

Esercizio u.3

Web service 50 requests per second \rightarrow

Workload is equally distributed across

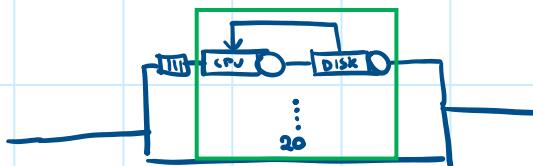
Max concurrent requests is $\boxed{4}$

10 msec

Requests require 8 CPU msec and 5 disks read

Average request service time? Throughput? Percentage of rejected requests?

Performance improvement with cache (access time of 100 μsec) but 70%?



$$S_{CPU} = 8 \text{ msec}$$

$$D_{Disk} = 50 \text{ msec} = \frac{s}{V_i} \cdot S_{Disk}$$

Caso 1: ($u=1$)

(MVA)

$$R_{CPU_1} = D_{CPU} = 8 \text{ msec}$$

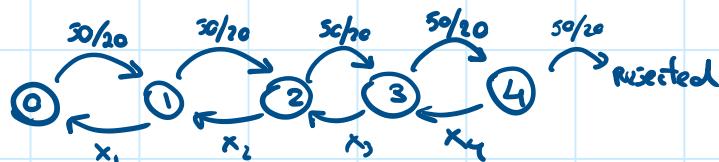
$$R_{Disk_1} = D_{Disk} = 50 \text{ msec}$$

$$R_{TOT_1} = 58 \text{ msec}$$

$$x(1) = \frac{1}{R_{TOT_1}}$$

$$N_{CPU_1} = R_{CPU_1} \cdot X_0(1) = 0,14$$

$$N_{Disk_1} = R_{Disk_1} \cdot X_0(1) = 0,86$$



Caso 2 ($u=2$):

$$R_{CPU_2} = D_{CPU_2} = 16 \text{ msec}$$

$$R_{Disk_2} = 73$$

$$R_{TOT_2} = 102$$

$$x(2) = \frac{2}{R_{TOT_2}}$$

$$\parallel D_{CPU}(1 + N_{CPU_{i-1}})$$

$$\parallel 50 (1 + N_{CPU_{i-1}})$$

$$N_{CPU_2} = R_{CPU_2} \cdot X(2) = 0,18$$

$$N_{Disk_2} = R_{Disk_2} \cdot X(2) = 1,82$$

Caso u.3

$$R_{CPU_3} = 9,44$$

$$R_{Disk_3} = 141$$

$$R_{TOT_3} = 150,44$$

$$X(3) = 19,94$$

$$N_{CPU_3} = 0,18$$

$$N_{Disk_3} = 2,81$$

$$\begin{cases} P_0 \cdot \frac{5}{2} = P_1 \cdot X_0 \\ P_1 \cdot \frac{5}{2} = P_2 \cdot X_0 \end{cases}$$

$$P_1 = \frac{P_0 \cdot \frac{5}{2}}{X_0(1)}$$

$$P_0 \left[1 + \frac{2.5}{R_{CPU}} \dots \right]$$

$$\left\{ \begin{array}{l} P_1 \cdot \frac{5}{2} = P_2 \cdot X_{02} \\ P_3 \cdot \frac{5}{2} = P_3 \cdot X_{03} \\ P_4 \cdot \frac{5}{2} = P_4 \cdot X_{04} \\ P_1 + P_2 + P_3 + P_4 + P_0 = 1 \end{array} \right.$$

$$P_2 = \frac{P_1 \cdot \frac{5}{2}}{X_{02}}$$

$$P_3 = \frac{P_2 \cdot \frac{5}{2}}{X_{03}}$$

$$P_4 = \frac{P_3 \cdot \frac{5}{2}}{X_{04}}$$

$$P_0 = \frac{P_4 \cdot \frac{5}{2}}{X_{04}}$$

P_0 è costante

$$\text{Probabilità Loss} = 1 - (P_0 + P_1 + P_2 + P_3 + P_4)$$

$$\text{Throughput media} = X(1) \cdot P_1 + X(2) \cdot P_2 + X(3) \cdot P_3 + X(4) \cdot P_4$$

$$\text{Average response time } \frac{N}{R} = \frac{P_1 \cdot 1 + P_2 \cdot 2 + P_3 \cdot 3 + P_4 \cdot 4}{P_1 \cdot X(1) + P_2 \cdot X(2) + P_3 \cdot X(3) + P_4 \cdot X(4)} = \frac{\bar{N}}{2.4}$$

Considering cache

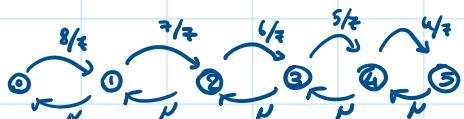
$$R_{disk,i} = P_{miss} (D_{disk} + D_{cache}) + (1 - P_{miss}) (D_{cache}) [1 + u_{i-1}]$$

Esercizio n.1

Server composed of a single unit with user 5 users in queue, limited 8 users

$$\lambda = 30 \text{ sec}$$

$$S = 20 \text{ sec}$$



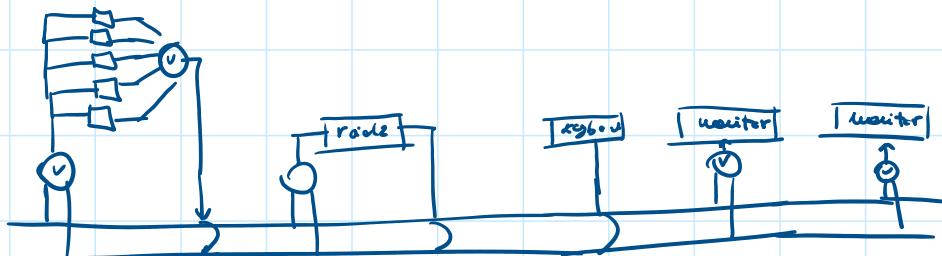
$$N = \frac{1}{S} = \frac{1}{20} = 0.05$$

$$\begin{cases} P_0 \frac{8}{S} = P_1 \mu \\ P_1 \frac{7}{S} = P_2 \mu \\ P_2 \frac{6}{S} = P_3 \mu \\ P_3 \frac{5}{S} = P_4 \mu \\ P_4 \frac{4}{S} = P_5 \mu \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \end{cases} \Rightarrow \begin{cases} P_1 = 3,95 \cdot 10^{-5} \\ P_2 = 18,43 \cdot 10^{-5} \\ P_3 = 73,73 \cdot 10^{-5} \\ P_4 = 215,77 \cdot 10^{-5} \\ P_5 = 655,40 \cdot 10^{-5} \end{cases} P_0 = 0,000742$$

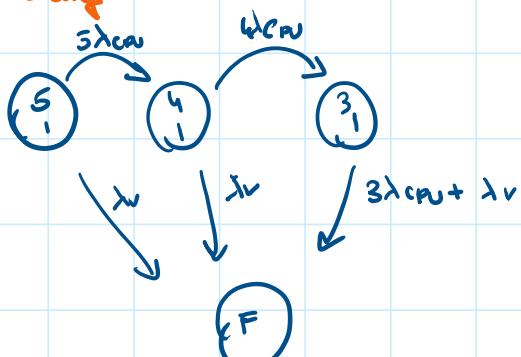
$$\bar{X} = \sum P_i \cdot M_i = P_1 \cdot M_1 + P_2 \cdot M_2 + P_3 \cdot M_3 + P_4 \cdot M_4 + P_5 \cdot M_5 = 49,85 \text{ riduzione/sec}$$

$$\bar{N} = \sum_{i=1}^5 i \cdot P_i \quad R = \frac{\bar{N}}{\bar{X}}$$

Esercizio 1



Reliability



$$\dot{P}_{S,1}(t) = P_{S,1}(t) \cdot (-5\lambda_{CPU} - \lambda_V)$$

$$\dot{P}_{U,1}(t) = P_{U,1}(t) \cdot (-4\lambda_{CPU} - \lambda_V) + P_{S,1}(t) \cdot 5\lambda_{CPU}$$

$$\dot{P}_{S,1}(t) = P_{S,1}(t) \cdot (-3\lambda_{CPU} - \lambda_V) + P_{U,1}(t) \cdot 4\lambda_{CPU}$$

$$\dot{P}_F(t) = -\lambda_V (P_{S,1}(t) + P_{U,1}(t) + P_{S,1}(t)) - 3\lambda_{CPU} P_{S,1}(t)$$

$$P_{S,1}(t) + P_{U,1}(t) + P_{S,1}(t) + P_F(t) = 1$$

$$P_{S,1}(0) = 1$$

$$R_{CPU} = 1 - P_F(t) = P_{S,1}(t) + P_{U,1}(t) + P_{S,1}(t)$$

Keyboard - Printer - Monitor



$$\dot{P}_I(t) = P_I(t) \cdot (-\lambda_I)$$

$$P_I(t) = P_I(t) \cdot \lambda_I$$

$$P_I(t) + P_F(t) = 1$$

$$P_I(0) = 1$$

$$R_{KPM} = 1 - P_F(t) = P_I(t)$$

Bus + Voters



$$R_{BV} = 1 - P_F(t) = P_{BV}(t)$$

$$\dot{P}_{BV}(t) = P_{CPU,V}(t) \cdot (-2\lambda_V - \lambda_{UV})$$

$$P_0(t) = 1$$

$$R_k P_k = 1 - P_F(t) = P_i(t)$$

Raid



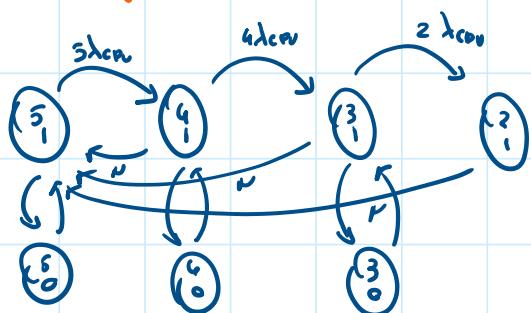
Availability: $\prod_i R_i$

$$\dot{P}_2(t) = -\lambda_{\text{DISK}} \cdot P_2(t)$$

$$P'_0(t) = +\lambda_{\text{DISK}} P_2(t) - 6 \lambda_{\text{DISK}} P_0$$

$$P'_F(t) = 6 \lambda_{\text{DISK}} P_0(t)$$

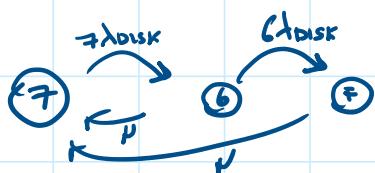
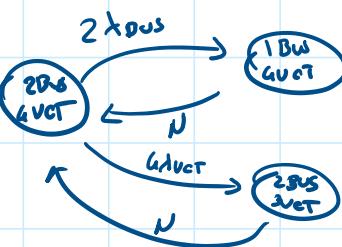
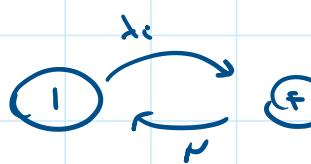
Availability



$$P_{S,1}(t) = -P_{S,1}(t)(5\lambda_{CPU} + \lambda_U)$$

$$+ \lambda(P_{S,0}(t) + P_{U,1}(t) + P_{3,1}(t) + P_{2,1}(t))$$

:



Availability totale:

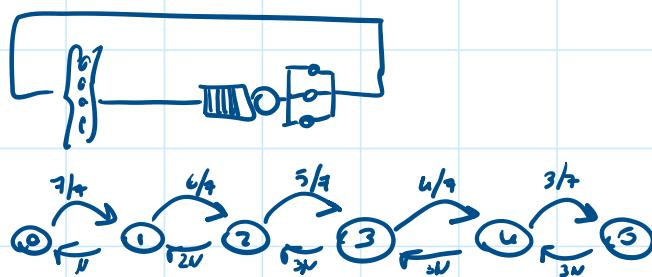
$$\prod_i A_i$$

Safety:

Se una segnale va con posso

Esercizio 11.3

Response time, throughput, probability of loss?



$$\left\{ \begin{array}{l} P_0 \frac{7}{2} = P_1 4 \\ P_1 \frac{6}{4} = P_2 24 \\ P_2 \frac{5}{4} = P_3 34 \\ P_3 \frac{4}{4} = P_4 34 \\ P_4 \frac{3}{4} = P_5 34 \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \end{array} \right. \Rightarrow \begin{array}{l} P_0 = 0,054 \\ P_1 = 0,183 \\ P_2 = 0,2835 \\ P_3 = 0,23625 \\ P_4 = 0,1575 \\ P_5 = 0,07875 \end{array}$$

$$\bar{x} = \sum_{i=1}^5 P_i \cdot u_i; \quad \text{Il throughput medio}$$

$$\bar{N} = \sum_i i \cdot P_i \quad \text{Il numero di utenti medi}$$

Esercizio 11.3

100 requests x secondi

4 identical servers (CPU+RAM)

Fileserver: CPU + 8 disks **connected with 8 channels**

connected with Router (no delay)

File dimension: 100 kB/s

Request: 500 bytes

$$D_{CPU}^{HTTP} = 5 \text{ micros (lat)}$$

$$D_{CPU}^{RAM} = 10 \text{ micros} \quad \text{HTTP} = 1000 \text{ hours}$$

$$D_{CPU+RS} = 10 \text{ micros} \quad \text{HTTR} = 1 \text{ hour}$$

$$D_{Router} = 10 \text{ micros}$$

average service time, throughput, final bottleneck

$$F_{HTTP}^{req} = \frac{500}{65535} = 1 \text{ fragments ETH}$$

$$F_{HTTP}^{req} = \frac{100.000}{65535} = 2 \text{ fragments ETH}$$

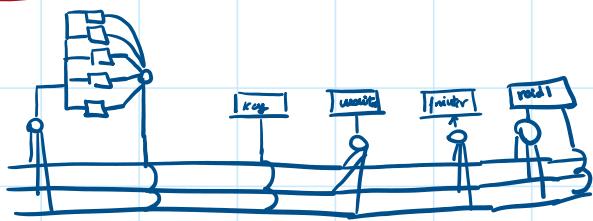
$$\text{Frames} = \text{datagrammi} = \frac{100.000}{1460} = 69 + 3$$

$$\text{Overhead} = (69+3) (20+20+16) = 4196$$

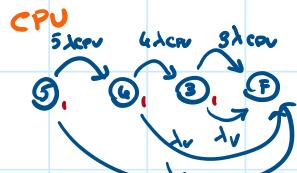
$$D_{ETH}^{req} = \frac{100.000 + 4196}{2 \cdot 10^3} = 5208 \cdot 10^{-5}$$

R,S

Esercizio n° 1



Reliability



$$\begin{aligned} \dot{P}_S(t) &= P_S(t) \cdot -3\lambda_{CPU} + P_F(t) \cdot \lambda_V \\ \dot{P}_U(t) &= P_S(t) \cdot 5\lambda_{CPU} - P_U(t) \cdot 4\lambda_{CPU} - P_U(t) \cdot \lambda_{Vout} \\ \dot{P}_F(t) &= P_U(t) \cdot 4\lambda_{CPU} - P_U(t) \cdot \lambda_V + P_U(t) \cdot \lambda_{Vout} \\ \dot{P}_F(t) &= P_S \lambda_V + P_U \lambda_V + P_3(t) \cdot 3\lambda_{CPU} \\ P_S(t) &= 1 \quad \text{Reliability}(t) = 1 - P_F(t) \end{aligned}$$

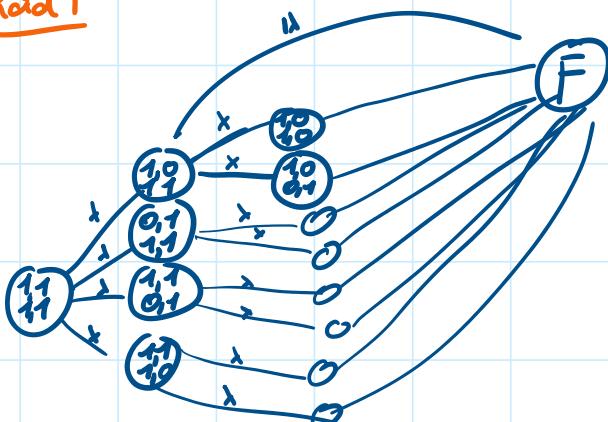
Keyboard / Monitor / Printer



$$\begin{cases} \dot{P}_I(t) = -P_I(t) \cdot \lambda_{Fi} \\ \dot{P}_F(t) = P_I(t) \cdot \lambda_{Fi} \\ P_I(t) = 1 \end{cases}$$

$$\text{Reliability}(t) = 1 - P_F(t)$$

Raid 1



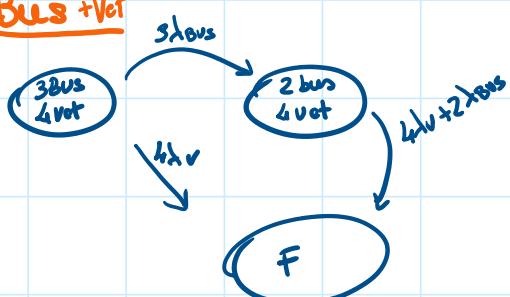
2-5

1 failure

Raid 0

0 failure

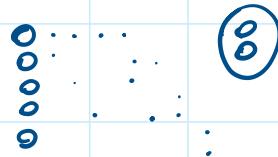
BUS + VOT



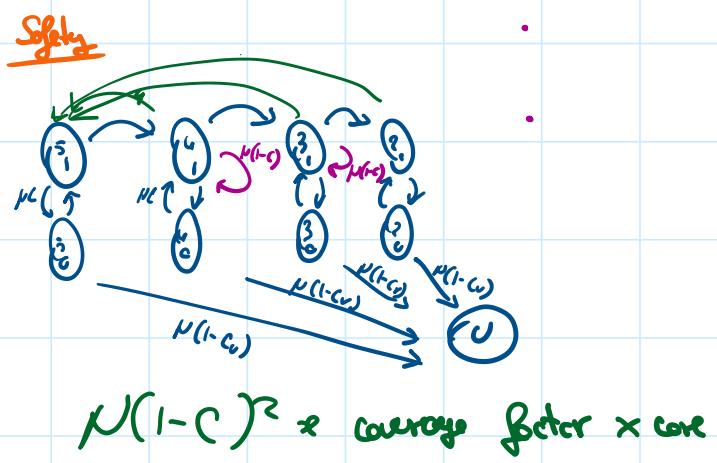
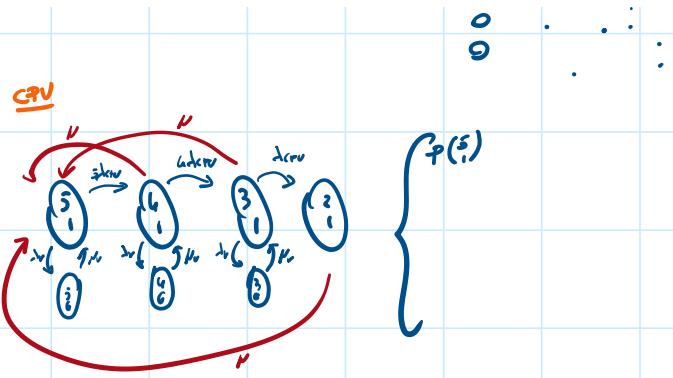
$$\begin{cases} \dot{P}(z_u)(t) = -P(z_u)(3\lambda_{bus} + 4\lambda_V) \\ \dot{P}(z_u)(t) = P(z_u)(t) \cdot (3\lambda_{bus}) + -P(z_u)(t) \cdot (4\lambda_V + 2\lambda_{bus}) \\ \dot{P}(F)(t) = (4\lambda_V) \cdot P(z_u)(t) + (4\lambda_V + 2\lambda_{bus}) P(z_u)(t) \\ P(z_u)(t) = 1 \end{cases}$$

Avoidability

N riparo tutto il componente



CPU



Esercizio 2

Sei p e q uguali per quanto riguarda
calcolo P $\bar{x} = \bar{N}$

Se tempo scontato delle rettifiche sono indipendenti

$$A = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}} = 0.38$$

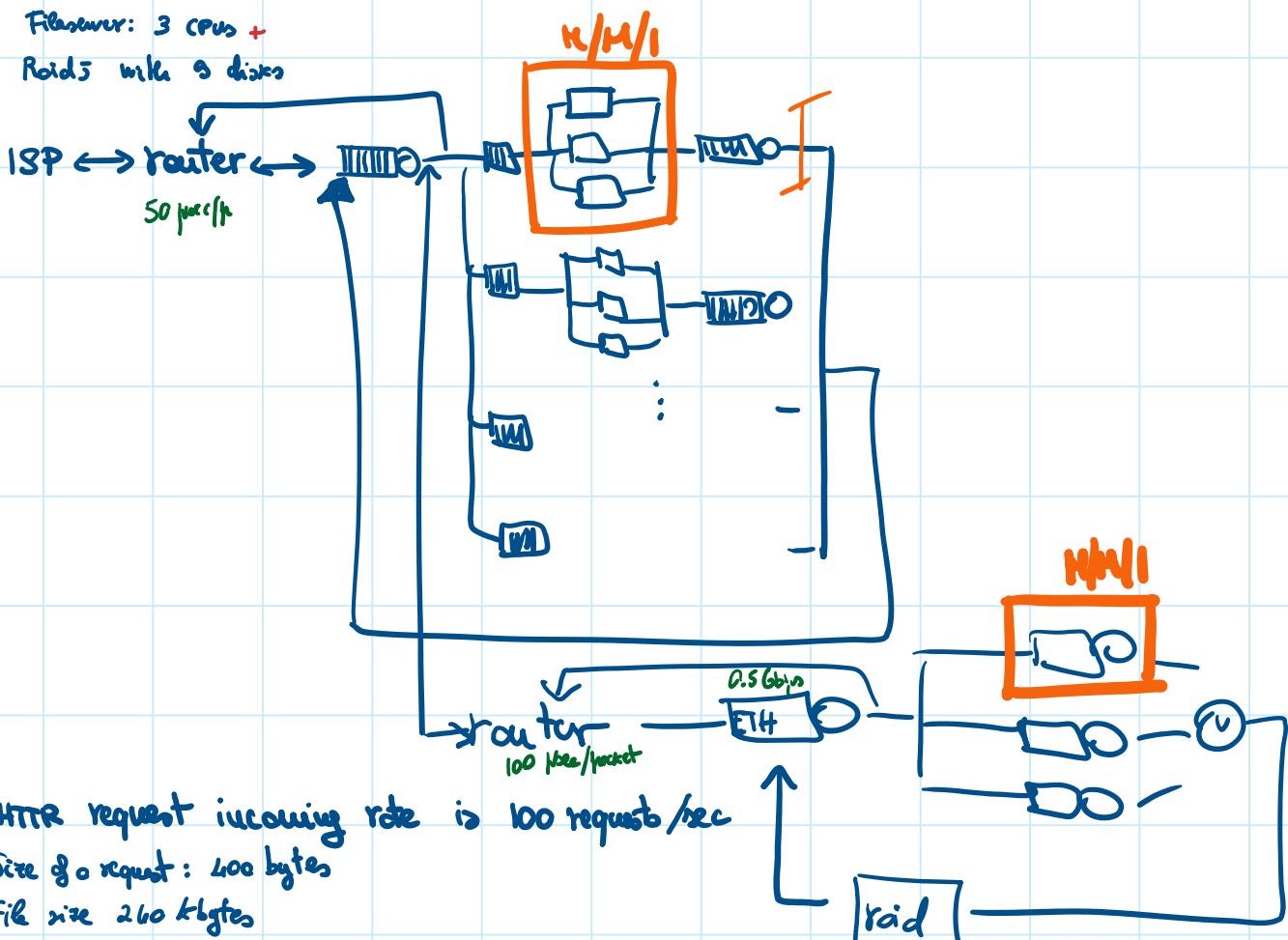
$$q_3 = 1 \cdot A^3 \quad q_2 = A^2(1-A) \quad q_1 = 3A(1-A)^2$$

$$q_0 = (1-A)^3$$

$$\bar{R} = q_1 \cdot R_1 + q_2 \cdot R_2 + q_3 \cdot R_3$$

$$\bar{x} = q_1 \bar{x}_1 + q_2 \bar{x}_2 + q_3 \bar{x}_3$$

1 Gbps FDDI
50 μsec/packet latency
FDDI-Router 1.66 Gbps
4 Web ~~Servers~~: 4 cores, local mem, local disk
Fileserver: 3 CPUs +
RAID5 with 3 disks



HTTP request incoming rate is 100 requests/sec

Size of a request: 400 bytes

File size 260 kbytes

$$D_{CPU} = 5 \text{ msec} \quad \text{if stored in main memory}$$

$$D_{CPU-DISK} = 10 \text{ msec} \quad \text{if local-disk}$$

$$D_{CPU-NODE} = 30 \text{ msec}$$

$$D_{DISK} = 10 \text{ msec}$$

$$\text{Single disk} = 10 \text{ msec} \times 20 \text{ kbytes} \\ \text{or RAID5}$$

$$D_{RAID5} = \frac{260}{20} \cdot 10 = 120$$

Networking Course MTU

$$\text{packets} = \frac{260,000}{1460} = 165$$

$$\text{ackets/b request} = \frac{600}{1460} = 1$$

$$D_{FDDI} = 1 + 3(20 + 20 + 28) + 400 + (165 + 3)(20 + 20 + 28) + 260,000 = 200,224$$

↑ 165 - 20 - 28 →

rate generated

$$D_{Router} = 172 \cdot 50 \text{ μsec} = 8,6 \text{ μ/sec}$$

IP: choose 2 packets, optimize jacks

Rete Immobiliare

$$D_{\text{Router}} = 172 \cdot 50 \text{ } \mu/\text{sec} = 8,6 \text{ } \mu/\text{sec}$$

D_{Router} = uguale o quello minimo con $16 \text{ Gbit} = 2,0 \text{ } \mu/\text{sec}$

$$P_{\text{idle}} = \sum_{i=1}^5 k_i \quad 20\% \text{ hit rate}, 80\% \text{ miss}$$

$$P_{\text{disk}} = \sum_{i=1}^5 k_i \quad 60\% \text{ hit disk}, 40\% \text{ miss}$$

Average response time:

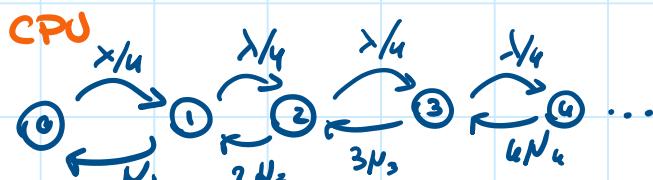
$$\bar{R}_i = \frac{D}{1-U_i}$$

$R_{\text{tot}} = \sum R_i$

U = X · D

diff:

~~$\frac{D}{1-U_i}$~~



$$P_0 = \left[1 + \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} p(k) + \frac{\lambda^I}{I!} \frac{p^I}{1-p} \right]$$

$$\begin{aligned} x_1 &= P_1 \\ x_2 &= P_2 \\ x_3 &= P_3 \\ x_4 &= P_4 \end{aligned}$$

$$\Rightarrow \bar{x}$$

Disk

Router:

ipercapte

Esercizio n. 1

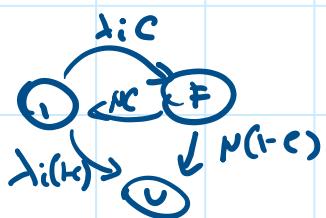
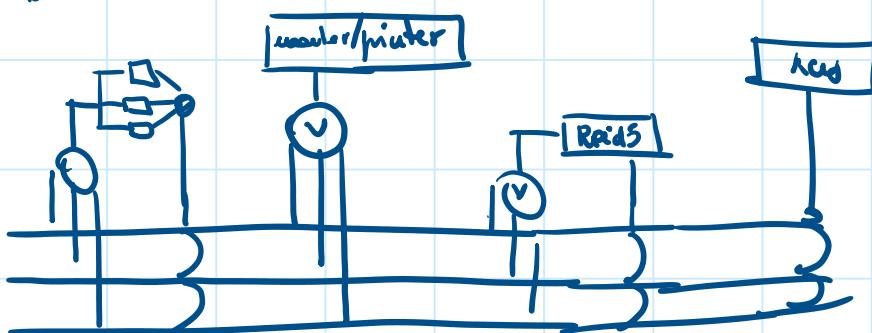
3 CPU in parallelo with voter

Raid 5 with 8 disks

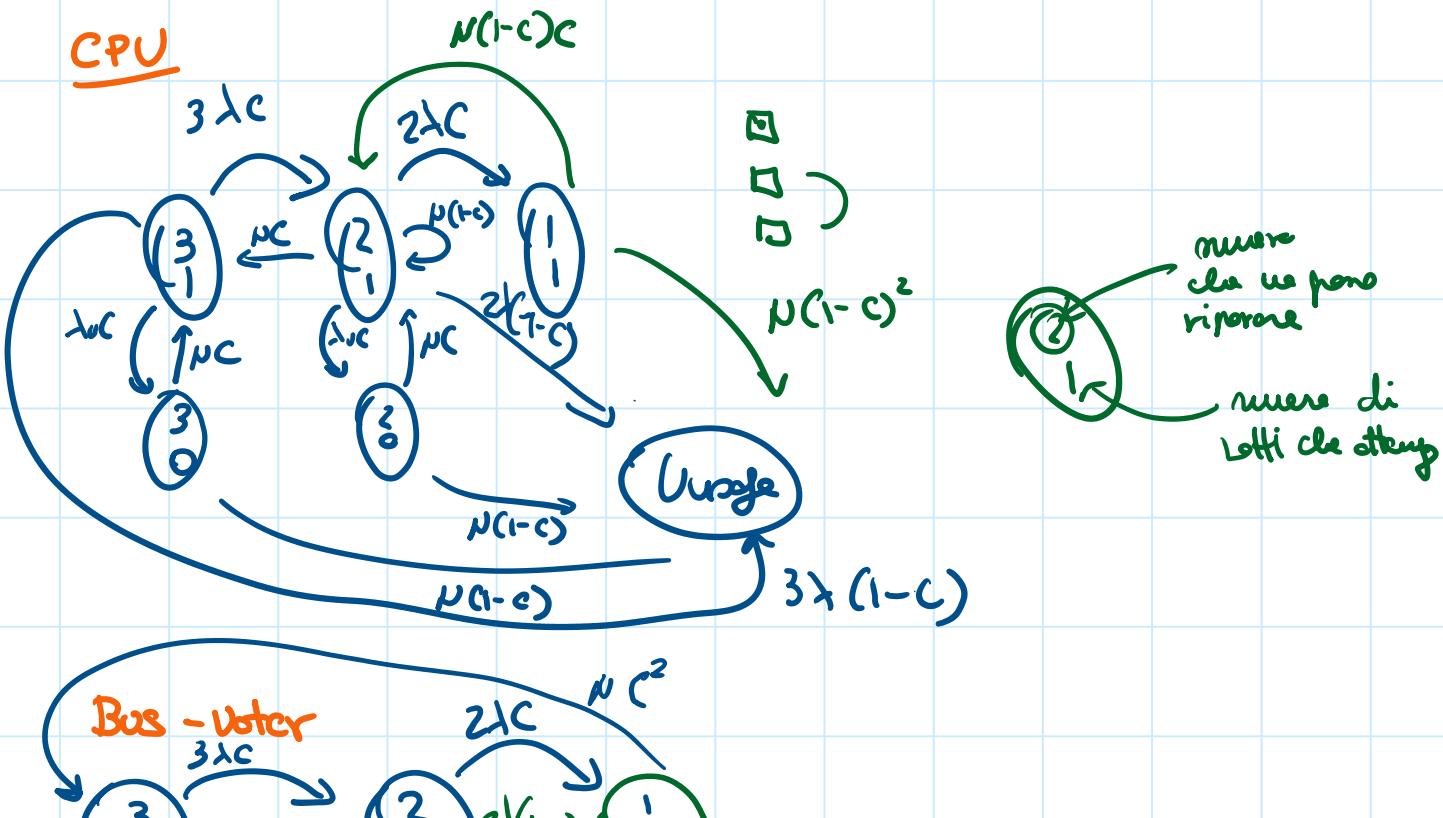
3 bus

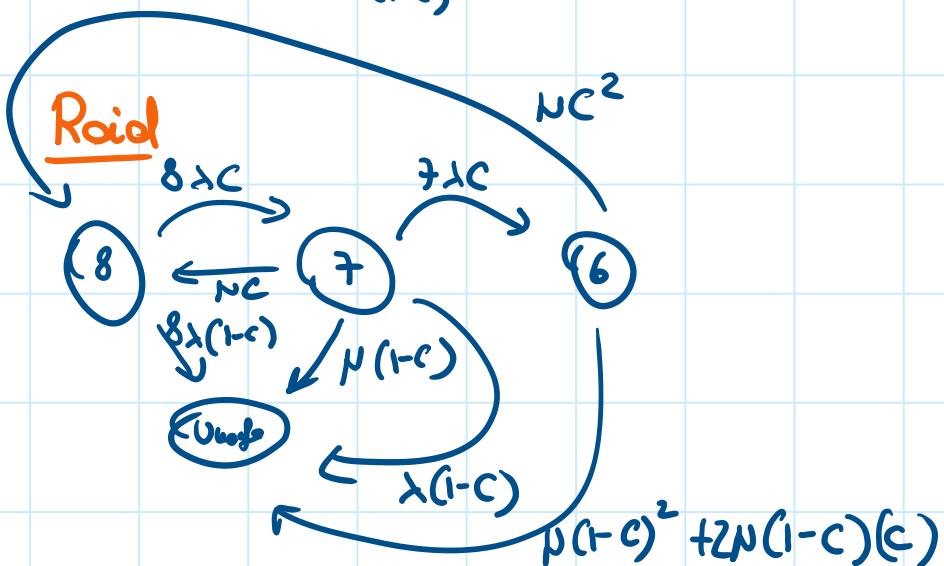
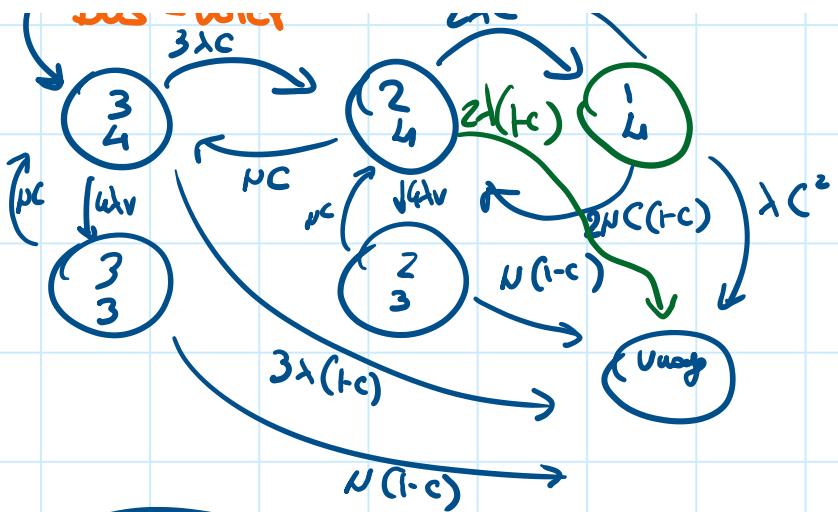
1 keyboard

1 monitor



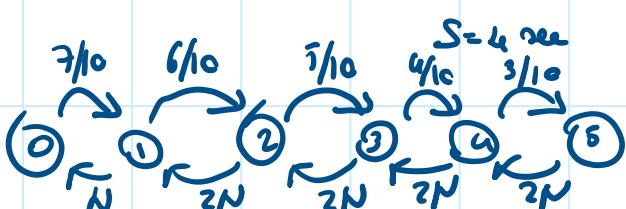
$$\begin{cases} P_i(t) = P_i(0) \cdot (-\lambda_i C) + P_f(0) \\ P_f(t) = P_f(0) \cdot (-\mu(C) - N(1-C)) + P_i(t) \lambda_i C \\ V(t) = P_f(t) N(1-C) + P_i(t) \lambda_i (1-C) \end{cases}$$





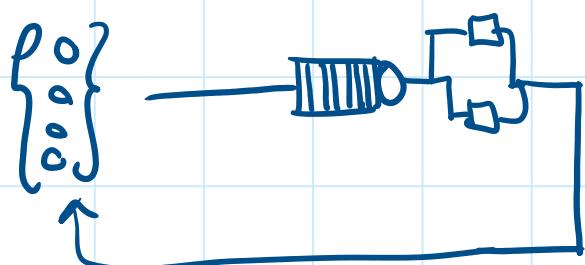
Esercizio n. 2

\bar{R} \bar{x} ?



10 sec think time

$S = 4$ sec



2. Funzionanti

$$\left\{ \begin{array}{l} P_0 \frac{7}{10} = P_1 2N \\ P_1 \frac{6}{10} = P_2 2N \\ P_2 \frac{5}{10} = P_3 2N \\ P_3 \frac{4}{10} = P_4 2N \\ P_4 \frac{3}{10} = P_5 2N \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \end{array} \right.$$

\Rightarrow

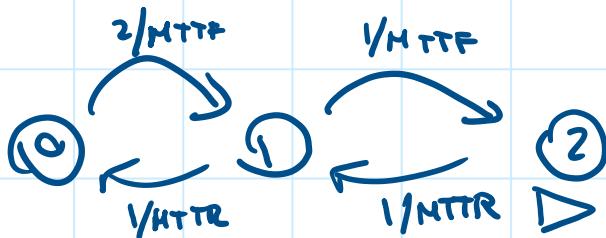
$$\begin{aligned} P_0 &= x_{1,2} \\ P_1 &= \alpha_{1,2} \\ P_2 &= \alpha_{2,2} \\ P_3 &= \alpha_{3,2} \\ P_4 &= \alpha_{4,2} \end{aligned}$$

X

1. Funzionanti

1. Fundamentals

$$\left\{ \begin{array}{l} P_0 \cdot \frac{2}{10} = P_1 \cdot 1 \\ P_1 \cdot \frac{6}{10} = P_2 \cdot 1 \\ P_2 \cdot \frac{5}{10} = P_3 \cdot 1 \\ P_3 \cdot \frac{4}{10} = P_4 \cdot 1 \\ P_4 \cdot \frac{3}{10} = P_5 \cdot 1 \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \end{array} \right. \Rightarrow \begin{array}{l} P_0 = \alpha_{1,1} \\ P_1 = \alpha_{1,1} \\ P_2 = \alpha_{2,1} \\ P_3 = \alpha_{3,1} \\ P_4 = \alpha_{4,1} \\ P_5 = \alpha_{5,1} \end{array}$$



Ip: T ripetutore + archiubi

$$\left\{ \begin{array}{l} P_0 \cdot \frac{2}{MTTF} = P_1 \cdot \frac{1}{MTTR} \\ P_1 \cdot \frac{1}{MTTF} = P_2 \cdot \frac{1}{MTTF} \\ P_0 + P_1 + P_2 = 1 \end{array} \right. \Rightarrow \begin{array}{l} P_0^F, P_1^F, P_2^F \end{array}$$

$$X_0^2 = \sum w_i P_i = \mu R + 2N P_2 \dots$$

$$X_0' = \sum p_i P_i = \mu R_1 + 2 \mu P_2 \dots$$

$$N^2 = \sum_i P_i = P_1^2 + 2P_2^2 + 3P_3^2$$

$$\bar{R}^2 = \frac{\bar{N}^2}{X_0^2}$$

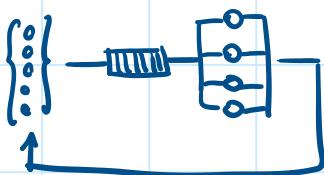
$$\bar{R}' = \frac{\bar{N}'}{X_0}$$

$$\bar{R} = P_0 \cdot \bar{R}^2 + P_1 \cdot \bar{R}'$$

Esercizi 19/02/15

lunedì 20 maggio 2019

14:47



$$z = 10 \text{ sec}$$

$$S = 2 \text{ sec}$$

$$\mu = \frac{1}{S} = 0.5$$

$$\text{MTTF} = 10 \text{ sec} \rightarrow \text{MTTRE} = 10 \cdot 0.5 = 5 \text{ sec}$$

$$\text{MTTR} = 2 \text{ sec}$$



Caso 4/4

$$\left\{ \begin{array}{l} P_0 \cdot 0.5 = P_1 \cdot 0.5 \\ P_1 \cdot 0.2 = P_2 \\ P_2 \cdot 0.1 = P_3 \cdot 0.5 \\ P_3 \cdot 0.2 = P_4 \cdot 0.2 \\ P_4 \cdot 0.5 = P_5 \cdot 0.2 \\ P_5 \cdot 0.2 = P_6 \cdot 0.2 \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1 \end{array} \right.$$

$$P_0 = 0.292$$

$$P_1 = 0.391$$

$$P_2 = 0.279$$

$$P_3 = 0.103$$

$$P_4 = 0.025$$

$$P_5 = 5 \cdot 10^{-3}$$

$$P_6 = 7 \cdot 10^{-4}$$

$$\bar{N}_4 = \sum i P_i = 1,327$$

$$\bar{x}_4 = \sum N_i P_i = 0,66$$

$$\bar{R} = \frac{\bar{N}_4}{\bar{x}_4} = 2,01$$

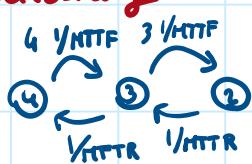
Caso 3/4



Caso 3/4

$$\left\{ \begin{array}{l} P_0 \cdot 0.5 = P_1 \cdot 0.5 \\ P_1 \cdot 0.2 = P_2 \\ P_2 \cdot 0.1 = P_3 \cdot 0.5 \\ P_3 \cdot 0.2 = P_4 \cdot 0.5 \\ P_4 \cdot 0.5 = P_5 \cdot 0.5 \\ P_5 \cdot 0.5 = P_6 \cdot 0.5 \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1 \end{array} \right.$$

Availability



I_{fp} : Riparazione e guasti dipendenti

I_{fp} : Sposta dopo z

$$P(B)_{\text{time}} = P_0 \cdot \frac{1}{\text{MTTF}}$$

$$P_u = P_3 \cdot \frac{\text{MTTF}}{L \cdot \text{MTTR}}$$

$$P_3 = \frac{4 \cdot \text{MTTR}}{\text{MTTF}} \cdot P_4$$

MTTR = 1/MTTF

$$\begin{cases} P_3/MTTF = P_u/MTTF \\ P_2/MTTF = P_3/MTTF \\ P_3 + P_2 + P_u = 1 \end{cases}$$

$$P_u = P_3 \cdot \frac{MTTF}{4 MTTF}$$

$$P_3 = P_2 \cdot \frac{MTTF}{3 MTTF}$$

$$P_3 + P_2 + P_u = 1$$

$$P_3 = \frac{4 MTTR}{MTTF} \cdot P_u$$

$$P_2 = \frac{3 MTTR}{MTTF} \cdot \cancel{P_3} \quad \frac{4 MTTR}{MTTF} \cdot P_u$$

$$\left(\frac{4 MTTR}{MTTF} + \frac{3 MTTR}{MTTF} \cdot \frac{4 MTTR}{MTTF} + 1 \right) P_u = 1$$

 ~~$\frac{8}{1680} + \frac{6}{1680} \cdot \frac{8}{1680} + 1$~~

$$P_u = 0.557$$

$$P_3 = 4,76 \cdot 10^{-3}$$

$$P_2 = 2,256 \cdot 10^{-3}$$

$$\bar{R} = P_u \bar{R}_u + P^* \bar{R}_s$$

Esercizio 06/06/13

lunedì 20 maggio 2019 15:25

Requests 5 workstations (users)

$$Z = 5000 \text{ msec}$$

$$R_{CPU} = 20 \text{ usec}$$

$$D_{Disk} = 10 \cdot 5 = 50 \text{ msec}$$

At most 3 users

R ? $\bar{x}=?$ Percentage of refined requests = ?



Caso N=1

$$R_{CPU} = 20 \text{ usec}$$

$$D_{Disk} = 50 \text{ msec}$$

$$R_{TOT} = 70 \text{ msec}$$

$$\bar{x}_1 = \frac{1}{R_{TOT}} = 0,014$$

$$N_{CPU} = R_{CPU} \cdot X_0(1) = 0,28$$

$$N_{Disk} = R_{Disk} \cdot X_0(1) = 0,72$$

Caso N=2

$$R_{CPU} = R_{CPU}(1 + M_{CPU,i-1}) = 26,6 \text{ msec}$$

$$R_{Disk} = D_{Disk}(1 + M_{Disk,i-1}) = 86 \text{ msec}$$

$$\bar{x}_2 = \frac{2}{R_{TOT}} = \frac{2}{16,6} = 0,018$$

$$M_{CPU_2} = 25,6 \cdot 0,018 = 0,46$$

$$M_{Disk_2} = 86 \cdot 0,018 = 1,54$$

Caso u=3

$$R_{CPU} = R_{CPU}(1 + M_{CPU,2}) = 29,2$$

$$R_{Disk} = D_{Disk}(1 + M_{Disk,2}) = 127$$

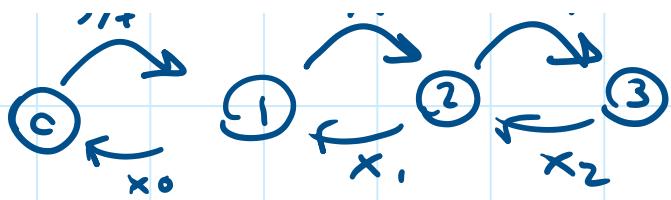
$$R_{TOT} = 156,2$$

$$N_{CPU_3} = 0,554$$

$$N_{Disk_3} = 2,413$$

$$X_3 = \frac{3}{R_{TOT}} = 0,0192$$





$$\begin{cases} P_0 \cdot x_0 = P_1 \cdot x_0 \\ P_1 \cdot x_1 = P_2 \cdot x_1 \\ P_2 \cdot x_2 = P_3 \cdot x_2 \\ P_0 + P_1 + P_2 + P_3 = 1 \end{cases}$$

$$P_0 = 0,930$$

$$P_1 = 0,066$$

$$P_2 = 2,953 \cdot 10^{-3}$$

$$P_3 = 9,328 \cdot 10^{-5}$$

$$\bar{x} = x_i - P_i = 9,79 \cdot 10^{-4} = 0.$$

$$\bar{R} = \frac{N}{x} = \frac{0,072}{9,79 \cdot 10^{-4}} = 73,55$$

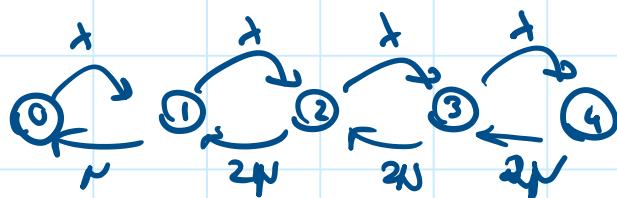
I controlli sono
di minor

0,013902 +

2 servers, max 4 users

$\lambda = 0.5$ (service rate)

$X = ?$ $V = ?$



Fluxin = flow out

$$\left\{ \begin{array}{l} p_0 \\ \vdots \\ p_N \end{array} \right. \Rightarrow \left\{ \begin{array}{l} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \right.$$

part by quantity

$$X = \sum_i \mu_i p_i =$$

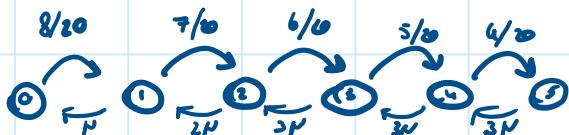
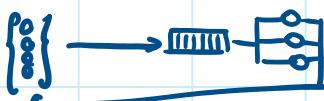
$$N = \sum_i i p_i =$$

$$p_0 = \frac{\lambda}{N}$$

$$V = 1 - p_0 = 0.24$$

Esercizio 2

$$\bar{R} = ?$$



$$N = 10$$

$$\frac{1}{MTTF} = \frac{1}{12} \text{ months} = \frac{1}{365 \cdot 24}$$

$$\frac{1}{MTTR} = \frac{1}{12} \text{ hours} =$$

$$P_0 = 0.03$$

$$\begin{cases} P_0 \cdot \frac{8}{20} = P_1 \cdot 0.1 \\ P_1 \cdot \frac{7}{20} = P_2 \cdot 0.2 \\ P_2 \cdot \frac{6}{20} = P_3 \cdot 0.3 \\ P_3 \cdot \frac{5}{20} = P_4 \cdot 0.3 \\ P_4 \cdot \frac{4}{20} = P_5 \cdot 0.3 \end{cases}$$

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

$$P_1 = \frac{P_0 \cdot \frac{8}{20}}{0.1} = 0.12$$

$$P_2 = \frac{P_1 \cdot \frac{7}{20}}{0.2} = 0.14$$

$$P_3 = \frac{P_2 \cdot \frac{6}{20}}{0.3} = 0.21$$

$$P_4 = \frac{P_3 \cdot \frac{5}{20}}{0.3} = 0.175$$

$$P_5 = 0.116$$

$$N_i = \sum_i P_i = 2.45$$

$$\bar{X}_i = \sum N_i \cdot P_i = 0.20$$

$$\bar{R}_S = \frac{N}{N_S} = 12$$

Caso 2 server

$$\begin{cases} P_0 \cdot \frac{8}{20} = P_1 \cdot 0.1 \\ P_1 \cdot \frac{7}{20} = P_2 \cdot 0.2 \\ P_2 \cdot \frac{6}{20} = P_3 \cdot 0.2 \\ P_3 \cdot \frac{5}{20} = P_4 \cdot 0.2 \\ P_4 \cdot \frac{4}{20} = P_5 \cdot 0.2 \\ P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \end{cases}$$

\Rightarrow

$$P_0 = \frac{1}{[P_1 + P_2 + P_3 + \dots]} = 0.02$$

$$P_{1T} = 4$$

$$P_{2T} = \frac{4 \cdot \frac{7}{20}}{0.2} = 7 \quad P_2 = 0.14$$

$$P_{3T} = 10.5$$

$$P_{4T} = 13.12$$

$$P_{5T} = 13.12$$

$$P_1 = 0.08$$

$$P_3 = 0.21$$

$$P_4 = 0.26$$

$$P_5 = 0.26$$

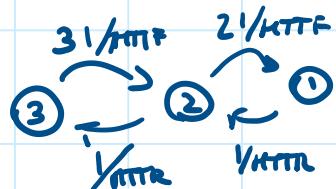
$$P_{ST} = 13,12$$

$$\bar{N}_2 = 3,33$$

$$x_2 = 0,18$$

$$R_2 = 18,29$$

Availability



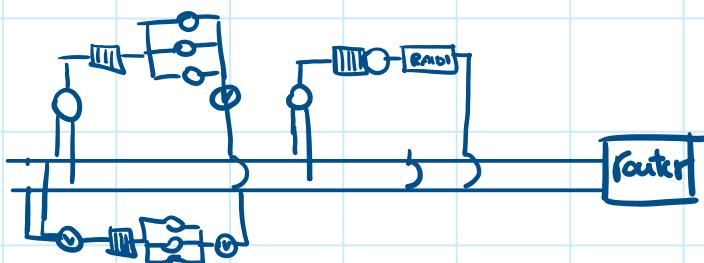
$$\begin{cases} P_3 Y_{MTTF} = P_2 \cdot \frac{1}{kMTTF} \\ P_2 Y_{MTTF} = P_1 \cdot \frac{1}{kMTTF} \\ P_3 + P_2 + P_1 = 1 \end{cases}$$

$$\begin{aligned} P_3 &= 0,995 \\ P_2 &= 4,08 \cdot 10^{-3} \\ P_1 &= 9,2 \cdot 10^{-6} \end{aligned}$$

$$\bar{x} = P_3 \cdot \bar{x}_3 + P_2 \cdot \bar{x}_2 = 0,203$$

$$\bar{R} = P_3 \cdot \bar{R}_3 + P_2 \cdot \bar{R}_2 = 12,01$$

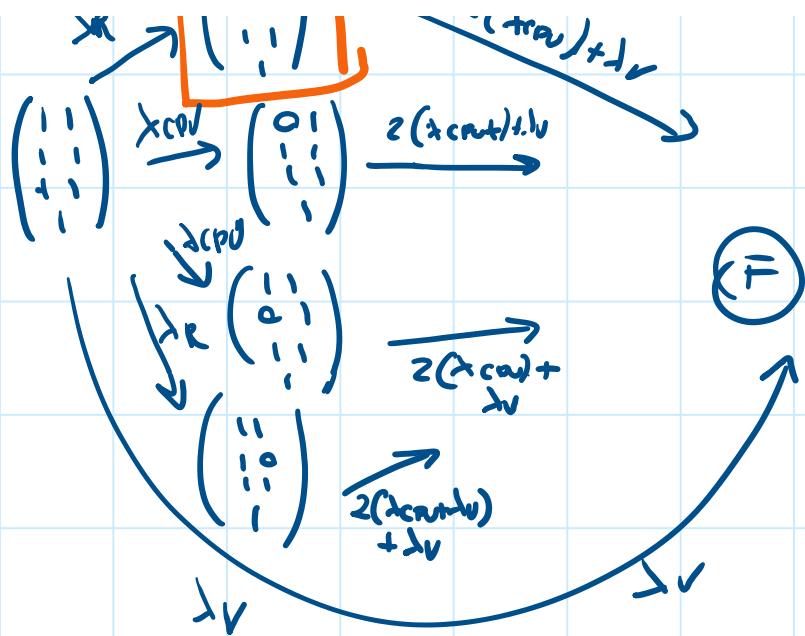
Esercizio 1



Reliability

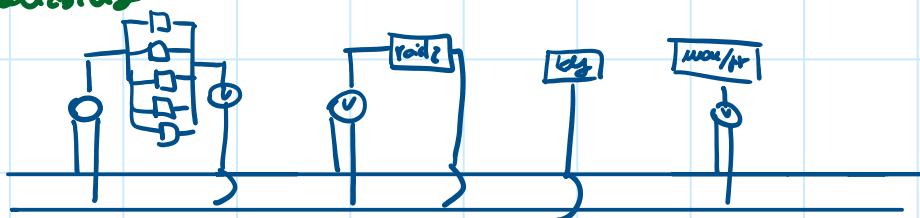
(3 2) $\rightarrow F$



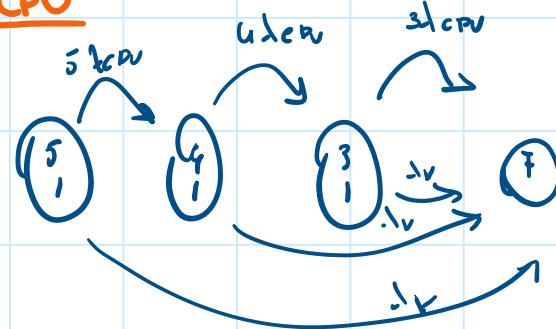


Esercizio 1

Reliability

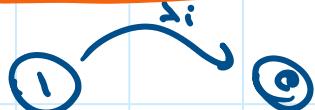


CPV



$$\left\{ \begin{array}{l} \dot{P}_{S,1}(t) = P_{S,1}(t) - (-\gamma)x_{CPV} \\ \vdots \\ P_{S,1}(0) = 1 \end{array} \right.$$

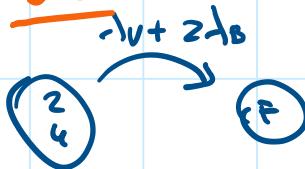
MONITOR PRINTER Keyboard



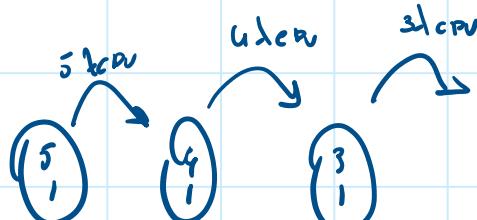
RAIO



Bus



Availability



Esercizio cosa

martedì 21 maggio 2019 16:08

$$SLA: \lambda = 100 \text{ requests/sec}$$

$$R = 1 \text{ sec}$$

$$D_{99} = 99\% \quad D_{95} = 2\%$$



$$S_{CPU} = 10 \text{ msec}$$

$$S_{Disk} = 20 \text{ msec}$$

max 4 utenti

$$MTTR_{CPU} = 1 \text{ week} = 52$$

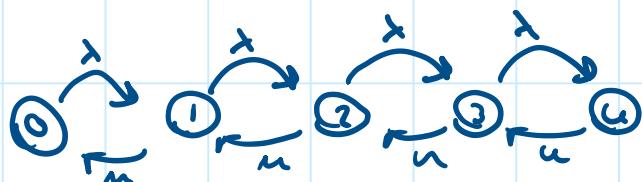
4 utenti max

$$MTTR_{Disk} = 6 \text{ weeks} = 26 \text{ sett}$$

$$MTTR_{CPU} = 1 \text{ week}$$

$$MTTR_{Disk} = 1 \text{ week}$$

maintenance indipendenti



$$A_{CPU} = \frac{MTTR}{MTTR_{CPU} + MTTR_{Disk}} = \frac{52}{1+52} = 0.98$$

$$A_{Disk} = \frac{MTTR}{MTTR_{Disk} + MTTR_{CPU}} = \frac{26}{1+26} = 0.96$$

$$A_{1/2 disk} = 2 A_{Disk} (1 - A_{Disk}) = 0.076$$

$$A_{2/2 disk} = 0.96 \cdot 0.96 = 0.92$$

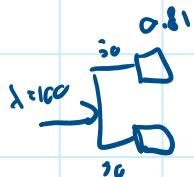
$$A_S = A_{CPU} \cdot A_{1/2 disk} = 0.076\% \quad \text{|| single disk, ignora}$$

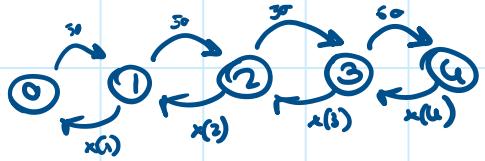
$$A_S = A_{CPU} \cdot A_{2/2 disk} = 0.90\% \quad \text{|| con doppio disk, come da SLA}$$

$$A_{Total} = 1 - (1 - 0.90)^2 = 0.53\% \quad \text{||}$$

$$Attivo e 2 = 0.81 \quad \text{P}_2$$

$$Attivo e 1 = 2(1 - 0.90)(0.50) = 0.18 \quad \text{P}_1$$





Caso u1

$$\begin{aligned} R_{CPU} &= 10 \\ R_{Disk} &= 20 \\ R_{TOT} &= 30 \\ X_1 &= \sqrt{3 \cdot 10^{-2}} \\ N_{CPU} = D_{CPU} & X_1 = 0,33 \\ N_{Disk} = D_{Disk} & X_1 = 0,66 \end{aligned}$$

Caso u2

$$R_{CPU} = 10(1 + N_{CPU_1}) = 13,3$$

$$R_{Disk} = 20$$

$$R_{TOT} = 33,3$$

$$X_2 = 1/33,3 = 3$$

$$N_{CPU_2} = D_{CPU} X_1 =$$

$$N_{Disk} = D_{Disk} \cdot X_1 =$$

! ! ? ?

Caso u3

$$\begin{aligned} R_{CPU} &= 10(1 + N_{CPU_2}) \\ R_{Disk} &= 20(1 + N_{Disk_2}) \\ X_3 &= \\ N_{CPU_3} & \end{aligned}$$

Caso u4

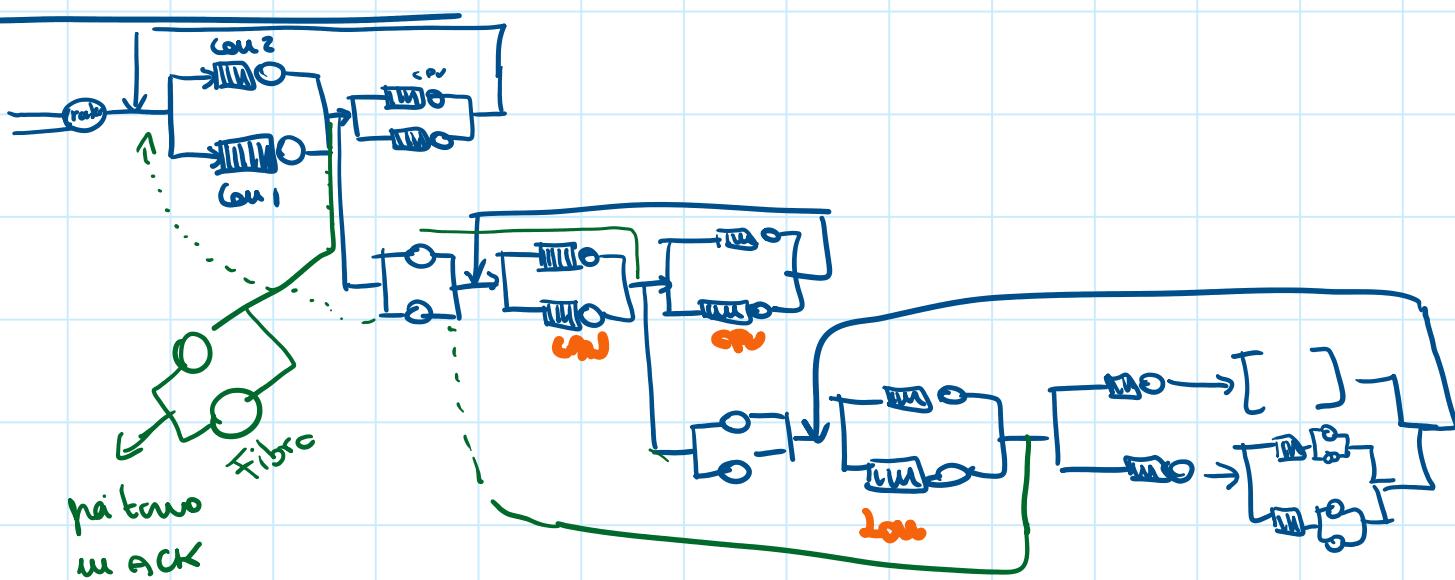
$$\begin{aligned} R_{CPU} &= \\ R_{Disk} &= R_{Disk_3} \\ X_4 & \end{aligned}$$

$$\bar{R}_2 = \sum P_i \cdot R_i$$

Rifore con 1 solo funzionante ($\lambda=100$)

→ Normalizzo con $(1 - P_0)$ se riduce solo al periodo funzionante

→ Non normalizzate altrimenti



Esercizio perfeability Alex

sabato 25 maggio 2019 17:29

$$\bar{R} = ? \quad \bar{x} = ?$$

6 server \rightarrow 1 CPU, RAM, RAID 1 (4+4)

$$MTTF_D = 500 \text{ ore} \quad MTTF_C = 1000 \text{ ore}$$

$$MTTR_D = 50 \text{ ore} \quad MTTR_C = 10 \text{ ore}$$

6 utenti con $\lambda = 10 \text{ user}$

$$A_{CPU} = \frac{1000}{1000} = 0.99$$

$$\mu = \frac{1}{5}$$

$$A_{Disk} = \frac{500}{500} = 0.90$$

$$A_{total} = (1 - (1 - A_{Disk})^6)^6 = 0.96$$

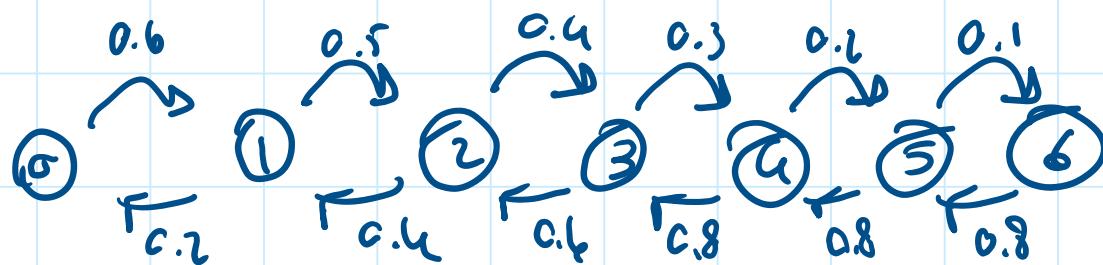
$$A_{server} = A_{CPU} \cdot A_{Disk}$$

$$q_0 = 0.95^6 = 0.81$$

$$q_1 = 6 \cdot 0.95^5 \cdot (1 - 0.95) = 0.17$$

$$q_2 = 6 \cdot 0.95^4 \cdot (1 - 0.95)^2 = 0.013$$

$${6 \choose 1} = \frac{6!}{1(5!)} = \frac{6}{6} = 1 \quad \begin{matrix} \text{in oggetti} \\ \text{per k o k} \end{matrix}$$



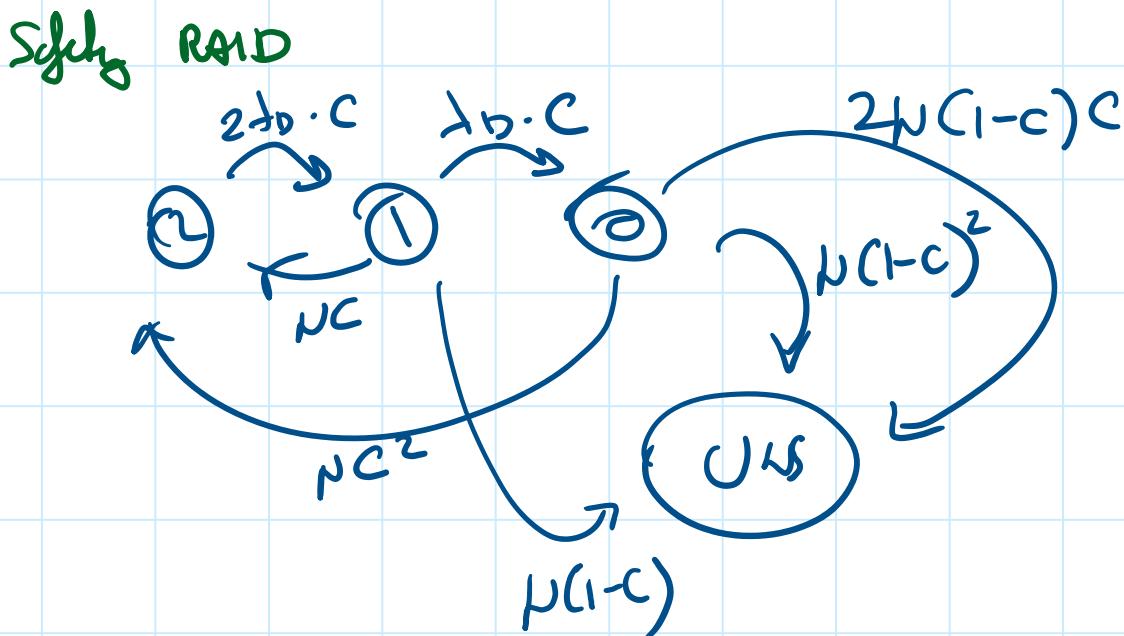
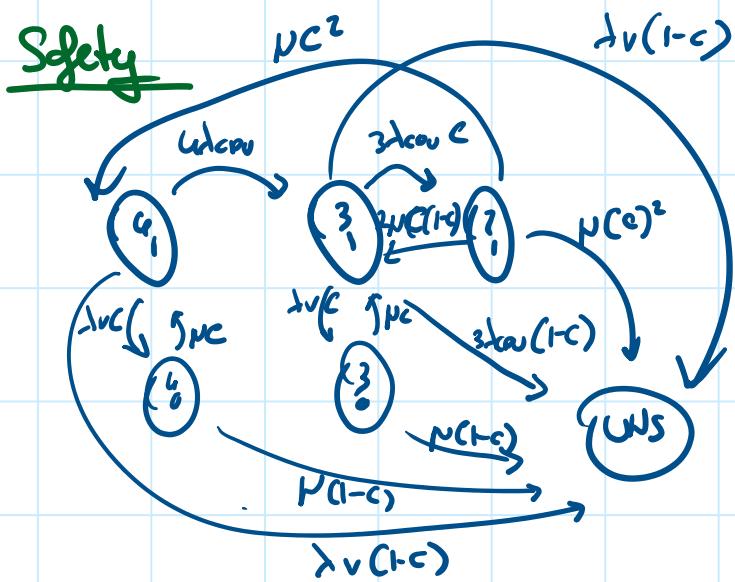
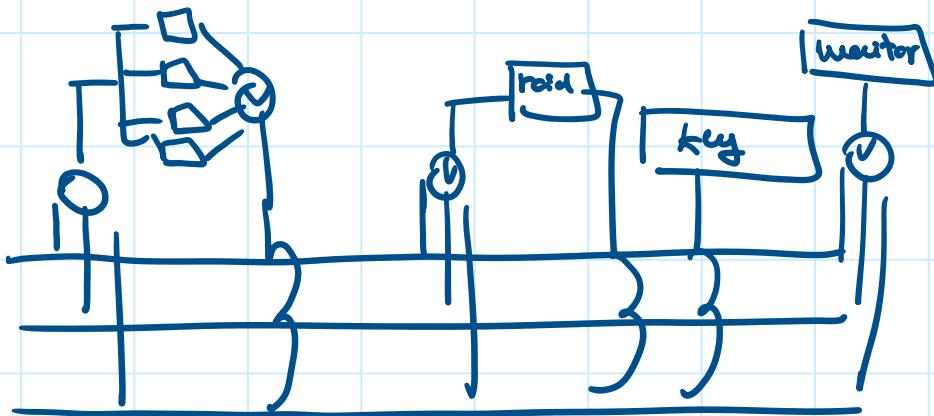
$$R_u = \sum i \cdot p_i = 1.87$$

$$X_u = \sum_N p_i = 0,39$$

$$\bar{R}_u = 5,05$$

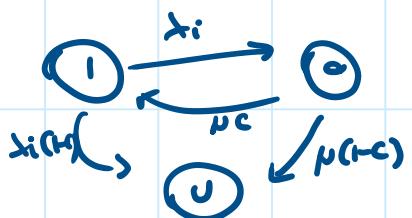
Fondo $\frac{1}{2} \cdot 3 \text{ gms}, \frac{1}{2} \text{ gms}, \frac{1}{2} \text{ gms}$

$$R_{\text{TOT}} = q_u \cdot R_u + q_3 \cdot R_3 + q_2 \cdot R_2 + q_1 \cdot R_1$$



$$\mu(1-c)$$

Safety Monitor/Printer



$$\bar{R} = ? \quad \bar{x} = ?$$

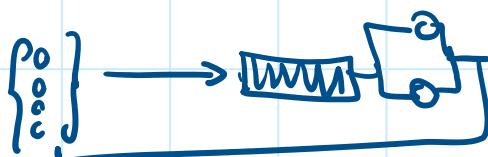
2 parallel systems

at most 4 users

$$\tau = 100 \text{ sec}$$

$$MTTF = 3 \text{ week}$$

$$MTTR = 1 \text{ hour}$$



$$P_2 \frac{2}{\text{sec}} = P_1 \frac{1}{\text{sec}}$$

$$P_1 \frac{1}{\text{sec}} = P_0 \frac{1}{\text{sec}}$$

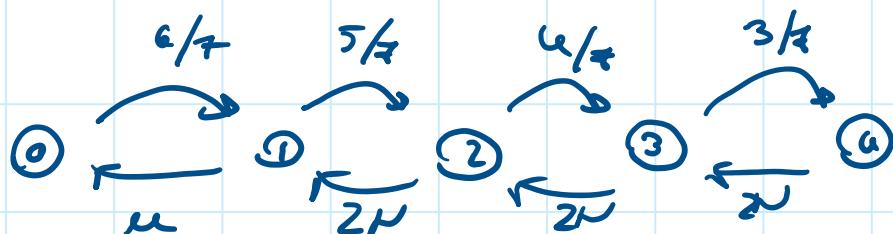
$$P_2 + P_1 + P_0 = 1$$

$$P_2 = \sqrt{1 + \frac{2}{\text{sec}} + \frac{2}{\text{sec}} \cdot \frac{1}{\text{sec}}} = 0,936$$

$$P_1 = 3,35 \cdot 10^{-3}$$

$$P_0 = 7,8 \cdot 10^{-6}$$

I_p : sistemi funzionano solo se entrambi funzionano



$$\left\{ \begin{array}{l} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \right.$$

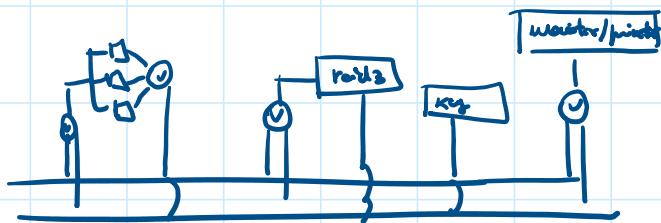
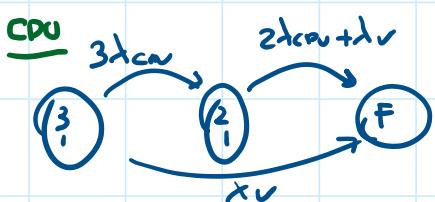
$$N = \sum_i P_i$$

$$X = \sum_i N_i \cdot P_i$$

$$R = \frac{N}{X}$$

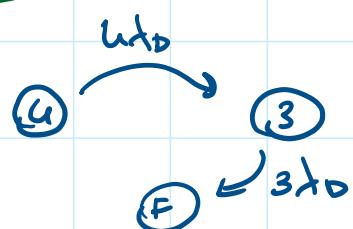
Esercizio 4.1.

3 CPU
Raid 3 4 disk
1 keyboard
1 Printer
1 monitor
2 bus

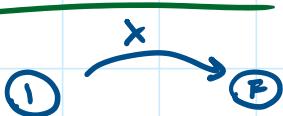
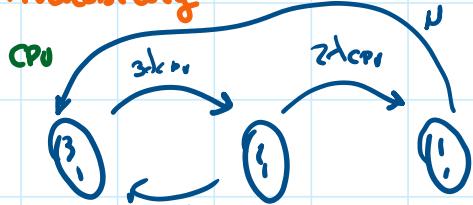
Reliability

$$\begin{cases} \dot{P}_{3,1}(t) = -P(3)(3\lambda_{CPU} + \lambda_V) \\ \dot{P}_{2,1}(t) = -P(2)(2\lambda_{CPU} + \lambda_V) + P(3) 3\lambda_{CPU} \\ \dot{P}_F(t) = +P(2)(t)(2\lambda_{CPU} + \lambda_V) + P(3)(t)(\lambda_V) \\ P(3)(0) = 1 \end{cases}$$

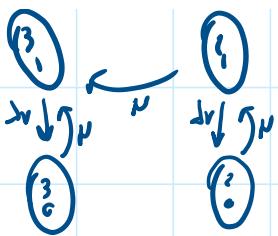
$$R_{CPU} = 1 - P_F(t)$$

DISK

$$\begin{cases} \dot{P}_4(t) = -P(4)(t) \lambda_D \\ \dot{P}_3(t) = +P(4)(t) \lambda_D - 3\lambda_D P_F(t) \\ \dot{P}_F(t) = P(F) 3\lambda_D \\ P(4)(0) = 1 \quad 0 \text{ otherwise} \end{cases}$$

MON / PRINTER / KEYBUSAvailability

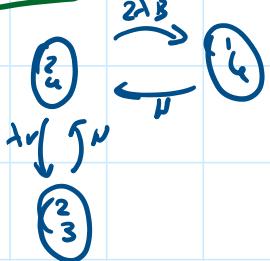
$$\begin{cases} P_{3,1}(3\lambda_{CPU} + \lambda_V) = P(3)\mu + P(1)N + P(3)N \\ P_{2,1}(2\lambda_{CPU} + \lambda_V) = P(2)\mu + P(3)3\lambda_C \\ P_{1,1}(N) = P(1)2\lambda_{CPU} \end{cases}$$



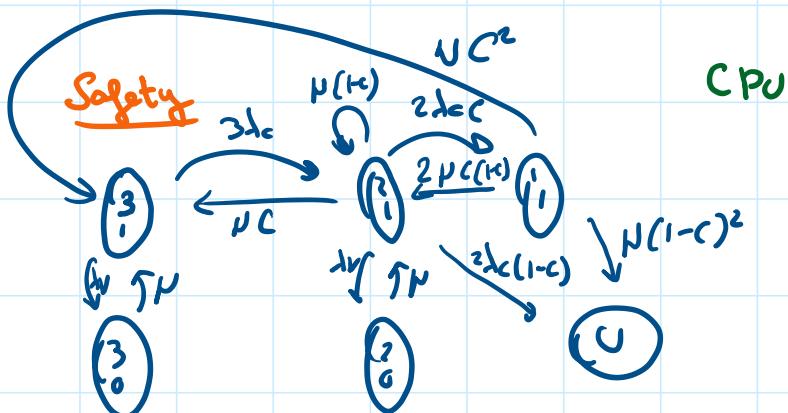
$$\left\{ \begin{array}{l} P_{1,0}(N) = P(1)2\lambda_{bus} \\ P_{3,0}(N) = P(3)\lambda_{bus} \\ P_{2,0}(N) = P(2)\lambda_{bus} \\ \sum_i P_i = 1 \end{array} \right.$$

$$A_C = P(3) + P(1)$$

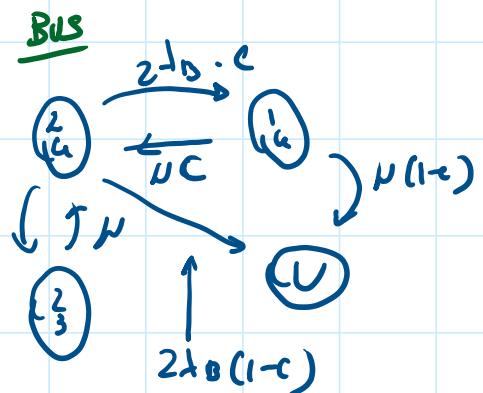
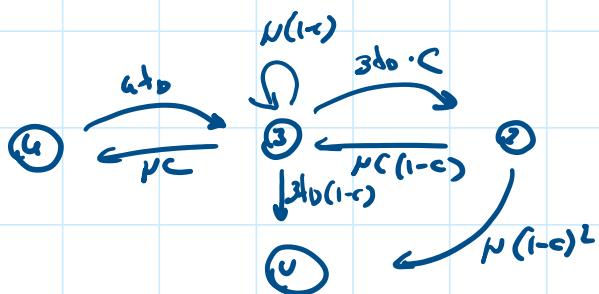
Bus



$$\left\{ \begin{array}{l} P(3)_N (2\lambda_B + \lambda_U) = P(1)_N + P(3)_N \\ P(1)_N = P(4)2\lambda_B \\ P(3)_N = P(1)\lambda_U \\ \sum_i P_i = 1 \end{array} \right.$$



DISK



Esercizio 11.3

$$R, \bar{x}$$

$$M/M/1/4/6$$

$$\tau = 30 \text{ sec}$$

$$S = 20 \text{ sec}$$

$$N = 1/S = 0,05 \text{ req/s}$$



$$\left\{ \begin{array}{l} 6/7 P_0 = \mu P_1 \\ 5/7 P_1 = \mu P_2 \\ 4/7 P_2 = \mu P_3 \\ 3/7 P_3 = \mu P_4 \\ \sum_i P_i = 1 \end{array} \right.$$

$$P_0 = 1 / \left[1 + \frac{6}{7\mu} + \frac{6}{7\mu} \cdot \frac{5}{7\mu} + \frac{6}{7\mu} \cdot \frac{5}{7\mu} \cdot \frac{4}{7\mu} + \frac{6}{7\mu} \cdot \frac{5}{7\mu} \cdot \frac{4}{7\mu} \cdot \frac{3}{7\mu} \right]$$

35,46

 P_0 P_1 P_2 P_3 P_4

$$\bar{x} = \sum i \cdot P_i$$

$$N = \sum i \cdot P_i$$

$$\bar{R} = \frac{N}{\bar{x}}$$

Esercizio 6

$$\lambda = 10 \text{ req/s}$$

$$SCN = 10 \text{ ms}$$

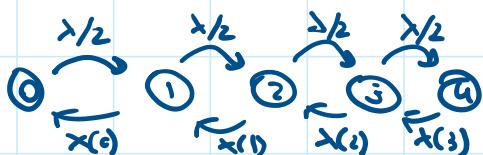
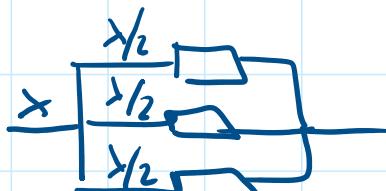
$$V = 2$$

$$S_{10} = 10 \text{ ms}$$

max 6 users

$$MTTF = 100000 \text{ s}$$

$$MTTR = 10 \text{ s}$$



Caso $\mu = 1$

Esercizio 2

Hip: queste e riparazioni indipendenti

2 parallel server

$\tau = 100 \text{ sec}$

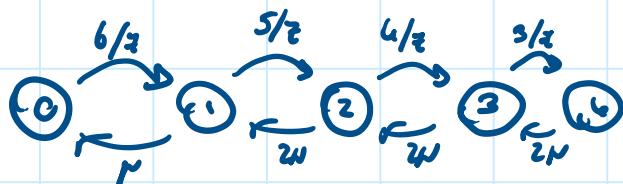
MTTF = 3 weeks

MTTR = 1 hour

$$A = \frac{\tau_2}{\tau_1 + \tau_2} = 0.38$$

$$A_2 = 0.38^2 = 0.36$$

$$A_1 = (0.58)(1 - 0.38) = 0.03$$



Due server online

$$\begin{cases} P_0 \cdot 6/\tau = P_1 \mu \\ P_1 \cdot 5/\tau = P_2 \mu \\ P_2 \cdot 4/\tau = P_3 \mu \\ P_3 \cdot 3/\tau = P_4 \mu \end{cases}$$

$$B = 1 / [1 + 0.06 + 1.5 \cdot 10^3 + 3 \cdot 10^{-5} + 4.5 \cdot 10^{-7}]$$

$$P_1 = 0.05$$

$$P_2 = 1.41 \cdot 10^{-3}$$

$$P_3 = 2.88 \cdot 10^{-5}$$

$$P_4 = 4.23 \cdot 10^{-7}$$

$$N = \sum i P_i = 0.05$$

$$X = \sum P_i \cdot \tau_i = 0.05$$

$$R = 1$$

Un server online

$$\begin{cases} P_0 \cdot 6/\tau = P_1 \mu \\ P_1 \cdot 5/\tau = P_2 \mu \\ P_2 \cdot 4/\tau = P_3 \mu \\ P_3 \cdot 3/\tau = P_4 \mu \end{cases}$$

$$P_0 = 1 / [1 + 0.06 + 3 \cdot 10^{-3} + 1.2 \cdot 10^{-4} + 3.6 \cdot 10^{-6}] = 0.96$$

$$P_1 = 0.05$$

$$P_2 = 2.5 \cdot 10^{-3}$$

$$P_3 = 1.10^{-4}$$

$$P_4 = 0.0006$$

$$\begin{cases} P_3 \cdot \frac{1}{2} = P_{4,1} \\ P_3 = 1 \cdot 10^{-6} \\ P_4 = 3 \cdot 10^{-6} \end{cases}$$

$$N = \sum i \cdot P_i = 0,05$$

$$X = \sum N \cdot P_i = 0,05$$

$$k \geq 1$$

$$R = (A_2)(R_2) + (A_{\text{angle}})(R_1) \approx 1$$

Esercizio 11/11/2014

lunedì 27 maggio 2019

10:47

SLA:

0.2 seconds Response time

2% request loss

$$\begin{cases} 10 \text{ ms} & 0:00 - 12:00 \\ 20 \text{ ms} & 12:00 - 24:00 \end{cases}$$

Prime provider

$$D_{CPU} = 10 \text{ ms}$$

$$D_{DISK} = 30 \text{ ms} \text{ of I/O disk}$$

$$3 \text{ clients max}$$

$$12 \text{ euro } \times \text{day}$$

Second provider

$$D_{CPU} = 20 \text{ ms}$$

$$D_{DISK} = 20 \text{ ms}$$

$$4 \text{ clients max}$$

$$0.5 \text{ euro}$$

Availability:

$$A = \frac{24}{25} = 0.96$$

$$A_{1/1} = 0.92$$

$$A_{1/2} = 0.0768$$

99%

Prime provider



Case u=1

$$D_{CPU} = 10$$

$$D_{DISK} = 30 \cdot 10^{-3}$$

$$R_{CPU} = 40 \cdot 10^{-3}$$

$$X_1 = 1/10 = 0.1$$

$$N_{CPU,1} = X \cdot R_{CPU} = 0.25$$

$$N_{DISK,1} = X \cdot R_{DISK} = 0.75$$

$$\begin{cases} P_0 \lambda/2 = P_1 \cdot X_1 \\ P_1 \lambda/2 = P_2 \cdot X_2 \\ P_2 \lambda/2 = P_3 \cdot X_3 \\ \sum_i P_i = 1 \end{cases}$$

Case u=2

$$D_{CPU} = R_{CPU} (1 + 0.25) = 12.5 \cdot 10^{-3}$$

$$D_{DISK} = 30(1 + 0.35) = 52.5$$

$$X_2 = 30,7$$

$$N_{CPU,2} = 30,7 \cdot 12,5 \cdot 10^{-3} = 0.38$$

$$N_{DISK} = 30,7 \cdot 52,5 \cdot 10^{-3} = 1,61$$

$$P_0 = 1 / [1 + 0.4 + 0.13 + 0.03] = 0.64$$

$$P_1 = 0.25$$

$$P_2 = 0.08$$

$$P_3 = 0.02$$

$$N = \sum_i P_i = 0.67$$

$$X = \sum_i P_i \cdot X_i = 9,35$$

$$R = u/X = 0.05$$

Case u=3

$$D_{CPU} = R_{CPU} (1 + 0.38) = 13.8 \cdot 10^{-3}$$

$$D_{DISK} = R_{DISK} (1 + 1.61) = 58,3 \cdot 10^{-3}$$

$$X_3 = 32,4 \Rightarrow 32$$

$$N_{CPU} = 32,4 \cdot 13,8 \cdot 10^{-3} = 0,45$$

$$N_{DISK} = 32,4 \cdot 58,3 \cdot 10^{-3} = 2,15$$

$$\begin{cases} P_0 \lambda = P_1 \cdot X_1 \\ P_1 \lambda = P_2 \cdot X_2 \\ P_2 \lambda = P_3 \cdot X_3 \\ \sum_i P_i = 1 \end{cases}$$

$$P_0 = 1 / [1 + 0.8 + 0.53 + 0.30] = 0.59$$

$$P_1 = 0.304$$

$$P_2 = 0.2$$

$$P_3 = 0.11$$

$$\left\{ \begin{array}{l} P_1 x = P_2 x_2 \\ P_2 x = P_3 x_3 \end{array} \right.$$

$$\begin{aligned} P_1 &= 0,300 \\ P_2 &= 0,2 \\ P_3 &= 0,11 \end{aligned}$$

$$N = 1,01$$

$$X = 17,11$$

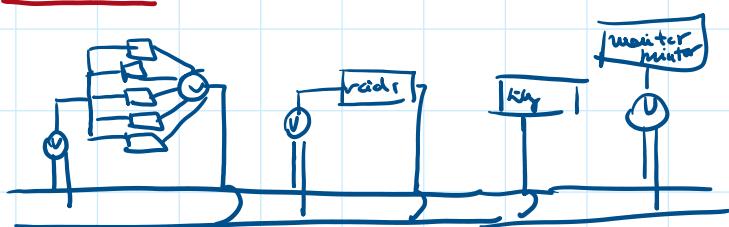
$$R = 0,06$$

$$\overline{P} = A_2(R_2) + A_{1/2}(R_1) = 0,05 \text{ RC}$$

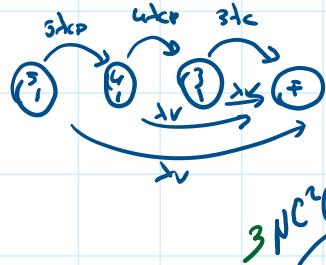
$$G_{\text{RC}} = A_{2/2} \cdot 0,02 + A_{1/2} \cdot 0,11 = 0,026$$

Costo 26 euro

Esercizio 1

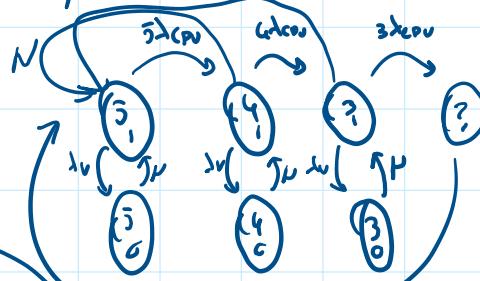


Reliability CPU



$$3\lambda U^2(1-c)$$

Availability CPU

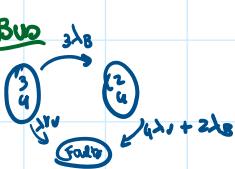


$$\binom{5}{2}$$

$$5 \cdot 4 \cdot 3 \cdot 2$$

$$\frac{5!}{(5-2)! \cdot 2!} = \frac{120}{12} = 10$$

$$\frac{5!}{(5-3)! \cdot 3!} = \frac{120}{2 \cdot 6} = 10$$

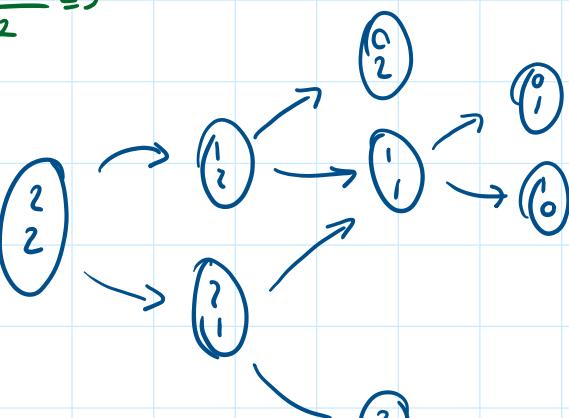
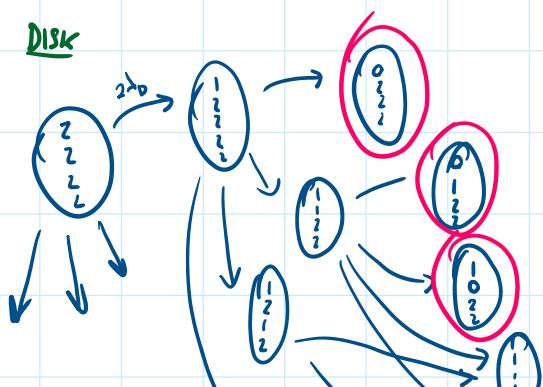


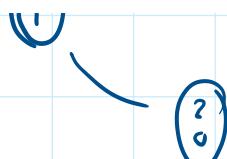
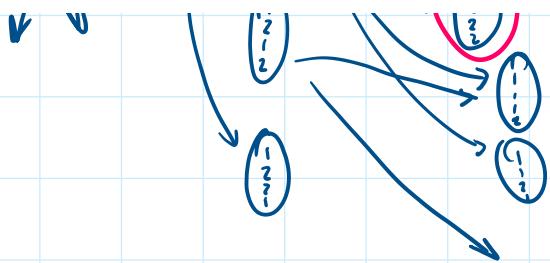
(2) quanti me vuo riporre
(1) quanti te de ve rompere

$$\binom{3}{2} = \frac{6}{2}$$

$$\binom{3}{1} = \frac{6}{2} = 3$$

DISK





Esercizio 2



$$\mu = \frac{1}{15} = 0.2$$

$$\begin{cases} P_0 \cdot \frac{7}{15} = P_1 \cdot \mu \\ P_1 \cdot \frac{6}{15} = P_2 \cdot \mu \\ P_2 \cdot \frac{5}{15} = P_3 \cdot \mu \\ P_3 \cdot \frac{4}{15} = P_4 \cdot \mu \\ P_4 \cdot \frac{3}{15} = P_5 \cdot \mu \end{cases}$$

$$P_0 = \frac{1}{[1 + 3.5 + 3.25 + 4.375]} = 0.05$$

$$\begin{aligned} P_1 &= 0.175 \\ P_2 &= 0.24 \\ P_3 &= 0.21 \\ P_4 &= 0.14 \\ P_5 &= 0.07 \end{aligned}$$

$$N = \sum_i i \cdot P_i$$

$$X = \sum_i \mu \cdot P_i$$