UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Assignment 3
Spring 2022

Lab 3: Spread of memes on a network

The deadline for this sheet is midnight 16th of May.

Please submit hand-ins on studium. All code should be included or uploaded to github. Please feel free to submit videos illustrating your results where appropriate, via studium or uploaded elsewhere such as vimeo or youtube. You may work in groups of size 1-5, and only one group member needs to submit the assignment. State clearly the members of the group. This exercise will be covered in lab session on 4th of May.

Spread of memes

Consider the following model of the spread of Internet memes. There are three states: resting (R), sharing (S) and bored (B). We will consider various rules for how memes spread between individuals. Use discrete time steps in the model.

1. Lets first assume the following rules for spreading of memes for each time step of the model.

   Resting With probability p a resting person will discover a new meme by themselves and become a sharer.

   Sharer With probability q a sharer will pick one person completely at random from the population. If that person is resting then they will now become a sharer. However, if the person they pick is bored, then the sharer will lose interest and become bored too.

   Bored With probability r a bored person will pick one person completely at random from the population. If that person is resting then the bored person will now become resting, otherwise they will continue to be bored.

   Write a simulation of this model and run it with $N = 1000$ people of whom only one is bored (i.e. $B(0) = 1$) and only one who is sharing ($S(0) = 1$). Simulate this model for different parameter values and describe the types of dynamics that can arise. Start by choosing a very small value of p. (e.g. $p = 0.001$, $q = 0.01$, $r = 0.01$).

   1.1. Either by plotting single runs of the model, or otherwise. Show that the model can exhibit at least two behaviours depending on the parameters p, q, r (or possibly also on numbers of bored, sharing and resting initially). **(2 points)**

1.2. Use heatmaps to show phase transitions in the system: i.e. varying a parameter (or function of parameters) the probability of the system showing certain behaviours changes from one mode to another. Find two such transitions and plot them using heatmaps (or another graphical format if you can find one). You may consider your own choice of parameters or one or two of the following.

- varying q and plotting the proportion of runs with certain numbers of sharers after a large time T (e.g. $p = 0.001$, $r = 0.01$, $T = 4000$ with 1 sharer, 1 bored, $N - 2$ resting initially and for q between 0 and 0.1).
- varying p and plotting the proportion of runs with certain numbers of sharers after a large time T (e.g. $q = 0.01$, $r = 0.01$, $T = 2000$ for p between 0 and 0.001).

(2 points)

2. The idea of this question is to compare the behaviour of the memes model on two different networks. You may choose those networks to be a $2d$ square lattice and a real network (and the question is written in this form). However you may also choose a different pair of networks e.g. a $2d$ lattice vs. the same lattice with some long range edges added or a real network vs. that same network with the node of maximum degree removed (or top 5 nodes removed).

   Implement the same model as the one above, but on (a) a two dimensional lattice (either square lattice, or another lattice of your choosing) and (b) a real network with at least 100 vertices (you may choose to modify a real network you find, or choose a subset of it - e.g. the largest connected component). When individuals pick people to interact with, they pick (still randomly) only from their neighbours[1] and not from the whole population. For the lattice use periodic boundary conditions, so that cells interact over the left and right and the top and bottom boundaries. Investigate properties of this model and write a short report, including pictures and/or film, explaining the patterns the model generates and explaining possible reasons/heuristics for why different network structures effect the model outcome. **(6 points)**

[1] In a graph we say that u is a neighbour of v if the edge/link between u and v is in the graph