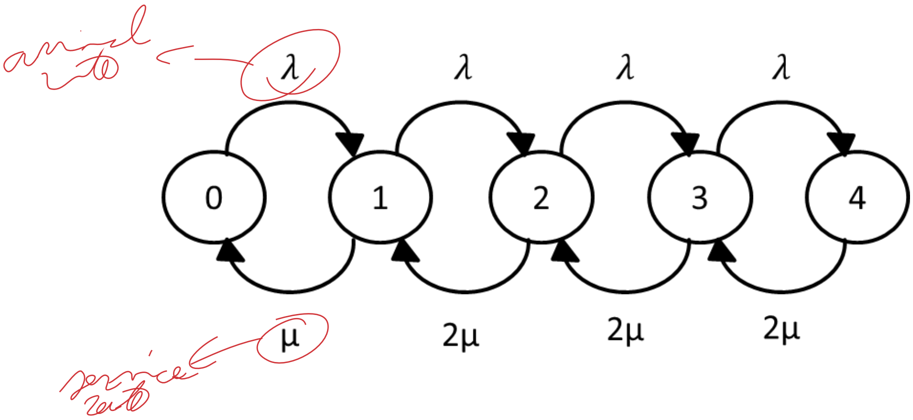
Obtain the markovian process of a queue M/M/2/4 (2 servers, 4 users in the system). Then calculate the probability that a user request is rejected, the throughput and the utilization factor, assuming that the arrival rate is $\lambda = 0.5$ req/sec and the service rate is $\mu = 0.66$ req/sec.



Flow-in = Flow-out :

$$\begin{cases} p_0\lambda = p_1 \cdot \mu \\ p_1\lambda = p_2 \cdot 2\mu \\ p_2\lambda = p_3 \cdot 2\mu \\ p_3\lambda = p_4 \cdot 2\mu \end{cases} \Rightarrow \begin{cases} p_1 = p_0 \left(\frac{\lambda}{\mu}\right) \\ p_2 = p_0 \left(\frac{\lambda}{\mu}\right)^2 \cdot \frac{1}{2} \\ p_3 = p_0 \left(\frac{\lambda}{\mu}\right)^3 \cdot \frac{1}{2} \cdot \frac{1}{2} \\ p_4 = p_0 \left(\frac{\lambda}{\mu}\right)^4 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \end{cases}$$

$$\begin{cases} p_1 = p_0 \left(\frac{0.5}{0.66}\right) \\ p_2 = p_0 \left(\frac{0.5}{0.66}\right)^2 \cdot \frac{1}{2} \\ p_3 = p_0 \left(\frac{0.5}{0.66}\right)^3 \cdot \frac{1}{4} \\ p_4 = p_0 \left(\frac{0.5}{0.66}\right)^4 \cdot \frac{1}{8} \end{cases} \Rightarrow \begin{cases} p_1 = p_0 \cdot 0.757 \\ p_2 = p_0 \cdot 0.2869 \\ p_3 = p_0 \cdot 0.108 \\ p_4 = p_0 \cdot 0.0410 \end{cases}$$

Using the constraint $\sum_{k=0}^{4} p_k = 1$, we can solve the above system of equations.

$p_0 = 0.454$

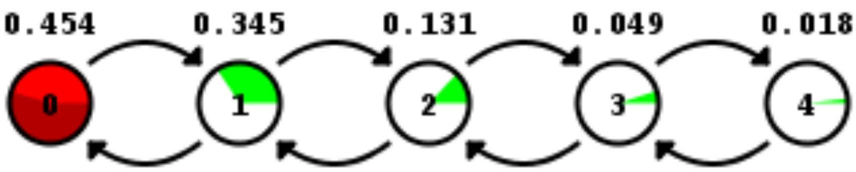
$p_1 = p_0 \cdot 0.7575 = 0.344$

$p_2 = p_0 \cdot 0.2869 = 0.131$

$p_3 = p_0 \cdot 0.108 = 0.049$

$p_4 = p_0 \cdot 0.0410 = 0.018$

Probability that a user request is rejected



$\boldsymbol{X} = \boldsymbol{p_1} \cdot \mu + \sum_{i=2}^{4} \boldsymbol{p_i} \cdot \boldsymbol{2}\mu = 0{,}23 + 0{,}172 + 0{,}064 + 0{,}023 = 0{,}49\ req/sec$