Ant Colony Optimization Algorithm for Vertical Alignment of Highways

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ABSTRACT: Based on discrete theory, this article tries to develop an optimization methodology to produce an optimum vertical highway profile for a pre-selected horizontal alignment. The aim of the program was to establish an initial vertical alignment according to discreet ground elevation of station. Considering the discreet characteristic of the ground elevation and the intersection point of grade line, a discrete model is presented. The automatic design problem is set to select the number, location and elevation of the intersection point of the grade line after considering several designing constraints. To solve the constrained nonlinear problem, an ant colony optimization algorithm is adopted to select the roadway grades that minimize the earthwork cost and satisfy the geometric specification. A numerical example is presented to illustrate the application of the program.

1. INTRODUCTION

The alignment design of roadway normally involves the vicissitudinary applications of two elements: the horizontal design and the vertical design. The two parts would be employed by turn for many times before the final alignment of the roadway could be established. Usually, the vertical alignment design is based on a pre-selected horizontal alignment. The horizontal alignment could provide the related data of road centerline and some necessary parameters to the automatic design of vertical alignment. Following a certain algorithm we could turn to the vertical alignment automatically with the help of computer by using the ground data and parameter provided by the horizontal alignment. To realize this idea, many scholars devoted to researches involving the following algorithm: enumeration, dynamic programming, linear programming, and genetic algorithms.

Enumeration has been employed by Easa (1988). The searching process is exhaustive. the dynamic programming (Goh et al., 1988; Fwa, 1989) is more effective and better than before. The method makes it possible to optimize the grade change point and the position of the elevation, obtaining the best railroad vertical section design. However, when it comes to three-or-more-dimension cases, the model could not cope with problems caused by the advanced time and space complexity. For the two-dimensional problem, the memory requirement for it is approximately of O(n·m), while the computational times is of O(n ·m^2). If applying it in a higher dimension situation, computation is not able to be carried out

Another alternative approach for the problem is to regard the road profile as a continuous function, such as a spline function to simulate alignments and turn it into a constrained nonlinear programming problem. After all, the function curve is quite different from the alignment of roadway.

The linear programming approaches (ReVelle et al., 1997; Chapra and Canale, 1988; Fwa, 1989) were used for vertical alignment optimization. The approaches employed much more simplified assumptions to establish a model which was adapted to linear programming approaches. However the computing ability was limited.

Numerical Search (Goh et al., 1988; Fwa, 1989)is effective ,but there exist some local optimal in the search space.

Genetic Algorithms: Jong (1998) employed genetic algorithms for vertical alignment optimization. Only in the condition that the preliminary design is given and the number of intersection points (grade change points) is determined, the algorithm could carry on. But actually, the computer, not given by an engineer, should calculate the number of intersection point out.

Ant colony optimization algorithm is a kind of important intellectual colony optimization algorithm, ant colony optimization algorithm was established initially by Dorigo (1997, 1999), an Italian scholar. Its basic idea is that ant individuals deliver the information by the pheromone. When an ant is crawling, it will not only release its own pheromone which would gradually disappear at a certain velocity on its route, but also detect the circumstance nearby to figure out whether the pheromone exists or not. An ant adopts a kind of positive feedback mechanism, if more ants had chosen this route, there will be more pheromone left on the route, and this route also will lead more ants to select it, creating a higher pheromone. So the final result is that ants form an optimum route.

Ant colony optimization algorithm can provide solution to many intractable NP-hard optimization problems. It has its own advantage in solving the complicate combination optimization, with huge parallelism, positive feedback and robustness. A great deal of literature shows that the ant colony optimization algorithm had successfully worked out the following combination optimization problem: Traveling salesman (TSP), Vehicle routing (VRP), Job shop scheduling (JSP) and so on.

If the vertical alignment automatic optimization is built into a model with discrete idea, it could be seen as the problem of combinational optimization essentially. Thus, we could use the ant colony optimization algorithm to seek a route effectively. This paper is set to solve the problem of vertical alignment automatic optimization with the help and development of ant colony optimization algorithm.

2. THE MODEL

The design parameter of road alignment includes the position and elevation of intersection points, the radius of vertical curve. Among the parameters, the earthwork coast is mainly determined by the position and elevation of intersection points. Therefore this paper only considers the position and elevation of intersection points as the parameter of optimization to illustrate this algorithm.

Once the centerline of the road is established, the elevation of every intermediate station is going to be given to decide the ground profile. This is a discrete procedure which also corresponds with the basic requirement of ant colony algorithm.

If we make a grid for the model, and along with the ground profile, we draw a vertical parallel line at each intermediate station with a certain horizontal space. The line is going to be equally divided in the bilateral direction. Thus, scattered grid points are formed along the ground profile, as shown in Figure 1.

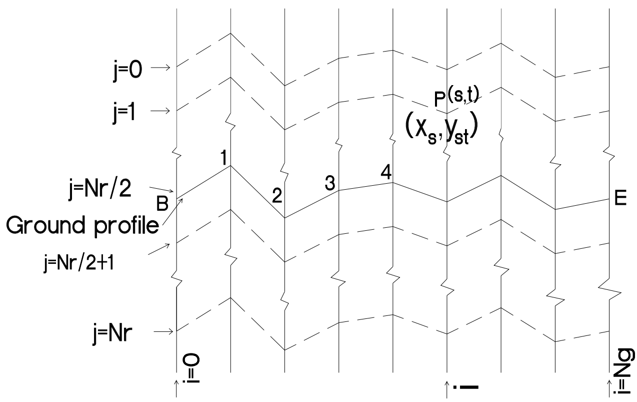

Figure 1. Form of Grid

The lines are indexed by 0,1,2, …,Ng (midddle stick station), the horizontal lateral lines are indexed by 0,1,2, …, Nr (from up to down).

Grid points $G_{ij} \in G=\{(i,j) \mid i\in[0, Ng], j\in[0, Nr]\}$; The horizontal and vertical coordinate of P_{ij} is (x_{ij}, y_{ij}) (i.e. intermediate stations and the elevation); The problem can be expressed by seeking the serial set of all the grade change points, to make the objective functions minimum, route= $\{P_0, P_1, P_2, \ldots, P_t, \ldots, P_{Np}\}$, $P_t=\{(i, j) \mid i\in Hroute, j\in Vroute\}$, Np+1 is the total number of the grade change points.

$$Z=\min C(P_0,\ P_1,\ P_2\ ,\ \ldots,\ P_t,\ \ldots,\ P_{Np}) \tag{1}$$

$$\begin{aligned}
s.t.\quad & \sum_{j=0}^{Nr} w_{ij} = 1, \quad i \in Hroute^k \\
& \sum_{j=0}^{Nr} w_{ij} = 0, \quad i \notin Hroute^k \\
& \sum_{i=0}^{Nd} w_{ij} < N_d, \quad j = 0,1,\cdots,Nr \\
& w_{ij} \in \{0,1\}, \qquad (i,j) \in G \\
& \text{Lmin} \le x_{i+1} - x_i \le \text{Lmax}, \quad i \in Hroute^k, \\
& \text{Im}in \le \frac{y_{i+1},{}_{j+1} - y_i,{}_j}{x_{i+1} - x_i} \le \text{Im}ax, \quad (i,j) \in route^k \\
& j = Nb, \qquad i = 0 \\
& j = Ne, \qquad i = Nd.
\end{aligned}$$

Hroute is a subset of the horizontal number in route set, while Vroute is a subset of the vertical number in the route set. The decision variant w_{ij} shows whether the ant passes the grid point; G_{ij}, w_{ij}=1 shows that the route includes the grid point $G_{ij;}$,and w_{ij}=0 shows that the route does not include the grid point $G_{ij.}$

(0,Nb) is the starting number, (Ng,Ne) is the ending number of the profile; generally: $Nb = \frac{N_r}{2}$, $Ne = \frac{N_r}{2}$.

Lmin is the allowed minimum slope length. Lmax is the allowed maximum slope length. Imin is the allowed minimum roadway grades. Imax is the allowed maximum roadway grades.

3. ANT ALGOTITHM TO THE PROBLEM

3.1 The way of choosing intersection point:

Placing the ants at point B, ants will then start from point B, and step toward the destination E, every ant can select only one point at one step, and which point would be chosen as the next step depends on the transition probability. "$p^k_{ij,uv}$" is the transition probability that an ant steps from point G^k_{ij} to point G^k_{uv} ,k={0,1, 2.….m}, and it could be work out by eqn (2). There are two factors: the pheromone ants have left behind on the route and the heuristic information between the two points, which are affecting the transition probability. Respectively, "$\tau_{ij,uv}$" and "ηij,uv" represent the two factors. The parameter "α" and "β" are used to show the level of relative influence on an ant's choice between point "G^k_{ij}" and point "G^k_{uv}". "α" is the pheromone transition factor that reflects the relative importance of pheromone. "β" is expectation transition factor that reflects the importance of "visibility".

$$p^k{}_{ij,uv}(t) = \frac{[\tau_{ij,uv}(t)]^{\alpha} \cdot [\eta_{ij,uv}(t)]^{\beta}}{\sum\limits_{(u,v)\in arrived^k} [\tau_{ij,uv}(t)]^{\alpha} \cdot [\tau_{ij,uv}(t)]^{\alpha}}, \quad if\ (u,v) \in arrived(i,j)^{\,k} \tag{2}$$

3.2 The construction of "set arrived"

The present point of ant "k" is assumed as P_t(i,j) (the ant k reaches a grade change point at the moment T). According to the usual way of dealing with TSP problems, the total number of visiting point is (Ng-i) ×Nr should start from column i+1 to column Ng. If the length of the road increases, the number of the grid points will also increase, which will make the calculation impossible. Therefore an arrived set of the visiting points should be chosen according to the current point to minimize the storage and searching range. The set is named "set arrived" (denoted by arrived (i,j))includes all the possible points that an ant may reach at point P_t(i,j).

As the design of the vertical alignment, from a grade change point to the next point, the next grade change point must be restricted in the space within the minimum roadway grades and the maximum roadway grades and the minimum slope length and the maximum roadway length. Therefore all the grid points in the space form a set Arrived. If an arbitrary point Q (u,v) is assumed to belong to the set Arrived (i,j), then the choice possibility $p^k{}_{ij,uv}$ between the point P_t(i,j) and the points within the set Arrived should be calculated in order. Using the method of roulette wheel selection according to the possibility, the next point P_{t+1} to be visited should be selected. And when the ant visits this grade change point, P_{t+1} should be stored in the set $Route^k$ of this cycle. In order to ensure that the ant goes from the start point to the ending point and each grid column is visited only once, the points visited should be stored in the set $Route^k$ immediately. Actually the set $Route^k$ is a kind of tabu set, and the points in the set $Route^k$ will not be chosen again. If the ant chooses to visit the next point, then actually a grade section is formed between this point and the next point. The route that the ant goes from the starting point to the ending point is the grade change point set which is a way of the vertical alignment, as shown in Figure 2.

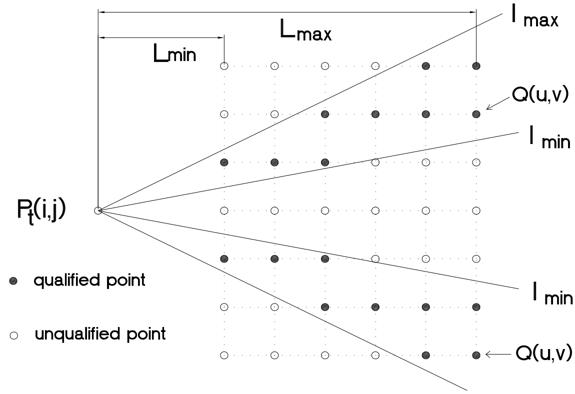

Figure 2. Set Arrived of Point Pt

3.3 The update of pheromone:

There are two aspects in the update of pheromone on each grade line. On one hand the pheromone will be increased if the ant selects to pass through that line, on the other hand, the pheromone is a kind of material of volatility, it could gradually disappear at a certain time. So the amount of pheromone changing at the moment "t+n" could be calculated as follow:

$$\tau^{k}{}_{ij,uv}(t+n) = (1-\rho)\cdot\tau^{k}{}_{ij,uv}(t) + \Delta\tau^{k}{}_{ij,uv}(t) \tag{3}$$

$$\Delta\tau_{ij,uv}(t) = \sum_{k=1}^{m}\Delta\tau^{k}_{ij,uv}(t) \tag{4}$$

$\Delta\tau_{ij,uv}(t)$ is the increment of pheromone between the points "P_t" and "P_{t+1}" in a cycle. (at the original moment, $\Delta\tau_{ij,uv}(t)=0$). $\tau^{k}_{ij.uv}(t)$ is the pheromone which is left by the ant "k" between the points "P_t" and "P_{t+1}" in a cycle. "ρ" is the volatile coefficient of pheromone, so "1-ρ" shows the remainder level of pheromone.

Ant colony algorithm could divide into ant-cycle algorithm、ant-quantity algorithm and ant-density algorithm according to the renew way of pheromone. Ant-cycle algorithm is the one that the feedback information used in searching process is overall, but the feedback information used in ant-quantity algorithm and ant-density algorithm is local. So the ant-cycle algorithm is more discreet than the other two algorithms, and

$\Delta\tau^{k}_{ij.uv}(t)$ can be worked out by the following function:

$$\Delta\tau^{k}{}_{ij,uv}(t)=\frac{Q}{\sum_{s=1}^{\mathrm{Ng}}(yg_s-yd_s)^2} \tag{5}$$

where yg_s = The ground elevation of the intermediate station “k”. s {0,1，2，…，Ng}; yd_s = The design elevation of the intermediate station “k”. s {0,1，2，…，Ng}; Q = The intensity of pheromone, affecting the convergence velocity to a certain extent.

3.4 The usage of heuristic function

The heuristic function is used to work out the visibility that could show the influence of guidance factor between the two points. So the function should be the total cost of all the roadway cost, including earthwork cost, vehicle operating cost, land cost and pavement cost. But this report is purposed to illustrate the application of ant colony algorithm in vertical alignment design, so it only regard the earthwork cost as the target of optimization in the objective function. The other three kinds of roadway cost are assumed to be excluded from the objective function. The earthwork cost C is mainly due to the relative difference between grade and ground elevations. The computation of the cutting and filling cost is summarized below according to C. J. GoH and E. P. CHEW and T. F. Fwa（1988）.Let “μ” and “υ” denote the filling and cutting cost per unit cross sectional area that may be location dependent. Let $\Delta_i = yd_i - yg_i, \Delta_{i+1} = yd_{i+1} - yg_{i+1}$, where h = the vertical distance of a grid; C_i = the earthwork cost between the station “i ”and “i +1” , i {0,1，2，…，Ng}.

(i) Cutting,no crossing of ground profile and road profile

$$If\ \Delta_i<0\ and\ \Delta_i\cdot\Delta_{i+1}\geq 0\quad then\ \ C_i=-\frac{\nu h}{2}(\Delta_i+\Delta_{i+1}). \tag{6}$$

(ii) Fitting, no crossing

$$If\ \Delta_i\geq 0\ and\ \Delta_i\cdot\Delta_{i+1}\geq 0\quad then\ \ C_i=\frac{\mu h}{2}(\Delta_i+\Delta_{i+1}). \tag{7}$$

(iii) Crossing, cutting, and filling

$$If\ \Delta_i\geq 0\ and\ \Delta_i\cdot\Delta_{i+1}<0\quad then\ \ C_i=\frac{h}{2}\cdot\frac{\mu\Delta_i^{\,2}+\nu\Delta_{i+1}^{\,2}}{\Delta_i-\Delta_{i+1}}. \tag{8}$$

(iv) Crossing, cutting, and filling

$$If\ \Delta_i<0\ and\ \Delta_i\cdot\Delta_{i+1}<0\quad then\ \ C_i=\frac{h}{2}\cdot\frac{\nu\Delta_i^{\,2}+\mu\Delta_{i+1}^{\,2}}{-\Delta_i+\Delta_{i+1}}. \tag{9}$$

So the heuristic function between point "G_{ij}"and "G_{uv}" is $\eta_{ij,uv} = \dfrac{1}{\sum_{s=i}^{u} C_s}$.

3.5 The processing of fixed points constrain

As for the constraints, we had set a set arrived which is made up of all optional points to make sure that the route is under our control. But actually we might have to set another kind of constraint. It is that we need some intermediate stations keeping a certain elevation fixed to satisfy the requirement. For example, the elevation of a bridge or a tunnel is not allowed to be modified. Supposed elevation of G_{ij} is yg_i, let $H_1 = yg_i - \varepsilon, H_2 = yg_i + \varepsilon$ (ε is a quite small real number). If we divide the space between "H_1"and "H_2"more accurately to obtain a finer grid, that we will force ants to select those accurate grid points to keep the elevation of this point change in a small range, as shown in Figure 3.

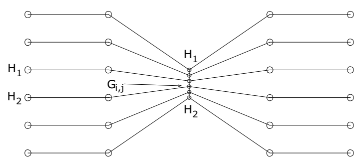

Figure 3. Fixed Point Constrain

3.6 A numerical example

The length of a highway in Hunan Province is 4360m.We set parameter as follows: $\alpha = 0.2, \beta = 4.0, \rho = 0.4, Q = 100, I_{\min} = 0.3\%, I_{\max} = 6.0\%, L_{\min} = 120, L_{\max} = 900$, Nr=20, Nc=219. The vertical space distance is 0.6m, the horizontal space distance is 20m.The whole region is divided by 219×20 grids in our experiment. The iteration number of ants is 60.The result alignment shows in Figure 4 (only a part of the alignment). The total running time is 6.72s on a computer with Intel(R) core(TM)2 Duo CPU T7250, 1G memory.

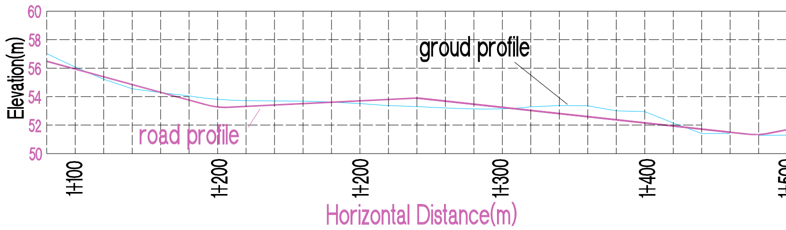

Figure 4. A numerical example

3.7 The selection of parameter

Ant colony algorithm is a kind of stochastic algorithm. The accuracy of the result has something to do with the optimum parameter. For instance, "m"(the total number of ants) exercises a power influence upon the result. Table 1 shows that the more the ants are, the better the result is.

Table 1. Cost about the Total Number of Ants

m	Worst	Average	Best
2	180.72	137.5764	110.562
12	146.491	120.8007	92.5396
100	110.37	100.8759	84.5539

The classic ant colony algorithm cannot supply the parameter. So only by a lot of experiments can we get more suitable parameters. We run the program 100 times in the case of changing a parameter to get some experimental results:

$$0 \le \alpha \le 5, 0 \le \beta \le 5, 0.1 \le \rho \le 0.9, 10 \le Q \le 10000.$$

4.CONCLUSIONS

According to the ant colony algorithm, we designed a program to carry out automatic design of roadway vertical section. This program has the following advantage:

(1) Creating the grade change point automatically: the program can automatically select the number, location and elevation of the grade change points considering several design constraints, while some other methodology can only select the elevation of the grade change points.

(2) Computing efficiently a large number unknown parameter, just as our experiment, the program is able to compute a long distance roadway with a large quantity of parameters in a high speed.

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