SAPIENZA Università di Roma – MSc. in Engineering in Computer Science Formal Methods – Feb. 13, 2017

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(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.

Alphabet: Student(x), Course(x), Class(x), Lab(x), Takes(x, y), Credit(x, y), Mark(x, y, z)

Axioms:

Forall x. Class(x) implies Course(x) ISA generalization
Forall x. Lab(x) implies Course(x) ISA generalization
Forall x. Class(x) implies not Lab(x) DISJOINTESS ISA
Forall x. Course(x) implies Class(x) OR Lab(x) COMPLETENESS ISA

Forall x, y. Takes(x, y) implies Student(x) and Course(y)
Forall x. Student(x) implies 1 <= #{y|Takes(x, y)} <= 30

Forall x, y. Credit(x, y) implies Student(x) and Class(y)
Forall x. Student(x) implies 1 <= #{y|Credit(x, y)} <= 30

Forall x, y, z. Mark(x, y, z) implies Credit(x, y) and Integer(z)
Forall x, y. Credit(x, y) implies 1 <= #{z|Mark(x, y, z)} <= 1

Exists z. Mark(x, y, z) and (Forall z, z'. Mark(x, y, z) and Mark(x, y, z') implies z=z')

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

1.Check whether the instantiation (once completed) is correct (and explain why it is or it is not).

To complete the given partial instantiation, there must be added a new table Course and add all the atoms contained in both Class and Lab tables, since there is an ISA relation between them. The following resulting table is:

Course:= { calculus, AI, FM, Algorithms, IoT Lab, DB Lab, Hacking Lab}

All other constraints are satisfied and all the multiplicities are not violated.

2.Express in FOL and evaluate the following queries:

(a) Return students that have taken at least 3 courses.

Student(x) and Exists y. takes(x, y) and Exists y'. Takes(x, y') and Exists y''. Takes(x, y'') and y!=y' and y'!=y''

Conjunctive Query

(b) Return students that have taken only classes.

Student(x) and Forall y. takes(x, y) implies Credit(x, y)

(c) Check if there exists a student that has taken all labs.

(d) Check if there is a student that has taken all classes, but not for credit.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \land [next]X) \lor [next]Y)$ and the CTL formula EF $(\neg a \supset (EX\ a \land EXAG\ b))$ (showing its translation in Mu-Calculus) against the following transition system:

$\Phi\ \nu X.\mu Y.((a \land [next]X) \lor [next]Y)$

$[|X_0|] = \{1, 2, 3, 4, 5\}$
$[|X_1|] = [|\ \mu Y.((a \land [next]X) \lor [next]Y)\ |] = \{2, 4, 5\}$
$[|Y_0|] = \{\}$
$[|Y_1|] = [|\ a \land [next]X_0 \lor [next]Y_0\ |] =$
$= [|a|]$ inter PreA(next, X_0) U PreA(next, Y_0) =
$= \{2, 4, 5\}$ inter $\{1, 2, 3, 4, 5\}$ U $\{\} = \{2, 4, 5\}$

$[|Y_2|] = [|\ a \land [next]X_0 \lor [next]Y_1\ |] =$
$= [|a|]$ inter PreA(next, X_0) U PreA(next, Y_1) =
$= \{2, 4, 5\}$ inter $\{1, 2, 3, 4, 5\}$ U $\{ 2, 4 \} = \{ 2, 4, 5 \}$

$[|X_2|] = [|\ \mu Y.((a \land [next]X_1) \lor [next]Y)\ |] = \{ 2, 4 \}$
$[|Y_{00}|] = \{\}$
$[|Y_{01}|] = [|\ a \land [next]X_1 \lor [next]Y_{00}\ |] =$
$= [|a|]$ inter PreA(next, X_1) U PreA(next, Y_{00}) =
$= \{2, 4, 5\}$ inter $\{ 2, 4 \}$ U $\{\} = \{ 2, 4 \}$

$[|Y_{02}|] = [|\ a \wedge [next]X_1 \vee [next]Y_{01}\ |] =$

$= [|a|]\ inter\ PreA(next, X_1)\ U\ PreA(next, Y_{01}) =$

$= \{2, 4, 5\}\ inter\ \{ 2, 4 \}\ U\ \{4\} = \{ 2, 4 \}$

$[|X_3|] = [|\ \mu Y.((a \wedge [next]X_2) \vee [next]Y)\ |] = \{ 4 \}$

$[|Y_{10}|] = \{\}$

$[|Y_{11}|] = [|\ a \wedge [next]X_2 \vee [next]Y_{10}\ |] =$

$= [|a|]\ inter\ PreA(next, X_2)\ U\ PreA(next, Y_{10}) =$

$= \{2, 4, 5\}\ inter\ \{ 4 \}\ U\ \{\} = \{ 4 \}$

$[|Y_{12}|] = [|\ a \wedge [next]X_2 \vee [next]Y_{11}\ |] =$

$= [|a|]\ inter\ PreA(next, X_2)\ U\ PreA(next, Y_{11}) =$

$= \{2, 4, 5\}\ inter\ \{ 4 \}\ U\ \{4\} = \{ 4 \}$

$[|X_4|] = [|\ \mu Y.((a \wedge [next]X_3) \vee [next]Y)\ |] = \{ 4 \}$

$[|Y_{20}|] = \{\}$

$[|Y_{21}|] = [|\ a \wedge [next]X_3 \vee [next]Y_{20}\ |] =$

$= [|a|]\ inter\ PreA(next, X_3)\ U\ PreA(next, Y_{20}) =$

$= \{2, 4, 5\}\ inter\ \{ 4 \}\ U\ \{\} = \{ 4 \}$

$[|Y_{22}|] = [|\ a \wedge [next]X_3 \vee [next]Y_{21}\ |] =$

$= [|a|]\ inter\ PreA(next, X_3)\ U\ PreA(next, Y_{21}) =$

$= \{2, 4, 5\}\ inter\ \{ 4 \}\ U\ \{4\} = \{ 4 \}$

$[|X_3|] = [|X_4|] = \{4\}$

$\Phi = \{4\}$

Is 1 in Φ? No, initial state of TS is not in the extension of Φ. Hence the Formula is not True in Transition System.

Decompose CTL formula EF (¬a ⊃ (EX a ∧ EX AG b))

Alpha = AG b
Beta = EX alpha
Gamma = EX a ∧ Beta
Delta = ¬a ⊃ (Gamma)
Theta = EF (Delta)

T(alpha) = νX. B ∧ [next] X
T(Beta) = <Next> alpha

$T(\mathrm{Gamma}) = \langle \mathrm{Next} \rangle\, a \wedge T(\mathrm{Beta})$
$T(\mathrm{Delta}) = a \wedge \neg T(\mathrm{Gamma})$
$T(\mathrm{Theta}) = \mu X.\ T(\mathrm{Delta}) \vee \langle \mathrm{Next} \rangle X$