Homework 2.1

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1 Finanzas Cuantitativas

1.1 Homework 2.1

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1.1.1 Find the best parameters for a ML for PDF of Poisson Distribution

$$f(x|\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$f(x|\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$L(\lambda) = f(x|\lambda) = \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}$$

$$l(\lambda) = Ln(L(\lambda)) = Ln(\frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!})$$

$$l(\lambda) = -n\lambda + Ln\lambda \cdot \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} Ln(x_i!)$$

$$\frac{\partial\ l(\lambda)}{\partial\lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i$$

$$-n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i = 0$$

$$\frac{1}{\lambda}\sum_{i=1}^{n} x_i = n$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \hat{\lambda}$$

$$\hat{x} = \hat{\lambda}$$

Showing that the maximum likelihood for lamda is the mean of the data

1.2 Homework 2.2

- Research the properties for random variables by distributions

The Poisson distribution of discrete variables has two important properties:

• The probability is never negative:

$$P(x = x0)0$$

• The sum of all probabilities is unity:

$$\sum_{i=} P(x = xi) = 1$$

Bernoulli experiment has the following properties:

- Each experiment consists of a number of identical trials ("coin flips"). The random variable, x, is the total number of "successful" trials ob- served after all the trials are completed.
- Each trial has only two possible results, "success" and "failure." The probability of success is p and the possibility of failure is q.

$$p + q = 1$$

- The probabilities p and q remain constant for all the trials in the experiment.

The mean, or expected value, E(x), of a random variable is defined as follows: for a discrete variable, it is E(x) = µx = while for a continuous variable, it is:

$$E(x) = \mu x = \sum x_i p(x_i)$$

while for a continuous variable, it is:

$$E(x) = \mu x = \int_{-}^{+} xp(x)dx$$

The means of binomial and Poisson distributions are given by the fol- lowing general formulas:

$$\mu x = np$$

Normal distributions have key characteristics that are easy to spot in graphs:

- The mean, median and mode are exactly the same.
- The distribution is symmetric about the mean—half the values fall below the mean and half above the mean.
- The distribution can be described by two values: the mean and the standard deviation.