Equitable Public Transit Network Extreme Weather Event Resilience

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Career Advising & Professional Development | Graduate Student Professional Development

Agenda

Peter:

- Public Transit (PT) equity
- Problem contextualization
- Problem formalization

Riccardo:

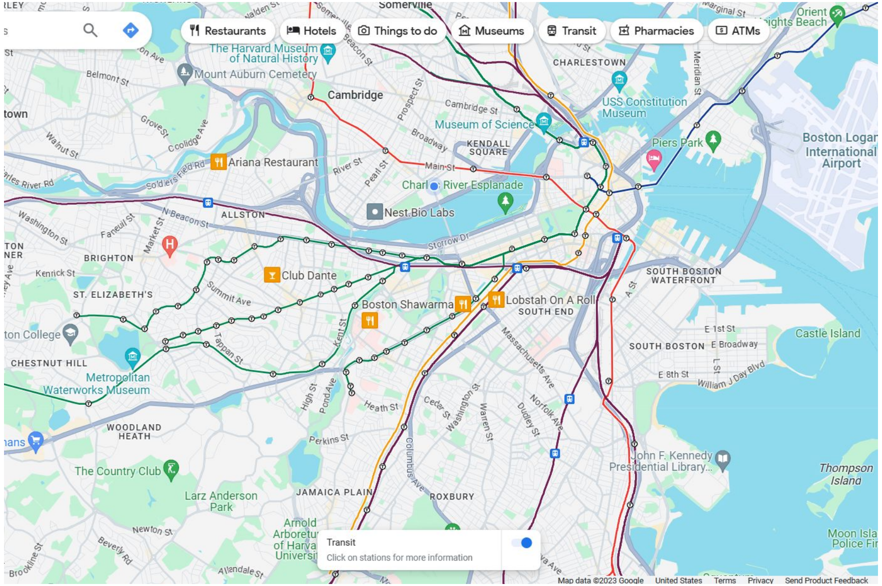
- Formalization validation
- MILP Approach / Experiments
- Conclusion & future work

Public Transit (PT) Equity

PT Equity

- Map of Boston
- Map of Boston with neighborhoods
- Neighborhood centroids and colored in with census data
- MBTA network on top
- LINK

PT Equity



Restaurants
Hotels
Things to do
Museums
Transit
Pharmacies
ATMs
Somerville
Cambridge
CHARLESTOWN
The Harvard Museum of Natural History
Mount Auburn Cemetery
USS Constitution Museum
Museum of Science
KENDALL SQUARE
Boston Logan International Airport
Piers Park
Ariana Restaurant
Charles River Esplanade
Nest Bio Labs
ALLSTON
BRIGHTON
Club Dante
SOUTH BOSTON WATERFRONT
ST. ELIZABETH'S
Boston Shawarma
Lobstah On A Roll
SOUTH END
SOUTH BOSTON
Castle Island
CHESTNUT HILL
Metropolitan Waterworks Museum
WOODLAND HEATH
The Country Club
Larz Anderson Park
JAMAICA PLAIN
ROXBURY
John F. Kennedy Presidential Library...
Thompson Island
Transit
Click on stations for more information
Map data ©2023 Google
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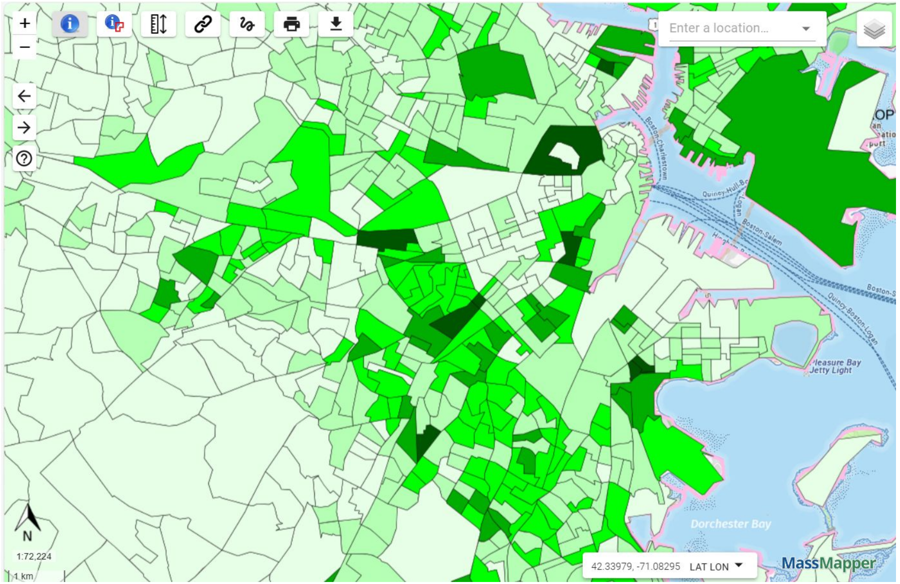
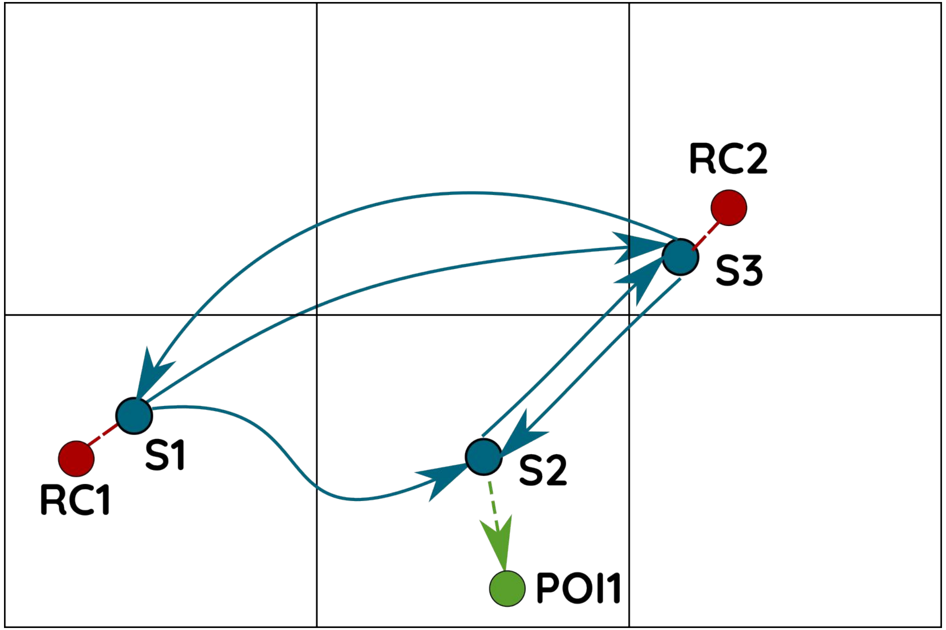
Privacy
Send Product Feedback



Enter a location...
Dorchester Bay
Pleasure Bay Jetty Light
1:72,224
1 km
42.33979, -71.08295
LAT LON
MassMapper

Abstraction



PT Equity

- PT is the backbone of socio-economic functioning
- Who are the most **transit critical individuals**?
  - Low income
  - Car-less individuals and households
  - Minorities/Migrants
- Why do we care?
  - Fairness / Equality / **Equity**
  - As a planner but also as a society
  - Economic benefit for all

PT Design

- Design/Planning (Equitable) PTNs
  - Design from scratch
    - “Designing Equitable Transit Networks” [1]
  - Expansion/Reduction
- Operations
  - Paper “Defending transportation networks” [2]
  - Disruptions
    - Extreme weather events (EWE)
      - Increasing in frequency and magnitude
      - Difficult to model (**outliers!**)
      - Impact on transit critical population

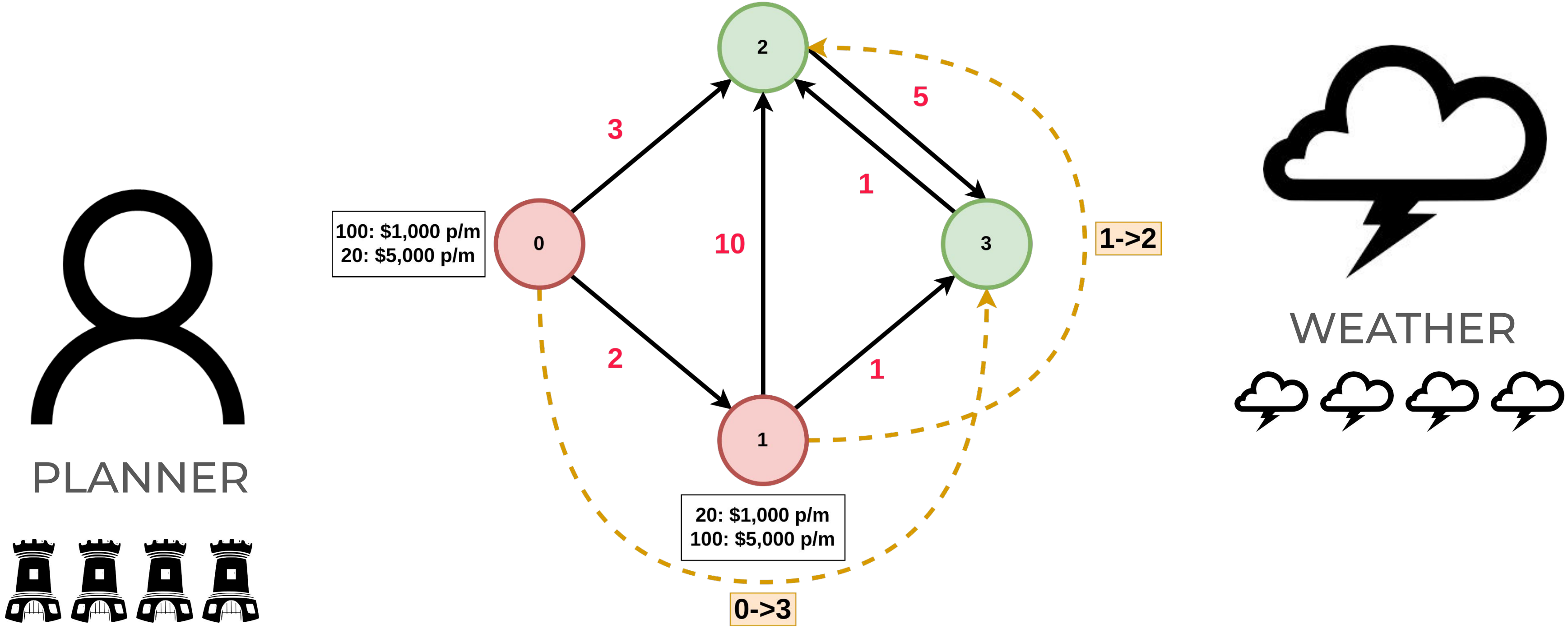
Examples

- Pictures of EWE affecting public transit
- Because EWE affect PTNs
- And because individuals from disadvantaged backgrounds are the most vulnerable (transit critical)
- We need to find equitable ways to plan for these disruptions, ensuring access for those that require it the most

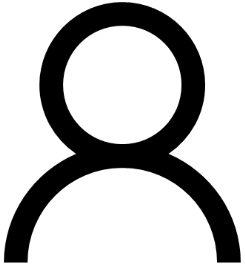
Plan for Disruptions Equitably

Problem Contextualization & Formulation

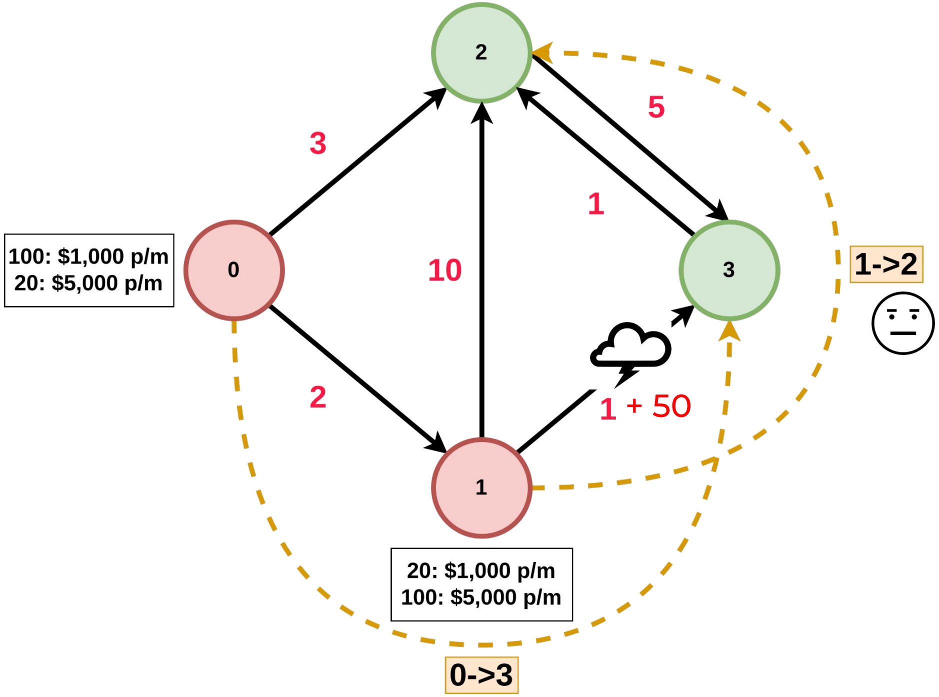
The Struggle



Disruption

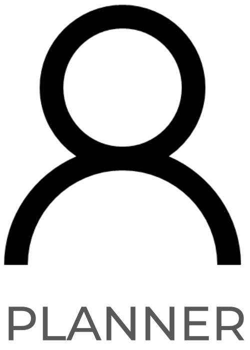


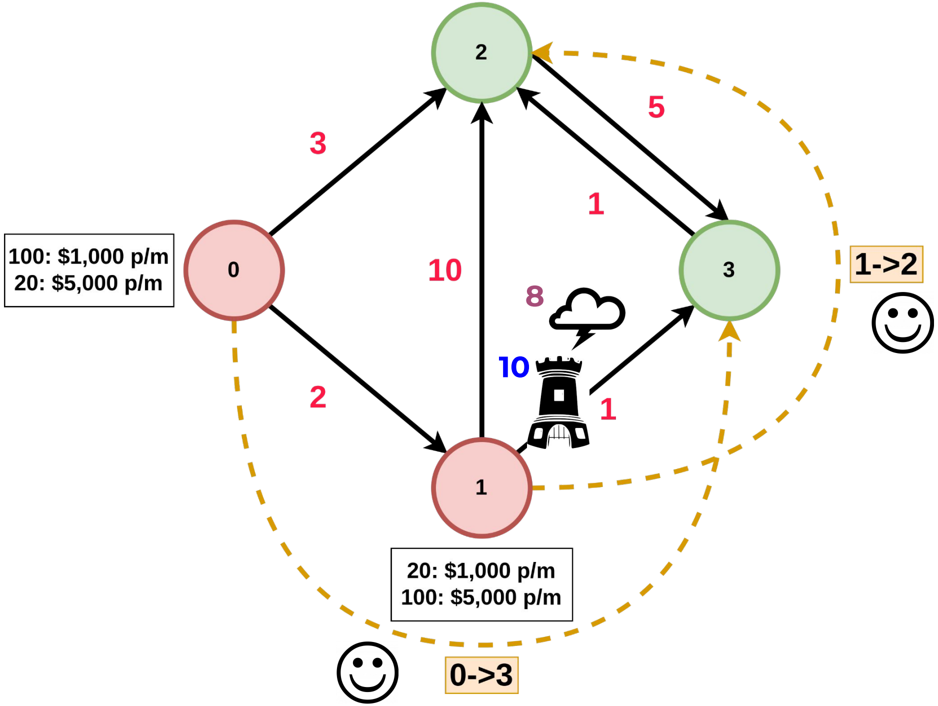
PLANNER



WEATHER

Fortifying Against Disruption





Problem Formalization

Fortification Allocation (Planner)

Interdiction (EWE)

Shortest Paths

$$\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0}} \max_{\boldsymbol{y}\in\mathbb{Z}_{\geq 0}} \min_{\boldsymbol{w}\in\mathbb{R}_{\geq 0}} \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot \boxed{z_a \cdot w_a^p}$$

$$\left.\begin{aligned}
&\sum_{(i,o^p)\in A} w^p_{io^p} - \sum_{(o^p,i)\in a} w^p_{o^p i} = -1\\
&\sum_{(i,d^p)\in A} w^p_{id^p} - \sum_{(d^p,i)\in a} w^p_{d^p i} = 1\\
&\sum_{(i,a)\in A} w^p_{iv} - \sum_{(v,i)\in a} w^p_{vi} = 0, \quad \forall v\in V \setminus \{p=\left(o^p,d^d\right)\in P\}
\end{aligned}\right\}\ \text{Flow constraints}$$

$$\left.\begin{aligned}
&y_a - x_a \leq M \cdot z_a\\
&y_a - x_a \geq -M \cdot (1 - z_a)
\end{aligned}\right\}\ \text{Interdiction}$$

$$\left.\begin{aligned}
&\sum_{a\in A} x_a \leq B_x\\
&\sum_{a\in A} y_a \leq B_y
\end{aligned}\right\}\ \text{Budget}$$

$$\left.\begin{aligned}
&w_a^p \in \{0,1\}, \quad \forall a\in A \quad \forall p\in P \quad \forall c\in C\\
&z_a \in \{0,1\}, \quad \forall a\in A
\end{aligned}\right\}\ \text{Variables}$$

Linearization: McCormick envelopes

$$\min_{x\in\mathbb{Z}_{\geq 0}} \quad \max_{y\in\mathbb{Z}_{\geq 0}} \quad \min_{w\in\mathbb{R}_{\geq 0}} \quad \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot \boxed{q_a^p}$$

$$\left.\begin{aligned}
&\sum_{(i,o^p)\in A} w^p_{io^p} - \sum_{(o^p,i)\in a} w^p_{o^p i} = -1\\
&\sum_{(i,d^p)\in A} w^p_{id^p} - \sum_{(d^p,i)\in a} w^p_{d^p i} = 1\\
&\sum_{(i,a)\in A} w^p_{iv} - \sum_{(v,i)\in a} w^p_{vi} = 0, \quad \forall v \in V \setminus \{p = (o^p, d^d) \in P\}
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&\sum_{a\in A} x_a \leq B_x\\
&\sum_{a\in A} y_a \leq B_y
\end{aligned}\right\}\text{Budget}$$

$$\left.\begin{aligned}
&w_a^p \in \{0,1\}, \quad \forall a \in A \quad \forall p \in P \quad \forall c \in C\\
&z_a \in \{0,1\}, \quad \forall a \in A
\end{aligned}\right\}\text{Variables}$$

$$\underbrace{\begin{aligned}
&q_a^p \leq z_a\\
&q_a^p \leq w_a^p\\
&q_a^p \geq z_a + w_a^p - 1\\
&q_a^p \in \{0,1\}, \quad \forall a \in A \quad \forall p \in P
\end{aligned}}_{\text{McCormick envelope constraints}}$$

Problem Formalization

$$\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0}} \; \max_{\boldsymbol{y}\in\mathbb{Z}_{\geq 0}} \; \min_{\boldsymbol{w}\in\mathbb{R}_{\geq 0}} \quad \sum_{p\in P} \boxed{u_p} \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot \; q_a^p$$

$$\left.\begin{aligned}
&\sum_{(i,o^p)\in A} w^p_{io^p} - \sum_{(o^p,i)\in a} w^p_{o^p i} = -1\\
&\sum_{(i,d^p)\in A} w^p_{id^p} - \sum_{(d^p,i)\in a} w^p_{d^p i} = 1\\
&\sum_{(i,a)\in A} w^p_{iv} - \sum_{(v,i)\in a} w^p_{vi} = 0, \quad \forall v \in V \setminus \{p = (o^p, d^d) \in P\}
\end{aligned}\right\} \text{Flow constraints}$$

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&y_a - x_a \leq M \cdot z_a\\
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&\sum_{a\in A} x_a \leq B_x\\
&\sum_{a\in A} y_a \leq B_y
\end{aligned}\right\} \text{Budget}$$

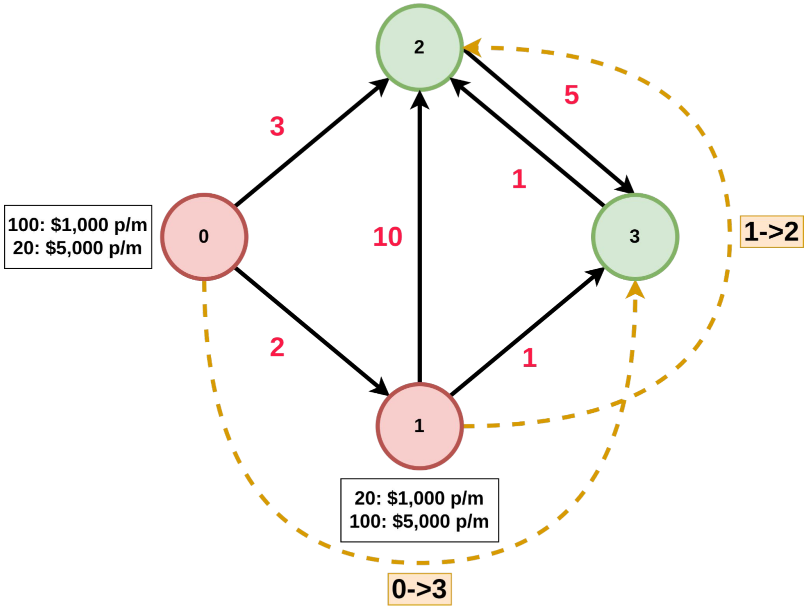
$$\left.\begin{aligned}
&w_a^p \in \{0,1\}, \quad \forall a \in A \quad \forall p \in P \quad \forall c \in C\\
&z_a \in \{0,1\}, \quad \forall a \in A
\end{aligned}\right\} \text{Variables}$$

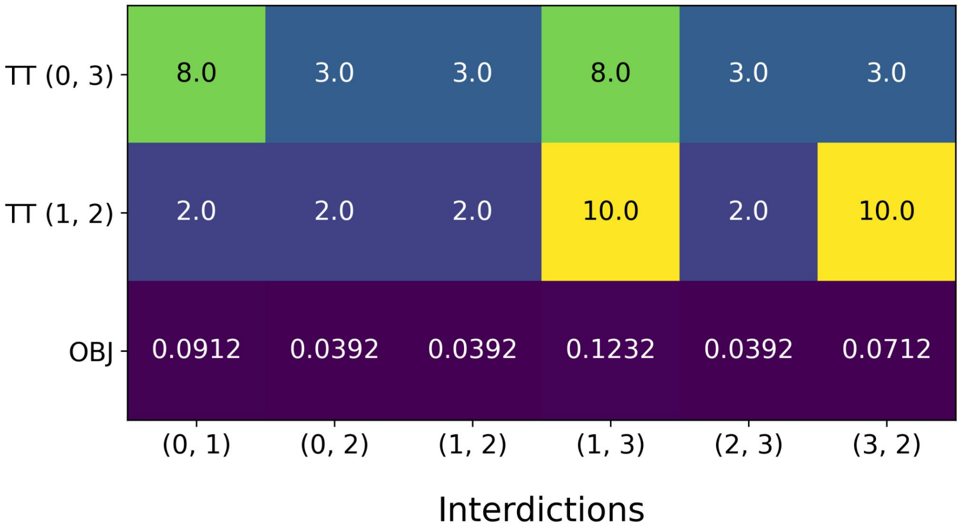
Group and neighborhood median household income

Number of inhabitants of that group in the neighborhood

$$\underbrace{u_p = \sum_{g\in G} \left(\frac{1}{s_g^o}\right) b_g^o \quad \forall p = (o,d) \in P}_{\text{Priority weighting}}$$

Formulation Evaluation





Tri/Bilevel MILP

Approach

$$\min_{\boldsymbol{x} \in \mathbb{Z}_{\geq 0}} \quad \max_{\boldsymbol{y} \in \mathbb{Z}_{\geq 0}} \quad \min_{\boldsymbol{w} \in \mathbb{R}_{\geq 0}} \qquad \sum_{p \in P} u_p \sum_{a \in A} t_a \cdot w_a^p + e_a \cdot q_a^p$$

Shortest Path Interdiction

$$\min_{\boldsymbol{w} \in \mathbb{R}_{\geq 0}} \quad \sum_{a \in A} (t_a \qquad + e_a \cdot z_a)\, w_a^p$$

$$\text{subject to} \quad \sum_{(i,o^p) \in A} w_{io^p}^p - \sum_{(o^p,i) \in a} w_{o^p i}^p = -1$$

$$\sum_{(i,d^p) \in A} w_{id^p}^p - \sum_{(d^p,i) \in a} w_{d^p i}^p = 1$$

$$\sum_{(i,a) \in A} w_{iv}^p - \sum_{(v,i) \in a} w_{vi}^p = 0, \quad \forall v \in V \setminus \{p = (o^p, d^d) \in P\}$$

Shortest Path Interdiction [3]

$$\max_{\boldsymbol{x}\in X} \min_{\boldsymbol{w}} \sum_{a\in A} (t_a + e_a \cdot x_a) w_a$$

$$\text{s.t.} \quad \sum_{a\in FS(i)} w_a - \sum_{a\in RS(i)} w_a = \left\{\begin{matrix} 1 & \text{for } i = o \\ 0 & \forall i \in N\{o,d\} \\ -1 & \text{for } i = d \end{matrix}\right\}$$

$$w_a \geq 0 \quad \forall a \in A,$$

Shortest **Multi** Path Interdiction

$$\max_{\boldsymbol{x}\in X}\min_{\boldsymbol{w}}\sum_{a\in A}(t_a+e_a\cdot x_a)w_a$$

$$\text{s.t.}\quad \sum_{a\in FS(i)} w_a-\sum_{a\in RS(i)} w_a=\left\{\begin{matrix}1 & \text{for } i=o\\ 0 & \forall i\in N\{o,d\}\\ -1 & \text{for } i=d\end{matrix}\right\}$$

$$w_a\geq 0\quad \forall a\in A,$$

$$\downarrow$$

$$\max_{\boldsymbol{x}\in X}\min_{\boldsymbol{w}}\sum_{p\in P}\sum_{a\in A}(t_a+e_a\cdot x_a)w_a^p$$

$$\text{s.t.}\quad \sum_{a\in FS(i)} w_a^p-\sum_{a\in RS(i)} w_a^p=\left\{\begin{matrix}1 & \text{for } i=o\\ 0 & \forall i\in N\{o^p,d^p & \forall p\in P\}\\ -1 & \text{for } i=d\end{matrix}\right\}$$

$$w_a^p\geq 0\quad \forall a\in A,$$

Shortest **Weighted** **Multi** Path Interdiction

$$\max_{\boldsymbol{x}\in X} \min_{\boldsymbol{w}} \sum_{p\in P} \sum_{a\in A} (t_a + e_a \cdot x_a) w_a^p$$

$$\text{s.t.} \quad \sum_{a\in FS(i)} w_a^p - \sum_{a\in RS(i)} w_a^p = \left\{ \begin{cases} 1 & \text{for } i = o \\ 0 & \forall i \in N\{o^p, d^p \\ -1 & \text{for } i = d \end{cases} \quad \forall p \in P\} \right\}$$

$$w_a^p \geq 0 \quad \forall a \in A,$$

$$\max_{\boldsymbol{x}\in X} \min_{\boldsymbol{w}} \sum_{p\in P} u_p \sum_{a\in A} (t_a + e_a \cdot x_a) w_a^p$$

$$\text{s.t.} \quad \sum_{a\in FS(i)} w_a^p - \sum_{a\in RS(i)} w_a^p = \left\{ \begin{cases} 1 & \text{for } i = o \\ 0 & \forall i \in N\{o^p, d^p \\ -1 & \text{for } i = d \end{cases} \quad \forall p \in P\} \right\}$$

$$w_a^p \geq 0 \quad \forall a \in A,$$

Shortest Weighted Multi Path Interdiction **Dual**

$$\max_{\boldsymbol{x}\in X} \ \min_{\boldsymbol{y}} \sum_{p\in P} u_p \sum_{a\in A} (c_a + x_a d_a) y_a^p$$

$$\text{s.t.} \quad \sum_{a\in FS(i)} y_a^p - \sum_{a\in RS(i)} y_a^p = \left\{\begin{matrix} 1 & \text{for } i = s \\ 0 & \forall i \in N\{s^p, t^p\} \\ -1 & \text{for } i = t \end{matrix} \quad \forall p \in P \right\}$$

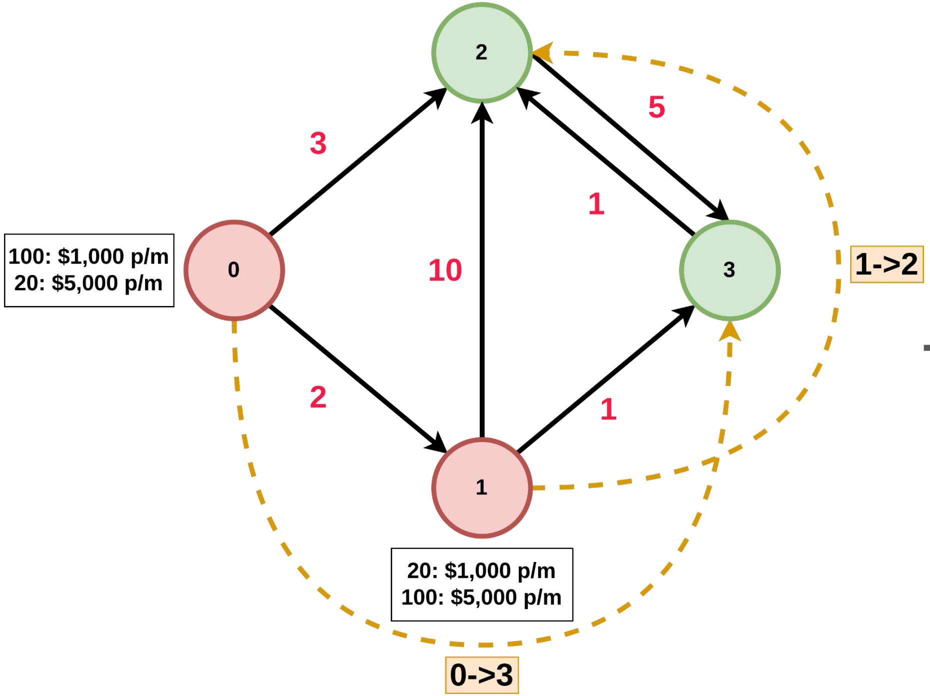
$$y_a^p \geq 0 \quad \forall a \in A \quad \forall p \in P,$$

$$\boxed{\begin{aligned} &\max_{\mathbf{x},\pi} \sum_{p\in P} \pi_t^p - \pi_s^p \\ &\text{s.t.} \quad u_p\left(\pi_j^p - \pi_i^p\right) - d_a x_a \leq c_a \quad \forall a = (i,j) \in A \quad \forall p \in P \\ &\pi_s^p = 0 \\ &\mathbf{x} \in X \end{aligned}}$$

Bilevel Optimization

$$
\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0}} \quad \max_{\boldsymbol{y}\in\mathbb{Z}_{\geq 0},\boldsymbol{\pi}\in\mathbb{R}} \quad & \sum_{p,a} \pi_d^p - \pi_o^p \quad \text{where } p=(o,d) \\
\text{subject to} \quad & \sum_{a\in A} y_a - B_y \leq 0 \\
& \sum_{a\in A} x_a - B_x \leq 0 \\
& u_p \cdot \left(\pi_j^p - \pi_i^p\right) - e_a z_a - t_a \leq 0 && \forall a=(i,j)\in A, \forall p \in P \\
& x_a - y_a - M \cdot z_a + \epsilon \leq 0 && \forall a=(i,j)\in A \\
& -M\left(1-z_a\right) - x_a + y_a \leq 0 && \forall a=(i,j)\in A \\
& z_a \in \{0,1\} && \forall a \in A \\
& e_a, t_a \in \mathbb{R}_{\geq 0} && \forall a \in A \\
& \pi_o^p = 0 && \forall p=(o,d)\in P
\end{aligned}
$$

Results with Fixed Fortifications



Interdiction Budget = 1

```
Shortest path (dual formulation | multi-path):
     For OD pair (0, 3)
           [0, 1, 2, 3] travel time: 18.0
     For OD pair (1, 2)
           [1, 2] travel time: 10.0
Interdicted: [(1,3)]
```

Interdiction Budget = 2

```
Shortest path (dual formulation | multi-path):
     For OD pair (0, 3)
           [0, 2, 3] travel time: 8.0
     For OD pair (1, 2)
           [1, 3, 2] travel time: 2.0
Interdicted: [(0,1), (1,2)]
```

What Next?

If you like it you should put an Expectation on it

$$\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0}} \quad \max_{\boldsymbol{y}\in\mathbb{Z}_{\geq 0}} \quad \min_{\boldsymbol{w}\in\mathbb{R}_{\geq 0}} \quad \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot q_a^p$$

$$\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0}} \quad \mathbb{E}_\xi[\quad \min_{\boldsymbol{w}\in\mathbb{R}_{\geq 0}} \quad \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot q_a^p]$$

$$\min_{\boldsymbol{x}\in\mathbb{Z}_{\geq 0},\boldsymbol{w}\in\mathbb{R}_{\geq 0}} \quad \sum_{s\in S} \alpha(s) \sum_{p\in P} u_p \sum_{a\in A} t_a \cdot w_a^p + e_a \cdot q_a^p$$

...

$$y_a^\omega - x_a \leq M \cdot z_a$$
$$y_a^\omega - x_a \geq -M \cdot (1 - z_a)$$

...

Conclusion

- Formalizing real-world problems is hard
- Taking the dual is hard

- Stochastic approach is conceptually more appropriate
  - Particularly given real data
- But results suggest that equitable planning for EWE is
  - Quantifiable
  - Optimizable
- Novel weighted-multipath shortest path optimization
- Open questions
  - Quality of solutions?
  - Can we do better?

References

1. Pavia, Sophie, J. Mori, Aryaman Sharma, Philip Pugliese, Abhishek Dubey, Samitha Samaranayake, and Ayan Mukhopadhyay. "**Designing Equitable Transit Networks.**" arXiv preprint arXiv:2212.12007 (2022).

2. Lou, Yingyan, and Lihui Zhang. "**Defending transportation networks against random and targeted attacks.**" Transportation research record 2234, no. 1 (2011): 31-40.

3. Israeli, Eitan, and R. Kevin Wood. "Shortest-path network interdiction." Networks: An International Journal 40, no. 2 (2002): 97-111.