Normal Distribution, Functions & Random Variables: set-2

1) The work begins after 10 min, so the average time increase from 45min to 55min.
For normal distribution: -
X = 50
= 45
= 8
Z = (X-)/ = (X-45)/8
Z = (50-45)/8
P(X<=50)=P(Z<=(50-45)/8)
Z = 0.625 =73.4%
Probability that the service manager will not meet his demand will be = 100 – 73.4
= 26.6% or 0.2676

2) Mean = 38
SD = 6
Z score = (value – mean)/SD
a) Z score for 44 = (44 -38)/6 =1 =>84.13%
People above 44 ages = 100 – 84.13 = 15.87% = 63 out of 400
Z score for 38 = (38 – 38)/6 = 0 => 50%
Hence people between 38 & 44 age = 84.13 – 50 = 34.13% ~ 137 out of 4000
Hence more employees at the processing center are older than 44 than between 38 and 44.is
"FALSE"
b) Z scores for 30 =(30 – 38)/6 = -1.33 = 9.15% = 36 out of 400
Hence a training program for employees under the age at the center would be expected to
Attract about 36 employees - TRUE

3) The difference between $2X_1$ and $X_1 + X_2$ is $N(0,6^2)$.
According to Central Limit Theorem, the properties of normal random variables,
$X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are two independent distributed random variables.

Given to find, $2X_1$: $2X_1 \sim N(2,2^{22}) \Rightarrow 2X_1 \sim N(2,2^2)$
$X_1 + X_2 \sim N(,^2 + ^2) \sim N(2,2^2)$
And the difference between two is given by: $2X_1 - (X_1 + X_2) \sim N(2\mu - 2, 2_1^2 + 4\ {}_1^2) \sim N(0,6^2)$
The mean of $2X_1$ is 2 times more than the variance of $X_1 + X_2$.

4) The probability of getting value between a & b is 0.99

So, the probability of value getting outside a & b is 1-0.99 = 0.01

The probability towards left of a = -0.01/2 = -0.05

The probability towards right of b = 0.01/2 = 0.05

Since we have probabilities of a & b, we need to calculate the probability of X,

By finding Standard Normal Variable (z), need to calculate X:

Z=(X-Meu)/sigma

For a probability of 0.005, z values is -2.57

Z*+ = X

a – (-2.57) *20+100=151.4

b – (-2.57) *20+100=48.6

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5].

5) Division1 = Profit1 ~ N(5, 32) = N(X1=5, S11=32)

Division2 = Profit2 ~ N(7, 42) = N(X2=7, S22=42)

= Company = (Profit1 + Profit2) = Mean Profit of Division1 + Division2

= 5 + 7 = 12

Mean profit of company in rupees = 12*45 = 540

i) Variance of the company distribution = ^2 = 32=42 = 9+16 = 25 = 52

Standard Deviation of the company distribution = = 2) = 5

Confidence Level = 0.95

Therefore, Confidence Interval = ± za/2= 0.025()

= 540 ± 1.96(225) = (99,981)

ii) 5th percentile from z table= Za/2= 0.05 = -1.645

5th percentile = – Za/2= 0.05()

= 540 – 1.645(225)

= 169.87

iii) The division 2(Profit2 ~ N(7, 42)) has a larger probability of making a loss in a given year.