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Crosstalk Cancellation in xDSL Systems

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Abstract

Near-end crosstalk (NEXT) is one of the major impairments to the current ADSL downstream transmission. This paper presents two methods for an ADSL receiver to cancel one (dominant) NEXT signal from other types of services (such as HDSL, SDSL, T1, etc). The methods exploit the fact that the crosstalk signal has a large excess bandwidth and its spectra in the main lobe and in the excess band are strongly correlated. The principal idea is then to estimate the crosstalk in some frequency bands (e.g., excess band) and cancel it in other frequency bands (e.g., main lobe). The frequency-domain analysis in this paper provides an intuitive explanation of the crosstalk estimation and cancellation, as well as a guidance to select the right frequency bands to observe the crosstalk signal. Moreover, a fast algorithm is proposed for practical implementation. This algorithm avoids matrix inversion and large matrix multiplication in every transmission block. Simulation results show that one of the proposed methods, MMSE estimation and cancellation, is very effective to cancel one (dominant) NEXT and the improvement is significant in terms of the data rate and the line reach for the ADSL service. For example, using a real measured NEXT transfer function, the proposed method can increase the ADSL downstream data rate by 200% for some loops. The methods are extended to estimate and cancel two or more crosstalkers. The amount of improvement depends on the crosstalkers' characteristics and it is generally less than that of a single crosstalker case.

Keywords

DSL, crosstalk, excess bandwidth, decision-aided cancellation, MMSE estimation and cancellation.

I. Introduction

Digital subscriber line (DSL) technology uses the existing phone lines to offer high speed data transmission services to both residential and business customers. There are many types of DSLs [1], generically referred to as xDSL, including basic rate DSL (ISDN), HDSL (high-bit-rate DSL), ADSL (asymmetric DSL), HDSL2 (second generation HDSL), SDSL (single-pair, symmetric DSL), and VDSL (very-high-bit-rate DSL). Of these DSLs, ISDN, ADSL, and HDSL have been standardized by International Telecommunication Union (ITU). ITU-T Recommendation G.995.1 [3] provides a comprehensive overview of these standardized recommendations. HDSL2 and VDSL are currently in the process of being standardized. SDSL is not standardized but has been deployed to offer various data rate less than 1.536Mbps.

One of the major impairments of the xDSL systems is the severe crosstalk [2] among the telephone lines in the same or neighboring bundles. The crosstalk is classified into near-end crosstalk (NEXT) and far-end crosstalk (FEXT). In general, NEXT is much larger than FEXT because the interference source is closer to the receiver. Therefore, ADSL and VDSL use frequency division duplexing (FDD) to avoid NEXTs from the same services. However, other types of services (such as HDSL, SDSL, T1, etc.), which use different duplexing schemes and overlap in frequency with ADSL and VDSL, may produce detrimental NEXT. Mitigating the effect of NEXT in an ADSL receiver can dramatically increase the data rate, the line reach, or the system operational margin.

The optimum detector for the interference cancellation can be theoretically achieved by maximizing the *a posteriori* probability of the primary signal (MAP rule), which is unfortunately too complex in practice. Some suboptimal multiuser detectors [4][5][6][7] are proposed to mitigate or cancel the interference signal in the non-spreading system. These detectors decode each user's data using "soft" symbols and iterate the detection process until a certain criterion is reached (e.g., the maximum number of iterations). The convergence of this type of algorithms is an open problem. The algorithms are still very complex when the signals have large constellation sizes [7].

This paper presents new practical methods to cancel or mitigate one (dominant) NEXT for an ADSL receiver. The principal idea is to estimate the crosstalk signal in certain frequency bands and subtract it in other frequency bands. A similar idea [8][9] has been previously used to suppress *very* narrow band radio frequency interference (RFI) in VDSL systems. The crosstalk signal to an ADSL receiver has large excess bandwidth and its spectra in the main lobe and the excess band are strongly correlated, which gives the opportunity to cancel the crosstalk signal in some dependent frequency bands. For example, the crosstalk can be estimated in the excess band and cancelled in the main lobe; vice versa. This paper provides a guidance on how to select the best frequency bands to observe the crosstalk signal and an intuitive interpretation of the crosstalk cancellation process. Another important aspect of the proposed techniques is that they can be implemented with low computational complexity, without matrix inversion or large matrix multiplication in each transmission block.

Previously, the fractionally-spaced equalizer (FSE) was used to suppress cyclostationary NEXTs [10][11] if both the crosstalk signals and the primary signal are synchronized and have excess bandwidth. The FSE processes the signals' spectrum in both the main band and the excess band. The folded spectrum after resampling to the symbol rate then provides the flexibility to suppress NEXTs in the main band. The problem addressed in this paper is different mainly in the following two aspects. First, the primary signal (ADSL) and the crosstalk signal (such as NEXT from HDSL, SDSL, T1) have completely different modulation schemes and sampling rates. Second, the primary received signal is decoded in the frequency domain, thus the crosstalk signal suppression is also processed in the frequency domain. In fact, the frequency domain explanation in this paper gives an insight on how much NEXT can be suppressed.

This paper proceeds as follows. Section II describes the system model of the primary and crosstalk channels. Section III presents the methods to cancel the crosstalk signal and a fast computation scheme for practical implementation. The methods are then extended to estimate and cancel two or more crosstalkers. Simulation results are shown in Section IV to verify the proposed methods. Section V concludes the paper.

In this paper, the notations are arranged in the following convention. A small letter, a bold small letter, and a capital letter represent a scalar, a vector, and a matrix, respectively. The superscript symbols ^{T} and * represent "transpose" and "conjugate and transpose" operations, respectively.

II. System Model

ADSL [12] uses the discrete multiple tone (DMT) modulation scheme [13][14] for data transmission. DMT is an effective realization of multicarrier transmission [15][16][17], which partitions the intersymbol interference (ISI) channel into a large number of narrowband subchannels. There is no or little ISI in each subchannel if the bandwidth of the subchannel is sufficiently narrow. The data is then transmitted in each subchannel almost free of ISI. A subchannel is more often called a "tone" in DMT systems and this terminology will be used in the rest of this paper.

The crosstalk signal from HDSL, SDSL, T1, or ISDN has different modulation schemes. HDSL, SDSL, and ISDN use 2B1Q baseband transmission and T1 uses alternative mark inversion (AMI) baseband transmission.

Fig. 1 shows a general model of a primary DMT transmission system with one crosstalker.

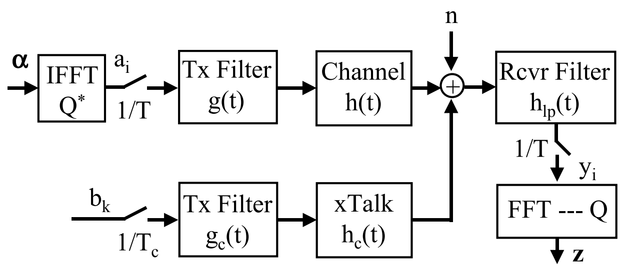

Fig. 1. **The primary and crosstalk channel model.**

The cyclic prefix is not shown in the figure. The primary channel has a sampling rate of $1/T$, but the interference signal has a sampling rate of $1/T_c$ at the transmitter. As a result, the received crosstalk signal is not stationary. With this general model, the received time-domain output is

$$y(t) = \sum_i a_i p(t - iT) + \sum_k b_k c(t - kT_c + \tau) + \underbrace{n(t) * h_{lp}(t)}_{\tilde{n}(t)} \tag{1}$$

where a_i and b_k are the primary transmitted signal and the crosstalk transmitted signal, respectively. τ $(0 < \tau < T_c)$ is a fractional timing difference between the transmitted primary signal and the crosstalk signal. $p(t)$ and $c(t)$ are the aggregated channel and crosstalk responses, respectively. Mathematically (refer to Fig. 1),

$$p(t) = g(t) * h(t) * h_{lp}(t) \tag{2}$$

$$c(t) = g_c(t) * h_c(t) * h_{lp}(t). \tag{3}$$

Both are causal and have finite impulse responses. After sampling, the discrete output is

$$y_m = \sum_i a_i p\left((m-i)T\right) + \sum_k b_k c(mT - kT_c + \tau) + \tilde{n}_m. \tag{4}$$

In DMT systems, the data is transmitted in a packet fashion. Suppose the packet size (FFT size) is M, the above equation in one block can be represented compactly in a matrix form:

$$\mathbf{y} = P\mathbf{a} + C\mathbf{b} + \tilde{\mathbf{n}}, \tag{5}$$

where $\mathbf{y} = [y_{M-1}, y_{M-2,} \cdots, y_0]^T$, $\mathbf{a} = [a_{M-1}, a_{M-2,} \cdots, a_0]^T$, $\mathbf{b} = [b_{L-1}, \cdots, b_{0,} \cdots, b_{-\mu}]^T$, $\tilde{\mathbf{n}} = [\tilde{n}_{M-1}, \tilde{n}_{M-2,} \cdots, \tilde{n}_0]^T$, P and C are the channel and crosstalk responses matrices, respectively. The DMT signal has a cyclic prefix ($a_{M-i} = a_{-i}$, $i = 1, \cdots, \nu$), therefore the channel response matrix P is circulant and has the following form,

$$P = \begin{bmatrix} p_0 & \cdots & p_{\nu-1} & p_\nu & 0 & \cdots & 0 \\ 0 & p_0 & \cdots & p_{\nu-1} & p_\nu & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & p_0 & \cdots & p_{\nu-1} & p_\nu \\ p_\nu & 0 & \cdots & 0 & p_0 & \cdots & p_{\nu-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ p_1 & \cdots & p_\nu & 0 & \cdots & 0 & p_0 \end{bmatrix}$$

where $\nu + 1$ is the number of taps of the channel response $p(t)$. The crosstalk matrix $C_{M\times(L+\mu)}$ is

$$C = \begin{bmatrix} c(\tau - (L-1)T_c + (M-1)T) & \cdots & c(\tau + (M-1)T) & \cdots & c(\mu T_c + \tau + (M-1)T) \\ c(\tau - (L-1)T_c + (M-2)T) & \cdots & c(\tau + (M-2)T) & \cdots & c(\mu T_c + \tau + (M-2)T) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c(\tau - (L-1)T_c + T) & \cdots & c(\tau + T) & \cdots & c(\mu T_c + \tau + T) \\ c(\tau - (L-1)T_c) & \cdots & c(\tau) & \cdots & c(\mu T_c + \tau) \end{bmatrix}$$

where $\mu + 1$ is the number of taps of the crosstalk response, L is the number of crosstalk symbols in one DMT block ($L = \lceil MT/T_c \rceil$). Note that there are many zero entries in the above matrix C. Since the delay τ changes block over block, the matrix C varies over different blocks.

With a cyclic prefix, the circulant matrix P can always be decomposed [18, p. 201-2] as

$$P = Q^* \Lambda Q \tag{6}$$

where Q is a fast Fourier transform (FFT) matrix, Λ is a diagonal matrix whose diagonal elements correspond to the frequency response of the channel. More specifically, the FFT matrix is

$$Q = \frac{1}{\sqrt{M}} \begin{bmatrix} e^{-j\frac{2\pi}{M}(M-1)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-1)} & 1 \\ e^{-j\frac{2\pi}{M}(M-2)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-2)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j\frac{2\pi}{M}(M-1)} & \cdots & e^{-j\frac{2\pi}{M}} & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}$$

and the diagonal elements of Λ from the top left to the right bottom are

$$diag(\Lambda) = Q \cdot \begin{bmatrix} 0 \\ \vdots \\ p_\nu \\ \vdots \\ p_0 \end{bmatrix}.$$

At the transmitter, the signal $\boldsymbol{\alpha}$ is modulated in the frequency domain and transformed into the time domain by an inverse FFT for transmission, i.e., $\mathbf{a} = Q^*\boldsymbol{\alpha}$. At the receiver, the received signal is transformed back to the frequency domain. Therefore, the whole system model in the frequency domain is

$$\mathbf{z} = Q\mathbf{y} = \Lambda\boldsymbol{\alpha} + QC\mathbf{b} + \mathbf{n}. \tag{7}$$

where $\mathbf{z} = [z_{M-1},\ z_{M-2,}\ \cdots, z_0]^T$, and the white Gaussian noise $\mathbf{n} = Q\tilde{\mathbf{n}}$ has the same variance as $\tilde{\mathbf{n}}$. The traditional system treats the crosstalk signal as Gaussian interference, which significantly limits the overall system performance. This paper presents new methods to cancel or suppress the crosstalk component $QC\mathbf{b}$ in the received signal. The channel and crosstalk responses $p(t)$ and $c(t)$ are assumed to be known in the ADSL receiver. The channel response is obtained through training sequences and the crosstalk response can be acquired by the method in [20].

III. Crosstalk Cancellation

The principal idea of crosstalk cancellation in this section is to first estimate the crosstalk signal by observing the output of certain frequency bands and then reconstruct the crosstalk signal in other bands to cancel out the interference. If the crosstalk signal has an excess bandwidth, like HDSL and SDSL crosstalkers in the xDSL systems, the cancellation is possible because the excess bandwidth provides more information about the crosstalk signal. In this section, the basic idea to cancel the crosstalk signal is described under the terminology of the above system model. A geometrical interpretation is introduced to explain the cancellation process intuitively and to select the right frequency bands to observe the crosstalk signal. Then, a fast computational method is proposed for practical implementation, which avoids the matrix inversion in the crosstalk signal estimation and cancellation. Finally, the method is extended to more than one crosstalker.

A. Crosstalk Signal Estimation and Cancellation

ADSL uses DMT modulation and the received signal is processed in the frequency domain. In order to detect and cancel the crosstalk signal, all the tones of interest[1] are partitioned into two disjoint sets, S_1 and S_2. In set S_1, the DMT system treats the crosstalk signal as a Gaussian noise, like a traditional system. The primary signal is detected and subtracted from the received signal. Then the crosstalk signal $\mathbf{b}$ is estimated by observing the residual signal in set S_1. With the estimated crosstalk signal $\tilde{\mathbf{b}}$, the interference in set S_2 can be constructed and subtracted from the received signal. If the interference in set S_2 can be completely eliminated, then the primary DMT system will have a significantly higher signal to interference and noise ratio (SINR) and more bits can be transmitted in this set of tones. The channel model representation in (7) is re-grouped according to the partition:

$$\begin{bmatrix} \mathbf{z_2} \\ \mathbf{z_1} \end{bmatrix} = \begin{bmatrix} \Lambda_2 \boldsymbol{\alpha_2} \\ \Lambda_1 \boldsymbol{\alpha_1} \end{bmatrix} + \begin{bmatrix} Q_2 C \mathbf{b} \\ Q_1 C \mathbf{b} \end{bmatrix} + \begin{bmatrix} \mathbf{n_2} \\ \mathbf{n_1} \end{bmatrix} \tag{8}$$

where every vector or matrix with the subscript belongs to the set with the same subscript. For example, $\mathbf{z_1}$ and $\mathbf{z_2}$ are the received data in sets S_1 and S_2, respectively. Assume the primary signal in set S_1 can be detected reliably and denote $\tilde{\mathbf{z}}_\mathbf{1} = \mathbf{z}_1 - \Lambda_1 \boldsymbol{\alpha_1}$, then crosstalk signal can be estimated by a linear minimum mean-squares error (MMSE) estimator [19, p. 95] as

$$\tilde{\mathbf{b}} = (R_{\mathbf{b}}^{-1} + C^* Q_1^* R_{\mathbf{n}}^{-1} Q_1 C)^{-1} C^* Q_1^* R_{\mathbf{n}}^{-1} \tilde{\mathbf{z}}_\mathbf{1} \tag{9}$$

[1]Only includes those tones where the crosstalk signal exists, not all the tones in the ADSL receiver.

where $R_{\mathbf{b}}$ and $R_{\mathbf{n}}$ are the signal and noise covariance matrices respectively. The transmitted crosstalk signal sequence $\mathbf{b}$ and the background noise are normally assumed to be white, and the crosstalk signal can be assumed to have unit variance without loss of generality. Then the above equation can be further simplified to

$$\tilde{\mathbf{b}} = (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_1^* \tilde{\mathbf{z}}_{\mathbf{1}} \tag{10}$$

where σ_n^2 is the background noise variance. There are several different approaches to construct the crosstalk signal in set S_2. Here are the two simple approaches.

1. Linear MMSE estimation and cancellation. The receiver directly uses the estimated signal from (10) to construct the interference signal:

$$\begin{aligned} X_c &= Q_2 C \tilde{\mathbf{b}} \\ &= Q_2 C (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_1^* \tilde{\mathbf{z}}_{\mathbf{1}}. \end{aligned} \tag{11}$$

and subtracts it from the received signal in set S_2. In fact, $\tilde{\mathbf{b}}$ can be considered as a special kind of "soft" decision for interference cancellation. The error covariance matrix of the estimated crosstalk signal from (10) is

$$\begin{aligned} \epsilon_b &= E(\tilde{\mathbf{b}} - \mathbf{b})(\tilde{\mathbf{b}} - \mathbf{b})^* \\ &= \sigma_n^2 (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1}. \end{aligned} \tag{12}$$

Then the error covariance of the constructed interference is

$$\epsilon_X = \sigma_n^2 Q_2 C (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_2^*. \tag{13}$$

2. Decision-aided cancellation. The receiver makes "hard" decision on $\tilde{\mathbf{b}}$ to decode the transmitted crosstalk signal in set S_1, and constructs the interference signal in set S_2 based on the decision. The disadvantage is that a decision error would double the detrimental impact on the constructed interference. The improvement can be achieved by making "soft" decision with extra computational complexity.

The first approach works better if the transmitted crosstalk signal can not be reliably decoded. For example, the HDSL crosstalker signal includes the low-frequency band $[0 - 26kHz]$, which is eliminated by the front-end filter of the ADSL receiver. As a result, the

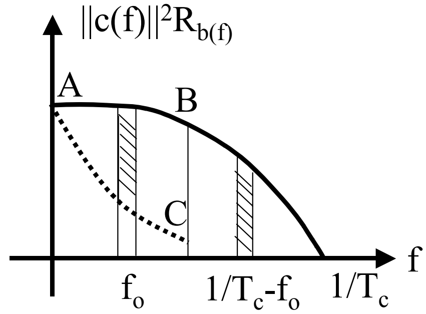

Fig. 2. **Crosstalk estimation and cancellation.**

transfer function for the crosstalk signal has a null response in this band, which would produce many incorrect decisions if the crosstalk signal were decoded. The decision-aided cancellation, however, works better if the interference to noise ratio in set S_1 is relatively high such that most of the decisions are correct. Inherently, this approach takes advantage of the known discrete constellation of the crosstalk source signal.

B. Geometrical Interpretation and Tone Selection

A natural question arises on how to select the best frequency bands to estimate the crosstalk signal and to mitigate the interference in the other bands. The answers for the two approaches above will be described separately in the following. The crosstalk signal uses baseband transmission, therefore the signal in the negative frequency is a conjugate of the counterpart in the positive frequency. For simplicity, this paper describes the method in the single-sided positive frequency band.

B.1 MMSE estimation and cancellation

The right strategy, which will be justified later, is as follows:

1. calculate SINRs in frequencies f_o and $1/T_c - f_o$, where T_c is the crosstalk signal sampling period;
2. estimate the crosstalk signal in the frequency bin that has a smaller SINR;
3. cancel the interference in the other bin that has the larger SINR.

In Fig. 2, the solid curve shows the power spectral density (PSD) of the crosstalk signal, with 100% excess bandwidth. The crosstalk signal is estimated in the excess band and then used to cancel the interference in the main lobe $(0, 1/2T_c)$. The dotted curve $\overline{AC}$ shows the residual crosstalk PSD as a result of the crosstalk subtraction. If the smaller one of the interference to noise ratios in frequencies f_o and $1/T_c - f_o$ is denoted by g_s (the shaded zone in the figure), then the SINR gain in frequency f_o is approximately equal to g_s. Similarly, if the crosstalk signal is estimated in the main lobe and used to cancel the interference in the excess band, the SINR gain in frequency $1/T_c - f_o$ is also approximately equal to g_s. These statements will be proved later in this subsection.

In fact, the gain is the same no matter which frequency, either f_o or $1/T_c - f_o$, is used to estimate the crosstalk signal. However, the data rate increase is not the same because it depends on the original SINR. For example, if the original SINR in frequency $1/T_c - f_o$ is $-9dB$ and the gain resulting from crosstalk cancellation is also $9dB$, then the capacity is increased by about $0.5bits/s/Hz$; if the original SINR in frequency f_o is $10dB$ and the gain is the same, then the capacity is increased by about $1.5bits/s/Hz$. Therefore, the crosstalk signal should be estimated in frequency $1/T_c - f_o$ and cancelled in frequency f_o. Otherwise, the data rate improvement is smaller ($0.5b/s/Hz$ versus $1.5b/s/Hz$). This phenomenon could actually happen in an ADSL environment. When the line reach is very long, the ADSL signal in the excess band is weak compared to the NEXT signal, therefore, the crosstalk signal should be estimated in the excess band and cancelled in the main lobe. Correspondingly, if the primary signal has a smaller SINR in the main lobe for some reason (e.g., because of the bridge tap), the crosstalk signal should be first estimated in the main lobe and cancelled in the excess band.

The following provides mathematically the rationale of the above statements. The problem is easier to understand in the frequency domain instead of the matrix representation in (11). The short-term Fourier transform (STFT) of the system model in (1) is defined as

$$\begin{aligned} y(f) &= \frac{1}{\sqrt{MT}} \int_0^{MT} y(t) e^{-j2\pi f t} dt \\ &\approx p(f)a(f) + c(f)e^{j2\pi f \tau} b(f) + n(f) \end{aligned} \tag{14}$$

where $p(f)$ and $c(f)$ are the normal frequency responses of $p(t)$ and $c(t)$, respectively. The noise $n(f)$ is a stochastic process, which is the STFT of $\tilde{n}(t)$. The variance of $n(f)$ is equal to its PSD, i.e., $E(n(f)n^*(f)) = \sigma_n^2$. The approximation in (14) is due to the term of the crosstalk signal, because the crosstalk signal does not have the cyclic prefix and is not cyclostationary. This approximation is accepted for the following three reasons. First, the approximation error is small if M is relatively large and is exact if $M \to \infty$. Second, the analysis resulting from this approximation is used only for a guidance to select the right frequency bins to observe the crosstalk signal, not for the real crosstalk estimation. Third, this approximation makes the analysis much more succinct and provides an intuitive explanation of the estimation and cancellation process.

The primary and the crosstalk signals are

$$a(f) = \frac{1}{\sqrt{MT}} \sum_i a_i e^{-j2\pi f T i},$$
$$b(f) = \frac{1}{\sqrt{MT}} \sum_k b_k e^{-j2\pi f T_c k}.$$

The crosstalk signal has the following property: $b(f) = b^*(1/T_c - f)$, where T_c is the crosstalk symbol period. Therefore, if the crosstalk component in frequency $1/T_c - f_o$ is observed, the interference component in frequency f_o can be reconstructed.

In the following analysis, $0 < f_o < 1/(2T_c)$ is assumed without loss of generality. The crosstalk signal b_k is assumed to be white and is uncorrelated with the primary signal a_i. For brevity, symbol f_d denotes frequency $1/T_c - f_o$. The linear MMSE estimation of $b(f_d)$, given the observation $y(f_d)$, is

$$\tilde{b}(f_d) = \frac{c^*(f_d) e^{-j2\pi f_d \tau} R_b}{||c(f_d)||^2 R_b + \sigma_n^2} \left(y(f_d) - p(f_d)a(f_d)\right) \tag{15}$$

where $R_b = E(b_k b_k^*)L/(MT) \approx \varepsilon_b/T_c$, and $||c(f_d)||^2 R_b$ is the PSD of the transmitted crosstalk signal at frequency f_d. The reconstructed interference in frequency f_o is then

$$x_c(f_o) = c(f_o) e^{j2\pi f_o \tau} \tilde{b}^*(f_d). \tag{16}$$

The variance of the cancellation residual error plus the background noise is

$$\sigma_t^2(f_o) = \sigma_n^2 \frac{||c(f_o)||^2 R_b}{||c(f_d)||^2 R_b + \sigma_n^2} + \sigma_n^2. \tag{17}$$

The original error variance without cancellation is

$$\sigma_x^2(f_o) = ||c(f_o)||^2 R_b + \sigma_n^2. \tag{18}$$

Therefore, the SINR gain is equal to

$$\begin{aligned} Gain &= \frac{\sigma_x^2(f_o)}{\sigma_t^2(f_o)} \\ &= 1 + \frac{||c(f_o)||^2||c(f_d)||^2 R_b^2}{\sigma_n^2(||c(f_o)||^2 R_b + ||c(f_d)||^2 R_b + \sigma_n^2)} \qquad (19) \\ &= 1 + \frac{PSD(f_o)\cdot PSD(f_d)}{\sigma_n^2\left(PSD(f_o) + PSD(f_d) + \sigma_n^2\right)} \qquad (20) \end{aligned}$$

where $PSD(f_o)$ is the interference PSD at frequency f_o.

Proposition 1: Suppose the PSDs of the primary signal and the background noise are fixed. Given the dual frequencies f_o and f_d, the SINR gain by the MMSE estimation and cancellation is the same no matter which frequency is used for crosstalk estimation. However, with respect to the data rate improvement, the crosstalk should be estimated in the frequency with a lower SINR and cancelled in its dual frequency.

Proof: The SINR gain in (20) is derived by estimating the crosstalk signal in frequency f_o and cancelling it in frequency f_d. Since the SINR gain is symmetric with respect to $PSD(f_o)$ and $PSD(f_d)$ as seen in (20), the same improvement in terms of SINR is obtained if the crosstalk component in frequency f_o is estimated first and the interference in frequency f_d is cancelled next.

The data rate increase ΔR is

$$\begin{aligned} \Delta R &= W \log_2\left(\frac{1+\frac{SINR\cdot Gain}{\Gamma}}{1+\frac{SINR}{\Gamma}}\right) \\ &= W \log_2\left(1 + \frac{(Gain-1)}{\frac{\Gamma}{SINR}+1}\right) \qquad (21) \end{aligned}$$

where Γ is the gap [21] from the capacity and W is the subchannel bandwidth. The data rate improvement ΔR is a monotonic increase function of SINR. The higher the original SINR is, the larger the data rate increase is. Therefore, the crosstalk should be estimated in the frequency with a lower SINR and cancelled in the frequency with a higher SINR. This is especially important when SINR is small in one frequency and is large in its dual frequency. ■

The following example illustrates two important special cases for the SINR gain.

Example 2: (a) The interference levels are the same in the dual frequencies and more than 10dB larger than noise, then the SINR gain is approximately

$$Gain \approx \frac{PSD(f_o)}{2\sigma_n^2},$$

which is equal to the interference to noise ratio minus $3dB$.
(b) The interference levels have large disparity, for example, $||c(f_o)||^2 > 10||c(f_d)||^2$, then the SINR gain is approximately

$$Gain \approx \frac{PSD(f_d)}{\sigma_n^2}$$

which is essentially equal to the interference to noise ratio of the smaller crosstalker.

In the xDSL systems, the crosstalk signals from HDSL or SDSL have a large percentage of excess bandwidth. Therefore, the crosstalk signal can be estimated first in the excess band and then be used to cancel the interference in the main lobe. The crosstalk in the excess band is generally smaller than those in the main lobe, which fits case (b) in the above example. The SINR gain in the main lobe is equal to the interference to noise ratio in the excess band. If there is no excess bandwidth, then there is no SINR gain at all!

B.2 Decision-based Approach

For a given pair of the dual frequencies (f_o and $1/T_c - f_o$), the best SINR gain by the MMSE approach above is equal to the smaller one of the interference to noise ratios in these two frequencies. The decision-based approach could do better if the crosstalk signal can be reliably detected. For example, if the crosstalk can be detected reliably in the excess band, then all the interference in the main lobe can be eliminated, as opposed to the residual interference (dotted line) shown in Fig. 2. In this approach, the best choices of the frequency bins for set S_1 are those with large enough interference to noise ratio for the reliable detection of the crosstalk signal. If there are no such choices, the MMSE approach should be used instead.

C. Fast Computation

The crosstalk channel response is changing block over block because the crosstalk signal and the primary signal have different sampling rates. Direct computation of the linear MMSE estimation in (10) requires matrix inversion in every block. A fast computation method is developed to avoid the large matrix inversion, with a slightly degraded performance. This method is based on the well-known fact that the delay in the time domain is equivalent to the phase shift in the frequency domain.

Denote C_0 as the crosstalk function matrix with zero delay ($\tau = 0$), then

$$Q_1 C \approx D_{\tau,1} Q_1 C_0 \tag{22}$$

where $D_{\tau,1}$ represents the phase shift components of the channel response,

$$D_{\tau,1} = \begin{bmatrix} e^{j2\pi \frac{k_1}{N} \frac{\tau}{T}} & 0 & \cdots & 0 \\ 0 & e^{j2\pi \frac{k_2}{N} \frac{\tau}{T}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & e^{j2\pi \frac{k_m}{N} \frac{\tau}{T}} \end{bmatrix}.$$

where $k_1, \cdots, k_m$ are the tone indices in set S_1. The approximation in (22) is because of the edge effect of the matrix. Asymptotically as $M \to \infty$, the approximation becomes an exact equation. The fractional delay τ is easy to infer block over block because the sampling rates of the primary and crosstalk signals are both known.

With this approximation, the estimated signal can be simplified as

$$\tilde{\mathbf{b}} \approx \Psi D^*_{\tau,1} \tilde{\mathbf{z}}_{\mathbf{1}} \tag{23}$$

where Ψ is a constant matrix

$$\Psi = (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* \tag{24}$$

which can be pre-computed and stored. This constant matrix avoids matrix inversion and large matrix multiplications in each transmission block. The multiplication $D^*_{\tau,1} \tilde{\mathbf{z}}_{\mathbf{1}}$ implies that the receiver adjusts the timing offset of the signal $\tilde{\mathbf{z}}_{\mathbf{1}}$. Multiplying the constant matrix Ψ roughly represents that the adjusted signal is passed through a linear MMSE filter. The reconstructed signal in (11) can also be simplified as

$$X_c \approx D_{\tau,2} \Phi D^*_{\tau,1} \tilde{\mathbf{z}}_{\mathbf{1}} \tag{25}$$

where Φ is another constant matrix

$$\Phi = Q_2 C_0 (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* \tag{26}$$

and $D_{\tau,2}$ is similar to $D_{\tau,1}$ except that it uses the tone indices of set S_2 in the diagonal, *e.g.*, $e^{j2\pi\frac{l}{N}\frac{\tau}{T}}, l \in S_2$. Similarly, the computational complexity is reduced dramatically because the constant matrix Φ can be pre-computed and stored.

D. Extension to Multiple Crosstalkers

The idea of estimating the crosstalk signal in one set of frequency bands and cancelling it in another set of frequency bands can be extended to more than one crosstalker. The system model remains the same as (8), but the crosstalk component $QC\mathbf{b}$ is modified slightly to include more crosstalkers, i.e.,

$$C = [C_1,\ C_2,\ \cdots, C_k]$$
$$\mathbf{b} = [\mathbf{b_1}, \mathbf{b_2}, \cdots, \mathbf{b_k}]^{\mathbf{T}}$$

where k is the number of the crosstalkers. The amount of performance improvement depends on the crosstalkers' characteristics and the choice of the cancellation approaches. The following example illustrates one particular approach of successive cancellation of the crosstalk signals by estimating the crosstalk signals in the excess band.

Example 3: The crosstalkers HDSL and SDSL have single-sided bandwidth of $192kHz$ and $520kHz$. Both of them have about 100% excess bandwidth. Therefore, SDSL can be estimated first in frequency band $[520-1040kHz]$ where the HDSL crosstalker signal does not exist. Then the SDSL crosstalker can be cancelled in the main lobe $[138-520kHz]$. After that, the same process is used to estimate and cancel the smaller-bandwidth HDSL crosstalk.

IV. Simulation Results

The ADSL downstream transmission is more vulnerable to NEXT, because the primary signal attenuates very rapidly while NEXT increases as frequency increases. The simulations are thus concentrated on the downstream receiver on the customer side. In the current deployment, the strong NEXT mainly comes from ISDN, HDSL, SDSL, T1 or their repeaters. For a given line, there is very likely only one dominant crosstalk because of the following two reasons. First, the crosstalk line should reside physically close to the victim line. Second, most services in the same bundle are deployed with ADSL. With the

xDSL	ADSL	HDSL
Line code	DMT	2B1Q
Sampling rate f_o $(ks/\sec)$	2208	392
Power (dBm)	19.0	13.6
Duplexing	$up: 26 - 138kHz$ $down: 138 - 1104kHz$	Dual

TABLE I

Main ADSL and HDSL Characteristics.

above two justifications, the simulations assume only one NEXT for a given line.

The main characteristics of ADSL and HDSL are summarized in Table I. For more information, refer to [1][3] and the references therein. SDSL [22] has the same characteristic as HDSL except that it offers variable symmetric data rate. The single-sided PSD of SDSL and HDSL is

$$PSD(f) = K \cdot \frac{2}{f_{sym}} \sin c^2 \left(\frac{\pi f}{f_{sym}} \right) \frac{1}{1 + \left(\frac{f}{\frac{240}{392} f_{sym}} \right)^8} \tag{27}$$

Watts/Hz, where f_{sym} is the symbol rate and $K = \frac{5}{9} \frac{2.7^2}{135}$. The PSD includes a 4th order lowpass butterworth filter whose $3dB$ attenuation occurs at frequency $240 f_{sym}/392$. For HDSL, $f_{sym} = 392kHz$ and $f_{3dB} = 240kHz$ (see [2]). The NEXT coupling functions are taken from the real measured data. Fig. 3 shows several dominant NEXT coupling functions for a given line. The thickest line in the figure is used here for simulation. The crosstalk transfer function is then a cascade response of the rectangular pulse, the butterworth filter, the NEXT coupling function, and the receiver lowpass filter. A linear phase is assumed in the NEXT coupling function.

The system parameters used to calculate the ADSL downstream data rate are summarized in Table II. The upstream and downstream bands for ADSL are shown in Table I. The total noise for the data rate computation is the sum of the background noise, 24

[2]In the original ADSL test procedures as specified in ITU-T G.996.1 Recommendations, $f_{3dB} = 192$kHz. This number has been changed in [22] to 240kHz.

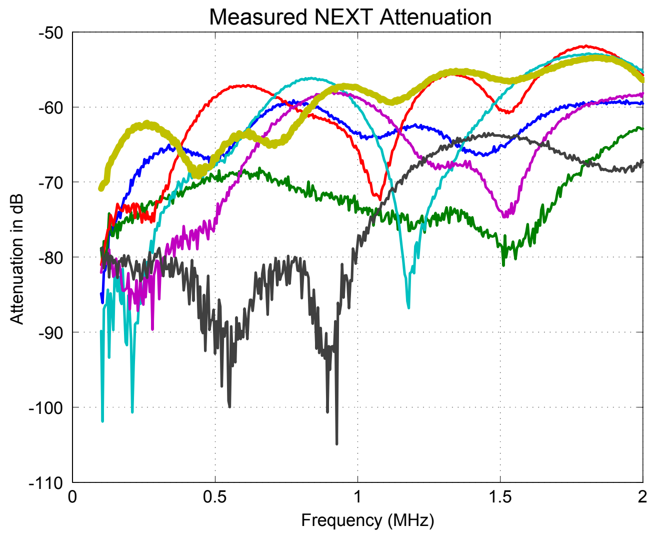

Fig. 3. **Measured NEXT attenuation for a given line.**

ADSL line type:	26 Gauge
AWGN (single-sided):	$-140dBm/Hz$
System margin:	$6dB$
Coding gain:	$3dB$
SNR-gap:	$\Gamma = 9.8dB$
# of self-FEXTs:	24

TABLE II

ADSL system parameters for data rate calculation.

FEXTs from ADSL services by the standard model [12], and one dominant NEXT from HDSL or SDSL.

A. HDSL NEXT Cancellation

HDSL has a relatively small bandwidth with respect to the ADSL signal. The crosstalk is estimated and cancelled by the proposed approaches in Section III-A. Fig. 4 shows three PSDs which correspond to the original NEXT, the residual NEXT using the decision-aided cancellation approach, and the residual NEXT using the MMSE estimation and cancellation approach. The crosstalk signal is observed in the HDSL excess band [198 −

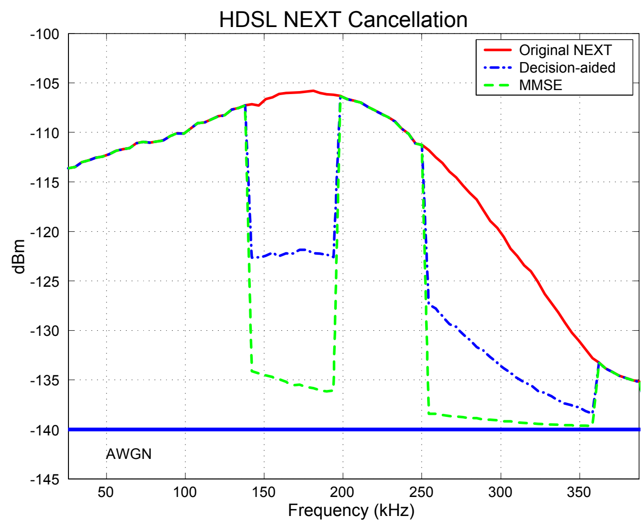

Fig. 4. **HDSL NEXT estimation and cancellation.**

$250kHz]$ and cancelled in the main band $[142 - 194kHz]$. In the observing band, the interference is treated as a Gaussian noise for the detection of the primary signal and no improvement has been achieved. However, in the main lobe $[142 - 194kHz]$, the improvement by the MMSE estimation and cancellation is very large as shown in Fig. 4, which is about +30 dB. This result fits very well with the analysis and geometric interpretation of the crosstalk cancellation in Section III-B. As expected in Example 2, the SINR gain in frequency $194kHz$ is approximately equal to the interference to noise ratio in the dual frequency $198kHz$ minus $3dB$. If self-NEXTs from other ADSL services in the upstream band $[26-138kHz]$ are small (this figure assumes no self-NEXTs), the HDSL crosstalk signal can also be estimated in the ADSL upstream band and then cancelled in the dual HDSL excess band $[254-366kHz]$. The interference in the excess band $[254-366kHz]$ can be almost eliminated because the crosstalk signal in band $[26 - 138kHz]$ has a much larger interference to noise ratio.

Interestingly, the MMSE estimation and cancellation technique works much better than the decision-aided technique for the HDSL crosstalker. The direct cause is that the HDSL signal are not detected reliably enough and the wrong decisions double the negative impact on the cancellation residual error. The decoding error is mainly because the HDSL crosstalk signal in the voice band [0-26kHz] is lost after it passes the ADSL receiver filter.

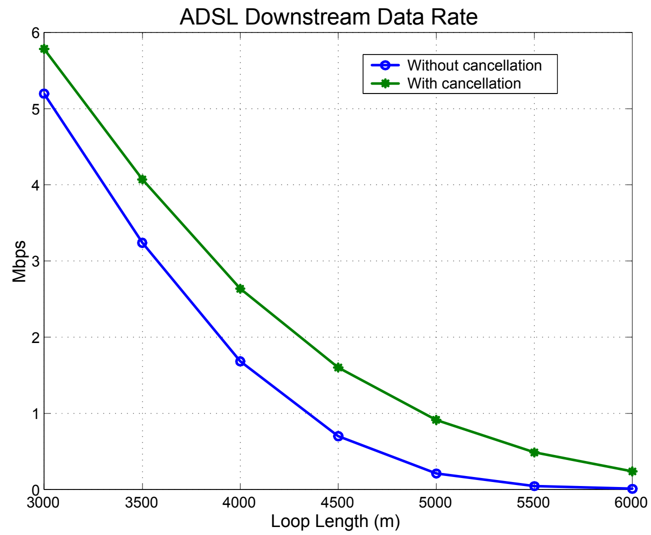

Fig. 5. **Downstream data rates with/without NEXT cancellation.**

The problem is even worse when the HDSL signal can not be estimated in the ADSL upstream band $[26 - 138kHz]$ if self-NEXTs from ADSL are stronger than the HDSL signal.

Fig. 5 shows the ADSL downstream data rate improvement as a result of the MMSE crosstalk estimation and cancellation. Given a line length of $4500m$, the data rate increases from $0.8Mbps$ to $1.7Mbps$. It is also interesting to note that, given the data rate of $0.8Mbps$, the line reach increases from $4500m$ to $5300m$, which is about 39% more coverage area for the service providers.

B. SDSL NEXT Cancellation

SDSL offers variable symmetric data rates by using different bandwidth. In the simulation, the symbol rate is chosen as $f_{sym} = 1040kHz$. Three PSDs of the SDSL NEXT are shown in Fig. 6. They correspond to the original NEXT, the residual NEXT using the decision-aided cancellation approach, and the residual NEXT using the MMSE cancellation approach, respectively. The crosstalk signal is estimated in the SDSL excess band $[522 - 902kHz]$ and cancelled in the dual main lobe $[138 - 518kHz]$. The improvement is very large in those frequency band around $f_{sym}/2$ where the excess band has very high interference to noise ratio. As the frequency increases towards f_{sym}, the crosstalk signal

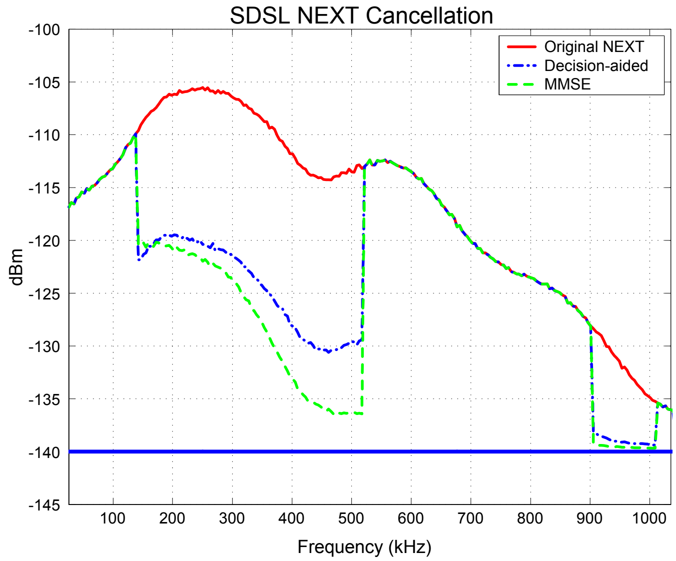

Fig. 6. **SDSL NEXT estimation and cancellation.**

becomes small, correspondingly the improvement of the cancellation in the dual frequency band becomes small. As shown in Fig. 6, the improvement of SINR is smaller in the bands closer to the minimum downstream frequency ($138kHz$). If the self-NEXTs from other ADSL services are small, the crosstalk signal can be estimated in the ADSL upstream band $[26 - 138kHz]$ and cancelled in the dual excess band $[904 - 1014kHz]$.

The MMSE estimation and cancellation approach performs better than the decision-aided approach. However, the difference from these two approaches is smaller than that of the HDSL service. The reason is that the SDSL service has a much larger bandwidth than the HDSL service and the loss of the signal in the low-frequency band $[0 - 26kHz]$ has a relatively smaller effect on the decoding error.

The ADSL data rate improvement is shown in Fig. 7. The improvement is most significant in the loops around $2500 - 4500m$. The improvement is smaller in the shorter loop ($< 2500m$) because the self-FEXTs are larger. Given a line length of $3500m$, the data rate increases from $0.9Mbps$ to $2.7Mbps$. Similarly, for a given data rate of $0.9Mbps$, the loop length increases from $3500m$ to $4600m$, which is about 73% more area coverage for a service provider. The SDSL NEXT is a very severe crosstalk to the ADSL downstream transmission because it has a large bandwidth. The cancellation method proposed in this paper can greatly improve the system performance if there is one (dominant) NEXT.

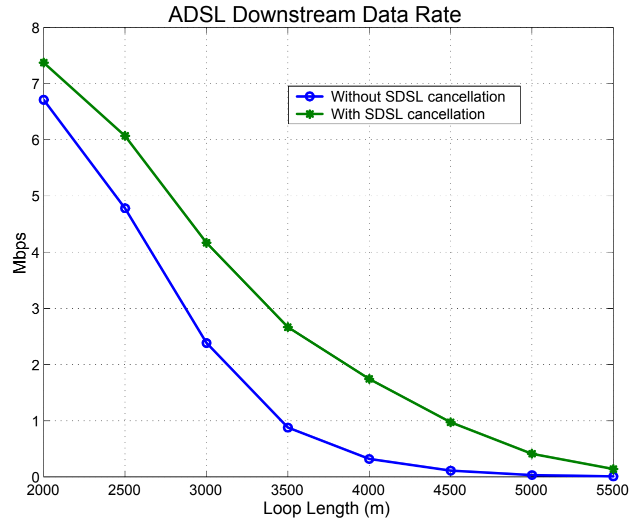

Fig. 7. **Downstream data rates with/without SDSL NEXT cancellation.**

C. Fast Computation

One important aspect of the proposed techniques is that there exists a fast computational scheme, which may be used for practical implementation. This fast algorithm degrades the system performance because of the edge effect in the crosstalk function matrix as described in Sec. III-C. Figs. 8 and 9 show the NEXT cancellation results for HDSL and SDSL, respectively. The three PSDs correspond to the original NEXT, the residual NEXT by the fast algorithm, and the residual NEXT by the accurate MMSE estimation and cancellation in Sec. III-A. The simulation results show that the SINR loss due to the fast algorithm is relatively small.

V. Conclusions

The crosstalk from other types of services, such as HDSL, SDSL, and T1, limits the data rate or the line reach of an ADSL service. This paper presents two new methods to mitigate such NEXT. Both methods are based on the idea of estimating the crosstalk signal in some frequency bands and canceling it in other frequency bands.

The decision-aided method decodes the crosstalk signal based on the observation in some frequency bands. The crosstalk signal is then reconstructed and cancelled in other bands. The method has the problem of error propagation. The MMSE estimation and

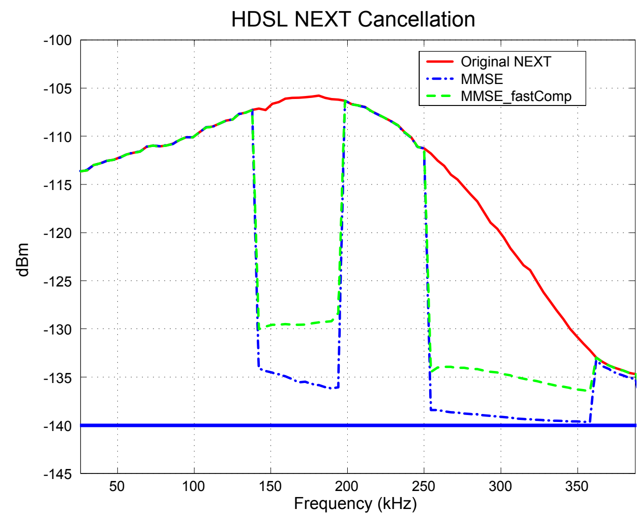

Fig. 8. **HDSL NEXT cancellation using the fast algorithm.**

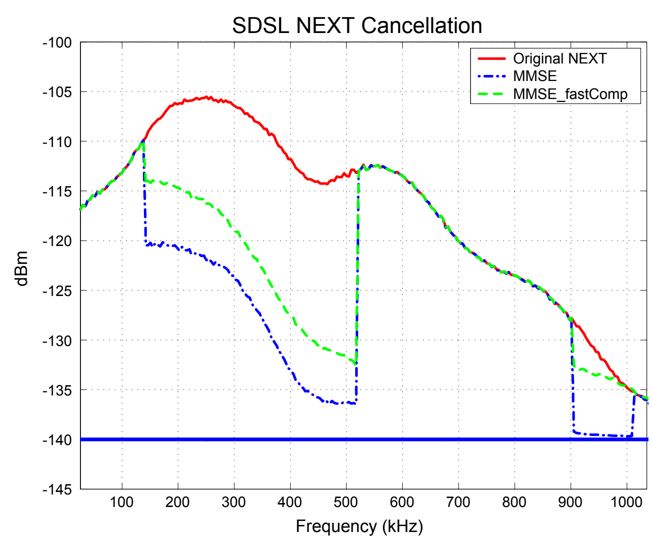

Fig. 9. **SDSL NEXT cancellation using the fast algorithm.**

cancellation method does not decode the crosstalk signal, but uses the MMSE estimation result directly to construct the interference and cancel it in other frequency bands. The latter approach is particularly better to suppress one HDSL or SDSL NEXT because the decision-aided approach can not decode the crosstalk signal reliably due to the signal loss in the low-frequency band $[0 - 26kHz]$. This conclusion is verified by the simulation results.

This paper also presents an intuitive explanation of crosstalk cancellation, which provides the guidance to select the right frequency bands to estimate the crosstalk signal. One interesting result of the MMSE estimation and cancellation is that the gain of interference suppression is the same no matter the crosstalk signal is estimated in the main lobe or in the excess band. In general, the main lobe has larger interference than the excess band. Therefore, if the crosstalk signal is estimated in the main lobe, the interference caused by the excess band can be eliminated. Conversely, if the crosstalk signal is estimated in the excess band, the SINR gain in the main lobe is equal to the interference to noise ratio in the excess band. The larger the interference in the excess band, the better the suppression of the interference in the main lobe.

Moreover, a fast computational algorithm is developed for practical implementation with slightly degraded performance. This method avoids matrix inversion and large matrix multiplication in every transmission block. In contrast, the traditional MMSE estimation requires high computational complexity since the crosstalk signal is non-stationary and large matrix inversion is need in every transmission block.

In conclusion, the MMSE estimation and cancellation scheme is very effective in cancelling the crosstalk if the crosstalk has a large percentage of excess bandwidth. In current xDSL systems, NEXT to an ADSL receiver has a large excess bandwidth, this method is very effective to cancel one dominant NEXT.

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