GEOMETRY OF (ANTI)-DE SITTER SPACE AND EXTENDED UNCERTAINTY PRINCIPLE

Abstract

On (anti)-de Sitter background, the Heisenberg uncertainty principle(HUP) should be modified by the introduction of a term proportional to the cosmological constant. So, in this paper I showed the derivation of HUP from the geometry of (anti)-de Sitter spacetime(ADSST). We extend our discussion to find the connection between the extended generalized uncertainty principle(EGUP) and triply special relativity(TSR).

RIDDHIMAN BHATTACHARYA

[riddhiman.butai2005@gmail.com ; Class-12(Purv International School)]

Introduction

The HUP($\Delta x_i \Delta p_j \geq \hbar/2$) of quantum mechanics(QM), plays an important role in protecting the conceptual structure and frame of quantum mechanics as mentioned by Feynman. and can be considered one of cornerstones the of the theory.

But in extreme cases from order of energy QM derived ,the Uncertainty Principle(UP) needs modification and has been called as Extended Uncertainty Principle(EUP). It is supposed to hold at quantum gravity scales and postulates discussed later on.

[In an (anti)-de Sitter background the HUP needs modification introducing corrections proportional to the cosmological constant $\Lambda = \frac{-3}{l_H^2}$ with l_H the (anti)-de Sitter radius, is

$\Delta x_i \Delta p_j \geq \frac{\delta_{ij}}{2}\left[1 + \frac{(\Delta x i)^2}{l_H^2}\right]$ (1)Where $\delta_{ij} = \hbar/2$] (1)

The mathematical framework of EUP is $\Delta x_i \Delta p_j \geq \frac{\hbar}{2[1+l_P^2(\Delta p_i)^2]}$ (2) with l_P the Planck length.

Though the correction to the Heisenberg formula given by eq^n (1) is negligible for cosmological values of l_H, as it can be used to derive the correct value of the temperature of an (anti)-de Sitter black hole.(**Hawking Temperature obtained with modification of UP using the dynamics of gravitational Field**).

This paper shows the derivation of EUP from geometry of ADSST unlike eq^n (1)'s derivation from Generalized Uncertainty Principle(GUP). We define quantum mechanics on a curved background with taking account of the symmetry. In ADSST, the translations do not satisfy the same algebra as in case of flat space. Jacobi identity follows the commutation relations between momentum and position coordinates. UP follows $\Delta x_i \Delta p_j \geq \frac{1}{2}\left|\langle [x_i, p_j] \rangle\right|$ (3) where < > denotes the expectation value.

GENERALISED AND EXTENDED UNCERTAINTY PRINCIPLE

GUP was proposed in context of string theories and then derived for gravity; non- commutative geometry. For $l_p^2 > 0$ eq^n 2implies the existence of absolute minimum in position uncertainty($x_{min}=2l_p$). If $l_p^2 < 0$ no lower limit on measurable length but bound on upper limit on attainable momentum according to DOUBLY SPECIAL RELATIVITY (DSR) theories based on definition of space time where action of Poincare' group nonlinearly defined in such a way that Planck Energy becomes observer independent setting an upper limit on the energy momentum of the elementary particles. Such deformation is not unique.

SYNDER MODEL(SM): An example of DSR defined on non-commutative spacetime under which Lorentz invariance follows commutation relations of SM is $[x_\mu, x_v]$=i$l_p^2 J_{\mu v}$ & $[x_\mu, P_v]$=i$(\eta_{\mu v} + l_p^2 P_\mu P_v)$ -(4)

Where $J_{\mu v}$ is Lorentz Generators; and μ, v = 0,1,2 , 3.

Usual DSR interpretation obtained for $l_p^2 < 0$ and $l_p^2 > 0$ the model shows different physical properties- existence of minimal length. We call $l_p^2 > 0$ anti-Snyder.

One can identify uncertainty in position of particle emitted by Hawking effect as $\Delta x_i = r_+$ where r_+ radius of horizon of black hole.[GUP is used to obtain corrections to the derivation of Hawking Temperature for Schwarzschild black holes] and Energy$_{uncertainty}$ $\Delta E \approx \Delta p_i \approx \frac{1}{4\pi R_+}$

T=$\frac{\Delta E}{2\pi} = \frac{1}{4\pi R_+}$ eqn(5) where T is Hawking's Temperature and $1/2\pi$ is due to normalization. If we had approached this by GUP, we would have used Δp_i from eq^n 2 and it would have been modified to T=$\frac{\Delta E}{2\pi} = \frac{r_+}{8\pi l_p^2}[1 - \sqrt{1 - \frac{4l_p^2}{r_+^2}}]$ -(6)

Giving rise to corrections of order $(lp \,/\, r_+)$ to the Hawking's Formula. With little more working $l_H^2 > 0$ (anti)-de Sitter Space and $l_H^2 < 0$ is de-sitter Space, We finally find the standard temp. of (anti)- de Sitter Black Hole. T=$\frac{\Delta E}{2\pi} = \frac{1}{4\pi}[\frac{1}{r_+} + \frac{3r_+}{l_H^2}]$ -(7)

QUANTUM MECHANICS IN (anti) De- Sitter Space

Quantum mechanics on ADSST poses nontrivial problems contrary to flat spacetime. There is no privileged reference frame like the one singled out Minkowski metric. Geodesic motion takes place with constant velocity along straight lines in the spatial metric section. ADSST can be thought as hyperboloid embedded in 5-D space. Coordinate $(\xi_A)^2$=$(I\alpha)^2$.

Isometrics of ADSST can be identified with the Lorentz algebraic method. The interpretation of 4-D ADSST is obtained by splitting the generators into Lorentz generator.$J_{\mu\nu}$ and translation generators $P_\mu = J_{4\mu}/l_H$. The ADSST can be written as

$$[J_{\mu\nu}, J_{\rho\sigma}]=i(\eta_{\nu\sigma}J_{\mu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho}] ; [J_{\mu\nu}, p_\lambda]=i\left(\eta_{\mu\lambda}p_\nu - \eta_{\nu\lambda}p_\mu\right); \ [p_\mu, P_\nu]=i(\frac{J_{\mu\nu}}{{l_H}^2}) \quad -(8)$$

[We can identify the generators of translation P_μ with the momentum operators as per definition of quantum theory on background of ADSST.]

The position operators X_μ depend instead on the parametrization of hyperboloid.

The Lorentz Algebra of 4D de sitter algebra is identical to Lorentz Algebra of flat space – and its generators have the usual commutation relations with the positions(x_μ) : $[J_{\mu\nu}, x_\lambda]=i(\eta_{\mu\lambda}x_\nu - \eta_{\nu\lambda}x_\mu)$ -(9)

But here relations of commutation of the translators depend on the choice of coordinates of hyperboloid. The natural parametrization of hyperboloid is given by use of $x_\mu = \ \xi_A/\xi_4$ -(10) by use of projective coordinates, and now from eq^n (2) and (3) EUP follows –$[x_\mu, P_\nu]=i(\eta_{\mu\nu} + \frac{x_\mu x_\nu}{{l_H}^2})$ -(11)

EUP AND SPECIAL RELATIVITY

We combine eq^n 1 and 2 to get Extended Uncertainty Principle (EGUP)$\Delta x_i \Delta p_j \geq \frac{\delta_{ij}}{2}\left[1 + l_P^2(\Delta p_i)^2 + \frac{(\Delta x_i)^2}{l_H^2}\right] - (12)$

UP considers commutation relations to a generalized DSR – Triply Special relativity(TSR)- a generalized DSR on curved space based on deformation of l_P& l_H are independent of position of observers. In this model, the commutation relation depends on parametrization of anti de sitter Hyperboloid on specific deformation. From Glikman and Smolin's paper we write the commutation relations- $[x_\mu, x_\nu]=il_P^2 J_{\mu\nu}$; $[x_\mu, P_\nu]=i(\eta_{\mu\nu} + \frac{x_\mu x_\nu}{{l_H}^2} + l_P^2\, p_\mu\, p_\nu + \frac{2l_p}{l_H}\, x_\mu\, p_\mu)$ -(13)

Depending on the signs of l_H^2 & l_P^2 we have different physical settings.

The temperature of black hole is modified and is corrected to give the new form T=$\frac{\Delta E}{2\pi} = \frac{r_+}{8\pi l_p^2}[1 - \sqrt{1 \frac{4l_p^2}{r_+^2}(1 - \frac{r_+^2}{2l_H^2}}]$ - (14)

CONCLUSION

I tried to find the EUP from geometric considerations on ADSST. We extended our process of the EGUP by Triply special relativity(TSR). Finally, we made little corrections in the temperature of Black Holes in (anti)-de Sitter background.

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