Foundations of Artificial Intelligence

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Exercise Sheet 6

Due: Friday, June 10, 2021

Exercise 6.1 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} =< \mathcal{D}, \cdot^{\mathcal{I}} >$ with

- $\mathcal{D} = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : \mathcal{D} \times \mathcal{D} \to \mathcal{D}, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$.
Explain your answer by formally applying the semantics.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x \ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y \ lessThan(x, y)$

(d) $\theta_4 = \forall x \ (even(x) \Rightarrow \exists y \ lessThan(x, y))$

(e) $\theta_5 = \forall x \ (odd(x) \Rightarrow even(plus(x, y)))$

Exercise 6.2 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} =< \mathcal{D}, \cdot^{\mathcal{I}} >$ and the variable assignment α:

- $\mathcal{D} = \{a, b, c\}$
- $P^{\mathcal{I}} = \{(a, a), (a, b), (b, a), (b, b), (b, c), (c, a)\}$
- $Q^{\mathcal{I}} = \{a, b\}$
- $R^{\mathcal{I}} = \{(a, a), (a, b), (a, c), (b, c), (c, b)\}$
- $\alpha = \{(v, a), (w, b)\}$

Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$.
Explain your answer by formally applying the semantics.

(a) $\theta_1 = \forall x \big(P(x, w) \Rightarrow Q(x)\big)$

(b) $\theta_2 = \exists x\big(R(v,x) \Rightarrow P(x,x)\big)$

(c) $\theta_3 = \forall x \forall y\big(R(x,y) \iff Q(y)\big)$

(d) $\theta_4 = \Big[\neg\forall x \forall y\big(Q(y) \vee P(x,y)\big)\Big] \wedge \Big[\exists z\big(Q(z) \vee P(w,z)\big)\Big]$

Note: The exercise sheets may be worked on in groups of up to three students.