TRANSIT ACCESS EQUITY AND NETWORK RESILIENCE IN CLIMATE CHANGE

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ABSTRACT

Climate change-induced natural disasters such as extreme weather events can impact Public Transit (PT) agencies where it matters most: Their service network. These events curtail access for urban populations to socio-economic facilities such as workplaces, schools, and hospitals. Such curtailments disproportionately affect disadvantaged groups that rely on them the most, exacerbating socio-economic disparities among varying urban demographics.

In this paper, we formulate a tri-level MILP optimization reflects the struggle between an equity-oriented PT network planner and the worst-case impacts of extreme weather events on the links of a PT network. Respectively, our setting encodes a resource-allocation problem with a nested interdiction and ultimate shortest multi-path problem, resulting in a tri-level min-max-min optimization. Here, the planner allocates fortifications across the network, which minimize the shortest paths between key origin-destination pairs, prioritizing those trips originating from low-income neighborhoods. Conversely, extreme weather events attempt to maximize those shortest paths by interdicting the network, adding delays on specific edges. We present our formulation and apply it to a toy network. Furthermore, we touch upon the possibility of a stochastic formulation of the problem. The accompanying code is available on the MIT GitHub[1].

Keywords: climate change, public transit equity, network resilience, interdiction

[1] `https://github.mit.edu/fiorista/1.208_project`

INTRODUCTION

Climate change is an omnipresent threat to our infrastructure. We need to adapt our networks, from electricity and pipelines to supply and transportation networks, to be resilient against ever-increasing variations in weather patterns promoting Extreme Weather Events (EWE). In the best case, such events can lead to increased TTs and, in the worst case, to partial or total network failures (*1*).

In the meantime, taking Greater Boston as an example, certain neighborhoods of a city or metropolitan area, such as the neighborhoods of Roxbury, Mattapan, Chelsea, and East Boston, are low-income. Unlike wealthy neighborhoods, where households usually own a car and can reach their socio-economic Points of Interest (POIs) even if public transportation is unavailable, people living in those neighborhoods often depend solely on public transportation. In the context of transportation equity, these *transit-critical* individuals are affected the most when EWEs impair certain parts of the network.

Focusing on Public Transportation Networks (PTNs), previous research has found that these types of networks display scale-free properties (*2*), i.e., a certain resilience against random attacks. Furthermore, these networks can be defended from climate-related attacks, both random and targeted, when planning accordingly (*3*). Finally, PTNs are central to socio-economic functioning, and we want to ensure that transit-critical individuals still have reliable transportation infrastructure to depend upon, even in the case of EWEs - part of the ideal of transportation equity. Thus, our approach is inspired by recent research on the equitable design of PTNs (*4–6*).

Building upon this research, this work fucses on unifying notions from both climate change-focused planning and equitable PTN design. We model the struggle between the equitable PTN planner and attacks from EWE as a tri-level mixed-integer min-max-min linear program. We assume that the planner can allocate fortifications to the network arcs to increase resilience and reduce the implicit likelihood that an EWE interdicts a transit line, i.e. adds a TT penalty to it. Given the fortifications allocated by the planner, nature attempts to maximize the Travel Time (TT) along Origin-Destination (OD) pairs. To ensure a Rawlsian (*7*) egalitarian approach, i.e., maximizing the utility of the least advantaged population groups first, the planner weights the TT of each population group inversely to their income.

To test our formulation, we present a toy network comprising two origins and two destinations, edges representing the modes of walking and PT with their respective TTs, and synthetic income level data for prioritization.

Objectives

Our contributions are meant to be two-fold:

1. Formulate a novel optimization formulation for the identified allocation/interdiction problem when defending PTNs against extreme weather events.
2. Apply and evaluate this formulation on an ad-hoc toy network.

LITERATURE REVIEW

Pavia et al. presented a mixed-integer linear program (MILP) to quantify the utility a passenger *reaps* from using the PTN compared to driving by car. Specifically, the linear program can be applied in a green-field setting, where an entire PTN is built from the ground up. Binary variables indicate edges of the network to be installed under budgetary constraints. The program then seeks to optimize a social welfare function which reflects the utility a transit network grants under a particular notion of welfare (*4*). A case study was done for the city of Chattanooga, Tennessee.

One interesting finding that pertains to the lexicographic maximization (leximax) formulation discussed is that improving the worst utility that any OD pair can achieve results in the cost reduction of the average utility across all OD pairs. This reflects the policy implication of the trade-off between maintaining a high average utility versus improving the utility of the worst-served OD pair. Here, the Rawlsian formulation of egalitarianism, which guides the distribution of public goods, aims to enhance the utility of the least advantaged population group. This approach has implications for urban planning, particularly in the allocation of transit resources. The findings indicate that prioritizing is crucial. It is important to identify which areas of the city most require transit services. Considering these priorities during planning can markedly enhance accessibility for transit-dependent or transit-critical residents. This is especially true if the city operates with a limited budget.

Lou and Zhang studied PTN resilience under directed and random attacks with a game structure formulation - a defender (planner) determines a network design, and an attacker then interdicts it. Network users who make their travel plans accordingly are considered third-level players. Under random, day-to-day, small-scale disruptive events, the network is assumed to remain connected and still has a feasible flow pattern to satisfy the travel demands. The authors believe the risks should not be taken too optimistically or conservatively. Thus, an uncertainty budget was introduced, with a more significant budget indicating a more conservative approach. The robust network Design Problem under Random attacks (DA-R) is then formulated to minimize the maximum system TT under the planner's expansion plan and the attacker's attack plan.

For the problem of targeted attacks, on the other hand, the authors separately consider the network's remaining capacity and unsatisfied demand as two vulnerability measures. With the assumption that a link cannot be attacked if protected or will be destroyed if unprotected and attacked, the authors used two sets of binary variables to denote whether a link is protected and whether a link is attacked, respectively. All three formulations above are approached as semi-infinite mini-max problems with complimentary constraints for the decision variables (*3*).

The authors conclude that under random attacks, because network connectivity is often unaffected, the reliability of the total system TT is one good measure to be addressed in the planning stage, and the uncertainty budget should be appropriately set for the model to produce robust plans that provide superior reliability. Both their formulations for the network defense against targeted attacks problem can generate network defense plans that substantially reduce the standard deviation of the total system TT and achieve a reasonable percentage of the network's original transport capacity, even under coordinated attacks.

Israeli and Wood studied a shortest path interdiction (or Maximize the Shortest Path, MXSP) problem that involves a leader destroying certain edges in a network, or increasing its effective length and thus increasing the follower's shortest path through the network. They formulated it as a mixed integer program (MIP). They devised several decomposition-based algorithms to solve the MXSP, including a Benders Decomposition algorithm with "supervalid inequalities" and two enhanced versions.

Recognizing the computational inefficiency of solving MXSP by taking its dual and solving that, their first decomposition algorithm involves iteratively generating interdiction-response pairs (x, y) where x denotes a specific interdiction plan and y denotes an optimal shortest path response. To strengthen the LP relaxation of their master problem, they introduce the notion of supervalid inequality, which intends to reduce the size of the LP relaxation's feasible region by making the most recent solution and a few other integer solutions infeasible but without eliminating any optimal solution(s). Their test results suggest that their decomposition algorithm significantly outperforms traditional LP-based branch and bound and straightforward Benders decomposition, especially with their supervalid inequality technique.

While the work by Pavia et al. provides us with a base for our optimization formulation, Lou and Zhang informed us on the potential attack vectors of climate change-related EWEs. Furthermore, we lend many concepts from Israeli and Wood's work on the MXSP problem, extending it to a weighted multi-path interdiction. We refer to all three of these works in the present paper, while also considering equity-related literature such as the work by Williams et al. (*9*) on the impact of elevated TT on specific demographic groups and modes.

METHODOLOGY

This section presents the formalization of the network, followed by our problem formulation. We present our toy network and illustrate the behavior of our objective function in different interdiction scenarios.

Contextualization

Assume a city's PTN is a graph of nodes and edges. The city's residents, who have varying income levels, travel through the network according to their needs, i.e. their OD pairs. For example, in the network represented by Figure 1, 100 low-income people and 20 high-income people travel from node 0 to node 3. Similarly, 20 low-income and 100 high-income people travel from node 1 to node 2. If nothing is done to fortify the network, an EWE will, for example, interdict edge $(1,3)$ by dramatically increasing the TT along it. To prevent this, a transportation planner may allocate some fortification resources to the network edges, such that, if they are strong enough, they do not get interdicted by EWEs and still allow people to pass through with the edge's nominal TT.

Preliminaries

First, we define the graph to which the problem formulation applies. Following, we introduce our tri-level optimization formulation and objective. Furthermore, we show how we adapted our objective to account for an introduced non-linearity. Finally, we take the dual of the inner problem, resulting in a bi-level optimization problem.

Problem Graph

For our setting, we consider a graph similar to those represented by Pavia et al. in (*4*). Thus, we assume $G = \langle N, A \rangle$ with node set N and arc set (edges) A. The graph is a weighted, directed multi-graph that can include cycles and multiple edges between nodes. The nodes can be Residential Centroids (RCs) $c \in C \subseteq N$, or Points Of Interest (POIs). RC nodes hold additional information on the number of individuals of each income group living there. The edges in $a \in A$ hold information on the nominal TT $t_a \forall A$. Finally, we have a set of OD pairs P, representing the desire to travel from one node to another. In our case, we only consider travel between nodes in C as source nodes and nodes in $N \setminus C$, i.e., POIs, as destination nodes.

During the optimization, the planner envisions a fortification plan which allocates fortification $x_a \in \mathbb{Z}_{\geq} 0$ on the network edges, up to a budget $\sum\limits_{a \in A} x_a \leq B_x$. Conversely, in the second stage, the EWE attempts to maximize the TTs along the edges by assigning link weakenings $y_a \in \mathbb{Z}_{\geq} 0$,

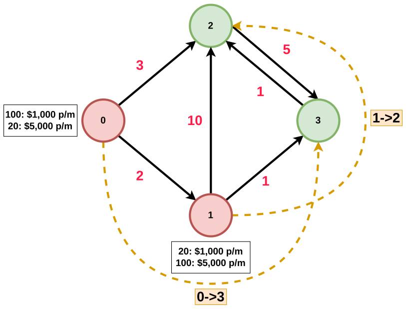

FIGURE 1 Toy graph with 4 nodes, 2 of which are Residential Centroids (RCs) (in red) with their respective socio-economic data and 2 POIs (in green). Between the nodes are the PTN edges with the respective TTs. The edges in yellow represent 2 OD pairs, indicating populations intending to travel from node 0 to 3 and node 1 to 2.

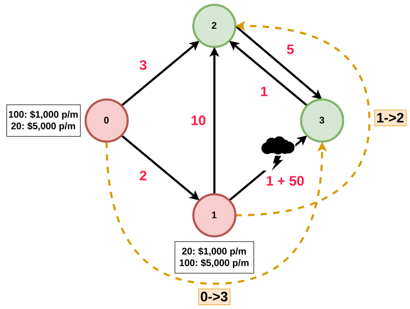

FIGURE 2 An EWE has interdicted the link between nodes 1 and 3, increasing the TT by 50 time units, e.g., minutes. In case the edge has been previously fortified, the fortification has not withstood the force of the EWE.

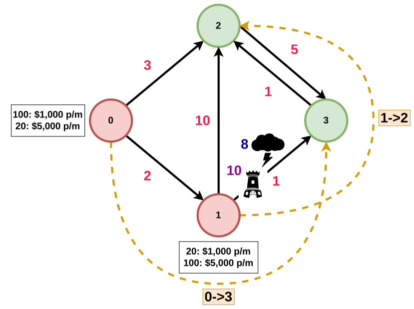

FIGURE 3 In this case, the planner has sufficiently fortified the link between nodes 1 and 3, repressing a potential interdiction from the EWE. Here, 8 indicates a measure of the force of the EWE, while 10 is the value of some fortification measure. As $8 < 10$, the fortification wins and prevents an interdiction.

up to a budget $\sum_{a \in A} y_a \leq B_y$. We consider an interdiction successful in the case of $x_a - y_a < 0$ for any given edge $a \in A$.

Problem Formulation

The optimization formulation of our problem can then be formalized as:

$$\min_{x \in \mathbb{Z}_{\geq 0}} \quad \max_{y \in \mathbb{Z}_{\geq 0}} \quad \min_{w \in \mathbb{R}_{\geq 0}} \quad \sum_{p \in P} u_p \sum_{a \in A} (t_a + e_a \cdot z_a) \cdot w_a^p \tag{1}$$

Here, we have the planner's resource allocation optimization as the outermost (green) term, which attempts to minimize the TT. Following, in the second stage, is the EWEs interdiction optimization (red), which, in turn, wants to find the weakening allocation regime to interdict the network's edges and maximize TT, considering the previously planned fortifications. Finally, we have a minimization (blue) of all the OD pair's TTs, weighted by a priority factor u_p based on the origin neighborhood's population groups, the number of individuals in each group, and their respective income. This priority factor is partially inspired by Pavia et al. (*4*), however builds on a more simplistic notion of equity priority which we have identified from various literature stating the inverse correlation between income and access to mobility. The TT objective considers all paths in the OD set P, and each one attempts to allocate the flow w_a^p to the shortest path. This allocation is subject to the TT t_a of each edge. However, the TT for an edge $a \in A$ is increased by a penalty e_a if the EWE's interdiction was successful, i.e., $z_a = 1$.

Below, we present the corresponding constraint set $\mathscr{P}$, and in Equation 4.4.1, we define the corresponding variables.

$$
\begin{aligned}
\mathscr{P}: & \sum_{(i,o^p)\in A} w^p_{io^p} - \sum_{(o^p,i)\in a} w^p_{o^p i} = -1 \\
& \sum_{(i,d^p)\in A} w^p_{id^p} - \sum_{(d^p,i)\in a} w^p_{d^p i} = 1 \\
& \sum_{(i,a)\in A} w^p_{iv} - \sum_{(v,i)\in a} w^p_{vi} = 0, \quad \forall v \in V \setminus \left\{ p = \left(o^p, d^d\right) \in P \right\} \\
& y_a - x_a \leq M \cdot z_a \\
& y_a - x_a \geq -M \cdot (1 - z_a) \\
& \sum_{a\in A} x_a \leq B_x \\
& \sum_{a\in A} y_a \leq B_y \\
& w^p_a \in \{0,1\}, \quad \forall a \in A \quad \forall p \in P \quad \forall c \in C \\
& z_a \in \{0,1\}, \quad \forall a \in A
\end{aligned} \tag{2}
$$

Accounting for Non-Linearity

The attentive reader will notice the non-linearity $e_a \cdot z_a$. To account for this nuisance, we make use of the McCormick envelope approach, allowing us to replace the non-linear $z_a \cdot w^p_a$ with a new variable q^p_a:

$$
\min_{x\in\mathbb{Z}_{\geq 0}} \quad \max_{y\in\mathbb{Z}_{\geq 0}} \quad \min_{w\in\mathbb{R}_{\geq 0}} \quad \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w^p_a + e_a \cdot q^p_a \tag{3}
$$

We can then add the following constraints to account for q^p_a:

$$q^p_a \leq z_a \tag{4}$$

$$q^p_a \leq w^p_a \tag{5}$$

$$q^p_a \geq z_a + w^p_a - 1 \tag{6}$$

$$\tag{7}$$

$\mathscr{G} = \langle V, A \rangle$	Directed graph representing the public transit network.
V	Set of nodes holding information on population data such as income group and number of individuals in that group.
A	Set of directed transit network arcs.
$a_{(i,j)} \in A$	Arc from node $i \in V$ to node $j \in V$ such that $(i, j) \in E$ holds information on the travel-time.
$c \in C \subseteq V$	RC $c \in C$, the areal centroid of a US census block in a metropolitan area.
$p = (o, d) \in P \quad o, d \in V \quad o \in C$	Origin-Destination pair (o^p, d^p) for path p which indicates travel from a RC c to a point of interest.
$g \in G$	Resident groups in the neighborhoods $\forall c \in C$.
$s \in S_g^c \quad \forall g \in G, \forall c \in C$	Median income of resident group $g \in G$ in neighborhood $c \forall c \in C$.
$b \in \mathscr{B}_g^c \quad \forall g \in G, \forall c \in C$	Number of residents of group $g \in G$ in neighborhood (or census block) $c \in C$.
$u_p = \sum_{g \in G} \left(\frac{1}{s_g^o} \right) b_g^o \quad \forall p = (o, d) \in P$	The priority weighting for each trip in the OD pairs P, based on the group's inverse income $\frac{1}{s_g^o}$ and the # of inhabitants $b_g^o \in \mathscr{B}$ of that group $g \in G$ in the origin RC $o \mid p = (o, d) \in P$
$x_a \in \mathbb{Z}_{\geq 0}$	Fortification of arc a by the public transit planner before extreme weather events occur.
$B_x \in \mathbb{Z}_{\geq 0}$	The fortification budget makes an edge more resilient against extreme weather events.
$y_a \in \mathbb{Z}_{\geq 0}$	Weakening of arc (i, j) caused by an extreme weather event.
$B_y \in \mathbb{Z}_{\geq 0}$	The attack budget is assigned to indicate a weakening or attack on edges during extreme weather events.
$z_a \in \{0, 1\}$	Binary variable which captures whether $x_a - y_a < 0$ is the case or not. If it is, then the interdiction (attack) was successful.
$t_a \in \mathbb{R}_{\geq 0}$	Travel time on arc $a \in A$
$e_a \in \mathbb{R}_{\geq 0}$	Travel-time penalty if the arc was successfully interdicted or disrupted, i.e. when $x_a - y_a < 0$.
$M \gg 1$	Reasonably *large* number for the (de)activation of z_a.

Taking the Inner Dual

As a first approach to feed our optimization formulation into a solver, we took the dual of the inner-most problem. Here, the work by Israeli and Wood has been seminal. Our work's first and second stages incorporate a similar structure of their shortest-path interdiction (MXSP) problem while extending it to a weighted multi-path setting. Thus, we adapted our primal formulation to match theirs. Accordingly, our formulation with the inner dual becomes a bi-level optimization with the following:

$$
\begin{array}{lll}
\min\limits_{x\in\mathbb{Z}_{\geq 0}} \quad \max\limits_{y\in\mathbb{Z}_{\geq 0},\pi\in\mathbb{R}} & \sum\limits_{p,a} \pi_d^p - \pi_o^p \quad \text{where } p=(o,d) & \\
\text{subject to} & \sum\limits_{a\in A} y_a - B_y \leq 0 & \\
& \sum\limits_{a\in A} x_a - B_x \leq 0 & \\
& u_p^{-1}\cdot\left(\pi_j^p - \pi_i^p\right) - e_a z_a - t_a \leq 0 \forall a=(i,j)\in A, \forall p\in P & \\
& x_a - y_a - M\cdot z_a + \varepsilon \leq 0 & \forall a=(i,j)\in A \\
& -M(1-z_a) - x_a + y_a \leq 0 & \forall a=(i,j)\in A \\
& z_a \in \{0,1\} & \forall a\in A \\
& e_a, t_a \in \mathbb{R}_{\geq 0} & \forall a\in A \\
& \pi_o^p = 0 & \forall p=(o,d)\in P
\end{array}
\tag{8}
$$

Featuring the following mapping of the constraints:

$$
d_k x_k \rightarrow \left(\sum_{g\in G^c}\left(\frac{b_g^c}{s_g^c}\right)\right)(t_a - e_a z_a) \tag{9}
$$

EXPERIMENTS & RESULTS

Our methodology in the section above is part of our main results. Thus, our first experiment aims at verifying the objective function by applying it to the toy network introduced in Figure 1. We fix all the fortifications of the planner to be $x_a = 1 \forall a \in A$ and experiment with the sensitivity of an increased weakening y on each edge of the network. Thus, we iteratively set $y_a = 2$ while keeping all other values $y_{a'} = 0$.

The outcome of this sensitivity-like analysis is visible in Figure 4 and clearly shows the expected behavior. Here, interdicting a non-critical edge is optimal, i.e., interdicting an edge not part of any shortest path between any OD pair. However, once the shortest path is interdicted, the path for OD pair $(1,2) = (1,3,2)$ is preferred due to the higher amount of high-income residents. Conversely, interdicting OD pair $(0,3)$ is sub-optimal as the lower-income, transit-critical population relies on it. These results reflect the expected preference to penalize OD pairs in P originating in high-income RCs, over those originating in low-income ones.

Subsequently, we implemented the primal and dual formulations of our inner two problems, leading to good results, i.e. identifying shortest multi-paths, until considering the optimization of interdictions. Here, we still face a software bug that causes the multi-path formulation of the dual inner stage (as presented in subsubsection 4.4.2) to return non-shortest paths. This error is likely due to a variable not being properly separated between paths, so flow variables w_a^p are considered for multiple paths. For completeness, we have included the console outputs of our different experiments in our repository's README[2].

Although our implementation results only provide partial progress toward our ultimate goal of devising reinforcement plans for PTNs, they show that it is possible to generate a formulation that prioritizes certain groups or neighborhoods that are transit-critical over others to plan for them accordingly.

[2] `https://github.mit.edu/fiorista/1.208_project/blob/main/README.md`

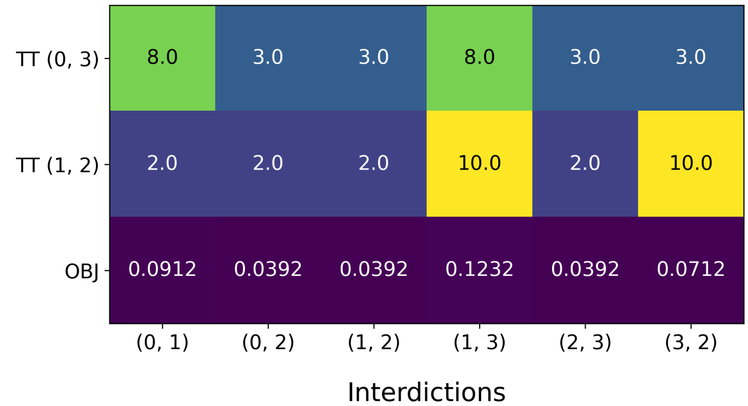

FIGURE 4 Interdicting one edge at a time with edges on the x-axis and the TT for the two OD paths $(0,3)$ and $(1,2)$ as well as the objective value on the y-axis. *No* interdiction on the edges that are part of OD paths is optimal. When interdiction results in a TT penalty, interdicting the shortest path for OD pair $(1,2)$ is preferred as its origin houses many high-income individuals.

LIMITATIONS

We recognize the limitations of our deterministic min-max-min formulation regarding assumptions and simplifications. First, while the assumption that a planner can allocate some investment is sound, we abstracted the nature and implementation of such investment. We do not quantify the associated cost per measure taken or distinguish how well a certain fortification protects against a certain kind of EWE. Another limitation is precisely this missing granularity of which EWE we consider. While extreme rain can affect edges representing underground transit more than busses, extreme winds can devastate over-ground infrastructure while the underground is safe from disruptions.

Moreover, our assumption that EWEs are explicitly targeting to maximize TTs, particularly those between OD pairs of transit critical riders, implies an unfounded targeted attack regime. Finally, the adoption of our inverse income priority weighting u_p is simplistic and, while under the assumption of low data availability, this is a good approximation, it fails to quantify equity issues between the groups and capture who requires a particular OD pair more than someone else or any other OD pair. A potential solution to the latter is to find better vectors to quantify equity and consider other equity-impacting factors, such as travel costs.

DISCUSSION & CONCLUSION

This paper presents our first approach to equitable resilience planning of PTNs against EWEs. Our problem formulation allows us to devise an edge fortification plan to ensure that the impacts of EWEs affect the transit-reliant population the least. The results show that our objective function captures this aspiration by the planner, assigning better (lower) values to travel-time increases for individuals traveling from wealthier neighborhoods. Furthermore, we show that the dual of the inner formulation can be taken and leads to similar results. However, we also find that reforming our tri-level optimization as a bi-level optimization, i.e., primal first stage and collapsed primal-dual of the second and third stages as the new second stage, still leads to faulty values when considering more than one OD pair. While this paper is a work-in-progress report, we are confident that a future continuation can lead to significant and publishable results.

OUTLOOK

We plan to pursue this problem further and consider a more apt formulation of the presented struggle between PTN planners and EWEs, which is increasing in frequency and magnitude. Thus, we started formulating and implementing a stochastic formulation of the problem at hand. Here, we substitute the EWE travel-time maximization with expectations, allowing us to better capture the probabilities, likelihoods, and distributions involved when EWEs arise.

$$\min_{x \in \mathbb{Z}_{\geq 0}} \quad \mathbb{E}_{\xi}[\mathscr{Q}(x, \xi)] \tag{10}$$

Here, $\mathscr{Q}(x, \xi)$ is the inner multi-path minimization problem which we introduced above in subsection 4.4. ξ represents the edges' interdictions optimally, sampled from previous data. However, approximations can be made by setting the probabilities of weather event intensities (no bad weather to EWE) and then sampling the interdictions accordingly from different, intensity-dependent distributions. Similarly, we can additionally model and sample the delays e_a, which we have assumed to be static but are very much dependent on the.

In our code, we provide a first implementation of the extensive form of this stochastic approach[3]. While the code exectues successfully, the fortification allocations $x \to 0$ as we increase the number of scenarios. There can be multiple explanations for this behavior. However, the two most likely at play here are a software bug and a potentially wrong sampling process for the random variable ξ, which promotes the averaging of x.

[3] `http://tinyurl.com/1208fps`

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