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Parameter Evaluation

Assignment - 6

(1.) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Let $x_1, x_2, — x_n$ by sample of size n

$$L(x_1, x_2, x_3 — x_n) = f(x_1) \cdot f(x_2) — f(x_n)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}$$

taking ln on both sides

$$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad (1)$$

Taking partial derivative wrt μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence $\boxed{\theta_1 = \bar{x}}$ is sample mean

Taking derative of eq (1) w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i - \mu)^2}{2\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i - \mu)^2\right)$$

hence $\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2}$

(2.) $L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log({}^{n}C_{x_i}) + \log \theta^{x_i} + \log(1-\theta)^{n-x_i} \right)$$

Diff w.r.t θ

$$\frac{\partial \log(L)}{\partial \theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{1}{1-\theta} \Sigma (n - x_i) = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \Sigma x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\Sigma x_i}{n^2}}$$