Assume for a contradiction that L is context-free. Then, by the pumping lemma for context-free languages, there exists a pumping length $p$. Let $w \in L$ be $a^{p+1}b^pc^{p+1}$. Clearly, $|w| \geq p$. By the pumping lemma, $w$ can be split into $uvxyz$ such that $|vxy| \leq p$, $|vy| > 0$ and $uv^ixy^iz \in L$ for any non-negative integer $i$. Use case distinction on $vy$ to show that this is not possible:
Case $vy$ contains only a's:
Taking $i = 0$ will result in $uv^0xy^0z = uxz$ which will have less than $p + 1$ a's, thus at most $p$ a's. However, the number of a's has to be greater than the number of b's, but there are $p$ b's in $uxz$.
Case $vy$ contains only b's:
Taking $i = 2$ will result in $uv^2xy^2z = uvvxyyz$ which will have more than $p$ b's and thus at least $p+1$ b's. However, the number of a's $(p+1)$ must be greater than the number of b's.
Case $vy$ contains only c's:
Taking $i = 0$ will result in $uv^0xy^0z = uxz$ which will have less than $p + 1$ c's, thus at most $p$ c's. However, the number of c's has to be greater than the number of b's, but there are $p$ b's in $uxz$.
Case $vy$ contains multiple different characters:
Since the a's and c's in $w$ are separated by $p$ b's, $vy$ must contain b and either a or c, but not both. Taking $i = 2$ will generate an extra b, so $uvvxyyz$ will have at least $p+1$ b's. However, the number of a's or c's will still be $p+1$ since $vy$ can't contain both characters. Thus the number of a's or the number of c's will not be greater than the number of b's.
Since there are only 3 possible characters and $|vy| > 0$, these were all cases. Since each case lead to a contradiction, the assumption that L is context-free must have been false. Thus L is not context-free.