

Let $\langle S, t \rangle$ be an arbitrary input for the SUBSET-SUM problem. Let $U = S \cup \{\text{sum}(S) - 2t\}$ where $\text{sum}(S)$ denotes the sum over the elements of S . Then $\langle S, t \rangle \in \text{SUBSET-SUM}$ iff $\langle U \rangle \in \text{SET-PARTITION}$ and thus $\text{SUBSET-SUM} \leq_P \text{SET-PARTITION}$.

Trivially, this reduction will run in polynomial time as it does nothing more than adding a single element to a set. I will prove that this reduction also works by proving both directions separately. First I will proof $\langle S, t \rangle \in \text{SUBSET-SUM} \rightarrow \langle U \rangle \in \text{SET-PARTITION}$:

Let S' be the subset of S such that $\sum_{x \in S'} x = t$. Then let U' be $S' \cup \{\text{sum}(S) - 2t\}$ so U/U' will be S/S' . Since the sum over S' equals t , the sum over U' will be $t + \text{sum}(S) - 2t = \text{sum}(S) - t$. Also, the sum over $U/U' = S/S'$ equals $\text{sum}(S) - t$. Thus indeed $\langle S, t \rangle \in \text{SUBSET-SUM} \rightarrow \langle U \rangle \in \text{SET-PARTITION}$.

Now the other direction: $\langle U \rangle \in \text{SET-PARTITION} \rightarrow \langle S, t \rangle \in \text{SUBSET-SUM}$:

Let U' be a subset of U such that $\sum_{x \in U'} x = \sum_{x \in U/U'} x$ and $\text{sum}(S) - 2t \in U'$. Such a U' must exist because $\langle U \rangle \in \text{SET-PARTITION}$ and $\text{sum}(S) - 2t \in S$. Since the sum over U' is the same as the sum over U/U' , it must be that the sum over U' is half of the sum over U . So the total sum over U is $2 * \text{sum}(S) - 2t$ so the sum over U' is $\text{sum}(S) - t$. Let S' be $U' / \{\text{sum}(S) - 2t\}$. Then the sum over S' will be $\text{sum}(S) - t - (\text{sum}(S) - 2t) = \text{sum}(S) - t - \text{sum}(S) + 2t = t$. Since S' contains only elements of S (so $S' \subseteq S$) and the sum over S' equals t , it holds that indeed $\langle S, t \rangle \in \text{SUBSET-SUM}$.

With both directions of the iff proven, it indeed holds that $\langle S, t \rangle \in \text{SUBSET-SUM}$ iff $\langle U \rangle \in \text{SET-PARTITION}$ and thus $\text{SUBSET-SUM} \leq_P \text{SET-PARTITION}$.