

1 **FORMALIZING THE EQUITABLE PUBLIC TRANSPORT REDUCTION PROBLEM**

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**ABSTRACT**

During the Covid-19 pandemic, public transport agencies globally have been experiencing exacerbated ridership declines and budget cuts. As a result, many public transport agencies responded by reducing their services. These cuts affect those who depend on public transport the most, furthering inequalities in socio-economic access between different demographic groups. This report formalizes the Equitable Public Transport Reduction Problem (EPTNR), providing a more equitable approach for necessary network reductions. We propose a framework for decision-makers to model and evaluate the equality effects of implementing specific transport network reductions and lay the foundations for their optimization. We demonstrate our method with a real-world example, assuming a required reduction of up to 5% of metro links in the Metropolitan Atlanta Rapid Transit Authority's (MARTA) network in Atlanta, Georgia, USA. The goal is to increase the equality of access in terms of average travel time between white and non-white residents while fulfilling the required reduction. We show the optimal configuration of the reduced network as found by an exhaustive search and present the data using a custom data dashboard.

*Keywords:* Public Transport, Service Reduction, Equitable Transit, Social Good

## 1 INTRODUCTION

2 Across North America, the COVID-19 pandemic led to a decline in Public Transport (PT) demand  
 3 (1). As a result, many local PT agencies adjusted their PT network (PTN) to the reduced demand  
 4 and available budget, which often meant reducing service frequency and cutting lines (2). In many  
 5 cities, these cuts further increased previously existing inequalities in access and service, affecting  
 6 mainly low-income, carless, and black populations (3, 4). While some previous research on the  
 7 Public Transport Network Design Problem (PTND) considered the equitable distribution of access  
 8 to socio-economic opportunities (5–7), studies focusing on planned, equitable PTN reductions  
 9 remain scant, primarily focusing on graph properties rather than experienced inequalities of access  
 10 (8–10).

11 This report introduces a novel formalization of the Equitable Public Transport Network  
 12 Reduction (EPTNR), a subset of the PTND, to address this gap. Moreover, we provide a dashboard  
 13 tool for PT decision-makers to evaluate the effects of implementing specific PTN reductions. The  
 14 considered effects are on the equality of access to socio-economic Points Of Interest (POIs), such  
 15 as educational and healthcare facilities, between groups with different demographics. Using an  
 16 exhaustive search algorithm, we determine the equitably optimal configuration of a reduced PTN  
 17 given a specific budget cut.

18 We employ our novel approach on a real-world example. Namely, we assume that the metro  
 19 lines of the Metropolitan Atlanta Rapid Transit Authority’s (MARTA) network need to be reduced  
 20 by up to 5% due to cost saving measured. Thus, we formalize this as an EPTNR problem and  
 21 attempt to identify the reduction that guarantees the best socio-economic access equality for the  
 22 city’s white and non-white population groups. The accompanying code is available on our GitHub  
 23 repository<sup>1</sup>.

## 24 LITERATURE REVIEW

25 Over the past years, the COVID-19 pandemic amplified the need for PT adaptability and resilience  
 26 (11–15). Furthermore, while by definition, PT should serve the public, differences in effectiveness,  
 27 inclusion and affordability have caused segregation of and discrimination against specific popula-  
 28 tion groups (16–18). Thus, the need for fast decision-support systems to plan under a lens of access  
 29 equity has been exacerbated.

30 To design our access equality quantification, we adopt the definition of equality of odds  
 31 by Zheng et al. (19), suggesting that all groups should have equal benefit or hardship in access-  
 32 ing socio-economic opportunities via PT. With this definition, we mathematically quantify access  
 33 equality based on the Average Travel Time (ATT) to socio-economic POIs (20), using the Theil  $T$   
 34 index as an (in)equality measure. Theil’s  $T$  fulfils the basic requirements of inequality metrics and  
 35 allows for additive decomposition into within- and between-group inequality components, provid-  
 36 ing a comprehensive quantification (21). Furthermore, it is bound in  $[0, \ln n]$ , with  $n$  being the  
 37 size of the observed population and 0 indicating perfect equality in the population while  $\ln n$  total  
 38 inequality.

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<sup>1</sup><https://github.com/RicoFio/eptnr-trb-competition>

## 1 METHODOLOGY

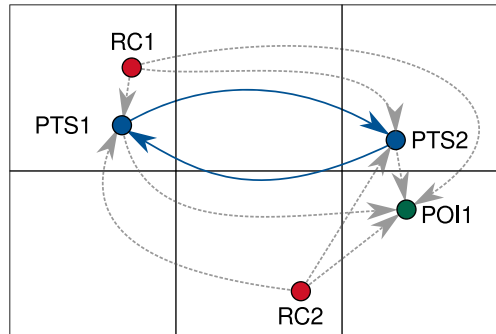
### 2 Definition of the Graph

3 For the EPTNR, we represent the PTN as a directed, cyclic, weighted multi-graph  $\mathcal{G} = \langle V, E \rangle$  with  
 4 vertex set  $V$  and edge set  $E$ . A directed multi-graph refers to a graph where the edge set is a  
 5 multi-set of pairs of vertices, such that any two vertices could be connected through multiple edges  
 6 (22).

7 Set  $V$  is composed of three types of vertices. First, the **population-weighted Residential**  
 8 **Centroids (RC)** of neighbourhoods  $V^{rc}$ . These serve as a simplified representation of the population  
 9 environment in the city. Second, the physical locations of the PTN's **stations (PTSs)**,  $V^{pt}$ . Third,  
 10 we summarize **socio-economic POIs** as the vertex set  $V^{poi}$ . Together, they form the complete set  
 11  $V = \{V^{rc}, V^{pt}, V^{poi}\}$ .

12 In Figure 1, we illustrate an example graph containing these vertex sets. Here, all edges  
 13 in grey represent the modality **WALK**, indicating a non-PT modality of transport and thus not  
 14 modifiable. For the EPTNR problem, we consider modifications on the PT edges only. These  
 15 edges represent transit between stations in  $V^{pt}$  by bus, tram, metro, or other modalities (23). For  
 16 simplicity, we refer to this set of edges as  $E^{pt} \subset E$ , which are extracted from PT providers' General  
 17 Transit Feed Specification (GTFS).

18 All vertices in  $V^{rc}$  are nodes of origin from which citizens of different population groups  
 19 commence their journey towards the POI nodes in  $V^{poi}$ . Journeys are conducted directly by walk-  
 20 ing or using the PTN, inherently visiting vertices in  $V^{pt}$ . For our definition of the EPTNR, we do  
 21 not consider the return journey from  $V^{poi}$  to  $V^{rc}$  for simplicity.



**FIGURE 1** An example graph, formalizing the key requirements of an EPTNR setup. Vertices are either Residential Centroid (RC), Public Transport Station (PTS), or Point of Interest (POI) vertices. The edges from RCs to PTSs, RCs to POIs, and PTSs to POIs represent the transport mode *walk*, while the edges between the PTSs can be multi-modal (*bus, tram, metro, ...*)

### 22 Objective Formulation

23 To solve the EPTNR, our objective is to identify the set  $\mathcal{K}$  of edges  $e_{i,j} \in E^{pt}$  with size  $|\mathcal{K}| \leq k$ ,  
 24 where  $k$  is a pre-defined maximum budget of edges to be removed, such that the inequality of access  
 25 between different demographic groups,  $g_i \in G$ , in a city is minimized. We thus require information  
 26 on the geographic neighbourhoods, their resident's demographic composition, the physical position

1 of the POIs, and the available travel options. We only consider PT modalities and *walking* for the  
 2 latter.

### 3 **Computation of Access and its Inequality**

4 We now formalize the computation of the equitable objective above. For this, we note that while  
 5 origin-destination journey surveys are available for the Transport Research Board challenge, such  
 6 and other data, e.g. demographic data with household granularity, are rarely available. Thus, we  
 7 make certain assumptions to facilitate our modelling and make our methodology generalizable.  
 8 For one, we consider a central point of departure in each neighbourhood, namely the residential  
 9 centroid, which is the areal centroid of the neighbourhood. Additionally, we posit that all residents  
 10 must reach all POIs from their respective RCs, omitting travel behaviour for applicability. As a  
 11 result, we compute the ATT to all POIs for each resident in the city, assuming their journey's start  
 12 in the individual's respective neighbourhood RC.

13 Based on the ATT, we compute the access inequality using the Theil  $T$  measure defined in  
 14 Equation 1. Here,  $n^g$  is the number of individuals in the observed population group with  $g \in G$ ,  
 15  $\mathbf{y}^g$  the vector of per-individual ATT in said population, and  $\mu^g$  the mean of  $\mathbf{y}$ . By definition, the  
 16 resulting measure is bound by  $0 \leq T(\mathbf{y}^g; n^g) \leq \ln n^g$  with 0 representing total equality of access and  
 17  $\ln n^g$  total inequality.

$$18 \quad T(\mathbf{y}^g; n^g) = \frac{1}{n^g} \sum_{i=1}^{n^g} \frac{y_i^g}{\mu^g} \ln \left( \frac{y_i^g}{\mu^g} \right) \quad (1)$$

20 As we consider the inequality between groups, Equation 2 defines the one-level *grouped*  
 21 Theil  $T$  index. Here, instead of a single vector  $\mathbf{y}$ , we pass the concatenation of the group ATT  
 22 vectors  $\mathbf{Y} = [\mathbf{y}^1 \mathbf{y}^2 \dots \mathbf{y}^G]$ . The measure is then bound by  $0 \leq T(\mathbf{Y}; n) \leq \ln n$ .

$$23 \quad T(\mathbf{Y}; n) = T([\mathbf{y}^1 \mathbf{y}^2 \dots \mathbf{y}^G]_{we}; n) \quad (2)$$

### 24 **Optimization Formulation**

25 With the above grouped Theil's  $T$ , the higher the measure's value, the higher the inequality. To  
 26 inform our search, we formulate the reward to be the negative grouped Theil's  $T$  measure. The  
 27 maximization of the reward thus leads to an increase in between-group access equality.

$$28 \quad r(\mathcal{G}) = -T(\hat{\mathbf{Y}}) \quad (3)$$

30 Our objective is to find a set of edges  $\mathcal{E} = \{e_1, e_2, \dots, e_i\} \subseteq E^{\text{pt}}, i \leq k$ , up to budget  $k$ , such  
 31 that the graph  $\mathcal{G}' = \mathcal{G} - \mathcal{E}$  maximizes the above reward. Hence, we can formalize the objective as  
 32 follows:

$$33 \quad J = \max_{s.t. |\mathcal{E}| \leq k} r(\mathcal{G}') \quad (4)$$

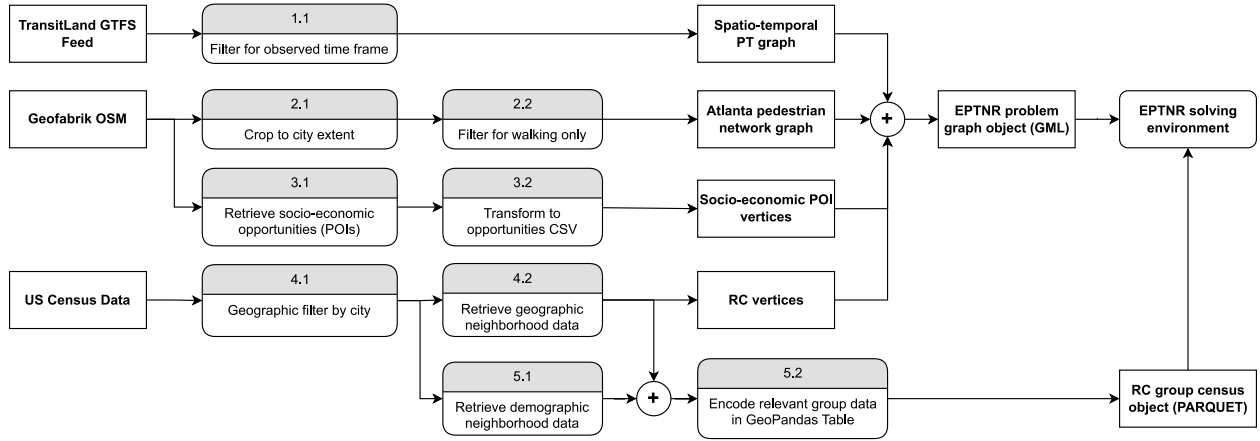
35 As our maximization formulation depends on  $\mathcal{E}$ , similarly to the formulation presented by  
 36 Ramachandran et al. (6), it is non-differentiable. It can, therefore, not be solved using gradient-  
 37 based optimization methods. Note that the order in which the edges are removed does not influence  
 38 the final reward value  $r(\mathcal{G})$ . Thus, the EPTNR is a combinatorial rather than a permutation-based

1 problem.

## 2 EXPERIMENTS

3 To construct the EPTNR problem graph described in section 4, we use openly accessible data pro-  
 4 vided by the Open Street Maps (OSM) project<sup>2</sup>, Transitland<sup>3</sup>, and the US Census<sup>4</sup>. We use the most  
 5 recent data, which, for the respective sources, have been published on 05/18/2023, 05/12/2023, and  
 6 2020 respectively.

7 In Figure 2, we illustrate how these different sources are transformed and composed to  
 8 generate an EPTNR graph. The final graph is stored using the Graph Modeling Language (GML)  
 9 specifications (24), while census and RC data are stored in a companion Apache PARQUET file  
 10 (25). The data generation notebook for Atlanta can be found on our repository<sup>5</sup>.



**FIGURE 2 Data-Flow diagram illustrating how different data sources contribute to the final EPTNR graph object, which is subsequently used in the EPTNR solving environment.**

11 Subsequently, we frame the 5% reduction of the city of Atlanta’s MARTA metro network  
 12 as an EPTNR problem and employ a full search to find the most equitable configuration.

## 13 RESULTS

14 The MARTA network consists of 3,591 network edges of type BUS, TRAM, and METRO. The  
 15 metro line consists of two routes servicing 42 stations and 46 network edges. Thus, a 5% reduction  
 16 of the metro network results in a removal of  $k = 2$  edges. Given the combinatorial complexity of  
 17 the EPTNR problem, this leads to 1081 possible configurations. For the 191053 amount of white  
 18 (w) and 283397 amount of non-white (nw) residents, the current . Notably, the nw group has an  
 19 advantage over the w group in terms of access through public transit.

20 The initial access equality, bound by  $[0, 13.0699]$ , 0 being the most equal, lies at 0.0404.  
 21 The optimal solution suggested by our full search of the 1081 possible configurations results in  
 22 access equality of 0.0362, a 0.0042 inequality decrease. The search suggests removing two edges,  
 23 namely the connection from ARTS Center Station to Lindbergh Center Station for the red and gold

<sup>2</sup><https://download.geofabrik.de/north-america/us/georgia.html>

<sup>3</sup><https://www.transit.land/feeds/f-dnh-marta>

<sup>4</sup>[https://censusreporter.org/user\\_geo/496b4a27911ad1d252830bf0cd1a55b3/](https://censusreporter.org/user_geo/496b4a27911ad1d252830bf0cd1a55b3/)

<sup>5</sup>[https://github.com/RicoFio/eptnr-trb-competition/blob/main/atlanta\\_data/atlanta\\_data\\_prep.ipynb](https://github.com/RicoFio/eptnr-trb-competition/blob/main/atlanta_data/atlanta_data_prep.ipynb)

1 lines. These statistics, as well as the option to further reduce the network manually, are provided  
2 by our data dashboard.

### 3 **DISCUSSION AND LIMITATIONS**

4 Our results show that using our equality formulation and the full search algorithm, we manage to  
5 reduce access inequality with the removal of two lines and thus find the optimal reduction strategy  
6 from the point of equality. We note that a full search is not advisable, given the combinatorial  
7 complexity of the EPTNR problem.

8       Limitations of our study stem primarily from the simplified modelling of the EPTNR prob-  
9 lem, which needs to capture the complexities of real-world PTN planning. The aggregation of  
10 journey origins in RCs, neighbourhood-level census data rather than data with higher resolution,  
11 and the need for more information on OD pairs narrow our equality quantification scope. We also  
12 recognize that ATT is not a complete access metric for our assessment. Finally, while the exhaus-  
13 tive search produced two edges which, when removed, increase access equality between the w  
14 and nw groups, we are not taking into account other factors contributing to inequality, e.g. higher  
15 dependence of some groups on PT due to lower car ownership.

16       Despite these limitations, our work underscores the importance of equitable public tran-  
17 sit, especially in challenging conditions like a global pandemic, where actions must be enabled  
18 quickly. We illustrate how numerical equality quantifications and machine learning approaches  
19 can contribute to tackling complex problems like the EPTNR. In a future version of our work,  
20 edge cost can be considered a joint optimization objective alongside equality. Furthermore, with  
21 an interactive version of our tool, decision-makers can get a quick overview of the expected con-  
22 sequences of a reduction in access equality

23       Although we did not use the General Modeling Network Specification (GMNS) for the  
24 TBR challenge, our EPTNR problem graph formulation, as shown in section 4, provides an equiv-  
25 alent macroscopic GMNS focused on PT. Without a suitable GMNS representation for editable  
26 PTN, we opted for this approach. We aimed to utilize *open source* GIS, census, and transit data  
27 from Atlanta, creating a reusable data pipeline for future work on other cities. Thus, we have not  
28 made use of the provided Atlanta Regional Commission’s (ARC) Activity-Based Model (ABM)  
29 data.

### 30 **CONCLUSION**

31 This work introduces a novel problem formalization, namely the Equitable Public Transport Net-  
32 work Reduction (EPTNR) problem. We show that when, due to budget or governmental restric-  
33 tions, public transit networks *must be* reduced, the most equitable configuration can be identified  
34 by using algorithmic approaches. To this end, we apply our formalization and an exhaustive search  
35 algorithm on a hypothetical 5% reduction in the Atlanta (GA, USA) MARTA metro network and  
36 show that public access equality can be increased even with necessary reductions. Our data dash-  
37 board allows quick insight for decision-makers on EPTNR problems and their solutions.

### 38 **ACKNOWLEDGEMENTS**

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