LAKSHYA
JEE 2025

MATHEMATICS | Lecture - 02

DETERMINANTS

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Topics *to be covered*

1. Minors & Co-Factors
2. Properties of Determinant
3. Question Practice
4.

Last Class Recap

Value of Determinant:

$\boxed{R = C}$

Order $\rightarrow 2$

$\hookrightarrow \underline{\underline{2}} \times \underline{\underline{2}}$

a_{ij}

$\Downarrow$

diag $\rightarrow i = j$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\# \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} x & y & z \\ p & q & r \\ l & m & n \end{vmatrix} \end{array} = x(qn - rm) - y(pn - lr) + z(pm - ql)$$

4×4:

$$\begin{vmatrix} a_{11} & \_ & \_ & \_ \\ \_ & \_ & \_ & a_{24} \\ \_ & a_{32} & \_ & \_ \\ \_ & \_ & a_{43} & \_ \end{vmatrix}$$

vilupt → नहीं आएगा!!

#

$$\begin{array}{cccc} + & - & + & - \\ \end{array}$$
$$\begin{vmatrix} 2 & -1 & 0 & 3 \\ 4 & 1 & 2 & 0 \\ -1 & 1 & 3 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix}$$

$$= +2\begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 2 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 4 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 & 2 \\ -1 & 1 & 3 \\ 0 & 1 & 1 \end{vmatrix}$$

$1(3-1) - 2(\cancel{1-1}) + 0(\ \ )$ → 2

$+4(3-1) - 2(-1-0)$ → $8+2$

$4(1-3) - 1(-1-0) + 2(-1-0)$

$-8+1-2 = -9$

$+2(2) + 1(10) - 3(-9)$

41

$= ???$ → 41 ✓

Chat!!

ACIDIC QUESTION

Drink & Drive

Show that $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$.

$$0\,|\ \ | - a\begin{vmatrix} -a & d & e \\ -b & 0 & f \\ -c & -f & 0 \end{vmatrix} + b\begin{vmatrix} -a & 0 & e \\ -b & -d & f \\ -c & -e & 0 \end{vmatrix} - c\begin{vmatrix} -a & 0 & d \\ -b & -d & 0 \\ -c & -e & -f \end{vmatrix}$$

$$-a\{-a(0-(-f^2)) - d(0-(-cf)) + e(bf)\}$$

$$-a((-af^2 - dcf + ebf))$$

$$-f$$

$$(af)\{af + cd - eb\}$$

$$+b(-a(ef) + e(eb - cd))$$

$$-be(+af - eb + cd)$$

$$-c\{-a(af - 0) + d(eb - cd)\}$$

$$-cd\{-af + eb - cd\}$$

$$+cd\{af - eb + cd\}$$

$$af(af + cd - eb)$$

$$-be(af + cd - be)$$

$$+cd(af + cd - eb)$$

$$(af + cd - eb)(af + cd - eb)$$

H.P.

Minors & Co-Factors

Minors :

Minors of an element is defined as the minor determinant obtained by deleting a particular row or column in which that element lies.

Cofactor :

It has no separate identity and is related to the minors as $C_{ij} = (-1)^{i+j} M_{ij}$, where '$i$' denotes the row and '$j$' denotes the column.

$$\begin{vmatrix} + & - \\ - & + \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$a_{11} \longrightarrow$ Minor of $a_{11} = M_{11} = -17$.

ex:-

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 5 \\ -7 & 3 & -2 \end{vmatrix}$$

$M_{13} = 19$.

$M_{23} = 27$.

Minor of $a_{22} = M_{22} = -11$

$M_{32} = 14$

$C_{32} = -14$.

cofactor

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

$i \to$ Row ka no.

$j \to$ column ka no.

ex:-

$$\begin{vmatrix} 2 & -1 \\ -4 & 5 \end{vmatrix}$$

$M_{11} = 5$

$M_{22} = -4$

$M_{21} = -1$

$M_{22} = 2$

$C_{23} = (-1)^{2+3} 27$

$= -27$.

$a_{13} = -1$

$M_{13} = 19$

$C_{13} = (-1)^{1+3}(19) = 19$.

QUESTION

Write the cofactors of the elements of the determinant $D = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$ and hence evaluate the determinant.

$i+j=2 \leftarrow a_{11}$

$$\begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$$

$2 \to 38$, $3 \to -13$, $2 \to -14$, $1 \to -12$, $4 \to 6$, $-1 \to +3$, $5 \to -11$, $6 \to 4$, $8 \to 5$

$a_{22} = 4$

$M_{22} = 6$

$C_{22} = (-1)^{2+2}(6) = 6.$

$a_{11} \longrightarrow M_{11} = 38.$

$C_{11} = (-1)^2 38 = 38.$

$a_{32} \longrightarrow M_{32} = -2-2 = -4.$

$C_{32} = (-1)^{3+2}(-4)$

$= -1 \times -4$

$= 4.$

QUESTION

$R_2 \rightarrow \ 1 \quad 4 \quad -1$

$\text{C.F. of } R_3 \rightarrow \ -11 \quad 4 \quad 5$

$-11 + 16 - 5 = 0$

$i+j=2 \quad a_{11}$

Write the cofactors of the elements of the determinant $D = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$ and hence evaluate the determinant.

C_1: $2 \rightarrow -13$, $1 \rightarrow 6$, $5 \rightarrow 4$

$\underbrace{-26+6+20}_{0}$

$\begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$

$2 \rightarrow 38$, $3 \rightarrow -13$, $2 \rightarrow -14$, $1 \rightarrow -12$, $-1 \rightarrow +3$, $4 \rightarrow 6$, $8 \rightarrow 5$, $5 \rightarrow -11$, $6 \rightarrow 4$

$\text{Row}_2 \rightarrow \ 1 \quad 4 \quad -1$

$\text{Cof. of } R_1 \rightarrow \ 38 \quad -13 \quad -14$

$38 - 52 + 14 = 0$

$3 \rightarrow -13$, $4 \rightarrow 6$, $6 \rightarrow 4$

$-39 + 24 + 24 = 9$

Value of determinant

$D = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix} = 38 \times 2 + (-13)(3) + (-14 \times 2) = 9$

Cofactors of R_1: $38 \quad -13 \quad -14$; cofactors of R_3: $-11 \quad 4 \quad 5$

$= (-11)5 + (6 \times 4) + (8 \times 5)$

$= -55 + 24 + 40$

$= 9$

$2(-14) - 1(3) + 8(5)$

$-28 - 3 + 40$

9

QUESTION

$R_2 \to \quad 1 \quad 4 \quad -1$

C.F. of $R_3 \to \quad -11 \quad 4 \quad 5$

$-11 + 16 - 5 = 0$

$i + j = 2 \quad a_{11}$

Write the cofactors of the elements of the determinant $D = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$ and hence evaluate the determinant.

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$$

38 $\quad$ −13 $\quad$ −14 $\quad$ −12 $\quad$ +3 $\quad$ 6 $\quad$ −11 $\quad$ 4 $\quad$ 5

DIBY!

$$\Rightarrow \Delta_C = \begin{vmatrix} 38 & -13 & -14 \\ -12 & 6 & 3 \\ -11 & 4 & 5 \end{vmatrix} = 81$$

$$\begin{array}{ccc} + & - & + \\ \end{array}$$

$$\begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix} = +2(32+6) - 3(13) + 2(6-20)$$

$$(-1)^{1+1} a_{11} M_{11} + (-1)^{1+2} a_{12} M_{12} + (-1)^{1+3} a_{13} M_{13}$$

$$\Downarrow$$

$$a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$$

$$\begin{vmatrix} x & y & z \\ p & q & r \\ l & m & n \end{vmatrix} = \boxed{x\underbrace{(qn - rm)}_{\text{Value}} - y(pn - lr) + z(pm - ql)}$$

$$l(qn - rm) - m(pn - lr) + n(pm - ql)$$

$$\cancel{lqn} - \cancel{lrm} - \cancel{mpn} + \cancel{lrm} + \cancel{pmn} - \cancel{qln} = 0$$

Summary.

$\left| \quad \right|_{3\times3}$ → value / expand → $\boxed{6 \text{ ways}}$

→ elem. kisi aur row ke
cof. kisi aur row ke

C.M. & add
⇓
Zero

Value of Determinant

 # Value of determinant = Sum of product of elements of corresponding row (or column) with their corresponding cofactors.

English

 # Note : C_{ij} is cofactor of a_{ij}

Maths

Value of determinant $(\Delta) = \boxed{\sum_{j=1}^{3} a_{ij}C_{ij}}$ for $i = 1, 2, 3$

Agar kisi row (or column) ke elements ko kisi aur row (or column) key cofactors se multiply krke add kre to jawab zero aata h.

NOTE :

Compact form.

Value of det $= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} =$

$= a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}$.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\sum_{i=1}^{3} a_{i1} C_{i1} = \underbrace{a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}}_{\text{value along column 1}}$ ✓

$j = 1$

$\sum_{j=1}^{3} a_{2j} C_{2j} = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$

↳ value along R_2

Value/Expansion of Determinant

➤ $\Delta = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = +6$

$\begin{vmatrix} 2 & 1 & -3 \\ \boxed{\overset{-}{0} \quad \overset{+}{1} \quad \overset{-}{1}} \\ 2 & 1 & 0 \end{vmatrix} = -0\{\quad\} + 1\underbrace{\begin{vmatrix} 2 & -3 \\ 2 & 0 \end{vmatrix}}_{6} - 1\cancel{\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}}^{0} = 6.$

➤ $\begin{array}{cc} & \Delta = \begin{vmatrix} 41 & 9 & 33 \\ 21 & 1 & 3 \\ 2 & 0 & 0 \end{vmatrix} \\ & \quad\quad + \quad - \quad + \end{array}$ $R_3 \rightarrow$ $= +2(27-33) - 0\cancel{(\quad)}^{0} + 0\cancel{(\quad)}^{0}$

$= -12.$

$C_3 \downarrow$

➤ $\Delta = \begin{vmatrix} 1021 & 226 & 0 \\ 932 & 854 & 0 \\ 518 & 2727 & 0 \end{vmatrix} = 0$

Important Note :

R.A.R.

If Δ_c is cofactor determinant which is formed by replacing all the elements of Δ their cofactors, then :

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

cofactor wala deter. = Δ_c

$$\Delta_c = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

Proof ⇓ In next chapter ⇓ Matrix.

$$\boxed{\Delta_c = (\Delta)^{n-1}}$$

, $n \equiv$ order of det.

for $n=2$: $\boxed{\Delta_c = \Delta}$

for $n=3$: $\boxed{\Delta_c = \Delta^2}$

QUESTION

Find $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 0 & -2 \\ -3 & 4 & 1 \end{vmatrix}$ also show that $\Delta_C = \begin{vmatrix} 8 & 7 & -4 \\ 14 & 10 & 2 \\ 4 & -1 & -2 \end{vmatrix} = 324$.

DIBY!!

Determinant used in Class XI

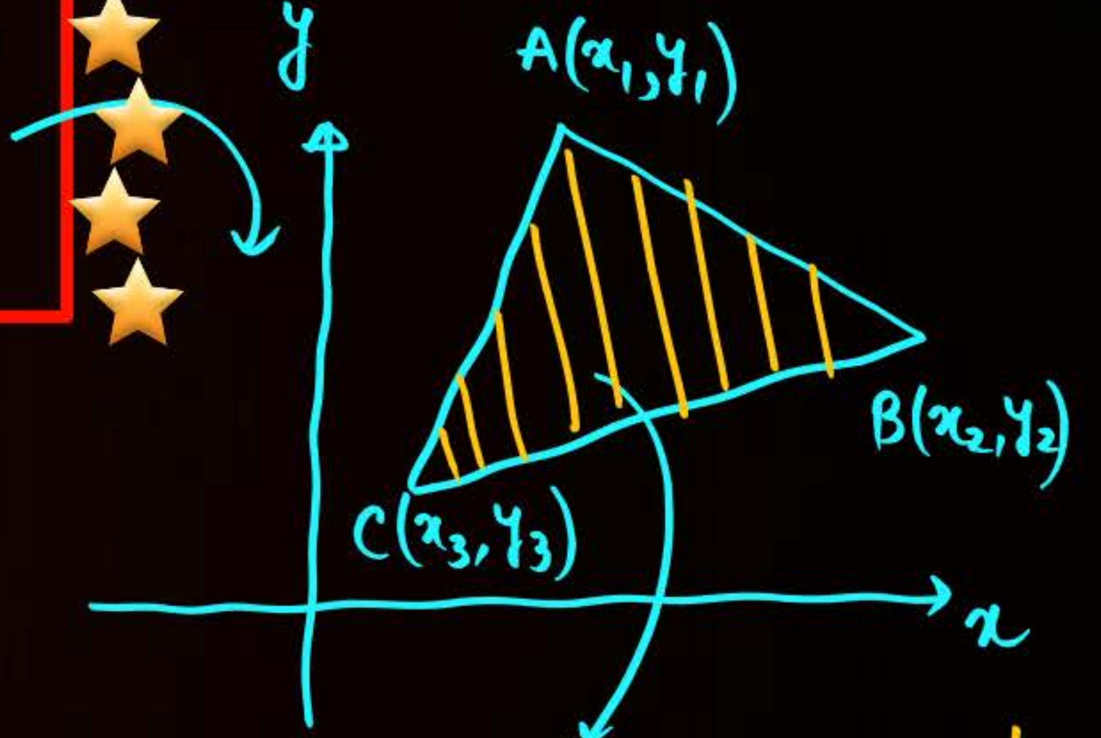
(i) Area of a triangle whose vertices are (x_r, y_r); $r = 1, 2, 3$ is:

$$D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If D = 0, then the three points are collinear.

(ii) Equation of a straight line passing through

(x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$



$$\text{area}(\Delta ABC) = \left| \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

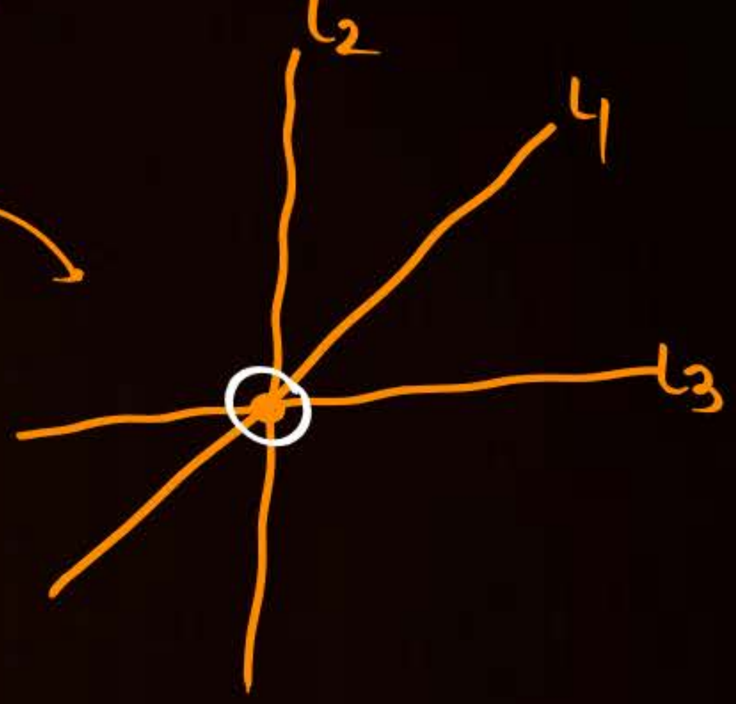
Mod.

Determinant used in Class XI

(iii) The lines : $a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$ ....(2)

$a_3x + b_3y + c_3 = 0$ ....(3)



are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_2 \end{vmatrix} = 0$

(iv) $\boxed{ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0}$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(v) **Cross product of two vectors.**

CHALLENGER QUESTION

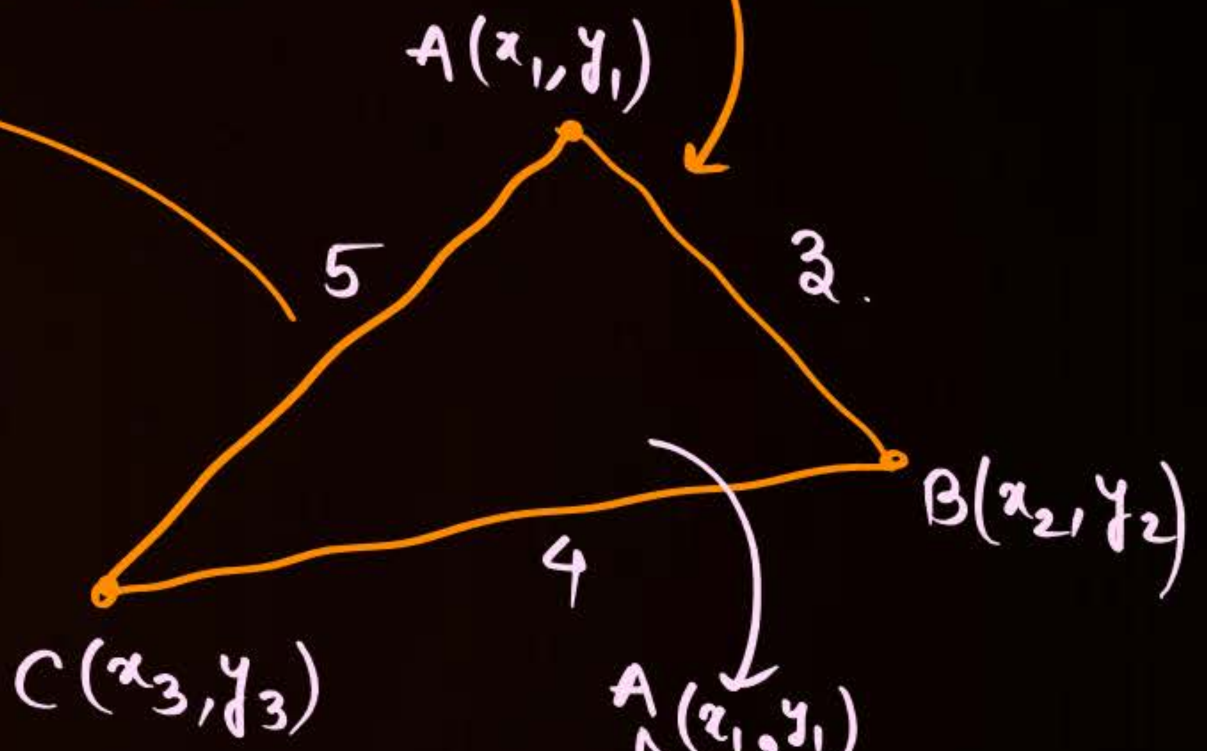
observe → decode

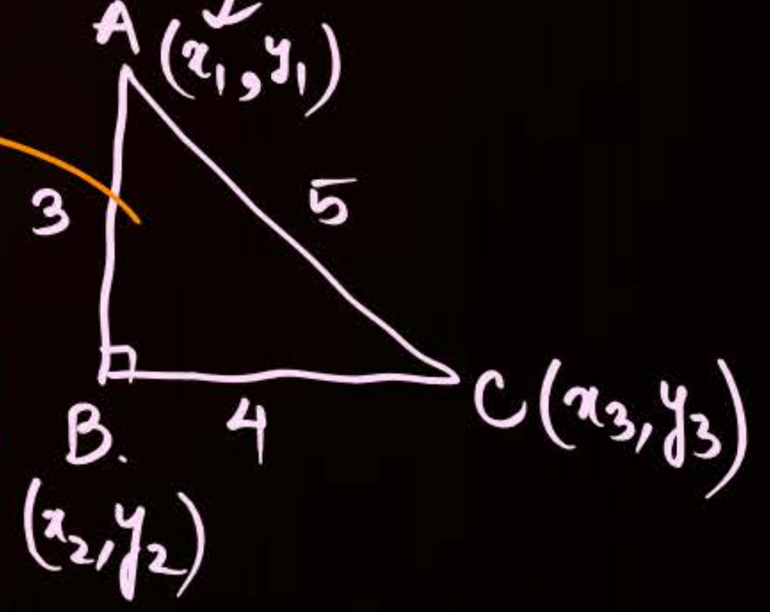
If $(x_1 - x_2)^2 + (y_1 - y_2)^2 = 9$,
$(x_2 - x_3)^2 + (y_2 - y_3)^2 = 16$ and $(x_1 - x_3)^2 + (y_1 - y_3)^2 = 25$,

then find the value of $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$. $= (12)^2$

$= 144$

Ans





$6 = \frac{1}{\cancel{2}} \times 3 \times \cancel{4}^{2} = ar(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

QUESTION (IIT-JEE-1983)

Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, the other two roots a and b, and then find ab.

HW

QUESTION (IIT-JEE-1981)

The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$, is

HW.

QUESTION (IIT-JEE-1981)

Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is

Q1BY!!

Homework

Re-attempt all the Questions - jo apke hisab se ache hai of Lecture.

DPP regular basis pr upload hogi, daily solve kre!!

Module:

Exercise (Prarambh) : Ques: 1, 2, 3, 5, 6, 7

Note: Jaha Matrix likha hua dikhe unn Questions ko leave kr de!

It’s not about End Result,
It is all about JOURNEY

THANK YOU

#futureIITians