Foundations of Artificial Intelligence

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Exercise Sheet 2 — Solutions

Exercise 2.1 (Formalizing problems)

Formalize the following problems as precisely as possible, by defining the initial state, the state space, the set of actions, the goal test and the path cost function:

- You want to solve Rubik's Cube.
  `http://en.wikipedia.org/wiki/Rubik%27s_Cube`

  Solution:

  A Rubik's Cube consists of 26 unique miniature cubes that can be moved to different locations on the cubes six faces by some rotation mechanic. The miniature cube's face plates are colored in one of six colors.

  state space: The states are $9 \cdot 6 = 54$-tuples representing the different color configurations (9 face plates per side, 6 sides).

  initial state: An arbitrary color configuration.

  goal test: Are all cube faces uniformly colored?

  actions: The rotations constitute actions. Each rotation/action changes the configuration of the cube from one state to another.

  path costs: Each rotation induces a cost, for instance 1 (or 2 for 180° rotations). The path cost is the sum of the rotations' costs.

- You have to color a map of Europe with only four colors. In order for the national borders to be recognizable, no two neighboring countries may be assigned the same color.

  Solution:

  state space: The states are 49-tuples of colors, e.g., *red*, *green*, *blue*, *yellow*, or *uncolored*. Every entry represents a country on the map. The colors can be encoded with values $0-4$, i.e., 0 (*red*) to 4 (*uncolored*), for compactness' sake.

  initial state: All countries are uncolored / all countries are colored red / each country is either assigned one of the four colores or "uncolored".

  goal test: Are all countries colored and are neighbouring countries colored differently?

  actions: Assigning a color to a country given no neighboring country is assigned the same color.

  path costs: Each action induces a uniform cost of 1.

Exercise 2.2 (Search algorithms)

Prove each of the following statements:

(a) Breadth-first search is a special case of uniform-cost search.

Solution:

Breadth-first search always expands an unexpanded node of minimal depth, while uniform-cost search (UCS) always expands an unexpanded node with minimal path costs. If the action costs for UCS are constant 1, then a node has depth k exactly when its path costs are k. In particular, a node has minimal depth exactly when it has minimum path costs. The expansion criteria are therefore identical.

(b) Breadth-first search, depth-first search, and uniform-cost search are special cases of best-first search.

Solution:

It was shown in the previous exercise that breadth-first search is a special case of uniform-cost search. Thus, it remains to be shown that depth-first search (DFS) and uniform-cost search (UCS) are special cases of best-first search (BFS):

(a) DFS is a special case of BFS where $f(n) := -depth(n)$.

(b) UCS is a special case of BFS where $f(n) := g(n)$.

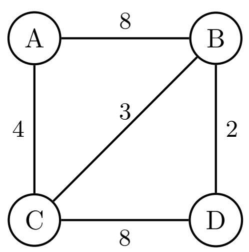
(c) Uniform-cost search is a special case of A* search.

Solution:

The evaluation function f of UCS is $f(n) = g(n)$. In contrast, the evaluation function of A* is $f(n) = g(n) + h(n)$. So UCS is a special case of A* where $h \equiv 0$.

Exercise 2.3 (Search)

Consider the graph depicted below. We are interested in a path from node **A** to node **D**. The cost of moving between two nodes is given by the respective edge weight. In the following, we refer to instances of the general TREE-SEARCH algorithm (not GRAPH-SEARCH).



	$h(x)$
A	8
B	3
C	4
D	0

(a) Perform a *Greedy Best-First Search* using heuristic h. In which order are the nodes expanded?

Solution:

Note: In the following tables, i denotes the iteration of the respective search algorithm. The search frontier contains the search nodes that are considered for expansion next. Each such node is a tuple, consisting of a label refering to a node in the problem graph and the respective f-value. Underlined nodes are expanded next.

i	search frontier
0	$\underline{(A, 8)}$
1	$\underline{(B, 3)}$ $(C, 4)$
2	$(C, 4)$ $(A, 8)$ $\underline{(D, 0)}$

Expansion order: A, B, D

(b) Perform a $A^\star$ search using heuristic h. In which order are the nodes expanded?

Solution:

i	search frontier
0	$\underline{\text{(A,0+8)}}$
1	(B,8+3) $\underline{\text{(C,4+4)}}$
2	(B,8+3) (A,8+8) $\underline{\text{(B,7+3)}}$ (D,12+0)
3	(B,8+3) (A,8+8) (D,12+0) (A,15+8) (C,10+4) $\underline{\text{(D,9+0)}}$

Expansion order: A, C, B, D

(c) Perform a *Uniform Cost Search*. In which order are the nodes expanded?

Solution:

i	search frontier
0	$\underline{\text{(A,0)}}$
1	(B,8) $\underline{\text{(C,4)}}$
2	(B,8) (A,8) $\underline{\text{(B,7)}}$ (D,12)
3	$\underline{\text{(B,8)}}$ (A,8) (D,12) (A,15) (C,10) (D,9)
4	$\underline{\text{(A,8)}}$ (D,12) (A,15) (C,10) (D,9) (A,16) (C,11) (D,10)
5	(D,12) (A,15) (C,10) $\underline{\text{(D,9)}}$ (A,16) (C,11) (D,10) (B,16) (C,12)

Expansion order: A, C, B, B, A, D

Note that there is a tie between nodes B and A in step 3 as both have a path-cost of 8. We chose to expand B before A but, as no tie breaking strategy was specified, it would also be correct if A was expanded in step 3 instead of B.

(d) Complete the following definition:

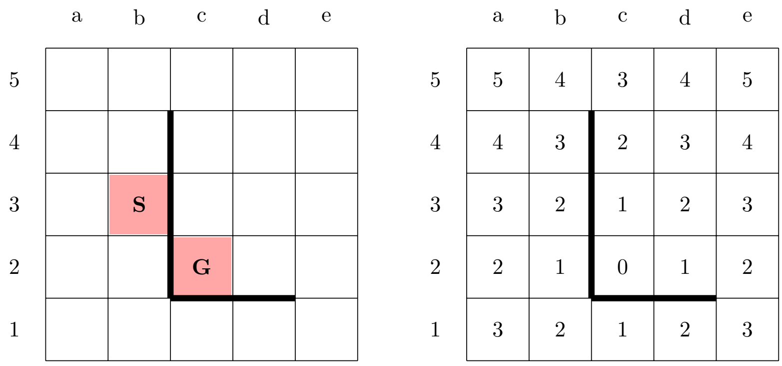
A heuristic h is *consistent* iff ...

Solution:

A heuristic h is *consistent* iff for all actions a leading from state s to state s' : $h(s) - h(s') \leq c(a)$, where $c(a)$ denotes the cost of action a.

Exercise 2.4 (A*)

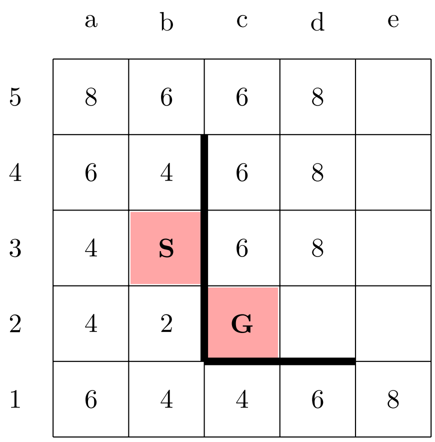
A house cleaning robot tries to find the shortest path from S (start) to G (goal). The robot can move between horizontally or vertically connected grid cells, one cell in each step. If a wall (thick black line) lies in between two cells, the robot cannot move between them. Each step incurs a uniform cost of 1. Figure (a) shows the initial state, Figure (b) the heuristic value estimates of each cell.



(a) Initial state (b) Heuristic values

(a) Perform an A* search to find the shortest path from S to G. For all generated[1] nodes, write down the respective g- and f-values in the corresponding grid cell. All other cells should be left blank. Use the graph-based variant of A* and re-open explored nodes when a better estimate becomes known (see slide 19 of 4-Informed-search-methods.pdf).

Solution:



($e1$ is optional)

[1]Generated nodes are all nodes that are visited by the search, i.e., all nodes that are added to the search frontier at some point. These nodes are not necessarily expanded.

(b) Is the heuristic from Figure (b) admissible?

Solution:

A heuristic h is admissible if $h(n) \leq h^{\star}(n)$ for all n, where $h^{\star}$ is the optimal heuristic. I.e. h is admissible if it never over estimates the cost of the cheapest solution from n to a goal. The heuristic shown in figure (b) is admissible. (It's the Manhattan Distance heuristic.)

(c) Let $h^{*}(n)$ be the actual cost of the optimal path from n to the goal G. How many nodes does A* expand when using the h^{*} heuristic?

Solution:

The implementation presented in the lecture needs 7 expansions (b3, b4, b5, c5, c4, c3, c2).