CTL Model Checking

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M. Benerecetti, A. Cimatti, M. Fisher, F. Giunchiglia, M. Pistore, M. Roveri, R.Sebastiani.

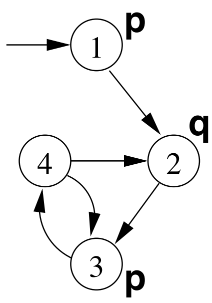
Summary

- CTL Model Checking: General Ideas.
- CTL Model Checking: The Labeling Algorithm.
- Labeling Algorithm in Details.
- CTL Model Checking: Theoretical Issues.

CTL Model Checking

CTL Model Checking is a formal verification technique s.t.

- The system is represented as a Kripke Model $\mathcal{KM}$:



- The property is expressed as a CTL formula φ, e.g.:

$$\mathbf{AG}(p \Rightarrow \mathbf{AF}q)$$

- The algorithm checks whether **all** the initial states, s_0, of the Kripke model satisfy the formula ($\mathcal{KM}, s_0 \models \varphi$).

CTL M.C. Algorithm: General Ideas

The algorithm proceeds along two macro-steps:

1. Construct the set of states where the formula holds:
   $[[\varphi]] := \{s \in S : \mathcal{KM}, s \models \varphi\}$
   ($[[\varphi]]$ is called the denotation of φ);

2. Then compare the denotation with the set of initial states:
   $I \subseteq [[\varphi]]$?

CTL M.C. Algorithm: General Ideas

To compute $[\![\varphi]\!]$ proceed “bottom-up” on the structure of the formula, computing $[\![\varphi_i]\!]$ for each subformula φ_i of φ.

For example, to compute $[\![\mathbf{AG}(p \Rightarrow \mathbf{AF}q)]\!]$ we need to compute:

- $[\![q]\!]$,
- $[\![\mathbf{AF}q]\!]$,
- $[\![p]\!]$,
- $[\![p \Rightarrow \mathbf{AF}q]\!]$,
- $[\![\mathbf{AG}(p \Rightarrow \mathbf{AF}q)]\!]$

CTL M.C. Algorithm: General Ideas

To compute each $[\![\varphi_i]\!]$ for generic subformulas:

- Handle boolean operators by standard set operations;
- Handle temporal operators $\mathbf{AX}$, $\mathbf{EX}$ by computing pre-images;
- Handle temporal operators $\mathbf{AG}$, $\mathbf{EG}$, $\mathbf{AF}$, $\mathbf{EF}$, $\mathbf{AU}$, $\mathbf{EU}$, by applying fixpoint operators.

Summary

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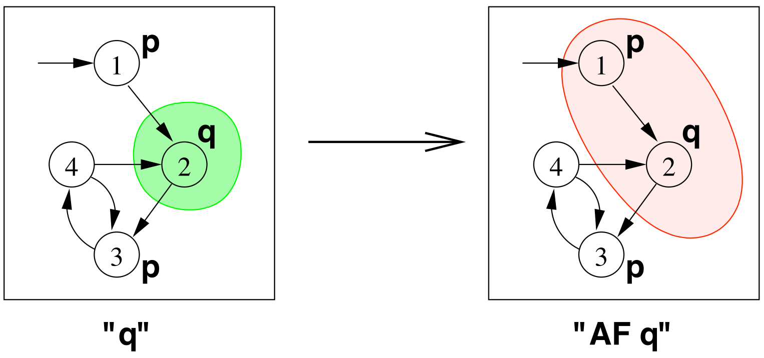
The Labeling Algorithm: General Idea

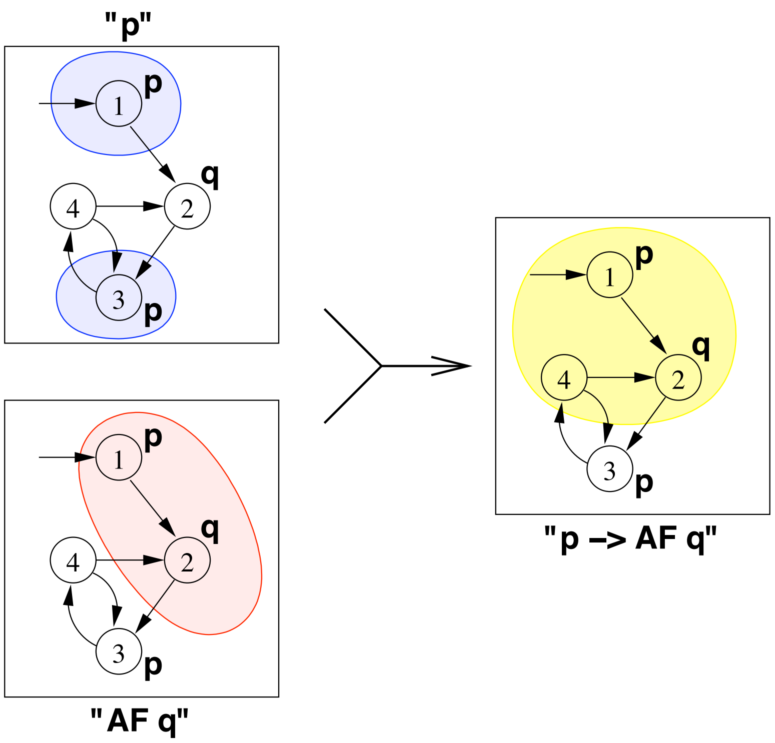
- The Labeling Algorithm given a Kripke Model and a CTL formula outputs the set of states satisfying the formula.
- Main Idea: Label the states of the Kripke Model with the subformulas of φ satisfied there.

The Labeling Algorithm: An Example

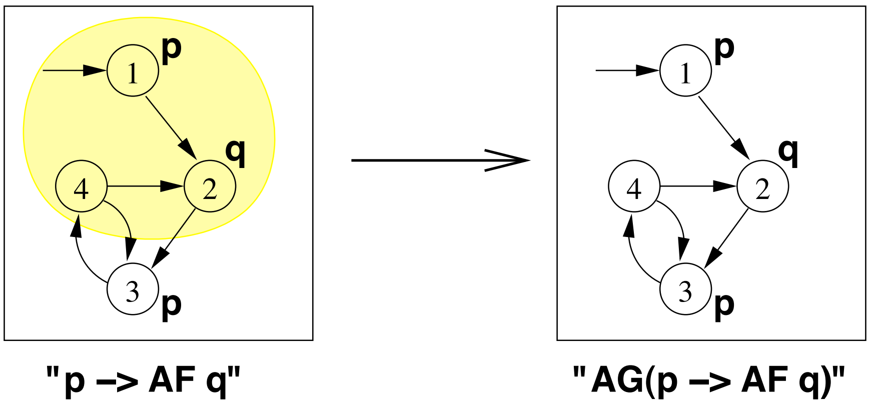


- ▷ $\mathbf{AF}q \equiv (q \vee \mathbf{AX}(\mathbf{AF}q))$
- ▷ $[\![\mathbf{AF}q]\!]$ can be computed as the union of:
  - $[\![q]\!] = \{2\}$
  - $[\![q \vee \mathbf{AX}q]\!] = \{2\} \cup \{1\} = \{1,2\}$
  - $[\![q \vee \mathbf{AX}(q \vee \mathbf{AX}q)]\!] = \{2\} \cup \{1\} = \{1,2\}$ (fixpoint).

The Labeling Algorithm: An Example



The Labeling Algorithm: An Example



- $\mathbf{AG}\varphi \equiv (\varphi \wedge \mathbf{AX}(\mathbf{AG}\varphi))$
- $[\![\mathbf{AG}\varphi]\!]$ can be computed as the intersection of:
  - $[\![\varphi]\!] = \{1,2,4\}$
  - $[\![\varphi \wedge \mathbf{AX}\varphi]\!] = \{1,2,4\} \cap \{1,3\} = \{1\}$
  - $[\![\varphi \wedge \mathbf{AX}(\varphi \wedge \mathbf{AX}\varphi)]\!] = \{1,2,4\} \cap \{\} = \{\}$ (fixpoint)

The Labeling Algorithm: An Example

- The set of states where the formula holds is empty, thus:
  - The initial state does not satisfy the property;
  - $\mathcal{KM} \not\models \mathbf{AG}(p \Rightarrow \mathbf{AF}q)$.
- Counterexample: A lazo-shaped path: $1, 2, \{3,4\}^\omega$ (satisfying $\mathbf{EF}(p \wedge \mathbf{EG}\neg q)$)

Summary

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The Labeling Algorithm: General Schema

▷ Assume φ written in terms of $\neg$, $\wedge$, **EX**, **EU**, **EG** – minimal set of CTL operators

▷ The Labeling algorithm takes a CTL formula and a Kripke Model as input and returns the set of states satisfying the formula (i.e., the *denotation* of φ):

1. For every $\varphi_i \in Sub(\varphi)$, find $[\![\varphi_i]\!]$;
2. Compute $[\![\varphi]\!]$ starting from $[\![\varphi_i]\!]$;
3. Check if $I \subseteq [\![\varphi]\!]$.

▷ Subformulas $Sub(\varphi)$ of φ are checked bottom-up

▷ To compute each $[\![\varphi_i]\!]$: if the main operator of φ_i is a
- *Boolean Operator*: apply standard set operations;
- *Temporal Operator*: apply recursive rules until a fixpoint is reached.

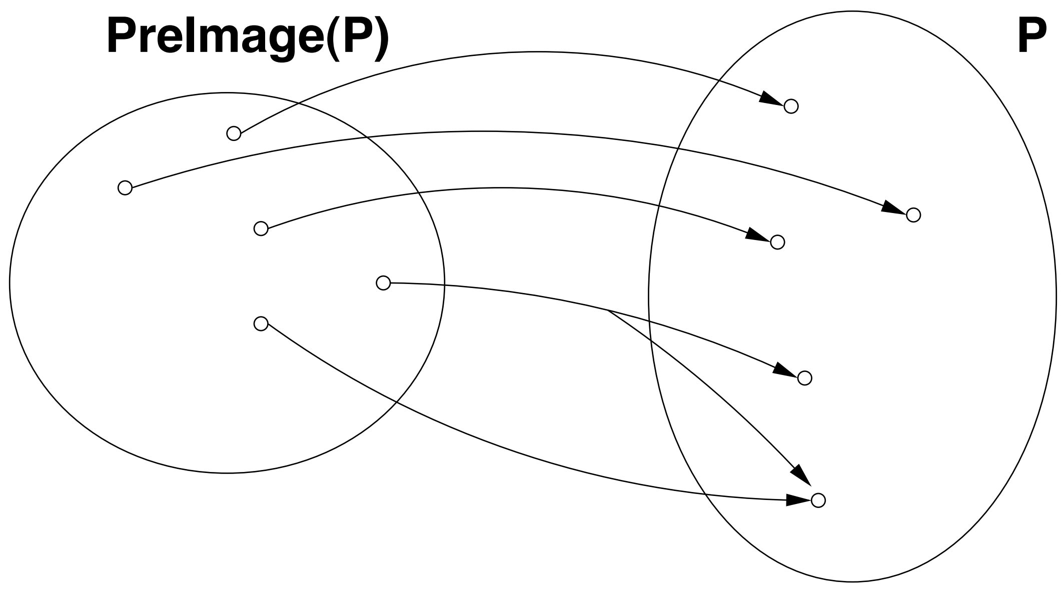
Denotation of Formulas: The Boolean Case

Let $\mathcal{KM} = \langle S, I, R, L, \Sigma \rangle$ be a Kripke Model.

$$\begin{array}{lcl} [\![false]\!] & = & \{\} \\ [\![true]\!] & = & S \\ [\![p]\!] & = & \{s \mid p \in L(s)\} \\ [\![\neg\varphi_1]\!] & = & S \setminus [\![\varphi_1]\!] \\ [\![\varphi_1 \wedge \varphi_2]\!] & = & [\![\varphi_1]\!] \cap [\![\varphi_2]\!] \end{array}$$

Denotation of Formulas: The EX Case

- $[\![\mathbf{EX}\varphi]\!] = \{s \in S \mid \exists s'.\langle s, s'\rangle \in R \text{ and } s' \in [\![\varphi]\!]\}$
- $[\![\mathbf{EX}\varphi]\!]$ is said to be the Pre-image of $[\![\varphi]\!]$ ($\textsc{Pre}([\![\varphi]\!])$).
- Key step of every CTL M.C. operation.



Denotation of Formulas: The EG Case

- From the semantics of the $\Box$ temporal operator:

  $$\Box\varphi \equiv \varphi \wedge \bigcirc(\Box\varphi)$$

- Then, the following equivalence holds:

  $$\mathbf{EG}\varphi \equiv \varphi \wedge \mathbf{EX}(\mathbf{EG}\varphi)$$

- To compute $[\![\mathbf{EG}\varphi]\!]$ we can apply the following recursive definition:

  $$[\![\mathbf{EG}\varphi]\!] = [\![\varphi]\!] \cap \textsc{Pre}([\![\mathbf{EG}\varphi]\!])$$

Denotation of Formulas: The EG Case

- We can compute $X := [\![\mathbf{EG}\varphi]\!]$ inductively as follows:

  $$\begin{array}{lll} X_1 & := & [\![\varphi]\!] \\ X_2 & := & X_1 \cap \textsc{Pre}(X_1) \\ \dots & & \\ X_{j+1} & := & X_j \cap \textsc{Pre}(X_j) \end{array}$$

- When $X_n = X_{n+1}$ we reach a fixpoint and we stop.
- **Termination.** Since $X_{j+1} \subseteq X_j$ for every $j \geq 0$, thus a fixed point always exists (Knaster-Tarski's theorem).

Denotation of Formulas: The EU Case

- From the semantics of the $\mathcal{U}$ temporal operator:
  $$\varphi\,\mathcal{U}\,\psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi\,\mathcal{U}\,\psi))$$
- Then, the following equivalence holds:
  $$(\varphi\mathbf{EU}\psi) \equiv \psi \vee (\varphi \wedge \mathbf{EX}(\varphi\mathbf{EU}\psi))$$
- To compute $[\![(\varphi\mathbf{EU}\psi)]\!]$ we can apply the following recursive definition:
  $$[\![(\varphi\mathbf{EU}\psi)]\!] = [\![\psi]\!] \cup ([\![\varphi]\!] \cap \textsc{Pre}([\![(\varphi\mathbf{EU}\psi)]\!]))$$

Denotation of Formulas: The EU Case

- We can compute $X := [\![(\varphi\mathbf{EU}\psi)]\!]$ inductively as follows:
  $$\begin{aligned} X_1 &:= [\![\psi]\!] \\ X_2 &:= X_1 \cup ([\![\varphi]\!] \cap \textsc{Pre}(X_1)) \\ &\dots \\ X_{j+1} &:= X_j \cup ([\![\varphi]\!] \cap \textsc{Pre}(X_j)) \end{aligned}$$
- When $X_n = X_{n+1}$ we reach a fixpoint and we stop.
- **Termination.** Since $X_{j+1} \supseteq X_j$ for every $j \geq 0$, thus a fixed point always exists (Knaster-Tarski's theorem).

The Pseudo-Code

We assume the Kripke Model to be a global variable:

```
FUNCTION Label($\varphi$) {
    case $\varphi$ of
        $true$:              return $S$;
        $false$:             return $\{\}$;
        an atom $p$:         return $\{s \in S \mid p \in L(s)\}$;
        $\neg\varphi_1$:     return $S \setminus$Label($\varphi_1$);
        $\varphi_1 \wedge \varphi_2$:   return Label($\varphi_1$)$\cap$Label($\varphi_2$);
        $\mathbf{EX}\varphi_1$:         return PRE(Label($\varphi_1$));
        $(\varphi_1 \mathbf{EU} \varphi_2)$:   return Label_EU(Label($\varphi_1$),Label($\varphi_2$));
        $\mathbf{EG}\varphi_1$:         return Label_EG(Label($\varphi_1$));
    end case
}
```

PreImage

$[\![\mathbf{EX}\varphi]\!] = \textsc{Pre}([\![\varphi]\!]) = \{s \in S \mid \exists s'.\langle s, s'\rangle \in R \text{ and } s' \in [\![\varphi]\!]\}$

```
FUNCTION PRE($[\![\varphi]\!]$){
    var $X$;
    $X := \{\}$;
    for each $s' \in [\![\varphi]\!]$ do
        for each $s \in S$ such that $\langle s, s'\rangle \in R$ do
            $X := X \cup \{s\}$;
    return $X$
}
```

Label_EG

$[\![\mathbf{EG}\varphi]\!] = [\![\varphi]\!] \cap \textsc{Pre}([\![\mathbf{EG}\varphi]\!])$

```
FUNCTION LABEL_EG($[\![\varphi]\!]$){
    var $X, OLD\text{-}X$;
    $X := [\![\varphi]\!]$;
    $OLD\text{-}X := \emptyset$;
    while $X \neq OLD\text{-}X$
    begin
        $OLD\text{-}X := X$;
        $X := X \cap \textsc{Pre}(X)$
    end
    return $X$
}
```

Label_EU

$[\![(\varphi\mathbf{EU}\psi)]\!] = [\![\psi]\!] \cup ([\![\varphi]\!] \cap \textsc{Pre}([\![(\varphi\mathbf{EU}\psi)]\!]))$

```
FUNCTION LABEL_EU($[\![\varphi]\!]$,$[\![\psi]\!]$){
    var $X, OLD\text{-}X$;
    $X := [\![\psi]\!]$;
    $OLD\text{-}X := S$;
    while $X \neq OLD\text{-}X$
    begin
        $OLD\text{-}X := X$;
        $X := X \cup ([\![\varphi]\!] \cap \textsc{Pre}(X))$
    end
    return $X$
}
```

Summary

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Correctness and Termination

- The Labeling algorithm works recursively on the structure φ.
- For most of the logical constructors the algorithm does the correct things according to the semantics of CTL.
- To prove that the algorithm is *Correct* and *Terminating* we need to prove the correctness and termination of both **EG** and **EU** operators.

Monotone Functions and Fixpoints

Definition. Let S be a set and F a function, $F : 2^S \rightarrow 2^S$, then:

1. F is monotone iff $X \subseteq Y$ then $F(X) \subseteq F(Y)$;
2. A subset X of S is called a fixpoint of F iff $F(X) = X$;
3. X is a least fixpoint (LFP) of F, written $\mu X.F(X)$, iff, for every other fixpoint Y of F, $X \subseteq Y$
4. X is a greatest fixpoint (GFP) of F, written $\nu X.F(X)$, iff, for every other fixpoint Y of F, $Y \subseteq X$

Example. Let $S = \{s_0, s_1\}$ and $F(X) = X \cup \{s_0\}$.

Knaster-Tarski Theorem

Notation: $F^i(X)$ means applying F i-times, i.e., $F(F(\ldots F(X)\ldots))$.

Theorem[Knaster-Tarski]. Let S be a finite set with $n+1$ elements. If $F : 2^S \rightarrow 2^S$ is a monotone function then:

1. $\mu X.F(X) \equiv F^{n+1}(\emptyset)$;
2. $\nu X.F(X) \equiv F^{n+1}(S)$.

Correctness and Termination: EG Case

The function $\textsc{Label\_EG}$ computes:

$$[\![\mathbf{EG}\varphi]\!] = [\![\varphi]\!] \cap \textsc{Pre}([\![\mathbf{EG}\varphi]\!])$$

applying the semantic equivalence:

$$\mathbf{EG}\varphi \equiv \varphi \wedge \mathbf{EX}(\mathbf{EG}\varphi)$$

Thus, $[\![\mathbf{EG}\varphi]\!]$ is the fixpoint of the function:

$$F(X) = [\![\varphi]\!] \cap \textsc{Pre}(X)$$

Correctness and Termination: EG Case

Theorem. Let $F(X) = [\![\varphi]\!] \cap \textsc{Pre}(X)$, and let S have $n+1$ elements. Then:

1. F is monotone;
2. $[\![\mathbf{EG}\varphi]\!]$ is the greatest fixpoint of F.

Correctness and Terminationpr: EU Case

The function LABEL_EU computes:

$$[\![(\varphi \mathbf{EU} \psi)]\!] = [\![\psi]\!] \cup ([\![\varphi]\!] \cap \text{PRE}([\![(\varphi \mathbf{EU} \psi)]\!]))$$

applying the semantic equivalence:

$$(\varphi \mathbf{EU} \psi) \equiv \psi \vee (\varphi \wedge \mathbf{EX}(\varphi \mathbf{EU} \psi))$$

Thus, $[\![(\varphi \mathbf{EU} \psi)]\!]$ is the fixpoint of the function:

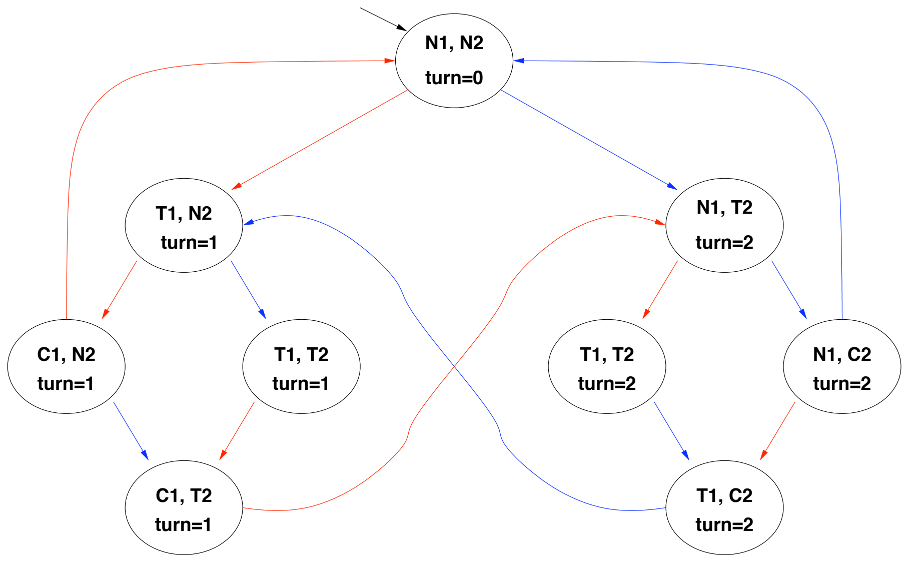
$$F(X) = [\![\psi]\!] \cup ([\![\varphi]\!] \cap \text{PRE}(X))$$

Correctness and Termination: EU Case

Theorem. Let $F(X) = [\![\psi]\!] \cup ([\![\varphi]\!] \cap \text{PRE}(X))$, and let S have $n+1$ elements. Then:

1. F is monotone;
2. $[\![(\varphi \mathbf{EU} \psi)]\!]$ is the least fixpoint of F.

Example 1: fairness

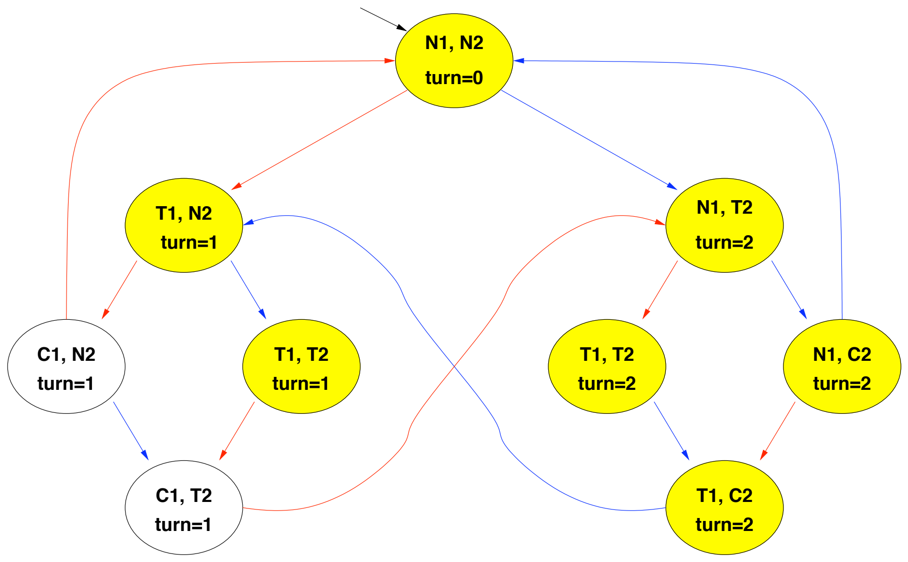


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$$

Example 1: fairness

$[\neg C_1]$



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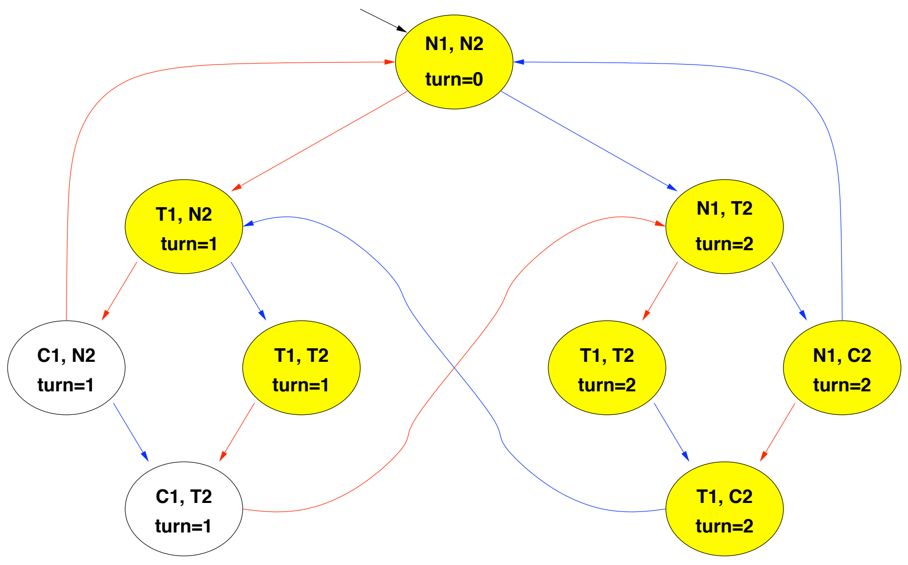
Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 0:



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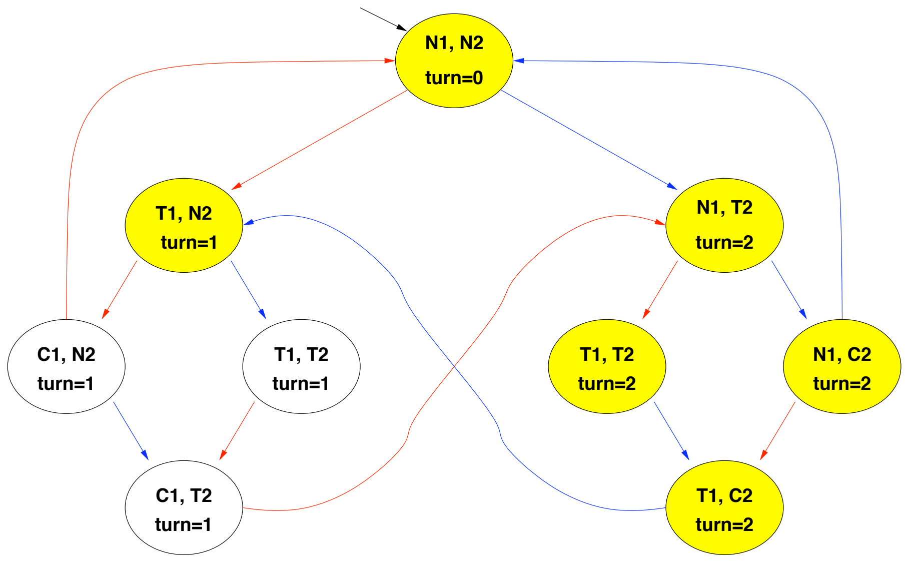
$$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 1:

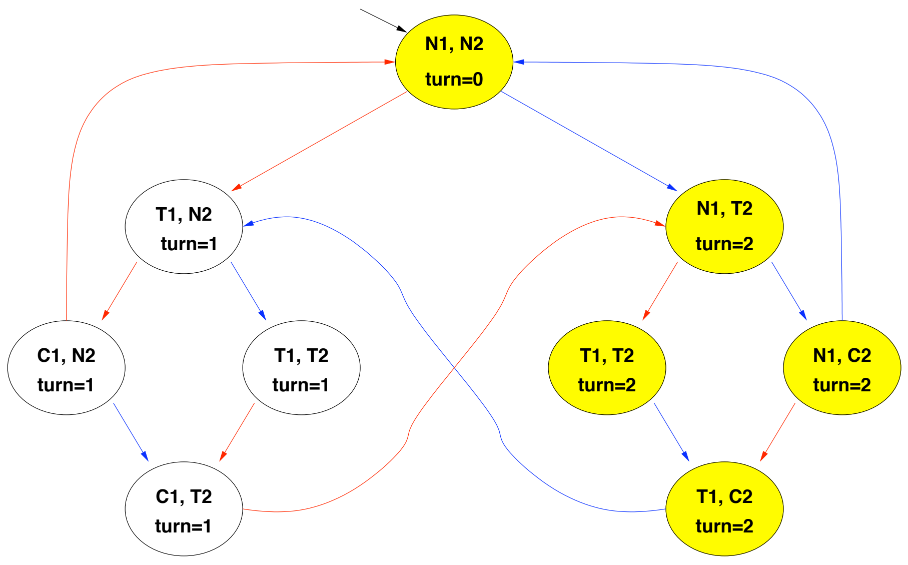


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 2:

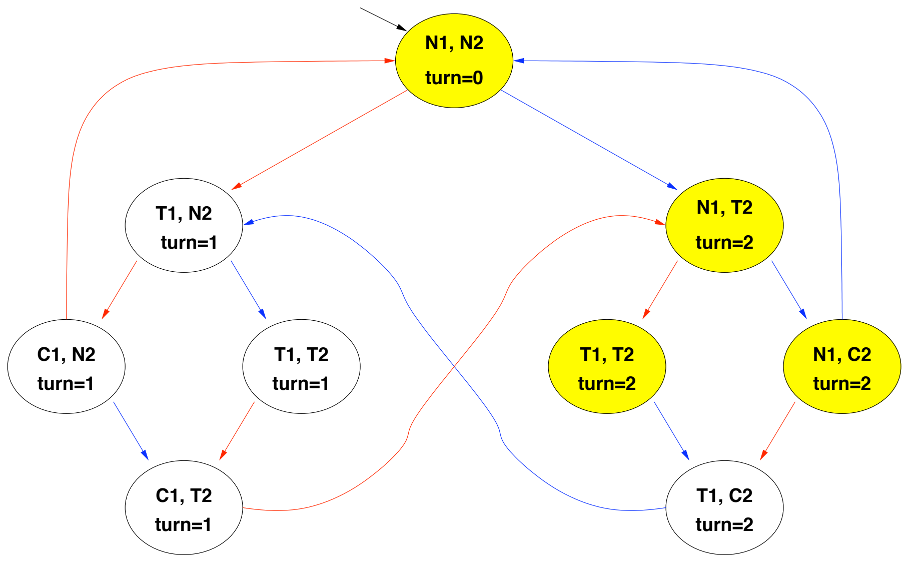


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 3:

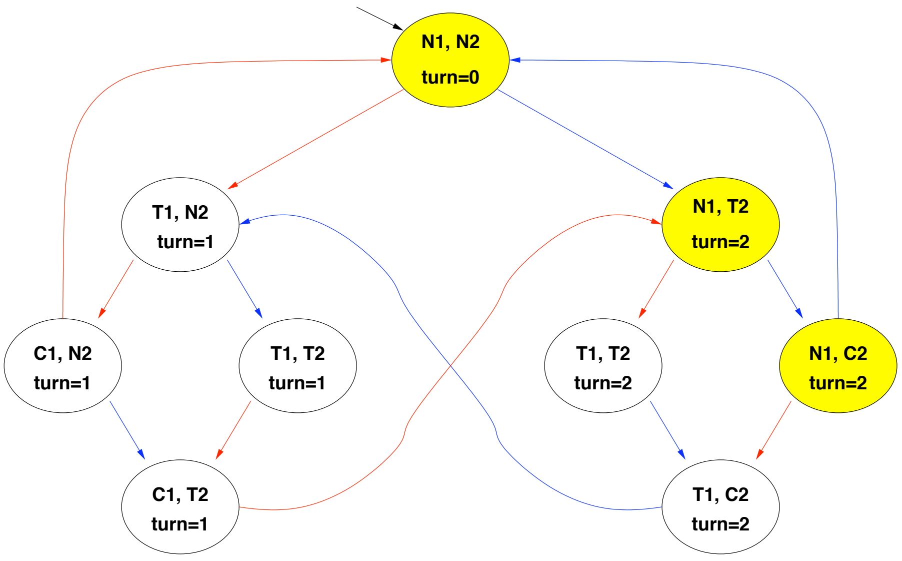


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 4:

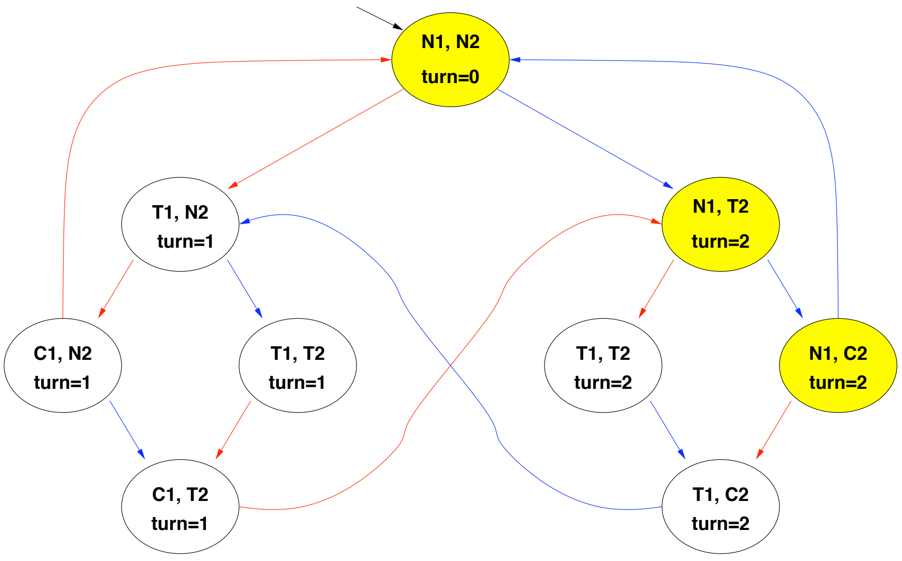


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, FIXPOINT!

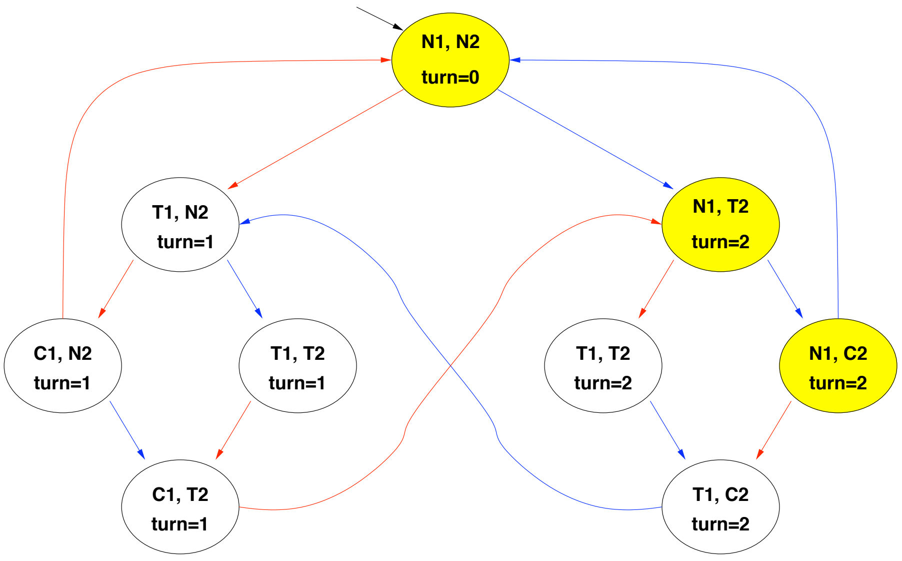


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 0

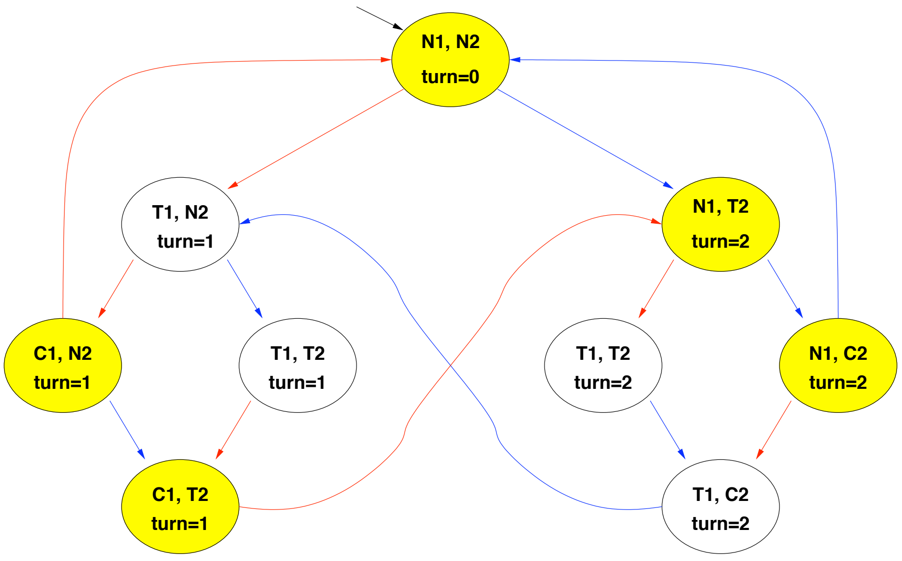


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 1



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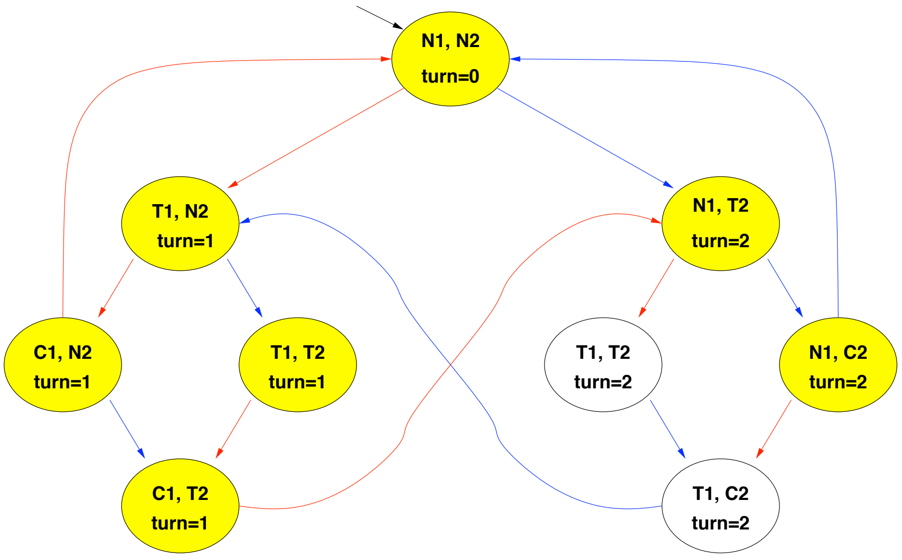
$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 2



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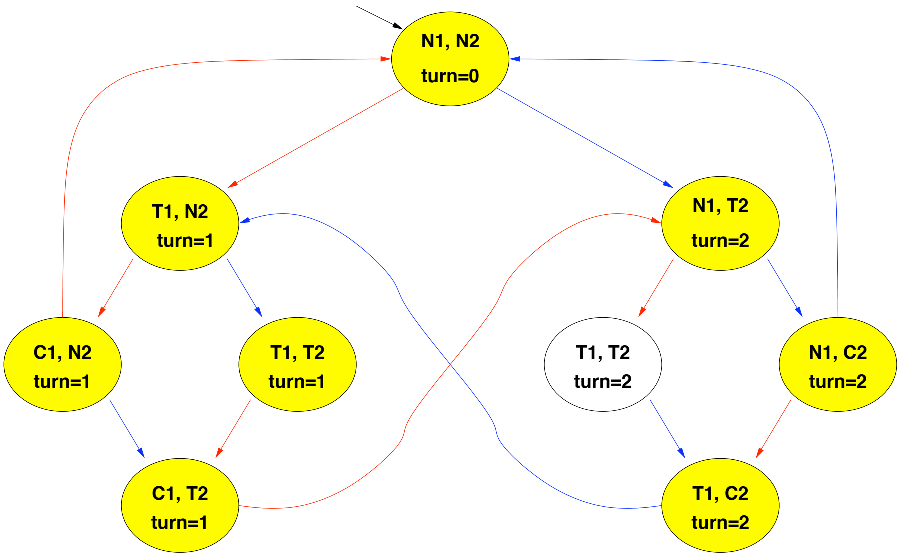
Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 3



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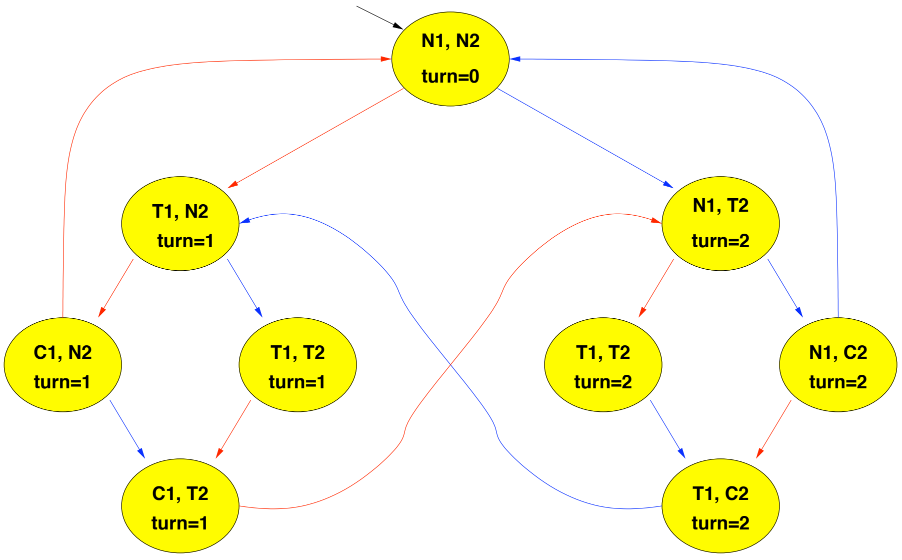
Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 4

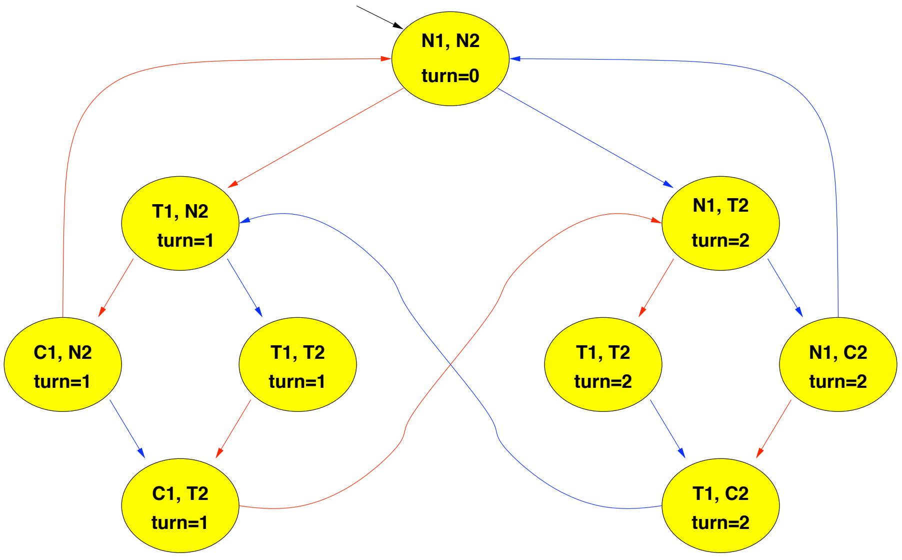


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, FIXPOINT!

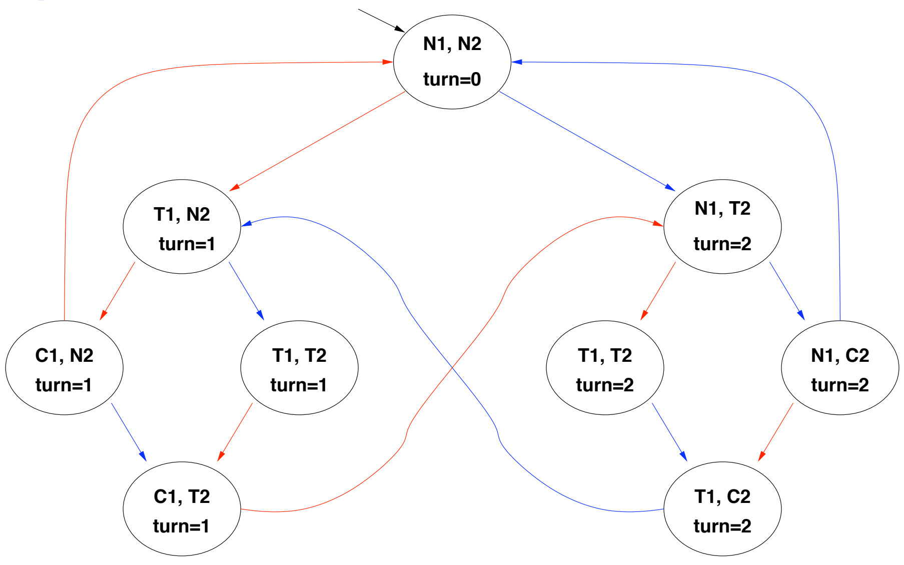


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$

Example 1: fairness

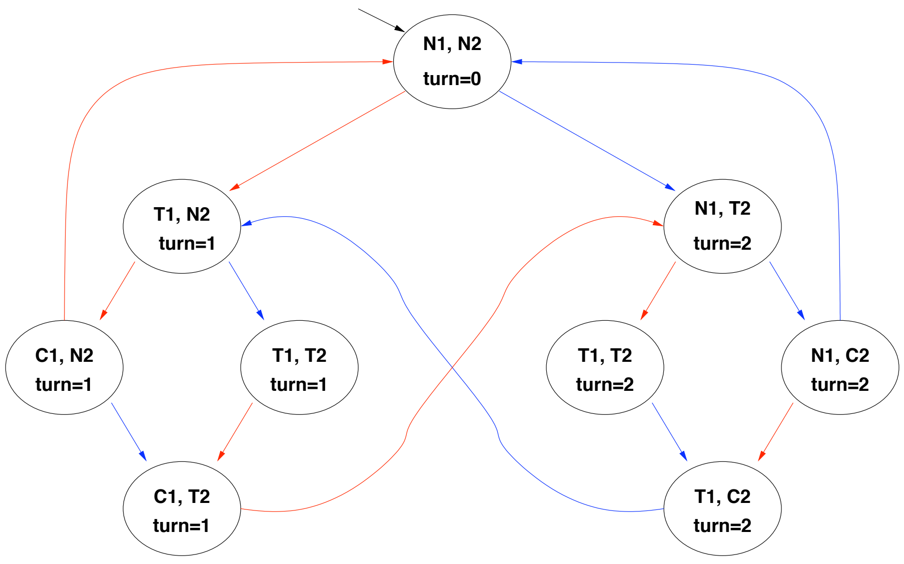
$[\neg \mathbf{EFEG}\neg C_1]$



N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ? \Longrightarrow$ NO!

Example 2: liveness

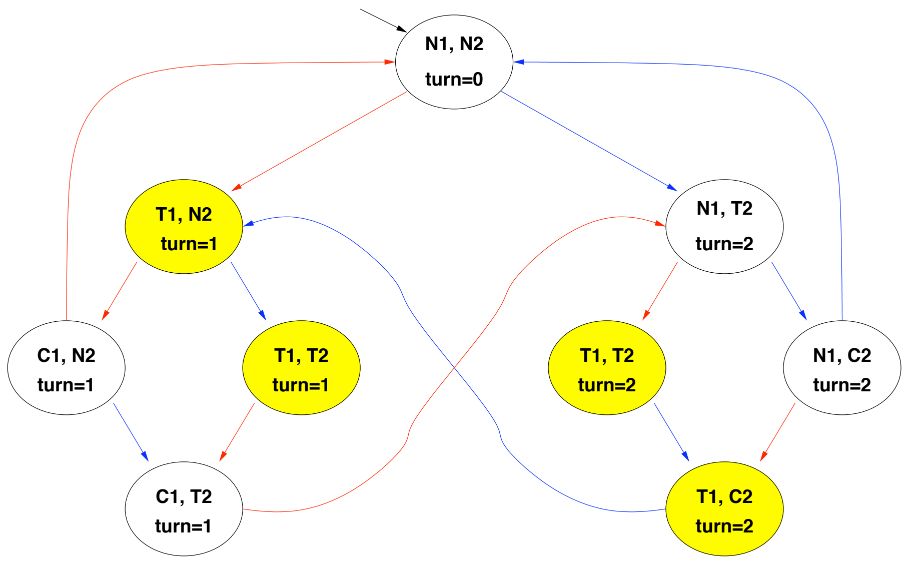


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1) \; ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1) \; ?$

Example 2: liveness

$[T_1]$:

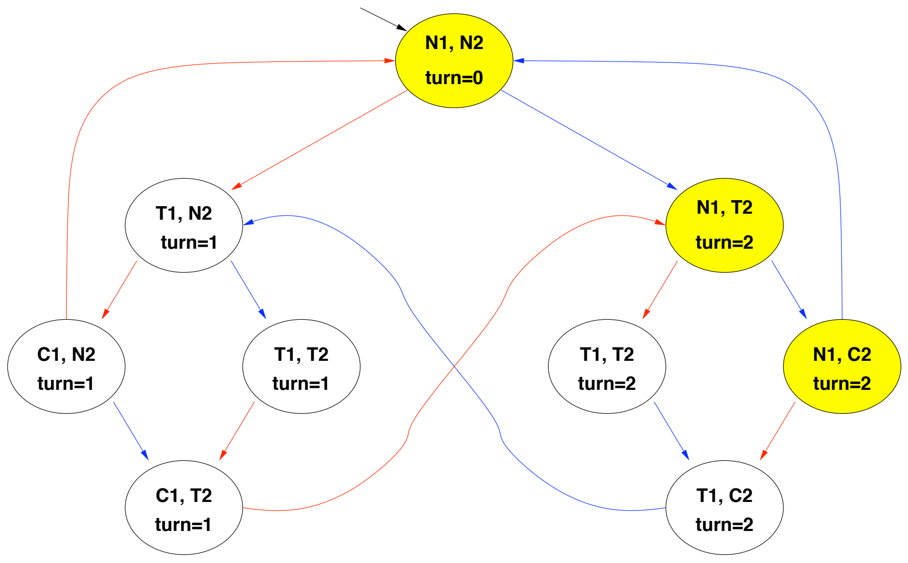


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)\ ?$

Example 2: liveness

$[\mathbf{EG}\neg C_1]$, STEPS 0-4: (see previous example)



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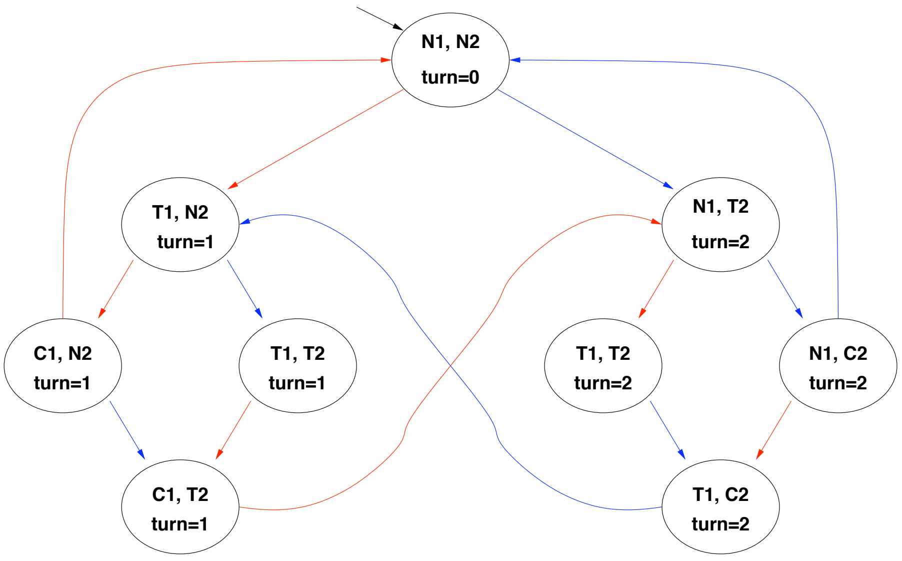
$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)\ ?$

Example 2: liveness

$[T_1 \land \mathbf{EG}\neg C_1]$:

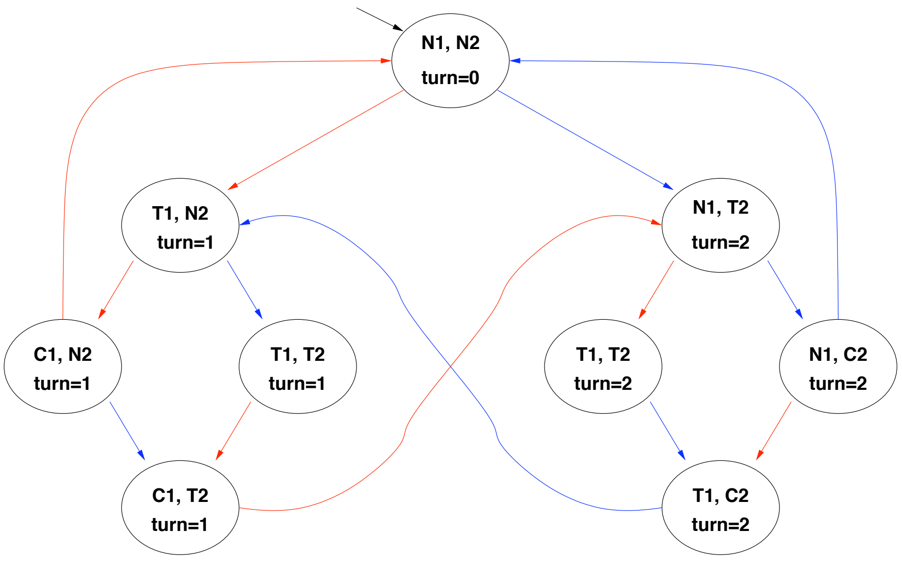


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \land \mathbf{EG}\neg C_1)\ ?$$

Example 2: liveness

$[\mathbf{EF}(T_1 \land \mathbf{EG}\neg C_1)]$:

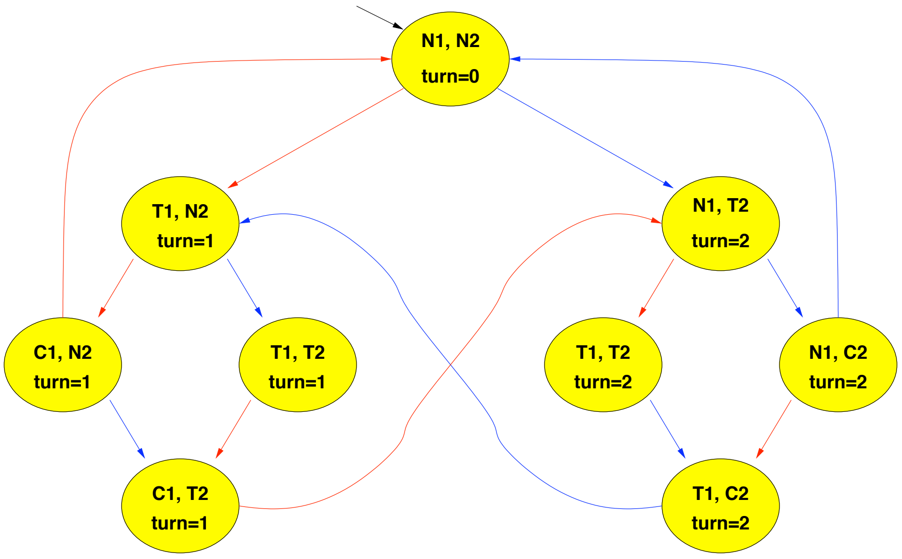


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \land \mathbf{EG}\neg C_1)\ ?$$

Example 2: liveness

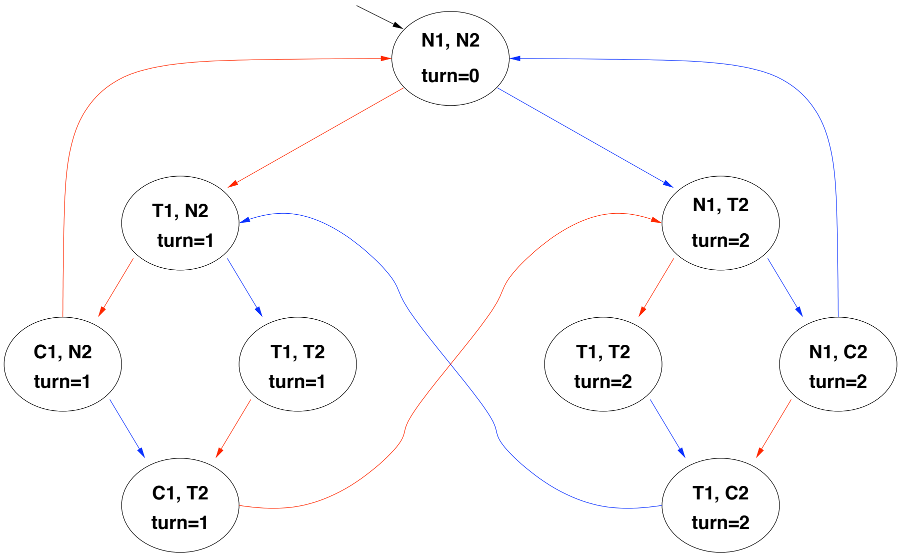
$[\neg \mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)]$:



N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)$? $\Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)$? YES!

Example 1: fairness

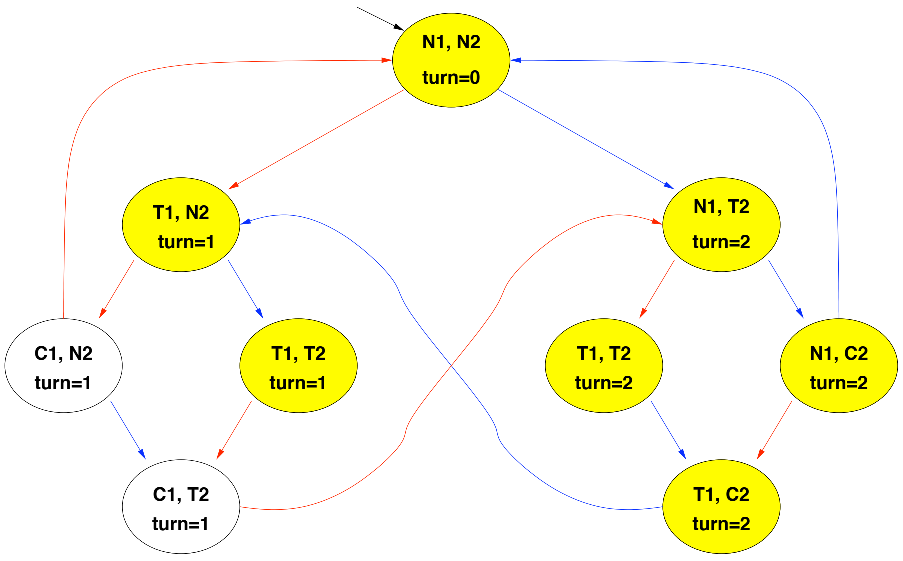


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1$? $\Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1$?

Example 1: fairness

$[\neg C_1]$

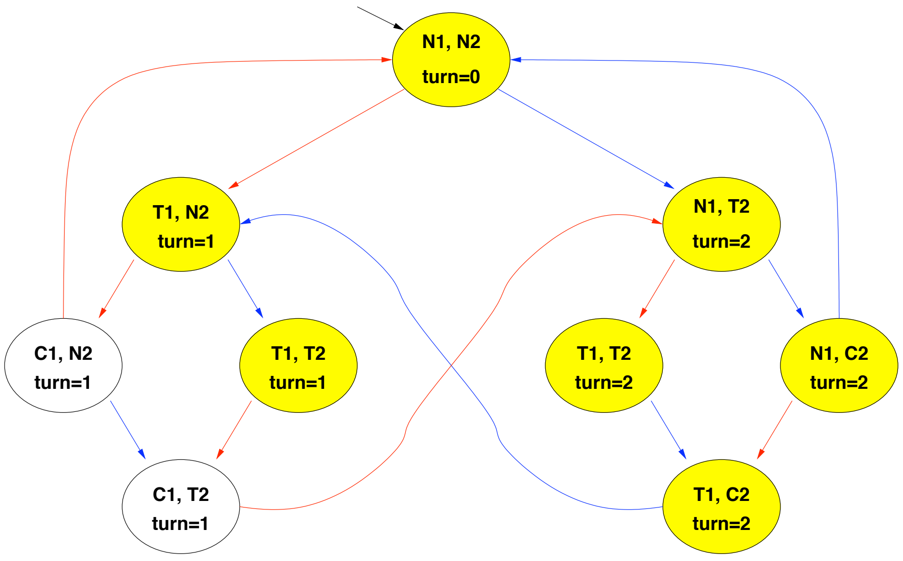


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 0:

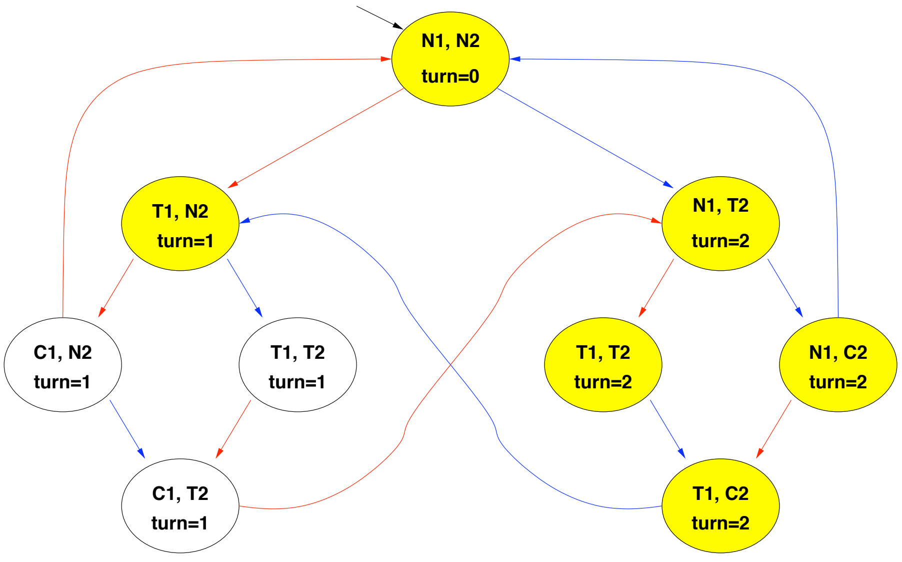


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 1:

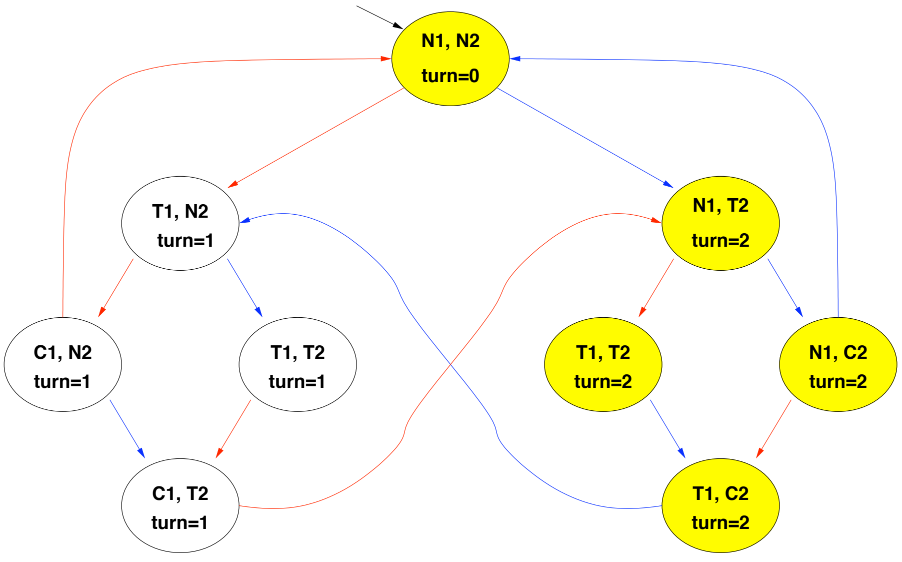


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 2:

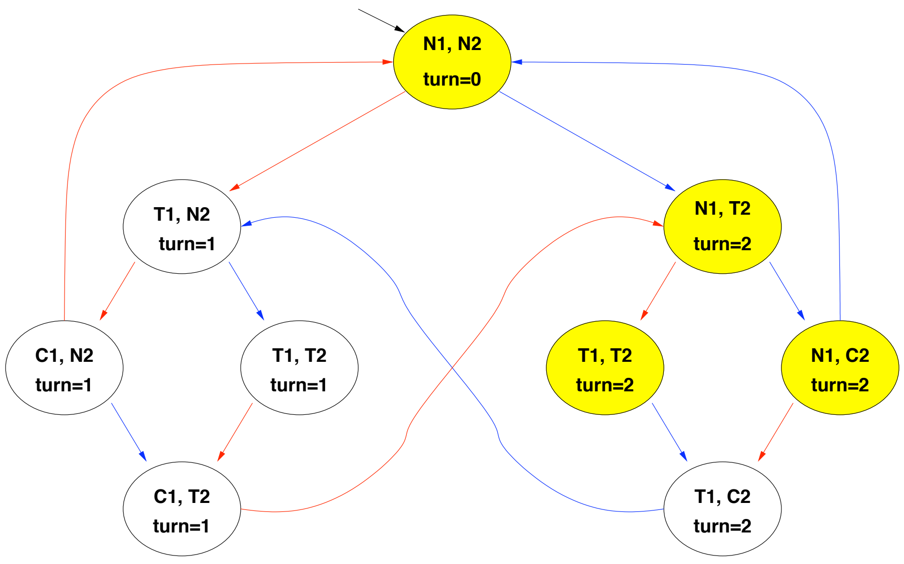


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 3:

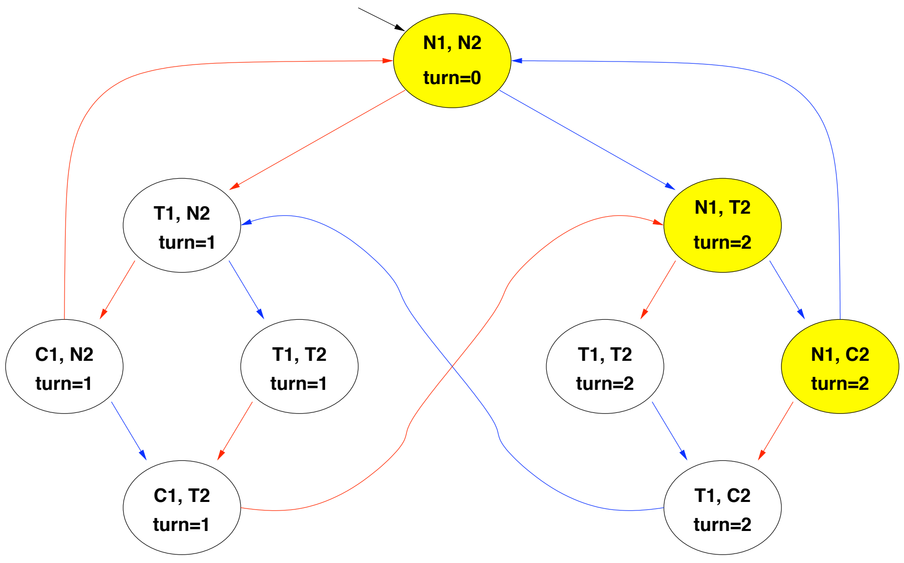


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, step 4:

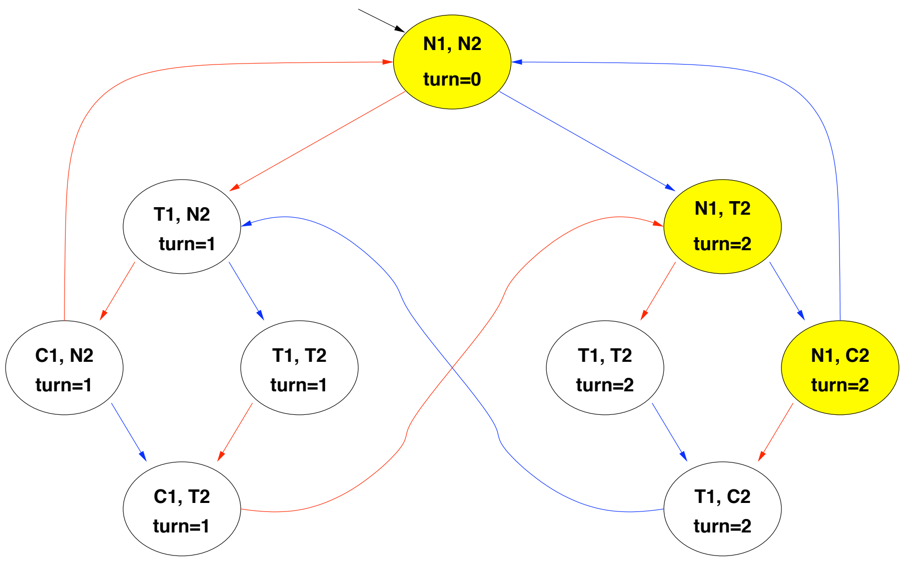


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EG}\neg C_1]$, FIXPOINT!

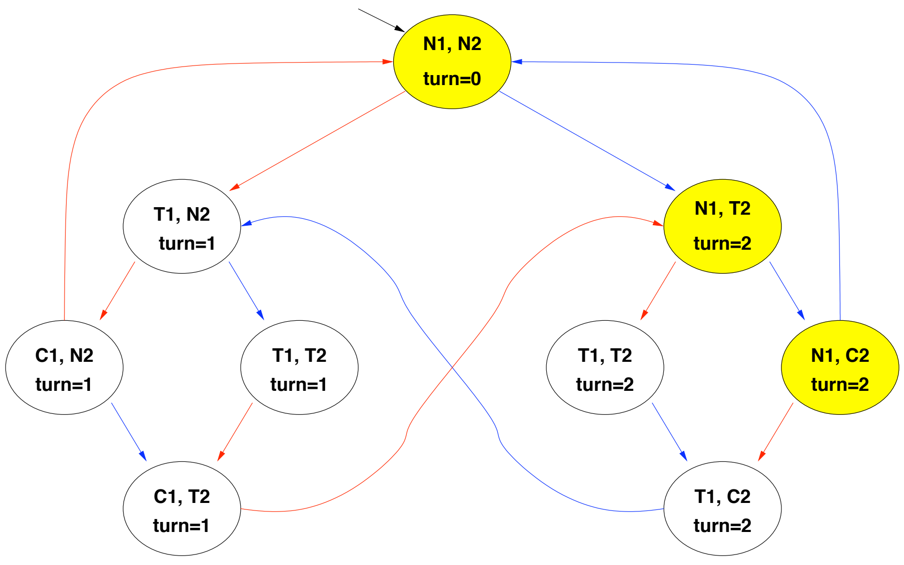


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1\ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1\ ?$$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 0

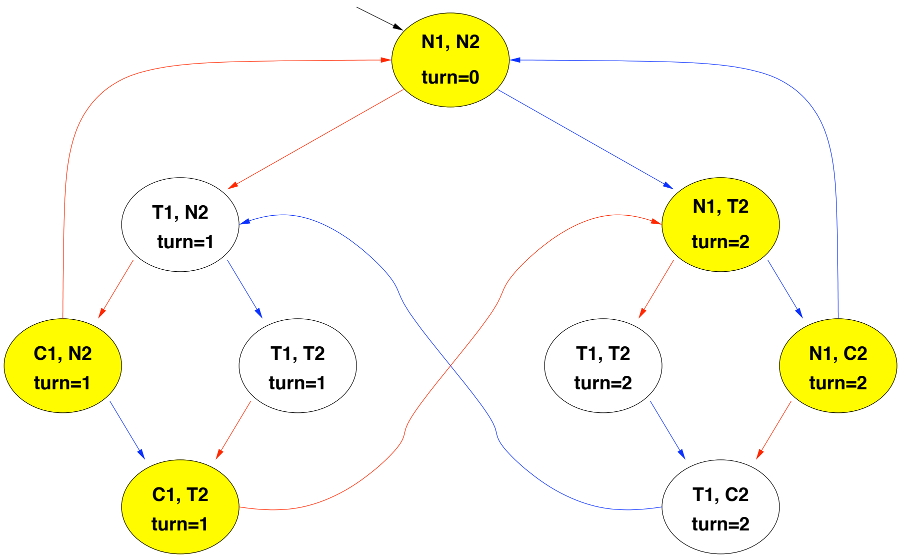


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1\ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1\ ?$$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 1

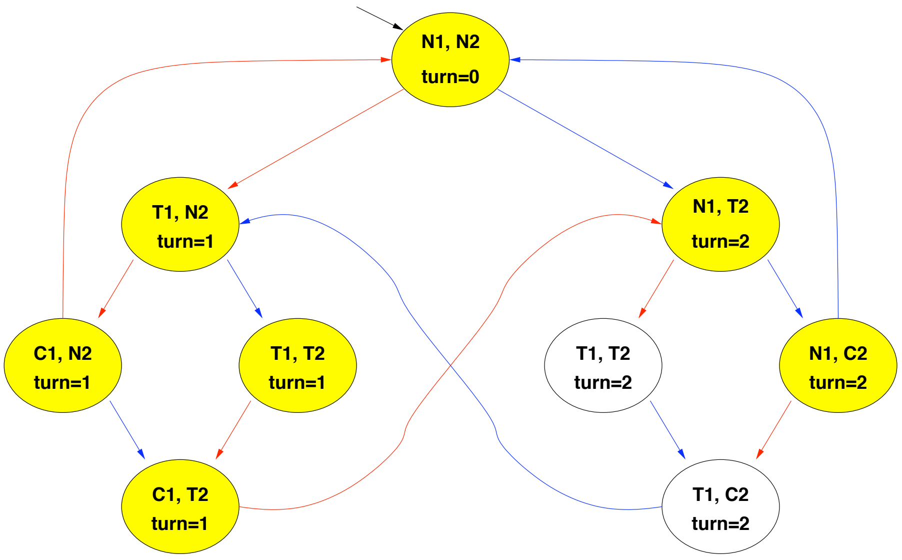


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 2

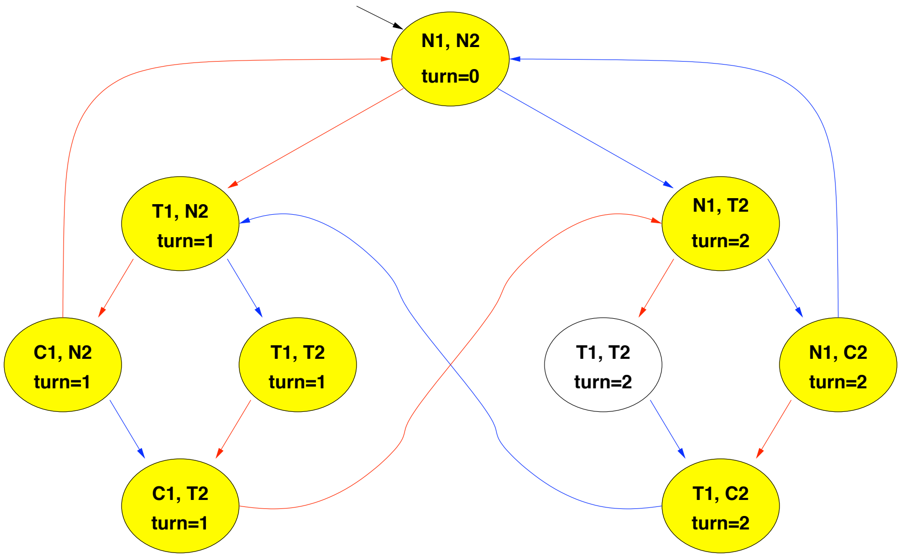


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AGAF}C_1 \ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \ ?$$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 3

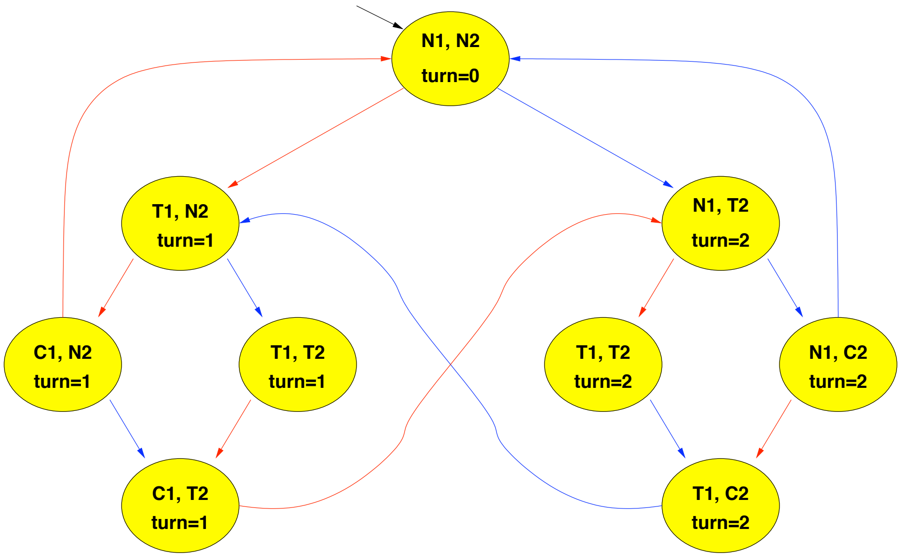


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

$[\mathbf{EFEG}\neg C_1]$, STEP 4

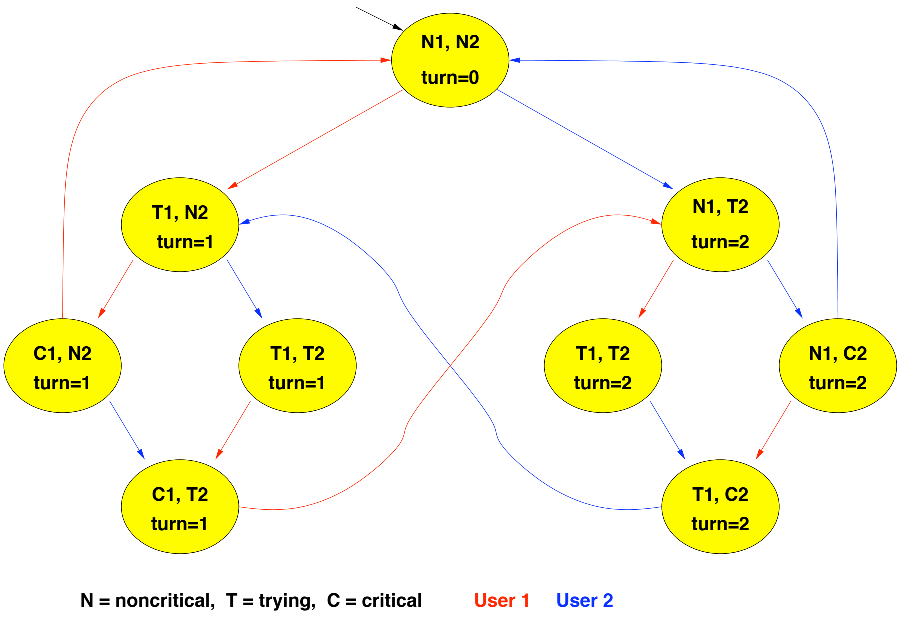


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AGAF}C_1 \; ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1 \; ?$

Example 1: fairness

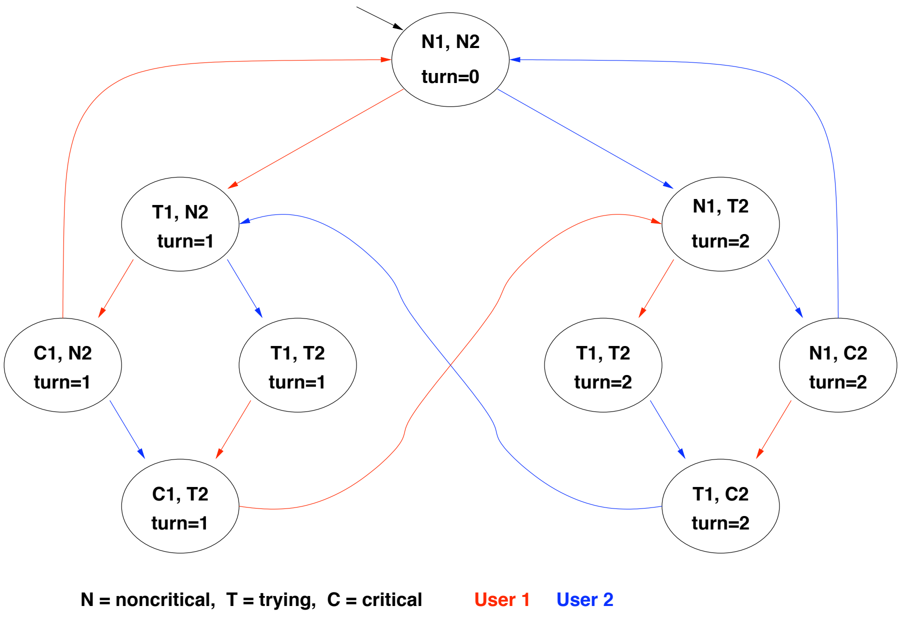
$[\mathbf{EFEG}\neg C_1]$, FIXPOINT!



$M \models \mathbf{AGAF}C_1\ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1\ ?$

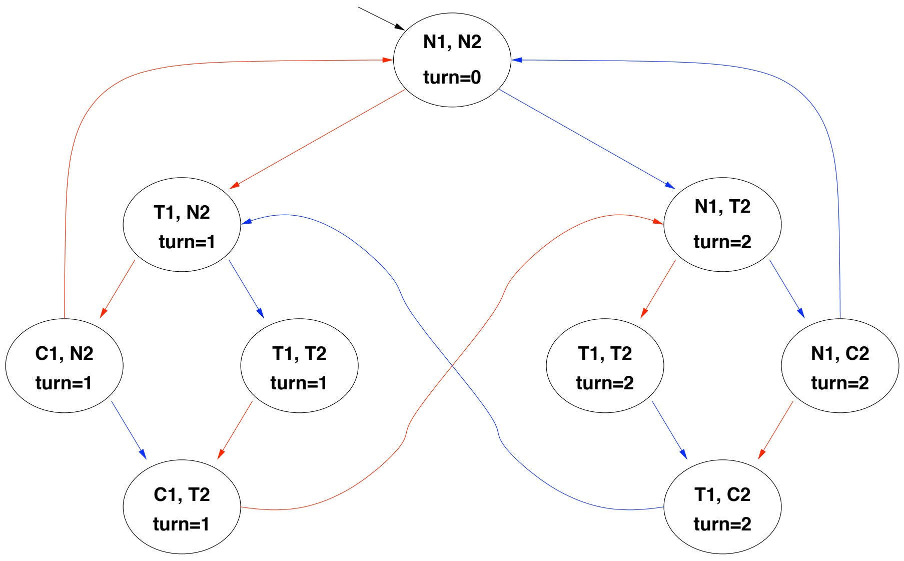
Example 1: fairness

$[\neg\mathbf{EFEG}\neg C_1]$



$M \models \mathbf{AGAF}C_1\ ? \Longrightarrow M \models \neg\mathbf{EFEG}\neg C_1\ ? \Longrightarrow$ NO!

Example 2: liveness

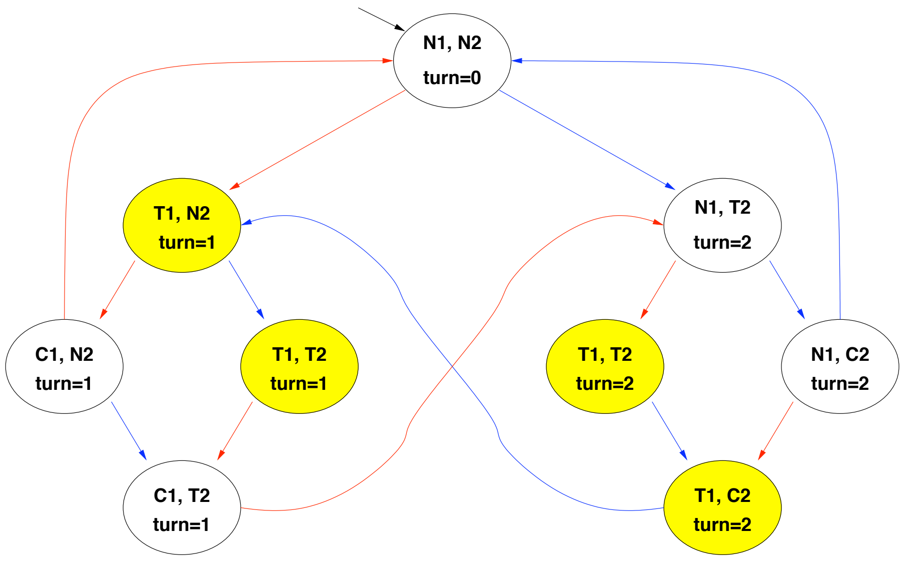


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)\ ?$$

Example 2: liveness

$[T_1]$:

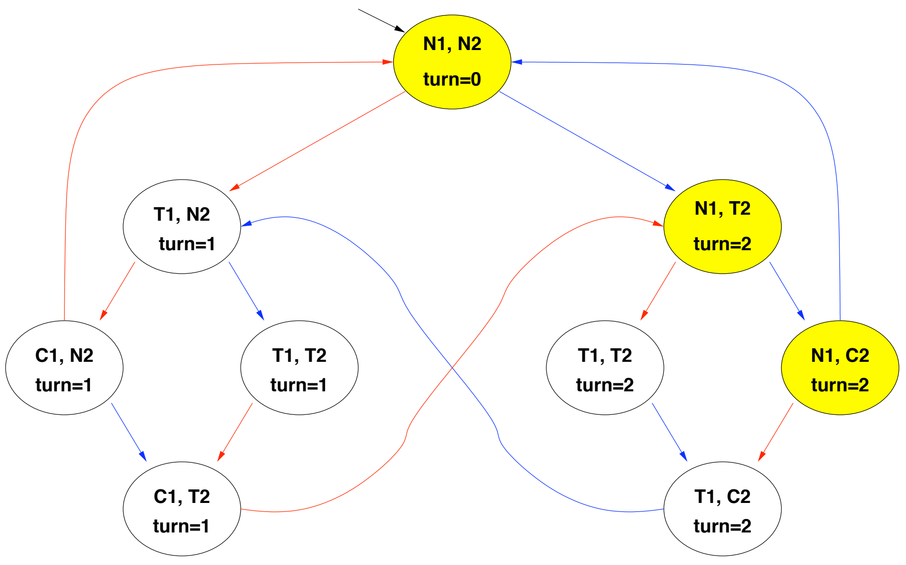


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)\ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)\ ?$$

Example 2: liveness

$[\mathbf{EG}\neg C_1]$, STEPS 0-4: (see previous example)

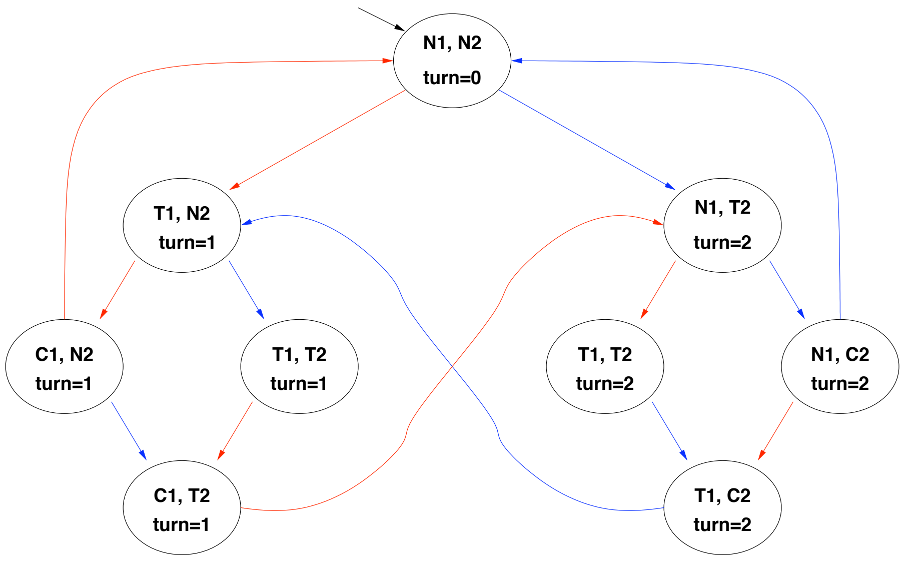


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1) \ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1) \ ?$$

Example 2: liveness

$[T_1 \wedge \mathbf{EG}\neg C_1]$:

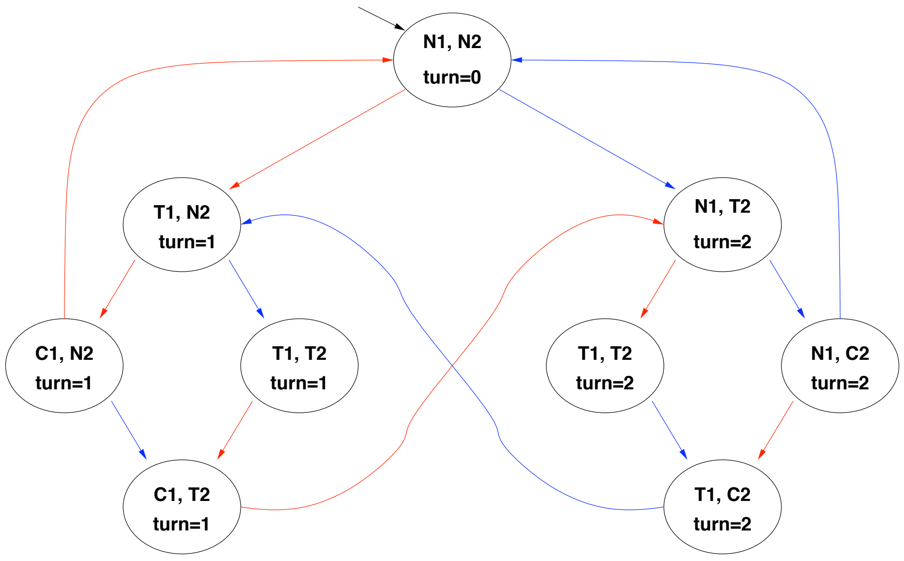


N = noncritical, T = trying, C = critical User 1 User 2

$$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1) \ ? \Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1) \ ?$$

Example 2: liveness

$[\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)]$:

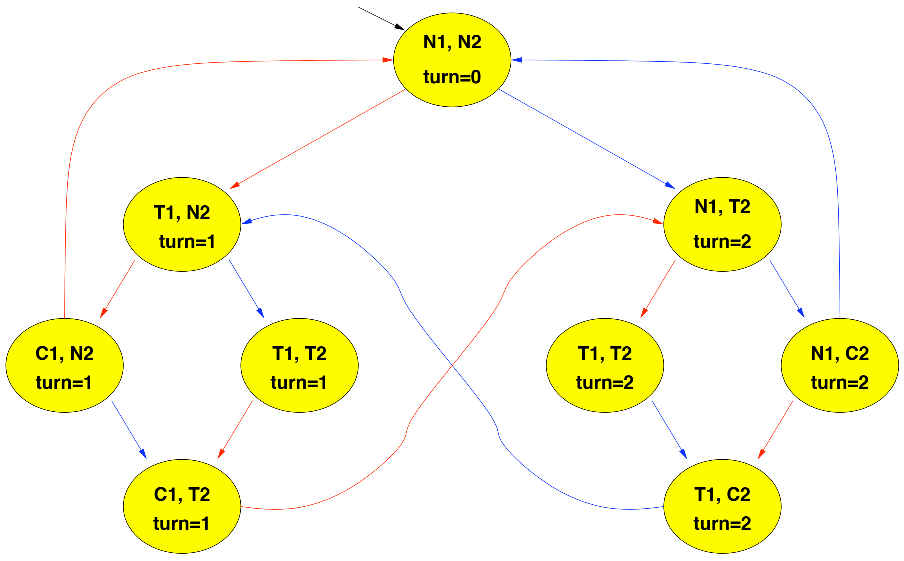


N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)$? $\Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)$?

Example 2: liveness

$[\neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)]$:



N = noncritical, T = trying, C = critical User 1 User 2

$M \models \mathbf{AG}(T_1 \rightarrow \mathbf{AF}C_1)$? $\Longrightarrow M \models \neg\mathbf{EF}(T_1 \wedge \mathbf{EG}\neg C_1)$? YES!