PRACTICE QUESTIONS EXPECTATION AND VARIANCE OF RANDOM VARIABLES

EXERCISE – 3.5

EXPECTATION AND VARIANCE OF DISCTRETE RANDOM VARIABLES

1. (a) The probability distribution for the random variable X is shown in the table.

x	0	1	2	3	4
$P(X = x)$	1/6	1/12	1/4	1/3	1/6

    Find $E(X)$.

    (b) The discrete random variable X has p.d.f $P(X = x) = kx \ for \ x = 1,2,3,4,5$
    Where k is constant. Find $E(X)$.

    $(Ans : 2.25, 11/3)$

2. A bag contains 5 black counters and six red counters. Two counters are drawn , one at a time, and not replaced. Let X be the number of red counters drawn.
    Find $E(X)$.

    $(Ans : 12/11)$

3. A

4. A discrete random variable X can take values 10 and 20 only. If $E(X) = 16$. write out the probability distribution of X.
    $(Ans : see table)$

x	10	20
$P(X = x)$	0.4	0.6

5. The discrete random variable X can take values $0, 1, 2 \ and \ 3 \ only$. Given that
    $P(X \le 2) = 0.9$, $P(X \le 1) = 0.5$ and $E(X) = 1.4$, find
    (a) $P(X = 1)$
    (b) $P(X = 0)$
    $(Ans : 0.3, 0.2)$

6. The table below shows the probability distribution for a random variable X.

x	0	1	2	3
$P(X = x)$	c	c^2	$c^2 + c$	$3c^2 + 2c$

    Calculate
    (a) c
    (b) $E(X)$
    $(Ans : 0.2, 2.08)$

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7. The discrete random variable X has p.d.f $P(X = x)$ for $x = 1, 2, 3$

x	1	2	3
$P(X = x)$	0.2	0.3	0.5

 Find
(a) $E(X)$
(b) $E(X^2)$
(c) $Var(X)$
($Ans : 2.3, 5.9, 0.61$)

8. The discrete random variable X has p.d.f $P(X = x) = k$ for $x = 1, 2, 3, 4, 5, 6$. Find
(a) $E(X)$
(b) $E(X^2)$
(c) $E(3X + 4)$
(d) $Var(X)$
(e) $Var(3X + 4)$
($Ans : 3.5, 15.67, 14.5, 2.92$)

9. The discrete random variable X has probability function given by

$$p(x) = \begin{cases} \left(\dfrac{1}{2}\right)^x & x = 1,2,3,4,5 \\ c & x = 6 \\ 0 & otherwise \end{cases}$$

Where c is a constant, than find
(a) the value of c
(b) mean of X
(c) Variance of X.
($Ans : 1/32, 1.96875, \qquad$)

10. The discrete random variable X has p.d.f $P(X = x) = k|x|$ where x takes the values $x = -3, -2, -1, 0, 1, 2, 3$, Than find
(a) the value of k
(b) $E(X)$
(c) $E(X^2)$
(d) Standard deviation of X
(e) $E(2X - 3) + Var(2X - 3)$
($Ans : 1/12, 0, 6, 2.45, \qquad$)

EXERCISE – 3.6

EXPECTATION AND VARIANCE OF CONTINUOUS RANDOM VARIABLE

1. Find $E(X)$ for each of the following continuous random variables.

    (a) $f(x) = \begin{cases} \dfrac{3}{4}(x^2 + 1) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$

    (b) $f(x) = \begin{cases} \dfrac{3}{4}x(2 - x) & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$

    (c) $f(x) = \begin{cases} \dfrac{3}{8} & \dfrac{2}{3} \le x < 2 \\ \dfrac{3}{32}x(4 - x) & 2 \le x \le 4 \\ 0 & otherwise \end{cases}$

    $(Ans : 9/16, 1, 49/24)$

2. If X is a continuous random variable with p.d.f $f(x)$. Find $E(X)$

    (a) $f(x) = \begin{cases} k\,x^2 & 0 \le x \le 4 \\ 0 & otherwise \end{cases}$

    (b) $f(x) = \begin{cases} k\,x^3 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$

    $(Ans : 3, 1.6)$

3. The continuous random variable X has p.d.f $f(x)$ where

    $$f(x) = \begin{cases} k\,x & 0 \le x < 1 \\ k & 1 \le x < 3 \\ k(4 - x) & 3 \le x \le 4 \\ 0 & otherwise \end{cases}$$

    (a) Find k
    (b) Find $E(X)$
    $(Ans : 1/3, 2)$

4. A random variable X has probability density function $f(x)$ given by

    $$f(x) = \begin{cases} c\,x(5 - x) & 0 \le x \le 5 \\ 0 & otherwise \end{cases}$$

    Show that $c = 6/125$ and find the mean of X. $(Ans : 2.5)$

5. A random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{c}{3}x & 0 \le x < 3 \\ c & 3 \le x \le 4 \\ 0 & otherwise \end{cases}$$

Than find
(a) the value of c
(b) the mean of X
(c) the value of a for there to be a probability of 0.85 that a randomly observed value of X will exceed a.
$(Ans : 0.4, 2.6, 1.5)$

6. Given the p.d.f of the function is denoted by $f(x)$. Than find the following
(a) $E(X)$ (b) $E(X^2)$ (c) $Var(X)$ (d) standard deviation of X for

(a) $f(x) = \begin{cases} \dfrac{3}{8}x^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$

$(Ans : 1.5, 2.4, 0.15, 0.387)$

(b) $f(x) = \begin{cases} \dfrac{1}{4}(4 - x) & 1 \le x \le 3 \\ 0 & otherwise \end{cases}$

$(Ans : 11/6, 11/3, 11/36, 0.553)$

(c) $f(x) = \begin{cases} \dfrac{3}{56}(x + 2)^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$

$(Ans : 17/14, 62/35, 291/980, 0.545)$

7. A random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{k}{x(4 - x)} & 1 \le x \le 3 \\ 0 & otherwise \end{cases}$$

(a) Show that $k = \dfrac{2}{\ln 3}$

(b) Calculate the mean and variance of X.

$(Ans : 2, 4 - \dfrac{4}{\ln 3})$

8. The continuous random variable X has p.d.f $f(x)$ given by where

$$f(x) = \begin{cases} \dfrac{1}{27}x^2 & 0 \le x < 3 \\[2mm] \dfrac{1}{3} & 3 \le x \le 5 \\[2mm] 0 & otherwise \end{cases}$$

(a) Find $E(X)$

(b) Find $E(X^2)$

(c) Find Standard Deviation of X.

$(Ans : 41/12, 571/45, 1.008)$

9. A random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} kx & 0 \le x < 1 \\ k(2-x) & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

Than find

(a) The value of k

(b) $E(X)$

(c) $Var(X)$

(d) $P\left(\dfrac{3}{4} \le X \le \dfrac{3}{2}\right)$

(e) The Mode

$(Ans : 1, 1, 1/6, 19/32, 1)$

10. The probability density function of a random variable X is given by

$$f(x) = \begin{cases} k\left(ax - x^2\right) & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

(a) Show that $a \ge 2$ and that $k = \dfrac{3}{6a - 8}$

(b) Given that $E(X) = 1$, calculate the value of a and k.

(c) Find $Var(X)$

$(Ans : a = 2, k = 0.75, \text{var}(x) = 0.2)$