STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE # 29

HYPOTHESIS TESTING

TESTING HYPOTHESIS ABOUT MEAN WHEN SIGMA IS UNKNOWN

PREPARED BY
HAZBER SAMSON
SCIENCES & HUMANITIES DEPARTMENT
FAST ISLAMABAD CAMPUS

INFERENCES ON A SINGLE POPULATION

INFERENCE ABOUT THE POPULATION MEAN WHEN SIGMA IS UNKNOWN

CASE-1 $If \quad H_0 : \mu = \mu_\circ \quad vs \quad H_1 : \mu \neq \mu_\circ$

- When σ is unknown then we use **test statistic**

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \quad with \quad d.f = n-1$$

- When σ is unknown then we use **P-Value**

$$P = 2[\Phi(|t_0|)]$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\bar{x} - t_{\frac{\alpha}{2}(n-1)} . \frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2}(n-1)} . \frac{S}{\sqrt{n}}$$

CASE-2 $If \quad H_0 : \mu \geq \mu_\circ \quad vs \quad H_1 : \mu < \mu_\circ$

- When σ is unknown then we use **test statistic**

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \quad with \ d.f = n-1$$

- When σ is unknown then we use **P-Value**

$$P = 1 - \Phi(|t_0|)$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\mu \leq \bar{x} + t_{\alpha(n-1)} . \frac{S}{\sqrt{n}}$$

CASE-3 $If \quad H_0 : \mu \leq \mu_0 \quad vs \quad H_1 : \mu > \mu_\circ$

- When σ is unknown then we use **test statistic**

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \quad with \quad d.f = n-1$$

- When σ is unknown then we use **P-Value**

$$P = \Phi(|t_0|)$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\mu \geq \bar{x} - t_{\alpha(n-1)} . \frac{S}{\sqrt{n}}$$

T-TEST

If $H_{\circ} :(=,\leq,\geq)$ then $H_1 :(\neq,>,<)$

Now for Critical Region

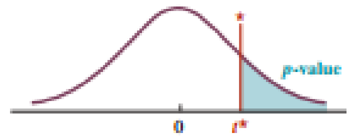
- If H_1 *contains* $\neq$ then C.R is $|t| \geq t_{\frac{\alpha}{2}(n-1)}$ (Two Sided)
- If H_1 *contains* $>$ then C.R is $t > t_{\alpha(n-1)}$ (One Sided)
- If H_1 *contains* $<$ then C.R is $t < -t_{\alpha(n-1)}$ (One Sided)

STUDENTS T-DISTRIBUTION TABLE

df	α = 0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
∞	1.282	1.645	1.96	2.326	2.576	3.091	3.291

Probability-Values for Student's t-distribution

The entries in this table are the p-values related to the right-hand tail for the calculated $t\star$ value for the t-distribution of df degrees of freedom.



$t\star$	3	4	5	6	7	8	10	12	15	18	21	25	29	35	df ≥ 45
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.463	0.463	0.462	0.462	0.462	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.460	0.460
0.2	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.422	0.422	0.422	0.422	0.422	0.421	0.421	0.421
0.3	0.392	0.390	0.388	0.387	0.386	0.386	0.385	0.385	0.384	0.384	0.384	0.383	0.383	0.383	0.383
0.4	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.348	0.347	0.347	0.347	0.346	0.346	0.346	0.346
0.5	0.326	0.322	0.319	0.317	0.316	0.315	0.314	0.313	0.312	0.312	0.311	0.311	0.310	0.310	0.310
0.6	0.295	0.290	0.287	0.285	0.284	0.283	0.281	0.280	0.279	0.278	0.277	0.277	0.277	0.276	0.276
0.7	0.267	0.261	0.258	0.255	0.253	0.252	0.250	0.249	0.247	0.246	0.246	0.245	0.245	0.244	0.244
0.8	0.241	0.234	0.230	0.227	0.225	0.223	0.221	0.220	0.218	0.217	0.216	0.216	0.215	0.215	0.214
0.9	0.217	0.210	0.205	0.201	0.199	0.197	0.195	0.193	0.191	0.190	0.189	0.188	0.188	0.187	0.186
1.0	0.196	0.187	0.182	0.178	0.175	0.173	0.170	0.169	0.167	0.165	0.164	0.163	0.163	0.162	0.161
1.1	0.176	0.167	0.161	0.157	0.154	0.152	0.149	0.146	0.144	0.143	0.142	0.141	0.140	0.139	0.139
1.2	0.158	0.148	0.142	0.138	0.135	0.132	0.129	0.127	0.124	0.123	0.122	0.121	0.120	0.119	0.118
1.3	0.142	0.132	0.125	0.121	0.117	0.115	0.111	0.109	0.107	0.105	0.104	0.103	0.102	0.101	0.100
1.4	0.128	0.117	0.110	0.106	0.102	0.100	0.096	0.093	0.091	0.089	0.088	0.087	0.086	0.085	0.084
1.5	0.115	0.104	0.097	0.092	0.089	0.086	0.082	0.080	0.077	0.075	0.074	0.073	0.072	0.071	0.070
1.6	0.104	0.092	0.085	0.080	0.077	0.074	0.070	0.068	0.065	0.064	0.062	0.061	0.060	0.059	0.058
1.7	0.094	0.082	0.075	0.070	0.066	0.064	0.060	0.057	0.055	0.053	0.052	0.051	0.050	0.049	0.048
1.8	0.085	0.073	0.066	0.061	0.057	0.055	0.051	0.049	0.046	0.044	0.043	0.042	0.041	0.040	0.039
1.9	0.077	0.065	0.058	0.053	0.050	0.047	0.043	0.041	0.038	0.037	0.036	0.035	0.034	0.033	0.032
2.0	0.070	0.058	0.051	0.046	0.043	0.040	0.037	0.034	0.032	0.030	0.029	0.028	0.027	0.027	0.026
2.1	0.063	0.052	0.045	0.040	0.037	0.034	0.031	0.029	0.027	0.025	0.024	0.023	0.022	0.022	0.021
2.2	0.058	0.046	0.040	0.035	0.032	0.029	0.026	0.024	0.022	0.021	0.020	0.019	0.018	0.017	0.016
2.3	0.052	0.041	0.035	0.031	0.027	0.025	0.022	0.020	0.018	0.017	0.016	0.015	0.014	0.014	0.013
2.4	0.048	0.037	0.031	0.027	0.024	0.022	0.019	0.017	0.015	0.014	0.013	0.012	0.012	0.011	0.010
2.5	0.044	0.033	0.027	0.023	0.020	0.018	0.016	0.014	0.012	0.011	0.010	0.010	0.009	0.009	0.008
2.6	0.040	0.030	0.024	0.020	0.018	0.016	0.013	0.012	0.010	0.009	0.008	0.008	0.007	0.007	0.006
2.7	0.037	0.027	0.021	0.018	0.015	0.014	0.011	0.010	0.008	0.007	0.007	0.006	0.006	0.005	0.005
2.8	0.034	0.024	0.019	0.016	0.013	0.012	0.009	0.008	0.007	0.006	0.005	0.005	0.005	0.004	0.004
2.9	0.031	0.022	0.017	0.014	0.011	0.010	0.008	0.007	0.005	0.005	0.004	0.004	0.004	0.003	0.003
3.0	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.006	0.004	0.004	0.003	0.003	0.003	0.002	0.002
3.1	0.027	0.018	0.013	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002
3.2	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.001
3.3	0.023	0.015	0.011	0.008	0.007	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001
3.4	0.021	0.014	0.010	0.007	0.006	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.5	0.020	0.012	0.009	0.006	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
3.6	0.018	0.011	0.008	0.006	0.004	0.004	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0+	0+
3.7	0.017	0.010	0.007	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0+	0+	0+
3.8	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.001	0.001	0.001	0.001	0+	0+	0+	0+
3.9	0.015	0.009	0.006	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0+	0+	0+	0+	0+
4.0	0.014	0.008	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0+	0+	0+	0+	0+	0+

For specific details about using this table to find p-values, see pages 484, 486.

P-VALUE FORMULAS IN Z-TEST

$$P = \begin{cases} 2[\Phi(|t_0|)] & if \quad H_1 : \mu \neq \mu_0 \\ \Phi(|t_0|) & if \quad H_1 : \mu > \mu_0 \\ 1 - \Phi(|t_0|) & if \quad H_1 : \mu < \mu_0 \end{cases}$$

EXAMPLES OF INFERENCE ABOUT MEAN WHEN SIGMA IS UNKNOWN

EXAMPLE-1 A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at a 0.05?

SOLUTION *Here* $n=10,\ \bar{x}=17.7,\ s=1.8,\ \alpha=0.05$

CRITICAL VALUE APPROACH

Step − 1: Formulation of Hypotheses

$H_0 : \mu = 16.3$

$H_1 : \mu \neq 16.3$

Step − 2: Level of Significance

$\alpha = 0.05$

Step − 3: Test Statistic

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{17.7-16.3}{1.8/\sqrt{10}} = 2.46$$

Step − 4: Critical Region

$$|t| \geq t_{\frac{\alpha}{2}(n-1)} \Rightarrow |t| \geq t_{(0.025)(9)} \Rightarrow |t| \geq 2.262$$

Step − 5: Conclusion

Since calculated value of z lies in CR so reject $H_{0.}$

P-VALUE APPROACH

Step − 1: Formulation of Hypotheses

$H_0 : \mu = 16.3$

$H_1 : \mu \neq 16.3$

Step − 2: Level of Significance

$\alpha = 0.05$

Step − 3: Test Statistic

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{17.7-16.3}{1.8/\sqrt{10}} = 2.46$$

Step − 4: P − value

$$p = 2[\Phi(|t_0|)] = 2[\Phi(|2.46|)] = 0.032 < P < 0.036$$

Step − 5: Conclusion

Since p − value $< \alpha$ *so reject* $H_{0.}$

CONFIDENCE INTERVAL APPROACH

Step − 1: Formulation of Confidence Interval

The $(1-\alpha)100\%$ *CI for* μ *is given by*

$$\bar{x} - t_{\frac{\alpha}{2}(n-1)} \cdot \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2}(n-1)} \cdot \frac{s}{\sqrt{n}}$$

$$17.7-(2.306)(1.8)/\sqrt{10} \leq \mu \leq 17.7+(2.306)(1.8)/\sqrt{10}$$

$$16.39 \leq \mu \leq 19.01$$

Step − 2: Conclusion

Since $\mu = 16.3$ *does not lie in CI so reject* $H_{0.}$

EXAMPLE-2 An educator claims that the average salary of substitute teachers in school districts in New York, is less than $60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at a 0.10?

60 56 60 55 70 55 60 55

SOLUTION $Here\ \ n=8,\ \bar{x}=\sum x/n=471/8=58.88,\ \ s=\sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}=\sqrt{\dfrac{180.875}{7}}=5.08,\ \alpha=0.10$

CRITICAL VALUE APPROACH

$Step-1: Formulation\ of\ \ Hypotheses$

$H_0:\mu\geq \$60$

$H_1:\mu< \$60$

$Step-2: Level\ of\ Significance$

$\alpha=0.10$

$Step-3: Test\ Statistic$

$$t=\frac{\bar{x}-\mu}{s/\sqrt{n}}=\frac{58.88-60}{5.08/\sqrt{8}}=-0.624$$

$Step-4: Critical\ \ \mathrm{Re}gion$

$t<-t_{\alpha(n-1)}\quad \Rightarrow t<-t_{(0.10)(7)}\Rightarrow t<-1.415$

$Step-5: Conclusion$

$Since\ calculated\ \ value\ of\ t\ does\ not\ lie\ in\ CR\ so\ do\ not\ reject\ H_{0.}$

P-VALUE APPROACH

$Step-1: Formulation\ of\ \ Hypotheses$

$H_0:\mu\geq \$60$

$H_1:\mu< \$60$

$Step-2: Level\ of\ Significance$

$\alpha=0.10$

$Step-3: Test\ Statistic$

$$t=\frac{\bar{x}-\mu}{s/\sqrt{n}}=\frac{58.88-60}{5.08/\sqrt{8}}=-0.624$$

$Step-4: P-value$

$p=1-\Phi(|t_0|)=1-\Phi(0.624)=1-0.284=0.716$

$Step-5: Conclusion$

$Since\ \ p-value>\alpha\ so\ do\ not\ reject\ H_{0.}$

CONFIDENCE INTERVAL APPROACH

$Step-1: Formulation\ of\ \ Confidence\ \ Interval$

$The\ (1-\alpha)100\%\ CI\ \ for\ \mu\ is\ given\ by$

$$\mu\leq \bar{x}+t_{\alpha(n-1)}.\frac{s}{\sqrt{n}}$$

$$\mu\leq 58.88+(1.415)(5.08)/\sqrt{8}$$

$$\mu\leq 61.42$$

$Step-2: Conclusion$

$Since\ \ \mu=60\ lie\ in\ CI\ so\ do\ not\ reject\ H_{0.}$

EXAMPLE-3 The Environmental Protection Agency (EPA) was suing the city of Rochester for noncompliance with carbon monoxide standards. EPA claimed that the mean level of carbon monoxide in downtown Rochester's air is dangerously high, higher than 4.9 parts per million. Does a random sample of 22 readings having sample mean 5.1 and sample standard deviation 1.17, present sufficient evidence to support the EPA's claim? Use 0.05.

SOLUTION *Here* $n = 22,\ \bar{x} = 5.1,\ s = 1.17,\ \alpha = 0.01$

CRITICAL VALUE APPROACH

$Step-1$: *Formulation of Hypotheses*

$$H_0 : \mu \le 4.9$$
$$H_1 : \mu > 4.9$$

$Step-2$: *Level of Significance*

$$\alpha = 0.05$$

$Step-3$: *Test Statistic*

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{5.1 - 4.9}{1.17/\sqrt{22}} = 0.80$$

$Step-4$: *Critical Region*

$$t > t_{\alpha(n-1)} \quad \Rightarrow t > t_{(0.05)(21)} \quad \Rightarrow t > 1.721$$

$Step-5$: *Conclusion*

Since calculated value of z does not lie in CR so do not reject $H_{0.}$

P-VALUE APPROACH

$Step-1$: *Formulation of Hypotheses*

$$H_0 : \mu \le 4.9$$
$$H_1 : \mu > 4.9$$

$Step-2$: *Level of Significance*

$$\alpha = 0.05$$

$Step-3$: *Test Statistic*

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{5.1 - 4.9}{1.17/\sqrt{22}} = 0.80$$

$Step-4$: $P-value$

$$p = \Phi(|t_0|) = \Phi(0.80) = 0.216$$

$Step-5$: *Conclusion*

Since $p-value > \alpha$ *so do not reject* $H_{0.}$

CONFIDENCE INTERVAL APPROACH

$Step-1$: *Formulation of Confidence Interval*

The $(1-\alpha)100\%$ *CI for* μ *is given by*

$$\mu \ge \bar{x} - t_{\alpha(n-1)} \cdot \frac{s}{\sqrt{n}}$$
$$\mu \ge 5.1 - (1.721)(1.17)/\sqrt{22}$$
$$\mu \ge 4.67$$

$Step-2$: *Conclusion*

Since $\mu = 4.9$ *lies in CI so do not reject* $H_{0.}$

Result Summary: At the 0.05 level of significance, the EPA does not have sufficient evidence to show that the mean carbon monoxide level is higher than 4.9.