STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE – 13

RANDOM VARIABLES

DISCRETE RANDOM VARIABLE

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RANDOM VARIABLES

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RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS

RANDOM VARIABLE A random variable is a variable whose value is determined by the outcome of a random experiment. (OR)

A random variable is a real valued function that associates a real number with each element in the Sample Space.

Random variables are denoted by capital letters X, Y, Z etc and their values are denoted by small letters x, y, z etc.

Exp: If 'X' denote the Number of Heads when a coin is tossed twice. Here $S = \{HH, HT, TH, TT\}$. So 'X' can take the values. $X = 0, 1, 2$ So 'X' associates a real number with each element in Sample Space So 'X' is Random Variable.

There are two types of Random variables.

① Discrete Random Variables.

② continuous Random Variables.

DISCRETE RANDOM VARIABLE.

A random variable that assumes countable values is called discrete random variable (DRV)

Exps ① Number of Houses in a Block

② Number of Fish caught on a fishing trip.

③ Number of Students in a class.

CONTINUOUS RANDOM VARIABLE.

A random variable that can assume any value contained in one or more intervals is called CRV.

Exps ① The Height of Person.

② Temperature at a place.

③ weight of a Person.

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PROBABILITY DISTRIBUTIONS

A Probability distribution gives the probability of each possible value of the variable.

DISCRETE PROBABILITY DISTRIBUTIONS

The Probability distribution of a discrete random variable lists all the possible values that the random variable can assume and their corresponding Probabilities. Simply A table or formula which consists all the values of a DRV with their respective Probabilities is known as Discrete Probability distribution.

Let $x_1, x_2, --, x_n$ be the values of a discrete random variable 'X' and $P(X=x_1), P(X=x_2), ----, P(X=x_n)$ are the probabilities of the values than the following table is known as Probability distribution of RV 'X'.

X	$P(X=x)$
x_1	$P(X=x_1)$
x_2	$P(X=x_2)$
x_3	$P(X=x_3)$
$\vdots$	$\vdots$
x_n	$P(X=x_n)$

Note that

① $P(X=x) \geq 0$

② $\sum_{x} P(X=x) = 1$

③ $P(X=x) = f(x)$ in DRV.

④ Probability dist. is also known as Probability Mass Function or Discrete Prob. Distribution.

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EXAMPLES OF DISCRETE PROBABILITY DISTRIBUTIONS

EXAMPLE 1 Find the Probability distribution of the Number of dots appeared when a die is rolled once.

SOLUTION when a die is rolled once then

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let 'x' denote the Number of dots appeared when a die is rolled once.

X	P(X=x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

which is the Required Prob. distribution of X.

clearly $\sum P(X=x) = 1$

EXAMPLE 2 Find the Probability distribution of the Number of Heads obtained when two fair coins are tossed once.

SOLUTION when Two coins are tossed once.

$$S = \{HH, HT, TH, TT\}$$

Let 'x' denote the Number of Heads appeared.

X	P(X=x)
0	1/4
1	2/4
2	1/4

which is the Probability distribution of X.

clearly $\sum P(X=x) = 1$

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EXAMPLE ③ Find the Probability distribution of Sum of dots appeared when two dice are thrown once.

SOLUTION when two dice are thrown once then

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Let 'x' denote the Sum of dots appeared when two dice are thrown once.

x	$P(X=x)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

which is the Required Probability distribution of X.

clearly

$\sum P(X=x) = 1$

EXAMPLE ④ A committee of 3 persons is selected from 3 boys and 4 Girls. construct a Probability distribution for the Number of Boys on the committee.

SOLUTION Here Boys = 3, Girls = 4, Select = 3.

Total = 7

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Let 'X' denote the Number of Boy's selected for committee.

X	$P(X=x)$
0	$\frac{{}^3C_0 \times {}^4C_3}{{}^7C_3} = 4/35$
1	$\frac{{}^3C_1 \times {}^4C_2}{{}^7C_3} = 18/35$
2	$\frac{{}^3C_2 \times {}^4C_1}{{}^7C_3} = 12/35$
3	$\frac{{}^3C_3 \times {}^4C_0}{{}^7C_3} = 1/35$

which is the Required Probability Distribution of X. clearly $\sum P(X=x)=1$

EXAMPLE 5 The Probability Mass function of random variable 'X' is given by $P(X=x) = cx^2$, for $x=0,1,2,3,4$. The find.

(a) the value of c.

(b) $P(X>2)$ (c) $P(X\leq 3)$ (d) $P(1<X<4)$ (e) $P(0\leq X\leq 2)$.

SOLUTION.

X	$P(X=x)$	$P(X=x)$
0	0	0
1	c	$1/30$
2	4c	$4/30$
3	9c	$9/30$
4	16c	$16/30$

(a) value of c = ?

As $\sum P(X=x) = 1$

$0+c+4c+9c+16c = 1$

$30c = 1$

$\boxed{c = 1/30}$

(b) $P(X>2)$

$= P(X=3)+P(X=4)$

$= \frac{9}{30}+\frac{16}{30} = \frac{25}{30} = \frac{5}{6}$

(c) $P(X\leq 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3) = 0+\frac{1}{30}+\frac{4}{30}+\frac{9}{30} = \frac{14}{30}$

(d) $P(1<X<4) = P(X=2)+P(X=3) = \frac{4}{30}+\frac{9}{30} = \frac{13}{30}$

(e) $P(0\leq X\leq 2) = P(X=0)+P(X=1)+P(X=2) = 0+\frac{1}{30}+\frac{4}{30} = \frac{5}{30} = \frac{1}{6}$

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DISTRIBUTION FUNCTIONS

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THE CUMULATIVE DISTRIBUTION FUNCTION

In many problems, we may be interested to know the probability that the value of random variable 'X' is less than or equal to some real number x. In this case we are looking at cumulative Probabilities ie for any random variable X, we may look at $P(X \leq b)$ for any real number b.

DEFINITION The distribution Function for a Random variable 'X' evaluated at 'b' is denoted by F(b) and is given by

$$F(b) = P(X \leq b)$$

Note that distribution Function is also called the cumulative distribution Function.

PROPERTIES OF DISTRIBUTION FUNCTION

Every distribution function must satisfy four Properties. given below

1. $\lim_{x \to -\infty} F(x) = 0$
2. $\lim_{x \to \infty} F(x) = 1$
3. The distribution function is a Non-decreasing function. ie if $a < b$, $F(a) \leq F(b)$.
4. The distribution function is Right-Hand continuous ie $\lim_{h \to 0^+} F(x+h) = F(x)$

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DISTRIBUTION FUNCTION FOR DISCRETE RANDOM VARIABLE

If 'x' is a discrete Random Variable, then

$$F(b) = P(X \le b)$$

$$F(b) = \sum_{x=-\infty}^{b} p(x)$$

where $p(x)$ is the probability function.

Finding Probabilities using CDF.

① $P(X > a) = 1 - P(X \le a) = 1 - F(a)$

② $P(X < a) = P(X \le a-1) = F(a-1)$

③ $P(X \le a) = F(a)$

④ $P(X \ge a) = 1 - P(X < a) = 1 - P(X \le a-1) = 1 - F(a-1)$

⑤ $P(a \le X \le b) = F(b) - F(a-1)$

⑥ $P(a < X < b) = F(b-1) - F(a)$

⑦ $P(X = a) = F(a) - F(a-1)$

EXAMPLES

EXAMPLE① Find distribution function when two coins are tossed once.

SOLUTION Here $S = \{HH, HT, TH, TT\}$

Let 'x' denote Number of Heads.

x	$f(x) = P(x)$	$F(x)$
0	$1/4$	$1/4$
1	$2/4$	$3/4$
2	$1/4$	1
	1	

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EXAMPLE-2 Find the Distribution Function of the number of dots appeared when a fair die is rolled once.

SOLUTION Here $S = \{1, 2, 3, 4, 5, 6\}$

Let 'x' denote the dots appeared.

X	P(x)	F(x)
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6

EXAMPLE-3 Find the Distribution Function of sum of dots appeared, when two dice are thrown once.

SOLUTION Here $S = \{(1,1), (1,2), \ldots, (6,6)\}$

Let 'x' denote sum of dots appeared.

X	P(x)	F(x)
2	1/36	1/36
3	2/36	3/36
4	3/36	6/36
5	4/36	10/36
6	5/36	15/36
7	6/36	21/36
8	5/36	26/36
9	4/36	30/36
10	3/36	33/36
11	2/36	35/36
12	1/36	36/36

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EXAMPLE-4 For a discrete Random variable 'X' the cumulative distribution function F(x) is given as

x	1	2	3	4	5
F(x)	0.2	0.32	0.67	0.9	1

Find the Probabilities using CDF

(a) $P(X=3)$

(b) $P(X>2)$

(c) $P(X<4)$

SOLUTION (a) $P(X=3) = F(3) - F(2) = 0.67 - 0.32 = 0.35$

(b) $P(X>2) = 1 - P(X \leq 2) = 1 - F(2) = 1 - 0.32 = 0.68$

(c) $P(X<4) = P(X \leq 3) = F(3) = 0.67$.

EXAMPLE 5 The cumulative Probabilities for a Random variable 'X' are given in the following table, use table to find

x	F(x)
0	0.0388
1	0.1756
2	0.4049
3	0.6477
4	0.8298
5	0.9327
6	0.9781
7	0.9941
8	0.9987
9	0.9998
10	1

(a) $P(X \leq 5)$

(b) $P(X>3)$

(c) $P(3 \leq X \leq 7)$

(d) $P(X=7)$

(e) $P(X \geq 8)$

SOLUTION

(a) $P(X \leq 5) = F(5) = 0.9327$

(b) $P(X>3) = 1 - F(3) = 1 - 0.6477 = 0.3523$

(c) $P(3 \leq X \leq 7) = F(7) - F(2) = 0.9941 - 0.4049 = 0.5892$

(d) $P(X=7) = F(7) - F(6) = 0.9941 - 0.9781 = 0.016$

(e) $P(X \geq 8) = 1 - P(X<8) = 1 - F(7) = 1 - 0.9941 = 0.0059$

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