Foundations of Artificial Intelligence
Exercises Sheet 8

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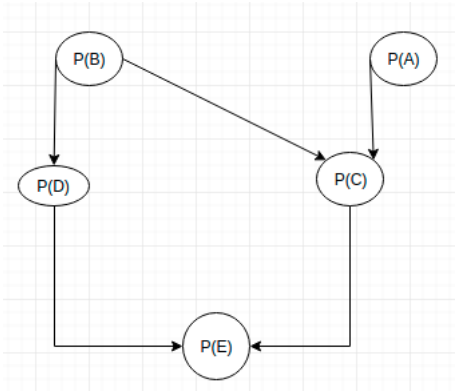
21th of June, 2021

- **Exercise 8.1**

- **(a)**

Some events were already independent (as said in the last sheet), respectively: E (even) - T (number ≥ 2) and O (odd) - T are independent. It follows also for the conditionally independence.
$P(E \wedge T|U) = P(E|T,U)P(T|U)$ No, you should check whether $P(E \cap T|U) = P(E|U) \cdot P(T|U)$ actually
Since, E and T are conditionally independent, it follows: holds, which is not the case → dependent
$= P(E|U)P(T|U)$, where U (die rolled ≥ 2). The same reasoning can be done for O and T. (same for O and T)
Instead, a case of conditionally dependence (that also follows from the previous sheet) is the following one:
$P(E \wedge O|U) = P(E|O,U)P(O|U) = 0 \cdot P(o|u) = 0 * \frac{2}{5}$, while, without the conditional event, it would have been: $P(E|U)P(O|U) = \frac{3}{5}\frac{2}{5}$ ✓

- **(b)**



✓

$P(B)$
0.8

$P(A)$
0.6

B	$P(D)$
F	0.5
T	0.1

A	B	$P(C)$
F	F	0.8
F	T	0.2
T	F	0.1
T	T	0.1

C	D	$P(E)$
F	F	0.1
F	T	0.3
T	F	0.9
T	T	0.5

$P(A, \neg B, \neg D, E)$
$= P(A|\neg B, \neg D, E) * P(\neg B|\neg D, E) * P(\neg D|E) * P(E)$ ← this does not work, because D is depedent of B and E is dependent of C and E (or which is hidden!)
$= P(A) * P(\neg B) * (\neg D) * P(E)$
$= 0.6 * 0.2 * 0.82 * 0.266 = 0.026$

$P(D) = P(D,B) + P(D,\neg B) = P(D|B)P(B) + P(D|\neg B)P(\neg B)$
$= 0.1 * 0.8 + 0.5 * 0.2 = 0.18$

$P(\neg D) = 1 - P(D) = 0.82$

$P(C) = P(C,A,B) + P(C,\neg A,B) + P(C,A,\neg B)$
$P(C) = P(C|A,B)P(A|B)P(B) + P(C|\neg A,B)P(\neg A|B)P(B) + P(C|A,\neg B)P(A|\neg B)P(\neg B) + P(C|\neg A,\neg B)P(\neg A|\neg B)P(\neg B)$
$P(C) = 0.1 * 0.6 * 0.8 + 0.2 * 0.4 * 0.8 + 0.1 * 0.6 * 0.2 + 0.8 * 0.2 * 0.4 = 0.188$

$P(E) = P(E,C,D) + P(E,\neg C,D) + P(E,C,\neg D) =$
$= P(E|C,D)P(C|D)P(D) + P(E|\neg C,D)P(\neg C|D)P(D) + P(E|C,\neg D)P(C|\neg D)P(\neg D) + P(E|\neg C,\neg D)P(\neg C|\neg D)P(\neg D)$
$P(E) = 0.5 * 0.188 * 0.18 + 0.3 * 0.812 + 0.18 + 0.9 * 0.188 * 0.82 + 0.1 * 0.812 * 0.82 = 0.266$

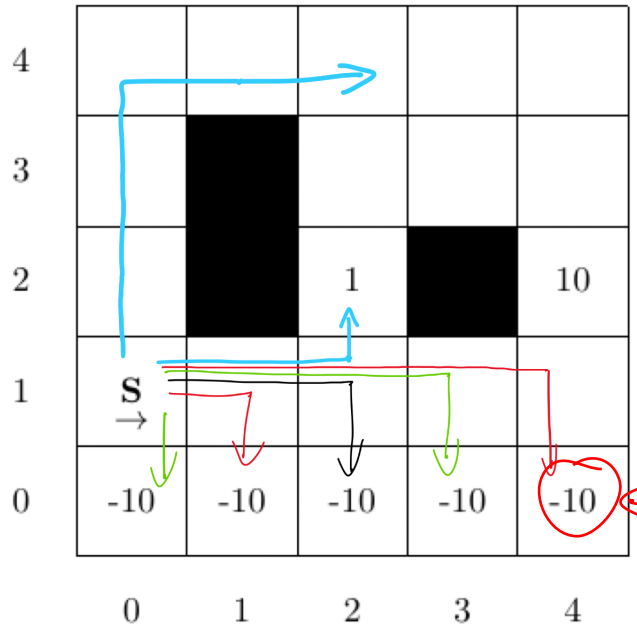
- **Exercise 8.2**

- **(a)**

  $P(E,E,E,E,N) = P(E|E,E,E,N) \cdot P(E|E,E,N) \cdot P(E|E,N) \cdot P(E|N) \cdot P(N)$
  Since every event is independent from the previous one:
  $= P(E) \cdot P(E) \cdot P(E) \cdot P(E) \cdot P(N)$
  These are the intended directions (P(intended direction) = 0.8), so:
  $= 0.8 \cdot 0.8 \cdot 0.8 \cdot 0.8 \cdot 0.8 = 0.328.$

- **(b)**

  With very low probability, every other direction is achievable. The agent will stop if has already done 5 steps, or is in a terminal state.



not reachable, because