CAPACITY PLANNING

6 JUNE 2013

Student's name and surname

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Exercise 1

A web site is connected to a 1 Gbps Ethernet, which is connected to an ISP through a router. The router has a latency of 200μsec/packet and connects the Ethernet to the ISP through a line of 1 Gbps full duplex. The web site is realized with 5 computing node (a CPU with a main memory and a local disk, used only for the OS's purpose) and with a file system, realized with a CPU and with a RAID 1 with 4 (2+2) disks. The incoming rate is 200 requests/sec and each HTTP request for a file is of 300 bytes, the dimension for the requested file is 50 Kbytes, the load is equally distributed among the server nodes.

The CPU service demand for each request is 5 msec in case of hit (file stored in the local main memory) and 20 msec in case of miss, the CPU-file-system service demand for each request is 5 msec, instead the service time of the each single disk for 10 Kbytes is 10 msec.

Compute the average response time in case all the components are fault-free, hypothesizing that the probability of hit is 50%.

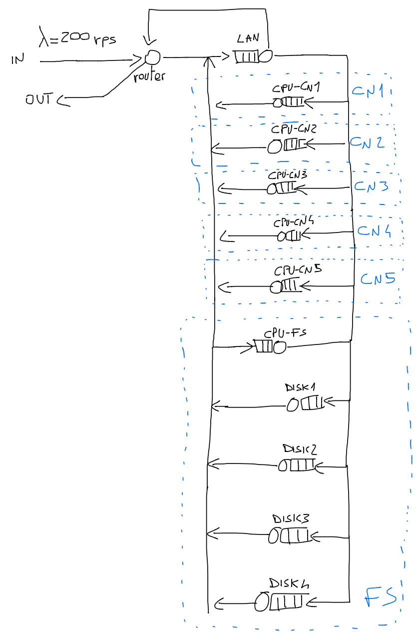
Moreover evaluate the availability of the system hypothesizing the following parameters (the system can work even in degraded mode):
MTTF (MTTR) for the CPU with its main memory: 5 years (1 week)
MTTF (MTTR) for the Ethernet: 10 years (2 weeks)
MTTF (MTTR) for each disk: 2 year (3 weeks)
MTTF (MTTR) for the router: 10 year (3 weeks)

Finally identify all the acceptable configurations and show the methodology to evaluate the average response time given the presence of faults and reparations.

Exercise 2

A file server receives requests from 5 workstations (users), that have a thinking time equal to 5 second. Every request needs 20 ms of CPU and 10 I/O requests to a disk, the time for each I/O is 5 ms. The server can manage at most 3 users at the same time. Evaluate the average service time, the average throughput and the percentage of refused requests. In case of refuse the client send another request with the same thinking time of 5 sec.

ex 1



$\lambda = 200$ rps

RouterLatency: 200 μsec/pkt

eth = line = 1 Gbps

request = 300 bytes

file = 50 Kbytes = 51200 bytes

$D^{h}_{CPU-CN} = 5$ msec $= 0{,}005$ s

$D^{m}_{CPU-CN} = 20$ msec $= 0{,}02$ s

$D_{CPU-FS} = 5$ msec $= 0{,}005$ s

$S_{DISK} = 10$ msec $\forall$ 10 kbytes $= 50$ msec $= 0{,}05$ s

$P_{hit} = 50\% = 0{,}5$

$MTTF_{CPU} = 5$ years $\quad MTTR_{CPU} = 1$ week

$MTTF_{eth} = 10$ years $\quad MTTR_{eth} = 2$ weeks

$MTTF_{DISK} = 2$ years $\quad MTTR_{DISK} = 3$ weeks

$MTTF_{router} = 10$ years $\quad MTTR_{router} = 3$ weeks

Response Time

$D_{CPU-CN} = 0{,}5 \cdot 0{,}2 \cdot 0{,}005 + 0{,}5 \cdot 0{,}2 \cdot 0{,}02 = 0{,}0025$ s

$D_{CPU-FS} = 0{,}5 \cdot 0{,}005 = 0{,}0025$ s

$D_{DISK} = 0{,}5 \cdot 0{,}25 \cdot 0{,}05 = 0{,}00625$ s

$\lambda < \frac{1}{\max D_i} < \frac{1}{0{,}0625} < \underline{\underline{160 \text{ rps}}}$ Saturation

a questo punto il prof. ha detto di continuare in maniera parametrica

$X_0 = \lambda$

$U_i = X_0 D_i$

$U_{CPU-CN} = X_0 \cdot D_{CPU-CN}$

$U_{CPU-FS} = X_0 \cdot D_{CPU-FS}$

$U_{DISK} = X_0 \cdot D_{DISK}$

$R'_i = \frac{D_i}{1 - U_i}$

$R'_{CPU-CN} = \frac{D_{CPU-CN}}{1 - U_{CPU-CN}}$

$R'_{CPU-FS} = \frac{D_{CPU-FS}}{1 - U_{CPU-FS}}$

$R'_{DISK} = \frac{D_{DISK}}{1 - U_{DISK}}$

Request

$NSegments_{req} = \frac{300}{65615} = 1$

$NDatagrams_{req} = \frac{300+20}{1500-20} = 1$

$Overhead_{req} = 20 + 20 + 18 = 58$ bytes

$S^{eth}_{req} = S^{line}_{req} = \frac{8 \cdot (300+58)}{1000 \cdot 10^{6}} = 2{,}864 \cdot 10^{-6}$ s

$S^{router}_{req} = 200 \cdot 10^{-6}$ s

Reply

$NSegments_{rep} = \frac{51200}{65515} = 1$

$NDatagrams_{rep} = \frac{51200+20}{1500-20} = 35$

$Overhead_{rep} = 20 + 35(20+18) = 1350$ bytes

$S^{eth}_{rep} = S^{line}_{rep} = \frac{8 \cdot (51200 + 1350)}{1000 \cdot 10^{6}} = 420{,}4 \cdot 10^{-6}$ s

$S^{router}_{rep} = 35 \cdot 200 \cdot 10^{-6} = 7 \cdot 10^{-3}$ s

TCP connection management

$\boxed{18 \,|\, 20 \,|\, 20} \times 6$

$Overhead_{cm} = 58 \cdot 6 = 348$ bytes

$S_{cm}^{eth} = S_{cm}^{line} = \frac{8 \cdot 348}{1000 \cdot 10^6} = 2{,}784 \cdot 10^{-6}\,s$

$S_{cm}^{router} = 6 \cdot 200 \cdot 10^{-6} = 1{,}8 \cdot 10^{-3}\,s$

$D_{eth} = S_{req}^{eth} + S_{rep}^{eth} + S_{cm}^{eth}$

$D_{line} = S_{req}^{line} + S_{rep}^{line} + S_{cm}^{line}$

$D_{router} = S_{req}^{router} + S_{rep}^{router} + S_{cm}^{router}$

$U_{req} = X_0 \cdot D_{req}$

$U_{rep} = X_0 \cdot D_{rep}$

$U_{cm} = X_0 \cdot D_{cm}$

$R'_{req} = \frac{D_{req}}{1 - U_{req}}$

$R'_{rep} = \frac{D_{rep}}{1 - U_{rep}}$

$R'_{cm} = \frac{D_{cm}}{1 - U_{cm}}$

$$\boxed{R'_{TOT} = R'_{CPU-CN} + R'_{CPU-FS} + R'_{DISK} + R'_{ETH} + R'_{LINE} + R'_{ROUTER}}$$

In case of faults

$A_{CPU} = \frac{MTTF_{CPU}}{MTTF_{CPU} + MTTR_{CPU}}$, $A_{ETH} = \frac{MTTF_{ETH}}{MTTF_{ETH} + MTTR_{ETH}}$, $A_{ROUTER} = \frac{MTTF_{ROUTER}}{MTTF_{ROUTER} + MTTR_{ROUTER}}$, $A_{DISK} = \frac{MTTF_{DISK}}{MTTF_{DISK} + MTTR_{DISK}}$

$P_r\{5\,CN, 2+2\,DISK\} = A_{CPU-CN}^5 \cdot A_{CPU-FS} \cdot A_{ETH} \cdot A_{ROUTER} \cdot A_{DISK}^4$

$P_r\{5, 1+2\} = A_{CPU-CN}^5 \cdot A_{CPU-FS} \cdot A_{ETH} \cdot A_{ROUTER} \cdot 2A_{DISK}(1-A_{DISK}) \cdot A_{DISK}^2$

$P_r\{5, 2+1\} =$ // // // // $A_{DISK}^2 \cdot 2A_{DISK}(1-A_{DISK})$

$P_r\{5, 1+1\} =$ // // // // $2A_{DISK}(1-A_{DISK}) \cdot 2A_{DISK}(1-A_{DISK})$

$P_r\{4, 2+2\} = \binom{5}{4} A_{CPU-CN}^4 \cdot (1-A_{CPU-CN}) \cdot A_{CPU-FS} \cdot A_{ETH} \cdot A_{ROUTER} \cdot A_{DISK}^4$

$P_r\{4, 1+2\} = \cdots$ $P_r\{3, 2+2\} = \cdots$ $P_r\{2, 2+2\} = \cdots$ $P_r\{1, 2+2\} = \cdots$

$P_r\{4, 2+1\} = \cdots$ $P_r\{3, 1+2\} = \cdots$ $P_r\{2, 1+2\} = \cdots$ $P_r\{1, 1+2\} = \cdots$

$P_r\{4, 1+1\} = \cdots$ $P_r\{3, 2+1\} = \cdots$ $P_r\{2, 2+1\} = \cdots$ $P_r\{1, 2+1\} = \cdots$

$P_r\{3, 1+1\} = \cdots$ $P_r\{2, 1+1\} = \cdots$ $P_r\{1, 1+1\} = \cdots$

$\boxed{5\,CN, 2+2\,DISK}$

no fault case

$R'(5,2+2) = R'_{TOT}$

$\boxed{5\,CN, 1+2\,DISK}$

$D_{DISK}(1+2) = 0{,}5 \cdot (0{,}25 \cdot 0{,}05 + 0{,}5 \cdot 0{,}05) = 0{,}01875\,s$

$U_{DISK}(1+2) = X_0 \cdot D_{DISK}(1+2)$

$R'_{DISK}(1+2) = \frac{D_{DISK}(1+2)}{1 - U_{DISK}(1+2)}$

$R'(5,1+2) = R'_{CPU-CN} + R'_{CPU-FS} + R'_{DISK}(1+2) + R'_{ETH} + R'_{LINE} + R'_{ROUTER}$

$\boxed{5\,CN, 2+1\,DISK}$

$D_{DISK}(1+2) = D_{DISK}(2+1)$

$\boxed{5\,CN, 1+1\,DISK}$

$D_{DISK}(1+1) = 0{,}5 \cdot (0{,}5 \cdot 0{,}05) \cdot 2 = 0{,}025\,s$

$U_{DISK}(1+1) = \cdots$

$R'_{DISK}(1+1) = \cdots$

$R'(5,1+1) = \cdots$

$\boxed{4\,CN, 2+2\,DISK}$

$D_{CPU-CN}(4) = 0{,}5 \cdot 0{,}25 \cdot 0{,}005 + 0{,}5 \cdot 0{,}25 \cdot 0{,}02 = 0{,}003125\,s$

$U_{CPU-CN}(4) = X_0 \cdot D_{CPU-CN}(4)$

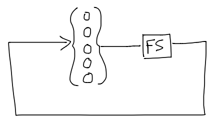
$R'_{CPU-CN}(4) = \frac{D_{CPU-CN}(4)}{1 - U_{CPU-CN}(4)}$

$R'(4,2+2) = \cdots$

... e così via per tutti i possibili stati

$$\bar{R} = \frac{P_{5,2+2} R'(5,2+2) + P_{5,1+2} \cdot R'(5,1+2) + P_{5,2+1} \cdot R'(5,2+1) + P_{5,1+1} \cdot R'(5,1+1) + P_{4,2+2} R'(4,2+2) + \cdots}{1 - P_{FAULT}}$$

ex2



$Z = 5\,s$

$S_{CPU} = 20\,ms$

$S_{DISK} = 5\,ms$

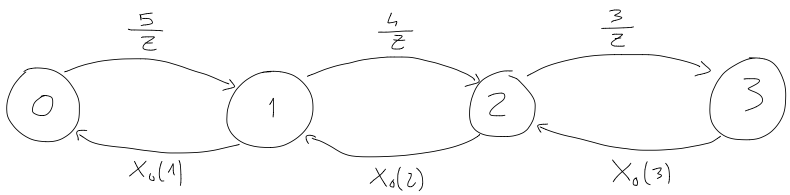
$V_{DISK} = 10$

$W = 3$

$\Longrightarrow$

$D_{CPU} = 0{,}02\,s$

$D_{DISK} = 50\,ms = 0{,}05\,s$



MVA

N=1	N=2	N=3
$R'_{CPU}(1) = 0{,}02\,s$	$R'_{CPU}(2) = 0{,}02\,(1+0{,}28) = 0{,}0256\,s$	$R'_{CPU}(3) = 0{,}0256\,(1+0{,}46) = 0{,}037\,s$
$R'_{DISK}(1) = 0{,}05\,s$	$R'_{DISK}(2) = 0{,}05\,(1+0{,}714) = 0{,}0857\,s$	$R'_{DISK}(3) = 0{,}0857 \cdot (1+1{,}54) = 0{,}22$
$X_0(1) = \frac{1}{0{,}07} = 14{,}28$	$X_0(2) = \frac{2}{0{,}0256+0{,}0857} = 17{,}97$	$X_0(3) = \frac{3}{0{,}037+0{,}22} = 11{,}67$
$n_{CPU}(1) = 0{,}02 \cdot 14{,}28 = 0{,}28$	$n_{CPU}(2) = 0{,}0256 \cdot 17{,}97 = 0{,}46$	
$n_{DISK}(1) = 0{,}05 \cdot 14{,}28 = 0{,}714$	$n_{DISK}(2) = 0{,}0857 \cdot 17{,}97 = 1{,}54$	

$$\begin{cases} \frac{5}{Z} P_0 = X_0(1)\, P_1 \\ \frac{4}{Z} P_1 = X_0(2)\, P_2 \\ \frac{3}{Z} P_2 = X_0(3)\, P_3 \\ P_0 + P_1 + P_2 + P_3 = 1 \end{cases}$$

$$\bar{X} = \sum_{k=1}^{3} X_0(k)\, P_k$$

$$\bar{N} = \sum_{k=1}^{3} k\, P_k$$

$$\bar{R} = \frac{\bar{N}}{\bar{X}}$$

$$\bar{F} = P_3$$