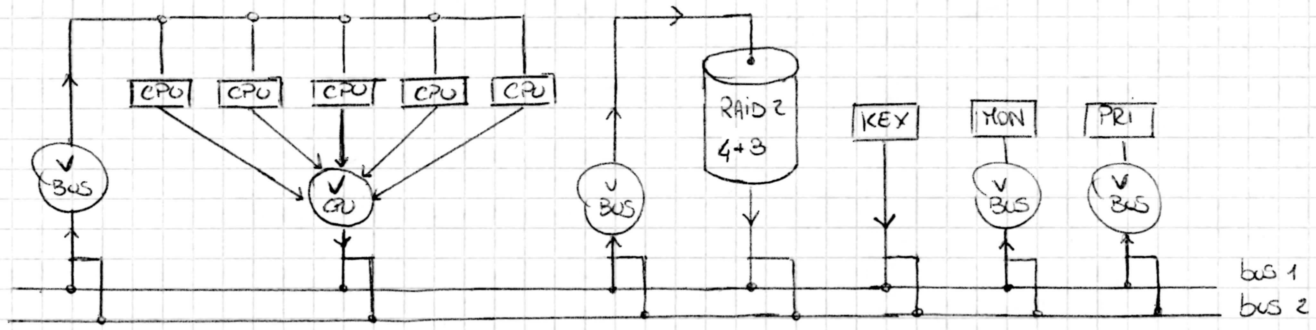
Exam 25.10.2013

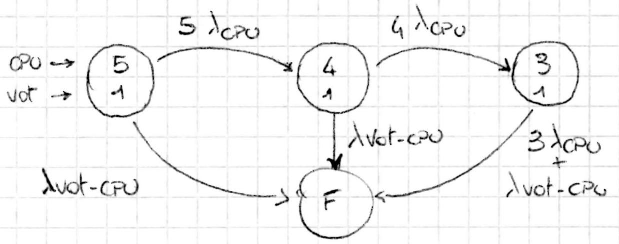
Exercise 1



Hypothesis: The system is switch[redacted] off when a component faults.

RELIABILITY

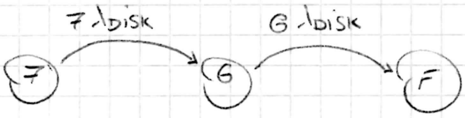
- CPU + VOTER CPU



$$\begin{cases} P'_{5,1}(t) = -P_{5,1}(t)\left[5\lambda_{CPU} + \lambda_{vot\text{-}CPU}\right] \\ P'_{4,1}(t) = +P_{5,1}(t)\left[5\lambda_{CPU}\right] - P_{4,1}(t)\left[4\lambda_{CPU} + \lambda_{vot\text{-}CPU}\right] \\ P'_{3,1}(t) = +P_{4,1}(t)\left[4\lambda_{CPU}\right] - P_{3,1}(t)\left[3\lambda_{CPU} + \lambda_{vot\text{-}CPU}\right] \\ P'_F(t) = +\lambda_{vot\text{-}CPU}\left[P_{5,1}(t) + P_{4,1}(t) + P_{3,1}(3\lambda_{CPU})\right] \\ P_{5,1}(t) + P_{4,1}(t) + P_{3,1}(t) + P_F(t) = 1 \\ P_{5,1}(\phi) = 1 \end{cases}$$

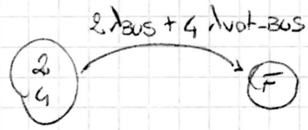
$$R_{CPU}(t) = 1 - P_F(t)$$

- RAID



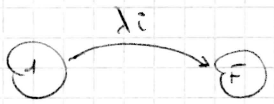
$$R_{RAID}(t) = 1 - P_F(t)$$

- BUS + VOTER BUS



$$R_{BUS}(t) = 1 - P_F(t)$$

- PRINTER/MONITOR/KEYBOARD



$R_i(t) = 1 - P_F(t)$

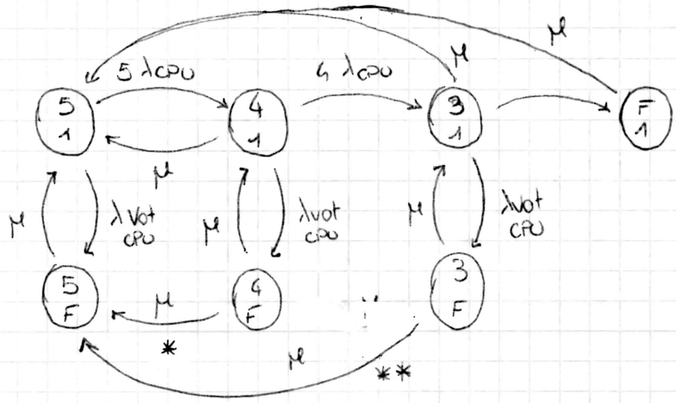
where i = pri/mon/key

Finally:

$$R_{tot}(t) = R_{CPU}(t) \cdot R_{RAID}(t) \cdot R_{BUS}(t) \cdot R_{PRI}(t) \cdot R_{MON}(t) \cdot R_{KEY}(t)$$

AVAILABILITY

- CPU + VOTER CPU

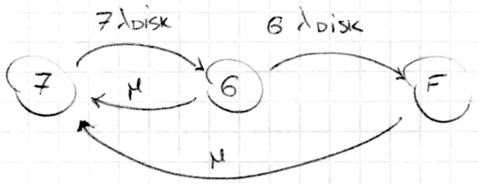


** According to the text:
"The technician is able to repair all faulty components contemporaneously, with the same repair rate μ"

$$A_{CPU}(t) = 1 - \left[P_{5,F}(t) + P_{4,F}(t) + P_{3,F}(t) + P_{F,1}(t) \right]$$

* This because, according to the hypothesis made, when the system goes in a "fault state", S cannot have others faults! But anyway, S can repair the ones that S already have!

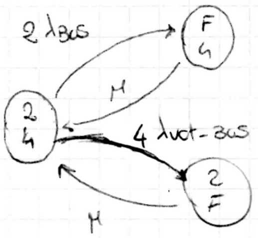
- RAID



$$A_{RAID}(t) = 1 - P_F(t)$$

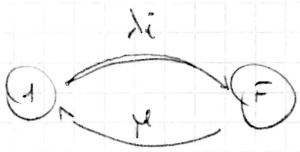
$$\begin{cases} P_7'(t) = \emptyset = -P_F[7\lambda_{DISK}] + \mu[P_6 + P_F] \\ P_6'(t) = \emptyset = -P_6[\mu + 6\lambda_{DISK}] + P_F[7\lambda_{DISK}] \\ P_F'(t) = \emptyset = +P_6[6\lambda_{DISK}] - P_F[\mu] \\ P_7(t) + P_6(t) + P_F(t) = 1 \\ P_7(\emptyset) = 1 \end{cases}$$

- BUS + VOTER BUS



$$A_{BUS} = 1 - \left[P_{F,4}(t) + P_{2,F}(t) \right]$$

- MONITOR/PRINTER/KEYBOARD



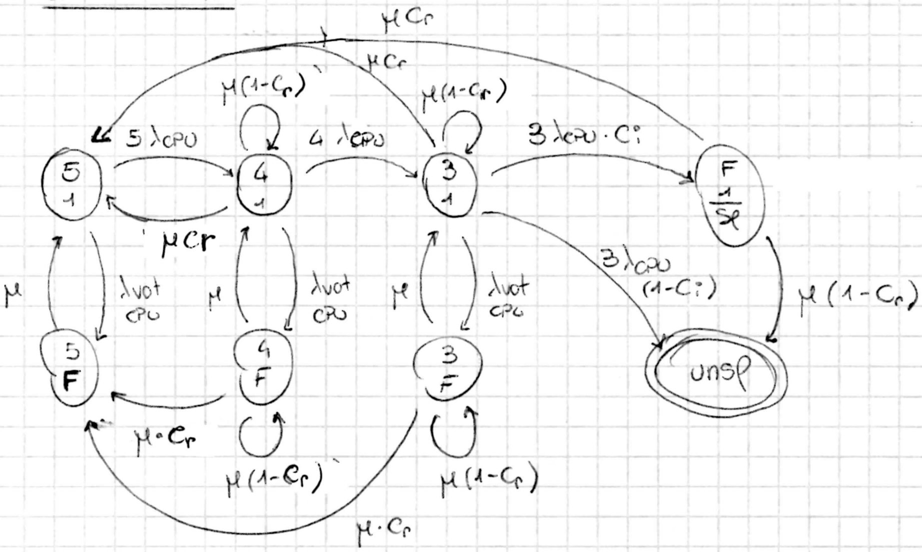
$A_i(t) = 1 - P_F(t)$ where i = pri/mon/key

Finally:

$$A_{tot}(t) = A_{CPU}(t) \cdot A_{BUS}(t) \cdot A_{RAID}(t) \cdot A_{PRI}(t) \cdot A_{MON}(t) \cdot A_{KEY}(t)$$

SAFETY

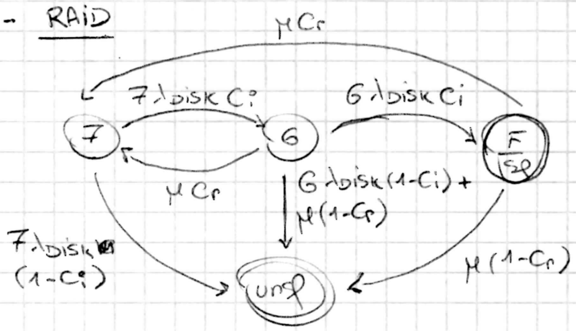
- CPU + VOTER CPU



I hipothesize that the coverage factor for the voter-CPU is 1 (so I always detect its fault and always repair it well). This for semplicity. (Otherwise I would have to distinguish even the cases when I don't detect a voter fault and an incorrect repair of it)

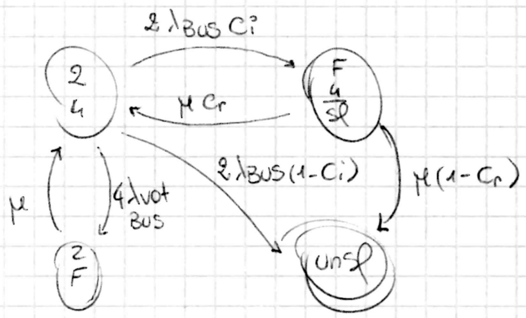
$S_{CPU}(t) = 1 - P_{unsf}(t)$

- RAID



$S_{RAID}(t) = 1 - P_{unsf}(t).$

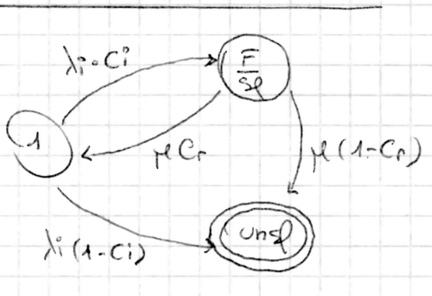
- BUS + voter bus



Also in this case, I have hypothesized that the coverage factor of the voter is 1 (for semplicity)

$S_{BUS}(t) = 1 - P_{unsf}(t)$

- MONITOR/KEYBOARD/PRINTER

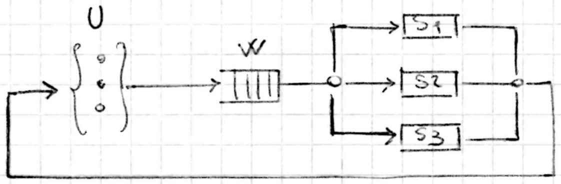


$$\begin{cases} P_1'(t) = -P_1(t)[\lambda_i C_i + \lambda_i(1-C_i)] + P_F(t)[\mu C_r] \\ P_F'(t) = +P_1(t)[\lambda_i C_i] - P_F(t)[\mu C_r + \mu(1-C_r)] \\ P_{unsf}'(t) = +P_1(t)[\lambda_i(1-C_i)] + P_F(t)[\mu(1-C_r)] \\ P_1(t) + P_F(t) + P_{unsf}(t) = 1; \\ P_1(\phi) = 1 \end{cases}$$

$S_i(t) = 1 - P_{unsf}(t)$
where i = pri, mon, key

Finally: $S_{tot}(t) = S_{CPU}(t) \cdot S_{RAID}(t) \cdot S_{BUS}(t) \cdot S_{PRI}(t) \cdot S_{MON}(t) \cdot S_{KEY}(t)$

Exercise 2



$U = 10$ (users);

$W = 6$ (users queue);

$Z = 20$ (sec) thinking time;

$S = 10$ (sec) service time; $\Rightarrow \mu = \frac{1}{S} = 0{,}1$ $\left(\frac{req}{sec}\right)$

$n = 3$ (n° of servers)

service rate (of a single server)

There aren't MTTF and MTTR $\Rightarrow$ no faulty servers!

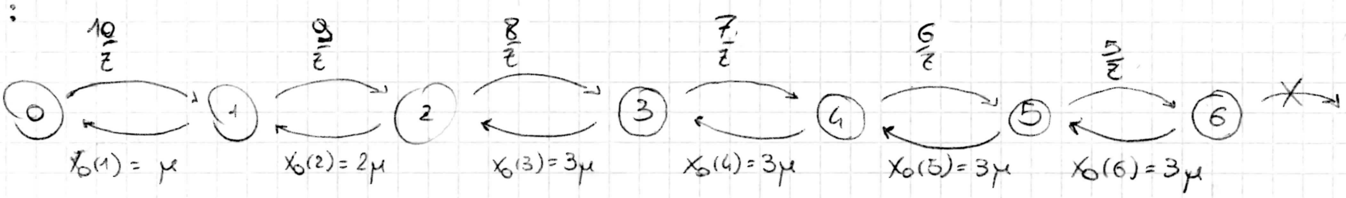
I can analyze directly the whole system according to the n° of requests present in it.

If "K" users are into the system, the others "U-K" users are in the think phase.

So, The rate with which these users submit new requests is given by: $\frac{U-K}{Z}$ for $K = 0, 1, \dots, 6$.

Instead the rate with which the request are served is given by: $X_0(K) = \begin{cases} K \cdot \mu & 1 \le K \le n \\ n \cdot \mu & K \ge n \end{cases}$ for $K = 1, \dots, 6$

So:



For the flow-in = flow-out principle:

$$\begin{cases} P_0 \cdot \frac{10}{Z} = P_1 \cdot \mu \\ P_1 \cdot \frac{9}{Z} = P_2 \cdot 2\mu \\ P_2 \cdot \frac{8}{Z} = P_3 \cdot 3\mu \\ P_3 \cdot \frac{7}{Z} = P_4 \cdot 3\mu \\ P_4 \cdot \frac{6}{Z} = P_5 \cdot 3\mu \\ P_5 \cdot \frac{5}{Z} = P_6 \cdot 3\mu \\ \sum_{i=0}^{6} P_i = 1 \end{cases}$$

The average throughput ~~for a single server~~ is given by:

$$X(0) = \sum_{i=1}^{W} P_i X_0(i) = (P_1 \mu) + (P_2 \cdot 2\mu) + (P_3 \cdot 3\mu) + (P_4 \cdot 3\mu) + (P_5 \cdot 3\mu) + (P_6 \cdot 3\mu)$$

~~Having three server, the average throughput of the system is given by:~~

$\cancel{\bar{X} = 3 \cdot X(0)}$

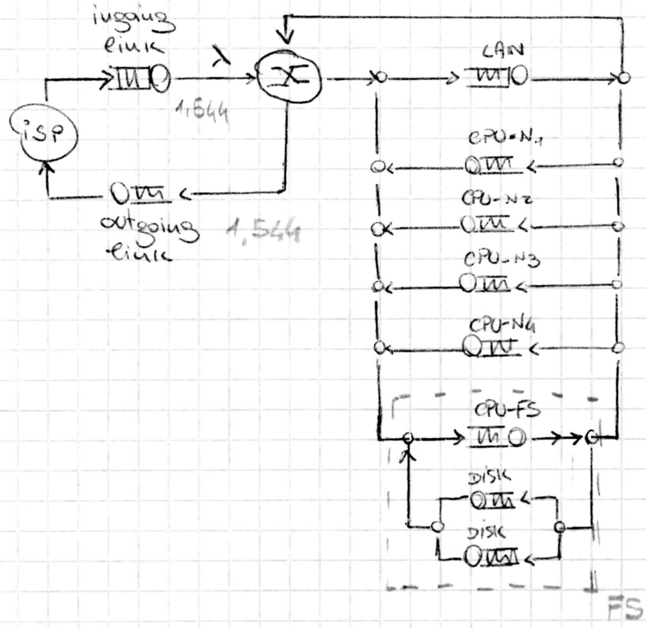
The average number of user ~~for a single server~~ is given by:

$$N(0) = \sum_{i=0}^{W} P_i \cdot i = 0 + P_1 + 2P_2 + 3P_3 + 4P_4 + 5P_5 + 6P_6$$

The average response time ~~for a single server~~ is given by the Little's law:

$R(0) = \frac{N(0)}{X(0)}$ ~~$\Rightarrow$ The average response time of the system is: $\bar{R} = R(0)$~~

Exercise 3



$\lambda = 10$ (req/sec)

$B_{eth} = 1$ Gbps;

$B_{isp} = 1{,}544$ Mbps;

$L = 50$ (μsec/pkt)

$R = 300$ (byte)

$F = 100$ (Kbyte) $= 100 \cdot 1024 = 102'400$ (byte)

$S_{CPU-N} = 10$ (ms)

$S_{CPU-FS} = 10$ (ms)

$S_{DISK} = 50$ (ms)

service time

- Average response time

Is an open network, so I have to use these formulas:

$D_i = \frac{S_i \cdot V_i}{n_i}$ (service demand) where V_i = average n° of visit to the i-th component; n_i = n° of i-th components (due to the load balancer!)

$U_i = X_0 \cdot D_i$ where $X_0 \overset{=\lambda}{\leq} \frac{1}{\max D_i}$ (utilization factor)

$R'_i = \frac{D_i}{1 - U_i}$ (residence time)

Thus:

$D_{CPU-N} = \frac{S_{CPU-N} \cdot 1}{4} = \frac{10\ (ms)}{4} = 2{,}5\ (ms) = 0{,}0025\ (sec)$

$D_{CPU-FS} = \frac{S_{CPU-FS} \cdot 1}{1} = 10\ (ms) = 0{,}010\ (sec)$

$D_{DISK} = \frac{S_{DISK} \cdot 1}{2} = \frac{50}{2}\ (ms) = 25\ (ms) = 0{,}025\ (sec)$

$X_0 = \frac{1}{\max D_i} = \frac{1}{D_{DISK}} = \frac{1}{25 \cdot 10^{-3}\ (sec)} = 40$ (req/sec) (⇒ Beyond this value, the system saturates)

For the steady state condition: $X_0 \overset{!}{=} \lambda = 10$

Then:

$U_{CPU-N} = X_0 \cdot D_{CPU-N} = 10 \left(\frac{req}{sec}\right) \cdot 0{,}0025\ (sec) = 0{,}025$ (2,5%)

$U_{CPU-FS} = X_0 \cdot D_{CPU-FS} = 10 \left(\frac{req}{sec}\right) \cdot 0{,}010\ (sec) = 0{,}1$ (10%)

$U_{DISK} = X_0 \cdot D_{DISK} = 10 \cdot 0{,}025 = 0{,}25$ (25%)

Finally:

$$R'_{CPU-N} = \frac{2,5\ (ms)}{0,975} = 2,56\ (ms)$$

$$R'_{CPU-FS} = \frac{10\ (ms)}{0,9} = 11,11\ (ms)$$

$$R'_{DISK} = \frac{25\ (ms)}{0,75} = 33,33\ (ms)$$

Regarding the networks: Hypothesis: The TCP doesn't know the ethernet MTU

because $MTU_{ETH} - TCP_{ovhd} = 1500 - 20 = 1480 = MSS$

$\boxed{\text{Request}}$

$$\#segment = \left\lceil \frac{R}{MTU_{TCP}} \right\rceil = \left\lceil \frac{300}{65'515} \right\rceil = 1;$$

$$\#datagram = \left\lceil \frac{R + \#segment\,(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{300 + 1\,(20)}{1480} \right\rceil = 1, = \#frame \text{ also.}$$

$$Overhead = \#segment\,(TCP_{ovhd}) + \#datagram\,(IP_{ovhd} + Eth_{ovhd}) =$$
$$= 1\,(20) + 1\,(20 + 18) = 58\ (byte)$$

$$S^{req}_{eth} = \frac{8 \cdot (R + Overhead)}{B_{eth}} = \frac{8 \cdot (358)\ (bit)}{1 \cdot 10^{9}\ (\frac{bit}{sec})} = 2'864 \cdot 10^{-9}\ (sec) = 0,0029\ (ms)$$

$$S^{req}_{isp} = \frac{8\,(R + Overhead)}{B_{isp}} = \frac{2864}{1,544 \cdot 10^{6}}\ (sec) = 1'854,92 \cdot 10^{-6}\ (sec) = 1,8549\ (ms)$$

$$S^{req}_{rout} = \#datagram \cdot L = 1\ (pkt) \cdot 50\ (\frac{\mu sec}{pkt}) = 50 \cdot 10^{-6}\ (s) = 0,05\ (ms)$$

$\boxed{\text{Reply}}$

$$\#segment = \left\lceil \frac{F}{MTU_{TCP}} \right\rceil = \left\lceil \frac{102'400}{65'515} \right\rceil = 2;$$

$$\#datagram = \left\lceil \frac{F + \#segment\,(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{102'440}{1480} \right\rceil = 70 = \#frame \text{ (for the same reason above)}$$

$$Overhead = \#segment\,(TCP_{ovhd}) + \#datagram\,(IP_{ovhd} + Eth_{ovhd}) = 2\,(20) + 70\,(20 + 18) = 2700$$

$$S^{rep}_{eth} = \frac{8\,(F + Overhead)}{B_{eth}} = \frac{8\,(105'100)}{1 \cdot 10^{9}}\ (sec) = 840'800 \cdot 10^{-9}\ (sec) = 0,84\ (ms)$$

$$S^{rep}_{isp} = \frac{8\,(F + Overhead)}{B_{isp}} = \frac{840'800}{1,544 \cdot 10^{6}}\ (sec) = 544'559,58 \cdot 10^{-6}\ (sec) = 544,55\ (ms)$$

$$S^{rep}_{rout} = \#datagram \cdot L = 70\ (pkt) \cdot 50\ (\frac{\mu sec}{pkt}) = 3500\ (\mu sec) = 3,5\ (ms)$$

HANDSHAKING

$\# segment = \# datagram = \# frame = 6$

$Overhead = \# frame \, (TCPovhd + IPovhd + Eth\,ovhd) = 6\,(20+20+18) = 348 \text{ (byte)}$

$S^{hs}_{eth} = \dfrac{8\,(\text{Overhead})}{B_{eth}} = \dfrac{8 \cdot 348}{1 \cdot 10^9} \text{ (sec)} = 2784 \cdot 10^{-9} \text{ (sec)} = 0{,}0028 \text{ (ms)}$

$S^{hs}_{isp} = \dfrac{8 \cdot 348}{1{,}544 \cdot 10^6} \text{ (sec)} = 1'803 \cdot 10^{-6} \text{ (sec)} = 1{,}803 \text{ (ms)}$

$S^{hs}_{rout} = \# datagram \cdot L = 6 \cdot 50 = 300 \,(\mu s) = 0{,}3 \text{ (ms)}$

Thus:

- $D_{eth} = (S^{req}_{eth} + S^{rep}_{eth} + S^{hs}_{eth}) \cdot 1 = 0{,}85 \text{ (ms)}$

  $D_{isp} = (S^{req}_{isp} + S^{rep}_{isp} + S^{hs}_{isp}) \cdot 1 = 548{,}21 \text{ (ms)}$

  $D_{rout} = (S^{req}_{rout} + S^{rep}_{rout} + S^{hs}_{rout}) \cdot 1 = 3{,}85 \text{ (ms)}$

- $U_{eth} = X_0 \cdot D_{eth} = 10 \left(\frac{req}{sec}\right) \cdot 0{,}00085 \text{ (sec)} = 0{,}0085 \; (0{,}85\,\%)$

  $U_{isp} = X_0 \cdot D_{isp} = 10 \cdot 0{,}54821 = 5{,}4821$

  $U_{rout} = X_0 \cdot D_{rout} = 10 \cdot 0{,}0003 = 0{,}003 \; (0{,}3\,\%)$

- $R'_{eth} = \dfrac{D_{eth}}{1 - U_{eth}} = \dfrac{0{,}85}{0{,}99} = 0{,}86 \text{ (ms)}$

  $R'_{isp} = \dfrac{D_{isp}}{1 - U_{isp}} =$

  $R'_{rout} = \dfrac{D_{rout}}{1 - U_{rout}} = \dfrac{3{,}85}{0{,}99} = 3{,}89 \text{ (ms)}$

Finally:

$\bar{R} = R'_{CPU-N} + R'_{CPU-FS} + R'_{Disk} + R'_{eth} + R'_{isp} + R'_{rout}$

2) The bottleneck of the system is given by the component that has the maximum residence time compared to everyone else.