SAPIENZA Università di Roma – MSc. in Engineering in Computer Science
Formal Methods – July 19, 2021
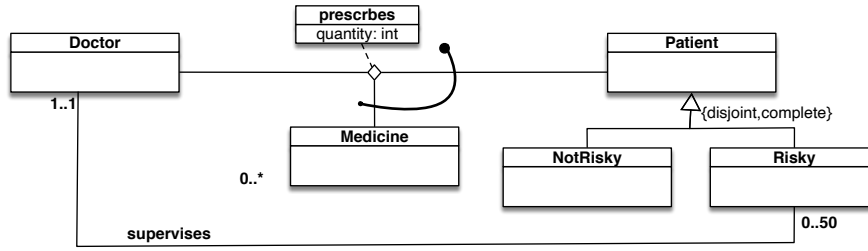*(Time to complete the test: 2 hours)*

**Exercise 1.** Express the following UML class diagram in FOL:

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation:

$NotRisky$
Norman
Nora

$Risky$
Robert
Rebecca

$Doctor$
Dory
Damian
Dana

$Medicine$
m1
m2
m3

$supervises$	
Dana	Robert
Damian	Rebecca

$prescribes/qnty$

Dory	Norman	m1	1
Dory	Nora	m1	1
Dana	Robert	m2	2
Dana	Robert	m3	2
Damian	Rebecca	m3	3

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.

2. Express in FOL the following queries, and evaluate them over the completed instantiation:

   (a) Return patients who have been prescribed at least two medicines.

   (b) Return risky patients who have been prescribed medicines by the doctor who supervises them.

   (c) Check that risky patients have medicines prescribed only by the doctor who supervises them.

   (d) Return doctors who have prescribed some medicine to all patients who they supervise.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge [next]X) \vee (b \wedge \langle next \rangle Y))$ and the CTL formula $AF(EG(a \supset AXEX\neg a))$(showing its translation in Mu-Calculus) against the following transition system:
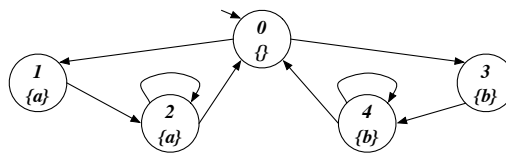
Exercise 4. Check whether the following FOL formula is valid, by using tableaux:

$$(\forall x.(P(x) \not\equiv (\exists y.Q(y)))) \supset ((\forall x.P(x)) \vee (\forall x.\neg P(x)))$$

Exercise 5. Consider the transition system TS below. Model check the LTL formula $\Box \Diamond a$, by considering that the Büchi automaton BA for $\neg \Box \Diamond a$ (i.e., $\Diamond \Box \neg a$) is the one below: