

Knowledge Representation and Semantic Technologies

Exercises on Datalog and ASP

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Exercise 1

Given the following positive Datalog program P:

$r(X,Y) \text{ :- } s(X,Y).$ ~~$s(X,Y)$~~

$r(X,Y) \text{ :- } r(X,Z), s(Z,Y).$

$t(X) \text{ :- } r(X,X).$

$q(Y) \text{ :- } t(X), r(X,Y).$

$s(a,b).$

$s(b,c).$

$s(c,a).$

$r' :- s, s$
 $s' :- s$

$\rightarrow s' :- s, r$
 $s' :- t, s$

1) compute the minimal model of P;

2) tell if atom $q(a)$ is entailed by P.

Exercise 1 - Solution

To compute the minimal model, we use the semi-naive evaluation method. We first define the program **P'** with **Δ -relations**:

$\Delta'r(X,Y) :- \Delta r(X,Z), s(Z,Y).$ [rule R1]

$\Delta't(X) :- \Delta r(X,X).$ [rule R2]

$\Delta'q(Y) :- \Delta t(X), r(X,Y).$ [rule R3]

$\Delta'q(Y) :- t(X), \Delta r(X,Y).$ [rule R4]

$s(a,b).$ $s(b,c).$ $s(c,a).$

Exercise 1 - Solution

We then execute the iterative computation of the intensional predicates r , t , q through semi-naive evaluation on P' :

Initialization:

$$\begin{aligned} I &= \{ s(a,b), s(b,c), s(c,a) \}, \\ I' &= T_P(I) = I \cup \{ r(a,b), r(b,c), r(c,a) \} \text{ (using 1st rule of } P), \\ \Delta'I &= \{ \Delta'r(a,b), \Delta'r(b,c), \Delta'r(c,a) \} \end{aligned}$$

1st execution of the repeat-until loop:

$$\begin{aligned} I &= I \cup \{ r(a,b), r(b,c), r(c,a) \}, \\ \Delta I &= \{ \Delta r(a,b), \Delta r(b,c), \Delta r(c,a) \} \\ \underline{\Delta'I} &= \underline{T_{\Delta P}(I \cup \Delta I)} = \{ \Delta'r(a,c), \Delta'r(b,a), \Delta'r(c,b) \} \text{ (using rule R1)} \end{aligned}$$

2nd execution of the repeat-until loop:

$$\Delta I = \{ \Delta r(a,c), \Delta r(b,a), \Delta r(c,b) \}$$

$$\Delta'I = T_{\Delta P}(I \cup \Delta I) = \{ \Delta'r(a,a), \Delta'r(b,b), \Delta'r(c,c) \} \text{ (using rule R1)}$$

3rd execution of the repeat-until loop:

$$I = I \cup \{ r(a,a), r(b,b), r(c,c) \},$$

$$\Delta I = \{ \Delta r(a,a), \Delta r(b,b), \Delta r(c,c) \}$$

$$\Delta'I = T_{\Delta P}(I \cup \Delta I) = \{ \Delta't(a), \Delta't(b), \Delta't(c) \} \text{ (using rule R2)}$$

Exercise 1 - Solution

4th execution of the repeat-until loop:

$$I = I \cup \{ t(a), t(b), t(c) \},$$

$$\Delta I = \{ \Delta t(a), \Delta t(b), \Delta t(c) \}$$

$$\Delta' I = T_{\Delta P}(I \cup \Delta I) = \{ \Delta' q(a), \Delta' q(b), \Delta' q(c) \} \text{ (using rule R3)}$$

5th execution of the repeat-until loop:

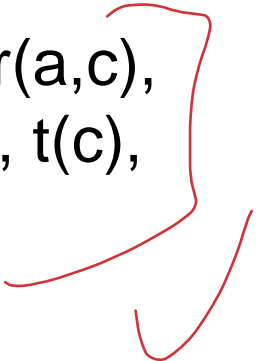
$$I = I \cup \{ q(a), q(b), q(c) \},$$

$$\Delta I = \{ \Delta q(a), \Delta q(b), \Delta q(c) \}$$

$$\Delta' I = T_{\Delta P}(I \cup \Delta I) = \{ \}$$

Exercise 1 - Solution

The minimal model of P is thus the following:

$$\text{MM}(P) = \{ s(a,b), s(b,c), s(c,a), r(a,b), r(b,c), r(c,a), r(a,c), \\ r(b,a), r(c,b), r(a,a), r(b,b), r(c,c), t(a), t(b), t(c), \\ q(a), q(b), q(c) \}$$


Exercise 1 - Solution

Finally, since atom $q(a)$ belongs to the minimal model of P , it is entailed by P .

Exercise 2

Given the following positive Datalog program with constraints P':

$r(X,Y) \text{ :- } s(X,Y).$

$r(X,Y) \text{ :- } r(X,Z), s(Z,Y).$

$t(X) \text{ :- } r(X,X).$

$q(Y) \text{ :- } t(X), r(X,Y).$

$\text{:- } t(X), q(X).$

Constraint.

$s(a,b). s(b,c). s(c,a).$

compute the minimal model of P'.

Exercise 2 - Solution

We notice that the program P' is the same as the positive program of Exercise 1, plus the constraint $\text{:- } t(X), q(X)$.
Namely, $P' = P \cup \{ \text{:- } t(X), q(X) \}$

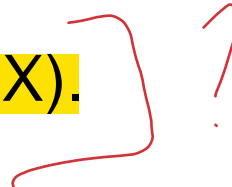
So, to answer the question we only have to **check whether the minimal model of P satisfies such a constraint.**

Exercise 2 - Solution

The minimal model M of P (see Exercise 1) is:

$MM(P) = \{ s(a,b), s(b,c), s(c,a), r(a,b), r(b,c), r(c,a), r(a,c),$
 $r(b,a), r(c,b), r(a,a), r(b,b), r(c,c), t(a), t(b), t(c),$
 $q(a), q(b), q(c) \}$

M does not satisfy the constraint :- $t(X), q(X)$.
(e.g., both $t(a)$ and $q(a)$ belong to M).



So, we conclude that there exists no (minimal) model for P.