Foundations of Artificial Intelligence

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Summer Term 2021

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Exercise Sheet 10 — Solutions

Exercise 10.1 (Decision Trees)

No	Age	Engine power [kW]	**Risk**
1	< 25	< 100	low
2	< 25	> 200	high
3	≥ 25	> 200	high
4	≥ 25	$100 - 200$	low
5	< 25	$100 - 200$	high
6	≥ 25	< 100	low

Consider the data on car insurance risk in the table above. Produce a decision tree, which correctly classifies the insurance risk for the examples given, using the attributes *Age* and *Engine Power* in order of decreasing *information gain*. Give detailed calculations that justify the order in which the attributes are tested.
You can make use of the following values:
$\log_2(\frac{1}{3}) \approx -\frac{3}{2}$, $\log_2(\frac{2}{3}) \approx -\frac{1}{2}$, $\log_2(\frac{1}{2}) = -1$, $\log_2(1) = 0$.

Solution:

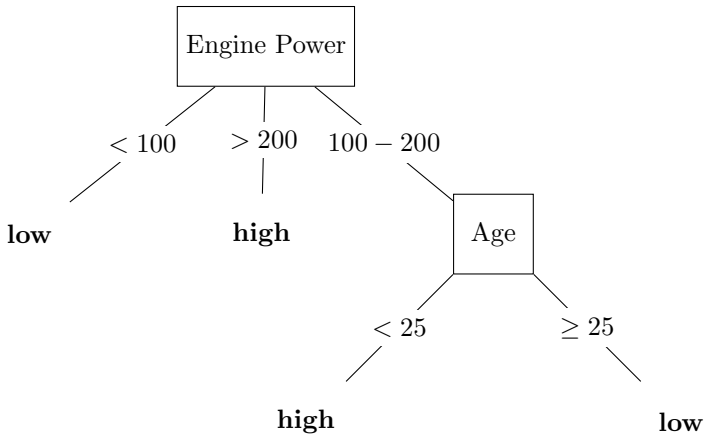
Entropy of the root node: $I(Risk) = I(\frac{1}{2}, \frac{1}{2}) = 1$
Remaining uncertainties after splitting on the different attributes:

$$R(EnginePower) = \frac{1}{3} \cdot I(1,0) + \frac{1}{3} \cdot I(0,1) + \frac{1}{3} \cdot I(\frac{1}{2}, \frac{1}{2}) = \frac{1}{3}$$
$$R(Age) = \frac{1}{2} \cdot I(\frac{1}{3}, \frac{2}{3}) + \frac{1}{2} \cdot I(\frac{2}{3}, \frac{1}{3}) = I(\frac{1}{3}, \frac{2}{3}) = -\frac{1}{3}\log_2(\frac{1}{3}) - \frac{2}{3}\log_2(\frac{2}{3}) = \frac{5}{6}$$

Gains are then:

$$Gain(EnginePower) = 1 - \frac{1}{3} = \frac{2}{3}$$
$$Gain(Age) = 1 - \frac{5}{6} = \frac{1}{6}$$

So the first split should be on attribute *Engine Power*. After that, splitting on *Age* will result in a clean split with no entropy left.



Exercise 10.2 (Best practices in ML)

When doing machine learning, it is good practice to split the dataset into a training/validation/test set.

- Which subset(s) should you use for the following tasks:
  (a) fitting models (R & D)[1]
  (b) guard against overfitting (R & D)
  (c) model selection (R & D)
  (d) progress reports (R & D)
  (e) evaluating the final model (product/publication)
- Which of these subsets should always be fixed a priori (before even looking at the data)?

Solution:

- (a) training set
  (b) validation set
  (c) validation set
  (d) validation set
  (e) test set

  Explanation: During research and development, the training set is used for fitting models, and the validation set is used to evaluate the fitted model (e.g., to detect overfitting, do model selection, in progress reports, etc.). The test set is only used to evaluate the final model (e.g., when releasing a product or publishing results). Side-note: When fitting this final model, one could technically use both training and validation data. However, this only makes sense when we have little training data (learning curves are still strongly increasing) since this final model can no longer be validated before testing.

[1] R & D: During research and development

- The test set should always be fixed a priori (and used only once, to evaluate the final model). Instances in validation/training sets may vary during R & D. However, having a sparsely-used, fixed validation subset that acts as a 'pseudo' test-set can be useful (e.g., for internal progress reports). Also, if examples in the validation set are frequently used (e.g during training, hyper-parameter tuning, etc.) failing to detect overfitting is a real risk.