$\int e^{-0t} e^{-j5t} dt$

$- \frac{1}{-s} [0 - 1]).$

Mid-Semester Exam

EE341 - Communication Systems - I

Examination: September 13th, 2025, 1:30 PM to 3:30 PM. Maximum score: 35

Note: Please note the following:

(a) $u(\cdot)$ denotes the continuous-time unit step signal.

(b) The continuous-time Fourier transform of $x(t)$ is defined as

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} dt$$

(c) The continuous-time Fourier series of $x(t)$ that is periodic with period T is defined as

$$c_k = \frac{1}{T} \int_{\langle T \rangle} x(t)\, e^{-j2\pi k t/T} dt$$

(d) Note that $\text{sinc}(t) = \sin(\pi t)/(\pi t)$.

(e) The power series for $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$.

Problem 1

(6 points) Find the Fourier transform of the following signals:

(a) $\exp(-t)\sin(2\pi f_c t)u(t)$. [2]

(b) $\frac{\cos(100\pi t)\sin(\pi t)}{\pi t}$. [2]

$F(\frac{1}{\pi t}) = -j\,\text{sgn}(f)$

$f(\frac{1}{\pi t}) = \frac{1}{j\,\text{sgn}\, f}$.

$\frac{1}{\pi t} \to -j\,\text{sgn}\, f$

(c) $\sum_{k=-\infty}^{\infty} (u(t+0.25+k) - u(t-0.25+k))$. [2]

Problem 2

(6 points) Consider the signal $m_1(t) = \text{sinc}(40,000t) - \text{sinc}(40,000t - 1)$, where t is measured in seconds. Answer the questions below with regard to this $m_1(t)$.

(a) Find the continuous-time Fourier transform of $m_1(t)$. [1]

(b) Find the energy in $m_1(t)$. [2]

(c) $m_1(t)$ is transmitted using double side-band suppressed carrier modulation using the carrier $\cos(2\pi 100 \times 10^6 t)$. Find the expression for the transmitted signal. [1]

(d) The signal from (c) is demodulated at the receiver using the local oscillator $\cos(2\pi 100 \times 10^6 t + \phi)$, followed by an ideal low-pass filter with cut-off at 1 MHz. Find the output signal. [2]

$\frac{1}{1+j}$

$f_a = f_c + \frac{k_p}{2\pi} m(t)$

$\Delta f = k_f A_m = \frac{1}{2\pi} \frac{dp}{dt}$

PROBLEM 3

(5 points) For this question, note that the standard PM modulated signal is $A\cos(2\pi f_c t + k_p m(t))$, while the standard FM modulated signal is $A\cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right)$, with $m(t)$ being the message signal.

Consider the following angle modulated signal:

$$s(t) = A\cos\left(2\pi f_c t + k\int_{-\infty}^{t} \operatorname{sinc}(W\tau)\,d\tau\right).$$

$e^{wt...}$

Answer the following questions with regard to $s(t)$.

(a) If $s(t)$ is an FM modulated signal with $k = k_f$, identify $m(t)$. [1]

check. (b) If $s(t)$ is an FM modulated signal with $k = k_f$, find its transmission bandwidth using Carson's rule. [2]

(c) If $s(t)$ is a PM modulated signal with $k = k_p$, find its transmission bandwidth using Carson's rule. [2]

PROBLEM 4

(4 points) Answer the following questions:

(a) Consider the FM signal $\cos(2\pi f_i(t)\,t)$. You are given that $f_i(t) = f_c - \Delta f$ for $|t| < T/2$, and $f_i(t) = f_c$ for $|t| > T/2$. Find the Fourier transform of this signal. [2]

(b) Let $m_1(t) = \operatorname{sinc}(Wt)$, where W is a positive number. You intend to transmit $m(t)$ using quadrature amplitude modulation at carrier frequency f_c (much larger than W) along with another real-valued signal $m_2(t)$ as follows:

$$s(t) = m_1(t)\cos(2\pi f_c t) - m_2(t)\sin(2\pi f_c t).$$

The spectrum of $s(t)$, denoted as $S(f)$, is known to be non-zero only for $f \in [f_c, f_c + W/2] \cup [-f_c - W/2, -f_c]$. What choice of $m_2(t)$ could produce this? [2]

PROBLEM 5

(6 points) Consider a narrow band FM signal defined as:

$$s(t) = A\cos(2\pi f_c t) - \beta A \sin(2\pi f_m t)\sin(2\pi f_c t)$$

$A \cos \cancel{A} + B \cos \phi$

where $\sin(2\pi f_m t)$ is the message signal.

(a) Determine the envelope of $s(t)$. What is the ratio of the maximum to the minimum value of this envelope? Plot this ratio versus β for $\beta \in [0, 0.3]$. [2]

(b) Determine the average power of the narrow band FM signal $s(t)$ as a fraction of the average power of the unmodulated carrier wave. Plot this ratio versus β for $\beta \in [0, 0.3]$. [2]

(c) By expanding the phase $\theta_i(t)$ of the narrow band FM signal $s(t)$ as a power series, and keeping $\beta \in [0, 0.3]$, we get

$\operatorname{sinc}(w\tau) = \frac{e^{w\tau} - e^{-w\tau}}{2}$

$$\theta_i(t) \approx 2\pi f_c t + a_1 \sin(2\pi f_m t) + a_2 \sin^3(2\pi f_m t).$$

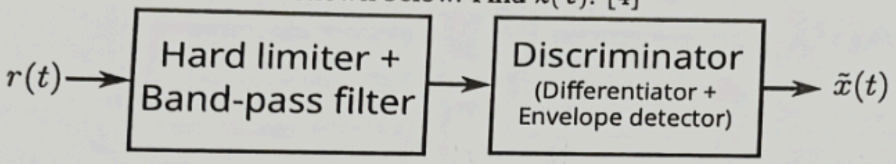
Find the values of a_1 and a_2 [2].

PROBLEM 6

(4 points) An FM radio in a car receives signals from two FM broadcasting transmitters. Thus, for a given carrier whose angular frequency is ω_c rad/s, the radio receives

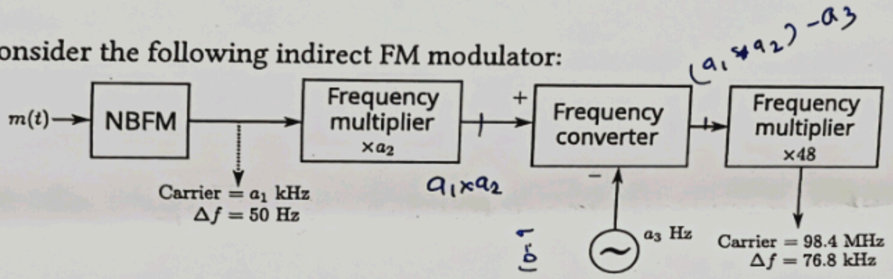
$$r(t) = A_1 \cos(\omega_c t + \phi_1(t)) + A_2 \cos(\omega_c t + \phi_2(t)).$$

The demodulator in the car radio is shown below. Find $\tilde{x}(t)$. [4]



PROBLEM 7

(4 points) Consider the following indirect FM modulator:



In the above, Δf is the maximum instantaneous frequency deviation of the signal. The frequency converter takes the input from the + terminal, and subtracts its frequency by a_3 Hz. In other words, the input $\cos(\omega_0 t + \phi)$ to this converter yields output $\cos(\omega_0 t - 2\pi a_3 t + \phi)$. Find the appropriate positive values of a_1, a_2 and a_3 to obtain the desired FM signal, given that a_3 is between 10^7 and 1.1×10^7 Hz.

$$\left[(a_1 \times 10^3 \times a_2) - a_3 \right] \times 48 = 98.4$$

$$50 \times a_2$$

$A_1 \cos \omega_1 t + A_2 \cos \omega$

$A \cos \alpha + B \cos \beta$