National University of Computer & Emerging Sciences Islamabad

FAST School of Computing | Fall-2024 | Islamabad Campus

MT1004 – Linear Algebra

Homework # 1

Question # 1

Find a sequence of elementary row operations that will convert A into B.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

Q1:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} R_1+R_2$$

$$\sim \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{Swap } R_2 \text{ and } R_3$$

$$\sim \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 2 & 2 & 0 \end{bmatrix} 2R_3$$

$$\sim \begin{bmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{bmatrix} R_2+2R_3$$

$\Rightarrow$ converted to matrix B.

Question # 2

What condition, if any, must a, b, and c satisfy for the linear system to be consistent?

$$x + 3y - z = a$$
$$x + y + 2z = b$$
$$2y - 3z = c$$

Q2:

$$x + 3y - z = a$$
$$x + y + 2z = b$$
$$2y - 3z = c$$

$$\tilde{A} = \begin{bmatrix} 1 & 3 & -1 & : & a \\ 1 & 1 & 2 & : & b \\ 0 & 2 & -3 & : & c \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & : & a \\ 0 & -2 & 3 & : & b-a \\ 0 & 2 & -3 & : & c \end{bmatrix} R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & : & a \\ 0 & 1 & -3/2 & : & \frac{-b+a}{2} \\ 0 & 2 & -3 & : & c \end{bmatrix} R_2 \times -\frac{1}{2}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & : & a \\ 0 & 1 & -3/2 & : & \frac{a-b}{2} \\ 0 & 0 & 0 & : & c-a+b \end{bmatrix} R_3 - 2R_2$$

The system will be consistent if $c - a + b = 0$

$$\boxed{c = a - b}$$

$\Rightarrow$ infinite solutions are possible in such case.

Question # 3

Determine whether the solution of the given system exists.

$$\frac{1}{2}x_1 + x_2 - x_3 - 6x_4 = 2$$
$$\frac{1}{6}x_1 + \frac{1}{2}x_2 - 3x_4 + x_5 = -1$$
$$\frac{1}{3}x_1 - 2x_3 - 4x_5 = 8$$

$$\tilde{A} = \left[\begin{array}{ccccc:c} 1/2 & 1 & -1 & -6 & 0 & 2 \\ 1/6 & 1/2 & 0 & -3 & 1 & -1 \\ 1/3 & 0 & -2 & 0 & -4 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc:c} 1 & 2 & -2 & -12 & 0 & 4 \\ 1 & 3 & 0 & -18 & 6 & -6 \\ 1 & 0 & -6 & 0 & -12 & 24 \end{array}\right] \begin{array}{l} R_1 \times 2 \\ R_2 \times 6 \\ R_3 \times 3 \end{array}$$

$$\sim \left[\begin{array}{ccccc:c} 1 & 2 & -2 & -12 & 0 & 4 \\ 0 & 1 & 2 & -6 & 6 & -10 \\ 0 & -2 & -4 & 12 & -12 & 20 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccccc:c} 1 & 2 & -2 & -12 & 0 & 4 \\ 0 & 1 & 2 & -6 & 6 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3 + 2R_2 \end{array}$$

$\Rightarrow$ Solution of this system exists because last column is not a pivot column

$\Rightarrow$ the solution is infinite because C_3, C_4, C_5 are also not pivot columns.