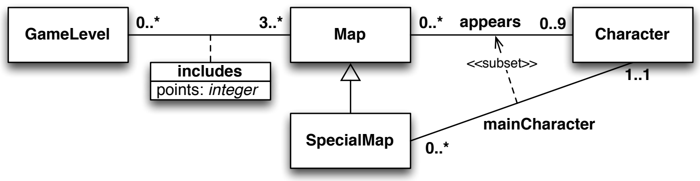
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – Jan. 29, 2021

(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

Map

artica
bush

$SpecialMap$

city
desert

$Character$

adrian
bob
charline

$appaears$

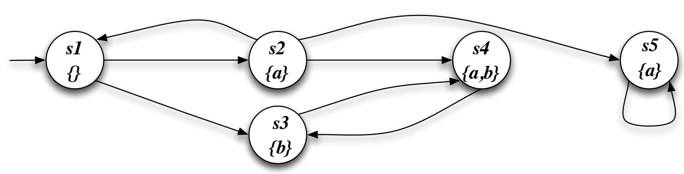
adrian	artica
adrian	bush
adrian	desert
bob	city
charline	artica

$mainCharacter$

adrian	city
charline	desert

1. Check whether the instantiation (once completed) is correct (and explain why it is or it is not).
2. Express in FOL and evaluate the following queries:
   (a) Return the maps with at least 3 distinct characters.
   (b) Return the characters that appear in maps only as main characters.
   (c) Check if there exists a map where all characters appears.
   (d) Return the maps that are not special maps such that a character that is the main character of some special map appears in them.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge \langle next\rangle X) \vee [next]Y)$ and the CTL formula $EF(\neg a \supset EXAGb)$ (showing its translation in Mu-Calculus) against the following transition system:



Exercise 4. Given the following conjunctive queries:

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q1(x) :- edge(x,y), edge(y,y), edge(x,z), edge(y,z), edge(z,y).
q2(x) :- edge(x,y), edge(y,z), edge(x,v), edge(v,z), edge(v,y).
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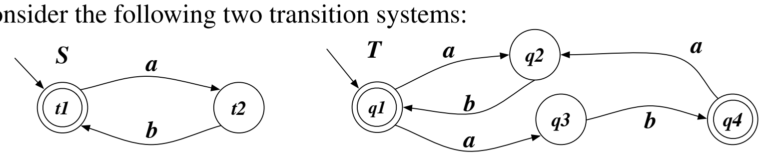
check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

Exercise 5. Check whether the following FOL formula is valid, by using tableaux, and if not, exhibit an interpretation that is a counter example:

$$(\forall x.P(x) \supset Q(x) \wedge R(X)) \equiv ((\forall x.P(x) \supset Q(x)) \ \wedge \ (\forall x.P(x) \supset R(x)))$$

Exercise 6 (optional). [1] Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

[1] *The student can get the maximum grade even without doing Exercise 6.*