

PRACTICE QUESTIONS CONTINUOUS RANDOM VARIABLE**EXERCISE – 3.3****PROBABILITY DENSITY FUNCTIONS**

1. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

- (a) value of constant k
 - (b) $P(X \geq 1)$
 - (c) $P(1/2 < X < 3/2)$
- (Ans : 3/8, 7/8, 13/32)

2. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} k & -2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

- (a) value of constant k
 - (b) $P(-1.6 \leq X \leq 2.1)$
- (Ans : 0.2, 0.74)

3. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} k(4-x) & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

- (a) value of constant k
 - (b) $P(1.2 \leq X \leq 2.4)$
- (Ans : 0.25, 0.66)

4. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} k(x+2)^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

- (a) value of constant k
 - (b) $P(0 \leq X \leq 1)$
 - (c) $P(X > 1)$
- (Ans : 3/56, 19/56, 37/56)

5. A continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} A(2-x)(2+x) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

(a) value of constant A

(b) $P(X \leq 1)$

(c) $P(X \geq 2)$

(d) $P(1 < X < 2)$

(e) $P(X = 1/2)$

(Ans : 3/16, 11/16, 0, 5/16, 0)

6. A continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $P(0 \leq X \leq 2) = 1$

(b) Find $P(X < 1.2)$

(c) Find $P(X \geq 0.5)$

(Ans : 0.68, 0.875)

7. The continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} kx^3 & 0 \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

and $P(X \leq 1/2) = 1/16$. Find the values of constants c and k .

(Ans : $c = 1, k = 4$)

8. Suppose a special type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The p.d.f that characterizes the proportion X that make a profit is given by

$$f(x) = \begin{cases} kx^4(1-x)^3 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) what is the value of k that renders the above a valid density function.

(b) Find the probability that at most 50% of the firms make a profit in the first year.

(c) Find the probability that at least 80% of the firms make a profit in the first year.

(Ans : $k = 280, 0.3633, 0.0563$)

9. A continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} k & 0 \leq x < 2 \\ k(2x-3) & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

(a) value of constant k

(b) $P(X \leq 1)$

(c) $P(X > 2.5)$

(d) $P(1 \leq X \leq 2.3)$

(e) $P(X = 2)$

(Ans : 0.25, 0.25, 0.3125, 0.3475, 0)

10. A continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} x/2 & 0 \leq x < 1 \\ (3-x)/4 & 1 \leq x < 2 \\ 1/4 & 2 \leq x < 3 \\ (4-x)/4 & 3 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

(a) $P(X \geq 3)$

(b) $P(X \leq 2)$

(c) $P(1 < X < 3)$

(d) $P(|x| < 1.5)$

(e) $P(|x| > 1.25)$

(Ans : 1/8, 5/8, 0.46875, 0.6328)

PRACTICE QUESTIONS CONTINUOUS RANDOM VARIABLE**EXERCISE – 3.4****DISTRIBUTION FUNCTIONS FOR CONTINUOUS RANDOM VARIABLE**

1. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the cumulative distribution function

$$(Ans : F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \leq x \leq 2 \\ 1 & x \geq 2 \end{cases})$$

2. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} \frac{1}{4} & 0 \leq x \leq 2 \\ \frac{1}{4}(2x-3) & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the cumulative distribution function

$$(Ans : F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{4} & 0 \leq x \leq 2 \\ \frac{1}{4}(x^2 - 3x + 4) & 2 \leq x \leq 3 \\ 1 & x \geq 3 \end{cases})$$

3. The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} c/\sqrt{x} & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the following

- (a) the value of c
 (b) The distribution function of X
 (c) $P(X > 1)$
 (d) $P(2 \leq X \leq 3)$

$$(Ans : c = 1/4, F(x) = \begin{cases} 0 & x < 0 \\ \frac{\sqrt{x}}{2} & 0 \leq x \leq 4, \\ 1 & x \geq 4 \end{cases}) (0.5, 0.159)$$

4. The continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{27} & 0 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Find the following

- (a) The p.d.f of X
 (b) $P(X \leq 2)$
 (c) $P(1 \leq X \leq 2)$

$$(Ans : p.d.f \ f(x) = \begin{cases} x^2/9 & 0 \leq x \leq 3 \\ 0 & otherwise \end{cases})$$

$$(Ans : 8/27, 7/27)$$

5. The continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{2x^2}{5} & 0 \leq x < 1 \\ -\frac{3}{5} + \frac{2}{5} \left(3x - \frac{x^2}{2} \right) & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

Find the following

- (a) The p.d.f of X
 (b) $P(|X| > 1.25)$

$$(Ans : p.d.f \ f(x) = \begin{cases} \frac{4x}{5} & 0 \leq x < 1 \\ 2(3-x)/5 & 1 \leq x < 2 \\ 0 & otherwise \end{cases})$$

$$(Ans : 0.4125)$$