Question No:- 1

Compute the column space, ... basis ... dim

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

- $\text{Col } C = \text{span}\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$ it is subspace of $\mathbb{R}^2$

OR

Each linear Combination of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is an element of Col C.

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} R_2 - 3R_1$$

- $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$ is basis for Col C

- dim Col C = 2

$$D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

- $\text{Col } D = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$ it is subspace of $\mathbb{R}^3$

OR

Each linear Combination of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ is in Col D.

$$D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix}$$

⊙ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ is basis for Col D

⊙ dim Col D = 1

Question No:- 2

Compute the row space, ... basis .... dim

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

⊙ Row C = span $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ from $\mathbb{R}^2$

Every linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is in Row C.

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} R_2 - 3R_1$$

⊙ $\{ [1 \ \ 2]^t, [0 \ \ -2]^t \}$ is basis of for Row C.

⊙ dim of row C = 2

$$D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

⊙ Row D = span $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$ from $\mathbb{R}^2$

each linear combination of $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix}$ and $\begin{bmatrix}3\\6\end{bmatrix}$ is in Row D.

$$D = \begin{bmatrix}1 & 2\\2 & 4\\3 & 6\end{bmatrix} \sim \begin{bmatrix}1 & 2\\0 & 0\\0 & 0\end{bmatrix}$$

- Basis of Row $D = \left\{ \begin{bmatrix}1 & 2\end{bmatrix}^t \right\}$
- dim Row $D = 1$

Question No:- 3

Find Null space .... Basis .... dim

$$C = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

- $\tilde{A} = \left[\begin{array}{cc|c}1 & 2 & 0\\3 & 4 & 0\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 2 & 0\\0 & -2 & 0\end{array}\right] R_2 - 3R_1$

$$-2y = 0 \qquad x + 2y = 0$$
$$y = 0 \qquad x = 0$$

Nul $C = \left\{ \begin{bmatrix}0\\0\end{bmatrix} \right\}$

- Basis of Nul $C = \{ \quad \}$
- dim Null $C = 0$

$$D = \begin{bmatrix}1 & 2\\2 & 4\\3 & 6\end{bmatrix}$$

$$\tilde{A} = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x + 2y = 0$$
$$x = -2y$$
$$y = y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2y \\ y \end{bmatrix} = y\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

- Every linear combination of $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is in Nul space of D.
- Basis for Nul $D = \left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$
- dim of Nul $D = 1$

Question No:- 4

Determine Left Null space -- Basis ... dim

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$C^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\tilde{A} = \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -2 & 0 \end{array}\right] \quad R_2 - 2R_1$$

- Nul $(C^t) = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- Basis of Nul $(C^t) = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- dim Nul $(C^t) = 0$

$$D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$D^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + 2y + 3z = 0$$

$$x = -2y - 3z$$
$$y = y$$
$$z = z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2y - 3z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

every linear Combination of $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ is in Nul (D^t)

Basis of Nul $(D^t) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

dim Nul $(D^t) = 2$