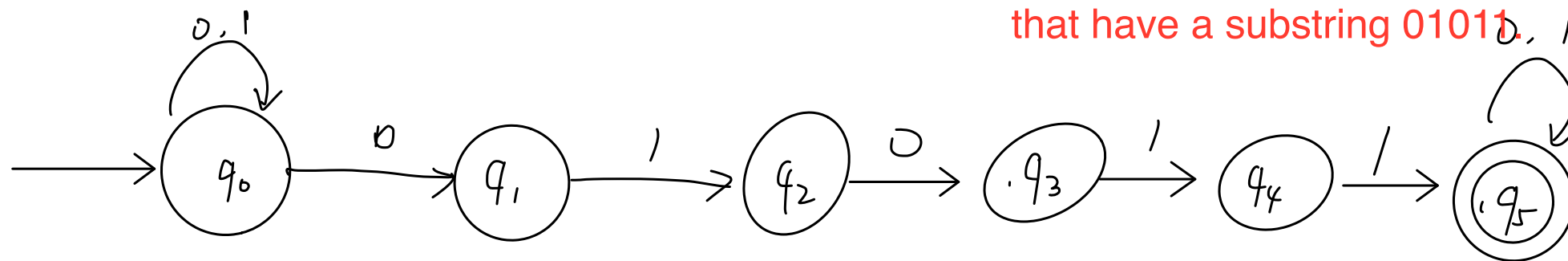


NFA :

Build DFA D and NFA N such that $L(D) = L(N)$, which consists of all binary strings that have a substring 01011.



Proof of correctness :

For string contains 01011:

Suppose any binary string w that consists substring 01011, ($w \in \Sigma^*$) denote it as $w = x01011y$ for some $x, y \in \Sigma^*$.

Let x be consumed by q_0 , then 01011 is consumed by states q_1, q_2, q_3, q_4, q_5 , q_5 is final state, it consume y .

Thus, we have a path that could consumes string $x01011y$ which starts at q_0 , ends at q_5 . i.e, This NFA accepts any strings that contains substring 01011.

For string does not contain 01011: (prove by induction)

BS: Suppose string with length 2, it must could not contain substring 01011. In this case, it could not be accepted since the path from q_0 to q_5 requires 5 input symbols.

IH: Suppose the NFA does not accept string does not contain 01011, the length of the string is n . Prove it also works for string with $(n+1)$ length that does not contain 01011.

IS: Denote the string with length n as w . ($w \in \Sigma^*$). Then add a binary number ^(0 or 1) to w , its length becomes $n+1$. There would be two cases: $w0, w1$.

For case w_0 : Since we already know string w does not contain 0101 , so w_0 would also does not contain 0101 . In this case, the NFA cannot accept w_0 as well.

For case w_1 :

The only possibilities that w_1 could be accepted is when w ends with " 0101 ", but in this case, w_1 contains substring 0101 , which conflicts with IH, so w cannot ends with " 0101 ". So w cannot ends at state q_4 .

Since w cannot ends at q_4 , and it is a string that does not contain 0101 , w_1 would also not be accepted.

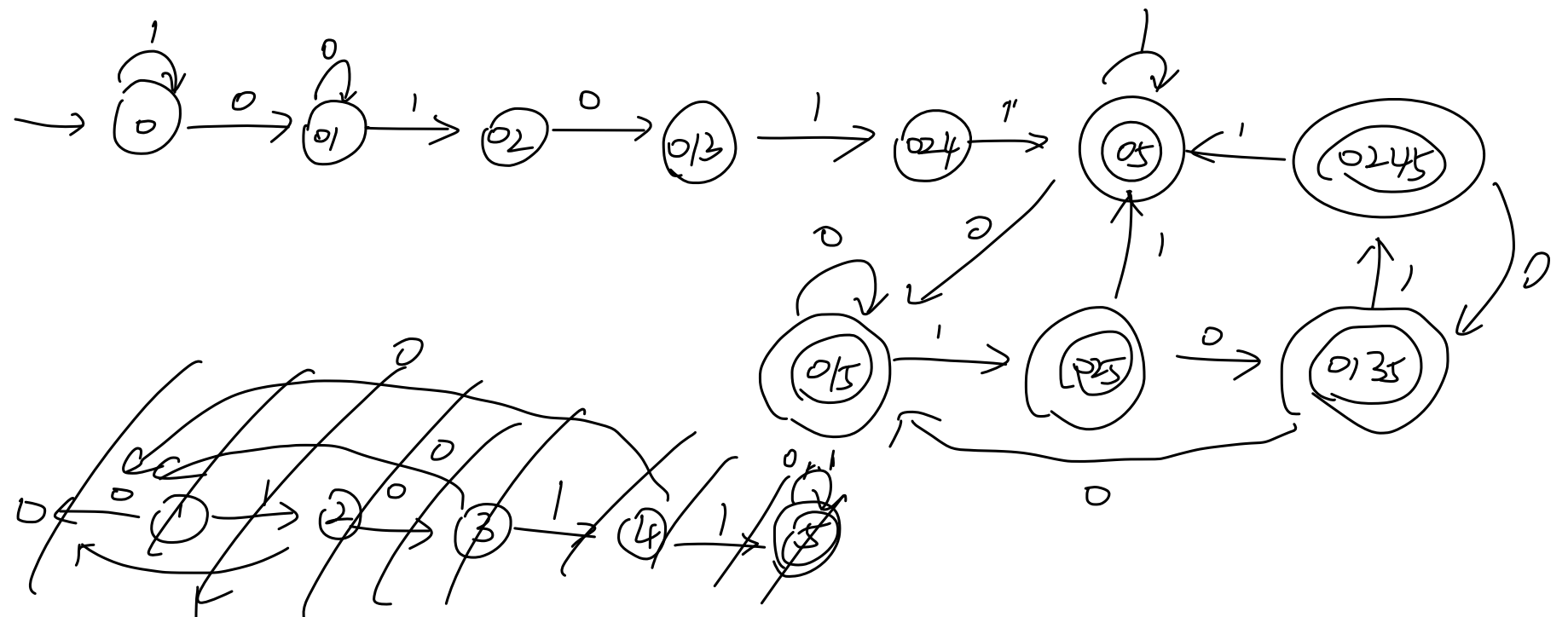
Therefore, this NFA does not accept any string does not contain 0101 .

Convert NFA to DFA :

$$N = \{ \Sigma, Q_N, \delta_N, q_{0N}, F_N \}$$

$$D = \{ \Sigma, 2^{Q_N}, S_D, q_{0D}, F_D \}$$

	0	1
q_0	$q_0 q_1$	q_0
q_1	ϕ	q_2
q_2	q_3	ϕ
q_3	ϕ	q_4
q_4	ϕ	q_5
q_5	q_5	q_5
$q_0 q_1$	$q_0 q_1$	$q_0 q_2$
$q_0 q_2$	$q_0 q_1 q_3$	q_0
$q_0 q_1 q_3$	$q_0 q_1$	$q_0 q_2 q_4$
$q_0 q_2 q_4$	$q_0 q_1 q_3$	$q_0 q_5$
$q_0 q_5$	$q_0 q_1 q_5$	$q_0 q_5$
$q_0 q_1 q_5$	$q_0 q_1 q_5$	$q_0 q_2 q_5$
$q_0 q_2 q_5$	$q_0 q_1 q_3 q_5$	$q_0 q_5$
$q_0 q_1 q_3 q_5$	$q_0 q_1 q_5$	$q_0 q_2 q_4 q_5$
$q_0 q_2 q_4 q_5$	$q_0 q_1 q_3 q_5$	$q_0 q_5$
ϕ	ϕ	ϕ



(\nwarrow Do not need it since these states are not connected with the start state)

Merge the final states $q_5, q_1q_5, q_2q_5, q_1q_3q_5, q_2q_4q_5$ as (F) we get the DFA:

