

1 **FORMALIZING THE EQUITABLE PUBLIC TRANSPORT REDUCTION PROBLEM**

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17 Submission Date: May 19, 2023

ABSTRACT

During the Covid-19 pandemic, public transport agencies globally have been experiencing exacerbated ridership declines and budget cuts. As a result, many public transport agencies responded by reducing their services. These cuts affect those who depend on public transport the most, furthering inequalities in socio-economic access between different demographic groups. This report formalizes the Equitable Public Transport Reduction Problem (EPTNR), providing a more equitable approach for necessary network reductions. We propose a framework for decision-makers to model and evaluate the equality effects of implementing specific transport network reductions and lay the foundations for their optimization. We demonstrate our method with a real-world example, assuming a required reduction of up to 5% of metro links in the Metropolitan Atlanta Rapid Transit Authority's (MARTA) network in Atlanta, Georgia, USA. The goal is to increase the equality of access in terms of average travel time between white and non-white residents while fulfilling the required reduction. We show the optimal configuration of the reduced network as found by an exhaustive search and present the data using a custom data dashboard.

Keywords: Public Transport, Service Reduction, Equitable Transit, Social Good

1 INTRODUCTION

2 Across North America, the COVID-19 pandemic led to a decline in Public Transport (PT) demand
 3 (?). As a result, many local PT agencies adjusted their PT network (PTN) to the reduced demand
 4 and available budget, which often meant reducing service frequency and cutting lines (1). In many
 5 cities, these cuts further increased previously existing inequalities in access and service, affecting
 6 mainly low-income, carless, and black populations (2, 3). While some previous research on the
 7 Public Transport Network Design Problem (PTND) considered the equitable distribution of access
 8 to socio-economic opportunities (4–6), studies focusing on planned, equitable PTN reductions
 9 remain scant, primarily focusing on graph properties rather than experienced inequalities of access
 10 (7–9).

11 This report introduces a novel formalization of the Equitable Public Transport Network
 12 Reduction (EPTNR), a subset of the PTND, to address this gap. Moreover, we provide a dashboard
 13 tool for PT decision-makers to evaluate the effects of implementing specific PTN reductions. The
 14 considered effects are on the equality of access to socio-economic Points Of Interest (POIs), such
 15 as educational and healthcare facilities, between groups with different demographics. Using an
 16 exhaustive search algorithm, we determine the equitably optimal configuration of a reduced PTN
 17 given a specific budget cut.

18 We employ our novel approach on a real-world example. Namely, we assume that the metro
 19 lines of the Metropolitan Atlanta Rapid Transit Authority’s (MARTA) network need to be reduced
 20 by up to 5% due to cost saving measured. Thus, we formalize this as an EPTNR problem and
 21 attempt to identify the reduction that guarantees the best socio-economic access equality for the
 22 city’s white and non-white population groups. The accompanying code is available on our GitHub
 23 repository¹.

24 LITERATURE REVIEW

25 Over the past years, the COVID-19 pandemic amplified the need for PT adaptability and resilience
 26 (10–14). Furthermore, while by definition, PT should serve the public, differences in effectiveness,
 27 inclusion and affordability have caused segregation of and discrimination against specific popula-
 28 tion groups (15–17). Thus, the need for fast decision-support systems to plan under a lens of access
 29 equity has been exacerbated.

30 To design our access equality quantification, we adopt the definition of equality of odds
 31 by Zheng et al. (18), suggesting that all groups should have equal benefit or hardship in access-
 32 ing socio-economic opportunities via PT. With this definition, we mathematically quantify access
 33 equality based on the Average Travel Time (ATT) to socio-economic POIs (19), using the Theil T
 34 index as an (in)equality measure. Theil’s T fulfils the basic requirements of inequality metrics and
 35 allows for additive decomposition into within- and between-group inequality components, provid-
 36 ing a comprehensive quantification (20). Furthermore, it is bound in $[0, \ln n]$, with n being the
 37 size of the observed population and 0 indicating perfect equality in the population while $\ln n$ total
 38 inequality.

¹<https://github.com/RicoFio/eptnr-trb-competition>

1 METHODOLOGY

2 Definition of the Graph

3 For the EPTNR, we represent the PTN as a directed, cyclic, weighted multi-graph $\mathcal{G} = \langle V, E \rangle$ with
 4 vertex set V and edge set E . A directed multi-graph refers to a graph where the edge set is a
 5 multi-set of pairs of vertices, such that any two vertices could be connected through multiple edges
 6 (21).

7 Set V is composed of three types of vertices. First, the **population-weighted Residential**
 8 **Centroids (RC)** of neighbourhoods V^{rc} . These serve as a simplified representation of the population
 9 environment in the city. Second, the physical locations of the PTN's **stations (PTSs)**, V^{pt} . Third,
 10 we summarize **socio-economic POIs** as the vertex set V^{poi} . Together, they form the complete set
 11 $V = \{V^{rc}, V^{pt}, V^{poi}\}$.

12 In Figure 1, we illustrate an example graph containing these vertex sets. Here, all edges
 13 in grey represent the modality **WALK**, indicating a non-PT modality of transport and thus not
 14 modifiable. For the EPTNR problem, we consider modifications on the PT edges only. These
 15 edges represent transit between stations in V^{pt} by bus, tram, metro, or other modalities (22). For
 16 simplicity, we refer to this set of edges as $E^{pt} \subset E$, which are extracted from PT providers' General
 17 Transit Feed Specification (GTFS).

18 All vertices in V^{rc} are nodes of origin from which citizens of different population groups
 19 commence their journey towards the POI nodes in V^{poi} . Journeys are conducted directly by walk-
 20 ing or using the PTN, inherently visiting vertices in V^{pt} . For our definition of the EPTNR, we do
 21 not consider the return journey from V^{poi} to V^{rc} for simplicity.

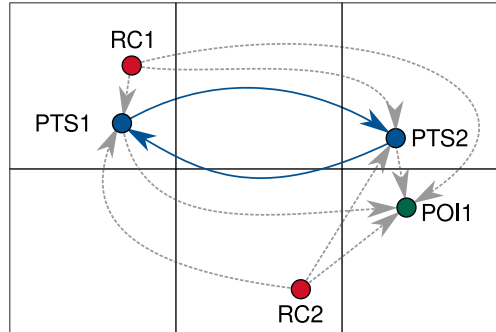


FIGURE 1 An example graph, formalizing the key requirements of an EPTNR setup. Vertices are either Residential Centroid (RC), Public Transport Station (PTS), or Point of Interest (POI) vertices. The edges from RCs to PTSs, RCs to POIs, and PTSs to POIs represent the transport mode *walk*, while the edges between the PTSs can be multi-modal (*bus, tram, metro, ...*)

22 Objective Formulation

23 To solve the EPTNR, our objective is to identify the set \mathcal{K} of edges $e_{i,j} \in E^{pt}$ with size $|\mathcal{K}| \leq k$,
 24 where k is a pre-defined maximum budget of edges to be removed, such that the inequality of access
 25 between different demographic groups, $g_i \in G$, in a city is minimized. We thus require information
 26 on the geographic neighbourhoods, their resident's demographic composition, the physical position

1 of the POIs, and the available travel options. We only consider PT modalities and *walking* for the
 2 latter.

3 **Computation of Access and its Inequality**

4 We now formalize the computation of the equitable objective above. For this, we note that while
 5 origin-destination journey surveys are available for the Transport Research Board challenge, such
 6 and other data, e.g. demographic data with household granularity, are rarely available. Thus, we
 7 make certain assumptions to facilitate our modelling and make our methodology generalizable.
 8 For one, we consider a central point of departure in each neighbourhood, namely the residential
 9 centroid, which is the areal centroid of the neighbourhood. Additionally, we posit that all residents
 10 must reach all POIs from their respective RCs, omitting travel behaviour for applicability. As a
 11 result, we compute the ATT to all POIs for each resident in the city, assuming their journey's start
 12 in the individual's respective neighbourhood RC.

13 Based on the ATT, we compute the access inequality using the Theil T measure defined in
 14 Equation 1. Here, n^g is the number of individuals in the observed population group with $g \in G$,
 15 \mathbf{y}^g the vector of per-individual ATT in said population, and μ^g the mean of \mathbf{y} . By definition, the
 16 resulting measure is bound by $0 \leq T(\mathbf{y}^g; n^g) \leq \ln n^g$ with 0 representing total equality of access and
 17 $\ln n^g$ total inequality.

$$18 \quad T(\mathbf{y}^g; n^g) = \frac{1}{n^g} \sum_{i=1}^{n^g} \frac{y_i^g}{\mu^g} \ln \left(\frac{y_i^g}{\mu^g} \right) \quad (1)$$

20 As we consider the inequality between groups, Equation 2 defines the one-level *grouped*
 21 Theil T index. Here, instead of a single vector \mathbf{y} , we pass the concatenation of the group ATT
 22 vectors $\mathbf{Y} = [\mathbf{y}^1 \mathbf{y}^2 \dots \mathbf{y}^G]$. The measure is then bound by $0 \leq T(\mathbf{Y}; n) \leq \ln n$.

$$23 \quad T(\mathbf{Y}; n) = T([\mathbf{y}^1 \mathbf{y}^2 \dots \mathbf{y}^G]_{we}; n) \quad (2)$$

24 **Optimization Formulation**

25 With the above grouped Theil's T , the higher the measure's value, the higher the inequality. To
 26 inform our search, we formulate the reward to be the negative grouped Theil's T measure. The
 27 maximization of the reward thus leads to an increase in between-group access equality.

$$28 \quad r(\mathcal{G}) = -T(\hat{\mathbf{Y}}) \quad (3)$$

30 Our objective is to find a set of edges $\mathcal{E} = \{e_1, e_2, \dots, e_i\} \subseteq E^{\text{pt}}, i \leq k$, up to budget k , such
 31 that the graph $\mathcal{G}' = \mathcal{G} - \mathcal{E}$ maximizes the above reward. Hence, we can formalize the objective as
 32 follows:

$$33 \quad J = \max_{s.t. |\mathcal{E}| \leq k} r(\mathcal{G}') \quad (4)$$

35 As our maximization formulation depends on \mathcal{E} , similarly to the formulation presented by
 36 Ramachandran et al. (5), it is non-differentiable. It can, therefore, not be solved using gradient-
 37 based optimization methods. Note that the order in which the edges are removed does not influence
 38 the final reward value $r(\mathcal{G}')$. Thus, the EPTNR is a combinatorial rather than a permutation-based

1 problem.

2 EXPERIMENTS

3 To construct the EPTNR problem graph described in section 4, we use openly accessible data
 4 provided by the Open Street Maps (OSM) project², Transitland³, and the US Census⁴. We use the
 5 most recent data, which, for the respective sources, have been published in 2023, 2023, and 2020
 6 respectively.

7 In Figure 2, we illustrate how these different sources are transformed and composed to
 8 generate an EPTNR graph. The final graph is stored using the Graph Modeling Language (GML)
 9 specifications (23), while census and RC data are stored in a companion Apache PARQUET file
 10 (24). The data generation notebook for Atlanta can be found on our repository⁵.

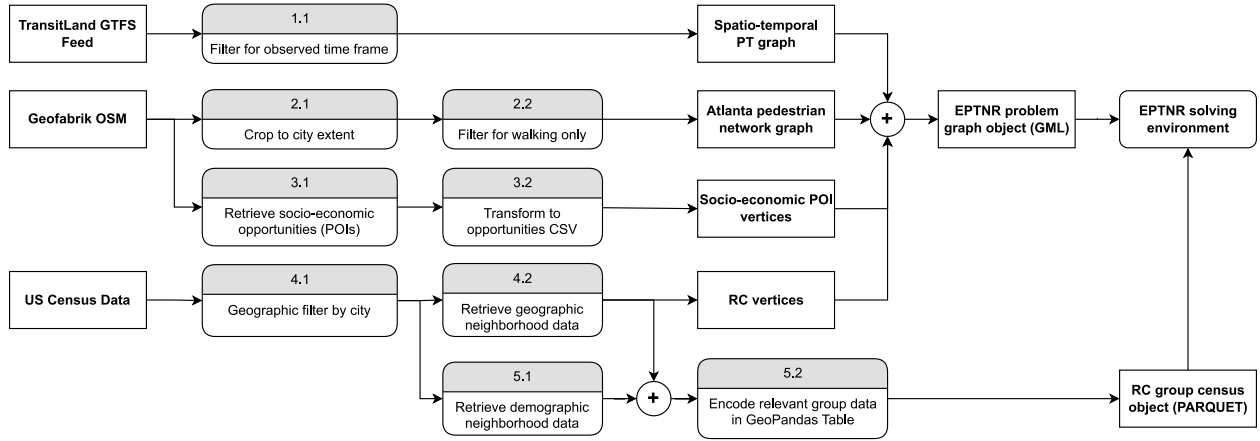


FIGURE 2 Data-Flow diagram illustrating how different data sources contribute to the final EPTNR graph object, which is subsequently used in the EPTNR solving environment.

11 Subsequently, we frame the 5% reduction of the city of Atlanta’s MARTA metro network
 12 as an EPTNR problem and employ a full search to find the most equitable configuration.

13 RESULTS

14 The MARTA network consists of 3,591 network edges of type BUS, TRAM, RAIL, and METRO.
 15 The metro line consists of two routes servicing 42 stations and 46 network edges. Thus, a 5%
 16 reduction of the metro network results in a removal of $k = 2$ edges. Given the combinatorial
 17 complexity of the EPTNR problem, this leads to 1081 possible configurations. For the 191053
 18 amount of white (w) and 283397 amount of non-white (nw) residents, the current travel time
 19 distribution to 119 primary and secondary schools is shown in ???. Notably, the nw group has an
 20 advantage over the w group in terms of access through public transit.

21 The initial access equality, bound by $[0, 13.0699]$, 0 being the most equal, lies at 0.0404.
 22 The optimal solution suggested by our full search of the 1081 possible configurations results in
 23 access equality of 0.0362, a 0.0042 inequality decrease. The search suggests removing two edges,

²<https://download.geofabrik.de/north-america/us/georgia.html>

³<https://www.transit.land/feeds/f-dnh-marta>

⁴https://censusreporter.org/user_geo/496b4a27911ad1d252830bf0cd1a55b3/

⁵

1 namely the connection from ARTS Center Station to Lindbergh Center Station for the red and gold
 2 lines. These statistics, as well as the option to further reduce the network manually, are provided
 3 by our data dashboard.

4 **DISCUSSION AND LIMITATIONS**

5 Our results show that using our equality formulation and the full search algorithm, we manage to
 6 reduce access inequality with the removal of two lines and thus find the optimal reduction strategy
 7 from the point of equality. We note that a full search is not advisable, given the combinatorial
 8 complexity of the EPTNR problem.

9 Limitations of our study stem primarily from the simplified modelling of the EPTNR prob-
 10 lem, which needs to capture the complexities of real-world PTN planning. The aggregation of
 11 journey origins in RCs, neighbourhood-level census data rather than data with higher resolution,
 12 and the need for more information on OD pairs narrow our equality quantification scope. We also
 13 recognize that ATT is not a complete access metric for our assessment. Finally, while the exhaus-
 14 tive search produced two edges which, when removed, increase access equality between the w
 15 and nw groups, we are not taking into account other factors contributing to inequality, e.g. higher
 16 dependence of some groups on PT due to lower car ownership.

17 Despite these limitations, our work underscores the importance of equitable public tran-
 18 sit, especially in challenging conditions like a global pandemic, where actions must be enabled
 19 quickly. We illustrate how numerical equality quantifications and machine learning approaches
 20 can contribute to tackling complex problems like the EPTNR. In a future version of our work,
 21 edge cost can be considered a joint optimization objective alongside equality. Furthermore, with
 22 an interactive version of our tool, decision-makers can get a quick overview of the expected con-
 23 sequences of a reduction in access equality

24 Although we did not use the General Modeling Network Specification (GMNS) for the
 25 TBR challenge, our EPTNR problem graph formulation, as shown in section 4, provides an equiv-
 26 alent macroscopic GMNS focused on PT. Without a suitable GMNS representation for editable
 27 PTN, we opted for this approach. We aimed to utilize *open source* GIS, census, and transit data
 28 from Atlanta, creating a reusable data pipeline for future work on other cities. Thus, we have not
 29 made use of the provided Atlanta Regional Commission’s (ARC) Activity-Based Model (ABM)
 30 data.

31 **CONCLUSION**

32 This work introduces a novel problem formalization, namely the Equitable Public Transport Net-
 33 work Reduction (EPTNR) problem. We show that when, due to budget or governmental restric-
 34 tions, public transit networks *must be* reduced, the most equitable configuration can be identified
 35 by using algorithmic approaches. To this end, we apply our formalization and an exhaustive search
 36 algorithm on a hypothetical 5% reduction in the Atlanta (GA, USA) MARTA metro network and
 37 show that public access equality can be increased even with necessary reductions. Our data dash-
 38 board allows quick insight for decision-makers on EPTNR problems and their solutions.

39 **ACKNOWLEDGEMENTS**

40 Thanks to Anson Stewart from the MIT JTL Lab for pointing out this competition and reviewing
 41 my drafts. Moreover, thanks to Dr. Cafer Avci for answering my questions over the past weeks.

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