Foundations of Artificial Intelligence

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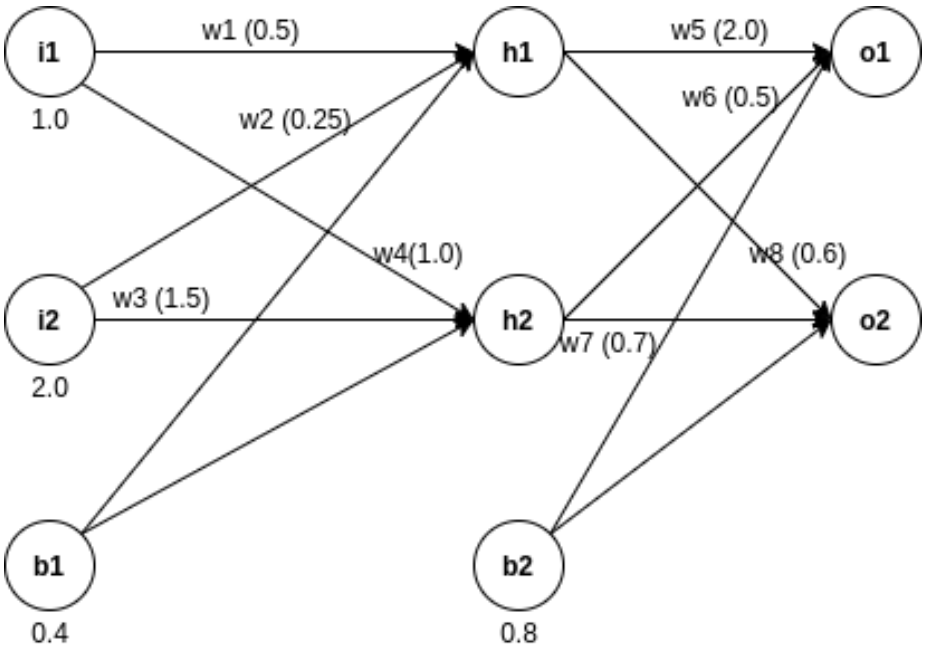
Exercise Sheet 11 — Solutions

Exercise 11.1 (Multi Layer Perceptron)

Given below is a structure of a multilayer perceptron with 2 inputs (i1 and i2), 2 hidden layers (h1 and h2), biases(b1 and b2) and one output layer(o). Each hidden and output layer output is activated using logistic sigmoid activation function:. :

a) Perform one forward pass with the values of parameters depicted with every variable in the network and calculate the outputs(o1,o2).

b) Calculate the mean square error given value of outputs (o1,o2) as (2.0,4.0).



Solution:

a) To solve the output of the network, the output of each node has to be calculated. Let the overall input of *h1* is denoted by *inh1* which is calculated as follows.

$inh1 = w1 * i1 + w2 * i2 + b1$
$inh1 = 0.5 * 1.0 + 0.25 * 2.0 + 0.4 = 1.4$

This is activated using logistic sigmoid activation function to give output, $outh1$.

$outh1 = 1/(1 + e^{-1.4}) = 0.8022$

Similarily we calculate the output of the node $h2$.
$inh2 = w4 * i1 + w3 * i2 + b1$
$inh2 = 1.0 * 1.0 + 1.5 * 2.0 + 0.4 = 4.4$

$outh2 = 1/(1 + e^{-4.4}) = 0.9878$

The output $outo_1$ is calculated as follows:
$ino_1 = w5 * outh1 + w6 * outh2 + b2$
$ino_1 = 2.0 * 0.8022 + 0.5 * 0.9878 + 0.8 = 2.8963$
$outo_1 = 1/(1 + e^{-2.8983}) = 0.9477$

The output oto_2 is calculated as follows:
$ino_2 = w8 * outh1 + w7 * outh2 + b2$
$ino_2 = 0.6 * 0.8022 + 0.7 * 0.9878 + 0.8 = 1.97278$
$outo_2 = 1/(1 + e^{-1.97278}) = 0.87791$

b) Mean sqaure error is defined as:

MSE $= \frac{1}{n}\sum_{i=1}^{n}(o_i - outo_i)^2$.
MSE $= (1/2) * ((o_1 - outo_1)^2 + (o_2 - outo_2)^2)$

Given values of o_1 and o_2 are 2.0 and 4.0 respectively.

MSE $= 0.5 * ((2.0 - 0.9477)^2 + (4.0 - 0.87791)^2)$
MSE $= 5.4273$

Exercise 11.2 (Convolutional Neural Network)

Given below is a sequence of operations in a small convolutional neural network(CNN) which takes input of shape (48 x 48 x 3). Calculate the output size and number of trainable paramters after each layer of the network.
conv1 and conv2 are the convolutional layers with given filter size f , stride s and output feature size o.

layer	**shape**	**parameters**
Input	(48,48,3)	0
conv1(f=3,s=1,o=8)		
conv2(f=5,s=1,o=16)		

Solution:

The output size of a convolution layer with input size nxn, filter size fxf, stride s and padding p is given by:

$$out = \frac{n-f-2p}{s} + 1$$

as no padiding is provided, the padding is 0.

For a filter size of fxf, previous feature size i, output feature size o, the number of learnable parameters are calculated as:

$$num_{param} = (i * f * f * o) + o.$$

For conv1, it will lead to:

$$num_{param} = (3 * 3 * 3 * 8) + 8 = 224.$$

Using these formulas, the complete table is as follows.

layer	**shape**	**parameters**
Input	(48,48,3)	0
conv1(f=3,s=1,o=8)	(46,46,8)	224
conv2(f=5,s=1,o=16)	(42,42,16)	3216