Foundations of Artificial Intelligence
Exercise Sheet 6

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Exercise 6.1

$D = \{0, 1, 2, 3\}$
$even^{\mathcal{I}} = \{0, 2\}$
$odd^{\mathcal{I}} = \{1, 3\}$
$lessThan^{\mathcal{I}} = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$
$two^{\mathcal{I}} = 2 plus^{\mathcal{I}} : D \ x \ D \rightarrow D, \ plus^{\mathcal{I}}(a, b) = (a + b) \ mod 4$

a)

$$\begin{aligned}
\Theta_1 &= odd(y) \wedge even(two) \\
&= odd(1) \wedge even(2) \\
&= T \wedge T \\
&= T
\end{aligned}$$

b)

$$\begin{aligned}
\Theta_2 &= \forall x (even(x) \vee odd(x)) \\
&= \forall x T \\
&= T
\end{aligned}$$

c)

$$\begin{aligned}
\Theta_3 &= \forall x \exists y \ lessThan(x, y) \\
&= \exists y \ lessThan(0, y) \wedge \exists y \ lessThan(1, y) \wedge \exists y \ lessThan(2, y) \wedge \exists y \ lessThan(3, y) \\
&= T \wedge T \wedge T \wedge F \\
&= F
\end{aligned}$$

d)

$$\Theta_4 = \forall x(even(x) \Rightarrow \exists y \; lessThan(x,y))$$
$$= (even(0) \Rightarrow \exists y \; lessThan(0,y)) \land (even(1) \Rightarrow \exists y \; lessThan(1,y))$$
$$\land (even(2) \Rightarrow \exists y \; lessThan(2,y)) \land (even(3) \Rightarrow \exists y \; lessThan(3,y))$$
$$= (T \Rightarrow T) \land (F \Rightarrow T) \land (T \Rightarrow T) \land (F \Rightarrow F)$$
$$= T \land T \land T \land T$$
$$= T$$

e)

$$\Theta_5 = \forall x(odd(x) \Rightarrow even(plus(x,y))$$
$$= (odd(0) \Rightarrow even(plus(0,1) \land (odd(1) \Rightarrow even(plus(1,1)) \land$$
$$(odd(2) \Rightarrow even(plus(2,1)) \land (odd(3) \Rightarrow even(plus(3,1))$$
$$= (F \Rightarrow F) \land (T \Rightarrow T) \land (F \Rightarrow F) \land (T \Rightarrow T)$$
$$= T \land T \land T \land T$$
$$= T$$

Exercise 6.2

a)

$$\Theta = \forall x \; (P(x,w) \Rightarrow Q(x))$$
$$= (P(a,b) \Rightarrow Q(a)) \land (P(b,b) \Rightarrow Q(b)) \land (P(c,b) \Rightarrow Q(c))$$
$$= (T \Rightarrow T) \land (T \Rightarrow T) \land (F \Rightarrow F)$$
$$= T \land T \land T$$

b)

$$\Theta_2 = \exists x(R(v,x) \Rightarrow P(x,x))$$
$$= [R(a,x) \Rightarrow (P(x,x))]$$
$$= [R(a,a) \Rightarrow (P(a,a))]$$
$$= [T \Rightarrow T]$$
$$= T$$

c)

$$\Theta_3 = \forall x \forall y \ (R(x,y) \Longleftrightarrow Q(y))$$
$$= (R(a,a) \Longleftrightarrow Q(a)) \wedge (R(a,b) \Longleftrightarrow Q(b)) \wedge (R(a,c) \Longleftrightarrow Q(c)) \wedge$$
$$(R(b,a) \Longleftrightarrow Q(a)) \wedge (R(b,b) \Longleftrightarrow Q(b)) \wedge (R(b,c) \Longleftrightarrow Q(c)) \wedge$$
$$(R(c,a) \Longleftrightarrow Q(a)) \wedge (R(c,b) \Longleftrightarrow Q(b)) \wedge (R(c,c) \Longleftrightarrow Q(c)) \wedge$$
$$= (T \Longleftrightarrow T) \wedge (T \Longleftrightarrow T) \wedge (T \Longleftrightarrow F) \wedge$$
$$(F \Longleftrightarrow T) \wedge (F \Longleftrightarrow T) \wedge (T \Longleftrightarrow F) \wedge$$
$$(F \Longleftrightarrow T) \wedge (T \Longleftrightarrow T) \wedge (F \Longleftrightarrow F)$$
$$= T \wedge T \wedge F \wedge T \wedge T \wedge F \wedge F \wedge T \wedge T$$
$$= F$$

d)

$$\Theta_4 = [\neg \forall x \forall y (Q(y) \vee P(x,y))] \wedge [\exists z (Q(z) \vee P(w,z))]$$
$$= [\forall y \exists x \neg (Q(y) \vee P(x,y))] \wedge [Q(a) \vee P(w,a)]$$
$$= [\forall y \exists x (\neg Q(y) \wedge \neg P(x,y))] \wedge T$$
$$= (\exists x (\neg Q(a) \wedge \neg P(x,a))) \wedge (\exists x (\neg Q(b) \wedge \neg P(x,b))) \wedge (\exists x (\neg Q(c) \wedge \neg P(x,c)))$$
$$= F \wedge F \wedge (\exists x (\neg Q(c) \wedge \neg P(x,c)))$$
$$= F$$