1. (40 pts) Is L = { $a^n b^m$; $n \geq 0$, $m \geq 0$ } a regular language?

   Prove that your answer is correct.

L is regualr.

Proof:

$a, b \in \sum$, a, b are regular expression.

So a^* is r.e., b^* is r.e..

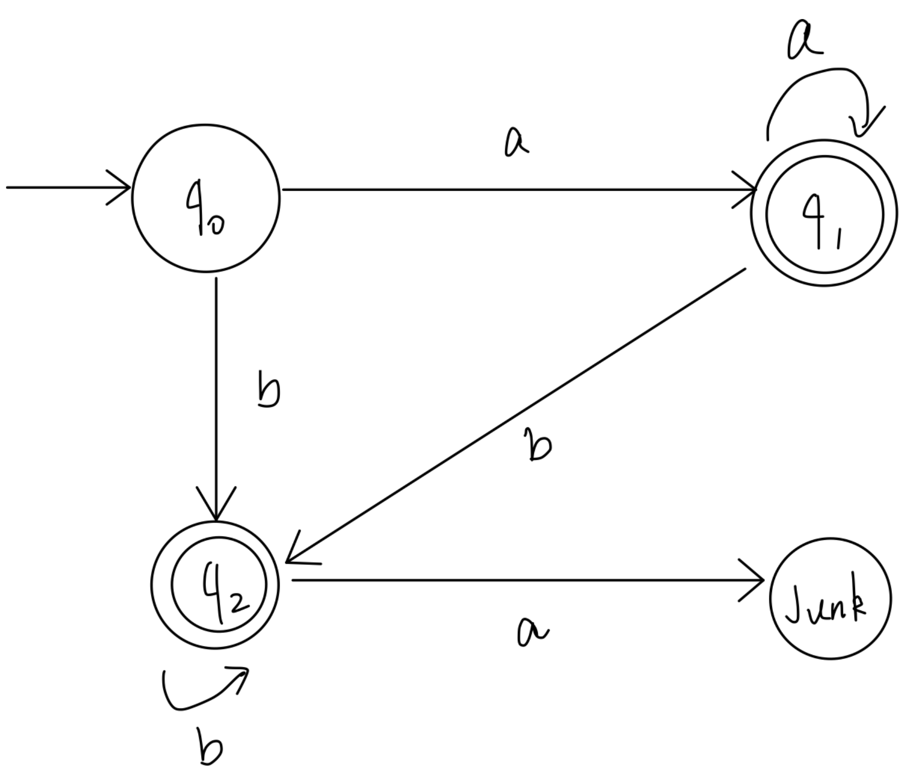
Hence $a^* b^*$ is r.e. as well.

For all word w generated by $a^* b^*$, $w = a^n b^m (m \geq 0, n \geq 0)$, $\Rightarrow w \in L$.

For all word $v \in L, v = a^n b^m (n \geq 0, m \geq 0). \Rightarrow w$ is generated by $a^* b^*$.

Hence L = { $a^* b^*$ }.

So the following regular expression $a^* b^*$ corresponds to L. Since we can write a regular expression for L, and a DFA could be constructed as follow,



In conclusion, L is regular.

2. (60 pts) Is L = { $a^n b^m c^{2(n+m)}$; $n \geq 0$, $m \geq 0$ } a regular language?

Prove that your answer is correct.

L is not regular.

Proof by contradiction.

Assume L is a regular language. Then pumping lemma must hold.

Let p be the pumping length given by the Pumping lemma. $p \geq 1$ depending only on L such that every string s in L of length at least p. Then $s \in L$ and $|s| \geq p$. Pumping lemma guranantees s can be split into 3 pieces, $s = xyz$, such that

- $|y| \geq 1,$
- $|xy| \leq h$
- for any $i \geq 0, xy^i z \in L.$

Let s = $a^n b^m c^{2(n+m)}$ where $n > h$.

then $s = a^p a^{n-p} b^m c^{2(n+m)}$ Where $xy = a^p$, $z = a^{n-p} b^m c^{2(n+m)}$.

$\Rightarrow x = a^{p-q}, y = a^q, z = a^{n-p} b^m c^{2(n+m)}$ since $|xy| \leq h$.

By case 3 of pumping lemma, $xyyz \in L$.

$xyyz$ = $a^{p-q} a^{2q} a^{n-p} b^m c^{2(n+m)} = a^{p+q} b^m c^{2(n+m)}$

$\Rightarrow xyyz \notin L$. A contradiction.

Due to the contradiction, the pumping lemma cannot be hold for L. Hence L is not regular.