COMBINED EVENTS

MULTIPLICATION LAW OF PROBABILITY FOR DEPENDENT EVENTS

If 'A' and 'B' are two events belonging to the sample space 'S', than.

$$P(A \cap B) = P(A).\ P(B/A).$$

or $$P(A \cap B) = P(B).\ P(A/B).$$

It states that Probability that both A and B occurs is equal to the Probability that 'A' occurs multiplied by the probability (conditional) that 'B' occurs given that 'A' has already occured.

EXAMPLES

EXAMPLE-1 Two cards are drawn without replacement from an ordinary deck of 52 playing cards, what is the Probability that both cards drawn are aces.

SOLUTION Here $n(S) = 52$

Let 'A_1' denote the event that first card is an Ace.
'A_2' " " " " 2^{ND} " " " Ace.

$P(A_1 \cap A_2) = ?$

$P(A_1 \cap A_2) = ?$

Now $P(A_1 \cap A_2) = P(A_1).\ P(A_2/A_1)$

$$= \frac{4}{52} \cdot \frac{3}{51}$$

$$= \frac{1}{221}$$

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EXAMPLE-2 Two cards are drawn from a Pack of 52 playing cards. Find the Probability that second card is a heart card.

SOLUTION Here $n(S) = 52$

Let 'H_1' denote the event that Heart card in I^{st} Draw.
and 'H_2' " " " " " " " 2^{ND} " .

Now $P(H_2) = P(H_1 \cap H_2) + P(H_1' \cap H_2)$

$= P(H_1) P(H_2/H_1) + P(H_1').P(H_2/H_1')$

$= \left(\frac{13}{52}\right)\left(\frac{12}{51}\right) + P\left(\frac{39}{52}\right)\left(\frac{13}{51}\right)$

$= \frac{1}{4}$

EXAMPLE-3 A Box contains 3 white and 2 Black balls. Two balls are drawn in succession. Find the Probability that both balls drawn are black balls when the balls are not replaced after being drawn.

SOLUTION White = 3 Total = 5, $n(S) = 5$
Black = 2

Let 'B_1' denote the event that First ball is black ball.
Let 'B_2' denote the event " 2^{ND} " " " " ".

Now $P(B_1 \cap B_2) = ?$

So $P(B_1 \cap B_2) = P(B_1).P(B_2/B_1)$

$= \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$

$= \frac{2}{20} = \frac{1}{10}$

EXAMPLE-4 A Box contains 15 items, 4 of which are defective and 11 are good. Two items are selected what is the probability that first is good and second defective.

SOLUTION Defective = 4
Good = 11
total = 15 , $n(S) = 15$

Let 'G' denote the event that selected is Good.
and 'D' " " " " " " Defective.

$$P(G \cap D) = ?$$

$$\text{So } P(G \cap D) = P(G)P(D/G)$$
$$= \left(\frac{11}{15}\right)\left(\frac{4}{14}\right) = \frac{44}{210} = \frac{22}{105}$$

GENERALIZED MULTIPLICATION RULE.

If in an experiment the events $A_1, A_2, \ldots, A_k$ can occur, then

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots A_k) = P(A_1).P(A_2/A_1).P(A_3/A_1 \cap A_2) \ldots\ldots P(A_k | A_1 \cap A_2 \ldots A A_{k-1})$$

EXAMPLE-⑤ Three cards are drawn in succession, without replacement from an ordinary deck of playing cards. Find the Probability that the first card is a red ace, the second card is a ten or jack and the third card is greater than 3 but less than 7.

SOLUTION let A_1: the first card is Red Ace.
A_2: the second card is 10 or Jack.
A_3: the third card is greater than 3

$$\text{Now } P(A_1 \cap A_2 \cap A_3) = P(A_1).P(A_2/A_1).P(A_3 | A_1 \cap A_2)$$
$$= \left(\frac{2}{52}\right).\left(\frac{8}{51}\right).\left(\frac{12}{50}\right) = \frac{8}{5525}$$

COMBINED EVENTS EXAMPLES

EXAMPLE-1 Events A and B are independent and $P(A) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{12}$, Find

(a) $P(B)$

(b) $P(A \cup B)$.

SOLUTION

(a) Since A and B are independent Events.

$$P(A \cap B) = P(A).P(B)$$

$$\frac{1}{12} = \frac{1}{3}.P(B)$$

$$\boxed{P(B) = \frac{1}{4}}$$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2} \Rightarrow \boxed{P(A \cup B) = \frac{1}{2}}$$

EXAMPLE-2 X and Y are two events such that $P(X|Y) = 0.4$, $P(Y) = 0.25$ and $P(X) = 0.2$, Find

(a) $P(Y|X)$. (b) $P(X \cup Y)$

SOLUTION (a) As $P(Y|X).P(X) = P(X|Y).P(Y)$

$$P(Y|X)(0.2) = (0.4)(0.25)$$

$$P(Y|X) = \frac{(0.4)(0.25)}{(0.2)}$$

$$= 0.5 = 0.5$$

(b) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= 0.2 + 0.25 - P(X \cap Y) \longrightarrow ①$$

As $P(X|Y) = 0.4$

$$P(X \cap Y) = (0.4).P(Y) = (0.4)(0.25) = (0.1)$$

So $P(X \cup Y) = 0.2 + 0.25 - 0.1 = 0.35$

EXAMPLE - 3 Two events A and B are such that $P(A) = 0.45$, $P(B) = 0.35$ and $P(A \cup B) = 0.7$, Find

(a) $P(A \cap B)$

(b) $P(A/B) + P(B/A)$

SOLUTION (a) WKT $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.7 = 0.45 + 0.35 - P(A \cap B)$$

$$P(A \cap B) = 0.45 + 0.35 - 0.7$$

$$\boxed{P(A \cap B) = 0.1}$$

(b) $P(A|B) + P(B|A) = \dfrac{P(A \cap B)}{P(B)} + \dfrac{P(A \cap B)}{P(A)}$

$$= \frac{0.1}{0.35} + \frac{0.1}{0.45} = 0.51$$

EXAMPLE-4 Three events E_1, E_2 and E_3 are defined in the Sample Space. The events E_1 and E_3 are Mutually Exclusive. The events E_1 and E_2 are independent. Given that $P(E_1) = \frac{2}{5}$, $P(E_3) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{5}{8}$ Find

(a) $P(E_1 \cup E_3)$

(b) $P(E_2)$

SOLUTION (a) $P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3)$

$$= \frac{2}{5} + \frac{1}{3} - 0 \quad (\text{AS } E_1, E_3 \text{ are ME})$$

$$= \frac{11}{15}$$

(b) $P(E_2) = ?$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\frac{5}{8} = \frac{2}{5} + P(E_2) - P(E_1) \cdot P(E_2)$. (AS E_1, E_2 are independent)

$\frac{5}{8} - \frac{2}{5} = P(E_2)(1 - P(E_1))$

$\frac{9}{40} = P(E_2)(1 - \frac{2}{5}) \Rightarrow P(E_2) = \frac{9}{40} \times \frac{5}{3} = \frac{3}{8}$

EXAMPLE-5 The events A and B are independent and are such that $P(A) = x$, $P(B) = x + 0.2$, $P(A \cap B) = 0.15$, Find

(a) x

(b) $P(A \cup B)$

(c) $P(A \cap B')$

(d) $P(B \cap A')$

(e) $P(A' | B')$

SOLUTION. (a) $P(A \cap B) = P(A).P(B)$ (As independent)

$$0.15 = (x)(x + 0.2)$$

$$x^2 + 0.2x - 0.15 = 0 \Rightarrow x = 0.3, -0.5$$

Since $x = -0.5$ (Not Possible) $\Rightarrow \boxed{x = P(A) = 0.3}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B) = x + 0.2$

$P(B) = 0.3 + 0.2 = 0.5$

$$= 0.3 + 0.5 - 0.15$$

$$P(A \cup B) = 0.65$$

(c) $P(A \cap B') = P(A) - P(A \cap B)$ or $P(A).P(B') = (0.3)(1 - 0.5)$

$= (0.3)(0.5)$

$= 0.15$

$$= 0.3 - 0.15$$

$$P(A \cap B') = 0.15$$

(d) $P(B \cap A') = P(B).P(A')$

$$= (0.5)(1 - 0.3)$$

$$= (0.5)(0.7) = 0.35$$

(e) $P(A' | B')$

$= P(A')$ as (A and B are independent)

$= 1 - P(A)$

$= 1 - 0.3$

$= 0.7$