

Numerische Mathematik Hausaufgaben

von Rico Kölling 192316 und Svaran Singh Chandla 193922

Aufgabe 13.1

Legendre-Polynom:

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

Stützstellen:

$$\frac{1}{2}(5x^3 - 3x) = 0$$

$$x_1 = -\sqrt{\frac{3}{5}}$$

$$x_2 = 0$$

$$x_3 = \sqrt{\frac{3}{5}}$$

$$w_i = \int_{-1}^1 \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j} dx$$

$$w_0 = \int_{-1}^1 \prod_{j=0, j \neq i}^3 \frac{x - x_j}{0 - x_j} dx$$

$$\prod_{j=0, j \neq i}^3 \frac{x - x_j}{0 - x_j} = \frac{x + \sqrt{\frac{3}{5}}}{0 + \sqrt{\frac{3}{5}}} \cdot \frac{x - \sqrt{\frac{3}{5}}}{0 - \sqrt{\frac{3}{5}}} = -\frac{5}{3} \left(x^2 - \frac{3}{5} \right)$$

$$w_0 = \int_{-1}^1 -\frac{3}{5} \left(x^2 - \frac{3}{5} \right) dx = \frac{8}{9}$$

$$w_1 = \int_{-1}^1 \prod_{j=0, j \neq i}^3 \frac{x - x_j}{-\sqrt{\frac{3}{5}} - x_j} dx$$

$$\prod_{j=0, j \neq i}^3 \frac{x - x_j}{-\sqrt{\frac{3}{5}} - x_j} = \frac{x - 0}{-\sqrt{\frac{3}{5}} - 0} \cdot \frac{x - \sqrt{\frac{3}{5}}}{-\sqrt{\frac{3}{5}} - \sqrt{\frac{3}{5}}} = \frac{5}{6} x \left(x - \sqrt{\frac{3}{5}} \right)$$

$$w_1 = \int_{-1}^1 \frac{5}{6} x \left(x - \sqrt{\frac{3}{5}} \right) dx = \frac{5}{9}$$

$$w_2 = \int_{-1}^1 \prod_{j=0, j \neq i}^3 \frac{x - x_j}{\sqrt{\frac{3}{5}} - x_j} dx$$

$$\prod_{j=0, j \neq i}^3 \frac{x - x_j}{\sqrt{\frac{3}{5}} - x_j} = \frac{x - 0}{\sqrt{\frac{3}{5}} - 0} \cdot \frac{x + \sqrt{\frac{3}{5}}}{\sqrt{\frac{3}{5}} + \sqrt{\frac{3}{5}}} = \frac{5}{6} x \left(x + \sqrt{\frac{3}{5}} \right)$$

$$w_2 = \int_{-1}^1 \frac{5}{6} x \left(x + \sqrt{\frac{3}{5}} \right) dx = \frac{5}{9}$$

$$w_1 = \frac{8}{9}$$

$$w_0 = w_2 = \frac{5}{9}$$

Aufgabe 13.2

a) $\int_0^1 \frac{\sqrt{x}}{1+x} dx = \int_0^1 \frac{\sqrt{t^2}}{1+t^2} dt = \int_0^1 \frac{t}{1+t^2} dt$

Durch diese Substituierung verkleinert sich der Fehler bei Quadraturmethoden.

b) ohne Substituierung

$h := \frac{b-a}{n} = \frac{1}{n}$

$T_n = \frac{h}{2}(f(a) + 2 \cdot \sum_{i=1}^{n-1} f(a + ih) + f(b))$

Für n=1

$h = 1$

$T(1) = 1(0 + \frac{1}{4} + 0) = \frac{1}{4}$

Für n=2

$h = \frac{1}{2}$

$T(\frac{1}{2}) = \frac{1}{2}(0 + \frac{1}{4} + \frac{\sqrt{2}}{\frac{3}{2}}) = \frac{1}{24}(3 + 4\sqrt{2})$

Für n=4

$h = \frac{1}{4}$

$T(\frac{1}{4}) = \frac{1}{4}(0 + \frac{1}{4} + (\frac{2}{5} + \frac{\sqrt{2}}{\frac{3}{2}} + \frac{\sqrt{3}}{\frac{7}{4}})) = \frac{1}{4}(\frac{13}{20} + \frac{\sqrt{2}}{3} + \frac{2\sqrt{3}}{7}) = 0.404069045$

Für n=8

$h = \frac{1}{8}$

$T(\frac{1}{8}) = \frac{1}{8}(0 + \frac{1}{4} + (\frac{\sqrt{2}}{\frac{9}{8}} + \frac{2}{5} + \frac{\sqrt{6}}{\frac{11}{8}} + \frac{\sqrt{2}}{\frac{3}{2}} + \frac{\sqrt{10}}{\frac{13}{8}} + \frac{\sqrt{3}}{\frac{7}{4}} + \frac{\sqrt{14}}{\frac{15}{8}}))$
 $= \frac{1}{8}(\frac{13}{20} + \frac{5\sqrt{2}}{9} + \frac{2\sqrt{3}}{7} + \frac{2\sqrt{6}}{11} + \frac{2\sqrt{10}}{13} + \frac{2\sqrt{14}}{15})$
 $= 0.42016244$

$P_{i,m} = P_{i,m-1} - \frac{1/k_i^2}{1/k_i^2 - 1/k_i^2}(P_{i,m-1} - P_{i-1,m-1})$

k_i	$P_{i,0}$	$P_{i,1}$	$P_{i,2}$	$P_{i,3}$
1	$\frac{1}{4}$			
2	0.3607022604	0.3976030139		
3	0.404069045	0.4185246399	0.4199194149	
4	0.42016244	0.4255269082	0.42059937261	0.4264443723

$\int_0^1 \frac{\sqrt{x}}{1+x} dx = 2 - \frac{\pi}{2} = 0.4292036732$

$|0.4292036732 - 0.4264443723| = 0.0027593009$

b) mit Substituierung

$f(0) = 0$

$f(1) = \frac{1}{2}$

Für n=1

$h = 1$

$T(1) = 1(0 + \frac{1}{4} + 0) = \frac{1}{4}$

Für n=2

$h = \frac{1}{2}$

$T(\frac{1}{2}) = \frac{1}{2}(0 + \frac{1}{4} + \frac{2}{5}) = \frac{13}{40}$

Für n=4

$h = \frac{1}{4}$

$T(\frac{1}{4}) = \frac{1}{4}(0 + \frac{1}{4} + (f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}))) = \frac{2321}{6800}$

Für n=8

$h = \frac{1}{8}$

$T(\frac{1}{8}) = \frac{1}{8}(0 + \frac{1}{4} + (f(\frac{1}{8}) + f(\frac{1}{4}) + f(\frac{3}{8}) + f(\frac{1}{2}) + f(\frac{5}{8}) + f(\frac{3}{4}) + f(\frac{7}{8}))) = 0.3452689484$

$P_{i,m} = P_{i,m-1} - \frac{1/k_i^2}{1/k_i^2-1/k_i^2}(P_{i,m-1} - P_{i-1,m-1})$

k_i	$P_{i,0}$	$P_{i,1}$	$P_{i,2}$	$P_{i,3}$
1	$\frac{1}{4}$			
2	$\frac{13}{40}$	$\frac{7}{20}$		
3	$\frac{2321}{6800}$	$\frac{1179}{3400}$	$\frac{8837}{25500}$	
4	0.3452689484	0.344285937	0.3465720469	0.3465724124

$\int_0^1 \frac{\sqrt{t^2}}{1+t^2} dt = \frac{ln(2)}{2} = 0.346573590$

$|0.346573590 - 0.3465724124| = 1.1776 \cdot 10^{-6}$

Aufgabe 13.4

$$\int_{-\infty}^0 te^{2t} dt \text{ substituiert mit } t = \ln\left(\frac{1}{2}(x+1)\right)$$

$$\int_{-1}^1 \ln\left(\frac{1}{2}(x+1)\right) e^{2 \cdot (\ln(\frac{1}{2}(x+1)))} dx = 0.2222222$$

$$x_1 = -\sqrt{\frac{3}{5}}, x_2 = 0, x_3 = \sqrt{\frac{3}{5}}$$

$$\gamma_0 = \gamma_2 = \frac{5}{9}, \gamma_1 = \frac{8}{9}$$

$$\sum_{i=1}^n f(x_i) \gamma_i$$

$$\rightarrow f\left(-\sqrt{\frac{3}{5}}\right) \frac{5}{9} + f(0) \frac{8}{9} + f\left(\sqrt{\frac{3}{5}}\right) \frac{5}{9}$$

$$f\left(-\sqrt{\frac{3}{5}}\right) \frac{5}{9} = -0.0154043757$$

$$f(0) \frac{8}{9} = -0.154032707$$

$$f\left(\sqrt{\frac{3}{5}}\right) \frac{5}{9} = -0.05230023$$

$$= -0.2217373127$$

Differenz:

$$\left| -\frac{2}{9} + 0.2217373127 \right| = 0.0004849095$$

Zusatz:

$$\int_{-\infty}^0 te^{2t} dt = \frac{1}{2} e^{2t} t \Big|_{-\infty}^0 - \frac{1}{2} \int_{-\infty}^0 e^{2t} dt$$

$$= - \lim_{a \rightarrow -\infty} \frac{1}{2} e^{2a} a - \frac{1}{2} \int_{-\infty}^0 e^{2t} dt$$

$$= 0 - \frac{1}{2} \int_{-\infty}^0 e^{2t} dt = -\frac{1}{4} \int_{-\infty}^0 e^u du \text{ für } u = 2t$$

$$= \left(-\frac{e^u}{4} \right) \Big|_{-\infty}^0 = \left(-\frac{e^0}{4} - \left(-\frac{e^{-\infty}}{4} \right) \right) = -\frac{1}{4}$$

$$\int_{-\infty}^0 te^{2t} dt = -\frac{1}{4}$$