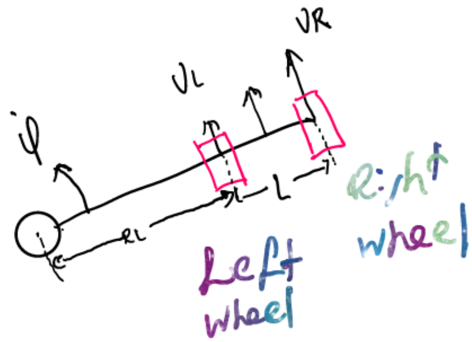

Figure 1: Motion of A Differential Drive

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Part 1 Figure 1 shows the motion of a differential drive. If we define the following:

(a) u_L, u_R are the commanded wheel velocities of the left and right wheels.
(b) ϕ is the heading of the robot, relative to the positive x axis.
(c) x, y are the x, y coordinates of the robot.

Then we can derive the following equations:

$$R_L * \dot{\phi} = u_L * r \tag{1}$$

$$(R_L + L)\dot{\phi} = u_R r \tag{2}$$

$$\dot{x} = \frac{u_L + u_r}{2} rcos(\phi) \tag{3}$$

$$\dot{y} = \frac{u_L + u_r}{2} rsin(\phi) \tag{4}$$

To sum up, the relationship between world frame velocities and wheel velocities are:

$$\begin{bmatrix}\dot{\phi}\\ \dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}-r/L & r/L\\ rcos(\phi)/2 & rcos(\phi)/2\\ rsin(\phi)/2 & rsin(\phi)/2\end{bmatrix}\begin{bmatrix}u_L\\ u_R\end{bmatrix} \tag{5}$$

Then, it is not difficult to derive the body frame twist and the wheel velocities:

$$\begin{bmatrix}\dot{\phi}\\ \dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}-r/L & r/L\\ rcos(0)/2 & rcos(0)/2\\ rsin(0)/2 & rsin(0)/2\end{bmatrix}\begin{bmatrix}u_L\\ u_R\end{bmatrix} = \begin{bmatrix}-r/L & r/L\\ r/2 & r/2\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_L\\ u_R\end{bmatrix} \tag{6}$$