

1 a

To show that $A \leq_P B$, one gives a function f that always runs in polynomial time such that $w \in A$ iff $f(w) \in B$ for any $w \in \Sigma^*$.

2 b

For this exercise, I will consider the set of all cnf formulas as the alphabet. Define f in the following way:

On input ϕ :

Add a fresh variable y .

Add two new clauses to ϕ : 1 containing only literal y and 1 containing only literal \bar{y} .

Return the modified ϕ .

Trivially, f will run in polynomial time. To prove that this reduction works, consider an arbitrary cnf formula ϕ and its corresponding $f(\phi)$. For any assignment of truth values, $f(\phi)$ will have one extra clause evaluating to true and one extra clause evaluating to false, namely the y and \bar{y} clause (not necessarily respectively). So $f(\phi)$ will always have at least 1 clause evaluating to false and it will have exactly 1 clause evaluating to false iff ϕ has none.

This proves that the reduction works and thus $\text{cSAT} \leq_P \text{PREsSAT}$.