

Foundations of Artificial Intelligence

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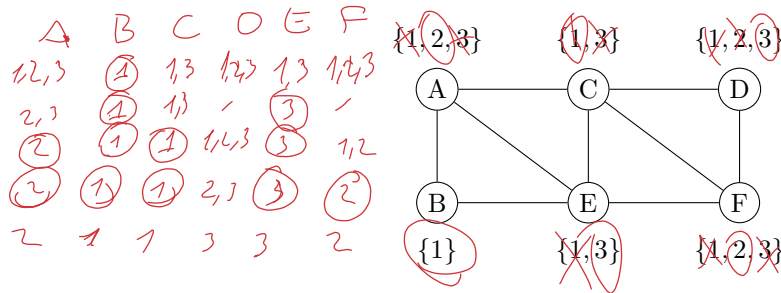
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Exercise Sheet 4

Due: Friday, May 21, 2021

Exercise 4.1 (Arc consistency)

Consider the constraint satisfaction problem given by the constraint graph below. The constraints are such that no two adjacent nodes have the same value. Establish arc-consistency.



Exercise 4.2 (Satisfiability, Models)

- (a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

- (a) $Smoke \Rightarrow Smoke$ *Valid*
(b) $Smoke \Rightarrow Fire$ *Satisfiable/Valid*
(c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Fire \Rightarrow \neg Smoke)$ *sat/Valid*
(d) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$ *Valid*
(e) $Spring \Leftrightarrow SunnyWeather$ *Unsat, sat.*

- (b) Consider a vocabulary with only four propositions, A , B , C , and D . How many models are there for the following formulae? Explain.

- (a) $(A \wedge B) \vee (B \wedge C)$ *16*
(b) $A \vee B$ *4*
(c) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 4.3 (CNF Transformation)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ , ψ , and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \quad (1)$$

$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \quad (2)$$

$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \quad (3)$$

$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \quad (4)$$

$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \quad (5)$$

Additionally, the operators \vee and \wedge are associative and commutative.

Consider the formula

$$(\neg X \rightarrow Y) \wedge ((X \wedge \neg Z) \leftrightarrow Y)$$

Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

Note: The exercise sheets may be worked on in groups of up to three students.