STATISTICS IS THE GRAMMAR OF SCIENCE

PROBABILITY AND STATISTICS

LECTURE – 17

PROBABILITY DISTRIBUTIONS

GEOMETRIC AND NEGATIVE BINOMIAL DISTRIBUTION

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GEOMETRIC DISTRIBUTION

The geometric probability distribution is one of the important discrete probability distribution. The Geometric Distribution is applied to experiments that satisfy the conditions of a Geometric Experiment, Lets study about Geometric Experiment first.

THE GEOMETRIC EXPERIMENT A geometric experiment is a probability experiment that satisfies the following four requirements

1- Number of trials are not finite.
2- Each trial has only two possible outcomes, either success or failure.
3- The trials are independent of each other.
4- The probability of success remains same for each trial.

EXAMPLES OF GEOMETRIC EXPERIMENT Some examples of Geometric experiment are as follows
1-Tossing a coin repeatedly until head appears.
2-Throwing a Die repeatedly until six appears.
3-Selecting a customer from a market until got first one who uses credit card.

THE GEOMETRIC PROBABILITY DISTRIBUTION If repeated independent trials can result in a success with probability p and a failure with probability $q=1-p$, then the probability distribution of the random variable X, the number of the trial on which the first success occurs, follows a geometric distribution.

THE GEOMETRIC FORMULA If random variable X follows geometric distribution then it is denoted by $X \sim g(n,p)$ and the geometric probability function is given by

$$P(X=x)=p\,q^{x} \qquad ,x=0,1,2,3,\ldots.$$

$$\begin{aligned} Where \quad p &= probability\ of\ success \\ q &= probability\ of\ failure \\ x &= number\ of\ failures\ prior\ to\ the\ first\ success \end{aligned}$$

AN ALTERNATE GEOMETRIC FORMULA If X is often defined as the number of trials required to obtain the first success then geometric probability function is given by

$$P(X=x)=p\,q^{x-1} \quad ,x=1,2,3,\ldots.$$

$$\begin{aligned} Where \quad p &= probability\ of\ success \\ q &= probability\ of\ failure \\ x &= number\ of\ trials\ required\ to\ obtain\ the\ first\ success \end{aligned}$$

PROPERTIES OF GEOMETRIC DISTRIBUTION

1- The Mean of Geometric Distribution is $E(X)=1/p$

2- The variance of Geometric Distribution is $V(X)=q/p^2$

EXAMPLES OF GEOMETRIC DISTRIBUTION

EXAMPLE-1 For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

SOLUTION Let *X denote t*he number of applicants required to obtain the first qualified one.

$$Here \quad X=5, \quad p=0.01, \quad q=1-p=1-0.01=0.99$$

Now $P(X=5)=p\,q^{x-1}=(0.01)(0.99)^{5-1}=0.0096$

EXAMPLE-2 A recruiting firm finds that 20% of the applicants for a particular sales position are fluent in both English and Spanish. Applicants are selected at random from the pool and interviewed sequentially. Find the probability that five applicants are interviewed before finding the first applicant who is fluent in both English and Spanish.

SOLUTION Let *X denote t*he number of applicants who are fluent in English and Spanish.

$$Here \quad X=6, \quad p=0.2, \quad q=1-p=1-0.2=0.8$$

Now $P(X=6)=p\,q^{x-1}=(0.2)(0.8)^{6-1}=0.066$

EXAMPLE-3 If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?

SOLUTION Let *X denote t*he number of tries for an applicant to finally pass the test.

$$Here \quad X=4, \quad p=0.75, \quad q=1-p=1-0.75=0.25$$

Now $P(X=4)=p\,q^{x-1}=(0.75)(0.25)^{4-1}=0.0117$

EXAMPLE-4 If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth measuring device tested will be the first to show excessive drift?

SOLUTION Let *X denote t*he number of measuring devices to show excessive drift

$Here \quad X=6, \quad p=0.05, \quad q=1-p=1-0.05=0.95$ Now

$$P(X=6)=p\,q^{x-1}=(0.05)(0.95)^{6-1}=0.0387$$

EXAMPLE-5 If the probability of engine malfunction during any one-hour period is $p=0.02$ and X denotes the number of one-hour intervals until the first malfunction, find the mean and standard deviation of *X.*

SOLUTION $Here \quad X=5, \quad p=0.02, \quad q=1-p=1-0.02=0.98,$

Now $E(X)=1/p=1/0.02=50$

$$V(X)=q/p^2=0.98/(0.02)^2=2450$$

$$S.D(X)=\sqrt{V(X)}=\sqrt{2450}=49.49$$

NEGATIVE BINIOMIAL DISTRIBUTION

The Negative Binomial Probability Distribution is one of the most widely used discrete probability distribution. The Negative Binomial Distribution is applied to experiments that satisfy the conditions of a Negative Binomial Experiment, Lets study about Negative Binomial Experiment first.

THE NEGATIVE BINOMIAL EXPERIMENT A negative binomial experiment is a probability experiment that satisfies the following four requirements
1- The trials are continued until we achieve a specific number of successes.
2- Each trial has only two possible outcomes, either success or failure.
3- The trials are independent of each other.
4- The probability of success remains same for each trial.

EXAMPLES OF NEGATIVE BINOMIAL EXPERIMENT Some examples of Negative Binomial experiment are as follows
1- Tossing a coin repeatedly until 5 heads appears.
2- Throwing a die repeatedly until 10 sixes appeared.
3- Selecting customers repeatedly until we get 7 customers who use credit card.

THE NEGATIVE BINOMIAL PROBABILITY DISTRIBUTION In geometric distribution models the probabilistic behavior of the number of failures prior to the first successin a sequence of independent Bernoulli trials. But what if we were interested in the number of failures prior to the second success, or the third success, or (in general) therth success? The distribution governing probabilistic behavior in these cases is called the negative binomial distribution.

THE NEGATIVE BINOMIAL FORMULA If random variable X follows negative binomial distribution then it is denoted by $X \sim NB(r, p)$ and the binomial probability function is given by

$$P(X = x) = {}^{x+r-1}C_{r-1} p^r q^x, \qquad x = 0,1,2,3,\ldots$$

$$Where \quad p = probability\ of\ success, \quad q = probability\ of\ failure$$

$$x = number\ of\ failures\ prior\ to\ the\ rth\ success$$

Notice that, if $r = 1$, we have the geometric distribution. Thus, the geometric distribution is a special case of the negative binomial distribution.

AN ALTERNATE NEGATIVE BINOMIAL FORMULA If X is often defined as the number of trials required to obtain the r^{th} success then negative binomial probability function is given by

$$P(X = x) = {}^{x-1}C_{r-1} p^r q^{x-r}, \qquad x = r, r+1, r+2, \ldots$$

$$Where \quad p = probability\ of\ success, \quad q = probability\ of\ failure$$

$$x = number\ of\ trials\ required\ to\ obtain\ the\ rth\ success$$

PROPERTIES OF BINOMIAL DISTRIBUTION
1- The Mean of Negative Binomial Distribution is $E(X) = r/p$
2- The Variance of Negative Binomial Distribution is $V(X) = rq/p^2$

EXAMPLES OF NEGATIVE BINOMIAL DISTRIBUTION

EXAMPLE-1 If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it?

SOLUTION Let *X* denote the number of children exposed to the disease.

$$Here \quad X = 10, \quad p = 0.40, \quad q = 1 - p = 1 - 0.40 = 0.60, \quad r = 3$$

We know that $P(X = x) = {}^{x-1}C_{r-1} p^r q^{x-r}$

$$\begin{aligned} So \quad P(X = 10) &= {}^{10-1}C_{3-1}(0.40)^3(0.60)^{10-3} \\ &= {}^9C_2(0.40)^3(0.60)^7 \\ &= 0.0645 \end{aligned}$$

EXAMPLE-2 A geological study indicates that an exploratory oil well drilled in a particular region should strike oil with probability .2. Find the probability that the third oil strike comes on the fifth well drilled.

SOLUTION Let *X* denote the number of oil strikes.

$$Here \quad X = 5, \quad p = 0.2, \quad q = 1 - p = 1 - 0.2 = 0.8, \quad r = 3$$

We know that $P(X = x) = {}^{x-1}C_{r-1} p^r q^{x-r}$

$$\begin{aligned} So \quad P(X = 5) &= {}^{5-1}C_{3-1}(0.2)^3(0.8)^{5-3} \\ &= {}^4C_2(0.2)^3(0.8)^2 \\ &= 0.0307 \end{aligned}$$

EXAMPLE-3 A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new childbirth regimen. She anticipates that 20% of all couples she asks will agree. What is the probability that 15 couples must be asked before 5 are found who agree to participate?

SOLUTION Let *X* denote the number of couples who agree to participate

$$Here \quad X = 15, \quad p = 0.2, \quad q = 1 - p = 1 - 0.2 = 0.8, \quad r = 5$$

We know that $P(X = x) = {}^{x-1}C_{r-1} p^r q^{x-r}$

$$\begin{aligned} So \quad P(X = 15) &= {}^{15-1}C_{5-1}(0.2)^5(0.8)^{15-5} \\ &= {}^{14}C_4(0.2)^5(0.8)^{10} \\ &= 0.034 \end{aligned}$$

EXAMPLE-4 In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams *A* and *B* face each other in the championship games and that team *A* has probability 0.55 of winning a game over team *B*.
(a) What is the probability that team *A* will win the series in 6 games?
(b) What is the probability that team *A* will win the series?

SOLUTION Let *X* denote the number of games won by team A.

We know that $P(X=x) = {}^{x-1}C_{r-1} p^r q^{x-r}$

(a) $Here\ \ X=6,\quad p=0.55,\quad q=1-p=1-0.55=0.45,\quad r=4$

$$\begin{aligned}
P(X=6) &= {}^{6-1}C_{4-1}(0.55)^4(0.45)^{6-4} \\
&= {}^{5}C_{3}(0.55)^4(0.45)^{2} \\
&= 0.1853
\end{aligned}$$

(b) $Here\ \ X=4,5,6,7,\quad p=0.55,\quad q=1-p=1-0.55=0.45,\quad r=4$

$$\begin{aligned}
P(X\ge 4) &= {}^{4-1}C_{4-1}(0.55)^4(0.45)^{4-4} + {}^{5-1}C_{4-1}(0.55)^4(0.45)^{5-4} + {}^{6-1}C_{4-1}(0.55)^4(0.45)^{6-4} \\
&\quad + {}^{7-1}C_{4-1}(0.55)^4(0.45)^{7-4} \\
&= {}^{3}C_{3}(0.55)^4(0.45)^0 + {}^{4}C_{3}(0.55)^4(0.45)^1 + {}^{5}C_{3}(0.55)^4(0.45)^2 + {}^{6}C_{3}(0.55)^4(0.45)^3 \\
&= 0.0915 + 0.1647 + 0.1853 + 0.1668 \\
&= 0.6083
\end{aligned}$$

EXAMPLE-5 Find the expected value and the variance of the number of times one must throw a die until the outcome 1 has occurred 4 times.

SOLUTION $Here\ \ r=4,\quad p=1/6,\quad q=1-p=1-1/6=5/6$

Now $E(X)=\dfrac{r}{p}=\dfrac{4}{1/6}=24$

$$V(X)=\frac{rq}{p^2}=\frac{4(5/6)}{(1/6)^2}=120$$