PRACTICE QUESTIONS BAYE'S THEOREM

EXERCISE – 2.10

BAYE'S THEOREM

1. A rare disease exists with which only 1 in 500 is affected. A test for the disease exists, but of course it is not infallible. A correct positive result (patient actually has the disease) occurs 95% of the time, while a false positive result (patient does not have the disease) occurs 1% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?
   (($Ans: 0.1599$) Walpole 2.120

2. A certain form of cancer is known to be found in women over 60 with probability 0.07. A blood test exists for the detection of the disease, but the test is not infallible. In fact, it is known that 10% of the time the test gives a false negative (i.e., the test incorrectly gives a negative result) and 5% of the time the test gives a false positive (i.e., incorrectly gives a positive result). If a woman over 60 is known to have taken the test and received a favorable (i.e., negative) result, what is the probability that she has the disease?
   ($Ans: 0.00786$) Walpole 2.118

3. A truth serum has the property that 90% of the guilty suspects are properly judged while, of course,10% of the guilty suspects are improperly found innocent. On the other hand, innocent suspects are misjudged 1% of the time. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime, and the serum indicates that he is guilty, what is the probability that he is innocent?
   ($Ans: 0.1743$) Walpole 2.103

4. A paint-store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a customer will purchase latex paint is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semigloss paint buyers purchase rollers. A randomly selected buyer purchases a roller and a can of paint. What is the probability that the paint is latex?
   ($Ans: 0.857$) Walpole 2.101

5. A firm is accustomed to training operators who do certain tasks on a production line. Those operators who attend the training course are known to be able to meet their production quotas 90% of the time. New operators who do not take the training course only meet their quotas 65% of the time. Fifty percent of new operators attend the course. Given that a new operator meets her production quota, what is the probability that she attended the program?
   ($Ans: 0.581$) Walpole 2.124

6. WALPOLE
$(Ans: 0.224)$

7. A certain federal agency employs three consulting firms (*A*, *B*, and *C*) with probabilities 0.40, 0.35, and 0.25, respectively. From past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. Suppose a cost overrun is experienced by the agency.
(a) What is the probability that the consulting firm involved is company *C*?
(b) What is the probability that it is company *A*?
$(Ans: 0.5515,\ 0.2941)$ Walpole 2.115

8. A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective.
(a) What is the probability that zero defective components exist in the lot?
(b) What is the probability that one defective exists in the lot?
(c) What is the probability that two defectives exist in the lot?
$(Ans: 0.6312,\ 0.2841, 0.0847)$ Walpole 2.119

9. Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John?
$(Ans: 0.1124)$ Walpole 2.99

10. A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunctions reported by each station and the causes are shown below.

Station	*A*	*B*	*C*
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station *C*?
$(Ans: 0.2632)$ WALPOLE 2.100

SOLUTIONS OF PROBLEMS

1.

Consider events:
D: a person has the rare disease, $P(D) = 1/500$,
P: the test shows a positive result, $P(P \mid D) = 0.95$ and $P(P \mid D') = 0.01$.
$P(D \mid P) = \frac{P(P \mid D)P(D)}{P(P \mid D)P(D)+P(P \mid D')P(D')} = \frac{(0.95)(1/500)}{(0.95)(1/500)+(0.01)(1-1/500)} = 0.1599.$

2.

Consider the events:
C: a woman over 60 has the cancer,
P: the test gives a positive result.
So, $P(C) = 0.07$, $P(P' \mid C) = 0.1$ and $P(P \mid C') = 0.05$.
$P(C \mid P') = \frac{P(P' \mid C)P(C)}{P(P' \mid C)P(C)+P(P' \mid C')P(C')} = \frac{(0.1)(0.07)}{(0.1)(0.07)+(1-0.05)(1-0.07)} = \frac{0.007}{0.8905} = 0.00786.$

3.

Consider the events:
G: guilty of committing a crime,
I: innocent of the crime,
i: judged innocent of the crime,
g: judged guilty of the crime.
$P(I \mid g) = \frac{P(g \mid I)P(I)}{P(g \mid G)P(G)+P(g \mid I)P(I)} = \frac{(0.01)(0.95)}{(0.05)(0.90)+(0.01)(0.95)} = 0.1743.$

4.

Consider the events:
A: a customer purchases latex paint,
A': a customer purchases semigloss paint,
B: a customer purchases rollers.
$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A)+P(B \mid A')P(A')} = \frac{(0.60)(0.75)}{(0.60)(0.75)+(0.25)(0.30)} = 0.857.$

5.

Consider the events:
T: an operator is trained, $P(T) = 0.5$,
M an operator meets quota, $P(M \mid T) = 0.9$ and $P(M \mid T') = 0.65$.
$P(T \mid M) = \frac{P(M \mid T)P(T)}{P(M \mid T)P(T)+P(M \mid T')P(T')} = \frac{(0.9)(0.5)}{(0.9)(0.5)+(0.65)(0.5)} = 0.581.$

6. A

7.

Consider the events:
O: overrun,
A: consulting firm A,
B: consulting firm B,
C: consulting firm C.

(a) $P(C \mid O) = \frac{P(O \mid C)P(C)}{P(O \mid A)P(A)+P(O \mid B)P(B)+P(O \mid C)P(C)} = \frac{(0.15)(0.25)}{(0.05)(0.40)+(0.03)(0.35)+(0.15)(0.25)} = \frac{0.0375}{0.0680} = 0.5515.$

(b) $P(A \mid O) = \frac{(0.05)(0.40)}{0.0680} = 0.2941.$

8.

Consider the events:
A: two nondefective components are selected,
N: a lot does not contain defective components, $P(N) = 0.6$, $P(A \mid N) = 1$,
O: a lot contains one defective component, $P(O) = 0.3$, $P(A \mid O) = \frac{\binom{19}{2}}{\binom{20}{2}} = \frac{9}{10}$,
T: a lot contains two defective components, $P(T) = 0.1$, $P(A \mid T) = \frac{\binom{18}{2}}{\binom{20}{2}} = \frac{153}{190}$.

(a) $P(N \mid A) = \frac{P(A \mid N)P(N)}{P(A \mid N)P(N)+P(A \mid O)P(O)+P(A \mid T)P(T)} = \frac{(1)(0.6)}{(1)(0.6)+(9/10)(0.3)+(153/190)(0.1)}$
$= \frac{0.6}{0.9505} = 0.6312$;
(b) $P(O \mid A) = \frac{(9/10)(0.3)}{0.9505} = 0.2841$;
(c) $P(T \mid A) = 1 - 0.6312 - 0.2841 = 0.0847$.

9.

Consider the events:
A: no expiration date,
B_1: John is the inspector, $P(B_1) = 0.20$ and $P(A \mid B_1) = 0.005$,
B_2: Tom is the inspector, $P(B_2) = 0.60$ and $P(A \mid B_2) = 0.010$,
B_3: Jeff is the inspector, $P(B_3) = 0.15$ and $P(A \mid B_3) = 0.011$,
B_4: Pat is the inspector, $P(B_4) = 0.05$ and $P(A \mid B_4) = 0.005$,
$P(B_1 \mid A) = \frac{(0.005)(0.20)}{(0.005)(0.20)+(0.010)(0.60)+(0.011)(0.15)+(0.005)(0.05)} = 0.1124$.

10.

Consider the events
E: a malfunction by other human errors,
A: station A, B: station B, and C: station C.
$P(C \mid E) = \frac{P(E \mid C)P(C)}{P(E \mid A)P(A)+P(E \mid B)P(B)+P(E \mid C)P(C)} = \frac{(5/10)(10/43)}{(7/18)(18/43)+(7/15)(15/43)+(5/10)(10/43)} =$
$\frac{0.1163}{0.4419} = 0.2632$.