## OPTIMAL PLANNING OF TRANSIT ROUTES FOR LARGE CITIES

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#### ABSTRACT

The developed system involves: selection of optimal hubs and delineation of their influence areas; estimation of demand; and generation of optimal paths and service frequencies. Fuzzy clustering and Self-organizing-map methods aggregate the stops into a number of clusters. A hub is located with the objective to minimize the 'Total Travel Distance' within a cluster. Optimal set of clusters is one that provides the minimum 'Total Passenger Time' for the network. Routing model generates reasonably direct routes with high frequency. Optimal path is selected on the basis of maximum 'Desire passenger-km per km'. Number of trips to be operated for a route depends upon the demand served along the route and the passenger flow on various links of the route. To estimate the optimal trips for a route, an iterative heuristic algorithm is adopted in the scheduling model. The devised methodology is successfully implemented for New Delhi, the capital city of India. The total daily transit demand generated at 1542 stops is 7.67 million passengers. A set of fifty optimal hubs is identified and 181 inter hub routes are generated. Eighty-eight of these inter-hub routes, which have very high demand, need to be operated with automated transit vehicles (APM). A total of 305 secondary routes (intra-hub) are also generated. Optimal scheduling is done for all the 486 transit routes. For peak period, a total fleet of 11497 standard size buses is required at the desired level of service.

#### INTRODUCTION

With the rapid urbanization in developing countries, the cities are expanding away from the central business district (CBD) area and the travel pattern of commuters is changing drastically. Due to the heavy demand, the destination oriented design methodology results in large number of very long transit bus routes. These zigzag routes overlap on certain corridors resulting in bunching on various sections of the network. This bunching leads to high concentration of buses, thereby causing

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irregular distribution of headways on the stops of the route. This type of transit network in a metropolitan city is very complex to understand, inefficient and also difficult to operate.

To minimize some of the major limitations encountered in the destination oriented bus routes, the large networks in a city area need to be designed on a different approach. Mass Rapid Transit System (Metro) is being provided only on few corridors due to high cost and long gestation period. Hub and spoke system, which combines the traditional destination oriented approach along with direction-oriented approach, is best suited for such type of large networks.

In hub and spoke system, the important terminals of the network constitute the hubs and are widely spread over the network. These hubs are inter-connected with each other through direction-oriented routes basically termed as inter-hub routes. Hubs are surrounded by stops within a certain geographical area, generally known as influence area of hubs. Stops within influence area can be connected to each other based on certain constraints of demand and length or to the hubs through secondary routes. Commuters with large trip length can avail the facility of inter-hub routes along with secondary routes. For shorter trip lengths within the influence area of a hub, only secondary routes can serve the purpose.

Inter-hub routes are reasonably direct, avoid unnecessary meandering, have high frequency of operation and reduced travel time between hubs. Provisions of Automated People Movers (like automated road transit vehicles) on some of the designed inter-hub routes improve the efficiency of operation. With the dynamic changes in the city structure and increase in population, more number of inter-hub routes can be operated with automated transit vehicles (APM).

## **DESIGN METHODOLOGY**

Transit network design is a combinatorial optimization problem involving non-linearity, non-convexity with multiple objective functions. Most of the past studies deal in routing and scheduling of bus transit networks of small to medium size cities, where routes have been constructed through destination-oriented approach. The effort is limited in case of large cities of metropolitan status.

The mathematical programming approaches (Laporte et al.1992, Spasovic and Schonfeld 1993, Nes Van and Bovy 2000) are theoretically rigorous but fail to handle any network of realistic size. Simulation of the transit system (Dhingra 1980, Santhakumar 1987), though a powerful tool, has been restricted mostly to individual routes or small size transit network. Knowledge based expert system approach (Dashora 1994) involves capturing the domain knowledge of one or more experts and using it to structure to a knowledge base. Collection of actual knowledge is extremely difficult. Non-conventional techniques (Kidwai 1998, Patnaik and Tom1998) like Genetic algorithms and neural networks though analytically optimize the system but their applicability and adaptability to real life integrated operations desires much more and leaves a big question mark on their application. Heuristic interactive graphic method (Parti 2002) that allows online interaction between the user and machine can be successfully employed in transit network design for large real life networks.

The adopted methodology for planning of transit network involves the following stages.

- (i) Selection of optimal Hubs and delineation of Influence area for each Hub
- (ii) Estimation of Inter-hub and Intra-hub demand
- (iii) Generation of the path for Inter-Hub routes
- (iv) Generation of the path for Secondary (Intra-hub) routes
- (v) Determination of service frequencies for Inter-hub routes
- (vi) Determination of service frequencies for secondary routes

# Selection of Optimal Hubs and their Influence Areas

The properties, which a hub should fulfill, are: generate high passenger demand, provide good transfer opportunities, have sufficient influence area to generate high demand, and should be separated from other hubs by a certain minimum travel distance/ travel time. Past studies on hub location (Shih et al 1998) deal the problem through intuitive procedure or on the basis of user knowledge of distribution of bus stops, the passenger demand at the bus stops and other relevant parameters. Such methods have no mathematical basis and therefore may be misleading.

In this work, optimal selection of hubs is attempted through a mathematical model and the objective is to minimize the 'Total Passenger Time' for the Transit System.

Minimize: 
$$\sum_{j=1}^{N} \sum_{i=1}^{N} dem_{ij} (t_{tt,ihb1} + t_{tt,hb1hb2} + t_{tt,hb2j} + t_{wt,i} + t_{wt,hb1} + t_{wt,hb2})$$

$$\forall i \in Inf_{hb1} \text{ and } \forall j \in Inf_{hb2}$$

where dem  $_{ij}$  is demand between node pair (i,j),  $t_{tt,ihb1}$  is travel time between stop 'i' and hub  $hb_1$ ,  $t_{tt,hb1hb2}$  is travel time between hub  $hb_1$  and hub  $hb_2$ ,  $t_{tt,hb2j}$  is travel time between hub  $hb_2$  and stop j,  $t_{wt,i}$  is waiting time at stop 'i',  $t_{wt,hb1}$  is waiting time at hub  $hb_1$ ,  $t_{wt,hb2}$  is waiting time at hub  $hb_2$ ,  $t_{wt,hb2}$  is influence area of hub  $t_1$ ,  $t_{wt,hb2}$  is influence area of hub  $t_2$ . N is total number of stops.

In a large city, the number of stops is very large and the hubs are to be selected among these stops on the basis of the generated demand as well as the required infrastructure at the stops. The hubs are to be well spread up subject to the certain operational constraints like minimum distance between the hubs. All these considerations lead to a very complex design problem and heuristic models are therefore involved in the design process.

The location of optimal hubs is generated by the following three-step procedure.

- (i) Partitioning of stops in clusters
- (ii) Location of hub within a cluster and its influence area.
- (iii) Selection of optimal hubs

### **Partitioning of Stops in Clusters**

Two clustering methods are introduced in this study to partition the stops into a predefined number of clusters. One method 'Fuzzy-c-means clustering' belongs to the class of fuzzy logic and the other 'Self-organizing-map' is chosen from the neural

networks. These clustering methods offer generalized procedure for computing the clusters center and stops present in the clusters. The data set to obtain the stops in the clusters depends on the location of bus stops and the generated demand at the bus stops.

**FuzzyC- Means Clustering.** Clustering refers to identifying the number of subclasses of c clusters in a data universe Z comprised of N data samples, and partitioning Z into c clusters  $(2 \le c < N)$ . To introduce this method, we define a set of 'N' bus stops and three features i.e x-coordinate, and y-coordinate and demand define each stop (zk).  $Z_k = \{z_{kl}, z_{k2}, z_{k3}\}$ 

Since the three features have different units, in general, we have to normalize each of the features to a unified scale before classification. In a geometric sense, each  $z_i$  is a point in three-dimensional feature space, and the universe of the bus stops sample, Z, is a point set with N elements in the sample space. The objective function is developed so as to do two things simultaneously: First, minimize the Euclidean distance between each data point in a cluster and its cluster center (a calculate point), and second, maximize the Euclidean distance between cluster centers. The objective function takes the form

$$J(Z,U,V) = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^{m} ||Z_{k} - v_{i}||^{2}$$
(1)

Where Z is the data set of bus stops consisting of feature vectors  $z_k$ ,  $k = \{0,1,2\}$  and  $U = [u_{ik}] \in [0,1]$  is the fuzzy partition matrix of Z subject to the constraints:

$$0 \le \sum_{k=1}^{n} \mu_{ik} \le N, 1 \le i \le c \quad \text{and} \quad \sum_{i=1}^{c} \mu_{ik} = 1, 1 \le k \le N$$
 (2)

The vector  $V = [v_1, v_2, ....., v_c]$ ,  $v_i \in R^n$  is a cluster of prototypes (centers), which have to be determine and  $d_{ik} = \|Z_k - v_i\|^2$  is the squared inner product norm (typically Euclidean ) and  $m \in [0,\infty]$  is weighting exponent that determines the fuzziness of the resulting clusters. The basic idea is to find those 'c' locations in the database of bus stops such that the sum of the distances from all the bus stops to those 'c' points is minimized. The term  $\mu_{ik}$ , which is an element of the partition matrix U, plays a very important role. Without this term the point of minima would simply lie at the mean of the entire distribution of bus stops. If one were to have only one cluster, then the center of cluster would simply be at the mean of the distribution of bus stops i.e.

$$v = \sum_{k=1}^{N} x_k / N$$

Since 'c' such minimum points need to be found without all the point collapsing to the mean, assigning a weight or membership to each cluster for every bus stop becomes mandatory. This is done through  $\mu_{ik}$ , which denote the membership of bus stops k in cluster 'i'. Assigning Membership values to each bus stop with respect to 'c' centers assures that all the centers do not collapse to the mean of the entire distribution of bus stops. Minimization of the objective function (1) with respect to  $v_i$ 's yields the clusters and minimizing with respect to  $\mu_{ik}$  yield the optimal membership value of each bus stop in the particular cluster.

The stationary points of the objective function can be found by adjoining the constraints of (2) by means of Lagrange multipliers:

$$J_m(Z;U,V,\lambda) = \sum_{k=1}^n \sum_{i=1}^c (\mu_{ik})^m \|Z_k - v_i\|^2 + \sum_{k=1}^n \lambda_k \left[ \sum_{i=1}^c \mu_{ik} - 1 \right]$$
(3)

$$\partial \mathbf{j}_{\mathrm{m}}/\partial \mathbf{v}_{\mathrm{i}} = 0; \qquad \Rightarrow -2\sum_{k=1}^{n} (\mu_{ik})^{m} (Z_{k} - \mathbf{v}_{i}) = 0$$

$$\Rightarrow v_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^m Z_k}{\sum_{k=1}^{n} (\mu_{ik})^m}$$

$$J_{m} = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^{m} d_{ik}^{2} + \sum_{k=1}^{n} \lambda_{k} \left[ \sum_{i=1}^{c} \mu_{ik} - 1 \right]$$
(4)

$$\begin{split} \partial j_m / \, \partial \mu_{ik} &= 0; \qquad \Longrightarrow \quad m(\mu_{ik})^{m\text{-}1} \, d^2_{\ ik} + \lambda_k = 0 \qquad \Longrightarrow \quad (\mu_{ik})^{m\text{-}1} &= - \, \lambda_k / \, m d^2_{\ ik} \\ \text{or} \quad \mu_{ik} &= \left[ - \, \lambda_k / \, m d^2_{\ ik} \right]^{\ 1 / \ m\text{-}1} \end{split} \tag{5}$$

$$\partial \mathbf{j} / \partial \lambda_{k} = 0 \quad \Rightarrow \quad \sum_{i=1}^{c} \mu_{ik} = 1 \quad \Rightarrow \quad \sum_{i=1}^{c} \left[ \frac{-\lambda_{k}}{m d_{ik}^{2}} \right]^{\frac{1}{m-1}} = 1$$

$$\mu_{ik} = \frac{1}{\sum_{i=1}^{c} (1/d_{jk}^{2})^{1/m-1} * 1/(d_{ik}^{2})^{1/m-1}} \quad \text{or} \quad \mu_{ik} = \frac{1}{\sum_{j=1}^{c} (d_{ik}^{2} / d_{jk}^{2})^{1/m-1}}$$
(6)

The algorithm converges to its optimal fuzzy partition matrix and cluster centers through an iterative procedure till the difference in the norm between successive partition matrices is less than a prescribed tolerance  $\varepsilon$ .

## Algorithm

- Fix c  $(2 \le c < N)$  and select a value for parameter m. Initialize the partition matrix,  $U^{(0)}$ . Each step in the algorithm will be labeled rx, where rx = 0,1,2....
- ii.
- Calculate the c centers  $\{v_i^{(rx)}\}$  for each step. Update the partition matrix for the  $rx^{th}$  step,  $U^{(rx)}$  as follows. iii.

$$\begin{split} \mu_{ik}^{(r+1)} &= \left[ \sum_{j=1}^{c} (d_{ik}^{(r)} \, / \, d_{jk}^{(r)})^{2 \, l(m'-1)} \right]^{-1} \quad \text{for } I_k = \varphi \\ \text{or} \qquad & \mu_{ik}^{(rx+1)} = 0 \qquad \qquad \text{for all classes I where } i \in I_k \\ \text{where} \quad & I_k = \{ \ I \ \big| \ 2 \leq c < N \ ; \ d_{ik}^{(\ rx \ )} = 0 \ \} \qquad \text{and} \qquad & I_k = \{ \ 1, 2, \dots, c \ \} - I_k \\ \text{and} \qquad & \sum_{i \in I_c} \mu_{ik}^{rx+1} = 1 \end{split}$$

iv. If  $\parallel~U^{(~rx+1)}~$  -  $U^{(~r~x)}~\parallel$   $\leq \epsilon$  , stop; otherwise set rx = rx + 1 and return to step ii.

- **Self Organizing Map** The principal goal of the self-organizing map (SOM) is to transform an incoming signal pattern of arbitrary dimension into a one or two-dimensional discrete map, and to perform this transformation adaptively in a topology conserving fashion. The algorithm responsible for the formation of the self-organizing map proceeds first by initializing the synaptic weights in the network. This can be done by assigning them small values picked from a random number generator, in so doing; no prior order is imposed on the feature map. Once the network has been properly initialized, there are three essential processes involved in the formation of the self-organizing map.
  - i. *Competition*: For each input pattern, the neurons in the network compute their respective values of the discriminant function. This discriminant function provides the basis for competition among the neurons. The particular neuron with the largest value of discriminant function is declared winner of the competition.
  - ii. *Cooperation:* The winning neuron determines the spatial location of a topological neighborhood of excited neurons, thereby providing the basis for cooperation among such neighboring neurons.
- iii. Synaptic Adaptation: This last mechanism enables the excited neurons to increase their individual values of the discriminant function in relation to the input pattern through suitable adjustments applied to their synaptic weights. The adjustments made are such that the response of the winning neuron to the subsequent application of a similar input pattern is enhanced.

# Algorithm

The inputs are the number of bus stops n, each represented as  $X_k = \{x_{k1}, x_{k2}, x_{k3}\}$ , where k = 1,2,...n are the number of centers that needs to be obtained. The algorithm processes the data by capturing the topology of the input space through a two-dimensional lattice of neurons of size  $N=\xi 1$  x  $\xi 2$ , where  $\xi 1$  and  $\xi 2$  are the number of neurons in each dimension. The weights of the neuron denote the centers of the clusters from which the terminals will be obtained. The steps of algorithm are as follows.

- i. *Initialization:* Choose random values for the initial weight vectors  $w_j(0)$ . The only restriction here is that the  $w_j(0)$  will be different for  $j = 1, 2, ..., \xi$ , where  $\xi$  is the number of neurons in the lattice. It may be desirable to keep the magnitude of the weights small. Another way of initializing the algorithm is to select the weight vectors  $\{w_j(0)\}_{j=1}^{\xi}$  from the available set of input vectors  $\{x_i\}_{i=1}^n$  in a random manner.
- *ii.* Sampling: Draw a sample x from the input space with a certain probability; the vector x represents the activation pattern that is applied to the lattice. The dimension of vector x is equal to k.
- *iii.* Similarity Matching: Find the best matching (winning) neuron i(x) at time step n by using the minimum distance Euclidean distance criterion:

$$i(x) = \arg \|\min x - w_i\|, \quad j = 1, 2, \dots, \xi.$$

iv. *Updating:* Adjust the synaptic weight vectors of all neurons by using the update formula  $w_j(nx+1) = w_j(nx) + \eta(nx)h_{j,i(x)}(nx)(x - w_j(nx))$  where  $\eta(nx)$  is the learning rate parameter, and  $h_{j,i(x)}(nx)$  is the neighborhood function centered around

the winning neuron i(x); both  $\eta(nx)$  and  $h_{j,i(x)}(nx)$  are varied dynamically during learning for best results.

- v. Annealing: Decreasing the neighborhood parameter  $(\sigma)$  and learning rate  $\eta$  with time is found to give better results. They are annealed as  $\sigma = \sigma_0(1-\omega_0/\omega)$ ;  $\eta = \eta_0(1-\omega_0/\omega)$ , where  $\sigma_0$  and  $\eta_0$  are the initial values of  $\sigma$  and  $\eta$ ,  $\omega$  is the maximum number of iterations and  $\omega_0$  is the current count of iteration. Another formulation for the annealing process  $\eta = \eta_0(n_f/n_0)^{t/T}$ ;  $\sigma = \sigma_0(\sigma_f/\sigma_0)^{t/T}$ , where  $\sigma_f$  and  $\sigma_0$  are final and initial value of neighborhood parameter and  $\eta_f$  and  $\eta_0$  are similarly for learning rates. Decreasing the width of gaussian neighborhood function has a key role in the formation of clusters. Initially a large width involve all the neurons in the lattice to get close to a particular input, however with passage of time as the width of gaussian decreases, clusters of influence are formed and only those within the cluster participate in updation.
- vi. *Continuation:* Continue with step ii until no noticeable changes in the feature map is observed.

### Location of Hub within a Cluster/Influence Area

Fuzzy-c-means clustering method and Self-organizing-map method partition the stops of the large city network into a number of clusters, each cluster representing some geographical area of the city. Hub is to be located in each cluster from the stops present in the cluster. This is achieved with the objective to minimize the total travel distance (passenger-km) for all stops within a cluster.

Objective function: Minimize 
$$\sum_{\substack{all \text{ stops in} \\ \text{inf luence area}}} dis_{ihb} \times dem_i$$
 (7)

where dis <sub>ihb</sub> is shortest distance from stop 'i' to a selected stop 'hb' in the cluster, and dem<sub>i</sub> is total generated demand of stop 'i'

Due to the computational complexity for a large network in a metropolitan area, an algorithmic approach is applied for the location of hubs within a selected cluster. For each cluster, five bus stops of highest demand are considered as probable terminal stops or hubs. Due to the high demand, one of them is likely be the optimal choice as a hub. Shortest distance between the bus stops of the total transit network is obtained. Choosing one bus stop among these five bus stops as the probable terminal stop, total passenger-km is calculated for all the stops in a cluster. Passenger-km of a stop is the product of the bus stop demand and the shortest distance between bus stop and probable terminal stop. Total Passenger-km is the sum of the passenger-km for all stops in the cluster. The probable terminal stop having minimum total passenger-km will be selected as a hub for the cluster. All the stops in the cluster are bounded in a geographical area, which is defined as the influence area of the selected hub. The same procedure is to be applied for all the clusters and hubs are obtained for the entire network.

# **Selection of Optimal Hubs**

Optimum number of clusters is based on minimizing the total passenger travel time of the entire network. The originating stop and the destination stop for any traveler may lie within the influence area of same cluster or in different clusters of the transit network. When both origin and destination stops lie within the influence area of same cluster, then there is no need of any transfer through the hubs. Travel time for all OD pairs through Hubs are to be estimated. Passenger-Time for traveling from bus stop i to j is calculated depending upon whether the originating and destination stops lie in the same cluster or in different clusters. Total passenger time for the network is also estimated.

Passenger\_Time 
$$(i,j)$$
 = Int\_stop\_demand  $(i,j)$  \* Travel Time  $(i,j)$ 

Total Passenger\_Time = 
$$\sum_{i} \sum_{j} Passenger_{Time}(i, j)$$
 (8)

The set of clusters, which provide the minimum 'Total Passenger Time', is treated as the optimum number of clusters/hubs for the entire bus transit network.

#### **Estimation of Demand for Transit Network**

Inter-stop demand matrix is adjusted to estimate Inter-hub demand matrix and Intra-hub demand matrix. Inter – hub demand matrix represents the estimated demand between the hubs and is of interest in planning inter-hub routes. Considering a stop pairs (i and j), where these stops lie in the influence areas of different hubs hb1 and hb2, then the total demand obtained from such node pairs lying in the influence area of a particular hub will be accumulated on the hub. Similar will be the case for the demand on other hub.

Inter\_hub\_demand (hb1,hb2) = 
$$\sum_{j} \sum_{i} int_{stop} _{dem}(i, j)$$
 (9)

for  $i \in Inf_{hb1}$  and  $j \in Inf_{hb2}$ ; where  $Inf_{hb1}$  and  $Inf_{hb2}$  are the influence areas of hub hb1 and hb2 respectively.

The intra hub demand matrix consists of demand obtained from two components.

(i) Feeder demand: The demand of stops lying in the influence area of a particular hub 'hb' and going to the influence area of some other hub. Feeder\_demand (i,hb) =  $\sum_{i}$  int\_ $stop\_dem(i,j)$  for j $\notin$  Inf<sub>hb</sub> (10)

(ii) Demand between stop to stop within the influence area of a hub 'hb'.

Intra\_hub\_demand (i,hb)= feeder\_demand (i,hb) + 
$$\sum_{j}$$
 int\_ $stop\_dem(i,j)$  (11)

for  $i,j \in Inf_{hb}$ : where  $Inf_{hb}$  is influence area of hub 'hb'.

#### Generation of Inter hub routes

Inter-hub routes are generated between the terminal hubs and inter-hub demand matrix heavily guides the inter-hub route identification. The model generates the different sets of routes corresponding to different trade-offs among conflicting objectives. The model consists of the following steps: Identification of terminal hubs, Generation of alternative paths between terminal hubs, Evaluation of alternative paths by a predefined criteria, and Selection of optimal path

The various feasible alternative paths generated between the two terminal hubs are evaluated on the basis of 'Route utilization coefficient' and 'Desire-passenger-km per km' criteria. Let there be an alternative path with 'n' nodes (1,2...n) and 'n-1' links. Let i be the intermediate node and L is a link in the path. The alternative paths are analyzed for determining the following parameters.

Flow on link L = 
$$\sum_{L+1}^{n} \sum_{i=1}^{L} Nodal\_demand(i)$$
 (12)

RouteUtilization Coefficient (RUC) = 
$$\frac{\sum_{i=1}^{n-1} flow \ on \ link(L) * length \ of \ link(L)}{Max. \ link \ flow * \sum_{i=1}^{n-1} length \ of \ link(L)}$$
(13)

Desire\_passenger\_km = 
$$\sum_{i=1}^{n} [Nodal\_demand(i)] * shortest distance(i,n)$$
 (14)

Desire\_passenger\_km per km = 
$$\frac{Desire_passenger_km}{Route \, length}$$
 (15)

This study considers combined effect of two criteria – RUC and 'Desire passenger-km per Km'. Alternative paths having route utilization coefficient (RUC) not less than that of the shortest path are firstly short-listed. In this way, the alternatives with low RUC are ignored. From amongst these short listed paths, the one having maximum 'Desire-passenger-km per km' is selected as the optimal route path between the selected terminals. For the selected optimal path, the total demand satisfied along the path is estimated. Assuming the minimum frequency of operation for the route, the proportion of demand satisfied is identified and the travel demand matrix is updated for further generation of routes. The above-mentioned procedure from step-I to step-IV for generation of inter-hub routes is continued till no further terminal pairs, based on specified criteria, could be identified and significant portion of the total demand is satisfied.

# **Generation of Secondary (Intra-Hub) Routes**

Intra-hub or secondary routes are to be generated in the influence area of the hubs. These routes may be classified under two categories: Feeder routes - which originate from the terminal stops within the influence area to the hub terminal; and Routes connecting the pair of stops, within the influence area of the same hub.

The model for generating secondary routes consists of (i) Identification of terminals, (ii) Generation of alternative paths, and (iii) Evaluation of alternative paths and selection of optimum path. An alternative that maximizes the desired-pass-km/km is taken as the optimal path. The procedure from step (i) to (iii) is repeated till all stops in the influence area are considered and optimally generated routes within the influence area for a particular hub are obtained.

# **Scheduling Model for Inter-hub Routes**

Scheduling can be defined in the following general terms: Given the origin destination matrix for the bus trips of design period, the underlying transit network characterized by the overlapping routes, how optimally to allocate the vehicles among these routes? The number of vehicle trips to be operated for a route depends upon the demand served along the route and the passenger flow on various links of the route. The demand served along the route can give only little idea about the desired vehicle trips, as some inter-nodal demand may be shared by more than one route.

**Estimation of Optimal Bus Trips** To estimate the optimal bus trips for a route, an iterative heuristic algorithm with following steps is adopted: (i) Estimation of passenger flow on each link, (ii) Determination of desired trips for each link, (iii) Assignment of minimum trips for each route (iv) Estimation of additional trips on each route and (v) Revised trips on each route of transit network

The passenger flow on each link of the network is estimated as

Passenger\_link\_flow(L) = 
$$\sum_{j} \sum_{i} int\_stop\_dem(i, j)$$
; (16)

if shortest path from i to j passes over link L.

Desired bus trips on link(L) = 
$$\frac{Passenger\_link\_flow(L)}{Desired\_average\_bus\_load(LOS)}$$
 (17)

where, LOS defines the specified level of service in terms of average vehicle load.

Minimum required daily trips on a route is estimated based on the weighted equivalent peak hours and policy headway. In the first iteration, each route is assigned these trips: Assigned bus trips (r) = Minimum required trips on route (r) The total assigned bus trips contributed by each of the route moving on the link(i,i+1) is determined as: Assigned bus trips on link(i,i+1) =  $\sum$  Assigned bus trips(r); for all routes passing over link(i,i+1). The difference between the desired bus trips for link(i,i+1) as estimated in step-ii and the already assigned bus trips on link(i,i+1) give the additional required bus trips on link(i,i+1). These additional required bus trips on link(i,i+1). These proportioned among the overlapping routes passing on the link(i,i+1). These proportional bus trips for route 'r' on link 'L' can be estimated:

Proportional bus trips(r,L) =

$$\frac{Additional\ required\ bus\ trips\ on\ link\ (i,i+1)*Assigned\ bus\ trips\ (r)}{Assigned\ bus\ trips\ on\ links\ (i,i+1)} \tag{18}$$

Estimated proportional bus trips on a route are determined for each link of the path. Minimum of these trips is considered as the additional trips for the route. Additional bus trips(r)=Minimum of [Proportional bus trips(r,L)] for all links The revised bus trips for each of the overlapping routes moving on the link are determined as the sum of already assigned trips and the additional bus trips for the route. Revised bus trips(r) = assigned bus trips(r) + additional bus trips(r) Determination of revised bus trips for a route 'r' completes an iteration of the model. In the next iteration, these revised bus trips act as the already assigned bus trips. Assigned bus trips (r) = Revised bus trips (r)

and the process of step iv is repeated. Additional bus trips are again determined for all the routes. This iterative process is continued till no more additional bus trips are required on each route.

#### APPLICATION OF MODELS FOR NEW DELHI

The developed models are successfully applied to New Delhi, the capital city of India. The public transport system of Delhi is primarily road based catering to a daily demand of about 7.7 million passengers. The existing bus transit network, spread over a road length of 1650 Km, has over 1100 bus routes with an operating fleet of over 7000 buses. A new MRTS network for Delhi, covering a route length of 33 Km, is under construction to ease out the public transport problems.

The city of New Delhi, represented by 1542 stops of the bus transit network, is to be partitioned into a number of clusters. To identify the optimum number of clusters, experiments are made for different groups varying from 25 to 60. For each group, clusters are identified by both the methods of Fuzzy-c-means and Self-organizing map. Hub is to be located in a cluster so as to minimize the 'Total passenger-Km' from all the stops to the hub in the cluster.

Optimum number of clusters is based on minimizing the total passenger travel time of the entire network. The estimated total passenger-time of the transit system is obtained through the Self-organizing map and fuzzy-c-means clustering method for five sets of waiting times and for clusters varying between 25 and 60. Based on the results of both methods, an optimal set of 50 clusters is identified. Some operational constraints like minimum inter-hub demand, minimum and maximum route length etc. are incorporated for the selection of terminal hubs. To test the sensitivity of the model to generate routes, the limit of inter terminal daily demand is varied from 1000 to 6600 passengers (corresponding to headway of 5 minutes).

For the daily demand constraint of 6600, a total of 88 inter hub optimal routes are selected. The demand satisfied along the paths of these 88 routes is 85.65 percent of the total daily demand. The average length of these routes is 13.78 km with a maximum of 38.9 km. These high frequency routes need to be operated with automated transit vehicles (APM). For demand limit of 1000, a total of 181 optimal routes, including 88 routes with APM are generated and these satisfy 98.81 percent of the total demand. The maximum route length from these routes is 54.9 Km and the average route length is 21.90 Km. A total of 305 secondary routes (intra-hub) are generated for all the 50 hubs. Highest numbers of 15 secondary routes are generated in one hub, while some hubs have only 1-2 routes. Six hubs, lying primarily on the outskirts of the city and with large influence area, have routes longer than 10 Km.

Designed methodology is adopted to schedule vehicles for all the generated inter-hub routes. For peak period, a total fleet of 9943 buses is required for inter-hub routes at the desired level (LOS-I). The required fleet size is 7203 for a poor level of service (LOS-IV). During mid day period (11 A.M. to 5 P.M.), the fleet size required for the LOS-I, is only 6148 buses. The total fleet size to be commissioned will be based on the requirement for the peak period. During the peak period of 3 hours, the buses operate on an average about 51 km, while for the mid day period of 6 hours, the average operating distance is of the order of 103 km. The average passenger waiting

time for the best level of service (LOS-I) is 1.4 minutes for the peak period and 2.3 minutes for mid day period. The passenger waiting time increases with the deterioration of the level of service, being 2.0 minutes for the peak period and 3.2 minutes for the mid day period at the worst level of service (LOS-IV).

For peak period, a total fleet of 1554 buses for secondary routes is required at the desired level (LOS-I). For the level of service LOS-IV, the required fleet size is only 1428. During mid day period, the fleet size required for the LOS-I, is only 1460 buses. During the peak period of 3 hours, the buses operate on an average about 44 km, while for the mid day period of 6 hours, the average operating distance is of the order of 91 km. The average passenger waiting time for the best level of service (LOS-I) is 2.1 minutes for the peak period and 2.5 minutes for mid day period. The passenger waiting time increases with the deterioration of the level of service, being 2.4 minutes for the peak period and 2.8 minutes for the mid day period at the worst level of service (LOS-IV).

The total fleet size of standard size buses required to operate 486 routes is 11497 at LOS-I, which gets reduced to 8631 at LOS-IV. For the large automated transit vehicles, which may be run on 88 routes, fleet size may be adjusted, based on vehicle capacity. For the total transit system, a vehicle operates for 234 km per day. The average waiting time during peak period varies from 2.4 minutes at LOS-I to 3.0 minutes at LOS-IV.

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