Foundations of Artificial Intelligence

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Exercise Sheet 8 — Solutions

Exercise 8.1 (Conditional Independence and Bayes Networks)

(a) Consider exercise 7.3 (a) in exercise sheet 7. You are given additional information that you rolled a number ≥ 2 (let's name this event U). Are any of the events E, O and T conditionally independent given this information? Show the probability values and reasoning.

Solution:

When given that U has occurred, there are 5 remaining possible outcomes for the roll of the die. Then

$P(E\|U) = 0.6$	3 out of 5 possibilities are covered under the event
$P(O\|U) = 0.4$	2 out of 5 possibilities are covered under the event
$P(T\|U) = 0.8$	4 out of 5 possibilities are covered under the event
$P(E \cap O\|U) = 0 \neq P(E\|U) * P(O\|U) = 0.24$	E and O are disjoint events
$P(E \cap T\|U) = 0.4 \neq P(E\|U) * P(T\|U) = 0.48$	E and T cover 2 out of 5 possibilities
$P(O \cap T\|U) = 0.4 \neq P(O\|U) * P(T\|U) = 0.32$	O and T cover 2 out of 5 possibilities

We see that, by definition, none of the events are conditionally independent given U.

(b) Consider the following probability tables (all variables are binary, thus they can be either true or false):

$P(B)$
0.8

$P(A)$
0.6

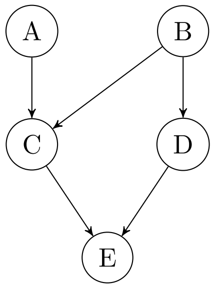
B	$P(D)$
F	0.5
T	0.1

A	B	$P(C)$
F	F	0.8
F	T	0.2
T	F	0.1
T	T	0.1

C	D	$P(E)$
F	F	0.1
F	T	0.3
T	F	0.9
T	T	0.5

Draw the corresponding Bayesian network and compute the probability $P(A, \neg B, \neg D, E)$.

Solution:

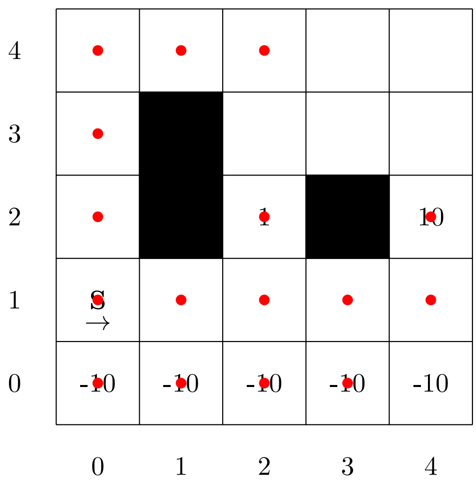


$$P(A, \neg B, \neg D, E)$$

$$\begin{aligned}
P(A, \neg B, \neg D, E) &= P(A)P(\neg B)P(\neg D|\neg B)\left(\sum_{c\in\{\neg C, C\}} P(c|A, \neg B)P(E|c, \neg D)\right) \\
&= 0.6 \cdot 0.2 \cdot 0.5 \cdot (0.1 \cdot 0.9 + 0.9 \cdot 0.1) \\
&= 0.6 \cdot 0.2 \cdot 0.5 \cdot (0.09 + 0.09) \\
&= 0.06 \cdot (0.18) = 0.0108
\end{aligned}$$

Exercise 8.2 (Sequential Decision Problems)

Consider the sequential decision problem in the following grid world. The agent wants to reach one of the goal states (terminal states with positive rewards) and wants to avoid falling of the cliffs (terminal states with negative rewards). The agent can carry out actions N (go north), E (go east), S (go south), or W (go west). The agent moves in the intended direction with a probability of 0.8, and with a probability of 0.1 each, it moves at right angles to the intended direction. If the agent bumps into a wall, it remains in its current state. The start state is marked **S** with the 1st action (E) marked under it. States are enumerated as in a Cartesian coordinate system. So, **S** $= (0, 1)$.



(a) How large is the probability that the sequence of actions (E,E,E,E,N) leads to the goal state with a reward of 10? Which other states can be reached with this sequence of actions? Mark the corresponding states in the grid above.

Solution:

0.8^5