Sapienza University of Rome

Master in Artificial Intelligence and Robotics
Master in Engineering in Computer Science

Machine Learning

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Sapienza University of Rome, Italy - Machine Learning (2020/2021)

5. Bayesian Learning

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Outline

- Bayes Theorem
- MAP, ML hypotheses
- MAP learners
- Bayes optimal classifier
- Naive Bayes learner
- Example: Learning over text data

References
T. Mitchell. Machine Learning. Chapter 6

Two Roles for Bayesian Methods

Provides practical learning algorithms:

- Naive Bayes learning (examples affect prob. that a hypothesis is correct) You can update the knowledge

features:
- Combine prior knowledge (prior probabilities) with observed data
- Make probabilistic predictions (new instances classified by weighted combination of multiple hypotheses) Explicitly represent probability of most probable class
- Requires prior probabilities (often estimated from available data)

e.g.: Given that day, could you play or not?

Provides useful conceptual framework

- Provides "gold standard" for evaluating other learning algorithms

Use to compare algorithms to evaluate

Basic Formulas for Probabilities

- *Product Rule*: probability of conjunction of A and B:

$$P(A \wedge B) = P(A|B)P(B) = P(B|A)P(A)$$

- *Sum Rule*: probability of disjunction of A and B:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

- *Theorem of total probability*: if events $A_1, \ldots, A_n$ are mutually exclusive with $\sum_{i=1}^{n} P(A_i) = 1$, then

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

- *Bayes theorem*:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \rightarrow \frac{P(A \wedge B)}{P(B)} = \frac{P(A|B) \cdot P(B)}{P(B)}$$

See Classification as Probabilistic estimation

to learn

based on which we will learn.

Given target function $f : X \to V$, dataset D and a new instance x', best prediction $\hat{f}(x') = v^*$

$$v^* = \operatorname*{argmax}_{v \in V} P(v|x', D)$$

v that maximizes probability.

Seven at 10:39

More general formulation: given D and x', compute the probability distribution over V

Data

$$P(V|x', D)$$

fixed not variable.

Instance

Learning as Probabilistic estimation

Given dataset D and hypothesis space H, compute a probability distribution over H given D.

$$P(H|D)$$ → distribution.

Bayes rule

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

Data: some days play other not
Prob that h:don't play tennis never generated the data we have? 0 the hypothesis is never, the data in sometimes yes and sometimes not

So if you don't have data you can estimate it

- $P(h)$ = prior probability of hypothesis h
- $P(D)$ = prior probability of training data D
- $P(h|D)$ = probability of h given D
- $P(D|h)$ = probability of D given h

See from 10:49

MAP Hypotheses

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

Generally we want the most probable hypothesis h given D

MAP

Maximum a posteriori hypothesis h_{MAP}:

$$\begin{aligned} h_{MAP} &\equiv \arg\max_{h \in H} P(h|D) = \arg\max_{h \in H} \frac{P(D|h)P(h)}{P(D)} \\ &= \arg\max_{h \in H} P(D|h)P(h) \end{aligned}$$

→ h that maximizes this,

max also this.

hypothesis that maximizes probability.

ML Hypotheses

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

If assume $P(h_i) = P(h_j)$, we can further simplify, and choose the *Maximum likelihood* (ML) hypothesis

the generating data you are observing.

$$h_{ML} = \arg\max_{h \in H} P(D|h)$$

Brute Force MAP Hypothesis Learner

1. For each hypothesis h in H, calculate the posterior probability

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

2. Output the hypothesis h_{MAP} with the highest posterior probability

$$h_{MAP} = \operatorname{argmax}_{h \in H} P(h|D)$$

Most Probable Classification of New Instances

h_{MAP}: most probable *hypothesis* given data D.

Given a new instance x', what is its most probable *classification* of x'?

$h_{MAP}(x')$ may not be the most probable classification !!!

We take the class returned by hmap

Most Probable Classification of New Instances

Consider:

- Three possible hypotheses h_1, h_2, h_3:
  $$P(h_1|D) = 0.4, \; P(h_2|D) = 0.3, \; P(h_3|D) = 0.3$$
- Given a new instance x,
  $$h_1(x) = \oplus, \; h_2(x) = \ominus, \; h_3(x) = \ominus$$

H1 classifies the example x and positive

- What is the most probable classification of x ?

Bayes Optimal Classifier

Class returned by map is not in general the most probable class.

Consider target function $f : X \mapsto V$, $V = \{v_1, ..., v_k\}$, data set D and a new instance $x \notin D$:

$$P(v_j|x, D) = \sum_{h_i \in H} P(v_j|x, h_i)P(h_i|D)$$

Given new example and data, the new example is classified as vj

Seen at 11:02 (learns one + and -)

total probability over H

$P(v_j|x, h_i)$: probability that $h_i(x) = v_j$ is independent from D given h_i

$\Rightarrow P(v_j|x, h_i) = P(v_j|x, h_i, D)$ for each h_i

h_i does not depend on $x \notin D \Rightarrow P(h_i|x, D) = P(h_i|D)$

We are independent from the dataset, because, hi are given, so they are independent from other hypothesis based on dataset.

Bayes Optimal Classifier

Bayes Optimal Classifier

Class of a new instance x:

Computes most prob class, Vob,

$$v_{OB} = \arg\max_{v_j \in V} \sum_{h_i \in H} P(v_j|x, h_i)P(h_i|D)$$

Bayes Optimal Classifier

Example:

$$\begin{aligned}
&P(h_1|D) = 0.4, \quad P(\ominus|x, h_1) = 0, \quad P(\oplus|x, h_1) = 1 \\
&P(h_2|D) = 0.3, \quad P(\ominus|x, h_2) = 1, \quad P(\oplus|x, h_2) = 0 \\
&P(h_3|D) = 0.3, \quad P(\ominus|x, h_3) = 1, \quad P(\oplus|x, h_3) = 0
\end{aligned}$$

therefore

$$\begin{aligned}
\sum_{h_i \in H} P(\oplus|x, h_i)P(h_i|D) &= 0.4 \\
\sum_{h_i \in H} P(\ominus|x, h_i)P(h_i|D) &= 0.6
\end{aligned}$$

and

$$v_{OB} = \arg\max_{v_j \in V} \sum_{h_i \in H} P(v_j|x, h_i)P(h_i|D) = \ominus$$

Bayes Optimal Classifier

Optimal learner: no other classification method using the same hypothesis space and same prior knowledge can outperform this method on average.

It maximizes the probability that the new instance x is classified correctly, i.e., $\mathrm{argmax}_{v_j \in V} P(v_j|x, D)$.

Very powerful: labelling new instances x with $\mathrm{argmax}_{v_j \in V} P(v_j|x, D)$ can correspond to none of the hypotheses in H.

X the labeling it returns may not be due to any of the hypothesis taken in isolation

e.g.: [illegible]

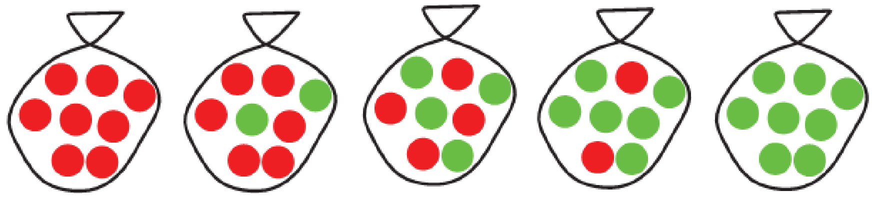
Bayesian Learning Example

Five kinds of bags of candiers: hypothesis

Screen at 11:33

1. 10% are h_1: 100% cherry
2. 20% are h_2: 75% cherry, 25% lime
3. 40% are h_3: 50% cherry, 50% lime
4. 20% are h_4: 25% cherry, 75% lime
5. 10% are h_5: 100% lime

5 classes of bags.



Bayesian Learning Example

We choose a random bag (not knowing which type it is) and extract some candies from it.

What kind of bag is it? What is the probability of extracting a candy of a specific flavor next?

Bayesian Learning Example

Prior probability distribution:

$$\mathrm{P}(H) = < 0.1, 0.2, 0.4, 0.2, 0.1 >$$

Likelihood for lime candy:

$$\mathrm{P}(l|H) = < 0, 0.25, 0.5, 0.75, 1 >$$

Probability of extracting a lime candy (without data set):

$$\sum_{h_i} P(l|h_i)P(h_i) = 0 \cdot 0.1 + 0.25 \cdot 0.2 + 0.5 \cdot 0.4 + 0.75 \cdot 0.2 + 1 \cdot 0.1 = 0.5$$

Bayesian Learning Example

1. First candy is lime: $D_1 = \{l\}$

$P(h_i|\{d_1\}) = \alpha P(\{d_1\}|h_i)P(h_i)$ (Bayes rule)

$$\begin{aligned}\mathrm{P}(H|D_1) &= \alpha < 0, 0.25, 0.5, 0.75, 1 > \cdot < 0.1, 0.2, 0.4, 0.2, 0.1 > \\ &= \alpha < 0, 0.05, 0.2, 0.15, 0.1 > \\ &= < 0, 0.1, 0.4, 0.3, 0.2 >\end{aligned}$$

The probability of hypothesis with more lime increas

Bayesian Learning Example

2. Second candy is lime: $D_2 = \{l, l\}$ Put the first candy in back and extract again

$P(h_i|\{d_1, d_2\}) = \alpha P(\{d_1, d_2\}|h_i)P(h_i)$ (Bayes rule)
$= \alpha P(\{d_2\}|h_i)\, P(\{d_1\}|h_i)P(h_i)$ (independent data samples)

$$\begin{aligned}\mathrm{P}(H|D_2) &= \alpha < 0, 0.25, 0.5, 0.75, 1 > \cdot < 0, 0.1, 0.4, 0.3, 0.2 > \\ &= \alpha < 0, 0.025, 0.2, 0.225, 0.2 > \\ &= < 0, 0.038, 0.308, 0.346, 0.308 >\end{aligned}$$

Bayesian Learning Example

3. Third candy is lime: $D_3 = \{l, l, l\}$

$P(h_i|\{d_1, d_2, d_3\}) = \alpha P(\{d_1, d_2, d_3\}|h_i)P(h_i)$ (Bayes rule)
$= \alpha P(\{d_3\}|h_i)\, P(\{d_2\}|h_i)\, P(\{d_1\}|h_i)P(h_i)$ (independent data samples)

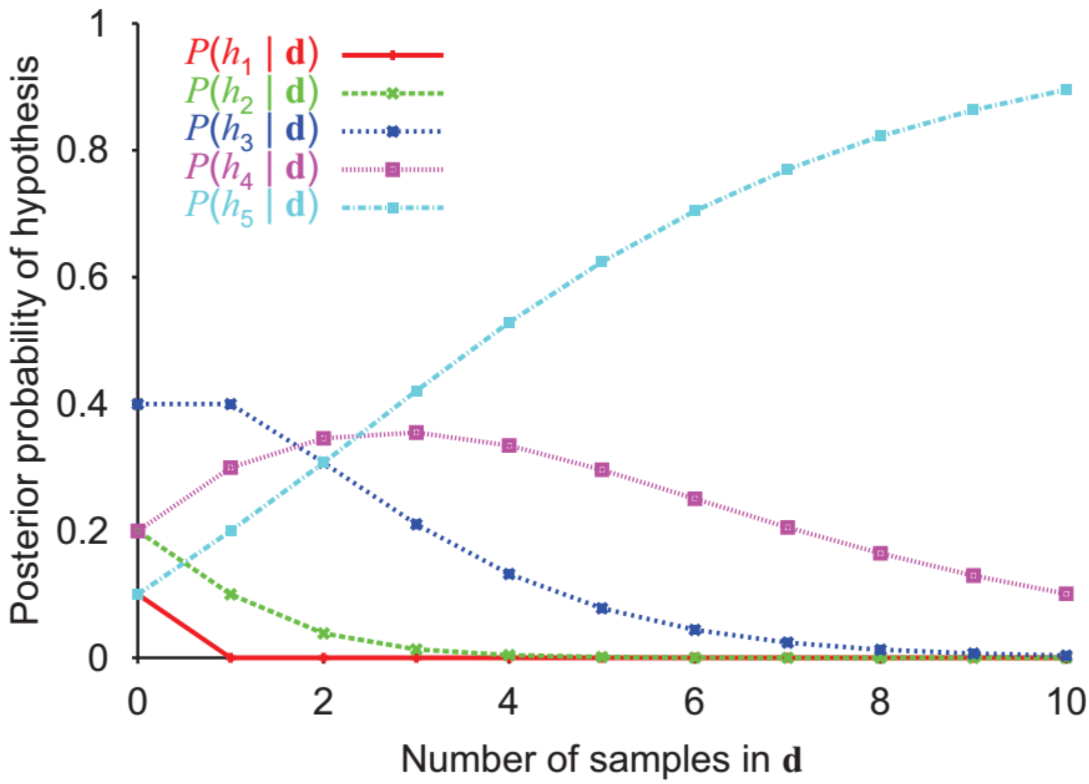
$$\begin{aligned}\mathrm{P}(H|D_3) &= \alpha < 0, 0.25, 0.5, 0.75, 1 > \cdot < 0, 0.038, 0.308, 0.346, 0.308 > \\ &= \alpha < 0, 0.01, 0.154, 0.260, 0.308 > \\ &= < 0, 0.013, 0.211, 0.355, 0.421 >\end{aligned}$$

Bayesian Learning Example

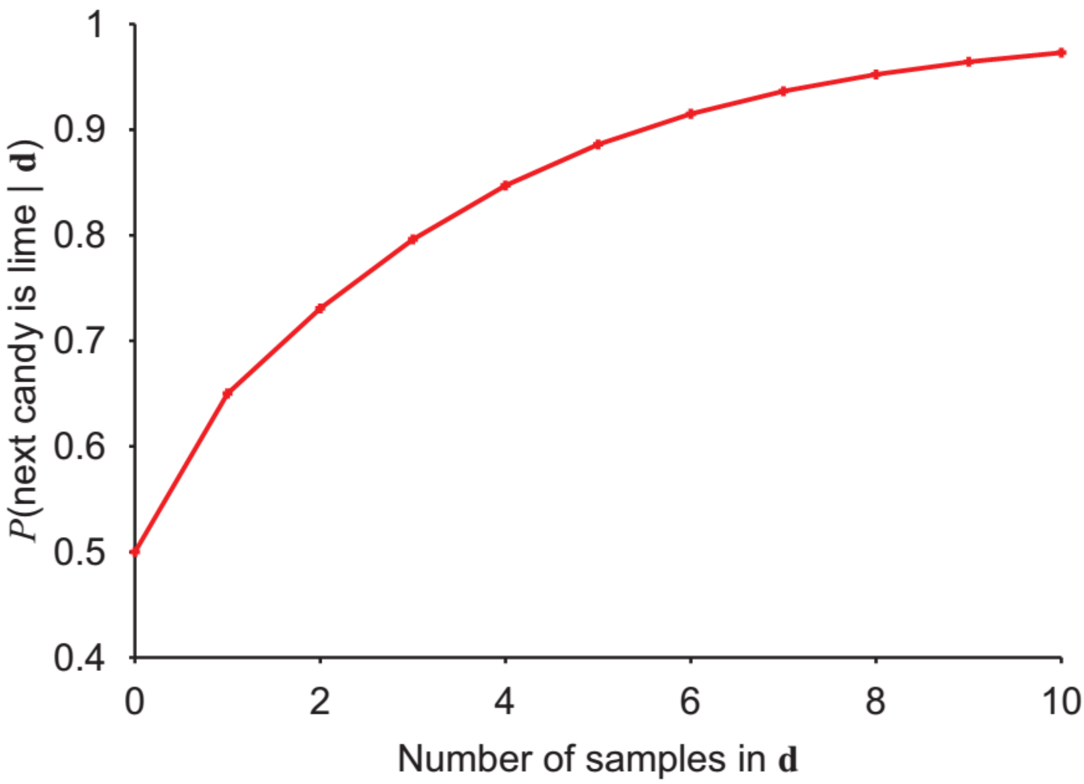
What is probability of having another lime candy after $D_3 = \{l, l, l\}$?

$$
\begin{aligned}
P(l|D_3) &= \sum_{h_i} P(l|h_i)P(hi|D_3) \\
&= 0 \cdot 0 + 0.25 \cdot 0.013 + 0.5 \cdot 0.211 + 0.75 \cdot 0.355 + 1 \cdot 0.421 \\
&= 0.8
\end{aligned}
$$

Bayesian Learning Example



Bayesian Learning Example



Bayesian Learning Example 2

Consider a new manufacturer producing bags with an arbitrary choice of cherry/lime candies. $\theta \equiv \frac{nr. of\ cherry\ candies}{N} \in [0,1]$.
Continuous space for hypotheses: h_θ

Data set: $D = \{c \text{ cherries}, l \text{ lime}\}$, $N = c + l$

$P(c|h_\theta) = \theta$
$P(l|h_\theta) = 1 - \theta$

- What is the ML hypothesis?

Bayesian Learning Example 2

is the max prob.

$= c/N$

$$h_{ML} = \operatorname*{argmax}_{h_\theta} P(D|h_\theta) = \operatorname*{argmax}_{h_\theta} L(D|h_\theta)$$

maximizes

log likelihood.

with $L(D|h_\theta) = \log P(D|h_\theta)$

$$\boxed{P(D|h_\theta)} = \prod_{j=1\dots N} P(d_j|h_\theta) = \theta^c \cdot (1-\theta)^l$$

Screen at: 12:12
and: 12:14

$$L(D|h_\theta) = c \log \theta + l \log(1-\theta)$$

$$\frac{dL(D|h_\theta)}{d\theta} = \frac{c}{\theta} - \frac{l}{1-\theta} = 0 \Rightarrow \theta_{ML} = \frac{c}{c+l} = \frac{c}{N}$$

Theta ml: is the proportion of that best explains the data you are observing, ratio cherries over total in your bag and it's the number that maximizes the probability of seeing the data you are observing given the hypothesis Htheta
Theta ml is the most probable class you can obtain

General approach

Given dataset $D = \{d_i\}$ with $d_i \in \{0, 1\}$,
assuming a probability distribution $P(d_i; \Theta)$

Vector of parameters.

Maximum likelihood estimation

$$\Theta_{ML} = \operatorname*{argmax}_{\Theta} \log P(d_i|\Theta)$$

Example: for Bernoulli distribution $P(X = k; \theta) = \theta^k (1-\theta)^{1-k}$

$$\theta_{ML} = \dots = \frac{|\{d_i = 1\}|}{|D|}$$

Bernoulli distribution

Probability distribution of a binary random variable $X \in \{0, 1\}$

$$P(X = 1) = \theta \quad P(X = 0) = 1 - \theta$$

(e.g., observing head after flipping a coin, extracting a lime candy, ...).

$$P(X = k; \theta) = \theta^k (1 - \theta)^{1-k}$$

Multi-variate Bernoulli distribution

Joint probability distribution of a set of binary random variables $X_1, \ldots X_n$, each random variable following Bernoulli distribution

indipendent.

$$P(X_1 = k_1, \ldots, X_n = k_n \,;\, \theta_1, \ldots, \theta_n)$$

$k_i \in \{0, 1\}$

(e.g., observing head after flipping a coin **and** extracting a lime candy, ...).

Under the assumption that random variables X_i are mutually independent, Multi-variate Bernoulli distribution is the product of n Bernoulli distributions

$$P(X_1 = k_1, \ldots \,;\, \theta_1, \ldots, \theta_n) = \prod_{i=1}^{n} P(X_i = k_i; \theta_i) = \prod_{i=1}^{n} \theta_i^{k_i} (1 - \theta_i)^{1-k_i}$$

Binomial distribution

Probability distribution of k outcomes from n Bernoulli trials

Combinations (give formula).

$$P(X = k; n, \theta) = \binom{n}{k} \theta^k (1-\theta)^{n-k}$$

(e.g., flipping a coin n times and observing k heads, extracting k lime candies after n extractions, ...).

Multinomial distribution

for > 2

Generalization of binomial distribution for discrete valued random variables with d possible outcomes.

Probability distribution of k_1 outcomes for X_1, ..., k_d outcomes for X_d, after n trials (with $\sum_{i=1 \dots d} k_i = n$)

parameters

$$P(X_1 = k_1, \dots, X_d = k_d; n, \theta_1, \dots, \theta_d) = \frac{n!}{k_1! \dots k_n!} \theta_1^{k_1} \cdot \dots \cdot \theta_d^{k_d}$$

(e.g., rolling a d-sided dice n times and observing k times a particular value, extracting k lime candies after n extractions form a bag containing d different flavors, ...).

>2

Naive Bayes Classifier

Works under the assumptions that the features attributes are independent

Bayes optimal classifier provides best result, but it is not a practical method when hypothesis space is large.

Naive Bayes Classifier uses conditional independence to approximate the solution.

X is *conditionally independent* of Y given Z

$$P(X, Y|Z) = P(X|Y, Z)P(Y|Z) = P(X|Z)P(Y|Z)$$

Naive Bayes Classifier

Assume target function $f : X \rightarrow V$, where each instance x is described by attributes $\langle a_1, a_2 \ldots a_n \rangle$.

Compute

$$\underset{v_j \in V}{\operatorname{argmax}} P(v_j|x, D) = \underset{v_j \in V}{\operatorname{argmax}} P(v_j|a_1, a_2 \ldots a_n, D)$$

without explicit representation of hypotheses.

Naive Bayes Classifier

Given a data set D and a new instance $x = \langle a_1, a_2 \ldots a_n \rangle$, most probable value of $f(x)$ is:

$$
\begin{aligned}
v_{MAP} &= \underset{v_j \in V}{\operatorname{argmax}} P(v_j | a_1, a_2 \ldots a_n, D) \\
&= \underset{v_j \in V}{\operatorname{argmax}} \frac{P(a_1, a_2 \ldots a_n | v_j, D) P(v_j | D)}{P(a_1, a_2 \ldots a_n | D)} \\
&= \underset{v_j \in V}{\operatorname{argmax}} P(a_1, a_2 \ldots a_n | v_j, D) P(v_j | D)
\end{aligned}
$$

Bayes rule.

we can eliminate because we know is positive

(Bayes rule)

Naive Bayes Classifier

Naive Bayes assumption: - independents given the class of the data.

Conditional.

$$P(\underbrace{a_1, a_2, \ldots, a_n}| v_j, D) = \prod_i P(a_i | v_j, D)$$

Naive Bayes classifier

Class of new instance x:

$$v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j | D) \prod_i P(a_i | v_j, D)$$

Naive Bayes Algorithm

Target function $f : X \mapsto V$, $X = A_1 \times \ldots \times A_n$, $V = \{v_1, ..., v_k\}$, data set D, new instance $x = \langle a_1, a_2 \ldots a_n \rangle$.

Naive_Bayes_Learn(A, V, D)

for each target value $v_j \in V$

$\boxed{\hat{P}(v_j|D)} \leftarrow$ estimate $P(v_j|D)$

How many times this class occurs in your data

for each attribute A_k

for each attribute value $a_i \in A_k$

$\hat{P}(a_i|v_j, D) \leftarrow$ estimate $P(a_i|v_j, D)$

Classify_New_Instance(x)

$$v_{NB} = \operatorname*{argmax}_{v_j \in V} \hat{P}(v_j|D) \prod_{a_i \in x} \hat{P}(a_i|v_j, D)$$

Naive Bayes estimation

Proportion of times you see that class above the total.

$$\hat{P}(v_j|D) = \frac{|\{< \ldots, v_j >\}|}{|D|}$$

$$\hat{P}(a_i|v_j, D) = \frac{|\{< \ldots, a_i, \ldots, v_j >\}|}{|\{< \ldots, v_j >\}|}$$

Note: if none of the training instances with target value v_j have attribute value a_i, then $\hat{P}(a_i|v_j, D) = 0$ and thus $\hat{P}(v_j|D) \prod_i \hat{P}(a_i|v_j, D) = 0$

Naive Bayes estimation

Typical solution is Bayesian estimate with prior estimates

$$\hat{P}(a_i|v_j, D) = \frac{|\{< \ldots, a_i, \ldots, v_j >\}| + mp}{|\{< \ldots, v_j >\}| + m}$$

where

- p is a prior estimate for $P(a_i|v_j, D)$
- m is a weight given to prior (i.e. number of "virtual" examples)

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Naive Bayes: Example

Consider *PlayTennis* again, and new instance

$$\langle Outlook = sun, Temp = cool, Humid = high, Wind = strong \rangle$$

We want to compute:

$$v_{NB} = \operatorname*{argmax}_{v_j \in V} P(v_j|D) \prod_i P(a_i|v_j, D)$$

without making any hypothesis space explicit.

Naive Bayes: Example

Note: easy notation with conditioning on D omitted.

$$P(PlayTennis = yes) = P(y) = 9/14 = 0.64$$
$$P(PlayTennis = no) = P(n) = 5/14 = 0.36$$
$$\begin{aligned} P(Wind = strong|y) &= 3/9 = 0.33 \\ P(Wind = strong|n) &= 3/5 = 0.60 \\ &\ldots \end{aligned}$$

The strong wind influence more to say no

$$P(y)\ P(sun|y)\ P(cool|y)\ P(high|y)\ P(strong|y) = .005$$
$$P(n)\ P(sun|n)\ P(cool|n)\ P(high|n)\ P(strong|n) = .021$$

$$\rightarrow v_{NB} = n$$

Naive Bayes Remarks

Conditional independence assumption is often violated

$$P(a_1, a_2 \ldots a_n|v_j, D) = \prod_i P(a_i|v_j, D)$$

...but it works surprisingly well anyway.

Note: don't need estimated posteriors $\hat{P}(v_j|x, D)$ to be correct; need only that

$$\operatorname*{argmax}_{v_j \in V} \hat{P}(v_j|D) \prod_i \hat{P}(a_i|v_j, D) = \operatorname*{argmax}_{v_j \in V} P(v_j|D) P(a_1 \ldots, a_n|v_j, D)$$

Issue: Naive Bayes posteriors often unrealistically close to 1 or 0

Learning to classify text

Input: set of documents (sequences of words)

Learn target function $f : Docs \mapsto \{c1, \ldots, c_k\}$

Examples:

- spam classification (e-mail, SMS, ...)
- sentiment analysis (facebook/twitter posts, web reviews, ...)
- ...

Bag of words representation

Vocabulary $V = \{w_k\}$: set of all the words appearing in any document of the data set.
$n = |V|$: size of the vocabulary

Bag of words representation of a text: n-dimensional feature vector

Note: BoW representation looses information (order of words in a text is important!)

Bag of words representation

Two options for representing each feature:

1. boolean features: 1 if word appears in the text, 0 otherwise (multivariate Bernoulli distribution)
2. ordinal features: number of occurrences of the words in the text (multinomial distribution)

Learning to Classify Text: Naive Bayes approach

Classification of documents $Docs$ in classes C.

Target function $f : Docs \mapsto C$, $C = \{c_1, \ldots, c_k\}$

Data set $D = \{< d_i, c_i >\}$

Given a new document d_i, compute

$$c_{NB} = \operatorname*{argmax}_{c_j \in C} P(c_j|D) \prod_i P(d_i|c_j, D)$$

Learning to Classify Text: Naive Bayes approach

Naive Bayes conditional independence assumption

$$P(d_i|c_j, D) = \prod_{i=1}^{length(d_i)} P(a_i = w_k|c_j, D)$$

where $P(a_i = w_k|c_j)$ is probability that word in position i is w_k, given c_j

one more assumption: $P(a_i = w_k|v_j, D) = P(a_m = w_k|v_j, D), \forall i, m$,
thus consider only $P(w_k|c_j, D)$.

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Multi-variate Bernoulli Naive Bayes distribution

Feature vector for document d: n-dimensional vector 1 if word w_k appears in document d, 0 otherwise

function return 1 if the word occur in the document

$$P(d|c_j, D) = \prod_{i=1}^{n} P(w_i|c_j, D)^{I(w_i \in d)} \cdot (1 - P(w_i|c_j, D))^{1-I(w_i \in d)}$$

$I(w_i \in d) = 1$ if $w_i \in d$, 0 otherwise

$$\hat{P}(w_i|c_j, D) = \frac{t_{i,j} + 1}{t_j + 2}$$

$t_{i,j}$: number of documents in D of class c_j containing word w_i
t_j: number of documents in D of class c_j
1, 2: parameters for Laplace smoothing

Multinomial Naive Bayes distribution

Feature vector for document d: n-dimensional vector with number of occurrences of word w_i in document d

$$P(d|c_j, D) = Mu(d; n, \theta) = \ldots$$

$$\hat{P}(w_i|c_j, D) = \frac{\sum_{d \in D} tf_{i,j} + \alpha}{\sum_{d \in D} tf_j + \alpha \cdot |V|}$$

$tf_{i,j}$: term frequency (number of occurrences) of word w_i in document d of class c_j
tf_j: all term frequencies of document d of class c_j
α: smoothing parameter ($\alpha = 1$ for Laplace smoothing)

Naive Bayes Text Classification algorithm

Estimate $\hat{P}(c_j)$ and $\hat{P}(w_i|c_j)$ using *Bernoulli distribution*.

LEARN_NAIVE_BAYES_TEXT_BE(D, C)

- $V \leftarrow$ all distinct words in D
- for each target value $c_j \in C$ do
  - $docs_j \leftarrow$ subset of D for which the target value is c_j
  - $t_j \leftarrow |docs_j|$: total number of documents in c_j
  - $\hat{P}(c_j) \leftarrow \frac{t_j}{|D|}$
  - for each word w_i in V do
    - $t_{i,j} \leftarrow$ number of documents in c_j containing word w_i
    - $\hat{P}(w_i|c_j) \leftarrow \frac{t_{i,j}+1}{t_j+2}$

Naive Bayes Text Classification algorithm

Estimate $\hat{P}(c_j)$ and $\hat{P}(w_i|c_j)$ using *multinomial distribution*.

$\textsc{Learn\_Naive\_Bayes\_text\_Mu}(D, C)$

$V \leftarrow$ all distinct words in D
for each target value $c_j \in C$ do
  $docs_j \leftarrow$ subset of D for which the target value is c_j
  $t_j \leftarrow |docs_j|$: total number of documents in c_j
  $\hat{P}(c_j) \leftarrow \frac{t_j}{|D|}$
  $TF_j \leftarrow$ total number of words in $docs_j$ (counting duplicates)
  for each word w_i in V do
    $TF_{i,j} \leftarrow$ total number of times word w_i occurs in $docs_j$
    $\hat{P}(w_i|c_j) \leftarrow \frac{TF_{i,j}+1}{TF_j+|V|}$

Sapienza University of Rome, Italy - Machine Learning (2020/2021)

Naive Bayes Text Classification algorithm

Use estimated $\hat{P}(c_j)$ and $\hat{P}(w_i|c_j)$ to classify a new document.

$\textsc{Classify\_Naive\_Bayes\_text}(d)$

remove from d all words not included in vocabulary V
return

$$v_{NB} = \operatorname*{argmax}_{c_j \in C} \hat{P}(c_j) \prod_{i=1}^{length(d)} \hat{P}(w_i|c_j)$$

Text Classification improvements

- Stop words: remove from all the documents common words ("the", "a", etc.)
- Stemming: replace words with basic forms ("likes" $\rightarrow$ "like", "liking" $\rightarrow$ "like", etc.)
- Bi-gram, n-gram: token is a sequence of words
- ...

look for details on a book.