Assignment No 9

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Subject: Linear Algebra

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Question No:- 1

1 Eignvalues $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

Solution:-

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 4 & -1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(-1-\lambda) - 4 = 0$$

$$-2 - 2\lambda + \lambda + \lambda^2 - 4 = 0$$
$$\lambda^2 - \lambda - 6 = 0$$
$$\lambda^2 - 3\lambda + 2\lambda - 6 = 0$$
$$\lambda(\lambda - 3) + 2(\lambda - 3) = 0$$
$$\lambda = 3 \qquad \lambda = -2$$

2 Eign Vectors

Solution:-

for $\lambda = 3$

$$(A - 3I)\bar{v} = \bar{0}$$

$$\left[\begin{array}{cc|c} 2-3 & 1 & 0 \\ 4 & -1-3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 4 & -4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad R_2 + 4R_1$$

$$-V_1 + V_2 = 0$$

$$V_1 = V_2$$

$$V_2 = V_2$$

$$\bar{V}_1 = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_2 \\ V_2 \end{bmatrix} = V_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for $\lambda = -2$

$$(A + 2I)\bar{v} = \bar{0}$$

$$\left[\begin{array}{cc|c} 2+2 & 1 & 0 \\ 4 & -1+2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ 4 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad R_2 - R_1$$

$$4\bar{v}_1 + V_2 = 0$$

$$V_1 = \frac{-V_2}{4}$$

$$\bar{v} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -V_2/4 \\ V_2 \end{bmatrix} = V_2 \begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$$

eign vectors $= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$

s eign space $=$ span $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1/4 \\ 1 \end{bmatrix} \right.$

(a) Compute $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

Solution:-

write $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ as linear Combination of eign vectors

$$= \left[\begin{array}{cc|c} 1 & -1/4 & 5 \\ 1 & 1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & -1/4 & 5 \\ 0 & 5/4 & -4 \end{array}\right] \quad R_2 - R_1$$

$$\frac{5}{4} x_2 = -4$$

$$x_2 = \frac{-16}{5}$$

$$x_1 - \frac{x_2}{4} = 5$$

$$x_1 + \frac{4}{5} = 5$$

$$x_1 = \frac{21}{5}$$

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \frac{21}{5}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{16}{5}\begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10} \left(\frac{21}{5}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{16}{5}\begin{bmatrix} -1/4 \\ 1 \end{bmatrix} \right)$$

$$\frac{21}{5}\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{16}{5}\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10}\begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$$

$$\therefore \; A^n v_1 = \lambda_1^n v_1$$

$$= \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (3)^{10} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 59049 \\ 59049 \end{bmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10} \begin{bmatrix} -1/4 \\ 1 \end{bmatrix} = (-2)^{10} \begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -256 \\ 1024 \end{bmatrix}$$

$$= \frac{21}{5} \begin{bmatrix} 59049 \\ 59049 \end{bmatrix} - \frac{16}{5} \begin{bmatrix} -256 \\ 1024 \end{bmatrix}$$

$$= \begin{bmatrix} 248005.8 \\ 248005.8 \end{bmatrix} + \begin{bmatrix} 819.2 \\ -3276.8 \end{bmatrix}$$

$$= \begin{bmatrix} 248825 \\ 244729 \end{bmatrix}$$

(iv) (b) Diagonalization

Solution:-

$$A = PDP^{-1}$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & -1/4 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{\begin{bmatrix} 1 & 1/4 \\ -1 & 1 \end{bmatrix}}{5/4} = \begin{bmatrix} 4/5 & 1/5 \\ -4/5 & 4/5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1/4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 4/5 & 1/5 \\ -4/5 & 4/5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1/2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 4/5 & 1/5 \\ -4/5 & 4/5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$ is similar with $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}^{10}$

Solution:-

$A^n = PD^n P^{-1}$

$$A^{10} = PD^{10}P^{-1}$$

$$= \begin{bmatrix} 1 & -1/4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 59049 & 0 \\ 0 & 1024 \end{bmatrix} \begin{bmatrix} 4/5 & 1/5 \\ -4/5 & 4/5 \end{bmatrix}$$

$$\begin{bmatrix} 59049 & -256 \\ 59049 & 1024 \end{bmatrix} \begin{bmatrix} 4/5 & 1/5 \\ -4/5 & 4/5 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 47444 & 11605 \\ 46420 & 12629 \end{bmatrix}$$