$ (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal)

P_3:(Mortal $\wedge$ $\neg$Mammal) $\vee$ Horned $\equiv$ (Mortal $\vee$ Horned) $\wedge$ ($\neg$ Mammal $\vee$ Horned)

P_4: $\neg$ Horned $\vee$ Magical

CNF: ($\neg$ Mythical $\vee$ $\neg$ Mortal) $\wedge$ (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal) $\wedge$ (Mortal $\vee$ Horned) $\wedge$ ($\neg$ Mammal $\vee$ Horned) $\wedge$ ($\neg$ Horned $\vee$ Magical)

c)

Mytical : Y

Mortal : O

Mammal : A

Horned : H

Magical : G

Assume the unicorn is not mythical, append $\neg Y$ as new clause in the CNF

$\neg Y \vee \neg O$	0	
$Y \vee A$	1	
$Y \vee O$	2	Δ
$O \vee H$	3	
$\neg A \vee H$	4	
$\neg H \vee G$	5	
$\neg Y$	6	Query : α
$O \wedge A$	7 (5, 3)	
$A \vee \neg O$	8 (0, 1)	
A	9 (7, 8)	
H	10 (9, 4)	
G	11 (5, 10)	
O	12 (2, 6)	

prove $\Delta \wedge \neg \alpha$ is unsat

Canpt apply resolution anymore.

Assign Y : false, O : True, A : true, H : true; then Δ is true. i.e., it is satisfied.

$\therefore$ We cannot prove unicorn is mythical.

Hence, we cannot prove the unicorn is mythical.

At the same time, if we insert $\neg A$ or $\neg H$ and support the unicorn is not magical or it is not horned. And we need to prove they are unsatisfible. We can easily find contradiction due to 9 and 10 clause in the above image. So we know they are unsatisfible, the unicorn is magical and horned.

Or we can prove in another way:

Resolving P_1 and P_2, we can get

P_5: $\neg$ Mortal $\vee$ (Mortal $\wedge$ Mammal) $\equiv \neg$ Mortal $\wedge$ Mammal

Resolving P_3 and P_5, we can get a unit clause

P_6 : Horned

Resolving P_6 and P_4, we can get a unit clause

P_7: Magical

Therefore, we also proved both magical and horned are true. But we cannot prove it mythical.

4. (20 pts) Consider the two NNF circuits in Figure 1 and Figure 2. Identify whether they are decomposable, deterministic, smooth and why.

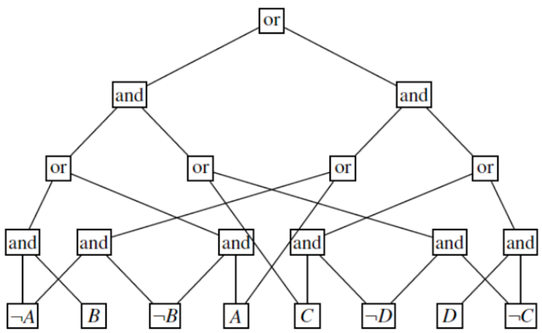


Figure 1

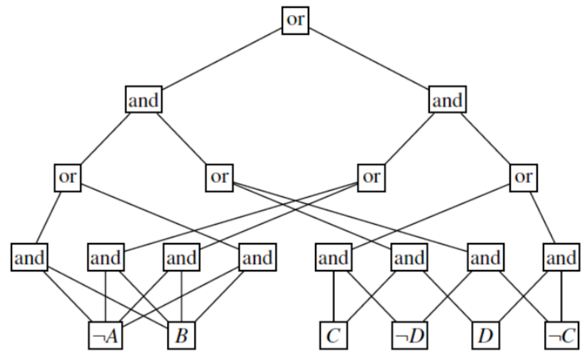


Figure 2

Figure 1:

Decomposable, Since for each and gate, the subcircuits do not share the same variables, (i.e., no duplicated variables.) The children have disjoint nodes.

Not deterministic, since the two OR gate in the mid level, they do not have two childrem that mutually exclusive. Consider assgin A , C as true, and B, D as false. And both sub-circuits into the OR gate would have the same value. They are not mutually exclusive.

Not smooth. Since the two or gate in the mid level do not mention the same variables for two children.

Figure2:

Decomposable. Since for each AND gate, the subcircuits do not share the same variables. (i.e., no duplicated variables.) The children have disjoint nodes.

Not deterministic. The OR gate in the mid-level of the left sub-circuits, it does not have two childrens with variables that mutual exclusive. For example, they share the same varibale assignments $\neg A, B$. If we assign A as false, B as true, then the both subcircutis of the OR gate will be true, not mutually exlcusive.

Smooth. As for each or gate, two children of it mention the same variables .

5. (20 pts) Given a propositional formula, where each literal has a weight ω in [0,1], the weight of a truth assignment is defined as the product of its literals weights. For example, $\omega(A, \neg B, C) = \omega(A)\,\omega(\neg B)\,\omega(C)$. The Weighted Model Count (WMC) of a propositional formula is defined as the added weight of its satisfying assignments (i.e., models).

Suppose we have the following literal weights: $\omega(A)=0.1$, $\omega(\neg A)=0.9$, $\omega(B)=0.3$, $\omega(\neg B)=0.7$, $\omega(C)=0.5$, $\omega(\neg C)=0.5$, $\omega(D)=0.7$, $\omega(\neg D)=0.3$.

(a) Compute the Weighted Model Count for formula $(\neg A \wedge B) \vee (\neg B \wedge A)$ by enumerating its models, computing their weights, then adding them up.

(b) Consider the decomposable, deterministic and smooth NNF circuit in Figure 3. If we assign the weights of literals to all the leaf nodes, the count of each $\wedge$ node is computed as the product of the counts of its children, and the count of each $\vee$ node is computed as the sum of the counts of its children. What is the relation between the count on the root with the Weighted Model Count for the formula?

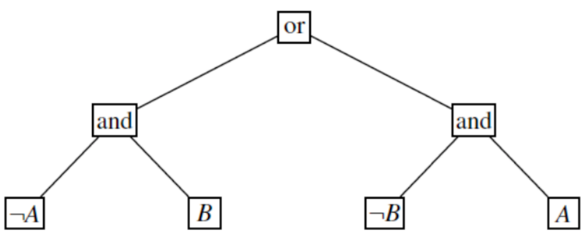


Figure 3

(c) Compute the Weighted Model Count for the formula associated with the decomposable, deterministic and smooth NNF circuit in Figure 4.

a)

Model Counting = #SAT

From the below table we know the modes are

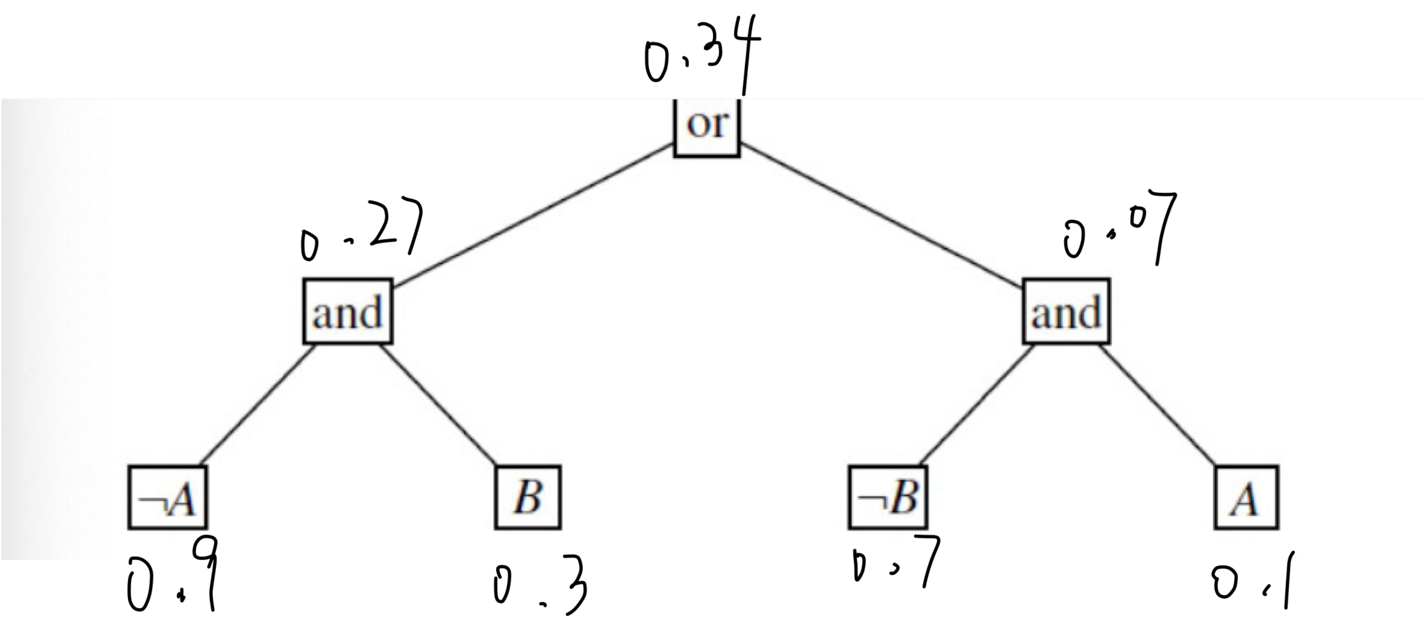
A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$	
T	T	F	F	F	
T	F	F	T	T	✓
F	T	T	F	T	✓
F	F	F	F	F	

$\# SAT = 2$

Two models $(A, \neg B)$ and $(\neg A, B)$

$$
\begin{aligned}
WAC &= w(\neg A, B) + w(A, \neg B) \\
&= w(\neg A)\, w(B) + w(A)\, w(\neg B) \\
&= 0.9 \times 0.3 + 0.1 \times 0.7 \\
&= 0.34
\end{aligned}
$$

b)



The count on the root will be same as the Weighted Model Count (WMC) for the formula in (a).

If we have a decomposable, deterministic, smooth (Three properites holds) NNF circuit for a particular propositional formula, we can be able to linearly compute do #SAT on the circuit. i.e., the number of assignments that satisfy the proposition formula, and get the WMC in linear time.

c)

Since the given NNF circuit is decomposable , deterministic, smooth (Three properties)

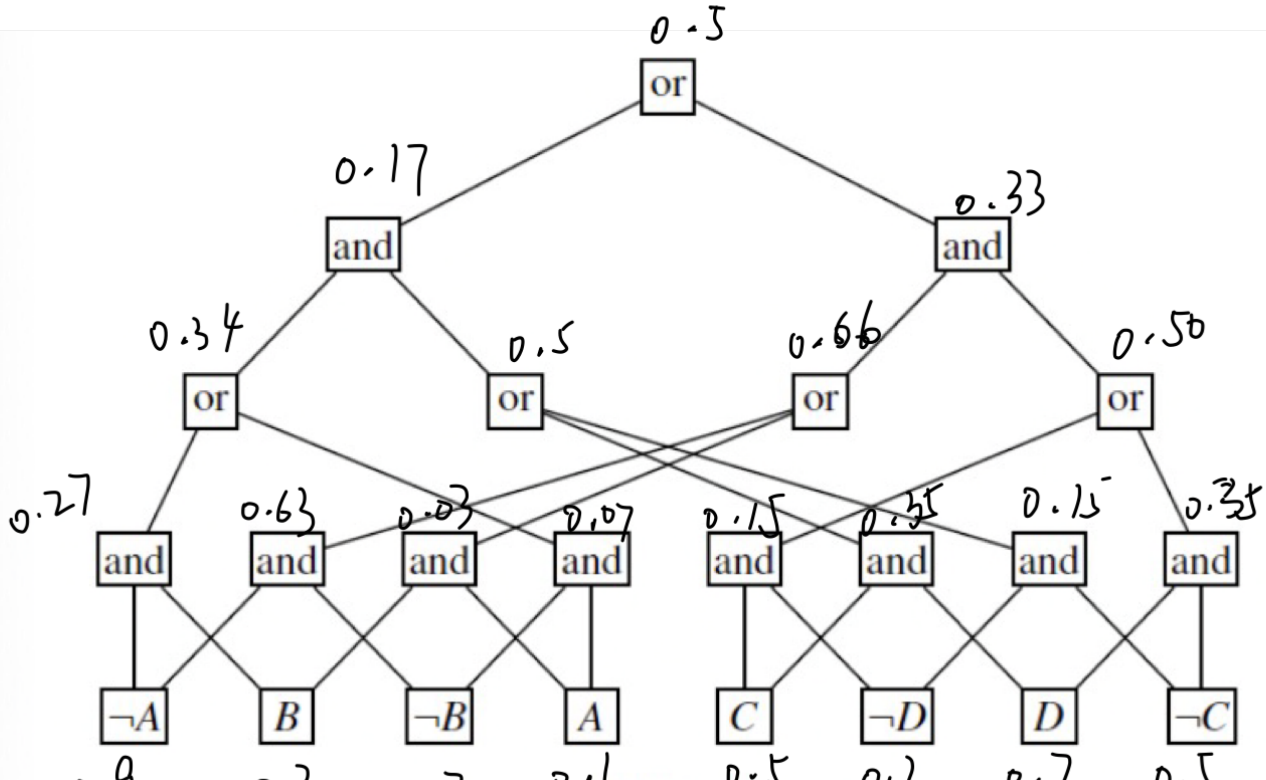


Figure 4

∴ Weight Model Count = 0.5