National University of Computer & Emerging Sciences Islamabad

FAST School of Computing | Fall-2024 | Islamabad Campus

MT1004 – Linear Algebra

Homework # 4

Question # 1

Either show that the given set, H, is a vector space, or find a specific example to the contrary, where a, b and c are any real numbers.

i) $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 0 \right\}$

The set H is the set of all solutions to the homogeneous system of equations $a + b + c = 0$. Thus $H = \text{Nul}\, A$, where $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$. Thus H is a subspace of $\mathbb{R}^3$.

ii) $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 1 \right\}$

Not a subspace since there isn't any zero vector in H.

iii) $H = \left\{ \begin{bmatrix} a - 2b \\ c \end{bmatrix} \right\}$

$H = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$. So it is a subspace of $\mathbb{R}^2$.

iv) $H = \left\{ \begin{bmatrix} a - 2 \\ c \end{bmatrix} \right\}$

Not a subspace since there isn't any zero vector in H.

Question # 2

Let $A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$.

i) Is $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ in Col A? Is $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ in Nul A?

With four entries, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ could not possibly be in Col A, since Col A is a subspace of $\mathbb{R}^2$.

$$A\boldsymbol{x} = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ is not a solution of $A\boldsymbol{x} = \mathbf{0}$, so $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ is not in Nul A.

ii) Is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ in Col A? Is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ in Nul A?

Let $\boldsymbol{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\left[\begin{array}{cccc:c} 1 & -3 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 & -1 \end{array}\right]$$

The system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent, so $\boldsymbol{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is in Col A.

With two entries, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ could not possibly be in Nul A, since Nul A is a subspace of $\mathbb{R}^4$.

iii) Find the vectors that span Nul A.

$$\tilde{A} = \left[\begin{array}{cccc|c} 1 & -3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right]$$

$$3x_3 + 0x_4 = 0$$

$$\boxed{x_3 = 0}$$

$$x_1 - 3x_2 + 2x_3 + 0x_4 = 0$$

$$x_1 - 3x_2 + 2(0) + 0x_4 = 0$$

$$\boxed{x_1 = 3x_2 - 0x_4}$$

$$x_2 = x_2$$
$$x_4 = x_4$$

Free variables

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Set $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ will span Nul A.