EE 301
Electromagnetic Waves
"Surprise" Quiz - 4
Time limit: 25 Mins

Instructions: Most questions are based on reasoning and concepts discussed in class. In case of reasoning questions, you need to describe your answer via simple mathematical models.
The maximum score is 15.

A) Fundamental vector operations:

1. Let $\mathbf{A}(\mathbf{r}) = (x^2 - y^2)\,\hat{\mathbf{x}} + 2xy\,\hat{\mathbf{y}} + z^2\,\hat{\mathbf{z}}$.

   (a) Compute $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$. [$2 \times 2 = 4$]

   (b) Verify whether $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ holds explicitly for this field. (Show calculation, not just the theorem.) [2]

2. Fill in the blanks (no explanation needed): [$4 \times 0.5 = 2$]

   (a) $\nabla \times (\nabla \phi) =$ _____

   (b) $\nabla \cdot (\nabla \times \mathbf{A}) =$ _____

   (c) $\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) =$ _____

   (d) $\nabla \times (\nabla \times \mathbf{A}) =$ _____

B) Vector Theorems in Action

1. Consider a circular loop C of radius R in the xy-plane centered at the origin.

   (a) Directly compute $\oint_C \mathbf{A} \cdot d\mathbf{l}$ for $\mathbf{A} = (-y/2)\,\hat{\mathbf{x}} + (x/2)\,\hat{\mathbf{y}}$. [2]

   (b) Compute $\nabla \times \mathbf{A}$ and then apply Stokes' theorem on the region bounded by C to evaluate the same integral. Comment on consistency. [3]

2. Match the following vector calculus identities to their physical interpretation. [$4 \times 0.5 = 2$]

	Identity	Interpretation
C	(i) $\nabla \cdot \mathbf{D} = \rho$	A. Absence of magnetic monopoles
A	(ii) $\nabla \cdot \mathbf{B} = 0$	B. Charge conservation
D	(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	C. Gauss' law for electricity
B	(iv) $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$	D. Faraday's induction law

$r\cos\theta \sin\phi \quad r\sin\theta \sin\phi \quad r\cos\phi$