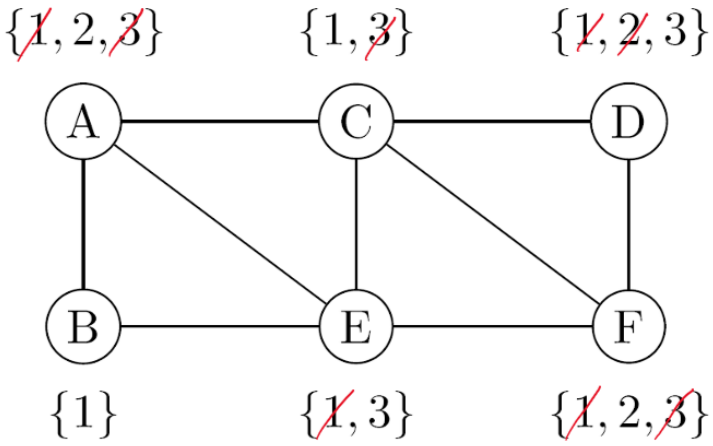
Foundations of Artificial Intelligence
Exercise Sheet 4

Robin Vogt, Riccardo Salvalaggio, Simon Lempp, Josephine Bergmeier

September 23, 2021

Exercise 4.1



Exercise 4.2

a)

a) valid and satisfiable

b) not valid but satisfiable

c) valid and satisfiable

d) not valid but satisfiable

e) not valid but satisfiable

b)

Truth table:

A	B	C	D	$A \wedge B$	$B \wedge C$	$(A \wedge B) \vee (B \wedge C)$	$A \vee B$	$A \leftrightarrow B$	$B \leftrightarrow C$	$(A \leftrightarrow B) \wedge (B \leftrightarrow C)$
0	0	0	1	0	0	0	0	1	1	1
0	0	1	0	0	0	0	0	1	0	0
0	0	1	1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	1	0	0	0
0	1	0	1	0	0	0	1	0	0	0
0	1	1	0	0	1	1	1	0	1	0
0	1	1	1	0	1	1	1	0	1	0
1	0	0	0	0	0	0	1	0	1	0
1	0	0	1	0	0	0	1	0	1	0
1	0	1	0	0	0	0	1	0	0	0
1	0	1	1	0	0	0	1	0	0	0
1	1	0	0	1	0	1	1	1	0	0
1	1	0	1	1	0	1	1	1	0	0
1	1	1	0	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	1	1	1

By reading the truth table you can see that there are 6 possible models for $(A \wedge B) \vee (B \wedge C)$ (a)), 12 possible models for $A \vee B$ (b)) and 4 possible models for $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$ (c))

Exercise 4.3

$$
\begin{aligned}
& (\neg X \rightarrow Y) \vee ((X \wedge \neg Z) \leftrightarrow Y) \\
\Leftrightarrow\ & (\neg X \rightarrow Y) \vee (((X \wedge \neg Z) \rightarrow Y) \wedge (Y \rightarrow (X \wedge \neg Z))) \\
\Leftrightarrow\ & (\neg X \vee Y) \vee ((\neg(X \wedge \neg Z) \vee Y) \wedge (\neg Y \vee (X \wedge \neg Z))) \\
\Leftrightarrow\ & (\neg X \vee Y) \vee ((\neg X \vee Z \vee Y) \wedge (\neg Y \vee (X \wedge \neg Z))) \\
\Leftrightarrow\ & (\neg X \vee Y) \vee ((\neg X \vee Z \vee Y) \wedge ((\neg Y \vee X) \wedge (\neg Y \vee \neg Z))) \\
\Leftrightarrow\ & ((\neg X \vee Y) \vee (\neg X \vee Z \vee Y)) \wedge ((\neg X \vee Y) \vee ((\neg Y \vee X) \wedge (\neg Y \vee \neg Z))) \\
\Leftrightarrow\ & (\neg X \vee Z \vee Y) \wedge ((\neg X \vee Y) \vee (\neg Y \vee X)) \wedge ((\neg X \vee Y) \vee (\neg Y \vee \neg Z)) \\
\Leftrightarrow\ & \neg X \vee Z \vee Y
\end{aligned}
$$