STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE – 18

PROBABILITY DISTRIBUTIONS

MUTINOMIAL DISTRIBUTION

PREPARED BY
HAZBER SAMSON
FAST NUCES ISLAMABAD

MULTINOMIAL DISTRIBUTION

Recall that in binomial distribution, each trial has two outcomes. But if each trial in an experiment has more than two outcomes, then we use **multinomial distribution.** *So the binomial experiment becomes a multinomial experiment if we let each trial have more than two possible outcomes.*

THE MULTINOMIAL EXPERIMENT A multinomial experiment is a probability experiment that satisfies the following four requirements

1- There must be a fixed number of trials. *i.e*. there are n identical trials.
2- Each trial has more than two possible outcomes.
3- The trials are independent of each other.
4- The probability of success for each outcome remains same for each trial.

EXAMPLES OF MULTINOMIAL EXPERIMENT Some examples of multinomial experiment are as follows

1. A survey might require the responses of “approve,” “disapprove,” or “no opinion.”
2. Samples of manufactured products are rated excellent, above average, average, or inferior
3. A person may have a choice of one of five activities for Friday night, such as a movie, dinner, baseball game, play, or party.

THE MUTINOMIAL DISTRIBUTION Multinomial experiment and its results give rise to a special probability distribution called the multinomial distribution.

Consider a sequence of n independent trials where each individual trial can have k outcomes that occur with constant probability values $p_1, p_2, p_3, \ldots, p_k$, then the probability distribution of the random variables $X_1, X_2, X_3, \ldots, X_k$, representing number of occurrences for the events $E_1, E_2, E_3, \ldots, E_k$ said to have a **multinomial distribution,** and their joint probability mass function is given by

$$P\left(X_1 = x_1, X_2 = x_2, \ldots, X_k = x_k\right) = \frac{n!}{x_1!.\,x_2!\ldots x_k!} \times p_1^{\,x_1} \times p_2^{\,x_1} \times \ldots p_k^{\,x_k}$$

$$where \;\; x_1 + x_2 + x_3 + \ldots + x_k = n \quad i.e. \; \sum_{i=1}^{k} x_i = n$$

$$and \;\; p_1 + p_2 + p_3 + \ldots + p_k = 1 \quad i.e. \; \sum_{i=1}^{k} p_i = 1$$

PROPERTIES OF MULTINOMIAL DISTRIBUTION

1- The Mean of Binomial Distribution is $E(X_i) = n\,p_i$

2- The variance of Binomial Distribution is $V\left(X_i\right) = n\,p_i\,q_i$

EXAMPLES OF MUTINOMIAL DISTRIBUTION

EXAMPLE-1 A die is rolled 5 times. Find the probability that 1 appears twice, 6 appears once and any other number appears twice. Also find expectation and variance of getting 6.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote the random variable that* 1 *appear.*

Let X_2 *denote the random variable that* 6 *appear.*

Let X_3 *denote the random variable that any number other than* 1 *and* 6 *appear.*

$$Here \quad n = 4, \quad X_1 = 2, \; X_2 = 1, \; X_3 = 2, \; p_1 = 1/6, \; p_2 = 1/6, \; p_3 = 4/6$$

$$Then \quad P(X_1 = 2, \; X_2 = 1, \; X_3 = 2) = \frac{5!}{2! \times 1! \times 2!}\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^1\left(\frac{4}{6}\right)^2 = 0.062$$

$$Then \quad E(X_2) = np_1 = (5)(1/6) = 0.83, \; V(X_2) = np_2q_2 = (5)(1/6)(5/6) = 0.69$$

EXAMPLE-2 In a large city, 50% of the people choose a movie, 30% choose dinner and a play, and 20% choose shopping as a leisure activity. If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a play, and 1 to a shopping mall.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote the people who choose movie*

Let X_2 *denote the people who choose dinner and play*

Let X_3 *denote the people who choose shopping*

$$Here \quad n = 5, \quad X_1 = 3, \; X_2 = 1, \; X_3 = 1, \; p_1 = 0.50, \; p_2 = 0.30, \; p_3 = 0.20$$

$$Then \quad P(X_1 = 3, \; X_2 = 1, \; X_3 = 1) = \frac{5!}{3! \times 1! \times 1!}(0.50)^3(0.30)^1(0.20)^1 = 0.15$$

EXAMPLE-3 A box contains 4 white balls, 3 red balls, and 3 blue balls. A ball is selected at random, and its color is written down. It is replaced each time. Find the probability that if 5 balls are selected, 2 are white, 2 are red, and 1 is blue.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote the number of white balls selected*

Let X_2 *denote the number of red balls selected*

Let X_3 *denote the number of blue balls selected*

$$Here \quad n = 5, \quad X_1 = 2, \; X_2 = 2, \; X_3 = 1, \; p_1 = 4/10, \; p_2 = 3/10, \; p_3 = 3/10$$

$$Then \quad P(X_1 = 2, \; X_2 = 2, \; X_3 = 1) = \frac{5!}{2! \times 2! \times 1!}\left(\frac{4}{10}\right)^2\left(\frac{3}{10}\right)^2\left(\frac{3}{10}\right)^1 = \frac{81}{625}$$

EXAMPLE-4 A certain city has 3 newspapers, A, B, and C. Newspaper A has 50 percent of the readers in that city. Newspaper B, has 30 percent of the readers, and newspaper C has the remaining 20 percent. Find the probability that, among 8 randomly-chosen readers in that city, 5 will read newspaper A, 2 will read newspaper B, and 1 will read newspaper C.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote readers who read newspaper A*

Let X_2 *denote readers who read newspaper B*

Let X_3 *denote readers who read newspaper C*

Here $n=8,\ X_1=5,\ X_2=2,\ X_3=1,\ p_1=0.5,\ p_2=0.3,\ p_3=0.2$

Then $P(X_1=5,\ X_2=2,\ X_3=1)=\dfrac{8!}{5!\times 2!\times 1!}(0.50)^5(0.30)^2(0.20)^1=0.0945$

EXAMPLE-5 If a pair of dice is rolled 6 times. What is the probability of obtaining a total of 7or 11 twice, a matching pair once and any other combination 3 times.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote the random variable that a total of* 7 *or* 11 *is obtained*

Let X_2 *denote the random variable that a matching pair is obtained*

Let X_3 *denote the random variable that any other combination is obtained*

Here $n=6,\ X_1=2,\ X_2=1,\ X_3=3,\ p_1=\dfrac{2}{9},\ p_2=\dfrac{1}{6},\ p_3=\dfrac{11}{18}$

Then $P(X_1=2,\ X_2=1,\ X_3=3)=\dfrac{6!}{2!\times 1!\times 3!}\left(\dfrac{2}{9}\right)^2\left(\dfrac{1}{6}\right)^1\left(\dfrac{11}{18}\right)^3=0.1127$

EXAMPLE-6 A small airport coffee shop manager found that the probabilities a customer buys 0, 1, 2, or 3 cups of coffee are 0.3, 0.5, 0.15, and 0.05, respectively. If 8 customers enter the shop, find the probability that 2 will purchase something other than coffee, 4 will purchase 1 cup of coffee, 1 will purchase 2 cups, and 1 will purchase 3 cups.

SOLUTION *Let us define the Random Variables first.*

Let X_1 *denote the customers who purchase no coffee.*

Let X_2 *denote the customers who purchase one cup of coffee.*

Let X_3 *denote the customers who purchase two cups of coffee*

Let X_4 *denote the customers who purchase three cups of coffee*

Here $n=8,\ X_1=2,\ X_2=4,\ X_3=1,\ X_4=1,\ p_1=0.3,\ p_2=0.5,\ p_3=0.15,\ p_4=0.05$

Then $P(X_1=2,\ X_2=4,\ X_3=1,\ X_4=1)=\dfrac{8!}{2!\times 4!\times 1!\times 1!}(0.3)^2(0.5)^4(0.15)^1(0.05)^1=0.0354$

EXERCISE – 4.4

MULTINOMIAL DISTRIBUTION

1. Use the multinomial formula to find the probabilities also find $E(X_1)$ *and* $V(X_1)$.

$(a)\ n=5,\ X_1=1,\ X_2=2,\ X_3=2,\ p_1=0.3,\ p_2=0.6,\ p_3=0.1$

$(b)\ n=6,\ X_1=3,\ X_2=2,\ X_3=1,\ p_1=0.5,\ p_2=0.3,\ p_3=0.2$

$(Ans:(a)\ 0.0324,\ 1.5,\ 1.05 \qquad (b)\ 0.135,\ 3,\ 1.5)$

2. In a factory producing certain items, 30 per cent of the items produced have no defect, 40 per cent have one defect, and 30 per cent have two defects. A random sample of 8 items is taken from a day's output. Find the probability that it will contain 2 items with no defect, 3 items with one defect, and 3 items with two defects. $(Ans:0.0871)$

3. The probabilities are 0.50, 0.40, and 0.10 that a trailer truck will have no violations, 1 violation, or 2 or more violations when it is given a safety inspection by state police. If 5 trailer trucks are inspected, find the probability that 3 will have no violations, 1 will have 1 violation, and 1 will have 2 or more violation. $(Ans:0.1)$

4. According to a genetics theory, a certain cross of guinea pigs will result in red, black, and white offspring in the ratio 8:4:4. Find the probability that among 8 offspring, 5 will be red, 2 black, and 1 white. $(Ans:21/256)$

5. The complexity of arrivals and departures of planes at an airport is such that computer simulation is often used to model the "ideal" conditions. For a certain airport with three runways, it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet:

Runway 1: $p_1=2/9$, *Runway* 2: $p_2=1/6$, *Runway* 3: $p_3=11/18$

What is the probability that 6 randomly arriving airplanes are distributed in the following fashion?

Runway 1: 2 *airplanes*, *Runway* 2: 1 *airplane*, *Runway* 3: 3 *airplanes*

$(Ans:0.1127)$

6. The probabilities are 0.4, 0.2, 0.3, and 0.1, respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. What is the probability that among 9 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by automobile, and 2 arrived by train? $(Ans:0.0077)$

7. According to Mendel's theory, if tall and colorful plants are crossed with short and colorless plants, the corresponding probabilities are $9/16,\ 3/16,\ 3/16$ *and* $1/16$ for tall and colorful, tall and colorless, short and colorful, and short and colorless, respectively. If 8 plants are selected, find the probability that 1 will be tall and colorful, 3 will be tall and colorless, 3 will be short and colorful, and 1 will be short and colorless. $(Ans:0.002)$

8. When people were asked if they felt that the laws covering the sale of firearms should be more strict, less strict, or kept as they are now, 54% responded more strict, 11% responded less, 34% said keep them as they are now, and 1% had no opinion. If 10 randomly selected people are asked the same question, what is the probability that 4 will respond more strict, 3 less, 2 keep them the same, and 1 have no opinion? $(Ans:0.0016)$

9. Find the probability of obtaining 2 ones, 1 two, 1 three, 2 fours, 3 fives and 1 six in 10 rolls of a balanced die. $(Ans:0.0025)$

10. According to the manufacturer, M&M's are produced and distributed in the following proportions: 13% brown, 13% red, 14% yellow, 16% green, 20% orange, and 24% blue. In a random sample of 12 M&M's, what is the probability of having 2 of each color?
$(Ans:0.0025)$