Discussion on Methods of Traffic Routes' Alignment Survey and Application of Principle of Least Squares

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ABSTRACT

Traffic route's alignment survey is among the important works required in location survey stage. This paper concisely introduces both conventional and contemporary techniques of traffic route's alignment survey. It also analyzes and discusses the question that while traffic route's alignment survey is being conducted by conventional diagrammatic method (or called offset method), how to improve the accuracy of diagrammatized centerline stations' coordinate parameters for staking-out by the principle of least squares to reduce the workload of repeatedly threading, checking on and adjusting the laid-out centerline stations. Finally, the authors draw the conclusion that the laid-out traffic route's center line stations based on staking-out coordinate parameters obtained by the principle of least squares are able to satisfy the required tolerance of location survey of traffic route's centerline stakes.

Key Words: traffic route, alignment, surveying method, least squares

1. INTRODUCTION

Alignment survey, one of the important works required in traffic route's location survey stage, is a process of staking out preliminarily-designed centerline in field. Its fundamental task is to stake out intersections, a series of straight lines separated by certain distances and directions of the centerline proposed by a plan.

In alignment survey, method used to acquire staking-out coordinates of centerline stations includes analytical method and diagrammatic method. With analytical method, coordinates of centerline stations (including starting point, ending point and intersections etc.) are provided by designing calculations, and these stations are staked out later in field by polar coordinate method and included-angle method. On the other hand, diagrammatic method means to take measurements of coordinates of centerline stations for staking-out directly from strip map or aerial map and lay out these stations by diagrammatic method. Analytical method always provides higher level of accuracy compared with diagrammatic method.

The reason for analytical method's higher level of accuracy is that the available staking-out data are provided by the designing plan and thus no contradiction exists between them. On the other hand, the reason for diagrammatic method's lower level of accuracy is that the deviation of diagrammatized coordinate parameters of centerline stations for staking-out is too large to ensure the laid-out stations are precisely in one straight line and therefore it is necessary for surveyors to repeatedly thread, check and adjust the laid-out center line stations (Zhang Kunyi 1999).

2. BRIEF INTRODUCTION ON METHODS OF TRAFFIC ROUTE'S ALIGNMENT SURVEY

Along with the rapid development of surveying techniques, especially the widely application of EDM instruments and total stations, the methods for alignment survey of traffic route have been improved greatly. The GPS RTK technique will be applied more extensively in alignment survey of traffic route and its construction.

2.1 Methods of Alignment Survey with Conventional Techniques and Instruments

Methods of alignment survey with conventional techniques and equipments include analytical method and diagrammatic method. Here the conventional instruments and equipments refer to theodolites, steel type and range finders.

In analytical method, the centerline of traffic route and coordinates of intersections of centerline straight segments (or any point) are both determined by designing plan. Therefore the lengths and directions of lines from traverse stations to centerline straight segments' intersections (or any point) can be inversely calculated from coordinates of laid-out traverse stations so that intersections of centerline straight segments can be laid out with polar coordinate method.

Diagrammatic method enables (Gong 1992) coordinates of intersections of centerline straight segments (or any point) to be taken directly from strip map or aerial map, and then laid out in field by polar coordinate method with inversely calculated lengths and directions of lines from traverse stations to centerline straight segments' intersections (or any point) and (2)centerline stations to be laid out in field by diagrammatic method (or called offset method) with staking-out offset parameters which have been diagrammatized directly from strip map or aerial map, such as d_4, d_5 (see Fig.1).

2.2 Methods of Alignment Survey with Modern Techniques and Instruments

(1) Application of Total Station in Alignment Survey of Route

Total station consists of electronic theodolite, electro-magnetic range finder, and micro-processor and is able to perform functions of measuring angles, distances, and elevations. Besides its function of measuring angle or distance as an electronic theodolite or electro-magnetic range finder respectively, total station supported by its program modules is able to accomplish the procedure of three-dimensional coordinate surveying, laying-out surveying, RDM and REM etc.

Based on designed coordinates of intersections of centerline straight segments (or any point), the location survey of traffic route can be carried out by applying total station's function of laying out coordinates. On the other hand, the distance between two adjacent intersections of centerline straight segments and included angle between two intersected centerline straight segments can be inversely calculated from formerly designed coordinates of intersections of centerline straight segments so that the location survey of traffic route is able to be accomplished by applying included-angle method and total station's function of laying out angles and distances.

Directly diagrammatized coordinates of centerline straight segments' intersections from strip map or aerial map can also be laid out by applying total station's function of laying out coordinates. Because of the lower accuracy of diagrammatized coordinates, it is necessary to thread, check and adjust the laid-out stations and then locate them by intersecting straight segments of centerline.

(2) Application of GPS RTK in Alignment Survey of Route

GPS RTK is developed from real-time dynamic differential techniques in phase measurement of carrier wave. It is able to provide three-dimensional coordinate in designated coordinate system of stations with centimeters accuracy. In RTK surveying, reference station conveys both their measurements and coordinates of themselves via data link to mobile station besides receiving data from reference station via data link. Mobile station collects GPS observational data and processes differential measurements real-timely to gain location measurements of centimeters accuracy.

The flow of work of alignment survey by GPS RTK is approximately as the following: firstly, make preparations of laid-out data; secondly, set up RTK reference station at known points and start GPS receiver to ensure differential signals are sending off from reference station; then start GPS receiver at mobile station, and with RTK setting to alignment surveying mode conduct alignment survey by selecting proper method.

Before, during and after staking-out traffic route, it is advisable for surveyors to perform necessary checks at known points.

Alignment survey by GPS RTK is composed of office work and field work. The former mainly includes designing centerline straight segments for later staking-out in GPS electronic field book with formerly designed coordinate data of intersections of centerline straight segments, starting and ending points of curves, centers and radii of circles. The latter includes locating the designed route in field with GPS RTK set to alignment surveying mode.

When applied in route's alignment surveys, especially for long traffic routes, RTK technique has unique advantages compared with conventional surveying techniques, which include (a) being handled expediently and operated flexibly. With

RTK technique, alignment survey can be conducted either consecutively or to start from any position of a route at any moment. This is convenient for several instruments to be operated jointly; (b) being direct-viewing, clear-displaying and convenient. The designed route can be displayed directly on RTK operating screen and the operator can lay out the points accordingly; (c) demanding less workers and thus resulting in higher efficiency; (d) measurements can be checked on in real time so the reliability of measurements is enhanced. Beware to check on positions of known points at any moment during alignment survey; (e) the errors of measurements are not cumulative and disseminative thus results in a higher standard of accuracy (Zhang Xiao et al. 2007).

3. APPLICATION OF LEAST SQUARES IN ALIGNMENT SURVEY OF ROUTE BY DIAGRAMMATIC METHOD

3.1 Procedure of Laying-out Centerline Stations Diagrammatically

Procedures or steps to be followed in laying-out centerline stations by diagrammatic method include

(1) diagrammatizing staking-out parameters, such as coordinates and offsets from designing plan;

(2) laying-out centerline stations in field by polar coordinate method or offset method;

(3) threading, which is equivalent to adjusting staked-out stations which have been displaced by errors stemming from diagrammatizing and laying-out back into the designed straight line;

(4) locating the intersections. The straight segments of centerline determined by preceding methods are checked to assure their validity. If they are extended, adjacent straight segments will intersect and an intersection is acquired. Therefore intersections of route's centerline straight segments are located. See Fig.1.

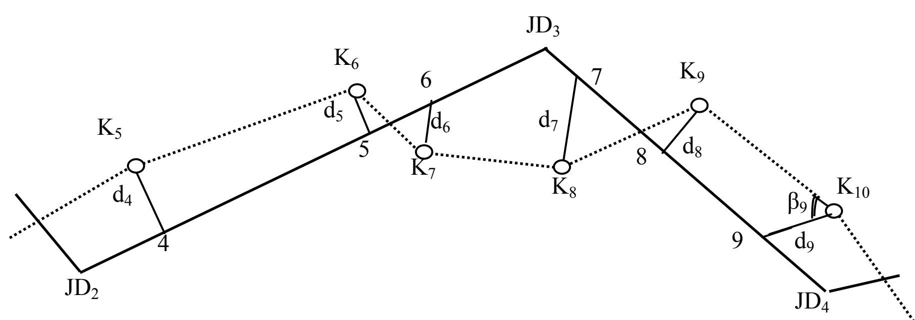

Figure 1.Sketch of alignment survey by diagrammatic method (or called offset method).

Note: In the above figure, JD_2, JD_3 and JD_4 are intersections of centerline straight segments of route and K_5、K_6 are traverse controls . d_4、d_5、β_9 are parameters for diagrammatizing.

While centerline stations are laid out either by polar coordinate method (see point 9 in Fig.1) or by offset method (see point 4, 5 and 6 in Fig.1), the diagrammatized pairs of coordinates X and Y of centerline stations, such as point 4, 5, 6, 7, 8, and 9, can be determined by applying formerly solved parameters. Because of the effects of errors stemming from diagrammatizing and laying-out, the obtained centerline stations are not in line and the displacements are large enough to guarantee repeatedly threading, checking on and adjusting the laid-out centerline stations. By extending the adjusted centerline straight segments, the intersections such as JD_2, JD_3 and JD_4 are established. This procedure is discussed in most of the present surveying textbooks (Ji Binde et al. 2002).

3.2 Procedure and Thread in Applying Principle of Least Squares

(1)Being centerline straight segments designed from strip map, JD_2 JD_3, JD_3 JD_4 are diagrammatized as coordinates of point 4, 5, 6 and point 7, 8, 9 respectively. See Fig.1. Take these diagrammatized coordinates for observational values, because point 4, 5, 6 and point 7, 8, 9 are in line respectively.

(2)List condition equations respectively for these two centerline segments in accordance with geometry features of three points in every centerline segment

$$\hat{\alpha}_{56} - \hat{\alpha}_{54} = 180^0 ;\ \hat{\alpha}_{89} - \hat{\alpha}_{87} = 180^0 .$$

(3)Solve the above condition equations with principle of least squares so that obtain the optimum estimations of centerline stations. Lay out these centerlines stations by polar coordinate method.

(4) Accuracy verification.

3.3 Steps in Applying Principle of Least Squares

3.3.1 Set up Condition Equation Model

Based on diagrammatized coordinate observational values, list condition equations about adjustments

$$\hat{\alpha}_{56} - \hat{\alpha}_{54} = 180^0 \tag{1}$$

$$\hat{\alpha}_{89} - \hat{\alpha}_{87} = 180^0 \tag{2}$$

Suppose adjustments of coordinate $\hat{X} = X + V_X$, $\hat{Y} = Y + V_Y$. So that

$$\arctan\frac{(y_6+v_6)-(y_5+v_5)}{(x_6+v_6)-(x_5+v_5)} - \arctan\frac{(y_5+v_5)-(y_4+v_4)}{(x_5+v_5)-(x_4+v_4)} - 180^0 = 0 \tag{3}$$

$$\arctan\frac{(y_9+v_9)-(y_8+v_8)}{(x_9+v_9)-(x_8+v_8)}-\arctan\frac{(y_8+v_8)-(y_7+v_7)}{(x_8+v_8)-(x_7+v_7)}-180^0=0 \tag{4}$$

Where $\hat{\alpha}$ stands for azimuth of straight line; $\hat{X}$, $\hat{Y}$, X, Y stand for adjustments and measurements of diagrammatized coordinates; and V_X, V_Y stand for corrections of diagrammatized coordinates coordinates (Wuhan University Geodesy Adjustment Group 2005).

3.3.2 Solution of Condition Equations to Obtain Corrections

(1) Linearization of Condition Equations

From equations 3 and 4, we know that condition equations are nonlinear. Expand them by method of Taylor series, omitting quadratic term and terms after it. The following equations are obtained by rearrangement

$$\rho''(\frac{\Delta Y_{56}^0}{(S_{56}^0)^2}-\frac{\Delta Y_{54}^0}{(S_{54}^0)^2})V_{X_5}-\rho''(\frac{\Delta X_{56}^0}{(S_{56}^0)^2}-\frac{\Delta X_{54}^0}{(S_{54}^0)^2})V_{Y_5}-\rho''\frac{\Delta Y_{56}^0}{(S_{56}^0)^2}V_{X_6}+\rho''\frac{\Delta X_{56}^0}{(S_{56}^0)^2}V_{Y_6}+\rho''\frac{\Delta Y_{54}^0}{(S_{54}^0)^2}V_{X_4}-$$

$$\rho''\frac{\Delta X_{54}^0}{(S_{54}^0)^2})V_{Y_4}+W_1=0 \tag{5}$$

$$\rho''(\frac{\Delta Y_{89}^0}{(S_{89}^0)^2}-\frac{\Delta Y_{87}^0}{(S_{87}^0)^2})V_{X_8}-\rho''(\frac{\Delta X_{89}^0}{(S_{89}^0)^2}-\frac{\Delta X_{87}^0}{(S_{87}^0)^2})V_{Y_8}-\rho''\frac{\Delta Y_{89}^0}{(S_{89}^0)^2}V_{X_9}+\rho''\frac{\Delta X_{89}^0}{(S_{89}^0)^2}V_{Y_9}+\rho''\frac{\Delta Y_{87}^0}{(S_{87}^0)^2}V_{X_7}-$$

$$\rho''\frac{\Delta X_{87}^0}{(S_{87}^0)^2})V_{Y_7}+W_2=0 \tag{6}$$

They are linear condition equations where ρ'' --206265"; ΔX^0, ΔY^0 stand for computed increments of X coordinate and Y coordinate from diagrammatized coordinates; S^0 stands for approximate corresponding side length computed from diagrammatized coordinates; and misclosure

$$W_1=\alpha_{56}^0-\alpha_{54}^0-180^0;\ \ W_2=\alpha_{89}^0-\alpha_{87}^0-180^0$$

Where α^0 stands for approximate value of straight line's azimuth reversely calculated from diagrammatized coordinates.

(2)Solution of Linear Condition Equations by Method of Least Squares

From linear condition equations 5 and 6, we know that the two equations contain twelve unknown variables and have indefinite solutions. It is a mathematical problem of solving function's extremum with conditions for surveyors to solve V from function $V^T PV = \min$. Basic equations are established and solved with the following result

$$V = -QA^T (AQA^T)^{-1} W \tag{7}$$

where Q stands for covariance factor of measurements and adopts the value of I (suppose diagrammatized coordinates are independent); A stands for Coefficient matrix of condition equations; and W stands for misclosure of condition equations.

After corrections being solved, the most probable value (or adjusted value) of observed coordinates can be obtained as the following

$$\hat{X} = X + V_X\ ; \quad \hat{Y} = Y + V_Y\ .$$

(3)Accuracy Estimation

Root mean square error of unit weight (RMSE)

$$\hat{\sigma}_0 = \sqrt{\frac{V^T PV}{r}} \tag{8}$$

Accuracy of adjusted value of diagrammatized coordinates

$$\hat{D}_{\hat{L}} = \sigma_0^2 (Q - QA^T (AQA^T)^{-1} AQ), \tag{9}$$

where P stands for weight of measurements , and adopts the value of I ; r stands for the number of condition equations; $\hat{D}_{\hat{L}}$ stands for variance-covariance matrix for adjusted value of diagrammatized coordinates.

3.4 Example for Demonstration

Diagrammatized coordinates of the three points on each straight segment of traffic route's centerline are shown in Table1. Adjusted coordinates based on least squares methods are listed in Table 1 (Liang Yubao et al. 2008).

Table 1. Data sheet of diagrammatized coordinates of traffic route stations.

Station number	Diagrammatized coordinates (m)		Station number	Diagrammatized coordinates (m)	
	X	Y		X	Y
4	3740500.0	38434719.3	7	3740000.0	38436364.0
5	3740413.5	38435000.0	8	3739500.0	38436986.5
6	3740259.5	38435500.0	9	3739312.5	38437220.0

Related data are calculated and then substituted into linear condition Equations 5 and 6. After rearranging, we get

$$-1.835V_{X_4} - 0.566V_{Y_4} + 1.932V_{X_5} + 1.726V_{Y_5} - 3.768V_{X_6} - 1.160V_{Y_6} - 29.8'' = 0$$

$$-2.014V_{X_7} - 1.618V_{Y_7} + 7.385V_{X_8} + 5.930V_{Y_8} - 5.371V_{X_9} - 4.312V_{Y_9} - 27'' = 0$$

Solve the above condition equations, then we get

$$\underset{6\,1}{V_X} = \left[V_{x_4}, V_{x_5}, V_{x_6}, V_{x_7}, V_{x_8}, V_{x_9} \right]^T = \begin{bmatrix} -2.10 \\ 2.22 \\ -4.33 \\ -0.38 \\ 1.39 \\ 1.01 \end{bmatrix} cm \qquad \underset{6\,1}{V_y} = \left[V_{y_4}, V_{y_5}, V_{y_6}, V_{y_7}, V_{y_8}, V_{y_9} \right]^T = \begin{bmatrix} -0.65 \\ 1.98 \\ -1.33 \\ -0.30 \\ 1.11 \\ -0.81 \end{bmatrix} cm$$

Accuracy estimation can be performed by the following steps.

From Equation 8, root mean square error of unit weight

$$\hat{\sigma}_0 = \sqrt{\frac{V^T PV}{r}} = \sqrt{\frac{39.3}{2}} = \pm 4.43cm.$$

From Equation 9, accuracy of adjusted value and location error of diagrammatized coordinates are computed and shown in Table 3.

Table 2. Data sheet of diagrammatized coordinates of traffic route stations by least squares.

Station number	Diagrammatized coordinates (m)		Station number	Diagrammatized coordinates (m)	
	X	Y		X	Y
4	3740499.979	38434719.293	7	3739999.996	38436363.997
5	3740413.522	38435000.019	8	3739500.014	38436986.511
6	3740259.457	38435499.987	9	3739312.510	38437219.992

Table 3. Data sheet of accuracy of adjusted value of diagrammatized coordinates of traffic route stations by least squares

Station number	accuracy of adjusted value of diagrammatized coordinates (cm)		Error in station location (cm)	Station number	accuracy of adjusted value of diagrammatized coordinates (cm)		Error in station location (cm)
	σ_X	σ_Y	σ		σ_X	σ_Y	σ
4	±4.13	±4.40	±6.03	7	±4.36	±4.38	±6.82
5	±4.09	±4.16	±5.83	8	±3.49	±3.84	±5.19
6	±2.97	±4.31	±5.23	9	±3.96	±4.13	±5.72

Regulations for highway surveying specified tolerance of deviation of stake locations for high order highway's centerline stakes as followings: in longitudinal direction, S/200 +0.1m (S stands for location measurements of centerline stakes, in meters); in transverse direction, 10cm (China Ministry of Communications 1992).

Take half the tolerance of deviation of centerline stake locations for tolerance of RMSE in lengthwise and transverse directions of centerline stake locations, i.e.

$$\sigma_L = \frac{1}{2} \times 10 = \pm 5\text{cm}; \qquad \sigma_T = \frac{1}{2} \times 10 = \pm 5\text{cm}$$

So that tolerance of RMSE of centerline stake locations

$$\sigma_C = \sqrt{\sigma_L^2 + \sigma_T^2} = \sqrt{5^2 + 5^2} = \pm 7.07\text{cm}$$

4. CONCLUSION

The following conclusion can be drawn from the above discussions:

If the designed coordinates of intersections of route's centerline straight segments are known, surveyors can conduct alignment survey for traffic route by either conventional surveying techniques or contemporary surveying techniques. The choice depends on actual situation. If the coordinates of intersections of route's centerline segments have not been designed, surveyors may directly diagrammatize coordinates from strip map or aerial map, and conduct alignment survey for traffic route by diagrammatic method (or offset method). When using diagrammatizing method, beware to diagrammatize coordinates of at least three evenly distributed centerline stations for each centerline segment. Then eliminate contradictions within diagrammatized coordinates and work out the optimum estimation by applying method of least squares.

From estimated accuracy of adjusted value of diagrammatized coordinates, we can see that location accuracy of diagrammatized coordinates which have been handled by method of least squares is within the range of tolerance of RMSE of centerline stake locations specified by regulations($\sigma_C = \pm 7.07\text{cm}$). Therefore surveyors can directly apply this method in laying out centerline stakes, such as starting point, ending point and intersections of centerline's straight segments without any troubles of repeatedly threading, checking on and adjusting the laid-out centerline stations. The related complicated computational process conforms to strong regularity and can be handled by computer programs that can be operated conveniently and practically.

In brief, this paper concisely introduces both conventional and contemporary techniques of traffic route's alignment survey. It also analyzes and discusses the question that while traffic route's alignment survey is being conducted by conventional diagrammatic method (or called offset method), how to improve the accuracy of diagrammatized centerline stations' coordinate parameters for staking-out by principle of least squares in order to reduce the workload of repeatedly threading, checking on and adjusting the laid-out centerline stations. Finally, the authors draw the conclusion that the laid-out traffic route's centerline stations based on staking-out coordinate parameters obtained by principle of least squares are able to satisfy the required tolerance of location survey of traffic route's centerline stakes.

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