Applicability of highway alignment optimization models

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ARTICLE INFO

ABSTRACT

This paper presents an intelligent optimization tool that assists planners and designers in finding preferable highway alignments, connecting specified endpoints or zones. It integrates genetic algorithms with a geographic information system (GIS) for optimizing highway alignments and processes massive amounts of relevant data associated with highway design and alternative evaluation. To show the applicability of the proposed model to a real-world problem, two actual highway projects in the state of Maryland have been analyzed using the model. An extensive analysis of sensitivity to key model parameters is also conducted to describe the model capabilities. The analysis results show that the model can effectively optimize highway alignments in an area combining complex terrain and various types of natural and cultural land-use patterns, and provide detailed information of optimized alignments as a model output. It is also found that the alignments optimized by the model are quite similar to those obtained through conventional manual methods by a state agency, but the model can greatly reduce the time required for highway planning and design as well as produce lower cost solutions. Finally, the results confirm that all dominating and alignment-sensitive costs should be simultaneously evaluated in the alignment optimization process because many trade-off opportunities exist among those costs. The proposed model can greatly contribute to the productivity of highway planners as well as to the quality of the resulting infrastructure.

1. Introduction

Increasing highway traffic and safety concerns often justify the construction of new highways and bypass routes or the realignment and expansion of existing highways. Highway agencies are then challenged to locate and design the best possible alternatives. However, unlike the rapid developments in the automobile and construction industries, search and design processes for highway location are still carried out in a traditional process which is more than 50 years old.

Finding preferred highway alternatives with existing methods requires considerable resources (e.g., manpower and time). Furthermore, the agencies often face complex decisions in aligning a road and estimating its cost because the project should be based on comprehensive analyses of many relevant factors, such as land availability, earthwork, maintenance, life-cycle cost, demand, land-use, user travel time, environmental impacts, safety, effects on the performance of other transportation modes, and effects on regional development. A survey of the procedures adopted by highway agencies (e.g., US Departments of Transportation (USDOT)) reveals that a combination of engineering judgment and manual cost-benefit analysis is

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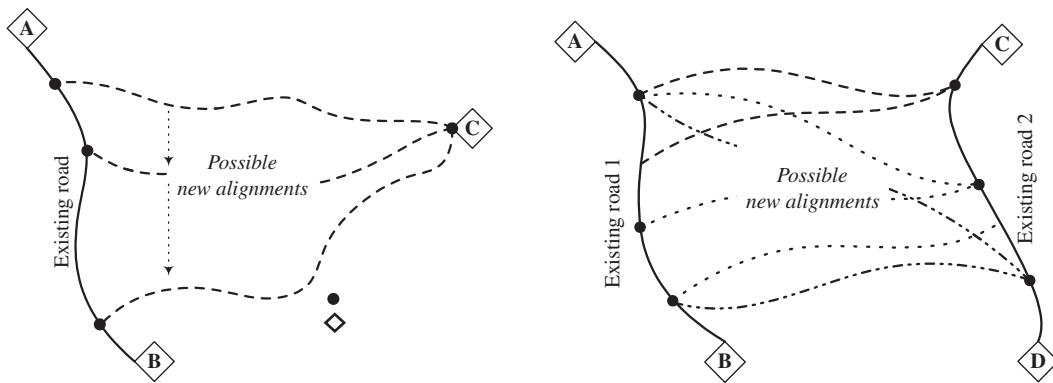[2] Tel.: +1 301 405 1954; fax: +1 301 405 2585.

Fig. 1. Possible highway alignments connecting existing roads.

generally followed for evaluating various alternatives for new highway construction, realignment, or expansion. The alternatives are ranked through a weighed-criteria analysis in which a set of viable alternatives are manually identified and ranked based on a total score. This score consists of weights assigned for various criteria, such as economic and environmental impacts, cost and safety. The alternatives are then presented to the stakeholders in town meetings, in which public support is sought. The alternative that wins the maximum support is usually chosen for the actual construction, and proceeds to the detailed design stage. A preliminary estimate of total cost for planning, design, right-of-way acquisition, environmental impact mitigation and construction is then prepared using historical unit cost data, engineering judgment, and trial–error.

It should be noted that many possible alternatives which are much better than the one selected for final design may exist but get no consideration (see Fig. 1). Furthermore, it has often been observed that many critical issues, including unusual topographic and geographic issues as well as design requirements arise at later stages, often inflating the preliminary cost estimates of the final alternative. Seeking additional funding at later stages often delays the scheduled completion of the project since funds are typically approved on an annual basis and are too scarce to support all worthwhile projects. Thus, an intelligent computer model that finds the best highway alignment for given requirements, and which can be easily re-run for changing objectives, constraints or preferences, can save billions of dollars in resources and life-cycle costs of capital, operation and user time.

For new highway construction, an optimized road connecting specified points or sections on existing highways is desired. In expansion projects, an optimized road between tightly specified bounds may be desired. The determination of the best available option for new highway construction, realignment, or expansion falls in the general area of "Highway Route Optimization" or "Highway Alignment Optimization."

This paper is organized as follows: after the introduction, Section 2 reviews the literature on methods previously developed for optimizing highway alignments. Section 3 discusses major highway costs and constraints associated with highway construction. The methodology developed for the highway alignment optimization (HAO) model is introduced next; Section 4 describes how we represent the HAO problem in genetic algorithms (GAs). Section 5 discusses our approach to modeling highway alignments and structures. Section 6 presents the integration of GAs and GIS. Section 7 describes a basic model formulation. In Section 8, we demonstrate the model's capability and usability through its application to real-world problems. Factors that are important in comprehensively optimizing highway alignments are considered in Section 9 through extensive analyses of sensitivity to critical model parameters. Section 10 concludes with the summary and possible extensions of this study. A list of notations used in this paper is provided in Appendix C.

2. Literature review

Optimizing highway alignments is a complex combinatorial problem that finds the best alternatives (usually the most economical path) of a new highway connecting specified points or locations. The development of effective methods for optimizing highway alignments (including processing massive amounts of relevant data and evaluating multiple highway impacts), has posed major challenges to researchers because various tangible and intangible factors are associated with the problem. These factors include, for example, highway design specifications, earthwork cost, right-of-way cost, land-use, terrain profiles, and political and environmental concerns. The alignment optimization problem is normally formulated as a minimization problem, and its objective functions and constraints are non-differentiable, noisy, unsmooth, and implicit. Theoretically, there may be an infinite number of alternatives to be evaluated.

Many mathematical models (at least ten so far, as shown in Table 1) have been developed either for optimizing horizontal alignments or vertical alignments or for three-dimensional (3D) alignments (i.e., optimizing both horizontal and vertical alignments). These are *calculus of variations*, *network optimization*, *dynamic programming*, *enumeration*, *linear programming*,

Table 1
Studies on highway alignment optimization.

Target for optimizing	Optimization approach	References
Horizontal alignment only	Calculus of variations	Howard et al. (1968), Shaw and Howard (1981, 1982), Thomson and Sykes (1988), and Wan (1995)
	Network optimization	Turner and Miles (1971), OECD (1973), Athanassoulis and Calogero (1973), Parker (1977), and Trietsch (1987a,b)
	Dynamic programming	Hogan (1973) and Nicholson et al. (1976)
	Mixed integer programming	Easa and Mehmood (2008)
	Neighborhood search heuristic with mixed integer programming	Lee et al. (2009)
	Genetic algorithms	Jong (1998) and Jong et al. (2000)
Vertical alignment only	Enumeration	Easa (1988)
	Dynamic programming	Puy Huarte (1973), Murchland (1973), Goh et al. (1988), Fwa (1989)
	Linear programming	Chapra and Canale (2006) and ReVelle et al. (1997)
	Numerical search	Hayman (1970) and Robinson (1973)
	Genetic algorithms	Jong (1998), Fwa et al. (2002), and Jong and Schonfeld (2003)
Three-dimensional alignment	Numerical search	Chew et al. (1989)
	Distance transform	Mandow and Perez-de-la-Cruz (2004) and de Smith (2006)
	Neighborhood search heuristic with mixed integer programming	Cheng and Lee (2006)
	Genetic algorithms	Jong (1998), Jha (2000), Kim (2001), Jong and Schonfeld (2003), Tat and Tao (2003), Jha and Schonfeld (2004), Kim et al. (2004a,b, 2005, 2007), Kang et al. (2007, 2009, 2010), and Kang (2008)

numerical search, *distance transformation, neighborhood search heuristic with mixed integer programming* (*MIP*), and *genetic algorithms* (*GAs*). Among them the GA has been the most popularly adopted method for optimizing highway alignments since it was first used for this problem by Jong (1998). It is an adaptive search method based on the principles of natural evolution and survival of the fittest, and can avoid getting trapped in local optima through a pool-based search rather than single solution comparison as in other heuristics (e.g. simulated annealing and Tabu search). In general, GAs are considered especially suitable for problems with huge numbers of local optima. While they do not guarantee that the globally optimal solution for a large problem is found within a reasonable time, their population-based search is very helpful in preventing the search from getting trapped in local optima. The effectiveness of GA-based approaches for the alignment optimization can be described in terms of the following key advantages:

a. Can yield realistic alignments.
b. Can search in a continuous search space.
c. Can find globally or near globally optimal solutions.
d. Can deal with most of the important highway costs and constraints.
e. Can jointly optimize horizontal and vertical alignments.
f. Can incorporate through customized operators much of the highway designers' intuition and experience about efficient searches for alignments.

A comprehensive review of the existing methods for the alignment optimization has been conducted by the authors (Kang et al., 2007, 2009). It concludes that all considered methods other than the GA-based approach have some flaws for alignment optimization. For example, they (i) cannot model discontinuous cost items (e.g., right-of-way cost and property areas actually affected by the alignment), (ii) cannot yield realistic highway alignments, (iii) use discrete solution sets rather than continuous search space, (iv) formulate too few cost components and constraints, (v) may produce multiple local optima, and (vi) do not jointly optimize horizontal and vertical alignments (i.e., use a conditional approach in which they sub-optimizes a horizontal alignment first, and then sub-optimize the vertical alignment based on the horizontal alignment created). Interested readers may consult the authors' previous publications (Kang et al., 2007, 2009) for further details on the advantages and disadvantages of the existing alignment optimization methods.

Two additional related studies should also be mentioned. Mandow and Perez-de-la-Cruz (2004) and de Smith (2006) tried a distance transformation (DT)-based heuristic for optimizing highway alignments. This method uses a (discrete) grid space as a search space of the alignment optimization, and requires a two stage process for generating alignments; (i) a sequence of grid points connecting two endpoints is searched, and then (ii) a highway alignment is approximated by the grid path with given design codes. Thus, there may be a significant approximation in converting a set of grid points to actual highway elements. Furthermore, it would be difficult to realistically represent actual geographic entities with the grid search space. Finally, important highway cost items (e.g., right-of-way cost) are also neglected in this method.

3. Highway costs and constraints

3.1. Major highway cost items

Many cost components directly or indirectly affect the construction of new highways. Besides the initial construction costs, which are directly counted for highway construction (e.g., earthwork, land acquisition, pavement and drainage costs), user costs and environmental impact should also be considered. It is important to note that all dominating and alignment-sensitive costs should be considered and precisely formulated for a good highway optimization model; dominating costs are those which make up significant fractions of the total cost of a new highway alignment, and alignment-sensitive costs are those which vary with relatively slight changes in alignment geometries. Normally, highway user costs (such as travel time cost and vehicle operating cost) are the most dominating ones as they persist over the entire design life time of the highway and the users' value of time is usually higher than other costs associated with highway construction. Structure costs (e.g., bridges and interchanges costs) and earthwork costs may dominate if a highway is constructed in a mountainous area. A highway passing through an urban area may have a high percentage of right-of-way cost, since the required land acquisition cost of that area may be relatively higher than other costs.

A number of studies (Winfrey, 1968; Moavenzadeh et al., 1973; OECD, 1973; Wright, 1996; Jong, 1998; Jha, 2000; Kang, 2008) have discussed the costs associated with highway construction, and identified major costs that should be considered for optimizing highway alignments. The major costs are summarized in Table 2. Note that costs required in the highway planning and design stages (such as consulting and data collection costs) are not included here among the major costs since they may not be sensitive to alignments of highway alternatives.

3.1.1. Construction costs

The construction costs are the major agency costs that directly affect highway agencies (e.g., local and federal governments) or highway construction companies. Normally, costs required for earthwork, pavement, right-of-way, structures (e.g., bridges and interchanges), and miscellaneous items (such as fencing and guardrails) are included in this category. Jong (1998) and Jha (2000) reclassified them into four sub-categories based on the characteristics of each cost component: (a) volume-dependent cost, (b) length-dependent cost, (c) location-dependent cost, and (d) structure cost. Such a classification is quite useful for quantifying the construction costs and representing them in the alignment optimization process.

The earthwork cost is a volume-dependent cost since it is quantifiable based on the amount of earthwork volume required for highway construction. Some unit costs related to the earthwork (such as unit embankment and excavation costs) are needed to estimate that cost. The right-of-way cost, which includes land acquisition costs and property damage and compensation costs, can be classified as the location-dependent costs. The length-dependent cost is defined as the cost proportional to alignment length. Pavement cost and road superstructure and substructure costs (such as fencing, guardrails, and drainage costs) can be included in this category. In highway engineering, structures normally include bridges, tunnels, interchanges, intersections, and overpasses or underpasses. Costs required for building those structures belong to the structure cost category. All costs that are dominating and sensitive to the alignment should be included in the alignment optimization process.

3.1.2. Highway maintenance costs

The maintenance costs occur throughout the design life of the road. Therefore, for life-cycle cost estimates, these maintenance costs are generally discounted over the road's life at an appropriate interest rate. These costs may include preventive maintenance costs (such as costs required for repairing pavements, guardrails, and medians) and even road rehabilitation costs.

3.1.3. User costs

The highway user costs usually include travel time, vehicle operating, and accident costs. These costs also occur throughout the design life of the road, and thus should be estimated as a life-cycle cost. In a highway improvement project, user costs are typically used for a user benefit analysis, by comparing their values estimated before and after the project. The user cost items are the dominating costs. They are also sensitive to the highway length as well as to the locations where the new

Table 2
Classification of major highway costs and impacts.

Classification	Examples
Construction costs	Earthwork, pavement, right-of-way, Structures
Management costs	Pavement mowing, lighting
User costs	
Vehicle operating cost	Fuel, tire wear, depreciation of vehicles
Travel time costs	Vehicle hours × unit value of time
Accident costs	Predicted numbers of accidents and their costs
Environmental and socio-economic impacts	Loss of environmentally sensitive areas, Loss of socio-economic areas, noise, air pollution

highway is connected to existing road networks. Therefore, the user cost should also be considered in the alignment optimization process. Methods for estimating these costs may be found in "User Benefit Analysis for Highways" by AASHTO (2003).

3.1.4. Environmental and socio-economic impacts

Construction of a new highway may also significantly affect environmentally sensitive areas (such as wetlands and historic areas) and human activities of the existing land-use system, and even may cause air pollution and increased noise level. These impacts are often the most important issues in the modern highway construction projects; hence, they should also be accounted for in the alignment optimization process. Jha (2000) provides more detailed discussions on the environmental issues associated with the new highway construction. He comprehensively formulates highway environmental costs in the alignment optimization process with a GIS-based application.

3.2. Constraints in highway construction

Normally two types of constraints are considered in highway planning and design. These are (i) design constraints and (ii) environmental and geographical constraints. The former constraints are usually based on recommended design standards (e.g., AASHTO, 2004); the latter ones are sensitive to many complex factors associated with topology, land-use of the project area, and even preferences of decision makers.

3.2.1. Design constraints

Basically, the geometric design of a highway determines the horizontal alignment, vertical alignment, and cross-section of that highway. The horizontal alignment of a highway, which is a projection of a 3D highway onto a two-dimensional horizontal space (i.e., *XY* coordinate system), generally consists of three types of design elements: tangent segments, circular, and transition curves. On the other hand, the vertical alignment is a projection of a design line on a vertical plane as if all horizontal curves were stretched to straight, and composed of a series of graded-tangents joined to each other by parabolic curves. According to AASHTO (2004), the most important design constraints required for constructing the horizontal and vertical alignments are:

- Minimum horizontal curve radius.
- Sight distance on horizontal curves.
- Minimum transition curve length (only if transition curves are considered as part of the horizontal curved section).
- Maximum gradient of vertical alignments.
- Minimum length of crest and sag vertical curves.
- Minimum vertical clearance for highway crossing and bridge construction.

3.2.2. Geographical and environmental constraints

Besides the design constraints stated above, geographical and environmental constraints should also be considered in the highway design process. Nowadays, these constraints are considered to be among the most important criteria in real highway construction projects, and vary with the different land-uses affected by projects. These are categorized as follows:

- Environmentally sensitive areas (e.g., wetlands and historic districts).
- Socio-economically sensitive areas (e.g. residential and commercial areas).
- Control areas defined by highway decision makers (e.g., political issued areas).
- Fixed point (or area) constraints through which the alignment must pass.

4. Representation of highway alignments in GA

In the proposed highway alignment optimization (HAO) model, a horizontal alignment is defined by the tangents, circular curves, and the connecting transition curve sections. A vertical alignment is defined by the graded-tangents connected with parabolic curves. The configuration of these elements depends on the points of intersections (PI's); thus generating a highway alignment can be reduced to determining its corresponding series of PI's.

To find the best candidate highway alignments, the model employs genetic algorithms (GAs). In the model, the alignments are represented with chromosomes, and each chromosome has a series of genes defined by the *xyz* coordinates of PI's (see Fig. 2); $PI_i = (x_i, y_i, z_i)$ for all $i = 1, \ldots, n_{PI}$. It is important to note that the genes are not independent of each other because if a coordinate of one PI is changed, the alignment configuration at other PI's may change. It should be also noted that the alignment optimization process is based on a pool-based search of the GA rather than single solution comparison, and thus a set of possible highway alignments (i.e., chromosomes) is treated as the population in the model. About 40–100 alignments are generated in each generation depending on the complexity of the chromosome (i.e., the number of genes), and the individual alignments within each generation compete with each other to reproduce offspring based on their "fitness" (i.e., objective function value). After enough generations, the fittest individuals should survive, whereas poor solutions get discarded, and the population will finally converge to an optimized solution.
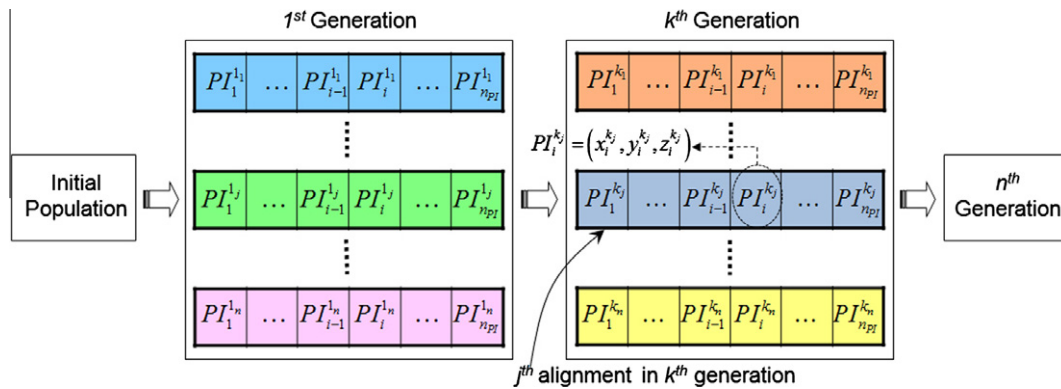
Fig. 2. Representation of highway alignments in GA.

The stopping criterion of the GA optimization method is based on the improvement in the objective function values of the alignments reproduced. Thus, if there is no significant improvement in the objective function value during a certain number of generations (e.g., less than 0.05% improvement during 50 generations for the examples shown in this paper), the alignment optimization process is terminated. Eight customized genetic operators have been developed for reproducing highway alignments. The role of these genetic operators is summarized in Table 3; further details may be found in Jong and Schonfeld (2003).

Fig. 3 shows orthogonal cutting planes where the PI's of a highway alignment are generated. The orthogonal cutting planes are equally spaced between the endpoints of the alignment. The total number of the cutting planes (denoted as n_{oc}) and the total number of the PI's (denoted as n_{PI}) are both prespecified as model parameters, and the former can be greater than or equal to the latter. This means that not all the cutting planes are selected for generating PI's. For example, n_{oc} is greater than n_{PI} in Fig. 3. A series of PI's of the alignment is generated only on the cutting planes selected, and each PI has a unique set of *xyz* coordinates in its corresponding search space (i.e., selected cutting plane). The selection of the cutting planes is randomly processed at every iteration of alignment generation during the optimization procedure.

In the proposed model, a horizontal PI (HPI) is determined by *xy* coordinates of a PI whose deflection angle is non-zero (note that no horizontal curve is needed for zero deflection angle at a PI). Thus, the number of HPI's that actually produce horizontal curves can be less than or equal to the total number of PI's specified for generating the highway alignments in the model. As with the horizontal curve, there is no vertical curve in the middle PI among three consecutive PI's if they have the same elevation (i.e., *z* value). There is also no vertical curve at the middle PI if the three PI's are aligned in a sloping straight line. Thus, the number of vertical PI's (VPI's) that actually produce vertical curves can also be less than or equal to the total number of PI's specified for the alignment generation. It is important to note that (1) both a horizontal and a vertical curve, (2) either one of them, or (3) neither of them may be needed at each PI generated. Therefore, a subset of the PI's used for generating horizontal curves can differ from that used for the vertical curves.

It is also important to note that the number of PI's in the model is a key input parameter that affects the configuration and total cost of the highway alignments generated. In dense urban areas and areas with significant variation in topography and/ or land-use a higher PI density will improve the precision of the alignment generation and cost evaluation, whereas in areas with slight variation in topography or land-use, fewer PI's will suffice. Thus, PI density (i.e., the number of PI's) should be

Table 3
Description of customized genetic operators for highway alignment optimization.

| Operator	Description
Uniform mutation	Randomly select a PI of a parent alignment (i.e., a gene of a chromosome), and then replace its values with randomly selected numbers to maintain genetic diversity in the population
Straight mutation	Randomly select two PI's, and then make the deflection angles of other PI's in between the selected PI's zero in order to straighten the alignment between two randomly selected PI's
Nonuniform mutation	Randomly select a PI of a parent alignment, and then replace its values with numbers randomly selected from the adjusted (i.e., reduced) mutation range. This operator is applied at latter generations to refine the alignment
Whole nonuniform mutation	This operator applies the nonuniform mutation operator to all the PI's of a randomly selected parent alignment. The resulting offspring will be totally different from its parent
Simple crossover	Generate a random integer number k between 1 and n_{PI}, and then cross kth PI of two randomly selected parent alignments
Two-point crossover	Exchange the PI's between two randomly generated positions k and l for two randomly selected parent alignments
Arithmetic crossover	Randomly select two parent alignments to be crossed, and then generate the offspring as a linear combination of two parent alignments, which guarantees the offspring is always feasible
Heuristic crossover	Randomly select two parent alignments, and generate a single offspring according to a heuristic rule based on the comparison of their objective function values
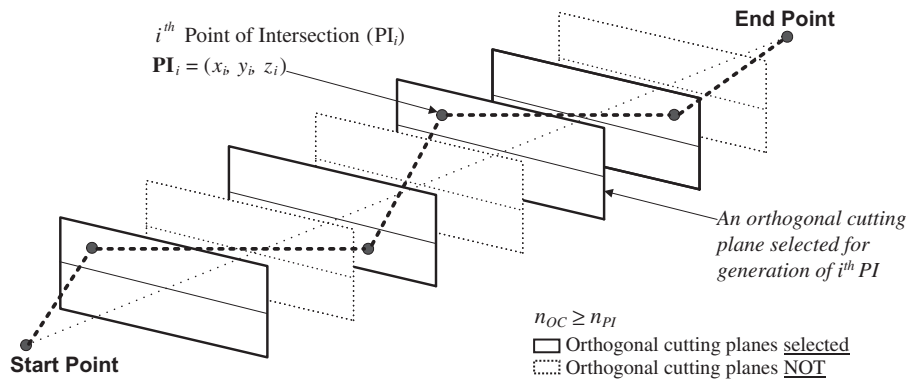
Fig. 3. A series of PI's generated on corresponding orthogonal cutting planes.

decided case-by-case in the alignment optimization process with careful consideration of land-use complexity and topography of the project area. An example case study for selecting the preferable number of PI's is presented in Section 9.1.

5. Modeling highway alignments and structures

5.1. Modeling highway alignments

In a highway alignment, a series of tangents and curved sections are adjoined. Circular curves and transition curves are typically combined to form the horizontal curved sections. Some kind of transition curve is often applied between a tangent and a circular curve for mitigating a sudden change in degree of curvature and hence, in lateral acceleration and force, from the tangent to the circular path. Particularly for high-speed highway alignments, spiral transition curves are strongly recommended in horizontal curved sections. In the previous version of the HAO model (Jong and Schonfeld, 2003; Jha and Schonfeld, 2004; Kim et al., 2005) only tangents and circular curves were used to generate the horizontal alignment. In this paper, however, we incorporate transition curves in the horizontal alignments. This enables the model to produce more realistic and desirable alignments during the optimization process. Spiral transition curves, which are widely used in practice, are chosen to model the transition curves. For most design standards, the model allows users to easily specify their own input values, but the default values are taken from the relevant AASHTO design manual (AASHTO, 2004).

Fig. 4a presents an example of a typical horizontal alignment with a series of reference points, representing intersection points between tangents, circular curves, and transition curves. For notational convenience, we let EP_1 and EP_2 be the start and end points, respectively, of a highway alignment, and its initial and final PI's (i.e., PI_0 and PI_{nPI+1} respectively) correspond to the start and end points. As shown in the figure, ST_i and TS_{i+1} are linked by a straight-line connecting PI_i and PI_{i+1} for all

Fig. 4. Geometric specification of typical horizontal alignments used in the HAO model.
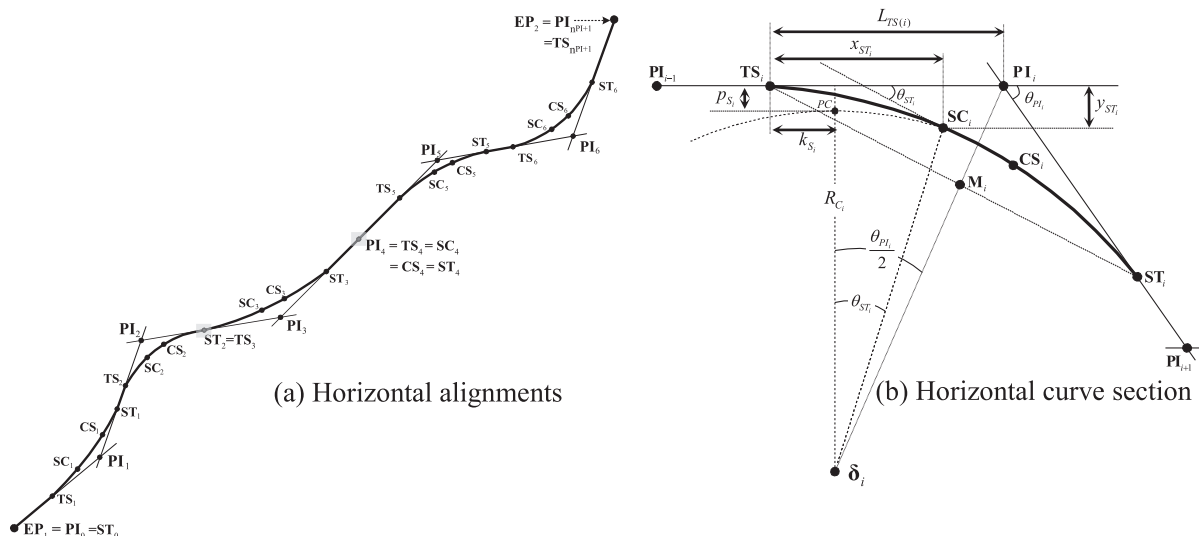
$i = 0, \ldots, n_{PI}$ whereas $\mathbf{TS}_i$ and $\mathbf{SC}_i$ and $\mathbf{CS}_i$ and $\mathbf{ST}_i$ are connected by a spiral transition curve and $\mathbf{SC}_i$ and $\mathbf{CS}_i$ are connected by a circular curve for all $i = 1, \ldots, n_{PI}$. Note that in an extreme case, where an alignment tangent section between two consecutive intersection points (e.g., between $\mathbf{PI}_2$ and $\mathbf{PI}_3$ in Fig. 4a) is completely eliminated by two spiral transition curves, the point of change from spiral to tangent section pertaining to one intersection point will coincide with the point of change from tangent to spiral curve pertaining to the next intersection point (e.g., $\mathbf{ST}_2$ and $\mathbf{TS}_3$ are the same point in Fig. 4a). Furthermore, if an intersection angle at $\mathbf{PI}_i$ (denoted as θ_{PIi}) becomes zero, all the reference points pertaining to $\mathbf{PI}_i$ are the same; for example, the locations of $\mathbf{TS}_4$, $\mathbf{SC}_4$, $\mathbf{CS}_4$, $\mathbf{ST}_4$, and $\mathbf{PI}_4$ shown in Fig. 4a would then all be the same. A list of notations used in this paper is provided in Table C.1.

Fig. 4b shows a typical horizontal curved section of the alignment with two spiral transition curves connecting the central circular curve to the adjoining tangents. Without violating the AAHTO design standards (AASHTO, 2004) the minimum radius can be used to fit the radius of a circular curve (R_{Ci}), and minimum superelevation run-off length can be used to fit the length of spiral transition curve (l_{STi}). We now realistically represent the centerline of the horizontal alignment given the coordinates of $\mathbf{PI}_i$ (for $i = 1, \ldots, n_{PI}$) and specified design codes. The coordinates of $\mathbf{TS}_i$, $\mathbf{SC}_i$, $\mathbf{CS}_i$, and $\mathbf{ST}_i$ can be found with simple vector operations shown in Appendix A.

In the HAO model, the horizontal and vertical alignments of a new highway are generated and evaluated jointly; the vertical alignment is determined by fitting parabolic curves to graded-tangents at vertical points of intersections (VPI's) while its corresponding horizontal alignment is being created. The evaluation of the highway (i.e., its total cost and environmental impact calculation) is then processed after its horizontal and vertical alignments are generated.

Let the *HZ* plane be a coordinate system designed to represent ground and road elevation along the horizontal alignment. The *H* and *Z* axes represent road distance and elevation along the horizontal alignment, respectively. We now define a vertical alignment on the *HZ* plane. Let $Start_V = (H_0, Z_0)$ and $End_V = (H_{nPI+1}, Z_{nPI+1})$ be start and end points of the vertical alignment, respectively where $H_0 = 0$, and Z_0, H_{nPI+1}, and Z_{nPI+1} are known. Then, *i*th VPI can be defined with a pair of H_i and Z_i. As shown in Fig. 5, a set of VPI's are generated on the orthogonal cutting planes, and outlines the track of the vertical alignment. Linking each pair of successive points with a straight line produces a piecewise linear trajectory of the vertical alignment. The upper and lower bounds (i.e., z_{UBi} and z_{LBi}) of *i*th VPI are determined based on the elevation of preceding and subsequent VPI's and the maximum allowable gradient (g_{max}). More detailed discussion of the vertical alignment generation may be found in Kang et al. (2007).

5.2. Modeling highway structures

Many highway structures may also be included in constructing new highways. These may include bridges for crossing rivers or valleys, tunnels, and cross-structures for intersecting existing highways (e.g., at-grade intersections, grade separation, and interchanges). Especially in mountainous terrain and river valleys, construction of tunnels and bridges may dominate the highway design process. Hence, such structures should also be considered in the highway alignment optimization process.

The basic principle for locating highway bridges or tunnels (unless they are major) is that the highway location should normally determine the structure location, not the reverse. If the bridge or tunnel location is located first, in most cases the resulting highway alignment is not the best. Thus, the general procedure for the highway design should be to first determine the highway location and next determine the bridge and/or tunnel sites or consider both simultaneously (Garber and Hoel, 1998). In the HAO model, we consider circumstances where bridge or tunnel construction is more economical than earthwork. During the alignment optimization process, an economic break-even point between earthwork cost and construction cost for bridges or tunnels is determined based on the elevation difference between the ground and the centerline of the highway alignment, and used for evaluating the alignments generated. The characteristics affecting a small tunnel cost may include tunnel length, cross section, clearance, horizontal alignment, and grade. Factors affecting the cost of bridges for
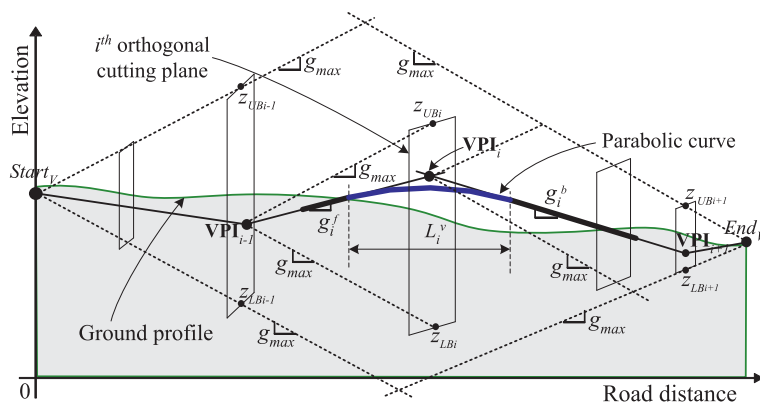

Fig. 5. Geometric specification of a typical vertical curve section used in the HAO model.

crossing rivers or valleys may include the bridge span length and pier height. A methodology that selects bridge or tunnel constructions in lieu of earthwork (if they are more economical) may be found in the authors' previous publication (Kim et al., 2007), and simple tunnel and bridge cost functions used in the model are presented in Appendix B.

Small highway bridges for grade separation and structures for highway junctions (e.g., interchanges and at-grade intersections) are also considered in the model to evaluate cases where the generated alignments cross existing roads. To model those structures, various design codes associated with the existing and new highways (e.g., minimum vertical clearances, widths of two cross roads, design speeds and design vehicles for turn roads), fill and/or cut slopes, intersection angle between the cross roads, horizontal and vertical PI's of the new alignment adjacent the crossing point, and elevation data of the crossing point are used. Modeling of these structures is further discussed below.

Retaining walls (which resist the lateral pressure of soil at highway cut sections) and noise barriers (which block traffic noise from the highway) should also be considered in optimizing highway alignments because locations where those structures are needed and their construction cost may significantly vary with a slight change of highway alignment geometry. Automated methods for determining where such structures are desirable are not yet incorporated in the HAO model.
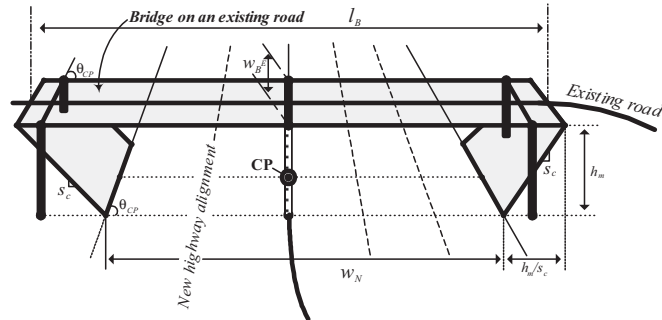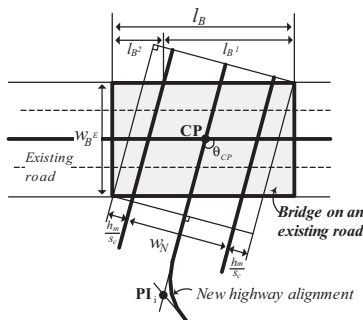
5.2.1. Small highway bridges for grade separation

A small highway bridge structure is used for grade separation where two highways cross. Such a structure can be used not only for grade separation, without any connection to the existing roads being crossed, but also for the part of interchange structures connecting the roads. Normally, for small highway bridges, few piers are used to support spans, and they are equally spaced. In addition, the pier heights may be considerably shorter than for bridges crossing rivers. The pier heights of small highway bridges should satisfy the minimum vertical clearance recommended by AASHTO (2004).

Fig. 6 shows two small highway bridges modeled in the HAO model for grade separation of new and existing roads: (1) a bridge on the new highway where it overpasses an existing road and (2) one on an existing road where the new highway is under-passed. To model the bridge section, various design codes associated with the existing and new highways (e.g., a minimum vertical clearance, fill or cut slope, width of two cross roads), intersection angle between the cross roads, horizontal and vertical PI's of the new alignment adjacent to the crossing point are used. Note that a decision on whether the new highway under-passes or overpasses the existing roads is made by comparing the total earthwork cost and structure cost of the entire alignment for each case. The length of the small highway bridge is calculated with Eq. (1), and can be used for estimating the bridge structure cost (see Eq. (B.7) in Appendix B).

$$l_B = \begin{cases} \left[\frac{w_E + 2h_m/s_f}{\cos(\theta_{CP} - \pi/2)}\right] + \left[w_{B^N} \tan\left(\theta_{CP} - \frac{\pi}{2}\right)\right] & \text{if overpassing an existing road} \\ \left[\frac{w_N + 2h_m/s_c}{\cos(\theta_{CP} - \pi/2)}\right] + \left[w_{B^E} \tan\left(\theta_{CP} - \frac{\pi}{2}\right)\right] & \text{if under-passing an existing road} \end{cases} \tag{1}$$
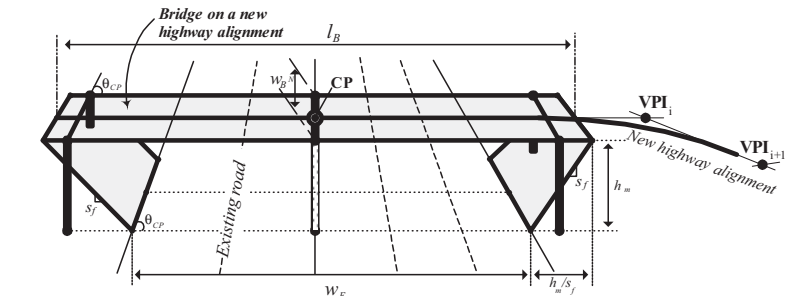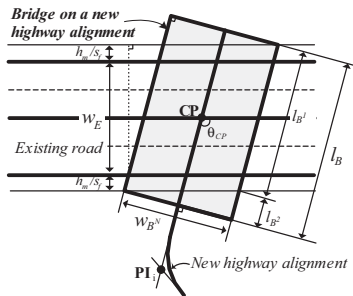
(a) New highway over-passing an existing road

(b) New highway under-passing an existing road

Fig. 6. Small highway bridges for grade separation of existing roads.

5.2.2. Structures for highway junction points with existing roads

For optimizing junction points of a new highway with existing roads, the HAO model allows its users to specify several preferred sub-segments along the existing roads. Such an assumption is realistic in the highway design process since there may be some critical points or locations along the existing roads where junctions between new and existing roads are not permitted. For instance, points near interchanges (or intersections), sharply curved sections in which drivers' sight-distances are insufficient, and bridge sections on existing roads may be unsuitable for junctions. These critical points should be pre-screened before the optimization process. Given such user preferences, design standards for structures and basic information about the existing road (e.g., a horizontal profile of the existing road and its corresponding elevation data), the HAO model generates simple highway structures that represent junctions of the new highway in addition to its alignments. Note that a piecewise linear data format is used to save and extract the coordinates of existing roads.

Among many types of three-leg cross structures where a new highway diverges from an existing road, a simple trumpet interchange and three-leg at-grade intersection (which are most commonly used in highway design process) are considered in the model. Fig. 7 shows the centerline of the two three-leg structures employed for representing the endpoint of a new highway on the existing road. Many reference points (at least 6 and 10 for the at-grade intersection and trumpet interchange, respectively), which include the endpoint and the first or last PI of the new highway alignment, are needed to draw the centerlines of the three-leg structures (see Fig. 7). Such reference points can also be found with simple vector operations as in the highway alignment generation process if design codes for the structures (e.g., minimum vertical clearance, width of the cross roads, design speed and design vehicle for turn roads, fill and/or cut slopes) and the profiles of the existing and new roads are provided.

The at-grade intersection has two separated right-turn ramps; the trumpet interchange comprises a small highway bridge for grade separation and several ramps. Pavement cost, right-of-way cost, and earthwork cost are major construction cost components of the at-grade intersection. For estimating the interchange structure cost, the cost of small highway bridges should be added to the cost items estimated for at-grade intersections. These cost items can be approximately estimated with the horizontal and vertical profiles of roadways associated with the three-leg structures and their cross section information. (see Appendix B). Note that the range of the intersection angle (θ_{EP}) at the crossing point is restricted to $\pi/3 \leqslant \theta_{EP} \leqslant 2\pi/3$ based on AASHTO (2004), which prohibits a sharp cross-angle below $\pi/3$.

Simple four-leg cross-structures, such as Diamond and Clover-typed interchanges and at-grade intersections are also considered in the model. Costs of those structures can also be estimated based on the roadways profiles associated with the four-leg structures. Other complex and large interchanges are omitted here since they require their own vast research areas.

6. Integration of GA and GIS

In the model, a genetic algorithm (GA) with a number of specialized genetic operators is used for optimizing 3D highway alignments. In addition, a geographic information system (GIS) is integrated with the GA to evaluate realistically and comprehensively the highway alignments generated. The primary roles of the GA-based optimization and the GIS module embedded in the model are summarized in Table 4.
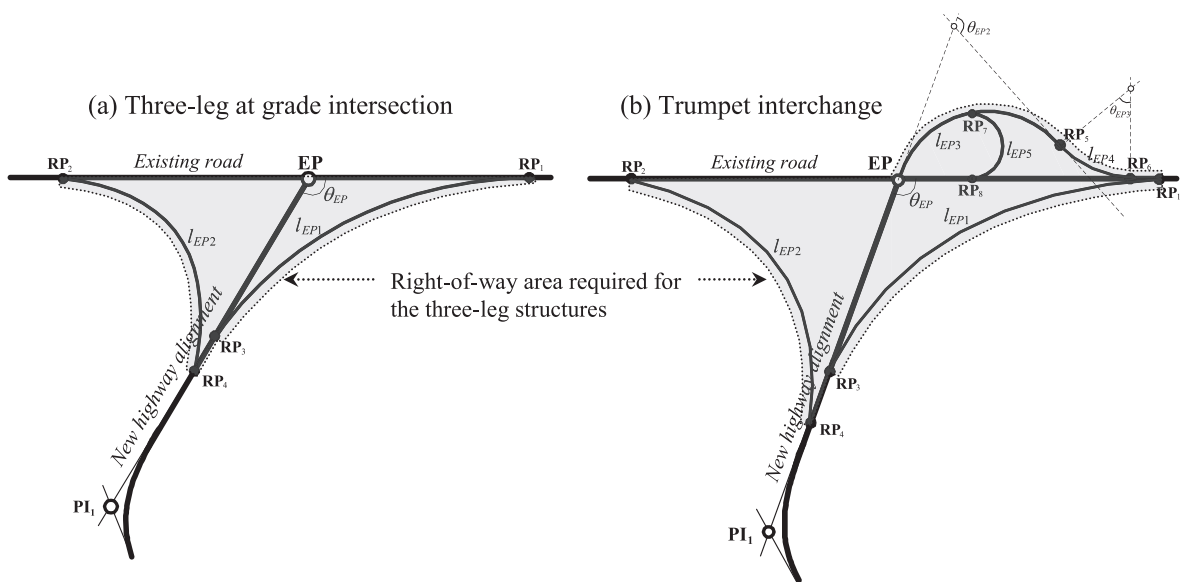

Fig. 7. Simple 3-leg structures represented with a set of reference points.

During the alignment search process, the GA and GIS communicate by exchanging their inputs and outputs (see Fig. 8). First, a set of new highway alignments (i.e., initial population) is generated from the GA. Then, spatial information about the alignments is transmitted to the GIS, and the alignments' right-of-way cost, environmental impact, and socio-economic impact are evaluated in the GIS, while the other alignment-sensitive costs (e.g., earthwork cost and maintenance cost) are evaluated in the GA module. After all the costs and impacts of the alignment set are estimated, they are ranked based on their fitness values (i.e., objective function value). Next, the fittest individuals (i.e., alignments ranked with higher fitness values) survive to reproduce new population of the next generation, whereas the least-fit individuals are eliminated. All these evolutionary steps (i.e., alignments generation, evaluation, ranking, and reproduction procedures) repeat until a specified stop-condition is reached.

Three main types of inputs are needed for optimizing the 3D highway alignments: (1) the design specifications, normally defined based on AASHTO design standards (2004), are needed for generating the highway alignments, which are evaluated based on several unit costs (e.g., unit pavement cost and unit earthwork cost) defined by the model users. (2) The GIS inputs are essential for computing an alignment's right-of-way cost as well as for evaluating its impacts on environmentally and socio-economically sensitive areas in the study region. The model users can also express their preferences, by specifying

Table 4
Summary of principal processes in the HAO model.

Principal process	Role of the principal process

1. GA-based optimization	■ Generating highway alignments
	■ Evaluating major alignment-sensitive costs
	  – Earth work cost/Length-dependent cost/Structure cost/Maintenance cost
	  – Travel time cost/Vehicle operation cost/Accident cost
	■ Searching optimized highway alignments based on the principles of natural evolution and survival of the fittest

2. GIS-based evaluation	■ Evaluating alignment's right-of-way cost ■ Evaluating alignment impacts on the study area – Environmental impact/Socio-economic impact
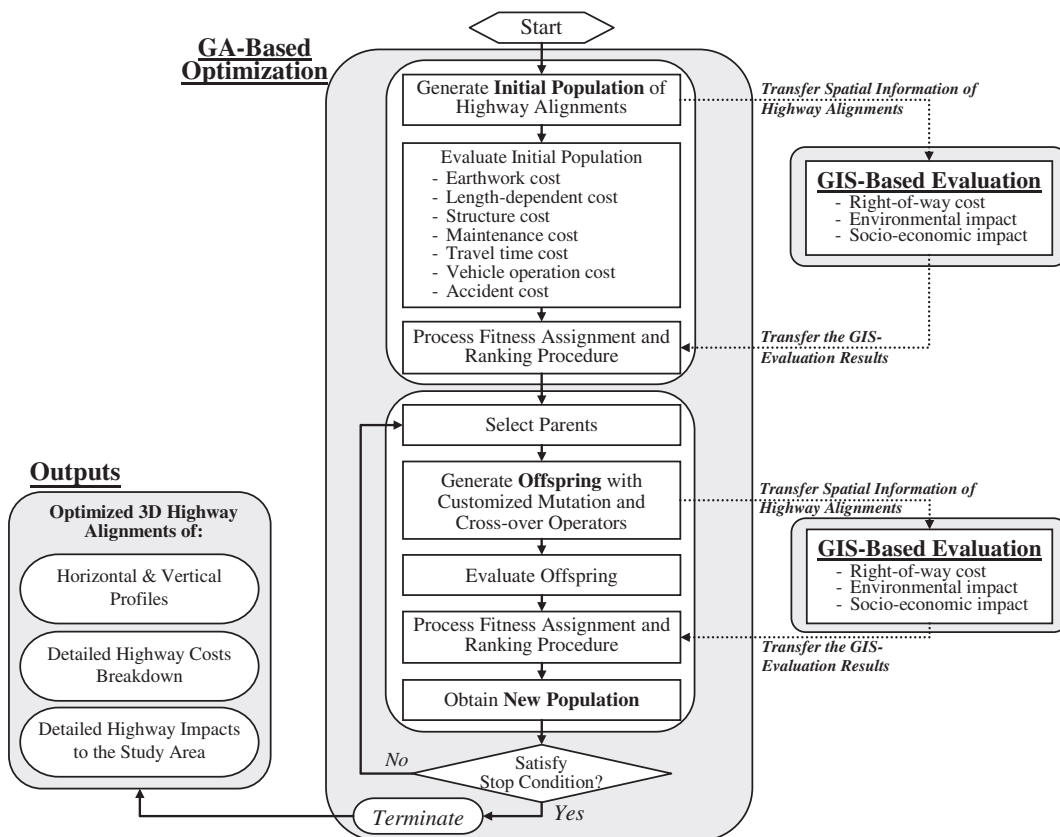
Fig. 8. HAO model structure.

their areas of interest and untouchable areas in the GIS layers. (3) Finally, information about current and future traffic on the new highway is also needed for user cost estimation.

Various practical and quantitative results of the optimized alignments are provided as model outputs. The model output includes horizontal and vertical profiles of the optimized highway alignments, impacts on the study areas, and various alignment-sensitive costs. These are quite useful to the decision makers for identifying and refining new highways. A graphical view of the optimized alignments is also provided on a GIS map as a model output.

7. Basic model formulation

The HAO model searches for the optimized solution by minimizing the comprehensively formulated objective function, while satisfying design, environmental and geographic constraints. The objective function is usually a total cost function (C_{Total}) that comprises ten major components: length-dependent cost (C_L), right-of-way cost (C_R), earthwork cost (C_E), structure cost (C_S), maintenance cost (C_M), travel time cost (C_T), vehicle operation cost (C_V), accident cost (C_A), penalty cost (C_P), and environmental cost (C_{EN}). All these costs are dominating and sensitive to the PI's of highway alignments, and they are formulated as functions of the PI's directly or indirectly in the model (i.e., PI's are decision variables in the HAO problem). A basic formulation for minimizing total cost of highway alignments can be expressed as follows:

$$\underset{x_1, y_1, z_1, \dots, x_{n_{PI}}, y_{n_{PI}}, z_{n_{PI}}}{Minimize} \quad C_{Total} = (C_L + C_R + C_E + C_S + C_M) + (C_T + C_V + C_A) + C_P + C_{EN} \tag{2}$$

$$\text{Subject to}: x_{LBi} \le x_i \le x_{UBi}; \quad y_{LBi} \le y_i \le y_{UBi}; \quad z_{LBi} \le z_i \le z_{UBi} \quad \text{for all } i = 1, \dots, n_{PI} \tag{3a}$$

$$R_{Hm} < R_{Hi}; \quad S_{Hm} < S_{Hi}; \quad S_{Tm} < S_{Ti} \quad \text{for all } i = 1, \dots, n_{HC} \tag{3b}$$

$$L_{Vm} < L_{Vi}; \quad S_{Vm} < S_{Vi}; \quad |g_i| < g_{maxi} \quad \text{for all } i = 1, \dots, n_{VC} \tag{3c}$$

$$A_k < MaxA_k \quad \text{for all } k = 1, \dots, n_{PC} \tag{3d}$$

Note:

$PI_i = (x_i, y_i, z_i)$ for all $i = 1, \dots, n_{PI}$

$R_{Hm}, S_{Hm}, S_{Tm}, L_{Vm}, S_{Vm}$, and g_{max} can be obtained and calculated with design codes.

L_{Vm} and S_{Vm} may be different for the crest and sag vertical curves.

The first five components of the objective function are mainly incurred by highway agencies, and thus can be classified as the highway agency cost. The next three items can be classified as the user cost, and the second from the last is a penalty cost for alignments that violate constraints. Environmental cost (C_{EN}) (which includes various environmental impacts expected along the proposed highway such as air, noise and water pollution costs) can also be added in the objective function if relevant information (such as traffic volume and unit environmental cost per vehicle mile traveled (VMT)) is available. The definitions and underlying concepts for these components are discussed in the next subsections. Their mathematical formulations are not discussed here (but presented in Appendix B) since they may be found in the authors' earlier publications (Jong and Schonfeld, 1999; Jha and Schonfeld, 2000, 2003; Kim, 2001; Kim et al., 2004, 2007; Kang et al., 2007; Kang, 2008).

7.1. Highway agency cost

The length-dependent cost (C_L) is defined as the cost proportional to alignment length. Initial highway pavement cost and costs of miscellaneous highway facilities for vehicle operation (such as guardrails, lighting poles, and medians installation costs) may be included in this category. The right-of-way cost (C_R) can be estimated by computing total cost of areas affected by the highway within its required right-of-way boundary. Thus, values of the affected land parcels as well as their actual areas needed for the right-of-way of the highway are necessary for the right-of-way cost estimation. The earthwork cost (C_E) is calculated based on (i) terrain profile (i.e., ground elevation) of the study area and (ii) road heights at each major break point (e.g., every 100 foot station) in the terrain surface along its vertical alignment. The ground elevation of the study area should be provided as a model input, and the vertical alignment is generated from the model. The costs required for those structures (C_S) are also sensitive and dominating in highway construction, and thus they should also be included in the objective function. Finally, besides the cost components that are initially required for a new highway construction (e.g., C_L, C_R, C_E, and C_S), the HAO model also considers the highway maintenance cost (C_M). To realistically evaluate this cost, the entire section of the new highway is subdivided into multiple subsections (e.g., highway basic segments and highway bridge sections), and their corresponding maintenance costs are calculated separately. Detailed highway agency cost functions used in the model are presented in Appendix B.

7.2. User cost

The travel time cost (C_T) is calculated based on the amount of time spent for traveling and the drivers' perceived value of time. In the model, it is assumed that two types of user classes (auto and truck) operate on the new highway, and they have

different values of travel time for different trip purposes. Note that a detailed trip purpose factor for each user class (such as, home-based work and non-home-based trips) is not considered in the model, since it may not be available in the initial stage of the highway project due to time and/or money constraints. Vehicle occupancy information is also used for evaluating travel time cost of highway users more precisely.

Another user cost component considered in the model is the vehicle operating cost (C_V) that can be directly perceived by the highway users as an out-of-pocket expense incurred while operating vehicles. This may include fuel consumption, maintenance, tire wear, and vehicle depreciation costs. However, since the vehicle depreciation cost is not sensitive to different highway alternatives, only the first three items, which are the most dominating and sensitive ones, are considered in the model. Generally, the vehicle operation cost can be calculated based on a per vehicle-mile basis, and thus length of highway alignment as well as traffic information (such as traffic flow, travel time, and speed) are necessary for estimating it.

Estimating highway accident cost (C_A) is relatively difficult since accidents are caused by combinations of various factors (such as traffic volume, highway geometry, and driving conditions of users operating on a highway). The accident cost can be estimated with unit accident cost (e.g., \$/accident) and the number of accidents predicted from an accident regression analysis. In the highway accident literature, it is obvious that highway geometric design elements affect road collisions. For instance, a sharp horizontal curve with insufficient tangent approach may cause a significant safety problem (Glennon et al., 1985). Thus, the new highway must satisfy highway design standards, and collision effects of various highway alternatives with different geometric design elements should be evaluated in the alignment optimization process. Note that the proposed highway alignment optimization model is designed with a modular structure in which various evaluation components can be easily replaced without changing the rest of the model structure. Thus, any available accident prediction relations or models can be incorporated in the model for estimating the accident frequency of new highways. The user cost functions employed in the model are presented in Appendix B.

7.3. Penalty cost

Various types of environmental and socio-economic areas may be included in the study area of a new highway construction. These are, for example, wetland, wild-life refuge, and residential areas and alignment's impacts on such land-use types should be as minimal as possible and if any, special care should be taken to replace and restore them. In the model, GIS maps containing various geographic entities are provided as a model input, and the highway alignment under evaluation is overlaid on the GIS maps for estimating its impact to the study area. Thus, the fractions of affected land parcels needed for the alignment is computed. If the area of the land parcel affected by the highway alignment exceeds its pre-defined maximum allowable limit (defined as *MaxA*), a penalty is applied to the excess area. A soft penalty function (Eq. (B.23) in Appendix B) is used for computing this penalty, and included in the total cost function to smoothly guide the search in the optimization process. The penalty function is also used in a case where a generated alignment violates the specified design constraint, for instance if the alignment is insufficient to accommodate minimum curve length (Kang et al., 2009).

7.4. Life-cycle cost

Road maintenance cost occurs throughout the entire design-life of the road. The user cost also persists over the system design life. Traffic demand fluctuates over daily, monthly, and yearly cycles, and tends to increase over the system design life. Thus, these costs should be estimated as a life-cycle cost, by being discounted over the estimated system life at an appropriate interest rate and traffic growth rate. Eqs. (B.15)–(B.21) in Appendix B show the formulations used in the model for estimating the present value of those costs. Environmental costs (C_{EN}), such as air, noise and water pollution costs triggered along the highway, may also be analyzed over the system design life because they are also affected by traffic characteristics as well as road condition (Eq. (B.26)).

8. Model application

Two actual highway construction projects in the state of Maryland have been analyzed and optimized using the proposed HAO model. These are: Case 1 for *MD 97 Brookeville Bypass project* in Montgomery County and Case 2 for *US 220 Alternative Alignments Development* in Allegany County. The model searched over 300 generations, thereby generating and evaluating about 10,000 alternative alignments for each case. Desktop Intel® Core™2 Duo PCs with 2 GB RAM were used to run the model.

To conduct the case studies, customized GIS maps, which include (1) a horizontal study area map and (2) an elevation map, were built through an input preparation process. The GIS inputs are customized to be directly accessed by the HAO model during the alignment optimization process. For preparation of the GIS inputs we used as-built plans of existing roads, a digital elevation raster dataset (or a study area elevation map in a CADD file format), and Maryland's GIS database (called MdProperty View), which includes various land-uses, land values, and information on natural resource and manmade features in the project area.

The horizontal study area map describes spatial location and value of various land-uses in the project area. The study area map should realistically represent the complex land-use system of the project area, and is eventually used for evaluating the

right-of-way cost and environmental impact of the alignments generated during the optimization process. Each property in the study area map must be a polygon which stores property value, area, land-use type, and decision maker's preference information (e.g., index to identify decision maker's area of interest and maximum allowable limit that can be affected by the alignment) in its attribute table. Note that the model users can also narrow down the search boundary (i.e., can define feasible regions) of solution alignments, in the process of preparing the study area map, for an efficient optimization process. Table 5 shows an example attribute table of the study area map used in the HAO model.

In the model the alignment earthwork cost is calculated based on the elevation map, whose preparation for the project area is also required. A high resolution elevation map (either in a digital elevation model (DEM) or a raster format) is preferred for precise earthwork volume calculation. If a DEM or raster format of the elevation map is not available, at least a CADD file which represents the elevation of the project area should be prepared for conversion to a DEM or raster file format.

Besides the customized GIS maps, various input parameters that describe unit costs and design specification of the proposed highway are needed to optimize highway alignments using the model. These are, for instance, unit costs for pavement and earthwork cost estimation, cut and fill slopes, design speed, maximum grade, and cross-section (including travel lane and shoulder widths) of the proposed highway. Since the optimized alignment varies depending on these inputs, the model users should carefully determine values of the input variables for particular circumstances.

8.1. Case 1

The Maryland State Highway Administration (MDSHA) has been working on the MD 97 Brookeville Bypass project in Montgomery County, Maryland. This area is listed on the National Register of Historic Places as a historic district, and is located approximately ten miles south of I-70 and three miles north of MD 108. The project objectives are to divert the increasing traffic volumes from the town of Brookeville by constructing a new bypass route so as to improve traffic operation and safety on existing MD 97, while preserving the historic character of the town. The HAO model has been used in this real highway construction project to assist the responsible government agencies in finding the best alternatives. Fig. 9 shows various natural and cultural land-use patterns in the project areas as well as the given start and end points of the proposed highway. The study area comprises about 650 geographic entities.

In this case study the start and end points of the proposed highway are given, and located on the south and north sections of MD 97 in Brookeville. The Euclidean distance between the two endpoints is about 1.22 km. The design speed is initially set at 80 kph, and the cross-section of the proposed alignment is assumed to represent a 2-lane road with 12 m width (3.3 m for travel lanes and 2.7 for shoulders). In addition, grade separation is assumed to be the only option for crossing the existing Brookeville Road. Baseline input parameters employed in the model application to this case study are presented in the table included in Fig. 9. Note that to ensure comparability with the normal evaluation criteria typically used by the responsible government agencies (i.e., MDSHA in this case), the user cost components and environmental cost are suppressed from the objective function, and thus $C_{Total} = C_L + C_R + C_E + C_S + C_M + C_P$.

8.1.1. Handling environmental issue

When considering roadway construction in a given project area, various geographically and/or environmentally sensitive regions may be encountered. These control areas should be avoided by the proposed highway alignment, whose impact on these regions should be minimized to the extent possible. Based on a previous Brookeville study by MDSHA (2001), we recognized residential properties, a community center, historic districts, and wetlands as primary sensitive areas that should be avoided by the proposed highway if at all possible (i.e., those are untouchable areas). In addition, parklands, floodplains, and streams, which are located in between the two endpoints and thus unavoidably be taken by the proposed highway, are con-

Table 5
Example attribute table of the GIS study area map used in the HAO model.

Shape	ID	Area (ft^2)	MaxA (ft^2)	Value ($/ft^2)	Land-use	Natural	Manmade	AOI
Polygon	1	3504	3504	0.15	Farm	0	1	1
Polygon	2	1000	0	0.01	Wetland	1	0	1
Polygon	3	2035	200	10.20	Resident	0	1	0
Polygon	4	4082	0	0.25	Park	1	0	1
Polygon	5	1730	0	0.12	Cemetery	0	1	1
Polygon	6	2150	0	13.44	Commercial	0	1	0
Polygon	7	1830	0	12.63	Resident	0	1	1
Polygon	8	1632	1632	0.02	Stream	1	0	0
Polygon	9	1024	0	2.16	Historic	1	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

MaxA: User-specifiable maximum allowable area affected by the alignment.
Natural: Index to identify whether the selected property is a natural resource.
Manmade: Index to identify whether the selected property is a manmade feature.
AOI: Index to identify whether the selected property is inside area of interest.

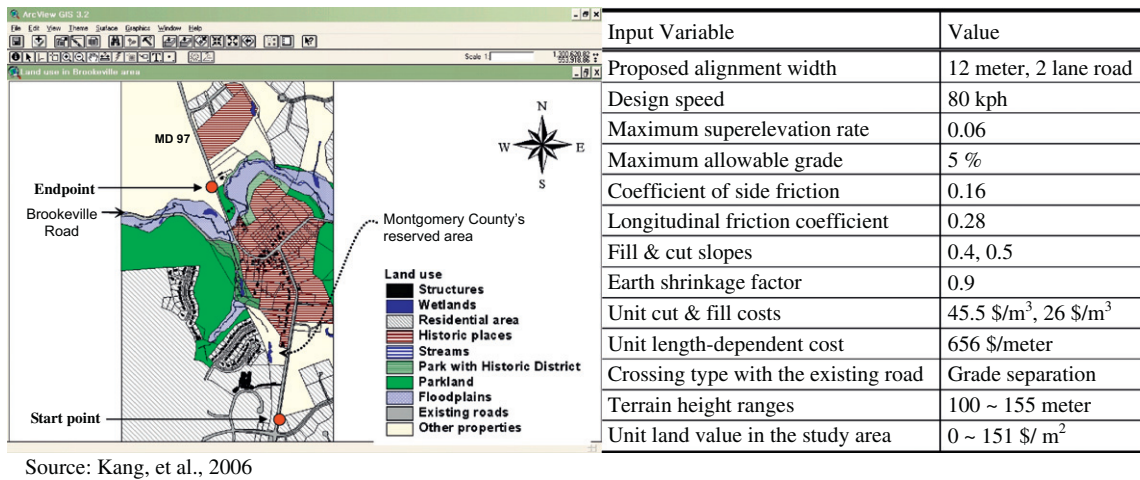| Input Variable	Value
Proposed alignment width	12 meter, 2 lane road
Design speed	80 kph
Maximum superelevation rate	0.06
Maximum allowable grade	5 %
Coefficient of side friction	0.16
Longitudinal friction coefficient	0.28
Fill & cut slopes	0.4, 0.5
Earth shrinkage factor	0.9
Unit cut & fill costs	45.5 \$/m^3, 26 \$/m^3
Unit length-dependent cost	656 \$/meter
Crossing type with the existing road	Grade separation
Terrain height ranges	100 ~ 155 meter
Unit land value in the study area	0 ~ 151 \$/ m^2

Source: Kang, et al., 2006

Fig. 9. Case 1 study area and baseline inputs used for the HAO model.

sidered secondary sensitive areas. To realistically represent such control areas in the model application, we divide them into two categories based on their land-use characteristics, as shown in Table 6: Type 1 areas that the proposed roadway alternatives can avoid, and Type 2 areas that the proposed alternatives cannot avoid.

To properly reflect these relevant issues in preparation of the customized study area map, tradeoff values with respect to the different land-use types must be carefully determined based on their relative importance, because these values may significantly affect the resulting alignments. Thus, the maximum allowable areas that can be affected by the proposed highway (denoted as *MaxA*) are set to be much stricter for Type 1 areas than for Type 2 areas; recall that a Type 1 area has primary control regions to be avoided by the alignments whereas a Type 2 area contains only secondary regions. The idea is to eliminate the alignments' impacts on Type 1 areas and minimize those on Type 2 areas, by guiding the alignments to take other less desirable properties, which have no restrictions. For this purpose, we discriminated between Type 1 and Type 2 areas by assigning different values of *MaxA*. For primary control regions classified as Type 1, their *MaxA* are set to be 0 (which means Type 1 areas must not be affected by new alignments), while the *MaxA* of secondary control areas defined as Type 2 can be interactively specified by the model users based their relative importance.

8.1.2. Optimized alignments

An optimized alignment found with the specified inputs is shown in Fig. 10. It took only about 4.5 h to obtain the optimized alignments through 300 generations because the Case 1 study area is small (2.5 km long and 1.1 km wide) and contains relatively few (about 650) geographic entities. As shown in Fig. 10, the profile of the optimized alignment does not affect any primary control regions (i.e., Type 1 area) without any residential, commercial, and historic site relocation. It fully utilizes a reserved area (by Montgomery County government) for the new bypass, while unavoidably affecting some fractions of parkland and floodplains in the study area.

Recall that the model objective function consists of the six cost components ($C_L + C_R + C_E + C_S + C_M + C_P$) in this case study. Thus, the model attempts to find a particular trade-off that minimizes the total sum of all the cost components, while satisfying the specified design and environmental constraints. It is noted however that this does not necessarily mean the distribution of cost will finally be balanced among the individual cost items. Several solutions may exist with similar total cost (i.e., objective function value) but quite different distributions of it.

The total cost of the optimized alignment for this case study does not include any penalty cost (see Fig. 10), indicating that no constraints are violated. The result also shows that (i) the earthwork cost and (ii) structure cost (for a small bridge and grade separation with the existing road) constitute the first and second highest fractions of the total cost, respectively. They dominate the other cost components included in the total cost. The length-dependent cost and the right-of-way cost also account for large fractions of the total cost. These results suggest that the appropriate objective function should be carefully chosen to reflect all important highway cost items in the alignment optimization process. This importance will be further addressed in Section 9.2.1, by analyzing the sensitivity to changes in the model's objective function.

Table 6
Types of spatial control areas in the Brookeville study area.

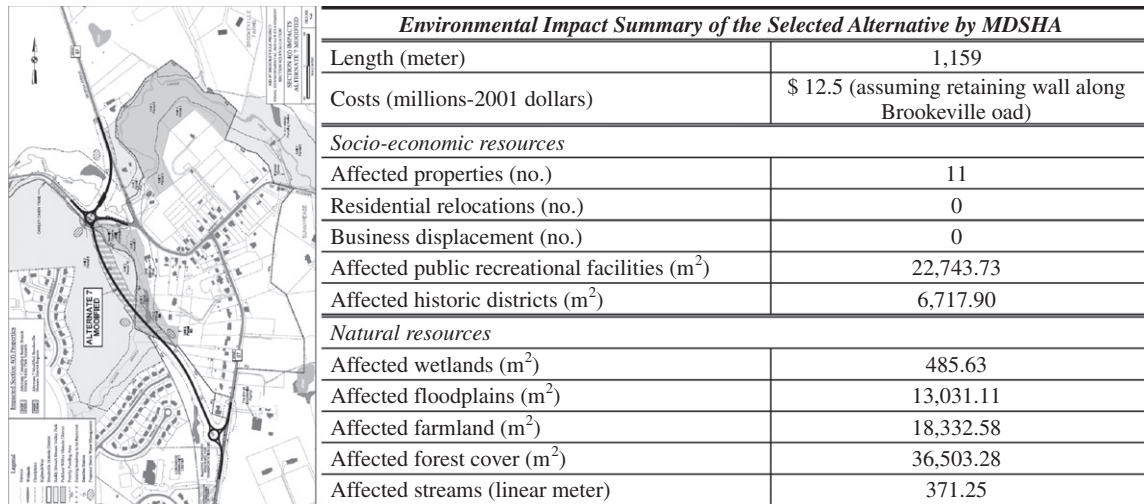| Type	Characteristics	Control areas	Allowable limit
Type 1	■ The control areas that the proposed alignment can avoid	■ Wetlands, historic places, residential areas, a community center, structures (houses, public facilities, etc.)	$MaxA_k = 0$
Type 2	■ The areas that the proposed alignment cannot avoid	■ Streams, floodplains, parklands	User-specifiable ($0 < MaxA_k \leqslant A_T$)
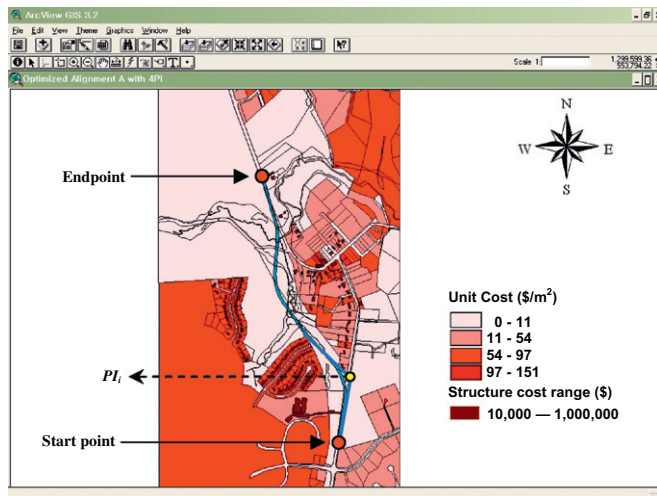
Alignment length (meter)		1,278
Total cost (million $)		4.63
Fraction of Total Cost	Length-dependent cost (%)	20
	Right-of-way cost (%)	12
	Earthwork cost (%)	37
	[1]Structure cost (%)	24
	Maintenance cost (%)	7
	Penalty cost (%)	0
Type 1 Area	Affected residential area (m^2)	0
	Residential relocations (no.)	0
	Affected community center (m^2)	0
	Affected historic districts (m^2)	0
	Affected wetlands (m^2)	0
Type 2 Area	Affected floodplains (m^2)	1,603
	Affected streams (m^2)	72
	Affected parkland (m^2)	4,177
Other	[2]Affected MC's reserved area (m^2)	4,206
	Affected existing roads (m^2)	2,749

[1] It only includes costs for small highway bridges.
[2] Montgomery County's reserved area for the new bypass.

Fig. 10. Case 1 optimized alignment obtained from the HAO model.

Environmental Impact Summary of the Selected Alternative by MDSHA	
Length (meter)	1,159
Costs (millions-2001 dollars)	$ 12.5 (assuming retaining wall along Brookeville oad)
Socio-economic resources	
Affected properties (no.)	11
Residential relocations (no.)	0
Business displacement (no.)	0
Affected public recreational facilities (m^2)	22,743.73
Affected historic districts (m^2)	6,717.90
Natural resources	
Affected wetlands (m^2)	485.63
Affected floodplains (m^2)	13,031.11
Affected farmland (m^2)	18,332.58
Affected forest cover (m^2)	36,503.28
Affected streams (linear meter)	371.25

Source: MDSHA, 2001

Fig. 11. MDSHA's selected alternative for the Brookeville bypass project.

Fig. 11 shows one of the most favored alternatives obtained manually by the MDSHA. The agency has obtained the alternative through a repetitive time-consuming process, while the proposed model can allow a quick alignment optimization process (within of several hours) with remarkable precision. A comparison of the MDSHA's alternative with the one optimized by our model indicates that the configurations of the two alternatives look similar; however, the total costs estimated and other important measures of effectiveness (MOEs) (e.g., total length and affected historic districts and wetlands) used for evaluating the alternatives are different. This is largely due to the resource limitations imposed by the manual method on the search for alternatives, sensitivity analysis and trade-off analysis. It should also be noted that the total cost of the optimized alignment is somewhat underestimated since it does not include several miscellaneous highway costs (e.g., drainage and other contingency cost) which are included in the manual solution.

8.2. Case 2

The HAO model has also been applied for the development of alternative alignments of the existing US 220 highway located in the western Maryland. The Case 2 project area is about 20 km-wide and 25 km-long (see Fig. 12), which is significantly larger than the Brookeville example. It is located in the Appalachian Mountains, and thus its ground elevation varies greatly.

Various land-uses (such as forest, river, agricultural and residential areas) exist in the project area, which is mostly covered by forest and cropland. In addition, the project area has many geographically sensitive regions (such as floodplains, state parks, protected lands, and wetlands) that must be carefully considered in selecting the highway location. Many priority funding areas (PFAs), designated by state and federal agencies, are also located in the project area, thus favoring (e.g., with low land cost) candidate alignments which use those areas for their rights-of-way.

The search for the optimized alternative alignment was made within a 4000 foot (1.2 km) wide buffer of the existing US 220 between I-68 near Cumberland, Maryland and the West Virginia state line, a distance of approximately 30 km. The search limit was recommended by a preliminary environmental impact study by MDSHA (2011) which had been prepared as a result of the *North South Appalachian Corridor Feasibility Study* by the West Virginia Department of Highways (WVDOH, 2008). Note that the WVDOH study showed that the improvement of the existing US 220 has great potential for benefiting the economic development of Appalachian region. Thus, in this case study we have applied the HAO model to search for the most economical alternative alignments of US 220 within the search limit (i.e., a 1.2 km-wide and 30 km-long buffer), while preserving environmentally sensitive areas. The proposed highway is a four-lane rural divided arterial with partial access controls as suggested in MDSHA (2011). Its typical section is 30 m-wide with a median in the middle and paved shoulders at the both sides of each direction. The design criteria and other important factors employed in this case study are also shown in Fig. 12. Design of interchange and/or intersection configurations at the northern and southern ends of the proposed highway was not included in the scope of this case study.

8.2.1. Protected land and narrow gate

There are many protected lands in the Case 2 study area. These areas are protected for the jurisdiction based on various federal, state, and local governments programs. Dan's Mountain State Park (which includes a wide variety of wildlife, mountain streams, and scenic overlooks), Selinger Marsh Preservation area, and environmentally preserved area by the Nature Conservancy are parts of the protected land. As shown in Fig. 12, many protected lands are located in the study area, and they make the alignment search boundary complex. The protected land is regarded as an untouchable region in this project, so that it should be excluded as a possible location of the proposed highway. Thus, the alignment search boundary is
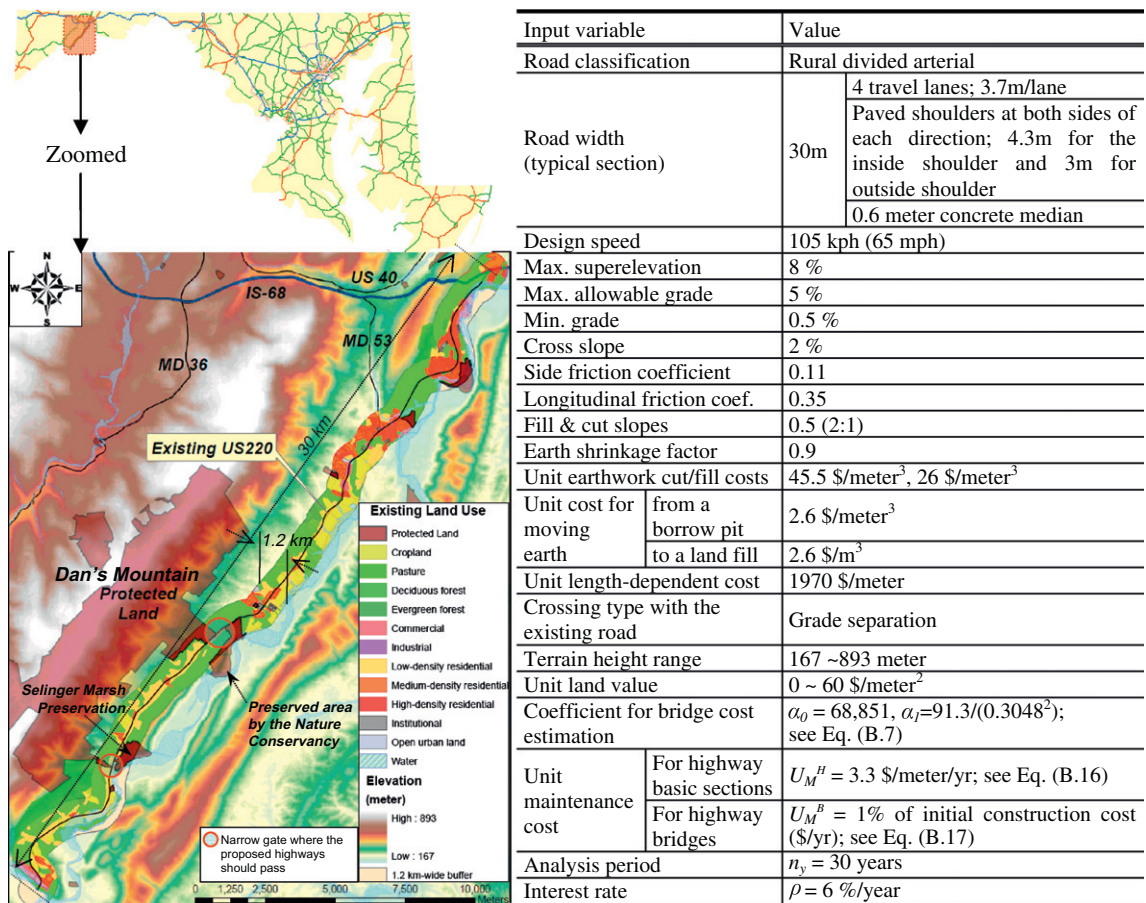

Input variable		Value	
Road classification		Rural divided arterial	
Road width (typical section)		30m	4 travel lanes; 3.7m/lane
			Paved shoulders at both sides of each direction; 4.3m for the inside shoulder and 3m for outside shoulder
			0.6 meter concrete median
Design speed		105 kph (65 mph)	
Max. superelevation		8 %	
Max. allowable grade		5 %	
Min. grade		0.5 %	
Cross slope		2 %	
Side friction coefficient		0.11	
Longitudinal friction coef.		0.35	
Fill & cut slopes		0.5 (2:1)	
Earth shrinkage factor		0.9	
Unit earthwork cut/fill costs		45.5 \$/meter^3, 26 \$/meter^3	
Unit cost for moving earth	from a borrow pit	2.6 \$/meter^3	
	to a land fill	2.6 \$/m^3	
Unit length-dependent cost		1970 \$/meter	
Crossing type with the existing road		Grade separation	
Terrain height range		167 ~893 meter	
Unit land value		0 ~ 60 \$/meter^2	
Coefficient for bridge cost estimation		$\alpha_0 = 68,851$, $\alpha_l = 91.3/(0.3048^2)$; see Eq. (B.7)	
Unit maintenance cost	For highway basic sections	$U_M^H = 3.3$ \$/meter/yr; see Eq. (B.16)	
	For highway bridges	$U_M^B = 1\%$ of initial construction cost (\$/yr); see Eq. (B.17)	
Analysis period		$n_y = 30$ years	
Interest rate		$\rho = 6\%$/year	

Fig. 12. Case 2 study area and baseline inputs used for the HAO model.

narrowed at several locations with reference to the spatial location of the protected land (see Fig. 12). The narrowed search boundary, called "narrow gate", guides the proposed model to avoid generating infeasible alignments that violate the spatial constraints, and thus to focus the search on the feasible solutions (Kang et al., 2007). The more complex in topography and land-use of the study area is, the more narrow gates are desirable in the alignment search space for an efficient search process.

8.2.2. Optimized alignments

The model objective function used in this case study also includes the six agency cost components (length-dependent, right-of-way, earthwork, structure, maintenance, and penalty costs) as used in Case 1. The assumed design life of the proposed highway is 30 years and a 6% annual interest rate is used to estimate the present value of the highway maintenance cost. Note that the penalty function, shown in Eq. (B.23), is employed to evaluate the amount of environmentally sensitive areas taken by the highway alignments generated by the model.

A search was made through 300 generations to obtain the optimized alignment. Computations for Case 2 (about 110 h with a Desktop Intel® Core™2 Duo PC with 2 GB RAM) took longer than for Case 1 due to the land-use complexity and large scale of the project. A Case 2 optimized alternative alignment found by the HAO model is shown in Fig. 13. Its total length is about 28.9 km, and consists of eight road segments (A-B-C-D-E-F-G-H) as displayed with different colors. Segments A, C, E, and H share the right-of-way of the existing US 220 but widen it, while segments B, D, F, and G are new bypasses. The horizontal profile of the optimized alignment successfully avoids environmentally sensitive areas (including the protected lands and state parks) and high land cost areas. However, it unavoidably affects some residential and commercial areas (see Table
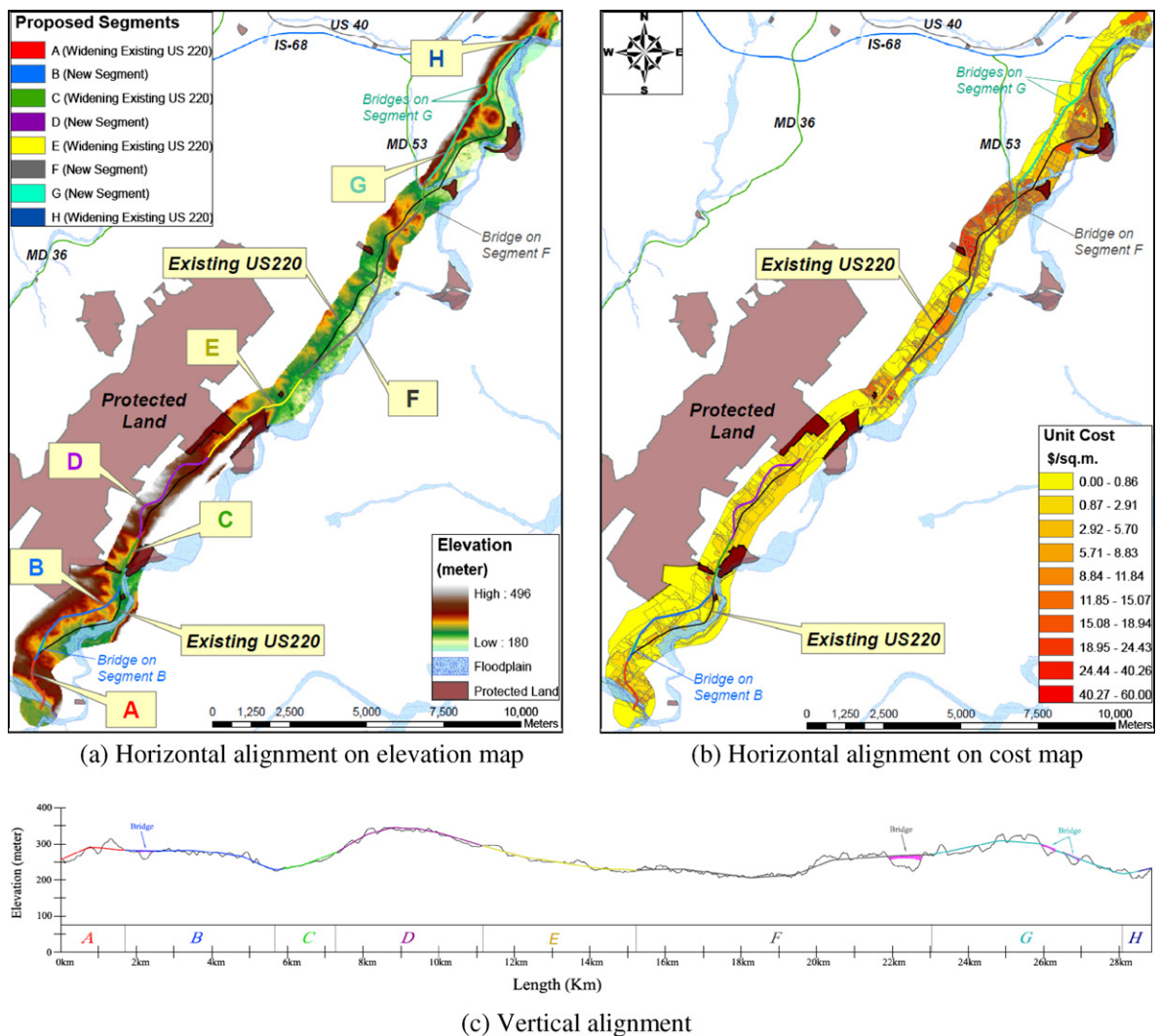

(a) Horizontal alignment on elevation map

(b) Horizontal alignment on cost map

(c) Vertical alignment

Fig. 13. Case 2 optimized alignment obtained from the HAO model.

Table 7
Total cost and areas for Case 2 optimized alignment by land-use type.

| Road segment	Total cost (million $)	Length (m)			Areas taken by the proposed road segment (m²) by land use type									Protected land (m²)	Flood-plain (m²)	Existing road (m²)	PFA (m²)
		Total	Basic section	Bridge	Total	Residential	Commercial	Industrial	Institutional	Start Park	Forest	Cropland	Pasture				
A	52	1703	1703	0	139,001	23,972	36,317	0	0	0	44,454	34,258	0	0	14,874	50,781	123,966
B	50	3967	3613	354	268,136	54,975	0	0	0	0	194,230	18,931	0	0	0	18,515	76,933
C	9	1593	1593	0	70,160	65,257	0	0	0	0	4903	0	0	0	8515	41,863	39,084
D	50	3895	3895	0	270,302	150,875	0	0	0	0	89,895	29,532	0	0	0	20,862	185,502
E	54	4064	4064	0	208,491	112,145	3174	0	0	0	92,857	0	314	0	0	63,845	141,853
F	179	7809	6986	823	639,955	230,762	22,107	0	0	0	129,682	204,352	53,053	0	39,265	88,895	621,234
G	222	5486	4785	701	649,170	26,091	59	196	0	0	622,824	0	0	0	30,588	8484	84,573
H	9	341	341	0	32,002	0	0	0	0	0	32,002	0	0	0	32,009	7365	25,598
Total	624	28,857	26,979	1878	2277,216	664,077	61,657	196	0	0	1210,846	287,073	53,367	0	125,252	300,609	1298,742
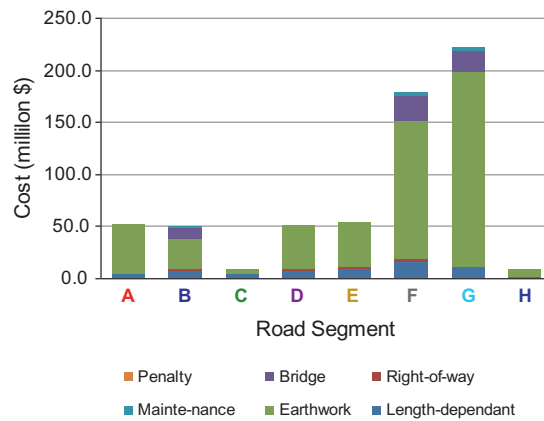
Fig. 14. Total cost breakdown for the Case 2 optimized alignment.

7) due to the complexity of the land-use within the 1.2 km-wide buffer. There are four bridges in the entire section of the optimized alignment: one on each of segments B and F, and two on segment G. Among them, the bridge on segment F provides grade separation for the existing US 220, while the others on segments B and G are chosen because bridge construction is more economical there than earthwork. No tunnel is considered in this project.

Fig. 14 shows the total cost breakdown for the Case 2 optimized alignment, which is based on the best trade-offs among the six cost components. It shows that the earthwork accounts for a significant fraction (about 80%) of the total cost, which is much more than in the Brookeville case. The bridge structure also accounts for the large fraction of the total cost; however, the fraction of the right-of-way cost is negligible, because the Case 2 project area is mountainous and property values in it are relatively low. Other costs such as user cost, contingency cost and utility relocation cost are not considered in this case study, and thus the total cost is underestimated. It is noted that among the eight road segments, segments B, F, and G account for more than 72% of the total cost, and cover about 60% of the entire section. Earthwork, bridge, and length-dependent costs are the three major agency costs for these segments; however, the maintenance cost also constitutes a substantial fraction.

9. Sensitivity analysis

9.1. Sensitivity of optimized alignments to the number of PI's

Selection of the preferable number of PI's is important in the model's application to individual projects since such a parameter may affect the precision of the solutions (i.e., total cost and configuration of generated alignments). Moreover, model computation time may vary with this parameter. To explore the preferable number of PI's in the alignment optimization, we ran the model several times with different numbers of PI's for the Brookeville example. Fig. 15a presents the optimized alignments resulting with varying number of PI's (from 4 to 7). No more than 8 PI's were considered, in order to avoid too many possible curves in this small-scale highway project.

It is found that rights-of-way of all four optimized alternatives are similar; however, detailed model outputs, such as total cost and the environmentally sensitive areas affected by the alignments, are slightly different. Among the four optimized alternatives, the least total cost is found with 5 PI's (i.e., alternative B). In terms of environmental impact, the sensitive areas taken by alternative B are the lowest, although the differences are negligible among the four alternatives. None of the four alternatives require any residential relocation and all have similar alignment lengths. In terms of computation efficiency, the model's computation time increases with the increase of the number of PI's because the PI's coordinates are the model's decision variables (defined as genes) that need to be mutated themselves as well as crossed over with others to reproduce new offspring. In the Brookeville example, however, the variation of model's computation time is negligible (from 4.4 h to 5) as the number of PI's increases from 4 to 7 because the project scale (e.g., the size of the project area and the number of geographic entities for evaluating environmental impact and right-of-way cost) is small. Fig. 15b shows changes in model's objective function value over successive generations for the four optimized alternatives. It is observed that most of the improvement is found in the early generations, and the improvement becomes negligible after a certain number of generations. This indicates that the model can provide reliable (though not guaranteed optimal) results quite quickly.

9.2. Sensitivity of optimized alignments to major input parameters

This section discusses the sensitivity of optimized alignments with respect to critical input parameters used in the model, such as components of the objective function, design speed, and elevation grid size. To examine the influence of such factors on
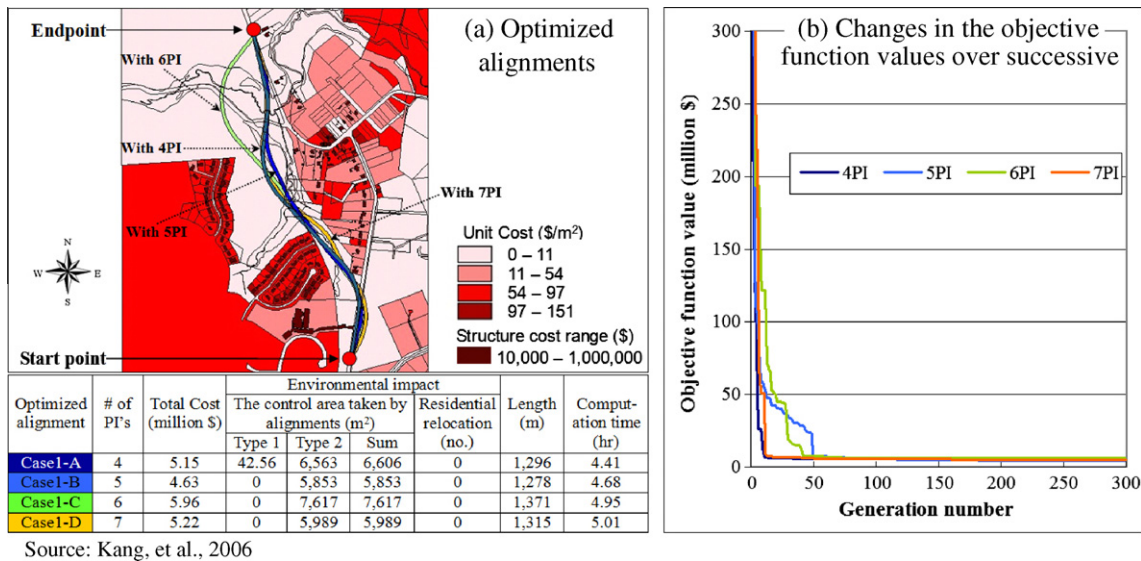
Fig. 15. Sensitivity of optimized alignments to the number of PI's.

the solution quality, the input parameter values used in Brookeville example (shown in Fig. 9) are used as the default values.

9.2.1. Sensitivity to model objective function

This analysis is intended to show that all the alignment-sensitive costs should be considered and precisely formulated for a good highway optimization model. Three different scenarios are designed to show how each cost items affects the resulting alignments. All input parameters used in the scenarios are identical except the cost items composed of the objective function as follows:

- Scenario 1: $C_{Total} = C_L + C_S + C_M$.
- Scenario 2: $C_{Total} = C_L + C_S + C_M + C_R + C_P$ (added right-of-way and penalty costs to Scenario 1).
- Scenario 3: $C_{Total} = C_L + C_S + C_M + C_R + C_E + C_P$ (added earthwork cost to Scenario 2).

The horizontal alignment being optimized with Scenario 1 is a straight line, as shown in Fig. 16a1, and it affects many high-cost and environmentally sensitive areas (e.g., residential and historic areas). In addition, Fig. 16b1 shows that its corresponding vertical alignment differs greatly from the ground profile without being optimized. Such results occur because the results of Scenario 1 are optimized with an objective function that does not represent the complexity of land-use system as well as topography of the study area. The objective function in this case does not include alignments' right-of-way cost, earthwork cost, and penalty cost (i.e., environmental impacts) on the sensitive areas. Fig. 16a2 shows the horizontal align-
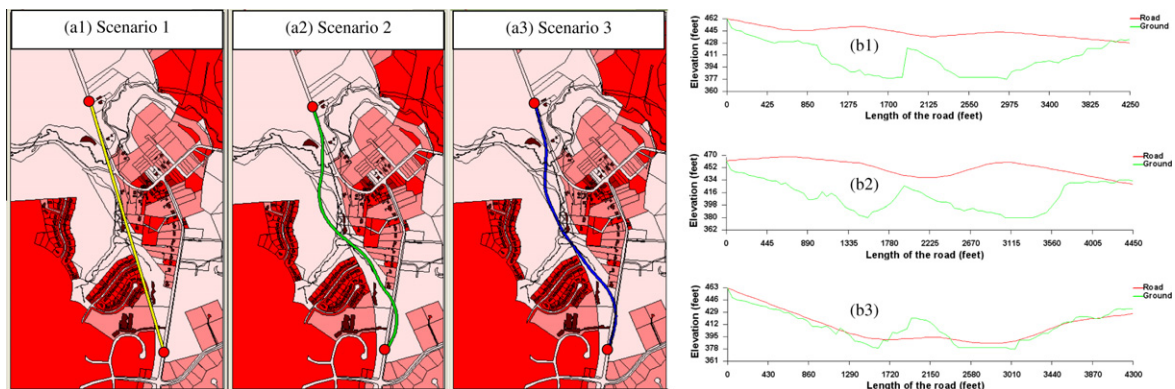

Fig. 16. Sensitivity of optimized alignments to decision criteria.

ment being optimized for Scenario 2. As shown in the figure, this hardly affects the area requiring high land-acquisition cost and is relatively circuitous due to its avoidance of environmentally sensitive areas. However, its vertical alignment (Fig. 16b2) is still not optimized (i.e., still quite far from the ground profile) since Scenario 2 does not consider the earthwork cost component. The horizontal and vertical profiles of the optimized alignment found with all the major six costs (Scenario 3) are presented in Figs. 16a3 and b3, respectively. Although the horizontal alignment of Scenario 3 is similar to that of Scenario 2, its vertical alignment is quite different. As shown in Fig. 16b3, its vertical profile closely follows the ground elevation. This occurs because horizontal and vertical alignments are optimized jointly while minimizing its earthwork cost as well as the other four cost components. These results indicate that all the major costs associated with road construction should be simultaneously evaluated for comprehensively optimizing highway alignments.

9.2.2. Sensitivity to design speed and related geometric design parameters

This analysis tests how the solution alignments optimized by the model are sensitive to critical geometric design parameters. It is important to note that the design speed is the key geometric control variable in highway design process, and interrelated with many other design features of a highway alignment (such as the horizontal curve radius, sight distance, and vertical curve length). In the model, the design speed is determined by model users as an input and other design features of the solution alignments are computed based on it. As shown in Fig. 17, the model generates similar horizontal alignments, but creates smoother and longer horizontal curves at higher design speeds. In addition, the higher design speed also forces the model to generate smoother and longer vertical curves. In the figure, vertical alignments (b1–b3) correspond to horizontal alignments (a1–a3), respectively. The result indicates that the model performs correctly in creating highway alignments as intended with input design specification, and shows its applicability to various highway design projects whose design standards would be different.
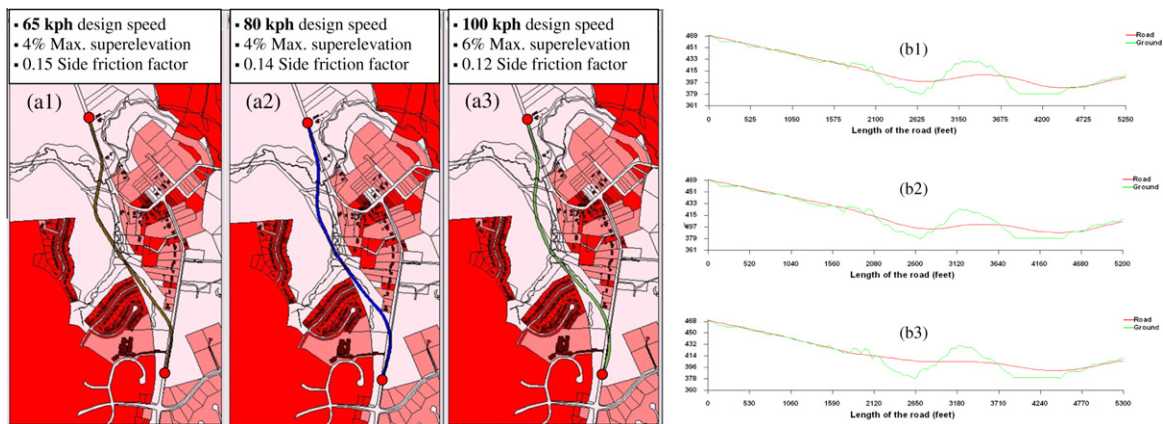

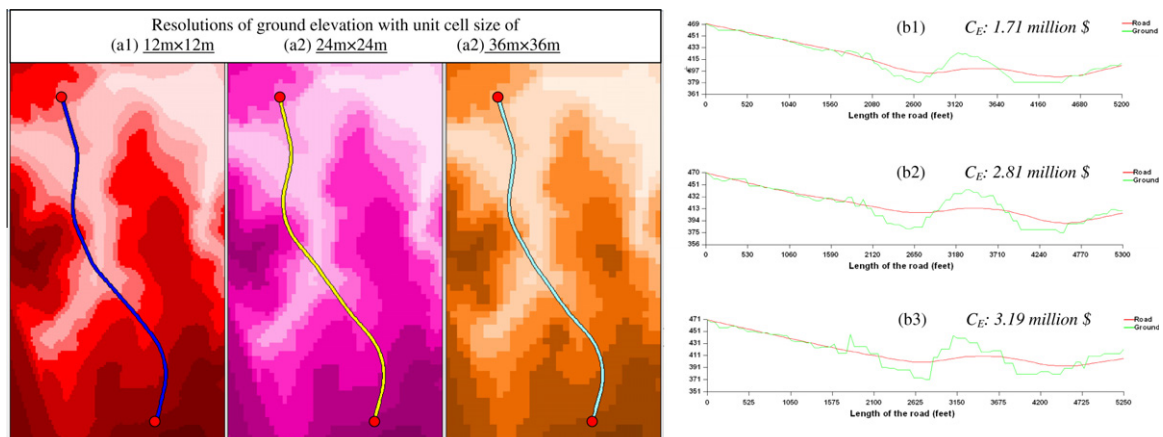Fig. 17. Sensitivity of optimized alignments to critical geometric design parameter.

Fig. 18. Sensitivity of optimized alignments to resolution of ground elevation.

9.2.3. Sensitivity to resolution of ground elevation

Resolution of ground elevation may also significantly affect the result of the optimized alignments because the earthwork quantity in the model is estimated based on the input elevation. As shown in Fig. 18, there are striking differences in earthwork cost estimation between three optimized alignments obtained with different resolutions of ground elevation, although they have very similar horizontal profiles. The earthwork cost significantly increases with a low resolution elevation data. This indicates that the model may produce unreliable earthwork estimates if the resolution of the ground elevation data is too rough. Thus, a high resolution of the input ground elevation is desirable.

10. Conclusions and discussion

This paper presents a highway alignment optimization (HAO) model, which has been developed for overcoming the limitations of repetitive manual procedures required in traditional highway planning and design. This paper also discusses (1) important costs and constraints associated with highway alignment optimization, (2) key roles of genetic algorithms (GAs) and a GIS evaluation process in the model, (3) representation of highway alignments and structures in the model, (4) model capabilities and applicability to real-world highway construction projects, and (5) critical factors that should be considered in comprehensively optimizing highway alignments through an extensive analysis of sensitivity to the key model parameters.

Throughout the model applications to two real-world projects, it has been shown that the model can not only effectively optimize highway alternative alignments which satisfy various user preferences and design standards, but also provide practical information about the resulting alignments to highway engineers and planners. The optimized alignments were found without any significant difficulties and within reasonable computation times, despite the complexity of land-use in the project area. In addition, it has been shown that the optimized alignments found by the model are quite similar with those obtained through conventional methods by the state agency, but the model can greatly reduce time required for highway planning and design. Finally, the analysis of sensitivity to key model parameters (such as the number of PI's and highway cost items included in the model objective function) confirms that all important alignment-sensitive costs can and should be jointly optimized for an effective highway alignment optimization and that many trade-off opportunities exist depending on the flexibility and preferences specified with the input parameters. It is expected that the model can be applied to many real-world projects and perform well in finding the most preferable alternative alignments during the initial stage of a road planning.

Despite demonstrated capabilities of the model, it can still benefit from many technical and methodological improvements. The following model improvements seem desirable:

- *Improve model evaluation criteria for cost-effective and sustainable highway design*: To ensure not only the cost-effectiveness but also sustainability in highway infrastructure planning and design, it is desirable to improve the model's evaluation criteria. Thus, in an upgraded version of the HAO model, the environmental sustainability, the road users' travel time, fuel efficiency and safety, and the highway agency's cost-effectiveness should be simultaneously evaluated for all the generated highway alignments through a trade-off analysis.
- *PI density sensitive to land-use and terrain complexity*: In the current version of the model, the number of PI's is a key input parameter affecting the precision of the generated highway alignments because it affects locations of horizontal and vertical curves as well as corresponding objective function components. In dense urban areas and areas with significant topographic variation, a higher PI density will improve the possibilities for alignment optimization, whereas in areas with slight variation in topography or land-use, fewer PI's will suffice. Thus, PI density should be related to the complexity of the model's search space and should be adjusted in the alignment optimization process.
- *Variable design speeds*: Different segments of a new highway may need different design speeds due to the terrain and land-use complexity of the surrounding environments. For instance the design speed may be reduced where a highway passes through mountains or a dense urban area. However, in the current version of the model, the generated highway alignments have a consistent design speed throughout their entire length. Thus, it is desirable to improve the model to reflect variable design speeds for different road segments in highway alignment optimization process.
- Employment of soil maps (which provide types and characteristics of various soils in the project area) and the use of proper cut/fill slopes and unit costs in the alignment optimization process should significantly improve the accuracy of the earthwork quantity estimation.
- Inclusion of structure costs for retaining walls and noise barriers should improve the model's capabilities.

It is noted that the structure cost of the Brookeville example (Case 1) only includes the cost of small highway bridges for grade separation of an existing road and for crossing streams. The analysis of sensitivity to highway evaluation criteria for this example shows that structure cost seems less sensitive to the geometry of the resulting alignments compared to the other major cost components. However, on a highway project with many major bridges and tunnels in a mountainous area, the structure cost would not only dominate the other costs but also decisively affect the highway location.

Acknowledgements

Appendix A

A.1. Horizontal alignment generation procedure

STEP 1: Connect $\mathbf{PI}_i$ and $\mathbf{PI}_{i+1}$ with a straight line for $i = 1,\ldots,n_{PI}$.
STEP 2: Find deflection angle at $\mathbf{PI}_i$.

$$\theta_{PI_i} = \cos^{-1}\left[\frac{(\mathbf{PI}_i - \mathbf{PI}_{i-1}) \cdot (\mathbf{PI}_{i+1} - \mathbf{PI}_i)}{\|\mathbf{PI}_i - \mathbf{PI}_{i-1}\|\|\mathbf{PI}_{i+1} - \mathbf{PI}_i\|}\right] \tag{A.1}$$

STEP 3: Find $\mathbf{TS}_i$ and $\mathbf{ST}_i$.

$$\mathbf{TS}_i = \mathbf{PI}_i + L_{TS(i)} \times \frac{(\mathbf{PI}_{i-1} - \mathbf{PI}_i)}{\|\mathbf{PI}_{i-1} - \mathbf{PI}_i\|} \tag{A.2}$$

$$\mathbf{ST}_i = \mathbf{PI}_i + L_{TS(i)} \times \frac{(\mathbf{PI}_{i+1} - \mathbf{PI}_i)}{\|\mathbf{PI}_{i+1} - \mathbf{PI}_i\|} \tag{A.3}$$

$$L_{TS(i)} = k_{S_i} + (R_{H_i} + p_{S_i}) \times \tan(\theta_{PI_i}/2) \tag{A.4}$$

STEP 4: Find $\mathbf{M}_i$ and δ_i.

$$\mathbf{M}_i = [\mathbf{TS}_i + \mathbf{ST}_i]/2 \tag{A.5}$$

$$\boldsymbol{\delta}_i = \mathbf{PI}_i + [(R_{H_i} + p_{S_i}) \times \sec(\theta_{PI_i}/2)] \times \frac{(\mathbf{M}_i - \mathbf{PI}_i)}{\|\mathbf{M}_i - \mathbf{PI}_i\|} \tag{A.6}$$

STEP 4: Find $\mathbf{SC}_i$ and $\mathbf{CS}_i$.

$$\mathbf{SC}_i = R_{H_i} \times \frac{(\mathbf{M}_i - \boldsymbol{\delta}_i)}{\|\mathbf{M}_i - \boldsymbol{\delta}_i\|} \times \mathbf{R}\left(\frac{\theta_{PI_i}}{2} - \theta_{ST_i}\right) \tag{A.7}$$

$$\mathbf{CS}_i = R_{H_i} \times \frac{(\mathbf{M}_i - \boldsymbol{\delta}_i)}{\|\mathbf{M}_i - \boldsymbol{\delta}_i\|} \times \mathbf{R}\left(-\frac{\theta_{PI_i}}{2} + \theta_{ST_i}\right) \tag{A.8}$$

$$\theta_{ST_i} = S_{T_i}/(2R_{H_i})$$

STEP 5: Connect $\mathbf{ST}_i$, $\mathbf{TS}_i$, $\mathbf{SC}_i$, and $\mathbf{CS}_i$ with spiral transition and circular curves.

Appendix B

B.1. Length-dependent cost (C_L)

$$C_L = u_P L_N w_P + u_L L_N \tag{B.1}$$

$$w_P \leq w_N \tag{B.2a}$$
$$w_N = w_L + w_S \tag{B.2b}$$

Note that Eq. (B.1) may also be useful for estimating the interchange length dependent cost since it can be roughly estimated with centerline distances and width of paved all approach roads.

B.2. Right-of-way cost (C_R)

$$C_R = \sum_{k=1}^{n_{PC}} u_{v_k} \times A_k \tag{B.3}$$

Note that the right-of-way cost function shown in Eq. (B.2) is just land-acquisition cost required for the right-of-way of the new highway. Besides the land-acquisition cost, reduction of property value due to a nearby highway as well as the usability

of the remaining lands or site improvements cost due to the new highway construction may be added for more precise right-of-way cost estimation (see Jha and Schonfeld, 2000).

B.3. Earthwork cost (C_E)

$$C_E = C_H + \frac{1}{2} \sum_{i=1}^{n_E} \left[\omega_0 u_{c_i} s_r A_{c_i} L_{E_i} + \omega_1 u_{f_i} A_{f_i} L_{E_i} + \omega_2 (u_{c_i} s_r A_{tc_i} + u_{f_i} A_{tf_i}) L_{E_i} \right]$$ (B.4)

$$\omega_0, \omega_1, \omega_2 = 0 \text{ or } 1; \omega_0 + \omega_1 + \omega_2 = 1$$ (B5.a)

$$\omega_0 = 1 \text{ for a cut section}, \quad \omega_1 = 1 \text{ for a fill section, and } \omega_2 = 1 \text{ for a transition section}$$ (B5.b)

Note that the earthwork cost function is formulated based on the average end area method, and C_H is the cost of moving earth between adjacent cut and fill sections to balance overall earthwork volume. For more detailed discussion of Eq. (B.3), see Jha (2000) and Jha and Schonfeld (2003).

B.4. Structure cost (C_S)

$$C_S = C_{S^B} + C_{S^{IC}} + C_{S^{IS}} + C_{S^T}$$ (B.6)

$$C_{S^B} = \alpha_0 + \alpha_1 \times l_B w_B$$ (B.7)

$$C_{S^{IC}} = C_{P^{IC}} + C_{R^{IC}} + C_{E^{IC}} + C_{B^{IC}}$$ (B.8)

$$C_{S^{IS}} = C_{P^{IS}} + C_{R^{IS}} + C_{E^{IS}}$$ (B.9)

$$C_{P^{IS}} = u_P A_{P^{IS}}$$ (B.10)

$$C_{E^{IS}} = u_f \times E_{V^{IS}}$$ (B.11)

$$C_{S^T} = C_{E^T} + C_{a^T}$$ (B.12)

$$C_{E^T} = \pi u_T l_T (R_T)^2$$ (B.13)

$$C_{a^T} = \gamma_0 + \gamma_1 (l_T) + \gamma_2 (l_T)^2$$ (B.14)

In the model, three types of interchanges (Clover, Diamond, and Trumpet) are used to represent structures for crossing existing roadways. Costs required for each structure consist of pavement cost, right-of-way cost, earthwork cost, and small bridge cost for grade separation of the existing road at the interchange. For estimating the interchange pavement cost, right-of-way cost, and earthwork cost, those for the highway basic segment (i.e., Eqs. (B.1), (B.3), and (B.4), respectively) can be used. Eq. (B.7) can be utilized for estimating the small bridge cost. The at-grade intersection cost also consists of pavement cost, right-of-way cost, and earthwork cost. Note that the intersection right-of-way cost can be roughly estimated with Eq. (B.3).

Tunnel cost consists of earthwork cost and additional tunnel cost which accounts for the cost of ventilation and lighting along the tunnel. The tunnel earthwork cost can be roughly estimated based on the radius and length of tunnel, while a quadratic function of tunnel length is used for estimating the additional tunnel cost (Kim et al., 2007). For estimating the cost of a large scale bridge for crossing rivers or valleys, linear cost functions based on the bridge span length and pier height may be used instead of Eq. (B.7).

B.5. Maintenance cost (C_M)

$$C_M = C_{M^H} + C_{M^B}$$ (B.15)

$$C_{M^H} = \left(L_N - \sum_{i=1}^{n_B} l_{B_i} \right) \left(u_{M^H} \sum_{k=1}^{n_y} \left(\frac{1}{(1+\rho)} \right)^k \right)$$ (B.16)

$$C_{M^B} = \sum_{i=1}^{n_B} \left[u_{M^B} l_{B_i} \sum_{k=1}^{n_y} \left(\frac{1}{(1+\rho)} \right)^k \right]$$ (B.17)

B.6. Travel time cost (C_T)

$$C_T = [\mathbf{x} \cdot \mathbf{t} \cdot \mathbf{H}] \cdot [\mathbf{v} \cdot \mathbf{T} \cdot \mathbf{o}] \left(\frac{e^{(r_t - \rho)n_y} - 1}{r_t - \rho} \right) \tag{B.18}$$

B.7. Vehicle operating cost (C_V)

$$C_V = L_T[\mathbf{x} \cdot \mathbf{H}] \cdot [\mathbf{u} \cdot \mathbf{T}] \left(\frac{e^{(r_t - \rho)n_y} - 1}{r_t - \rho} \right) \tag{B.19}$$

$$u_j = p_{F_j} f_j + m_j; \quad u_j \in \mathbf{u} \tag{B.20}$$

B.8. Accident cost (C_A)

$$C_A = u_A F_A \left(\frac{e^{(r_t - \rho)n_y} - 1}{r_t - \rho} \right) \tag{B.21}$$

Note that a variety of accident prediction models has been developed for predicting accidents on highway segments. Among them, theoretical models by Vogt and Bared (1998), Zegeer et al. (1992), are Chatterjee et al. (2003) can be adopted in the model to predict F_A.

B.9. Penalty cost (C_P)

$$C_P = C_{PE} + C_{PD^H} + C_{PD^V} \tag{B.22}$$

$$C_{PE} = \sum_{k=1}^{n_{PC}} \left[\left(\beta_{E^0} + \beta_{E^1} \times (A_k - MaxA_k)^{\beta_{E2}} \right) \times I_{PE_k} \right] \quad \text{only if } A_k > MaxA_k \tag{B.23}$$

$$C_{PD^H} = \sum_{i=1}^{n_{HC}} [C_{PD^{HR}} + C_{PD^{HS}} + C_{PD^{ST}}] \quad \text{only if } R_{H_i} < R_{H_m}, S_{H_i} < S_{H_m}, \quad \text{or } S_{T_i} < S_{T_m} \tag{B.24}$$

$$= \sum_{i=1}^{n_{HC}} \left[(\beta_{HR^0} + \beta_{HR^1} \times (R_{H_i} - R_{H_m})^{\beta_{HR2}}) + (\beta_{HS^0} + \beta_{HS^1} \times (S_{H_i} - S_{H_m})^{\beta_{HS2}}) + (\beta_{ST^0} + \beta_{ST^1} \times (S_{T_i} - S_{T_m})^{\beta_{ST2}}) \right]$$

$$C_{PD^V} = \sum_{i=1}^{n_{VC}} [C_{PD^{VL}} + C_{PD^{VS}} + C_{PD^{VG}}] \quad \text{only if } L_{V_i} < L_{V_m}, \ S_{V_i} < S_{V_m}, |g_i| > g_{max} \tag{B.25}$$

$$= \sum_{i=1}^{n_{VC}} \left[(\beta_{VL^0} + \beta_{VL^1} \times (L_{V_i} - L_{V_m})^{\beta_{VL2}}) + (\beta_{VS^0} + \beta_{VS^1} \times (S_{V_i} - S_{V_m})^{\beta_{VS2}}) + (\beta_{VG^0} + \beta_{VG^1} \times (g_i - g_{max})^{\beta_{VG2}}) \right]$$

B.10. Environmental cost (C_{EN})

$$C_{EN} = \frac{1}{5280} u_E[\mathbf{x} \cdot \mathbf{H}] \left(\frac{e^{(r_t - \rho)n_y} - 1}{r_t - \rho} \right) \tag{B.26}$$

In Eq. (B.26), the unit environmental cost per vehicle mile traveled (u_E) can derived by summing up the unit costs for different environmental impacts such as air, noise and water pollution costs.

Appendix C

See Table C.1.

Table C1
List of notations.

Notation	Description	Units
α_0, α_1	Coefficients used in bridge cost computation	
$\beta_{E^0}, \beta_{E^1}, \beta_{E^2}$	Coefficients used in computing C_{PE}	
$\beta_{HR^0}, \beta_{HR^1}, \beta_{HR^2}$	Coefficients used in computing $C_{PD^H R}$	
$\beta_{HS^0}, \beta_{HS^1}, \beta_{HS^2}$	Coefficients used in computing $C_{PD^H S}$	
$\beta_{ST^0}, \beta_{ST^1}, \beta_{ST^2}$	Coefficients used in computing $C_{PD^S T}$	
$\beta_{VG^0}, \beta_{VG^1}, \beta_{VG^2}$	Coefficients used in computing $C_{PD^V G}$	
$\beta_{VL^0}, \beta_{VL^1}, \beta_{VL^2}$	Coefficients used in computing $C_{PD^V L}$	
$\beta_{VS^0}, \beta_{VS^1} \beta_{VS^2}$	Coefficients used in computing $C_{PD^V S}$	
$\gamma_0, \gamma_1, \gamma_2$	Coefficients used in additional tunnel cost computation	
$\omega_0, \omega_1, \omega_2$	Binary integers used in earthwork cost computation;	
ρ	Annual interest rate	decimal
θ_{CP}	Intersection angle between two cross roads	radian
θ_{EP}	Intersection angle at the endpoint of the new highway	radian
θ_{PIi}	Deflection angle at the ith point of intersection (PI)	radian
θ_{STi}	Spiral angle at the ith horizontal curved section	radian
δ_i	The center point of the ith horizontal curved section	
$\cdot$	Inner (dot) product used in vector operation	
A_c, A_f, A_{tc}, A_{tf}	Cross-sectional areas under cut, fill, transitional cut, and transitional fill conditions	m^2
A_k	Affected area of the kth land parcel by the highway alignment generated	m^2
$A_{P^{IS}}$	Intersection pavement area	m^2
A_T	Total area of a land parcel affected by the highway alignment	m^2
C_A	Present value of total accident cost	$
C_{aT}	Additional tunnel cost which includes cost for ventilation and lighting	$
$C_{B^{IC}}$	Small bridge cost for grade separation of the existing road at the interchange	$
C_E	Earthwork cost of the highway alignment	$
$C_{E^{IC}}$	Interchange earthwork cost	$
$C_{E^{IS}}$	Intersection earthwork cost	$
C_{E^T}	Tunnel earthwork cost	$
C_H	Total haul cost	$
C_L	Length-dependent cost of the highway alignment	$
C_M	Present value of total maintenance cost of the highway alignment	$
C_{M^H}	Present value of the maintenance cost for highway basic segments	$
C_{M^B}	Present value of bridge maintenance cost	$
$C_{P^{IC}}$	Interchange pavement cost	$
$C_{P^{IS}}$	Intersection pavement cost	$
C_P	Total penalty cost	$
C_{PE}	Penalty associated with environmentally sensitive areas taken by the new highway	$
C_{PD^H}	Penalty cost for violating design constraints of horizontal alignments	$
C_{PD^V}	Penalty cost for violating design constraints of vertical alignments	$
$C_{PD^H R}$	Penalty cost for violating the minimum horizontal curve radius	$
$C_{PD^H S}$	Penalty cost for violating the minimum horizontal sight distance	$
$C_{PD^S T}$	Penalty cost for violating the minimum length of spiral transition curve	$
$C_{PD^V L}$	Penalty cost for violating the minimum length of vertical curve	$
$C_{PD^V S}$	Penalty cost for violating the minimum vertical sight distance	$
$C_{PD^V G}$	Penalty cost for violating the maximum allowable gradient	$
C_R	Right-of-way cost of the highway alignment	$
$C_{R^{IC}}$	Interchange right-of-way cost	$
$C_{R^{IS}}$	Intersection right-of-way cost	$
C_S	Structure cost	$
C_{S^B}	Bridge cost for grade separation	$
$C_{S^{IC}}$	Interchange cost	$
$C_{S^{IS}}$	Intersection cost	$
C_{S^T}	Tunnel cost	$
C_{Total}	Total construction cost (objective function value) of the highway alignment	$
C_T	Present value of total travel time cost	$
C_V	Present value of total vehicle operation cost	$
$\mathbf{CS}_i$	The point of change from circle to spiral pertaining to $\mathbf{PI}_i$; see Fig. 4b	
$\mathbf{EP}$	Endpoint of the new highway	
$E_{V^{IS}}$	Earthwork volume required for the at-grade intersection	m^3
F_A	Average accidents (frequency) estimated	accident/yr
f_j	Fuel consumption rate of jth mode operated on the new highway	liter/km
g_i	Forward or back tangent grade at the ith vertical curve section	%
g_{max}	Maximum allowable gradient	%
$\mathbf{H}$	A vector of duration of different time frames per year; $\mathbf{H} = [H_{peak}, H_{off\text{-}peak}]$	hours/yr
h_m	Minimum vertical clearance	m

Table C1 (*continued*)

Notation	Description	Units
I_{PE_k}	A dummy variable indicating if the kth parcel is the environmentally sensitive area	
k_{S_i}	Abscissa of the shifted PC referred to $\mathbf{TS}_i$; see Fig. 4b	m
L_N	Total length of the highway alignment generated	m
L_E	Length of a highway section for earthwork volume calculation	m
L_{TS_i}	Tangent distance from $\mathbf{TS}_i$ to $\mathbf{PI}_i$; see Fig. 4b	m
L_{V_i}	Vertical curve length at the ith vertical curve section	m
L_{V_m}	Minimum length of vertical curve	m
l_{B_i}	Length of the ith highway bridge	m
l_{E_P}	Length of roadways (e.g., ramps) associated 3-leg structures at the highway endpoint	m
l_T	Tunnel length	m
$\mathbf{M}_i$	The middle point of the line segment connecting $\mathbf{TS}_i$ to $\mathbf{ST}_i$; see Fig. 4b	
m_j	Vehicle maintenance cost of jth mode	\$/km
$MaxA_k$	Maximum allowable area of the kth land parcel for the new highway construction	m^2
n_B	Total number of bridges in the highway alignment generated	#
n_E	Total number of highway sections for earthwork volume calculation	#
n_{HC}	Total number of horizontal curve sections of the highway alignment generated	#
n_{PC}	Total number of land parcels affected by the highway alignment generated	#
n_{PI}	Total number of PI's that outlines the highway alignment generated	#
n_{VC}	Total number of vertical curve sections in the highway alignment generated	#
n_y	Analysis period or design life of the road	year
$\mathbf{o}$	A vector of average vehicle occupancies for different modes	decimal
p_{Fj}	Fuel price of jth mode operated on the new highway	\$/l
p_{Si}	Offset from the initial tangent to the PC of the shifted circle; see Fig. 4b	m
$\mathbf{PI}_i$	ith PI of the highway alignment generated; $PI_i = (x_i, y_i, z_i)$ for $i = 1,\ldots,n_{PI}$	
R_{H_m}	Minimum horizontal curve radius	m
R_{H_i}	Horizontal curve radius at the ith horizontal curve section	m
R_T	Tunnel radius	m
$\mathbf{RP}$	Reference point for representing highway structures	
$\mathbf{R}(\theta)$	Rotation matrix	
r_t	Annual traffic growth rate	decimal
s_c, s_c	Cut and fill slops, respectively	decimal
$\mathbf{SC}_i$	The point of change from spiral to circle pertaining to $\mathbf{PI}_i$; see Fig. 4b	
S_{H_i}	Horizontal sight distance at the ith horizontal curve section	m
S_{H_m}	Minimum horizontal sight distance	m
s_r	Earth shrinkage or swell factor	decimal
S_{T_i}	Spiral transition curve length at the ith horizontal curve section	m
S_{T_m}	Minimum length of spiral transition curve	m
$\mathbf{ST}_i$	The point of change from spiral to tangent pertaining to $\mathbf{PI}_i$; see Fig. 4b	
S_{Vi}	Vertical sight distance at the ith vertical curve section	m
S_{Vm}	Minimum vertical sight distance	m
$\mathbf{t}$	A vector of average travel time in different time frames; $\mathbf{t} = [t_{peak}, t_{off\text{-}peak}]$	h
$\mathbf{T}$	Traffic composition vector representing different modes on the new highway	decimal
$\mathbf{TS}_i$	The point of change from tangent to spiral pertaining to $\mathbf{PI}_i$; see Fig. 4b	
$\mathbf{u}$	A vector of unit vehicle operation costs for different modes; $u_j \in \mathbf{u}$	\$/km
u_A	Unit accident cost	\$/accident
u_c	Unit cut cost	\$/m^3
u_E	Unit environmental cost per vehicle mile traveled (VMT)	\$/VMT
u_f	Unit fill cost	\$/m^3
u_L	Unit length-dependent cost except pavement cost	\$/m
u_{M^B}	Unit bridge maintenance cost	\$/m/yr
u_{M^H}	Unit maintenance cost for highway basic segments	\$/m/yr
u_T	Tunnel earthwork unit cost	\$/m^3
u_P	Unit pavement cost	\$/m^2
u_{v^k}	Unit cost (property value) of the kth land parcel affected by the highway alignment	\$/m^2
$\mathbf{v}$	A vector of unit travel time values for different modes	\$/h
w_B	Bridge width; $w_B = w_L$ for the new highway	m
w_E	Width of an existing road intersected by the new highway	m
w_L	Travel lanes width of the new highway	m
w_N	Width of the new highway; $w_N = w_L + w_S$	m
w_P	Width of paved portion of the new highway; $w_P \leqslant w_N$	m
w_S	Shoulders width of the new highway	m
$\mathbf{x}$	A vector of average traffic volume in different time frames; $\mathbf{x} = [x_{peak}, x_{off\text{-}peak}]$	vehicle/h
x_{LB}, y_{LB}, z_{LB}	Lower bounds of x,y,z coordinates of PI_i	
x_{UB}, y_{UB}, z_{UB}	Upper bounds of x,y,z coordinates of PI_i	
x_{ST_i}	Total tangent distance from $\mathbf{TS}_i$ to $\mathbf{SC}_i$ with reference to initial tangent	m
y_{ST_i}	Total tangent offset at $\mathbf{SC}_i$ with reference to $\mathbf{TS}_i$ and initial tangent	m

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