Decision-Making Model of Highway Route Plan Based on Entropy and Entropy Weight Theory

Zhang Yingxue[1], Chunhua Han[2] and Zhang Qisen[1].

[1]School of Traffic and Transportation Engineering,Changsha University of Science and Technology, Changsha, Hunan, 410004, China, e-mail: cscuzyx@126.com
[2]Braun Intertec Corporation 11001 Hampshire Avenue S Minneapolis, MN 55438, U.S.A, e-mail:chan@braunintertec.com

ABSTRACT: It requires many factors to choose highway route plan. Different decision making aims will decide to select different decision making variables, namely indexes. There are also many different evaluation methods. This paper considers the general correlation of multiple objectives and multiple attributes based on Context Sensitive Design, and it presents the index system of decision-making route by investigating, analyzing, consulting experts and so on. In the decision-making problem of route plan which including indexes of M and plans of N, the paper gets the evaluation matrix of single object about multiple index by combining with the qualitative and quantitative principles, then gathers single index of different plans and forms a synthesis index based on entropy and entropy weight theory. At last, the best plan can be chosen by comparing the synthesis index value of different plans. It shows that the index system and decision-making model have reference value for engineering practice.

INTRODUCTION

It is a multiple objective and multi-attribute decision making problem for selecting highway route design plans. How to use reasonable making decision method to analysis indexes impersonality to reduce the synthesis impaction of highway on its context as natural environment, community, landscape, scenic resources, historic sites and so on are very important and practical. The paper introduces entropy and entropy weight into decision making route design plans of highway, and uses entropy to measure indexes and sets up a decision making model to select optimal design plan.

SET UP THE INDEXES SYSTEM OF HIGHWAY ROUTE DESIGN PLAN

In order to protect the natural environment, scenery and landscape resources, historic sites and so on along highway route, coordinate highway with its surroundings, we have made a lot of investigation about highway context and analyzed the existing route design decision making indexes of China, then put forward a new index system to evaluate highway route design based on context sensitive design concept. The indexes are shown in Table 1 and their criteria values are obtained by questionnaire investigation and are graded by experts. For the eight qualitative indexes as constructions aesthetic treatment, culture heritage integrality, effect of exploitation and using culture heritage, impaction on history and culture, impaction on community, development of tour trade, scenic resource preservation, sight status of the highway route in its area, it uses ten point system to get index value. When selecting and analyzing the indexes, we put highway project into its context, and regard it as one part of its context. Traditionally, we mainly consider highway construction from technology and cost and ignore other factors easily. So this can not only consider highway existing criterions and cost but also protect physical environment, aesthetic, scenic, historic, and cultural resources and the physical characteristics of the area that highway will travel along.

Table 1. Evaluation indexes system of highway route design

Indexes	Character	Indexes	Character
Maximum height of fill slope(m)	quantitative	culture heritage integrality	qualitative
Maximum depth of cutting slope (m)	quantitative	Exploitation & using culture heritage	qualitative
Average fill height(m)	quantitative	impact on history and culture	qualitative
Average cutting depth (m)	quantitative	Impact on community	qualitative
Average amount of each kilometer soil and rock square (10^4m3/km)	quantitative	development of tour trade	qualitative
Right of way acquisition area(the sum of all sorts of soil area (hm2))	quantitative	scenic resource preservation	qualitative
constructions aesthetic treatment	qualitative	sight status of the highway route in its area	qualitative

ENTROPY AND ENTROPY WEIGHT DECISION MAKING MODEL

Entropy concept comes from thermodynamics originally which describes the no-reversible phenomenon of movement process. It was introduced for the first time into information area by C.E.Shannon. But now it is applied widely in many areas as engineering technology, society, economy, and so on.

The process of the entropy weight decision making model has eight steps as below:

Step 1: There have n schemes and m evaluation indexes in the decision making system. If the eigenvector of the m evaluation indexes values of number j scheme are showed as the vector $X_j=(x_{1j},x_{2j},\cdots,x_{mj})^T$, then the eigenvectors of the m evaluation indexes values of n schemes can show as the matrix X which is

m by n : $X=(x_{ij})_{m\times n}$ .Where x_{ij} is the eigenvalue of number i index for number j objective, $i=1,2,\cdots,m$; $j=1,2,\cdots,n$. There are two types of index; one is the cost type which is better if it is less, and the other is the benefit which is better if it is larger. After normalizing the matrix X according to the following equations, (1) and (2), the matrix $R=(r_{ij})_{m\times n}$ can be deduced (Zuo1991, Song 2004), where r_{ij} ($0\le r_{ij}\le 1$) is the normalized eigenvalue of number i index for number j objective.

The cost type： $$r_{ij}=\frac{max\,x_{ij}-x_{ij}}{max\,x_{ij}-min\,x_{ij}} \qquad (1)$$

The benefit type： $$r_{ij}=\frac{x_{ij}-min\,x_{ij}}{max\,x_{ij}-min\,x_{ij}} \qquad (2)$$

Step 2: for the entropy of evaluation indexes, there have m evaluation indexes and n evaluation objectives in the evaluation problem ((m,n) for short as following), then the entropy H_i of number i index can be expressed as $H_i=-k\sum_{j=1}^{n} f_{ij}\ln f_{ij}$.

Where $i=1,2,\cdots,m$; $j=1,2,\cdots,n$; $f_{ij}=r_{ij}/\sum_{j=1}^{n} r_{ij}$, $k=1/\ln n$ .If $f_{ij}=0$, then $f_{ij}\ln f_{ij}=0$, the entropy value is smaller. This shows that the index is useful for decision makers. It is necessary for us to select k to ensure $0\le H_i\le 1$ when the value of H_i is more then 1.

Step 3: for the entropy weight of index, the entropy weight w_i of number i index in the (m,n) decision making problem can be defined as:

$$\omega_i=\frac{1-H_i}{m-\sum_{i=1}^{m} H_i} \qquad (3)$$

Where, the value range of w_i is from zero to 1, and the sum of w_i equals to 1.

According to the entropy theory, when the difference of the same index for different schemes is larger and the entropy weight value is smaller, it illustrates that the index carries useful information for decision makers and should be considered as an important factor for each scheme in that problem. If the index entropy is larger while the entropy weight is smaller, then the index is more unimportant. In order to evaluate more accurately and reliably for the evaluation objective, the indexes can be adjusted, added or reduced according to the values of entropy weight.

Step 4: Construct the weighted normalized matrix C

The weight value of m index firstly can be obtained by expert method, then we combine it with formula (3), the synthesis weight λ_i about index i can be deduced from the equation (4):

$$\lambda_i=\frac{\chi_i\omega_i}{\sum_{i=1}^{m}\chi_i\omega_i} \qquad (4)$$

After having introduced entropy and entropy weight, we add a synthesis entropy weight for each index, so the matrix $\boldsymbol{C}$ with synthesis entropy weight is as follows:

$$C=\begin{pmatrix} \lambda_1 r_{11} & \lambda_1 r_{12} & ..... & \lambda_1 r_{1n} \\ \lambda_2 r_{21} & \lambda_2 r_{22} & ..... & \lambda_2 r_{2n} \\ ..... & ..... & ..... & ..... \\ \lambda_m r_{m1} & \lambda_m r_{m2} & ..... & \lambda_m r_{mn} \end{pmatrix} \quad (5)$$

Then gathering each index of each scheme and forming one synthesis index, the optimal scheme can be obtained by comparing the synthesis index value (ZHANG 2005).

Step 5: Definite ideal point and negative ideal point. Negative ideal point means that it is a muster of all the worst points of each index. The muster is a zero matrix. It is maybe negative value if the indexes have not been normalized. The optimal ideal point of multi-objective is a muster matrix of the maximum point value of each index. By calculating the distance of each scheme from the ideal point and negative ideal point and the fidelity that describes in step 7, the scheme that is best close to the positive ideal point can be selected.

The ideal point: $P^*=\left(P_1^*,P_2^*,\cdots,P_m^*\right)^T$

In which $P_1^*=\max\limits_{j}\left\{c_{ij}\,\middle|\,j=1,2,\cdots,n;i=1,2,\cdots,m\right\}$

The negative ideal point: $P_0=(0,0,\cdots,0)^T$

Step 6: Distance calculation. The distance of the evaluation object point to the ideal point is calculated by using Euclidean distance.

$$d_j^*=\left\|c_j-p^*\right\|=\sqrt{\sum_{i=1}^{m}(c_{ij}-p_i^*)} \quad (6)$$

Step 7: The fidelity of the evaluation object to ideal point.

$$T_j=1-\frac{\sum_{i=1}^{m}(c_{ij}p_i)}{\sum_{i=1}^{m}(p_i^*)^2} \quad (7)$$

The fidelity indicates the ratio value that the distance of the evaluation object point to the ideal point divided by the distance of ideal point to negative point. If the ratio is smaller, the scheme is better.

Step 8: Scheme selection optimization priority

Prioritize and select each scheme according to the fidelity T_j value after it is calculated. If the value is small, the scheme is optimal. If T_j is equal, the value of d_j^* must be calculated and prioritized. The distance value of d_j^* is small, its scheme is optimal from the synthesis index.

APPLICATION EXAMPLE

There are three route design plans named A_1, A_2, A_3 to be selected of some freeway with design speed of 80km/h in mountainous area. The data and fraction value of each index and their expert weights show in Table 2. The decision making process of plan is as follows:

Table 2 The form of analysis and evaluation for route plans

Index	Expert weight χ_i	Plan A_1	Plan A_2	Plan A_3
Maximum height of fill slope (m)	0.04	16	15	19
Maximum depth of cutting slope (m)	0.04	30	70	60
Average fill height(m)	0.12	11	8	10
Average cutting depth (m)	0.20	8	23	16
Average amount of each kilometer soil and rock square ($10^4 m^3/km$)	0.02	10	16	22
Right of way acquisition area (hm^2)	0.22	5	8	7
constructions aesthetic treatment	0.08	8	6	7
culture heritage integrality	0.07	9	10	8
exploitation and using culture heritage	0.01	6	8	7
impact on history and culture	0.02	5	4	6
impact on community	0.05	4	6	5
development of tour trade	0.02	5	7	6
scenic resource preservation	0.04	9	7	8
sight status of the highway route in its area	0.06	4	3	5

According equations (1) and (2), the normalized matrix $\boldsymbol{R}$ is obtained.

$$\boldsymbol{R}=\begin{pmatrix} 0.75 & 1 & 0 & 1 & 1 & 1 & 0 & 0.5 & 1 & 0.5 & 0 & 1 & 0 & 0.5 \\ 1 & 0 & 1 & 0 & 0.5 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0.25 & 0.33 & 0.47 & 0 & 0.33 & 0.5 & 1 & 0.5 & 1 & 0.5 & 0.5 & 0.5 & 1 \end{pmatrix}^T$$

After calculating the entropy H_i, entropy weight ω_i and synthesis weight λ_i, the results are shown in Table 3.

Table 3 the entropy H_i, entropy weight ω_i and synthesis weight λ_i of each index

H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}
0.623	0	0	0.570	0.579	0.510	0.579	0	0.579	0.579	0.579	0.579	0.579	0.579
ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}	ω_{11}	ω_{12}	ω_{13}	ω_{14}
0.049	0.130	0.130	0.056	0.055	0.064	0.055	0.130	0.055	0.055	0.055	0.055	0.055	0.055
λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}
0.026	0.070	0.021	0.151	0.015	0.190	0.059	0.123	0.015	0.015	0.037	0.015	0.030	0.044

The ideal point and negative ideal point can be deduced when considering the synthesis entropy weight by formula (5) in the normalized matrix. The weighted matrix is as follows:

$$\boldsymbol{C}=\begin{pmatrix} 0.020 & 0.070 & 0 & 0.151 & 0.015 & 0.190 & 0 & 0.062 & 0.015 & 0.008 & 0 & 0.015 & 0 & 0.022 \\ 0.026 & 0 & 0.021 & 0 & 0.008 & 0 & 0.059 & 0 & 0 & 0 & 0.037 & 0 & 0.030 & 0 \\ 0 & 0.018 & 0.007 & 0.071 & 0 & 0.061 & 0.030 & 0.123 & 0.008 & 0.015 & 0.008 & 0.015 & 0.002 & 0.044 \end{pmatrix}^T$$

The ideal point:

$$\boldsymbol{P}^*=\left(0.026,0.070,0.021,0.151,0.015,0.190,0.059,0.123,0.015,0.015,0.037,0.015,0.030,0.044\right)^T$$

The negative ideal point: $P_0 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0)^T$

From equations (6) and (7), the distance d_j^* and fidelity T_j of each scheme can be calculated. The results are shown in Table 4.

Table 4 The distance, fidelity to ideal point and priority for plans

plans	A_1	A_2	A_3
distance d_j^*	0. 495	0. 794	0. 627
priority	1	3	2
fidelity T_j	0. 169	0. 921	0. 499
priority	1	3	2

Table 4 shows that the priority of distance is consistent with the fidelity. So the route plan A_1 is optimal.

CONCLUSIONS

1) Comparing with other methods, the weight of index by using the entropy weight method is related to objective data not subjective factors when select optimal route scheme. This makes the calculated result is more objective and realistic.

2) When making decision route scheme, it always consider the weight of each index separately and ignore the relationship of each other. The paper considers the relationship by introducing entropy weight and expert weight and calculates the synthesis weight, presents decision making model based on entropy weight theory to select optimal scheme. This can not only overcome the common methods shortage but also ensure that the selected route is more benefit to its context.

3) The index system for route design selection in this paper needs to be farther improved and perfected in engineering projects for the future.

ACKNOWLEDGMENTS

The authors appreciate the support of the West Transportation Construction Science and Technology of China (2003 318 798 05) and Hunan Provincial Natural Science Foundation of China (07JJ3111).

REFERENCES

Zuo Jun.(1991). Multiple Objective Decision Making Analysis. Zhejiang University Press.27-91.

Song Ximin, Zhou Shujing.(2004). Based on Entropy Weight Method on Investment Plan' Appraisal of Building Project. Building Technique Development. Vol.31, No.7, p113-114.

ZHANG Yang, WU Liu-yuan.(2005) Study on the design ideology and model of highway landscape environment. Journal of Xi'an University of Architecture &technology. Vol. 37, No.3, p366-369.