

# Foundations of Artificial Intelligence

## Exercise Sheet 8

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### Exercise 8.1

**a)**

E,T and O,T are independent. Therefore they're for sure conditionally independent.

$$P(e \wedge t|u) = P(e|t, u)P(t|u) \stackrel{e, t \text{ independent}}{=} P(e|u)P(t|u)$$

and therefore E,T are conditionally independent. Same follows for O,T.

$$P(e \wedge o|u) = P(e|o, u)P(o|u) = 0 \cdot P(o|u) = 0 \neq \frac{6}{25} = \frac{3}{5} \cdot \frac{2}{5} = P(e|u) \cdot P(o|u)$$

**b)**

$$P(a, \neg b, \neg d, e) = P(a|\neg b, \neg d, e)P(\neg b|\neg d, e)P(\neg d|e)P(e) = P(a)P(\neg b)P(\neg d)P(e)$$

$$P(A, \text{not} B, \text{not} D, E) = P(A)P(\text{not} B)P(\text{not} D|\text{not} B)P(E|A, \text{not} B, \text{not} D) = 0.6 \cdot 0.8 \cdot 0.5 \cdot P(\neg d) = 1 - (0.5 \cdot 0.2 - 8.1 \text{ a})$$

$$\begin{aligned} P(A, \neg B, \neg D, E) &= P(A|\neg B, \neg D, E) * P(\neg B|\neg D, E) * P(\neg D|E) * P(E) \\ &= P(A) * P(\neg B) * P(\neg D) * P(E) \\ &= 0.6 * 0.2 * 0.82 * 0.266 = 0.026 \end{aligned}$$

$$\begin{aligned} P(D) &= P(D, B) + P(D, \neg B) = P(D|B)P(B) + P(D|\neg B)P(\neg B) \\ &= 0.1 * 0.8 + 0.5 * 0.2 = 0.18 \end{aligned}$$

$$P(\neg D) = 1 - P(D) = 0.82$$

$$\begin{aligned} P(C) &= P(C, A, B) + P(C, \neg A, B) + P(C, A, \neg B) \\ P(C) &= P(C|A, B)P(A|B)P(B) + P(C|\neg A, B)P(\neg A|B)P(B) + P(C|A, \neg B)P(A|\neg B)P(\neg B) + \\ &P(C|\neg A, \neg B)P(\neg A|\neg B)P(\neg B) \end{aligned}$$

$$P(C) = 0.1 * 0.6 * 0.8 + 0.2 * 0.4 * 0.8 + 0.1 * 0.6 * 0.2 + 0.8 * 0.2 * 0.4 = 0.188$$

$$\begin{aligned} P(E) &= P(E, C, D) + P(E, \neg C, D) + P(E, C, \neg D) = \\ &= P(E|C, D)P(C|D)P(D) + P(E|\neg C, D)P(\neg C|D)P(D) + P(E|C, \neg D)P(C|\neg D)P(\neg D) + \\ &P(E|\neg C, \neg D)P(\neg C|\neg D)P(\neg D) \\ P(E) &= 0.5 * 0.188 * 0.18 + 0.3 * 0.812 * 0.18 + 0.9 * 0.188 * 0.82 + 0.1 * 0.812 * 0.82 = \\ &0.266 \end{aligned}$$

## Exercise 8.2