*STATISTICS IS THE GRAMMAR OF SCIENCE*

# PROBABILITY AND STATISTICS

# LECTURE – 11

## PROBABILITY TREES


PREPARED BY
**HAZBER SAMSON**
FAST NUCES ISLAMABAD

# PROBABILITY TREES

Some times in order to deal with situations having multiple cases, we use tree diagrams, we use multiplication law of probability and conditional probability to solve different problems. Let's understand Tree diagrams by doing different examples.

## EXAMPLES OF PROBABILITY

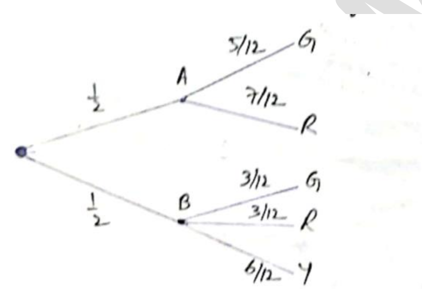
***EXAMPLE-1*** Box **A** contains 5 green and 7 red balls. Box **B** contains 3 green, 3 Red and 6 yellow balls. A box is selected at random and a ball is drawn from it. What is the probability that the ball drawn is a

$(a)$ *green ball*

$(b)$ *red ball*

$(c)$ *yellow ball*

***SOLUTION*** *Tree diagram for the given problem is as follows*




$(a)$ *Let G denote the event that selected ball is a green ball*

$$P(G)=\left(\frac{1}{2}\times\frac{5}{12}\right)+\left(\frac{1}{2}\times\frac{3}{12}\right)=\frac{8}{24}=\frac{1}{3}$$

$(b)$ *Let R denote the event that selected ball is a red ball*

$$P(R)=\left(\frac{1}{2}\times\frac{7}{12}\right)+\left(\frac{1}{2}\times\frac{3}{12}\right)=\frac{10}{24}=\frac{5}{12}$$

$(c)$ *Let Y denote the event that selected ball is a Yellow ball*

$$P(Y)=\left(\frac{1}{2}\times\frac{6}{12}\right)=\frac{3}{12}=\frac{1}{4}$$

***EXAMPLE-2*** Three urns of the same appearance are given as

*Urn A contains 5 red and 7 white balls*

*Urn B contains 4 red and 3 white balls*

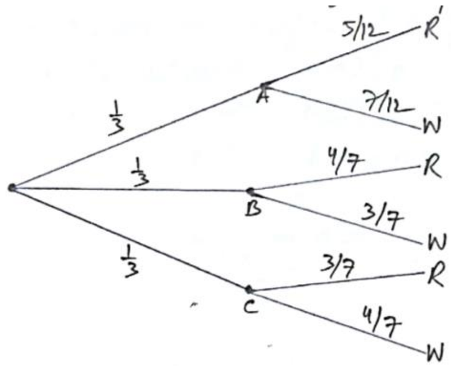
*Urn C contains 3 red and 4 white balls*

An Urn is selected at random and a ball is drawn from it.

$(a)$ *what is the probability that ball drawn is red.*

$(b)$ *If ball drawn is red what is the probability it came from Urn A.*

***SOLUTION*** *Tree diagram for the given problem is as follows*




$(a)$ *Let $R$ denote the event that selected ball is a red ball*

$$P(R)=\left(\frac{1}{3}\times\frac{5}{12}\right)+\left(\frac{1}{3}\times\frac{4}{7}\right)+\left(\frac{1}{3}\times\frac{3}{7}\right)=\frac{119}{252}$$

$(b)$ *Let $A$ denote the event $R$ denote the event that ball camefrom Urn $A$.*

*Let $R$ denote the event that selected ball is a red ball.*

*Now we have to find* $P(A|R)=\dfrac{P(A\cap R)}{P(R)}=\dfrac{1/3\times 5/12}{119/252}=\dfrac{5}{17}$

***EXAMPLE-3*** A factory has three machines **A, B, C** producing large number of certain item. Of the total daily production of the item 50% are produced on machine **A**, 30% on **B** and 20% on **C**. Records show that 2% of the items produced on **A** are defective. 3% of the items produced on **B** are defective. 4% of the items produced on **C** are defective. The occurrence of defective of defective items is independent of each other. One item is selected at random from a day's total output.

$(a)$ *Show that the probability of its being defective is* $0.027$.

$(b)$ *Given that it is defective, find the probability that it was produced on Machine A.*

***SOLUTION*** *Events are defined as follows*

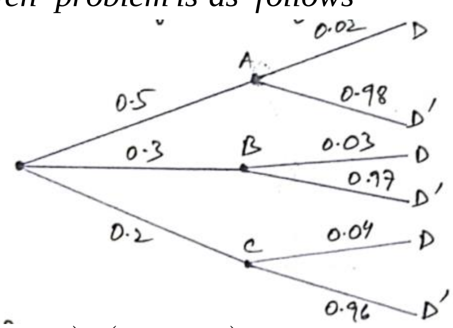
*Let $A$ denote the event that item is produced on machina $A$*

*Let $B$ denote the event that item is produced on machina $B$*

*Let $C$ denote the event that item is produced on machina $C$*

*Let $D$ denote the defective item.*

*Tree diagram for the given problem is as follows*




$(a)$ $P(D)=(0.5\times 0.02)+(0.3\times 0.03)+(0.2\times 0.04)=0.027$

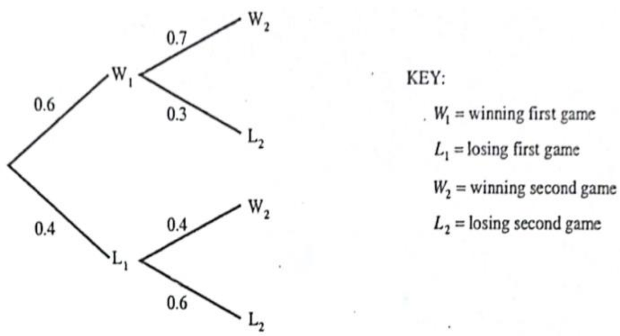
$(b)$ $P(A|D)=\dfrac{P(A\cap D)}{P(D)}=\dfrac{0.5\times 0.02}{0.027}=0.370$

***EXAMPLE-4*** Rachel and Anna play each other at badminton. Each game results in either a win for Rachel or a win for Anna. The probability of Rachel winning the first game is 0.6. If Rachel wins a particular game, the probability of her winning the next game is 0.7 but if she looses, the probability of her winning the next game is 0.4.

(a) *Find the conditional probability that Rachel wins the first game given that she looses the second.*

(b) *Find the probability that Rachel wins 2 games and looses one game out of first three games they play.*

***SOLUTION*** *Tree diagram for the given problem is as follows*




$$(a)\ P(W_1|L_2)=\frac{P(W_1\cap L_2)}{P(L_2)}=\frac{0.6\times 0.3}{(0.6\times 0.3)+(0.4\times 0.6)}=0.4285$$

$(b)\ P(\text{wins 2 games and losses 1 out of first three games})$

$$=P(W_1\cap W_2\cap L_3)+P(W_1\cap L_2\cap W_3)+P(L_1\cap W_2\cap W_3)$$

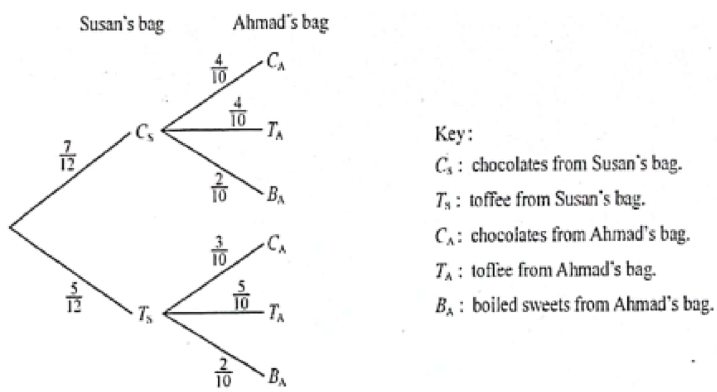
$$=(0.6\times 0.7\times 0.3)+(0.6\times 0.3\times 0.4)+(0.4\times 0.4\times 0.7)=0.31$$

***EXAMPLE-5*** Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and two boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

(a) *Find the probability that two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag.*

(b) *Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that sweet taken from Susan's bag was also a chocolate.*

***SOLUTION*** *Tree diagram for the given problem is as follows*




$$(a)\ P(T_S\cap B_A)=P(T_s)\times P(B_A)=\frac{5}{12}\times\frac{2}{10}=\frac{1}{12}$$

$$(b)\ P(C_S|C_A)=\frac{P(C_S\cap C_A)}{P(C_A)}=\frac{P(C_S\cap C_A)}{P(C_S\cap C_A)+P(T_S\cap C_A)}=\frac{7/12\times 4/10}{(7/12\times 4/10)+(5/12\times 3/10)}=\frac{28}{43}$$