Esame 17/06/2019
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Exercise 1.
Express the diagram in FOL:

Alphabet: C(x), P(x), S(x), SpecialS(x), SpecializedIn(x, y), Contract(x, y, z), Cost(x, y, z, w)

Axioms:

Forall x. SpecialS(x) implies S(x)

Forall x. P(x) implies 1 <= # {y | SpecializedIn(x, y)} <= 5

Forall x,y. SpecializedIn(x, y) implies P(x) AND S(y)

Forall x, y, z. Contract(x, y, z) implies C(x) AND P(y) AND S(z)

Forall x, y, z, z'. Contract(x, y, z) AND Contract(x, y, z') implies z=z'

Forall x, y, z, w. Cost(x, y, z, w) implies Contract(x, y, z) AND Real(w)

Forall x, y, z. Contract(x, y, z) implies 1 <= # {w | Cost(x, y, z, w)} <= 1

Exists w. Cost(x, y, z, w) AND (Forall w, w'. Cost(x, y, z, w) AND Cost(x, y, z, w') implies w=w')

Exercise 2.
Consider the above UML class diagram and the following (partial) instantiation:

1. **The instantiation is not correct, in order to complete it, all the instances of SpecialService must be either in the Service table. The resulting Table after the correction is the following:**

   Service: {S1, S2, S3, SS1, SS2}

2. Express in FOL the following queries and evaluate them over the completed instantiation:

   (a) Return those providers that have contracts with at least two customers.

   P(y) AND Exists x. Contract(x, y, z) AND Exists x’. Contract(x’, y, z) AND x!=x’

   ~~CONJUNCTIVE QUERY~~

   (b) Return those providers that have contracts only for services they are specialized in.

   P(x) AND Forall y. (Exists z. Contract(x, y, z) implies SpecializedIn(x, y))

   NOTE: X is instance of Provider, Y is instance of Service, Z is instance of Customer in this case (we existentially quantify z, customer because we don’t care about them)

   (c) Return those providers that have contracts for all services they are specialized in.

   P(x) AND Forall y. SpecializedIn(x, y) implies Exists z. Contract(x, y, z))

   (d) Check whether there exists a customer with contracts for all services.

   Exists x. C(x) AND Forall y. S(y) implies Exists z. Contract(x, y, z)

Exercise 3.
Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge \langle next\rangle X) \vee (\neg b \wedge \langle next\rangle Y)$ and the CTL formula $AG(AF\ a \wedge E\ F\ b \wedge E\ G\neg b)$ (showing its translation in Mu-Calculus) against the following transition system:

1.

$\Phi = \nu X.\mu Y.((a \wedge \langle next\rangle X) \vee (\neg b \wedge \langle next\rangle Y)$

$[|X_0|] = \{1, 2, 3, 4\}$

$[|X_1|] = [|\mu Y.((a \wedge \langle next\rangle X) \vee (\neg b \wedge \langle next\rangle Y)|] = \{1, 2, 3\}$

$[|Y_0|] = \{\}$

$[|Y_1|] = [|(a \wedge \langle next\rangle X_0) \vee (\neg b \wedge \langle next\rangle Y_0)|] =$
$[|a|] \wedge PreE(next, [|X_0|]) \vee [|\neg b|] \wedge PreE(next, Y_0) =$

$= \{2\}$ intersec $\{1, 2, 3, 4\}$ U $\{1, 2, 3\}$ intersec $\{\} = \{2\}$

$[|Y_2|] = [|(a \wedge \langle next\rangle X_0) \vee (\neg b \wedge \langle next\rangle Y_1)|] =$
$[|a|] \wedge PreE(next, [|X_0|]) \vee [|\neg b|] \wedge PreE(next, Y_1) =$

$= \{2\}$ intersec $\{1, 2, 3, 4\}$ U $\{1, 2, 3\}$ intersec $\{1\} = \{1, 2\}$

$[|Y_3|] = [|(a \wedge \langle next\rangle X_0) \vee (\neg b \wedge \langle next\rangle Y_2)|] =$
$[|a|] \wedge PreE(next, [|X_0|]) \vee [|\neg b|] \wedge PreE(next, Y_2) =$

$= \{2\}$ intersec $\{1, 2, 3, 4\}$ U $\{1, 2, 3\}$ intersec $\{1, 3, 4\} = \{1, 2, 3\}$

$[|Y_4|] = [|(a \wedge \langle next\rangle X_0) \vee (\neg b \wedge \langle next\rangle Y_3)|] =$
$[|a|] \wedge PreE(next, [|X_0|]) \vee [|\neg b|] \wedge PreE(next, Y_3) =$

$= \{2\}$ intersec $\{1, 2, 3, 4\}$ U $\{1, 2, 3\}$ intersec $\{1, 2, 3, 4\} = \{1, 2, 3\}$

Found a LFP -> $[|Y_3|] = [|Y_4|] = \{1, 2, 3\}$

$[|X_2|] = [|\mu Y.((a \wedge \langle next\rangle X_1) \vee (\neg b \wedge \langle next\rangle Y)|] = \{1, 2, 3\}$

$[|Y_{00}|] = \{\}$

$[|Y_{11}|] = [|(a \wedge \langle next\rangle X_1) \vee (\neg b \wedge \langle next\rangle Y_{00})|] =$
$[|a|] \wedge PreE(next, [|X_1|]) \vee [|\neg b|] \wedge PreE(next, Y_{00}) =$

$= \{2\}$ intersec $\{1, 2, 3, 4\}$ U $\{1, 2, 3\}$ intersec $\{\} = \{2\}$

$[|Y_{22}|]$ = $[|$(a ∧ ⟨next⟩X_1) ∨ (¬b ∧ ⟨next⟩Y_{11})$|]$ =
$[|a|]$ ∧ PreE(next, $[|X_1|]$) ∨ $[|$¬b $|]$ ∧ PreE(next, Y_{11}) =

= {2} intersec {1, 2, 3, 4} U {1, 2, 3} intersec {1} = {1, 2}

$[|Y_{33}|]$ = $[|$(a ∧ ⟨next⟩X_1) ∨ (¬b ∧ ⟨next⟩Y_{22})$|]$ =
$[|a|]$ ∧ PreE(next, $[|X_1|]$) ∨ $[|$¬b $|]$ ∧ PreE(next, Y_{22}) =

= {2} intersec {1, 2, 3, 4} U {1, 2, 3} intersec {1, 3, 4} = {1, 2, 3}

$[|Y_{44}|]$ = $[|$(a ∧ ⟨next⟩X_1) ∨ (¬b ∧ ⟨next⟩Y_{44})$|]$ =
$[|a|]$ ∧ PreE(next, $[|X_1|]$) ∨ $[|$¬b $|]$ ∧ PreE(next, Y_{44}) =

= {2} intersec {1, 2, 3, 4} U {1, 2, 3} intersec {1, 2, 3, 4} = {1, 2, 3}

Found a LFP -> $[|Y_{33}|]$ = $[|Y_{44}|]$ = {1, 2, 3}

Found a GFP -> $[|X_1|]$ = $[|X_2|]$ = {1, 2, 3}

2. AG(AF a ∧ EF b ∧ EG ¬b)

Alpha = EG ¬b
Beta = EF b ∧ Alpha
Gamma = AF a ∧ Beta
Delta = AG(Gamma)

T(Alpha) = νX. **¬b** ∧ ⟨next⟩X
T(Beta) = μX. b ∨ ⟨next⟩X ∧ T(Alpha)
T(Gamma) = μX. a ∨ [next]X ∧ T(Beta)
T(Delta) = νX. T(Gamma) ∧ [next]X

$[|$Alpha$|]$ = $[|$EG ¬b$|]$ = $[|$νX. **¬b** ∧ ⟨next⟩X**$|]$ = {1 , 2, 3}**
$[|X_0|]$ = {1, 2, 3, 4}
$[|X_1|]$ = $[|$¬b ∧ ⟨next⟩$X_0|]$ =
= $[|$¬b $|]$ intersec PreE(next, X_0) =
= {1, 2, 3} intersec (1, 2, 3, 4) = {1, 2, 3}

$[|X_2|]$ = $[|$¬b ∧ ⟨next⟩$X_1|]$ =
= $[|$¬b $|]$ intersec PreE(next, X_1) =

$= \{1, 2, 3\}$ intersec $(1, 2, 3, 4) = \{1, 2, 3\}$

$[|X_1|] = [|X_2|] = \{1, 2, 3\}$

$[|Beta|] = [|EF\ b \wedge Alpha\ |] = [|\ \mu X.\ b \vee \langle next \rangle X \wedge Alpha\ |] =$ **{1, 2, 3, 4}**

$[|X_0|] = \{\}$

$[|X_1|] = [|\ b \vee \langle next \rangle X_0|] \wedge [|\ Alpha\ |] =$

$= [|\ b\ |]$ U PreE(next, X_0) intersec $[|\ Alpha\ |]=$

$= \{4\}$ U $\{\}$ intersec $\{1, 2, 3\} = \{4\}$

$[|X_2|] = [|\ b \vee \langle next \rangle X_1|] \wedge [|\ Alpha\ |] =$

$= [|\ b\ |]$ U PreE(next, X_1) intersec $[|\ Alpha\ |]=$

$= \{4\}$ U $\{3\}$ intersec $\{1, 2, 3\} = \{3, 4\}$

$[|X_3|] = [|\ b \vee \langle next \rangle X_2|] \wedge [|\ Alpha\ |] =$

$= [|\ b\ |]$ U PreE(next, X_2) intersec $[|\ Alpha\ |]=$

$= \{4\}$ U $\{2, 3\}$ intersec $\{1, 2, 3\} = \{2, 3, 4\}$

$[|X_4|] = [|\ b \vee \langle next \rangle X_3|] \wedge [|\ Alpha\ |] =$

$= [|\ b\ |]$ U PreE(next, X_3) intersec $[|\ Alpha\ |]=$

$= \{4\}$ U $\{1, 2, 3\}$ intersec $\{1, 2, 3\} = \{1, 2, 3, 4\}$

$[|X_5|] = [|\ b \vee \langle next \rangle X_4|] \wedge [|\ Alpha\ |] =$

$= [|\ b\ |]$ U PreE(next, X_4) intersec $[|\ Alpha\ |]=$

$= \{4\}$ U $\{1, 2, 3, 4\}$ intersec $\{1, 2, 3\} = \{1, 2, 3, 4\}$

$[|X_4|] = [|X_5|] = \{1, 2, 3, 4\}$

$[|Gamma|] = [|AF\ a \wedge Beta\ |] = \mu X.\ a \vee [next]X \wedge Beta|] =$ **{1, 2, 4}**

$[|X_0|] = \{\}$

$[|X_1|] = [|\ a \vee [next]X_0|] \wedge [|\ Beta\ |] =$

$= [|\ a\ |]$ U PreA(next, $[|X_0|]$) intersec $[|\ Beta\ |]=$

$= \{2\}$ U $\{\}$ intersec $\{1, 2, 3, 4\} = \{2\}$

$[|X_2|] = [|\ a \vee [next]X_1|] \wedge [|\ Beta\ |] =$

$= [|\ a\ |]$ U PreA(next, $[|\ X_1\ |]$) intersec $[|\ Beta\ |]=$

$= \{2\}$ U $\{1\}$ intersec $\{1, 2, 3, 4\} = \{1, 2\}$

$[|X_3|] = [|\ a \vee [next]X_2|] \wedge [|\ Beta\ |] =$

= [| a |] U PreA(next, [|X_2|]) intersec [| Beta |]=
= {2} U {1, 4} intersec {1, 2, 3, 4} = {1, 2, 4}

[|X_4|] = [| a ∨ [next]X_3|] ∧ [| Beta |] =
= [| a |] U PreA(next, [|X_3|]) intersec [| Beta |]=
= {2} U {1, 4} intersec {1, 2, 3, 4} = {1, 2, 4}

[|X_3|] = [|X_4|] = {1, 2, 4}

[|Delta|] = [|AG(Gamma) |] = [| νX. Gamma ∧ [next]X |] = {}

[|X_0|] = {1, 2, 3, 4}

[|X_1|] = [| Gamma |] ∧ (next, [| X_0 |]) =
= [| Gamma |] intersec PreA(next, [|X_0|]) =
= {1, 2, 4} intersec {1, 2, 4} = {1, 2, 4}

[|X_2|] = [| Gamma |] ∧ (next, [| X_1 |]) =
= [| Gamma |] intersec PreA(next, [|X_1|]) =
= {1, 2, 4} intersec {1, 4} = {1, 4}

[|X_3|] = [| Gamma |] ∧ (next, [| X_2 |]) =
= [| Gamma |] intersec PreA(next, [|X_2|]) =
= {1, 2, 4} intersec {4} = {4}

[|X_4|] = [| Gamma |] ∧ (next, [| X_3 |]) =
= [| Gamma |] intersec PreA(next, [|X_3|]) =
= {1, 2, 4} intersec {} = {}

[|X_5|] = [| Gamma |] ∧ (next, [| X_4 |]) =
= [| Gamma |] intersec PreA(next, [|X_4|]) =
= {1, 2, 4} intersec {} = {}

[|X_4|] = [|X_5|] = {}

Exercise 4.
Check whether CQ q1 is contained in CQ q2, reporting canonical DBs and homomorphism:

q1() ← edge(r, g), edge(g, b), edge(b, r).
q2() ← edge(x, y), edge(y, z), edge(z, x), edge(z, v), edge(v, w), edge(w, z).

Check whether q1 is contained in q2:
Transform the containment into an evaluation.
Freeze the free variables, introducing fresh constants, in order to work on Boolean conjunctive queries.

Check if q1(a) implies q2(a) iff Iq1 models q2(a)

Build the canonical DB Iq1:

Iq1(a) = {Delta^{Iq1}, E^{Iq1}, C^{Iq1}} ->Composed by the domain of interest, the edges and constants

Delta^{iq1} = {r, g, b} Domain: all the terms that occur in the query q1
E^{iq1}= {(r, g), (g, b), (b, r)} all the tuples of edges of the query
C^{iq1}= {a} the constants; constant 'a' interpreted as itself

Tabula form of DB Iq1:

{R, g
G, b
B, r}

Check if q2 is True in q1 -> Iq1 models q2

Guess an assignment alpha for all the free variables of q2:
First, I look for constrained atoms.

Alpha(x) = r
Alpha(y) = g
Alpha(z) = b
Alpha(v) = r
Alpha(w) = g

This is a satisfying assignment.

From CM theorem, It is an homomorphism.
Check homomorphism:

Check if the two following properties are satisfied:

$$H(c^I) = H(c^J)$$

$$(h(x), h(y)) \text{ in } C^J$$

From CM theorem: Iq1(a) models Iq2(a) iff Iq2(a) implies Iq1(a)

To check homomorphism, I transform q2 in Iq2(a) and create its canonical DB in tabula form, then I map every atom of q2 to q1 to check that all the tuples of q2 are contained in q1:

Iq2 = {Delta^{Iq2}, E^{Iq2}, C^{Iq2}}
Delta^{Iq2} = {x, y, z, v, w}
E^{Iq2} = {(x, y), (y, z), (z, x), (z, v), (v, w), (w, z)}
c^{Iq2} = {a}

Tabula form:

{X, y
Y, z
Z, x
Z, v
V, w
W, z}

H(x) = alpha(x) = r
H(y) = alpha(y) = g
H(z) = alpha(z) = b
H(v) = alpha(v) = r
H(w) = alpha(w) = g

Check if the relation is maintained by the mapping, if it is still true:

(x, y) belong to Ej -> (h(x), h(y)) belong to Ei
(y, z) belong to Ej -> (h(y), h(z)) belong to Ei
(z, x) belong to Ej -> (h(z), h(x)) belong to Ei
(z, v) belong to Ej -> (h(z), h(v)) belong to Ei

(v, w) belong to Ej -> (h(v), h(w)) belong to Ei
(w, z) belong to Ej -> (h(w), h(z)) belong to Ei
All the properties are satisfied.