STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE – 20

NORMAL DISTRIBUTION

APPLICATIONS OF NORMAL DISTRIBUTION

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APPLICATIONS OF NORMAL DISTRIBUTION

EXAMPLE-1 The life of a certain make of electric bulbs is normally distributed with a mean life of 2000 hours and a standard deviation of 120 hours. Estimate the probability that the life of such bulb will be
(a) lesser then 1910 hours
(b) greater then 2150 hours
(c) within the range of 1850 hours to 2090 hours.

SOLUTION *Let X denote life of such bulbs.*

$Here\ X \sim N(2000{,}14400) \Rightarrow \mu = 2000,\ \ \sigma = 120$

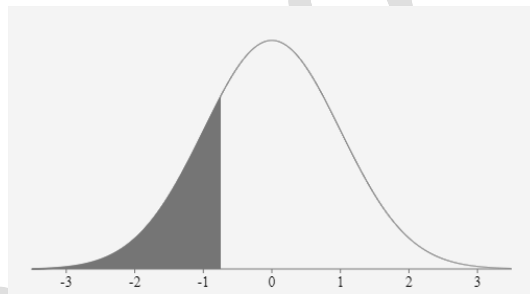
$(a)\ P(X < 1910) = ?$

$Z = \dfrac{X-\mu}{\sigma} = \dfrac{1910-2000}{120} = -0.75$

$So\ \ P(Z < -0.75)$

$= \Phi(-0.75)$

$= \ 0.2266$



$(b)\ P(X > 2150) = ?$

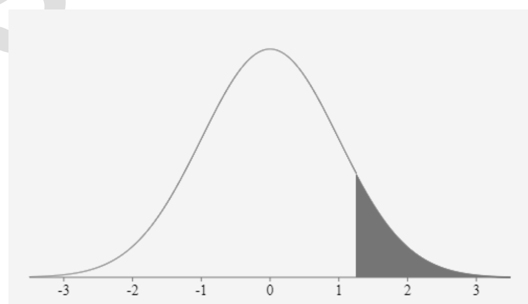
$Z = \dfrac{X-\mu}{\sigma} = \dfrac{2150-2000}{120} = 1.25$

$So\ \ P(Z > 1.25)$

$= 1 - P(Z < 1.25)$

$= 1 - \Phi(1.25) = 1 - 0.8944$

$= 0.1056$



$(c)\ P(1850 < X > 2090) = ?$

$Z_1 = \dfrac{X-\mu}{\sigma} = \dfrac{1850-2000}{120} = -1.25$

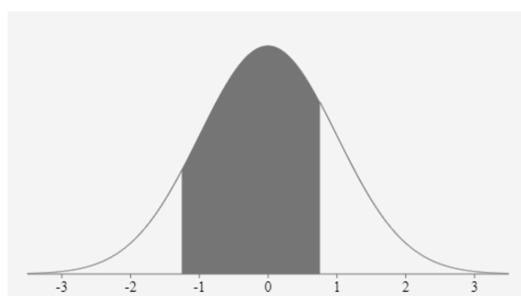
$Z_2 = \dfrac{X-\mu}{\sigma} = \dfrac{2090-2000}{120} = 0.75$

$So\ \ P(-1.25 < Z > 0.75)$

$= \Phi(0.75) - \Phi(-1.25)$

$= 0.7734 - 0.1056$

$= 0.6678$



EXAMPLE-2 Lengths of metal strips produced by a machine are normally distributed with mean length of 150 cm and a standard deviation of 10 cm. Find the probability that the length of a randomly selected strip is
(a) shorter than 165 cm.
(b) longer than 160 cm
(c) within 5 cm of the mean.

SOLUTION *Let X denote length of metal strips.*

$$Here\ X \sim N(150,100) \Rightarrow \mu = 150,\ \sigma = 10$$

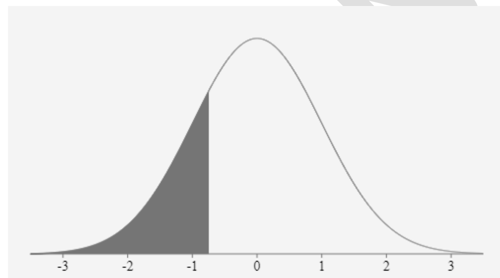
$$(a)\ P(X < 165) = ?$$

$$Z = \frac{X-\mu}{\sigma} = \frac{165-150}{10} = 1.5$$

$$So\ P(Z < 1.5)$$

$$= \Phi(1.5)$$

$$= 0.93$$



$$(b)\ P(X > 160) = ?$$

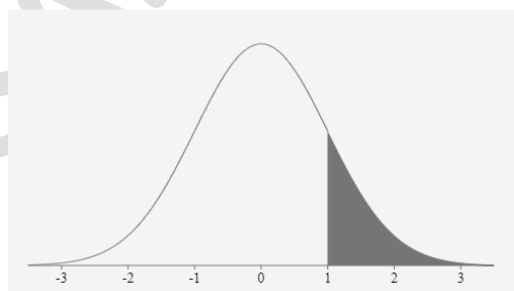
$$Z = \frac{X-\mu}{\sigma} = \frac{160-150}{10} = 1$$

$$So\ P(Z > 1)$$

$$= 1 - P(Z < 1)$$

$$= 1 - \Phi(1)$$

$$= 1 - 0.8413 = 0.1587$$



$$(c)\ P(145 < X > 155) = ?$$

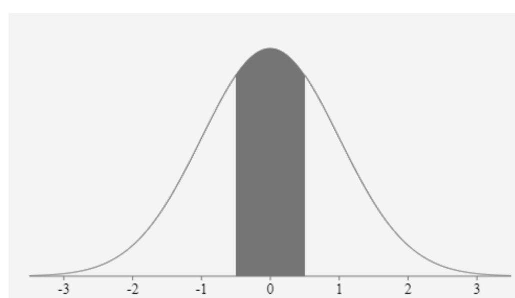
$$Z_1 = \frac{X-\mu}{\sigma} = \frac{145-150}{10} = -0.5$$

$$Z_2 = \frac{X-\mu}{\sigma} = \frac{155-150}{10} = 0.5$$

$$So\ P(-0.5 < Z > 0.5)$$

$$= \Phi(0.5) - \Phi(-0.5)$$

$$= 0.6915 - 0.3085 = 0.383$$



EXAMPLE-3 Time taken by the milk man to deliver milk to the high street is normally distributed with mean of 12 minutes and a standard deviation of 2 minutes. He delivers milk every day. Estimate number of days during the year when he takes
(a) longer than 17 minutes
(b) less than 10 minutes
(c) between 9 and 13 minutes.

SOLUTION *Let X denote time in minutes to deliver milk to High Street.*

$$Here\ X \sim N(12,4) \Rightarrow \mu = 12,\ \sigma = 2$$

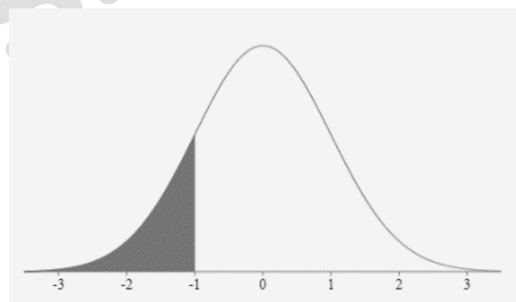
$$(a)\ P(X > 17) = ?$$

$$Z = \frac{X-\mu}{\sigma} = \frac{17-12}{2} = 2.5$$

$$So\ P(Z > 2.5)$$
$$= 1 - P(Z < 2.5)$$
$$= 1 - \Phi(2.5) = 1 - 0.9938$$
$$= 0.0062$$

So number of such days in a year $= 365 \times 0.0062 = 2.263 \equiv 2$ *days*

$$(b)\ P(X < 10) = ?$$

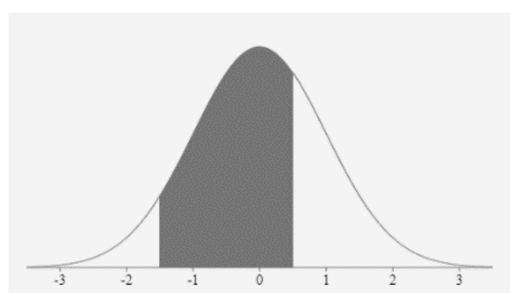


$$Z = \frac{X-\mu}{\sigma} = \frac{10-12}{2} = -1$$

$$So\ P(Z < -1)$$
$$= \Phi(-1) = 0.1587$$

So number of such days in a year $= 365 \times 0.1587 = 57.92 \equiv 58$ *days*

$$(c)\ P(9 < X > 13) = ?$$



$$Z_1 = \frac{X-\mu}{\sigma} = \frac{9-12}{2} = -1.5$$

$$Z_2 = \frac{X-\mu}{\sigma} = \frac{13-12}{2} = 0.5$$

$$So\ P(-1.5 < Z > 0.5)$$
$$= \Phi(0.5) - \Phi(-1.5)$$
$$= 0.6915 - 0.0668 = 0.6247$$

So number of such days in a year $= 365 \times 0.6247 = 228.01 \equiv 228$ *days*

EXAMPLE-4 The heights of female students at particular college are normally distributed with a mean of 169 cm and a standard deviation of 9 cm.
(a) Given that 80% of these female students have a height less than $h\ cm$, find the value of h.
(b) Given that 60% of these female students have a height greater than $g\ cm$, find the value of g.

SOLUTION *Let X denote height of female students in cm.*

$$Here\ X \sim N(169,81) \Rightarrow \mu = 169,\ \sigma = 9$$

$$(a)\ P(X < h) = 0.8$$

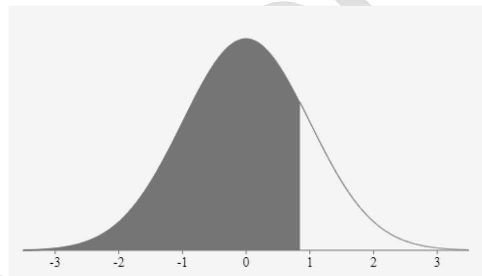
$$Z = \frac{h-169}{9} = a\ (say)$$

$$So\ P(Z < a) = 0.8$$

$$\Phi(a) = 0.8$$

$$a = \Phi^{-1}(0.8) = 0.842$$

$$So\ \frac{h-169}{9} = 0.84 \Rightarrow h = 176.58cm$$



$$(b)\ P(X > g) = 0.6$$

$$Z = \frac{g-169}{9} = b\ (say)$$

$$So\ P(Z > b) = 0.6$$

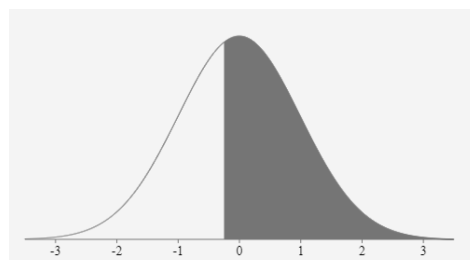
$$1 - P(Z < b) = 0.6$$

$$1 - \Phi(b) = 0.6$$

$$\Phi(b) = 0.4$$

$$b = \Phi^{-1}(0.4) = -0.25$$

$$So\ \frac{g-169}{9} = -0.25 \Rightarrow h = 166.72cm$$

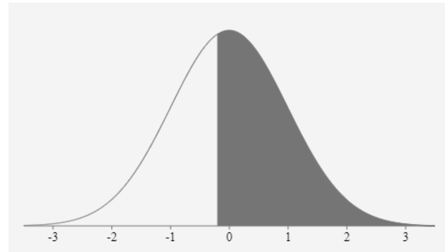


EXAMPLE-5 The marks of 500 candidates in an examination are normally distributed with a mean of 45 marks and a standard deviation of 20 marks.
(a) Given that the pass marks is 41, estimate the number of candidates who passed the examination.
(b) If 5% candidates obtain distinction by scoring ***a*** marks or more, estimate the value of a.
(c) estimate interquartile range of the distribution.

SOLUTION *Let X denote marks of the candidates in the examination.*

$$Here\ X \sim N(45,400) \Rightarrow \mu = 45,\ \sigma = 20$$

$(a)\ P(X \ge 41) = ?$



$$Z = \frac{X-\mu}{\sigma} = \frac{41-45}{20} = -0.2$$

$So\ P(Z \ge -0.2)$

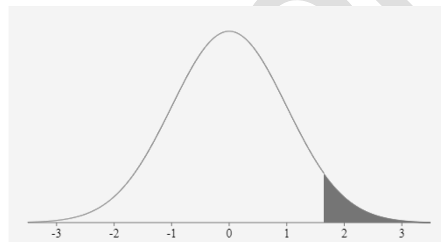
$= 1 - P(Z < -0.2)$

$= 1 - \Phi(-0.2) = 1 - 0.4207$

$= 0.5793$

$So\ number\ of\ candidates\ who\ pass\ = 500 \times 0.5793 = 289.65 \equiv 290$

$(b)\ P(X \ge a) = 0.05$



$$Z = \frac{a-45}{20} = b\ (say)$$

$So\ P(Z \ge b) = 0.05$

$1 - P(Z < b) = 0.05$

$1 - \Phi(b) = 0.05$

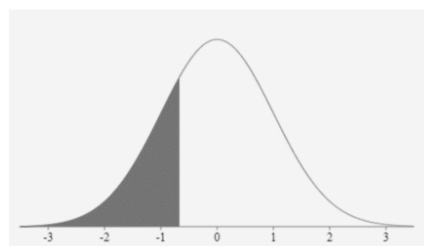
$\Phi(b) = 0.95$

$b = \Phi^{-1}(0.95) = 1.645$

$$So\ \frac{a-45}{20} = 1.645 \Rightarrow a = 78$$

$(a)\ Inter\ Quartile\,Range = Q_3 - Q_1$

$Solve\ \ P(X \le Q_1) = 0.25\ \ for\ Q_1$



$$Z = \frac{Q_1 - 45}{20} = a\ (say)$$

$P(Z \le a) = 0.25$

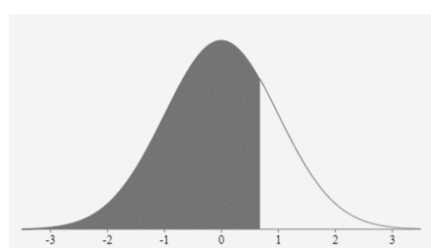
$\Phi(a) = 0.25$

$a = \Phi^{-1}(0.25) = -0.67$

$$So\ \ \frac{Q_1 - 45}{20} = -0.67$$

$\Rightarrow\ Q_1 = 31.6$

$Solve\ \ P(X \le Q_3) = 0.75\ \ for\ Q_3$



$$Z = \frac{Q_3 - 45}{20} = b\ (say)$$

$P(Z \le b) = 0.75$

$\Phi(b) = 0.75$

$b = \Phi^{-1}(0.75) = 0.67$

$$So\ \ \frac{Q_3 - 45}{20} = 0.67$$

$\Rightarrow\ Q_3 = 58.4$

$So\,Interquartile\,range\ = Q_3 - Q_1 = 58.4 - 31.6 = 26.8$