Exercise 2 - ASP

Exercise 2 Given the following ASP program P:

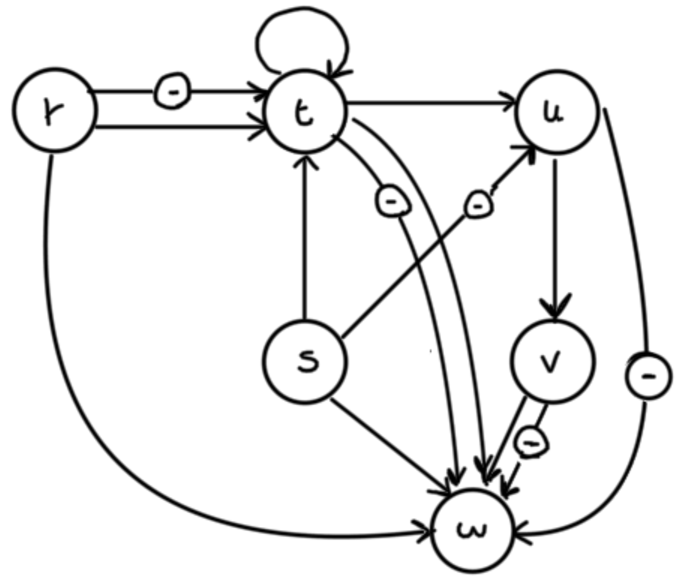
```
r(x,y) :- p(x,y,v).
s(x,y) :- p(v,x,y).
t(x,z) :- r(x,y), s(y,z), not r(x,z).
t(x,z) :- t(x,y), t(y,z), not r(x,z).
u(x,y) :- t(x,y), not s(x,y).
v(x,y) :- u(y,x).
w(x,z) :- r(x,y), s(y,z), not u(x,z).
w(x,y) :- t(x,y), not v(x,y).
w(x,y) :- v(x,y), not t(x,y).
p(a,b,c). p(c,d,e). p(e,f,f).
```

(a) tell whether P is stratified;

(b) compute the answer sets of P.

a)

Let's build the Labeled Dependency Graph of P:



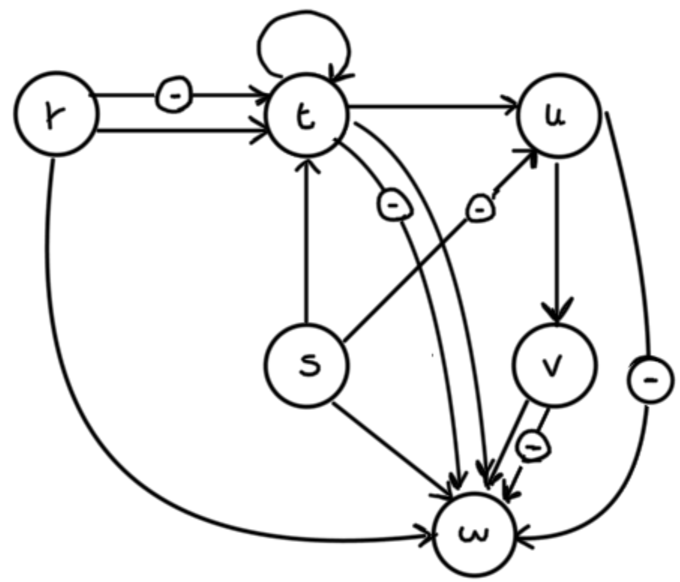
There are no cycle in the graph that contain a negative edge, that means that the program P **IS STRATIFIED**.

b)

Since the program is stratified the answer sets is unique and coincide with the minimal model.
To obtain the minimal model of P we need to

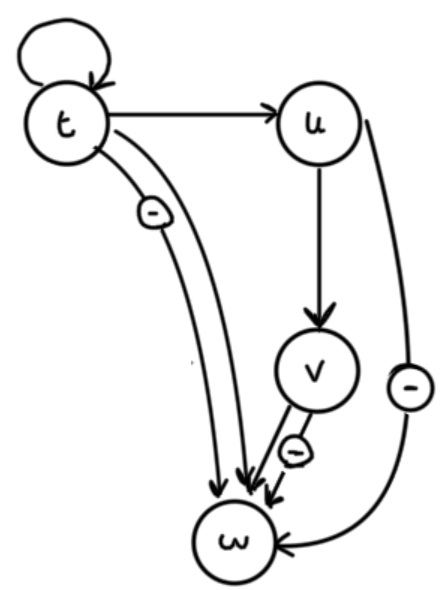
compute the stratas of r :

1.



$S_1 = \{ r, s \}$

2.



$S_2 = \{ t, u, v \}$

3.

w

$S_3 = \{w, \}$

$S_1 = \{r, s\}$, $S_2 = \{t, u, v\}$, $S_3 = \{w\}$

0 - $MM_0 = EDB(P) = \{ p(a,b,c), p(c,d,e), p(e,f,f) \}$

1 - $P(S_1) =$

$r(x,y) :- p(x,y,v)$

$s(x,y) :- p(v,x,y)$

$MM_1 = MM_0 \cup \{ r(a,b), r(c,d), r(e,f), s(b,c), s(d,e), s(f,f) \}$

2 - $P(S_2) =$

$t(x,z) :- r(x,y), s(y,z), \text{not } r(x,z)$

$t(x,z) :- t(x,y), t(y,z), \text{not } r(x,z)$

$u(x,y) :- t(x,y), \text{not } s(x,y)$

$$v(x,y) \text{ :- } u(y,x)$$

$$\Delta P(S_2) =$$

$$\Delta' t(x,z) \text{ :- } \Delta t(x,y),\ t(y,z),\ \text{not } r(x,z)$$

$$\Delta' t(x,z) \text{ :- } t(x,y),\ \Delta t(y,z),\ \text{not } r(x,z)$$

$$\Delta' u(x,y) \text{ :- } \Delta t(x,y),\ \text{not } s(x,y)$$

$$\Delta' v(x,y) \text{ :- } \Delta u(y,x)$$

$$I = MM_1 \cup T_{P_2}(MM_1) = MM_1 \cup \{ t(a,c),\ t(c,c),\ t(f,f) \}$$

$$\Delta I = \{ \Delta t(a,c),\ \Delta t(c,c),\ \Delta t(f,f) \}$$

$$\Delta' I = T_{\Delta P_{S_2}}(I \cup \Delta I) = \{ \Delta' u(a,c),\ \Delta' u(c,c) \}$$

$$I = I \cup \{ u(a,c),\ u(c,c) \}$$

$$\Delta I = \{ \Delta u(a,c),\ \Delta u(c,c) \}$$

$$\Delta' I = T_{\Delta P_{S_2}}(I \cup \Delta I) = \{ \Delta' v(c,a),\ \Delta' v(c,c) \}$$

$$I = I \cup \{ v(c,a),\ v(c,c) \}$$

$$\Delta I = \{ \Delta v(c,a),\ \Delta v(c,c) \}$$

$$\Delta' I = T_{\Delta P_{S_2}}(I \cup \Delta I) = \{\}$$

$$MM_2 = I = \{ p(a,b,c),\ p(c,d,e),\ p(e,f,f),\ r(a,b),$$

$r(c,d)$, $r(e,f)$, $s(b,c)$, $s(d,e)$, $s(f,f)$,
$t(a,c)$, $t(c,c)$, $t(f,f)$, $u(a,c)$, $u(c,c)$,
$v(c,a)$, $v(c,c)$ }

3 - $P(S_3)$:

$w(x,y)$:- $v(x,y)$, not $t(x,y)$

$$MM_3 = MM_2 \cup \{ w(c,a) \}$$

The answer sets of P consist in his minimal model which is :

$MM = \{ p(a,b,c), p(c,d,e), p(e,f,f), r(a,b),$
$r(c,d), r(e,f), s(b,c), s(d,e), s(f,f),$
$t(a,c), t(c,c), t(f,f), u(a,c), u(c,c),$
$v(c,a), v(c,c), w(c,a) \}$