STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE # 28

HYPOTHESIS TESTING

TESTING HYPOTHESIS ABOUT MEAN WHEN SIGMA IS KNOWN

PREPARED BY
HAZBER SAMSON
SCIENCES & HUMANITIES DEPARTMENT
FAST ISLAMABAD CAMPUS

INFERENCES ON A SINGLE POPULATION

INFERENCE ABOUT THE POPULATION MEAN WHEN SIGMA IS KNOWN

CASE-1 $If \quad H_1 : \mu \neq \mu_\circ$

- When σ is known then we use **test statistic**

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- When σ is known then we use **P-Value**

$$P = 2[1 - \Phi(|z_0|)]$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\bar{x} - Z_{\frac{\alpha}{2}} . \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}} . \frac{\sigma}{\sqrt{n}}$$

CASE-2 $If \quad H_1 : \mu < \mu_\circ$

- When σ is known then we use **test statistic**

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- When σ is known then we use **P-Value**

$$P = \Phi(z_0)$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\mu \leq \bar{x} + Z_\alpha . \frac{\sigma}{\sqrt{n}}$$

CASE-3 $If \quad H_1 : \mu > \mu_\circ$

- When σ is known then we use **test statistic**

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- When σ is known then we use **P-Value**

$$P = 1 - \Phi(z_0)$$

- The $(1-\alpha)$ 100% **confidence interval** for μ is

$$\mu \geq \bar{x} - Z_\alpha . \frac{\sigma}{\sqrt{n}}$$

Z-TEST

If $H_{\circ} : (=, \leq, \geq)$ then $H_1 : (\neq, >, <)$

CRITICAL REGIONS IN Z-TEST

- If H_1 *contains* $\neq$ then C.R is $|z| \geq Z_{\frac{\alpha}{2}}$ (Two Sided)
- If H_1 *contains* $>$ then C.R is $Z > Z_{\alpha}$ (One Sided)
- If H_1 *contains* $<$ then C.R is $Z < -Z_{\alpha}$ (One Sided)

FREQUENTLY USED CRITICAL VALUES OF Z

Significance Level (α)	**Confidence Level** $(1-\alpha)$	**One-Tailed Test**	**Two-Tailed Test**
$\alpha = 0.01$	99%	$Z_{\alpha} = 2.33$	$Z_{\alpha/2} = 2.58$
$\alpha = 0.02$	98%	$Z_{\alpha} = 2.05$	$Z_{\alpha/2} = 2.33$
$\alpha = 0.03$	97%	$Z_{\alpha} = 1.88$	$Z_{\alpha/2} = 2.17$
$\alpha = 0.04$	96%	$Z_{\alpha} = 1.75$	$Z_{\alpha/2} = 2.05$
$\alpha = 0.05$	95%	$Z_{\alpha} = 1.65$	$Z_{\alpha/2} = 1.96$
$\alpha = 0.06$	94%	$Z_{\alpha} = 1.55$	$Z_{\alpha/2} = 1.88$
$\alpha = 0.07$	93%	$Z_{\alpha} = 1.48$	$Z_{\alpha/2} = 1.81$
$\alpha = 0.08$	92%	$Z_{\alpha} = 1.41$	$Z_{\alpha/2} = 1.75$
$\alpha = 0.09$	91%	$Z_{\alpha} = 1.34$	$Z_{\alpha/2} = 1.70$
$\alpha = 0.10$	90%	$Z_{\alpha} = 1.28$	$Z_{\alpha/2} = 1.65$

P-VALUE FORMULAS IN Z-TEST

$$P = \begin{cases} 2[1-\Phi(|z_0|)] & if \quad H_1 : \mu \neq \mu_0 \\ 1-\Phi(z_0) & if \quad H_1 : \mu > \mu_0 \\ \Phi(z_0) & if \quad H_1 : \mu < \mu_0 \end{cases}$$

EXAMPLES OF INFERENCE ABOUT MEAN WHEN SIGMA IS KNOWN

EXAMPLE-1 A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At a 0.05, is there enough evidence to reject the claim? Use the *P*-value method.

SOLUTION *Here* $n = 32,\ \bar{x} = 8.2,\ \sigma = 0.6,\ \alpha = 0.05$

CRITICAL VALUE APPROACH

Step – 1: Formulation of Hypotheses

$$H_0 : \mu = 8$$
$$H_1 : \mu \neq 8$$

Step – 2: Level of Significance

$$\alpha = 0.05$$

Step – 3: Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89$$

Step – 4: Critical Region

$$|z| \geq z_{\frac{\alpha}{2}} \quad \Rightarrow |z| \geq 1.96$$

Step – 5: Conclusion

Since calculated value of z does not lies in CR so do not reject $H_{0.}$

P-VALUE APPROACH

Step – 1: Formulation of Hypotheses

$$H_0 : \mu = 8$$
$$H_1 : \mu \neq 8$$

Step – 2: Level of Significance

$$\alpha = 0.05$$

Step – 3: Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89$$

Step – 4: P – value

$$p = 2[1 - \Phi(|z_0|)] = 2[1 - \Phi(|1.89|)] = 0.0588$$

Step – 5: Conclusion

Since p – value > α so do not reject $H_{0.}$

CONFIDENCE INTERVAL APPROACH

Step – 1: Formulation of Confidence Interval

The $(1-\alpha)100\%$ *CI for* μ *is given by*

$$\bar{x} - Z_{\frac{\alpha}{2}}.\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}}.\frac{\sigma}{\sqrt{n}}$$

$$8.2 - (1.96)(0.6)/\sqrt{32} \leq \mu \leq 8.2 - (1.96)(0.6)/\sqrt{32}$$

$$7.99 \leq \mu \leq 8.41$$

Step – 2: Conclusion

Since $\mu = 8$ *lies in CI so do not reject* $H_{0.}$

EXAMPLE-2 A researcher wishes to test the claim that the average cost of tuition and fees at a four year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

SOLUTION $Here\ n = 36,\ \bar{x} = 5950,\ \sigma = 659,\ \alpha = 0.05$

CRITICAL VALUE APPROACH

$Step-1: Formulation\ of\ Hypotheses$

$$H_0 : \mu \le \$5700$$
$$H_1 : \mu > \$5700$$

$Step-2: Level\ of\ Significance$

$$\alpha = 0.05$$

$Step-3: Test\ Statistic$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{5950-5700}{659/\sqrt{36}} = 2.28$$

$Step-4: Critical\ Region$

$$z > z_\alpha \quad \Rightarrow z > 1.65$$

$Step-5: Conclusion$

$Since\ calculated\ value\ of\ z\ lies\ in\ CR\ so\ reject\ H_{0.}$

P-VALUE APPROACH

$Step-1: Formulation\ of\ Hypotheses$

$$H_0 : \mu \le \$5700$$
$$H_1 : \mu > \$5700$$

$Step-2: Level\ of\ Significance$

$$\alpha = 0.05$$

$Step-3: Test\ Statistic$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{5950-5700}{659/\sqrt{36}} = 2.28$$

$Step-4: P-value$

$$p = 1-\Phi(z_0) = 1-\Phi(2.28) = 0.0113$$

$Step-5: Conclusion$

$Since\ p-value < \alpha\ so\ reject\ H_{0.}$

CONFIDENCE INTERVAL APPROACH

$Step-1: Formulation\ of\ Confidence\ Interval$

$The\ (1-\alpha)100\%\ CI\ for\ \mu\ is\ given\ by$

$$\mu \ge \bar{x} - Z_\alpha . \frac{\sigma}{\sqrt{n}}$$
$$\mu \ge 5950 - (1.65)(659)/\sqrt{36}$$
$$\mu \ge 5768.78$$

$Step-2: Conclusion$

$Since\ \mu = 5700\ does\ not\ lie\ in\ CI\ so\ reject\ H_{0.}$

Summarize the results. There is enough evidence to support the claim that the tuition and fees at four-year public colleges are greater than $5700.

EXAMPLE-3 Rosie is an aging sheep dog in Montana who gets regular checkups from her owner, the local veterinarian. From past experience, the vet knows that Rosie's heart rate (beats per minute) has a normal distribution. The vet checked the Merck Veterinary Manual and found that for dogs of this breed $\mu = 115$ *and* $\sigma = 12$ beats per minute.

Over the past 6 weeks, Rosie's heart rate (beats/min) measured

93 109 110 89 112 117

The vet is concerned that Rosie's heaet rate may be slowing. Do the data indicate this is the case? Investigate using $\alpha = 0.01$.

SOLUTION *Here* $n = 6,\ \bar{x} = \sum x/n = 630/6 = 105,\ \sigma = 12,\ \alpha = 0.01$

CRITICAL VALUE APPROACH

Step-1: Formulation of Hypotheses

$$H_0 : \mu \geq 115$$
$$H_1 : \mu < 115$$

Step-2: Level of Significance

$$\alpha = 0.01$$

Step-3: Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{105 - 115}{12/\sqrt{6}} = -2.04$$

Step-4: Critical Region

$$z < -z_\alpha \quad \Rightarrow z < -2.33$$

Step-5: Conclusion

Since calculated value of z does not lie in CR so do not reject $H_{0.}$

P-VALUE APPROACH

Step-1: Formulation of Hypotheses

$$H_0 : \mu \geq 115$$
$$H_1 : \mu < 115$$

Step-2: Level of Significance

$$\alpha = 0.01$$

Step-3: Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{105 - 115}{12/\sqrt{6}} = -2.04$$

Step-4: P-value

$$p = \Phi(z_0) = \Phi(-2.04) = 0.0207$$

Step-5: Conclusion

Since $p-value > \alpha$ *so do not reject* $H_{0.}$

CONFIDENCE INTERVAL APPROACH

Step-1: Formulation of Confidence Interval

The $(1-\alpha)100\%$ *CI for* μ *is given by*

$$\mu \leq \bar{x} + Z_\alpha . \frac{\sigma}{\sqrt{n}}$$
$$\mu \leq 105 + (2.33)(12)/\sqrt{6}$$
$$\mu \leq 116.41$$

Step-2: Conclusion

Since $\mu = 115$ *lies in CI so do not reject* $H_{0.}$