Foundations of Artificial Intelligence
Exercise Sheet 7

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Exercise 7.1

a)

Applicable operators:	Resulting state:
A	$\{X, Y, Z\}$
B	$\{\neg X, Y, Z\}$

b)

$\pi = \langle A(\varnothing, \{X\}, \{Y, Z\}), F(\varnothing, \{Z\}, \{\neg Z, G\})\rangle$

Exercise 7.2

a)

List conditional and non conditional probabilities:
$P(\textit{is red}) = 0.8$
$P(\neg \textit{is red}) = 0.2$
$P(\textit{seen as red}|\textit{is red}) = 0.7$
$P(\neg \textit{seen as red}|\textit{is red}) = 0.3$
$P(\neg \textit{seen as red}|\neg \textit{is red}) = 0.9$
$P(\textit{seen as red}|\neg \textit{is red}) = 0.1$

b)

$$\begin{aligned}
P(\textit{seen as red}) &= P(\textit{seen as red}|\textit{is red}) \cdot P(\textit{is red}) + P(\textit{seen as red}|\neg \textit{ is red}) \cdot P(\neg \textit{ is red}) \\
&= 0.7 \cdot 0.8 + 0.1 \cdot 0.2 \\
&= 0.58
\end{aligned}$$

$$\begin{aligned}
P(\textit{is red}|\textit{seen as red}) &= \frac{P(\textit{seen as red}|\textit{is red}) \cdot P(\textit{is red})}{P(\textit{seen as red})} \\
&= \frac{0.7 \cdot 0.8}{0.58} \\
&= 0.97
\end{aligned}$$

Exercise 7.3

a)

$$\begin{aligned}
P(E) &= 1/6 + 1/6 + 1/6 \\
&= 3/6 \\
&= 0.5
\end{aligned}$$

$$\begin{aligned}
P(O) &= 1/6 + 1/6 + 1/6 \\
&= 3/6 \\
&= 0.5
\end{aligned}$$

$$\begin{aligned}
P(T) &= 1/6 + 1/6 + 1/6 + 1/6 \\
&= 4/6 \\
&= \frac{2}{3}
\end{aligned}$$

$P(E|T) = \frac{1}{4} + \frac{1}{4} = 0.5 = P(E) \Rightarrow$ E and T are independent.
$P(O|T) = \frac{1}{4} + \frac{1}{4} = 0.5 = P(O) \Rightarrow$ O and T are independent.
$P(E|O) = 0 \neq 0.5 = P(E) \Rightarrow$ E and O are dependent.

	E	$\neg E$
T	$\frac{1}{3}$	$\frac{1}{3}$
¬ T	$\frac{1}{6}$	$\frac{1}{6}$

b)

Joint probability distribution table for the events E and T:

$P(T \wedge E) \overset{T,E\ independent}{=} P(T) \cdot P(E) = \frac{2}{3} \cdot 0.5 = \frac{1}{3}$
$P(T \wedge \neg E) = P(T) \cdot P(\neg E) = \frac{2}{3} \cdot 0.5 = \frac{1}{3}$
$P(\neg T \wedge E) = P(\neg T) \cdot P(E) = \frac{1}{3} \cdot 0.5 = \frac{1}{6}$
$P(\neg T \wedge \neg E) = P(\neg T) \cdot P(\neg E) = \frac{1}{3} \cdot 0.5 = \frac{1}{6}$

c)

$$P(\neg e|t) \overset{T,E\ independent}{=} P(\neg e) = 0.5$$