Prove that the following statement is true or prove that it is false

For all sets, A \ (B \ C) = (A \ B) ∪ (A ∩ C)

True

Proof: (note that & means and, | means or)

1. (=>):

Suppose $\forall x,\ x \in A \setminus (B \setminus C)$

$x \in A \setminus (B \setminus C)$

$\Rightarrow x \in A \,\&\, x \notin (B \setminus C)$

$\Rightarrow x \in A \,\&\, \neg(x \in B \setminus C)$

$\Rightarrow x \in A \,\&\, \neg(x \in B \,\&\, x \notin C)$

By De Morgan's Laws,

$\Rightarrow x \in A \,\&\, (x \notin B \,|\, x \in C)$

Case 1:

$x \in A \,\&\, x \notin B \Rightarrow A \setminus B$

Case 2:

$x \in A \,\&\, x \in C \Rightarrow A \cap C$

by above steps,

$\Rightarrow x \in (A \setminus B) \cup (A \cap C)$,

therefore, we can conclude that

$A \setminus (B \setminus C) \subseteq (A \setminus B) \cup (A \cap C)$

2. (⇐):

Suppose $\forall z$, $z \in (A \setminus B) \cup (A \cap C)$

$z \in (A \setminus B) \cup (A \cap C)$

Case 1:

$z \in (A \setminus B) \Rightarrow z \in A \,\&\, z \notin B$

Case 2:

$z \in (A \cap C) \Rightarrow z \in A \,\&\, z \in C$

By above cases,

$\Rightarrow z \in A \,\&\, (z \notin B \,|z \in C)$

Apply De Morgan's law,

$\Rightarrow z \in A \,\&\, \neg(z \in B \,\&\, z \notin C)$

$\Rightarrow z \in A \,\&\, \neg(z \in (B \setminus C))$

$\Rightarrow z \in A \,\&\, z \notin (B \setminus C)$

$\Rightarrow z \in A \setminus (B \setminus C)$

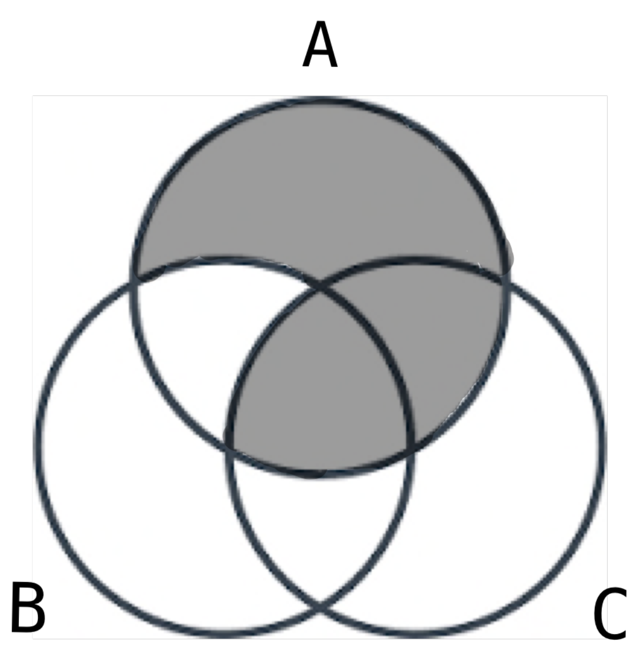
So we can conclude that

$(A \setminus B) \cup (A \cap C) \subseteq A \setminus (B \setminus C)$

Consequently, $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$

Proved.

Also, use Venn diagram to verify the identity:



This can be verified easily from above diagram that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$.