Assume for a contradiction that L is context-free. Then, by the pumping lemma for context-free languages, there exists a pumping length p. Let $w \in L$ be $a^{p+1}b^pc^{p+1}$. Clearly, $|w| \geq p$. By the pumping lemma, w can be split into $uvxyz$ such that $|vxy| \leq p$, $|vy| > 0$ and $uv^ixy^iz \in L$ for any non-negative integer i. Use case distinction on vy to show that this is not possible:
Case vy contains only a's:
Taking $i = 0$ will result in $uv^0xy^0z = uxz$ which will have less than $p + 1$ a's, thus at most p a's. However, the number of a's has to be greater than the number of b's, but there are p b's in uxz.
Case vy contains only b's:
Taking $i = 2$ will result in $uv^2xy^2z = uvvxyyz$ which will have more than p b's and thus at least $p + 1$ b's. However, the number of a's $(p + 1)$ must be greater than the number of b's.
Case vy contains only c's:
Taking $i = 0$ will result in $uv^0xy^0z = uxz$ which will have less than $p + 1$ c's, thus at most p c's. However, the number of c's has to be greater than the number of b's, but there are p b's in uxz.
Case vy contains multiple different characters:
Taking $i = 2$ will cause $uvvxyyz$ to be out of order, so it can't be in L.
Since there are only 3 possible characters and $|vy| > 0$, these were all cases. Since each case lead to a contradiction, the assumption that L is context-free must have been false. Thus L is not context-free.