

No,  $L$  is not context-free. Assume for a contradiction that it is. By the pumping lemma for context-free languages,  $L$  has a pumping length  $p$ . Let  $w \in L$  be  $a^p b^{2p} a^p$ . Clearly,  $|w| \geq p$ . By the pumping lemma,  $w$  can be split into  $uvxyz$  such that  $|vy| > 0$ ,  $|vxy| \leq p$  and  $uv^i xy^i z \in L$  for any non-negative integer  $i$ . Use case distinction to show that this is impossible.

Case  $\#_a(vy) = \#_b(vy)$ :

Since there are  $2p$  b's in the middle of  $w$  and  $|vxy| \leq p$ ,  $vy$  can't span a's on both sides of the b's. So taking  $i = 0$  will result in the string  $uxz$ , which will either start with less than  $p$  a's or end with less than  $p$  a's and thus can't start with the same amount of a's as it ends with, and thereby can't be in  $L$ , which is a contradiction.

Case  $\#_a(vy) \neq \#_b(vy)$ :

Taking  $i = 0$  will result in the string  $uxz$ , which can't have the same number of a's and b's since  $w$  had the same number of a's and b's and  $vy$  did not. So  $uxz$  is not in  $L$ , which is a contradiction.

Clearly, these were all cases and both lead to a contradiction, so the assumption that  $L$  is context-free must have been false. Thus  $L$  is not a context-free language.