EXERCISE – 6.1

INFERENCE ABOUT POPULATION MEAN WHEN SIGMA IS KNOWN

1. For each of the following, X follows a normal distribution with unknown mean and known standard deviation σ. A random sample of size n is taken from the population of X and the sample mean, $\bar{x}$, is calculated. For each of the following scenarios calculate the following for a given level of significance α. Test the Hypothesis Using

- Critical Value Approach
- P-Value Approach
- Confidence Interval Approach

$(a)\ n=30,\ \bar{x}=15.2,\ \sigma=3,\ \alpha=5\%,\ H_0:\mu=15.8,\ H_1:\mu\neq15.8$

$(b)\ n=10,\ \bar{x}=27,\ \sigma=1.2,\ \alpha=1\%,\ H_0:\mu\leq26.3,\ H_1:\mu>26.3$

$(c)\ n=49,\ \bar{x}=125,\ \sigma=4.2,\ \alpha=10\%\ H_0:\mu\geq123.5,\ H_1:\mu<123.5$

Answers

$(a)\ z=-1.095,\ P-value=0.2758,\ CI:(14.13\leq\mu\leq16.27),\ Conclusion:\ do\ not\ reject\ H_0)$

$(b)\ z=1.845,\ P-value=0.0325,\ CI:(\mu\geq26.11),\ Conclusion: do\ not\ reject\ H_0)$

$(c)\ z=2.5,\ P-value=0.9937,\ CI:(\mu\leq125.77),\ Conclusion: do\ not\ reject\ H_0)$

2. Assume the population is normally distributed. Test the claim about the population mean μ at the level of significance α using

- Critical Value Approach
- P-Value Approach
- Confidence Interval Approach

$(a)\ Claim:\ \mu=40,\ \alpha=0.05,\ \sigma=1.97,\ Sample\ Statistics:\ \bar{x}=39.2,\ n=25$

$(b)\ Claim:\ \mu>10,\ \alpha=0.01,\ \sigma=3,\ Sample\ Statistics:\ \bar{x}=12,\ n=36$

$(c)\ Claim:\ \mu\geq1475,\ \alpha=0.07,\ \sigma=29,\ Sample\ Statistics:\ \bar{x}=1468,\ n=26$

$(d)\ Claim:\ \mu\leq22500,\ \alpha=0.01,\ \sigma=1200,\ Sample\ Statistics:\ \bar{x}=23500,\ n=45$

$(e)\ Claim:\ \mu<220,\ \alpha=0.1,\ \sigma=15,\ Sample\ Statistics:\ \bar{x}=215,\ n=64$

$(f)\ Claim:\ \mu\neq5880,\ \alpha=0.03,\ \sigma=413,\ Sample\ Statistics:\ \bar{x}=5771,\ n=67$

Answers

$(a)\ z=-2.03.,\ P-value=0.0424,\ CI:(38.43.13\leq\mu\leq39.97), Conclusion:\ reject\ H_0$

$(b)\ z=4,\ P-value=0.000031,\ CI:(\mu\geq10.84),\ Conclusion: reject\ H_0$

$(c)\ z=-1.23,\ P-value=0.1093,\ CI:(\mu\leq1476.42),\ Conclusion: do\ not\ reject\ H_0$

$(d)\ z=5.59,\ P-value=0.0000,\ CI:(\mu\geq23083.19),\ Conclusion:\ reject\ H_0$

$(e)\ z=-2.67,\ P-value=0.0038,\ CI:(\mu\leq217.4),\ Conclusion:\ reject\ H_0$

$(f)\ z=-2.16,\ P-value=0.9692,\ CI:(5661.51\leq\mu\leq5880.49), Conclusion:\ do\ not\ reject\ H_0$

3. The mean lifetime of electric light bulbs produced by a company has in the past been 1120 hours with a standard deviation of 125 hours. A sample of 100 electric bulbs recently chosen from a supply of newly produced bulbs showed a mean lifetime 0f 1070 hours. Company claimed that lifetime of bulbs has not changed.
Test the claim at 5% level of significance using
(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach
$(Ans: H_0: \mu = 1120 \quad H_1: \mu \neq 1120,\ z = -4,\ P-value = 0.0002, CI: (1045.5 \le \mu \le 1094.5),\ Conclusion: reject\ H_0\ )$

4. A random sample of 100 medical school applicants at a university has a mean total score of 502 on the MCAT. According to a report, the mean total score for the school's applicants is more than 499. Assume the population standard deviation is 10.6. At $\alpha = 0.01$, is there enough evidence to support the report's claim? Test the claim at 1% level of significance using
(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach
$(Ans: H_0: \mu \le 499,\ H_1: \mu > 499,\ z = 2.83, P-value = 0.0023,\ CI: (\mu \ge 499..53), Conclusion: reject\ H_0\ )$

5. The mean salary of federal government employees on the General Schedule is \$59,593. The average salary of 30 state employees who do similar work is \$58,800 with $\sigma = \$1500$. At the 0.01 level of significance, can it be concluded that state employees earn on average less than federal employees?
Test the claim at 1% level of significance using
(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach
$(Ans: H_0: \mu \ge 59593, H_1: \mu < 59593, z = -2.89, P-value = 0.0019,\ CI: (\mu \le 59438.09),\ Conclusion: reject\ H_0\ )$

6. An LED lamp manufacturer guarantees that the mean life of a certain type of LED lamp is at least 25,000 hours. A random sample of 49 LED lamps has a mean life of 24,800 hours. Assume the population is normally distributed and the population standard deviation is 500 hours. At $\alpha = 0.05$, do you have enough evidence to reject the manufacturer's claim? Test the claim at 5% level of significance using
(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach
$(Ans: H_0: \mu \ge 25000,\ H_1: \mu < 25000, z = -2.8, P-value = 0.0026,\ CI: (\mu \le 24917.86),\ Conclusion: \text{Re}\ ject\ H_0\ )$

7. A random sample of 9 values from a normal population gave the values $66, 68, 59, 67, 64, 66, 63, 62, 61$. Discuss the suggestion that the mean in the population is 60, given that population standard deviation is 3 i.e. $\sigma = 3$. At 5% level of significance Test the hypothesis using
(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach

$(Ans: H_0: \mu = 60,\ H_1: \mu \neq 60,\ z = 4,\ CI: (62.04 \le \mu \le 65.96),\ P-value = 0.0002,\ Conclusion: reject\ H_0\ )$

8. From the previous records it is known that the duration of treating a disease by a standard medicine has a mean of 15 days and a standard deviation of 3 days. It is claimed that a new medicine can reduce the treatment time, To test this claim, the new medicine is tried on 70 patients and their mean time of recovery is recorded to be 14.5 days. Test the claim at 5% level of significance using

(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach

$(Ans\ \ H_0 : \mu \ge 15, H_1 : \mu < 15,\ z = -1.394,\ CI : (\mu \le 15.09),\ P - value = 0.0816,\ Conclusion : do\ not\ \ reject\ H_0)$

9. The breaking strength of cables produced by a company has a mean of 1800 pounds and standard deviation of 100 pounds. By a new technique in the production process, it is claimed that the breaking strength can be increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850 pounds. Test the claim at 1% level of significance using

(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach

$(Ans\ \ H_0 : \mu \le 1800,\ H_1 : \mu > 1800,\ z = 3.54,\ CI : (\mu \ge 1817.04),\ P - value = 0.0002,\ Conclusion :\ reject\ H_0)$

10. A recent national survey found that high school students watched an average of 6.8 movies per month with a population standard deviation of 1.8. The distribution of number of movies watched per month follows the normal distribution. A random sample of 36 college students revealed that the mean number of movies watched last month was 6.2. At the 0.05 significance level, can we conclude that college students watch fewer movies a month than high school students? Can we support the claim at 5% level of significance using

(a) Critical Value Approach
(b) P-Value Approach
(c) Confidence Interval Approach

$(Ans\ \ H_0 : \mu \ge 6.8,\ \ H_1 : \mu < 6.8,\ z = -2,\ CI : (\mu \le 5.705),\ \ P - value\ = 0.0228,\ Conclusion :\ reject\ H_0)$