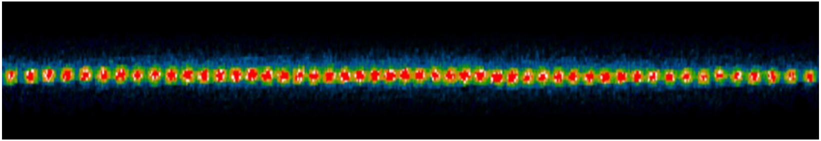
Quantum Computing With Trapped Ions

Riccardo Valperga

Imperial College London

February 14, 2020



Overview

1. Quantum computing, quantum gates and Deutsch-Josza algorithm
2. Paul trap
3. State initialization and state detection
4. Performing quantum gates with lasers: the Raman transition and the Mölmer-Sörensen gate
5. Conclusion: Benchmarks and scalability.

Quantum bits and quantum gates

The qubit:

$$|\Psi\rangle = a\,|0\rangle + b\,|1\rangle = |\psi\rangle = cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|1\rangle \tag{1}$$

Two-qubit state:

$$\sum_{i,j=0}^{1} c_{ij}\,|i\rangle \otimes |j\rangle \tag{2}$$

Single-qubit rotation:

$$R(\theta,\phi) = \begin{bmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \tag{3}$$

An entangling gate (the XX gate):

$$\begin{bmatrix} 1 & 0 & 0 & \pm i \\ 0 & 1 & \pm i & 0 \\ 0 & \pm i & 1 & 0 \\ \pm i & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Thie XX gate and the single-qubit rotation form a so-called **Universal set of gates**

The idea behind the Dutsch-Josza algorithm (1992)

Suppose we have $U_f : |i,j\rangle \to |i, j + f(i)\rangle$. By linearity of quantum evolution, the application of U_f to the input state:

$$\frac{1}{\sqrt{(m)}}(|0,0\rangle + \cdots + |m-1,0\rangle), \tag{5}$$

will evolve it as

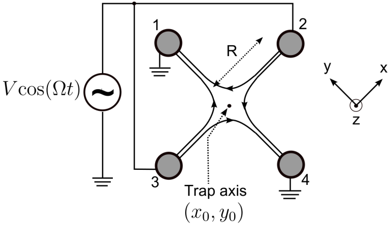
$$\frac{1}{\sqrt{(m)}}(|0,f(0)\rangle + \cdots + |m-1,f(m-1)\rangle), \tag{6}$$

In some sense, we have computed all the possible values of f by applying U_f only once.

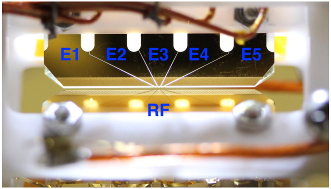
By measuring an observable that is not diagonal in the basis used, we can extract some global information of f

Physical realization: DiVincenzo Criteria (2000)

1. 'A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000...\rangle$
3. Long relevant decoherence times, much longer than the gate operation time
4. A universal set of quantum gates
5. A qubit-specific measurement capability.'



(a) *Schematic representation of a Paul Trap*



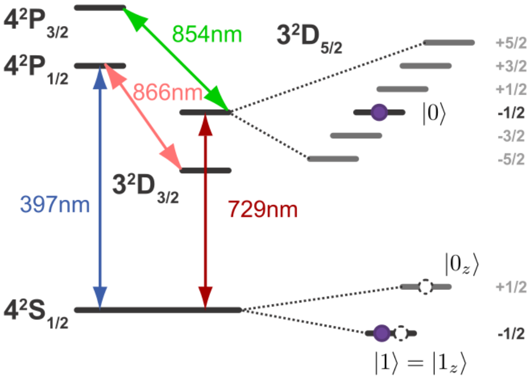
(b) *Trap with segmented blades used by IonQ*

$$V(x,y,t) = \frac{V_0}{2}\cos(\Omega t)\left(1 + \frac{x^2 - y^2}{R^2}\right). \tag{7}$$

Physical realization: qubits

Ions choice

We want to store the quantum information in two of the valence electron states of the ion. $^{40}Ca^{+}$ and $^{171}Yb^{+}$ are the most suitable.



State detection
Ciclic transitions

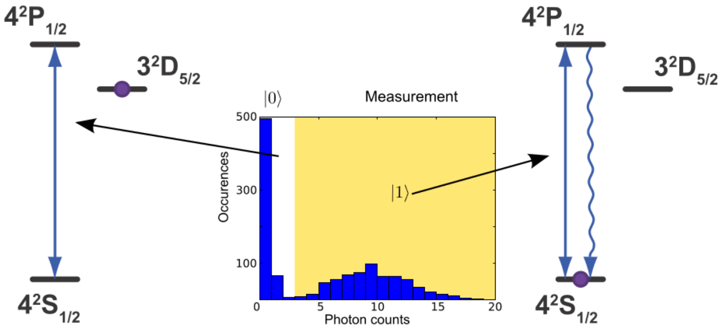
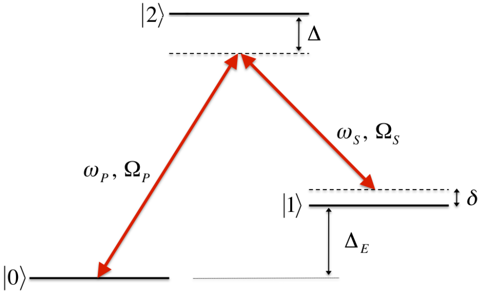


Figure: $^{40}Ca^{+}$ relevant transitions used for projective measurement, the colored area illustrates the treshold wether the state is detected as $|0\rangle$ or $|1\rangle$.

The aim is to coherently drive transitions between $|0\rangle$ and $|1\rangle$ using $|2\rangle$ as an intermediate state.



This is obtained using two lasers, both detuned from the intermediate state to avoid driving population into it.

The Hamiltonian of the system is:

$$\hat{H} = \begin{pmatrix} 0 & 0 & \Omega_p \\ 0 & -2\delta & \Omega_S \\ \Omega_p & \Omega_S & -2\Delta \end{pmatrix}, \tag{8}$$

Solving the time-dependent Schrödinger equation we find three coupled differential equations for the coefficients of

$$|\psi(t)\rangle = c_0(t)\,|0\rangle + c_1(t)\,|1\rangle + c_2(t)\,|2\rangle \tag{9}$$

If the detuning is much larger than the Rabi frequencies we find that

$$|c_0(t)|^2 = 1 + \frac{\Omega_R^2}{2\Omega_0^2}(\cos\Omega_0 t - 1) \tag{10}$$

with

$$\begin{aligned} \Omega_R &= \frac{\Omega_p \Omega_S}{2\Delta} \\ \Omega_0 &= \sqrt{\Omega_R^2 + \delta_{eff}^2}. \end{aligned} \tag{11}$$

Entangling gates use collective spatial vibrational modes for communication between ions. It is performed by coupling the states

$$|ggn\rangle \leftrightarrow \{|eg(n+1)\rangle\,,\;\; |ge(n-1)\rangle\} \leftrightarrow |een\rangle\,,$$

by means of two lasers both detuned from the resonance with a joint motional and internal excitation of the ion.

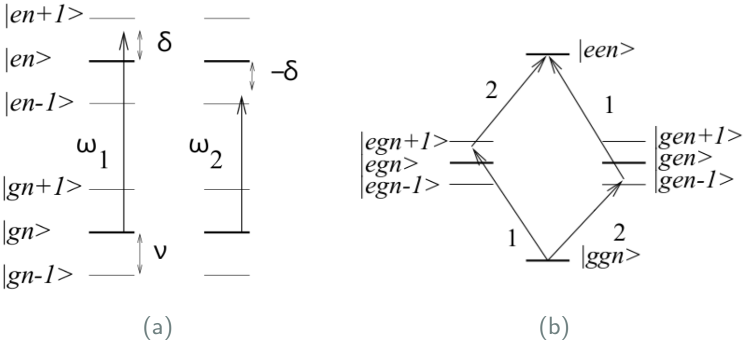


Figure: Schematic representation of the relevant (motional and internal) energy levels of the two ions addressed by the lasers

The hamiltonian of the system is:

$$
\begin{aligned}
H &= H_0 + H_{int} \\
H_0 &= \hbar\nu(a^\dagger a + \frac{1}{2}) + \hbar\omega_{eg}\sum_j \frac{\sigma_{zj}}{2} \\
H_{int} &= \sum_j \frac{\hbar\Omega}{2}(\sigma_{+j}e^{i(\eta(a+a^\dagger)-\omega_j)} + h.c.).
\end{aligned}
\tag{12}
$$

We are interested in the Rabi frequency for the transition $|gg\rangle \rightarrow |ee\rangle$

In the Lamb-Dicke limit the value of the Rabi frequency is

$$\tilde{\Omega} = -\frac{(\Omega\eta)^2}{\nu - \delta} \tag{13}$$

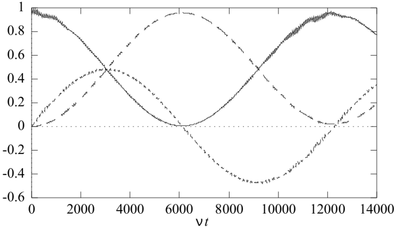


Figure: Rabi oscillations between $|gg\rangle$ and $|ee\rangle$.

Starting from the state $|gg\rangle$, by applying a radiation field pulse of the duration of $\frac{\pi}{2\tilde{\Omega}}$, the maximally entangled state $\frac{1}{\sqrt{2}}(|gg\rangle - i\,|ee\rangle)$ can be obtained.

Single-quibit gates: **79 Qubits** with average fidelty $> 99\%$

Two-qubit gates: **11 Qubits** with average fidelty $> 96\%$

The estimated number of qubits needed to break RSA-1024 is about 2000.

Move quantum information between modules by moving ions themselves

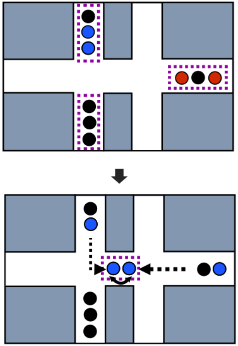


Figure: 2D Array with "memory" zones and "interaction reagion"

References

Monroe Christopher.
University of maryland department of physics, joint quantum institute.
http://iontrap.umd.edu/, 02 2020.

Shantanu Debnath.
A programmable five qubit quantum computer using trapped atomic ions.
PhD thesis, University of Maryland, College Park, 2016.

David Deutsch and Richard Jozsa.
Rapid solution of problems by quantum computation.
Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences, 439(1907):553–558, 1992.

Tony Hubert.
Stimulated Raman Transitions Between Hyperfine Ground States of Magnetically Trapped Rubidium-87 Atoms.
PhD thesis, Master's thesis, University of Amsterdam, Netherlands, 2015.

Kevin Antony Landsman.
Construction, optimization, and applications of a small trapped-ion quantum computer, 2019.

John Proos and Christof Zalka.
Shor's discrete logarithm quantum algorithm for elliptic curves.
arXiv preprint quant-ph/0301141, 2003.

Philipp Schindler, Daniel Nigg, Thomas Monz, Julio T Barreiro, Esteban Martinez, Shannon X Wang, Stephan Quint, Matthias F Brandl, Volckmar Nebendahl, Christian F Roos, et al.
A quantum information processor with trapped ions.
New Journal of Physics, 15(12):123012, 2013.

Anders Sørensen and Klaus Mølmer.
Quantum computation with ions in thermal motion.
Physical review letters, 82(9):1971, 1999.