Let 3cnf formula $\phi = \wedge_{i=1}^{m} \phi_i$ over the variables $v_1, ..., v_n$. I will construct a SOLITAIRE board B such that B is winning iff ϕ is satisfiable.
B will be an m x n board (m rows and n columns) with the following configuration: For every slot (x, y) on B with $1 \leq x \leq m$ and $1 \leq y \leq n$:
-Iff clause ϕ_x contains literal v_y and not $\bar{v_y}$, the slot will start with a blue stone.
-Iff clause ϕ_x contains literal $\bar{v_y}$ and not v_y, the slot will start with a red stone.
-Iff clause ϕ_x contains neither v_y nor $\bar{v_y}$, the slot will start empty.
-Iff clause ϕ_x contains both v_y and $\bar{v_y}$, row x will be removed from B.
To prove that indeed ϕ is satisfiable iff B is winning, consider what it means to reach an end configuration. In an end configuration, all rows have at least 1 stone, which means that at least 1 literal in its corresponding clause evaluates to true. Also, every column contains at most 1 color. If that color is blue, the corresponding variable should be set to true. If that color is red, the corresponding variable should be set to false. If the column doesn't contain any stones, the truth value of the variable doesn't matter. So if a column contains at most 1 color, the corresponding variable needs only a single truth value, which can be assigned and thus this leads to a possible satisfying assignment.
If B is not winning, it is not possible to reach any end configuration thus it is not possible to reach any satisfying assignment and thus ϕ is not satisfiable.
The construction of B should take O($m * n$) time, which is polynomial time. (It doesn't matter which of m and n you consider as variable and which as constant).