Kerr BH's solutions based on Naked Singularities and Cylindrical Wormholes

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Abstract

In Einstein's Theory of GR, Schwarzschild's solution gives a mathematical explanation of the gravitational potential around a spherically symmetric object with properties like zero charge, zero mass, and zero cosmological constants[2]. So, in this paper which is based on mathematics of BH, I focussed to create the mathematical structure and formulation of wormhole from Naked Singularity of Kerr BH without event Horizon and ergosphere, considering time as imaginary[11] to remove the concept of eternatilty of time. Also, through this paper I came to the conclusion that every parameter of a BH is altered by a "Naked Singularity" and spin angular momentum is necessary for the creation of a wormhole as well as the removal of the event horizon and singularity.

1 Introduction

Schwarzschild theory had an far reaching consequences but it lacked to explain the real-life BH with charge and spin angular momentum. So, this drawback was resolved by Roy Kerr, developing a metric for explanation and solution for charged rotating BH. This metric is known as Kerr-Newman Metric.[1,3] The Kerr Metric is the extensional generalisation for a rotating BH of the Schwarzschild Metric, and describes the vaccum geometry of Space-TIme around a rotating axially symmetrical BH with quasipotential event horizon. In Kerr metric we have inner & outer event horizons, two ergosphere and an ergosurface.

Also we know from **Cosmic Censor Hypothesis** [6,7]that a naked singularity can't exist in nature as nature always hides the singularity via the event horizon.

2 Hoop Conjecture

According to Prof. **Kip Thorne's hoop conjecture**[13], *an exploding object only creates a black hole if and only if a circular hoop with a particular critical circumference could be positioned around it and spun around its diameter.* To say it another way, a black hole cannot form unless the entire mass of the object is compressed to the point where it can fit inside a perfect sphere with radius equal to the object's Schwarzschild radius(R_s).

So, to explain it in another way, a sphere irrespective of charge, spin if has a critical radius, then it tends to collapse gravitationaly and that's when BH is formed. So, here in case above our critical radius is Schwarzschild Radius R_s.

So, this critical radius(R)that the sphere is getting transformed to, is called as Hoop Radius(R_h)

$$R = \frac{2GM}{c^2} \tag{1}$$

$$R_h = 4\pi R^2 = 4\pi \left(\frac{2MG}{R_s c^2}\right)^2 = 16\pi \left(\frac{2\sqrt{MG}}{R_s c^2}\right)^4 \tag{2}$$

3 Approach and Analysis

3.1 Making Time Imaginary and Removing the Event Horizon

In this part, we'll try to accomplish two important things-

- Making time imaginary to remove the eternality of time while reaching the singularity.
- Removing the event horizon to allow the particle to traverse the singularity without becoming entrapped there

Here we are going to add the term i with cdt to make time imaginary which will eliminate the singularity and lead to imaginary time instead of actual eternal time, lowering the issue of infinite time dilation while arriving at the singularity. So, I'll perform 'wick rotation' to make c^2dt^2 to $-i^2c^2dt^2$

$$\tau = -i^2 c^2 dt^2 \tag{3}$$

So, now if BH possess a vertical spin axis z, mass m, spin term J , then the affine parameter a of the spin along the spin axis z is given by [10]

$$a = \frac{J}{Mc} \tag{4}$$

Now using Boyer-Lindquist[8,9] coordinates and the standard spherical coordinates R_s, θ, ψ, we introduce the frame-dragging equation

$$\sum = r^2 + a^2 \cos^2 \theta \tag{5}$$

We've discussed *affine parameter* a has a constant value when the over an extreme solution of the metric is taken making the larger root of another grouping parameter Δ as 0, in the latter sections of the paper.

Now, we've two radiuses one is Hoop radius(R_{h}) and the other is Schwarschild Radius(R_s), we'll combine them to form the equation

$$\omega = R_{\mathrm{h}} R_s \tag{6}$$

When the limit of Δ parameter is taken to 0, the internal and exterior event horizons, as well as the ergospheres, will entirely vanish, opening the path for the development of a **NAKED SINGULARITY**, which is necessary for every particle passing through the singularity since there is no event horizon.

Now taking account of Δ factor. we write

$$\Delta = R_s^2 - R_{\mathrm{h}} R_s + a^2 \tag{7}$$

Now combining a with square of $\sin^2 \theta$ to get Ξ

$$\Xi = a \sin^2 \theta \tag{8}$$

Since wormhole is a cylinder[7], we'll use the cylindrical polar coordinate vectors $\hat{\rho}$, $\hat{\sigma}$, $\hat{z}$ and grouped these terms to get

$$\Omega = d\hat{\rho} + \rho d\hat{\sigma} + d\hat{z} \tag{9}$$

3.2 Final Approach

Now from the basic concepts, we can say (5) represents a circle. So, that means our singularity equation satisfies ring circularity which is a circle. A little more analysis helps us to see that value of θ making a wick rotation through a $\frac{\pi}{2}$ rotational plane paving the way for singularity to be zero.[4]

$$\sum = r^2 + a^2 \cos^2 \pi/2 \approx= r^2 \implies \cong 0|_{singularity} \tag{10}$$

Now substituting Hoop's radius and taking magnitude 1 for the constants of G, c we come to a conclusion that in case of $\Delta \neq 0$ event horizon will exist and it'll be spherical. But in case of naked singularity it vanishes[5]

$$r_h = 4\pi \left(\frac{2MG}{R_s c^2} \right)^2 \equiv R_s = 2M|_{G=1,c=1} \tag{11}$$

$$R_s = 2M = x^2 + y^2 + z^2 \cong R_s \equiv 2\beta|_{M=\beta} =^2 +y^2 + z^2 \tag{12}$$

$$x^2 + y^2 + z^2 = 2\beta \tag{13}$$

So, when $\Delta \to 0$, then a accepts the value as

We get **Extended Cauchy Horizon** when we manipulate (13), as affine parameter a along the z axis of motion and amplifying one side of z axis

$$z^2 = x^2 + y^2|_{extendedCauchyHorizon} \tag{14}$$

The spacelike timeline will exit the BH through a wormhole and enter a future-directed Cauchy horizon ($C(H^+)$), where it will then travel to another Kerr BH in a different universe and leave through yet another future Cauchy Horizon.[12,14]

$$a = R_s^2 - R_h R_s + a^2 \equiv 0 \implies R_h R_s - R_s^2 = a^2 \implies a = \sqrt{R_s(R_h - R_s)} \tag{15}$$

When the singularity changes to naked, the spin of BH changes according to -

$$a = \frac{J}{Mc} \tag{16}$$

Now, substituting value of (15) in (16) to get

$$J^\dagger = \sqrt{R_s(R_h - R_s)}Mc \tag{17}$$

So, the mass will also change-

$$M^\dagger = \frac{1}{\sqrt{R_s(R_h - R_s)}c} \tag{18}$$

As $\Delta \to 0$, event horizon along with ergosphere starts to vanish and event horizon starts forming, and reducing its mass and increasing the spin.

Substituting the value of $R_s = 2\Big|_{M=\beta}$ from (2) we get,

$$R_h = 16\pi\left(\frac{\sqrt{\beta}}{2}\right)^4\Bigg|_{c=1.M=\beta,G=1} = 4\pi\beta^2 \tag{19}$$

and assuming $R_h \approx R_s \approx m^\dagger$ we say $M^\dagger$ is less than M by a factor of $4\pi\beta^2$

3.3 Mass Energy Equivalence

One of the most simple, yet beautiful equations is

$$E = mc^2$$

We've seen in the earlier section of our studies that Mass M of BH got decreased by a factor of $4\pi\beta^2$ and the resultant mass is denoted by $M^\dagger$ and the spin $J^\dagger$ gets increased eventually. So, this means the E_{rot} gets increased by adding Mass to the BH?

So, here's a question arises- how this $M^\dagger$ is balanced/compensated by increasing mass due to E_{rot}?

Now, we'll spilt the mass M into two components

$$M = \begin{cases} M_{\text{apparent}}, & M^\dagger < M \\ M_{\text{absolute}}, & M^\dagger > M \end{cases} \tag{20}$$

Now we the rotation energy $E = \Delta mc^2$ where $\Delta = M - M^\dagger$, so the final expression is

$$E = (M - M^\dagger)c^2$$

so, here the M_{apparent} seems to decrease by the factor of $4\pi\beta^2$ but the M_{absolute} increases by a factor of $\sqrt{2}$ according to Penrose Mechanism where β^2 taking the value of $\frac{1}{2}\sqrt{2\pi}$, satisfy the increase of M_{absolute} from spin $J^\dagger$ by a factor of $\sqrt{2}$ according to Penrose's technique.

4 Conclusion

In this paper we've accomplised in removing the eternality of time by making it imaginary while reaching the singularity and able to remove the event horizon allowing the particle to travel through singularity without getting trapped, and can cross the singularity to enter another universe. Finally we found out that $M^\dagger$ is less than M by a factor of $4\pi\beta^2$

and also discussed how $M^\dagger$ get compensated by the increased due to E_{rot}.

And finally I've showed that every parameter of a BH is altered by a "Naked Singularity" and spin angular momentum is necessary for the creation of a wormhole as well as the removal of the event horizon and singularity.

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