Foundations of Artificial Intelligence
Exercise Sheet 6

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Exercise 6.1

$D = \{0, 1, 2, 3\}$
$even^{\mathcal{I}} = \{0, 2\}$
$odd^{\mathcal{I}} = \{1, 3\}$
$lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
$two^{\mathcal{I}} = 2 plus^{\mathcal{I}} : D\ x\ D \rightarrow D,\ plus^{\mathcal{I}}(a, b) = (a + b)\ mod 4$

tip: write down that you evaluate Θ under I, α
(for instance like I did in a))

a)

$$\begin{aligned}\Theta_1^{I,\alpha} &= (odd(y) \wedge even(two))^{I,\alpha} \\ &= odd^{I}(1) \wedge even^{I}(2) \\ &= T \wedge T \\ &= T\end{aligned}$$

✓

b)

$$\begin{aligned}\Theta_2 &= \forall x (even(x) \vee odd(x)) \\ &= \forall x T \\ &= T\end{aligned}$$

at least a little bit of formal reasoning why this is the case

c)

$$\begin{aligned}\Theta_3 &= \forall x \exists y\ lessThan(x, y) \\ &= \exists y\ lessThan(0, y) \wedge \exists y\ lessThan(1, y) \wedge \exists y\ lessThan(2, y) \wedge \exists y\ lessThan(3, y) \\ &= T \wedge T \wedge T \wedge F \\ &= F\end{aligned}$$

✓

for formulae $\exists x P(\ldots)$ it would be better to give an actual example, but I think it should be fine anyways. :)

d)

$$\begin{aligned}
\Theta_4 &= \forall x(even(x) \Rightarrow \exists y\ lessThan(x, y)) \\
&= (even(0) \Rightarrow \exists y\ lessThan(0, y)) \wedge (even(1) \Rightarrow \exists y\ lessThan(1, y)) \\
&\wedge (even(2) \Rightarrow \exists y\ lessThan(2, y)) \wedge (even(3) \Rightarrow \exists y\ lessThan(3, y)) \\
&= (T \Rightarrow T) \wedge (F \Rightarrow T) \wedge (T \Rightarrow T) \wedge (F \Rightarrow F) \\
&= T \wedge T \wedge T \wedge T \\
&= T
\end{aligned}$$

✓

e)

$$\begin{aligned}
\Theta_5 &= \forall x(odd(x) \Rightarrow even(plus(x, y)) \\
&= (odd(0) \Rightarrow even(plus(0, 1) \wedge (odd(1) \Rightarrow even(plus(1, 1))\wedge \\
&(odd(2) \Rightarrow even(plus(2, 1)) \wedge (odd(3) \Rightarrow even(plus(3, 1)) \\
&= (F \Rightarrow F) \wedge (T \Rightarrow T) \wedge (F \Rightarrow F) \wedge (T \Rightarrow T) \\
&= T \wedge T \wedge T \wedge T \\
&= T
\end{aligned}$$

✓

Exercise 6.2

a)

$$\begin{aligned}
\Theta &= \forall x\ (P(x, w) \Rightarrow Q(x)) \\
&= (P(a, b) \Rightarrow Q(a)) \wedge (P(b, b) \Rightarrow Q(b)) \wedge (P(c, b) \Rightarrow Q(c)) \\
&= (T \Rightarrow T) \wedge (T \Rightarrow T) \wedge (F \Rightarrow F) \\
&= T \wedge T \wedge T
\end{aligned}$$

✓

b)

$$\begin{aligned}
\Theta_2 &= \exists x(R(v, x) \Rightarrow P(x, x)) \\
&\overset{\exists x}{=} [R(a, x) \Rightarrow (P(x, x))] \\
&= [R(a, a) \Rightarrow (P(a, a))] \\
&= [T \Rightarrow T] \\
&= T
\end{aligned}$$

for this step you could either write down the replacement formally (I,α[x/a]) or simply write "let x be a".

c)

$$\begin{aligned}
\Theta_3 &= \forall x \forall y\ (R(x,y) \iff Q(y)) \\
&= (R(a,a) \iff Q(a)) \wedge (R(a,b) \iff Q(b)) \wedge (R(a,c) \iff Q(c)) \wedge \\
&\quad (R(b,a) \iff Q(a)) \wedge (R(b,b) \iff Q(b)) \wedge (R(b,c) \iff Q(c)) \wedge \\
&\quad (R(c,a) \iff Q(a)) \wedge (R(c,b) \iff Q(b)) \wedge (R(c,c) \iff Q(c)) \wedge \\
&= (T \iff T) \wedge (T \iff T) \wedge (T \iff F) \wedge \\
&\quad (F \iff T) \wedge (F \iff T) \wedge (T \iff F) \wedge \\
&\quad (F \iff T) \wedge (T \iff T) \wedge (F \iff F) \\
&= T \wedge T \wedge F \wedge T \wedge T \wedge F \wedge F \wedge T \wedge T \\
&= F
\end{aligned}$$

✓

this is wrong: $\neg\forall x \forall y \varphi \equiv \exists x \neg \forall y \varphi \equiv \exists x \exists y \neg \varphi$

d)

this is correct

$$\begin{aligned}
\Theta_4 &= [\neg\forall x \forall y (Q(y) \vee P(x,y))] \wedge [\exists z (Q(z) \vee P(w,z))] \\
&= [\forall y \exists x \neg (Q(y) \vee P(x,y))] \wedge [Q(a) \vee P(w,a)] \\
&= [\forall y \exists x (\neg Q(y) \wedge \neg P(x,y))] \wedge T \\
&= (\exists x(\neg Q(a) \wedge \neg P(x,a))) \wedge (\exists x(\neg Q(b) \wedge \neg P(x,b))) \wedge (\exists x(\neg Q(c) \wedge \neg P(x,c))) \\
&= F \wedge F \wedge (\exists x(\neg Q(c) \wedge \neg P(x,c))) \\
&= F
\end{aligned}$$